CENG 201 Veri Yapıları 5: Dengeli Ağaçlar

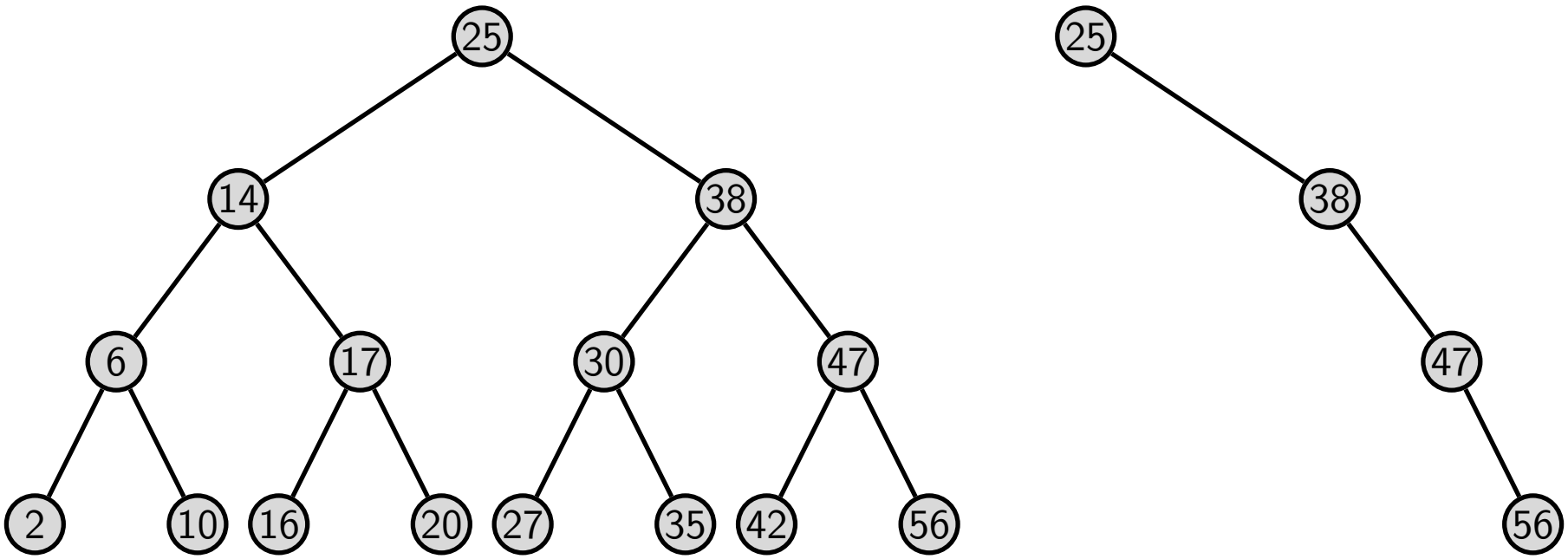
Öğr.Gör. Şevket Umut ÇAKIR

Pamukkale Üniversitesi

Hafta 5

Anahat

Dengeli Ağaçlar



- Yukardaki n elemanlı ikili arama ağaçlarda arama ve ekleme işlemleri en iyi ve en kötü durumda kaç adımda yapılır?

AVL Ağaçları

- İsmini geliştiricileri olan Adelson-Velskii ve Landis'den almıştır
- Dengeli bir ikili arama ağacıdır
- Dengeleme işlemi ekleme ve silme sırasında yapılır
- Her düğüm için denge faktörü -1'den küçük veya 1'den büyük olamaz
- Denge faktörü bir düğümün hangi yöne yatık olduğunu gösterir

Denge Faktörü

Yükseklik

Bir düğümün yüksekliği düğüm ile soyundan gelen yapraklardan en uzağı arasındaki mesafedir.

Denge Faktörü

Sağ çocuğun yüksekliği ile sol çocuğun yüksekliği arasındaki farktır.

Denge Faktörü/Balance Factor

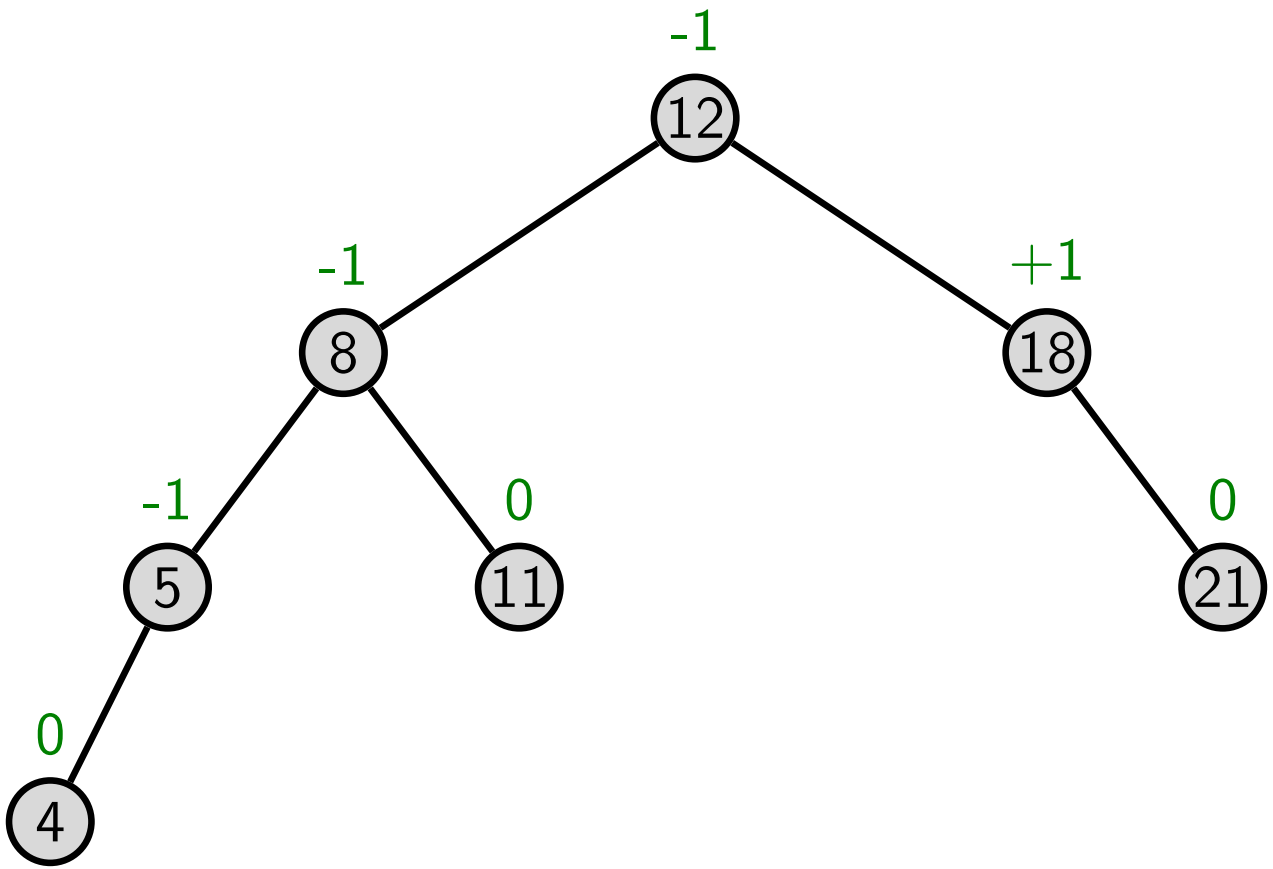


Figure: AVL Ağacı(Dengeli İkili Arama Ağacı)

Denge Faktörü/Balance Factor

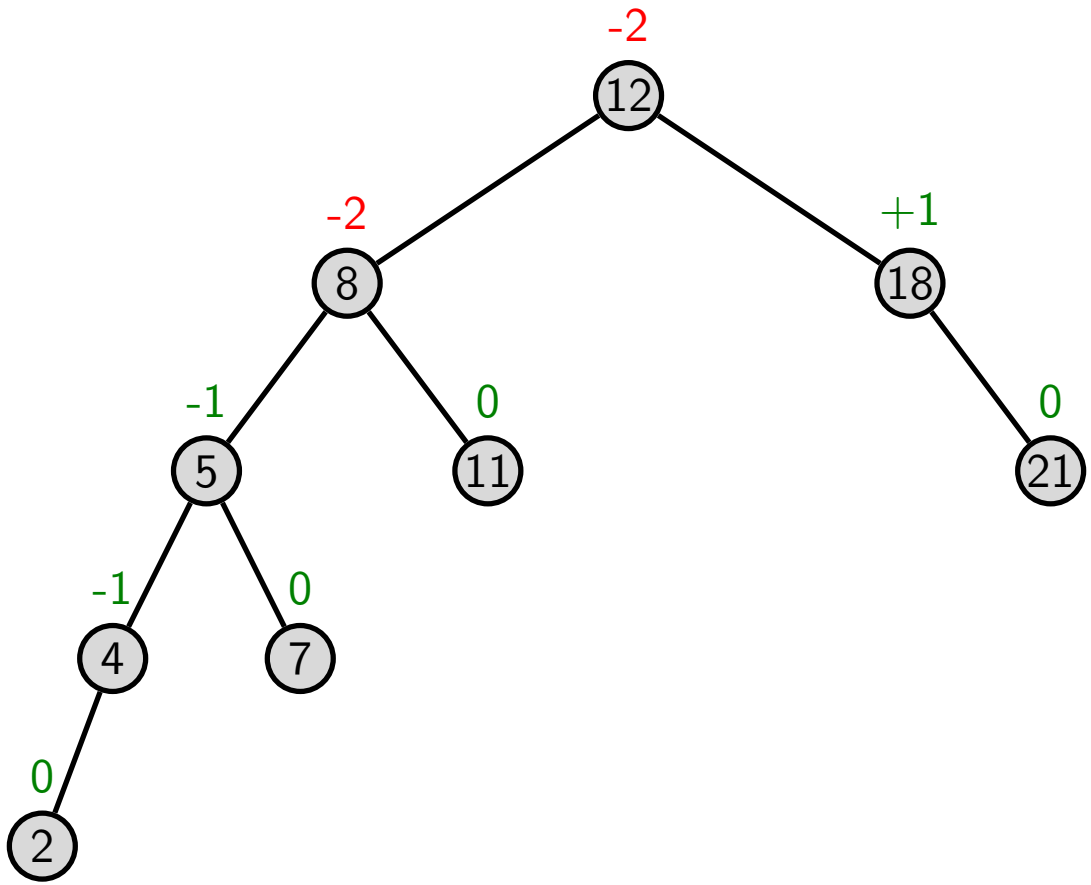
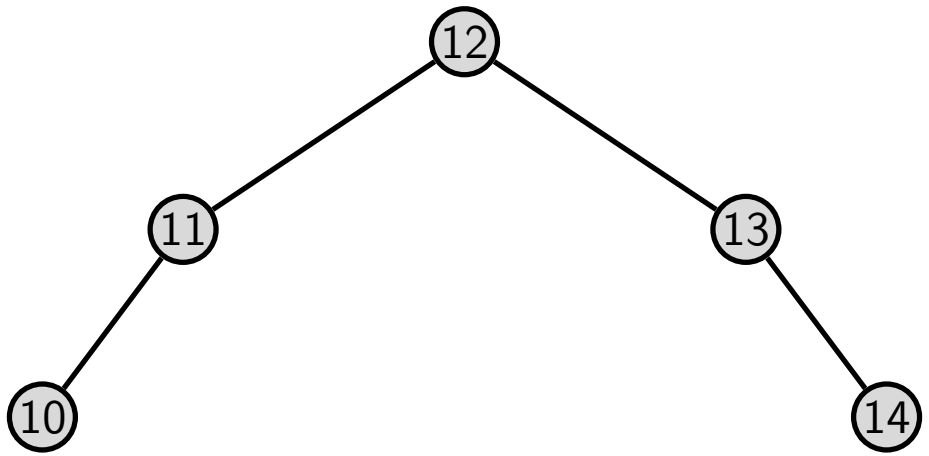


Figure: AVL Ağacı(Dengeli İkili Arama Ağacı)

AVL Ağaçları

Aşağıdaki ağaç bir AVL ağacı mıdır?



AVL Ağaçları

Aşağıdaki ağaç bir AVL ağacı mıdır?

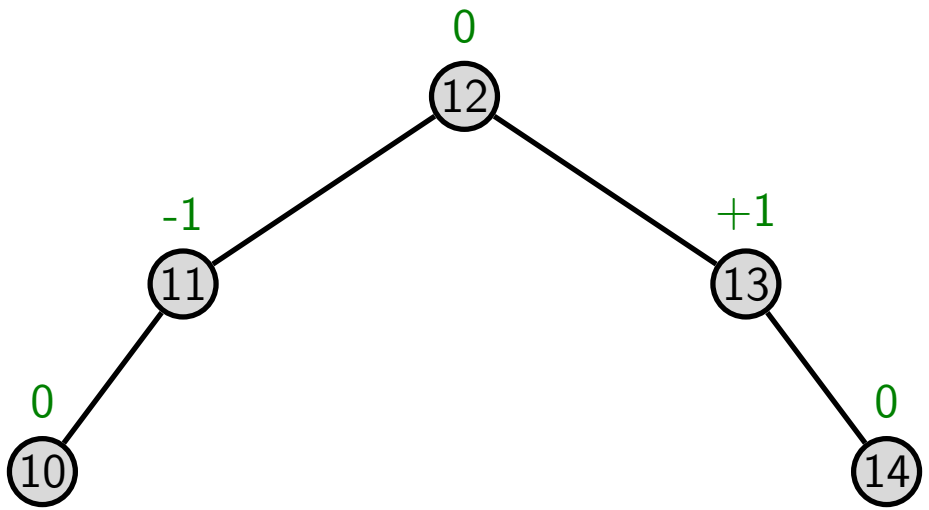
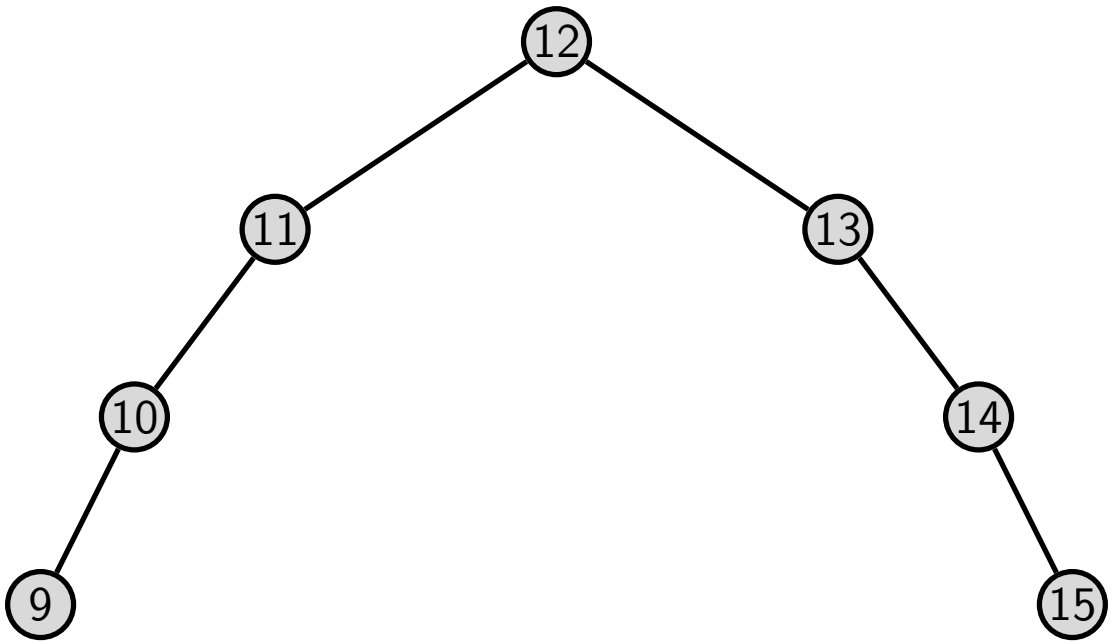


Figure: AVL Ağacıdır

AVL Ağaçları

Aşağıdaki ağaç bir AVL ağacı mıdır?



AVL Ağaçları

Aşağıdaki ağaç bir AVL ağacı mıdır?

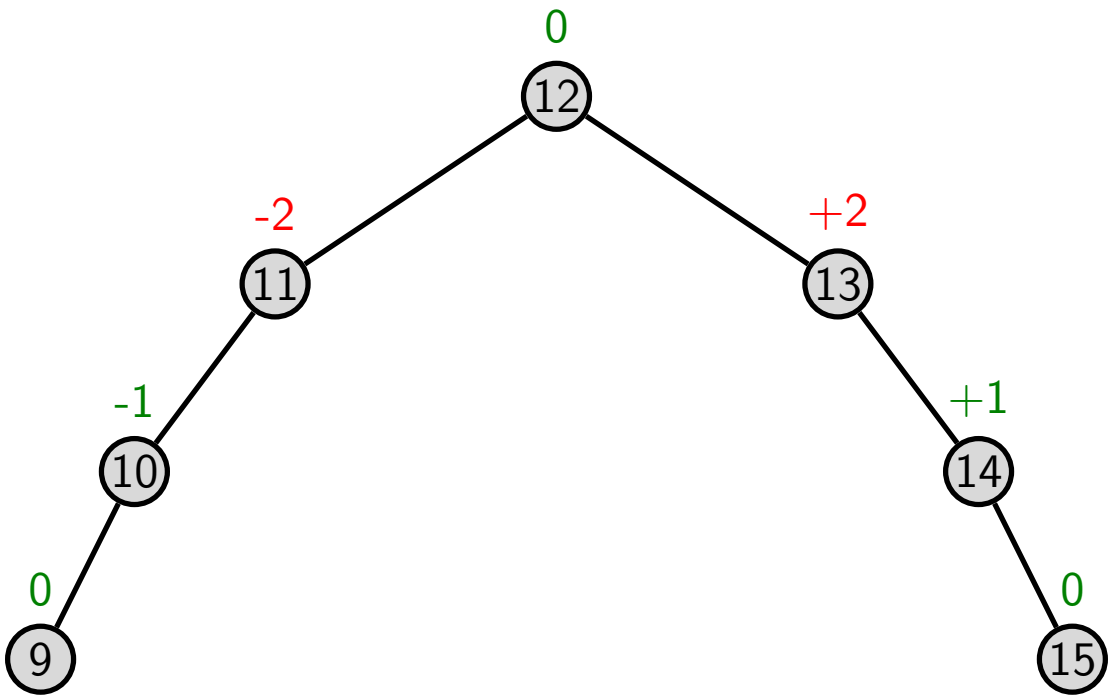


Figure: AVL Ağacı değildir

Ekleme

- Ekleme işlemi ikili arama ağaçlarındaki gibidir
- Ekleme işlemi sırasında denge bozulabilir
- Dengesi bozulan düğümler eklenen düğümden köke kadarki yolda olacaktır
- Dengenin tekrar sağlanması için döndürme işlemleri yapılır
- 4 farklı durum ortaya çıkabilir
- Tekli veya ikili döndürme ile ağaç tekrar dengelenir

Ekleme

- Yeni eklenen düğümden köke doğru gittikçe dengesi bozulan ilk düğüm α olsun
- 4 farklı ekleme biçiminden ötürü denge bozulmuş olabilir:
  1. α'nın sol çocuğunun soluna ekleme
  2. α'nın sol çocuğunun sağına ekleme
  3. α'nın sağ çocuğunun soluna ekleme
  4. α'nın sağ çocuğunun sağına ekleme
- Durum 1 ve 4 tekli döndürme ile düzeltilebilir
- Durum 2 ve 3 için çift döndürme gereklidir

Tekli Döndürme(Sağa)

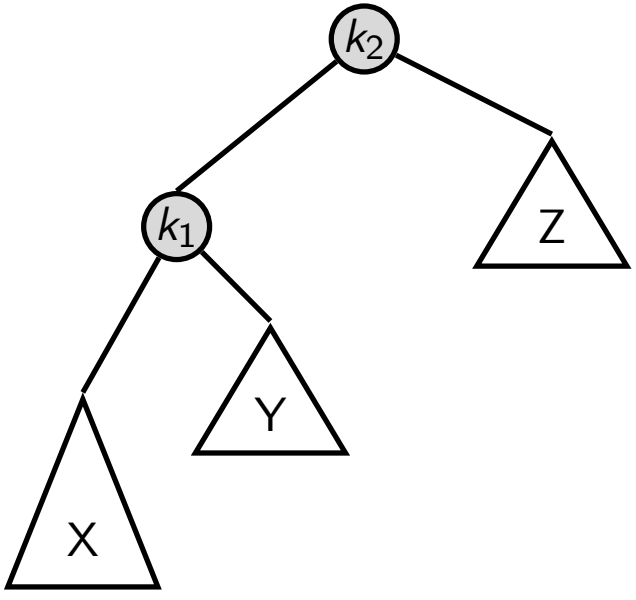


Figure: Durum 1'in çözümü

Tekli Döndürme(Sağa)

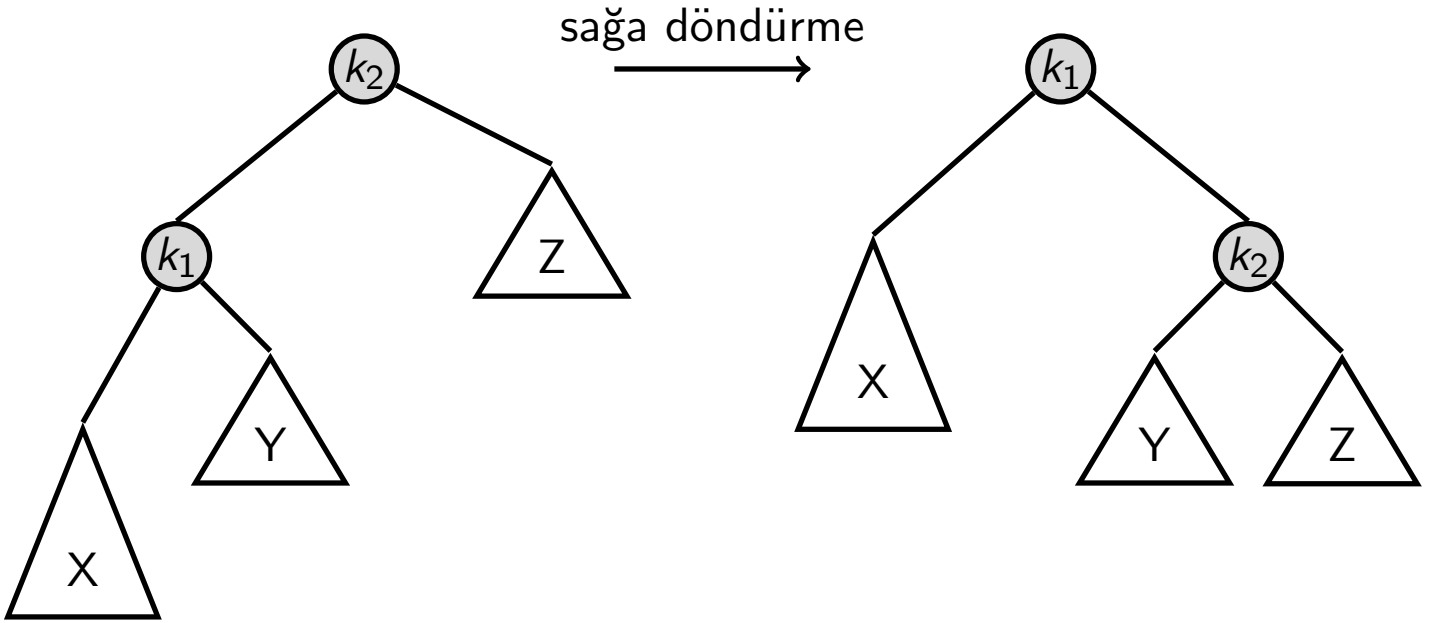
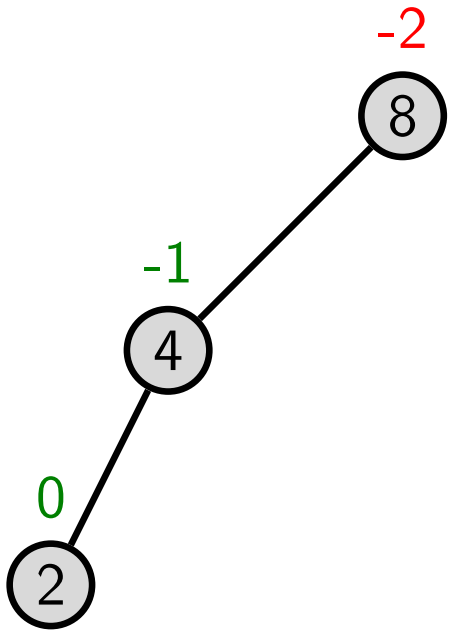
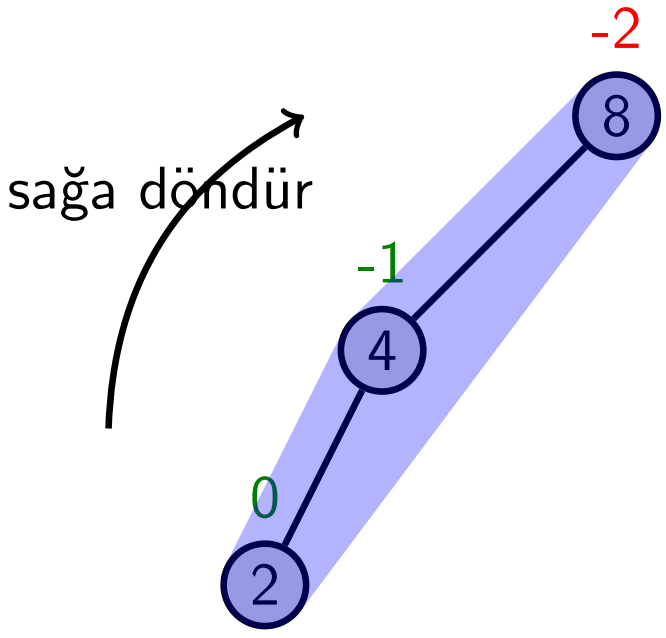


Figure: Durum 1'in çözümü

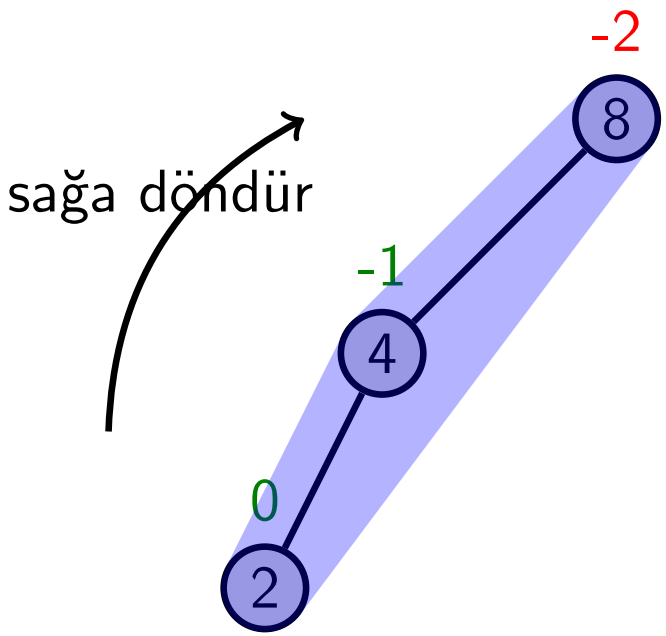
Durum 1 Örnek

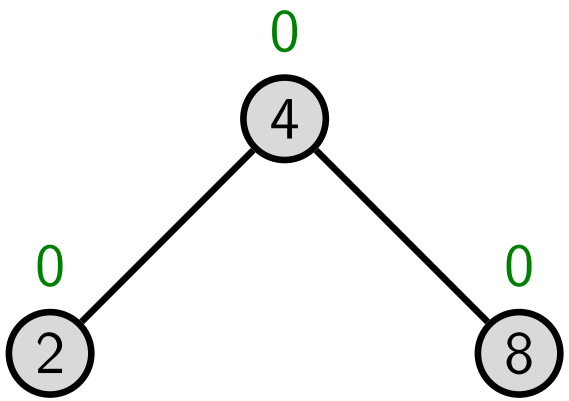


Durum 1 Örnek



Durum 1 Örnek





Tekli Döndürme(Sola)

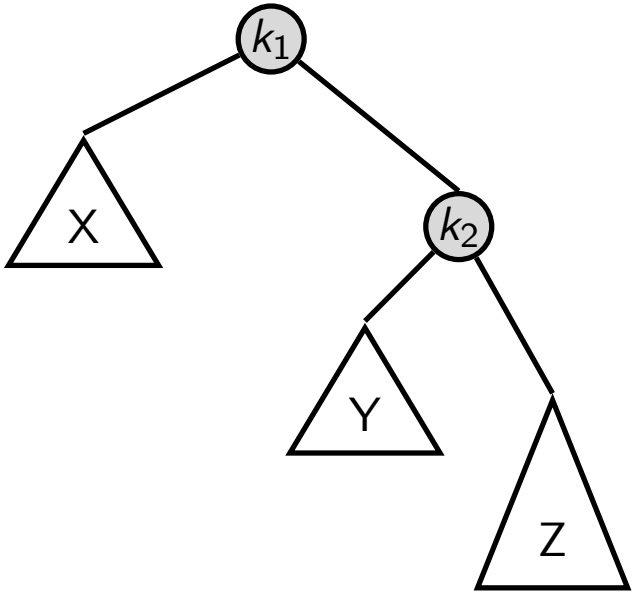


Figure: Durum 4'ün çözümü

Tekli Döndürme(Sola)

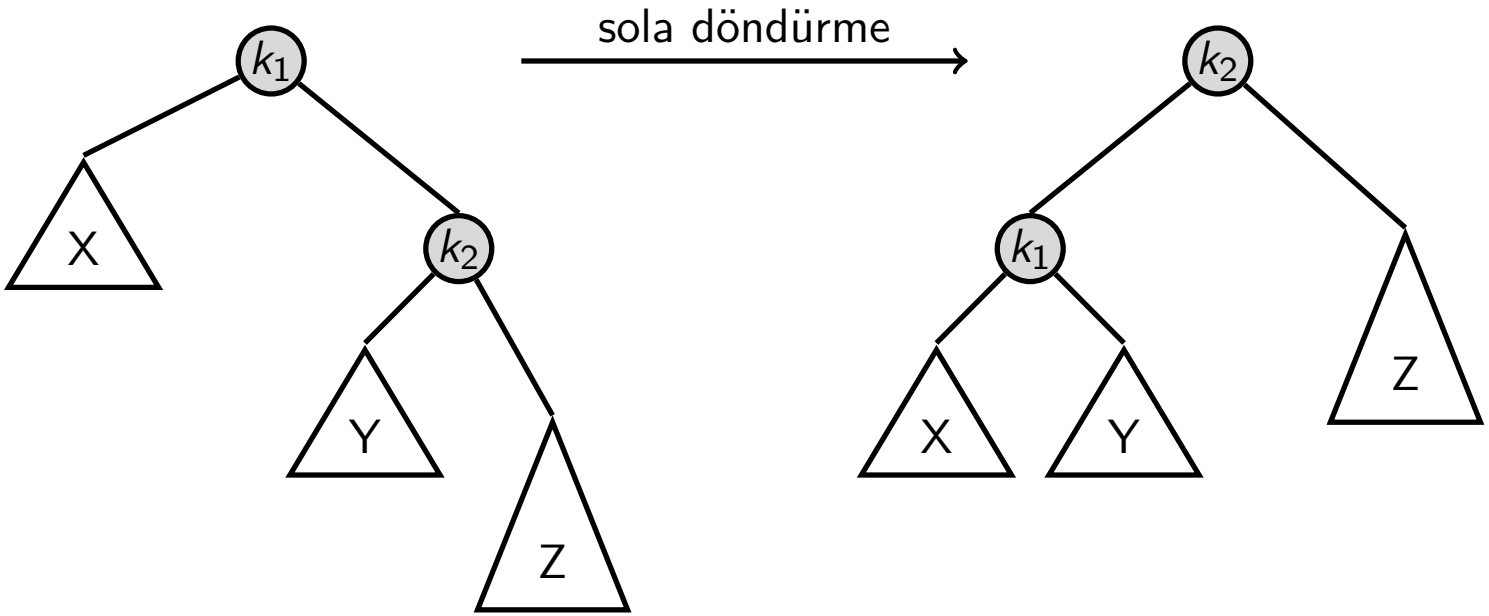
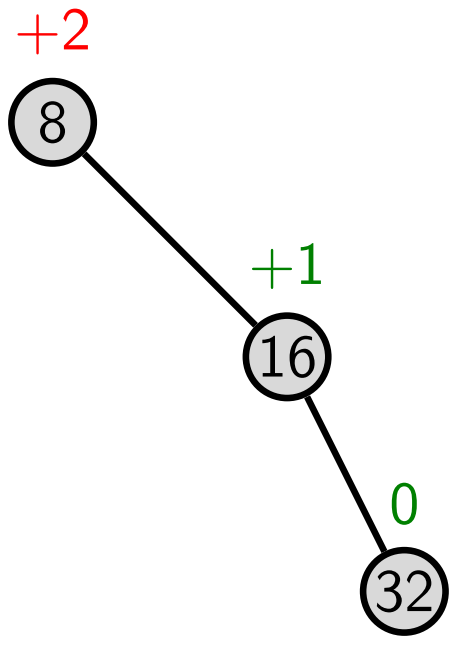
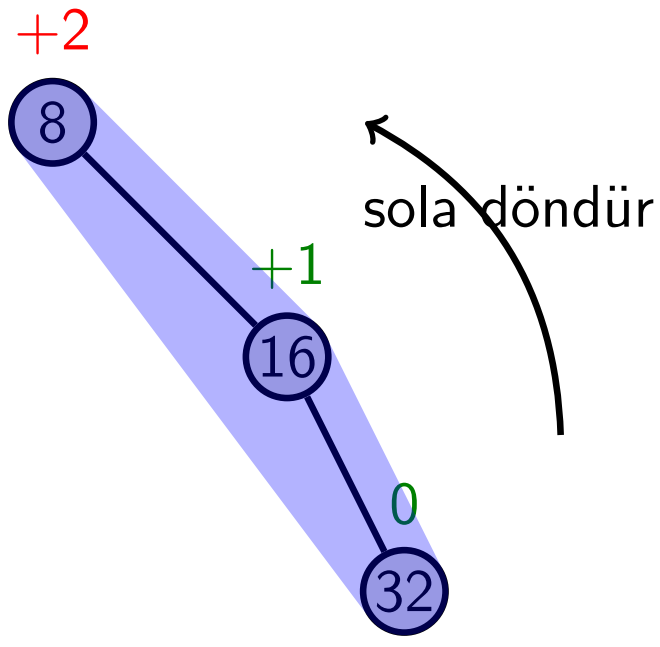


Figure: Durum 4'ün çözümü

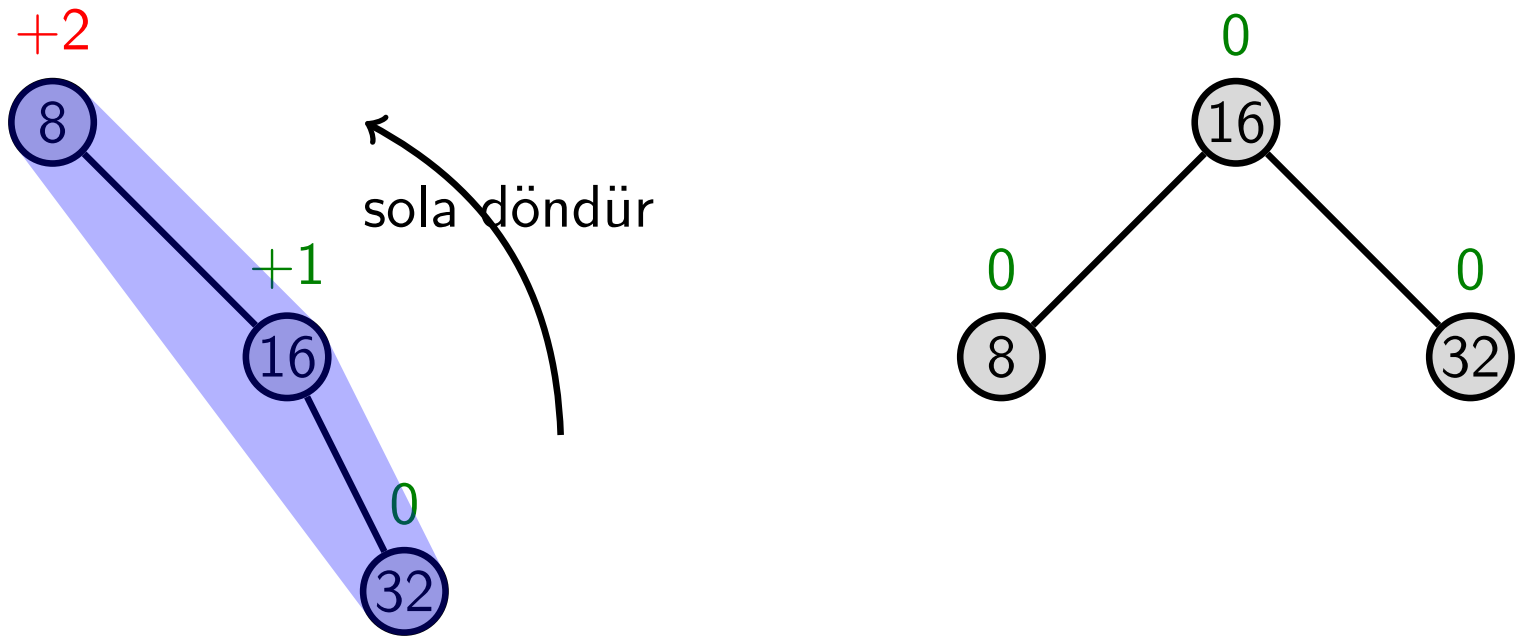
Durum 4 Örnek



Durum 4 Örnek



Durum 4 Örnek



Çift Döndürme(sol, sağ)

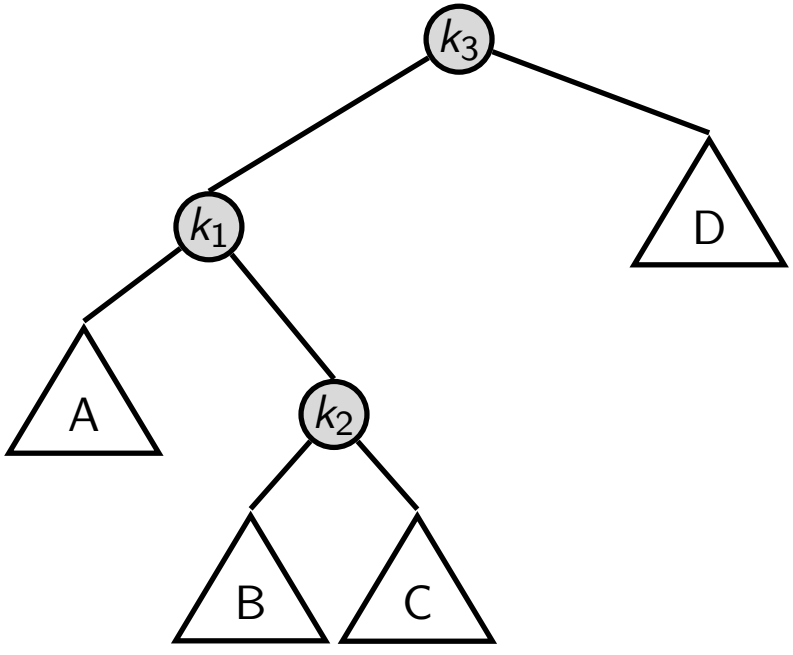


Figure: Durum 2'nin çözümü

Çift Döndürme(sol, sağ)

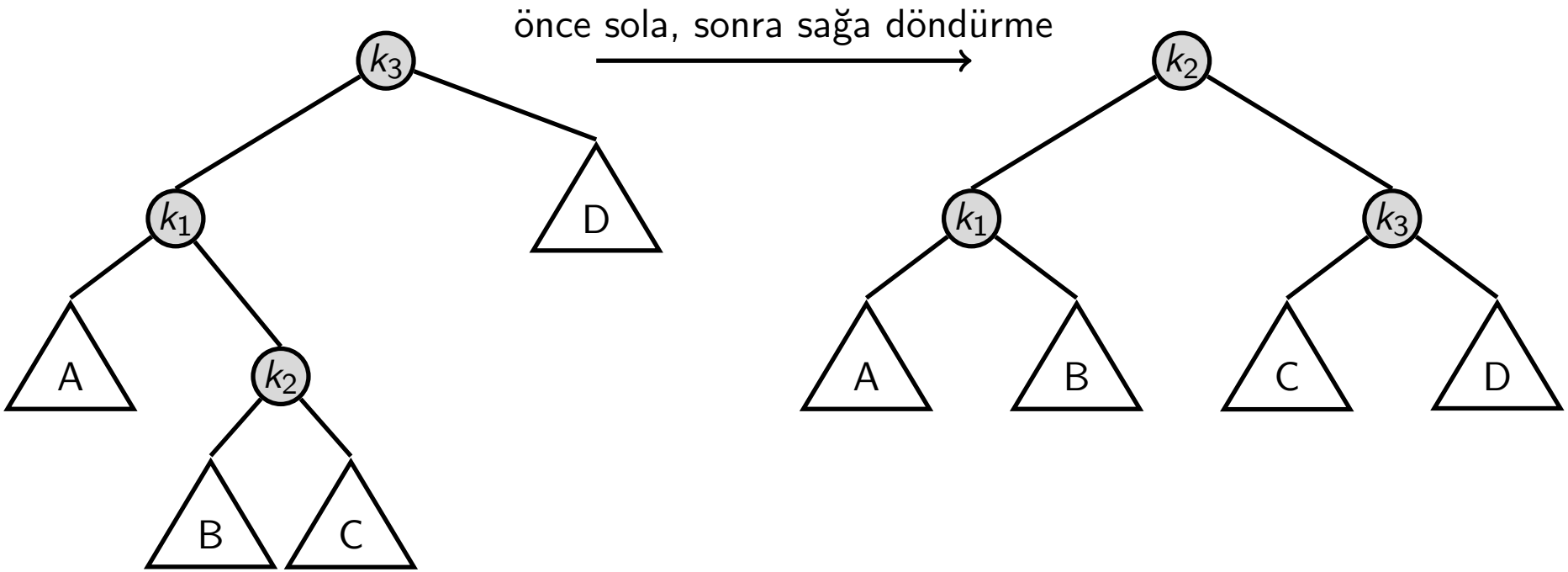
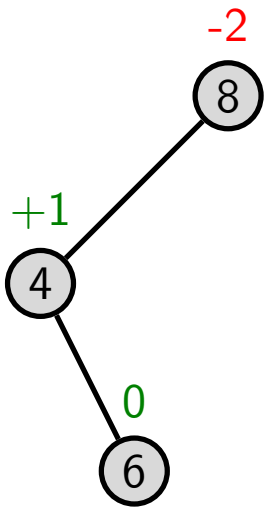
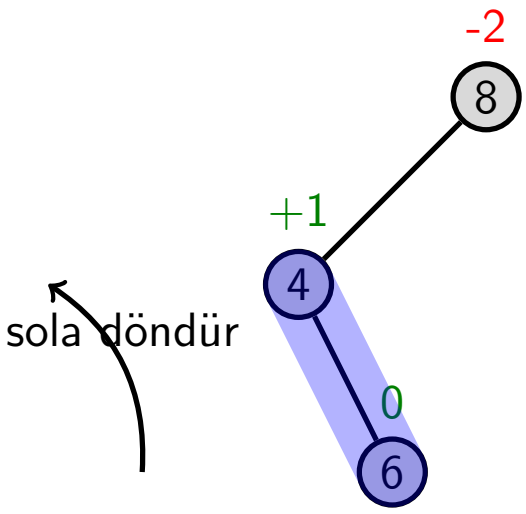


Figure: Durum 2'nin çözümü

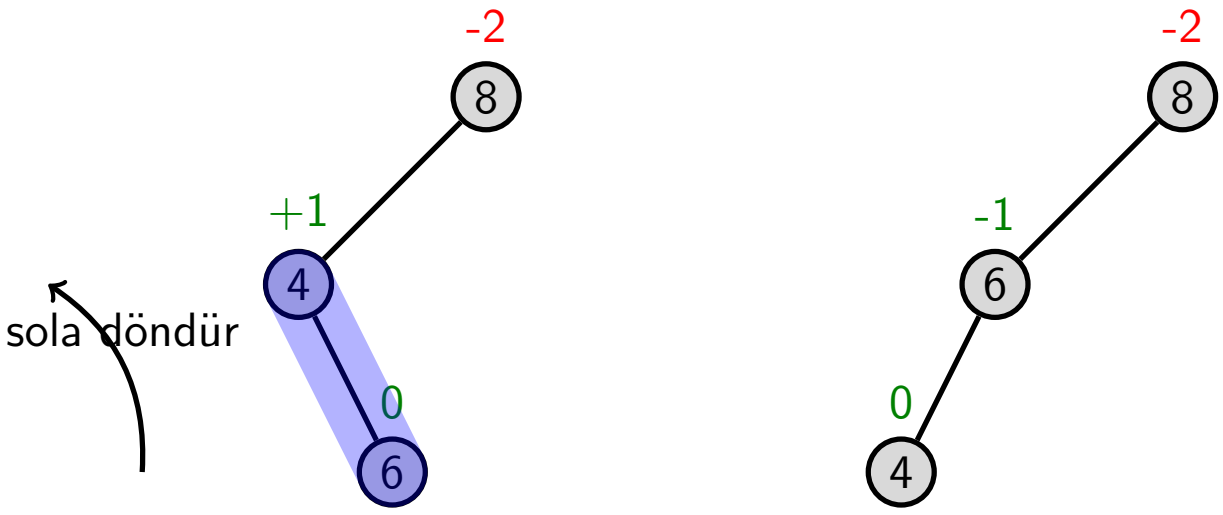
Durum 2 Örnek



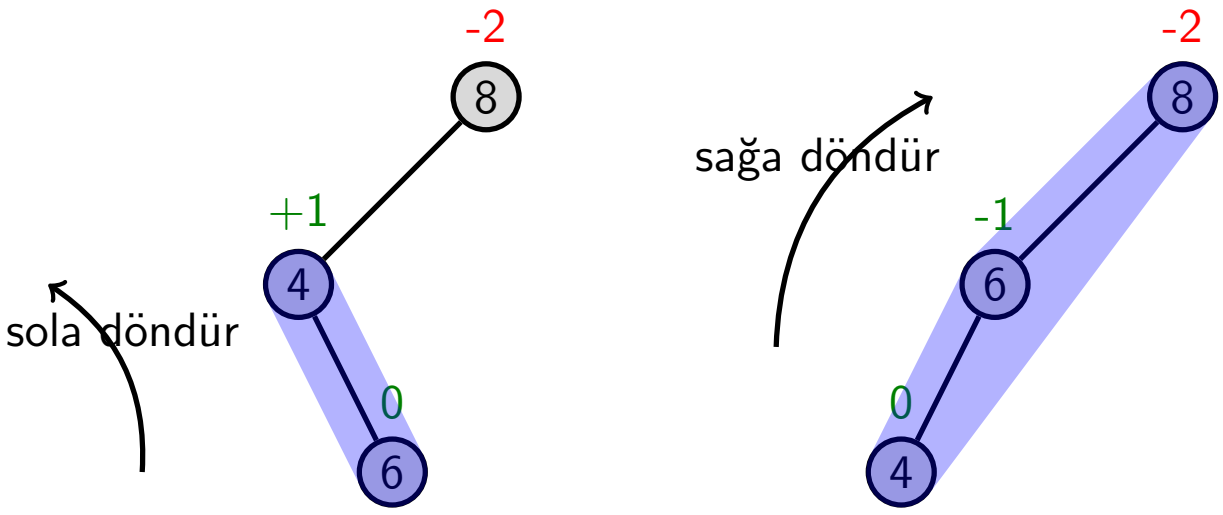
Durum 2 Örnek



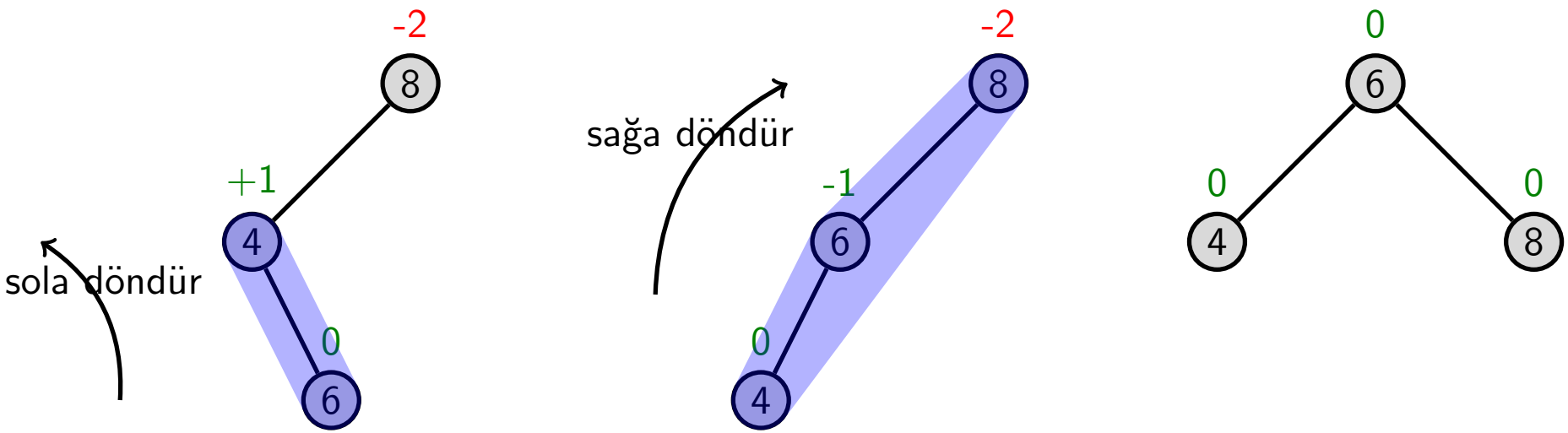
Durum 2 Örnek



Durum 2 Örnek



Durum 2 Örnek



Çift Döndürme(sağ, sol)

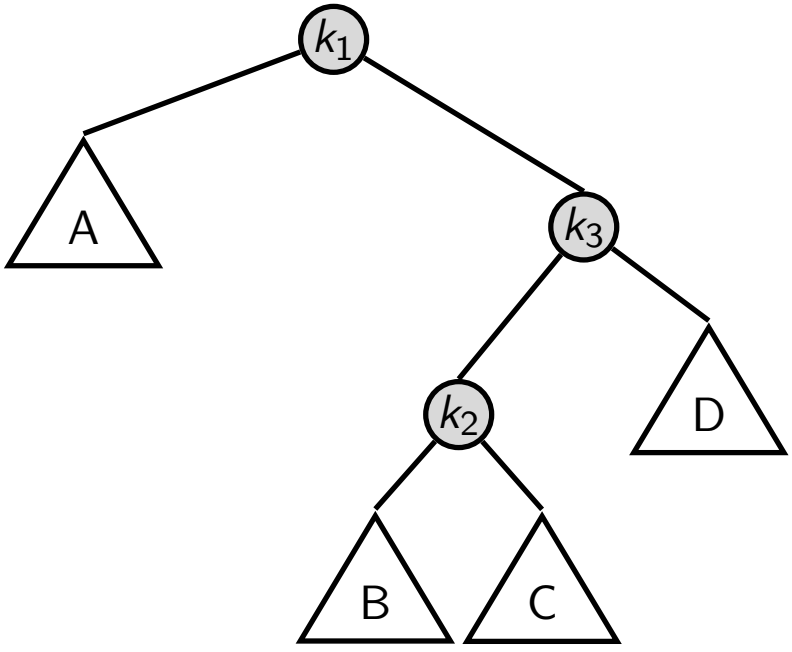


Figure: Durum 3'ün çözümü

Çift Döndürme(sağ, sol)

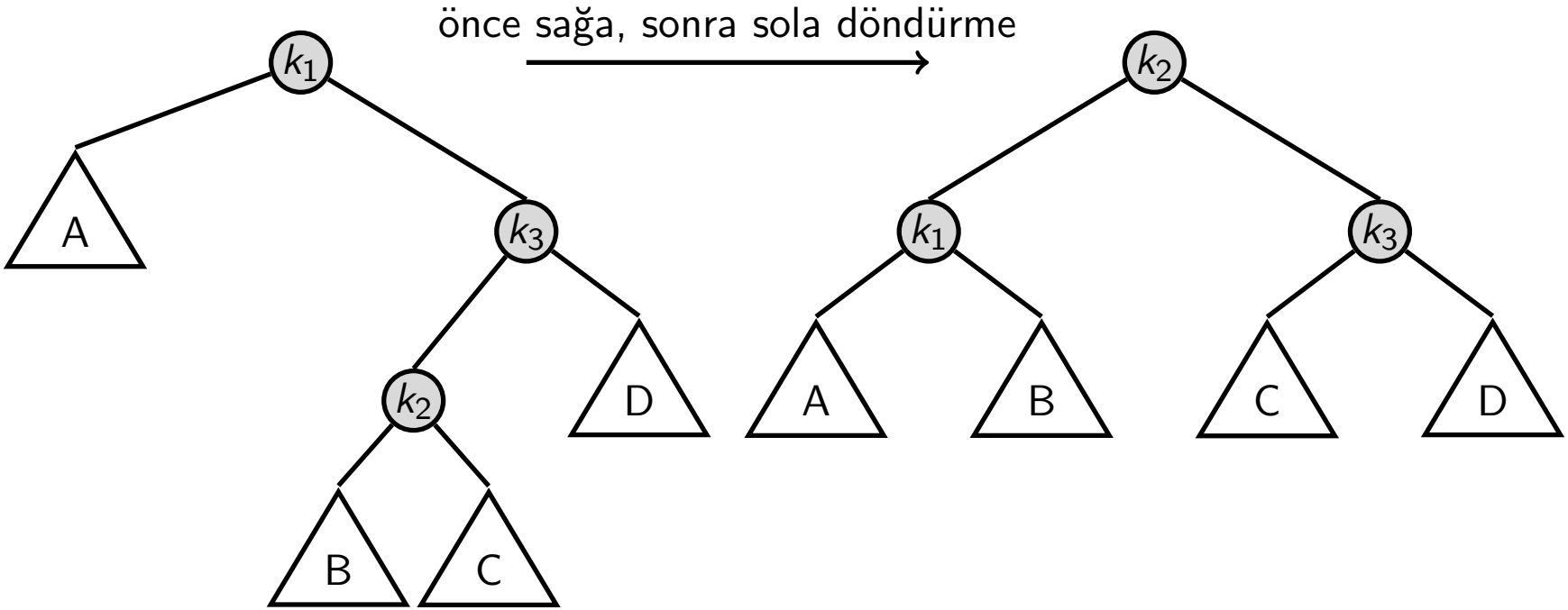
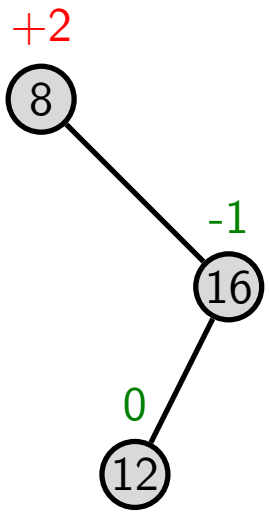


Figure: Durum 3'ün çözümü

Durum 3 Örnek



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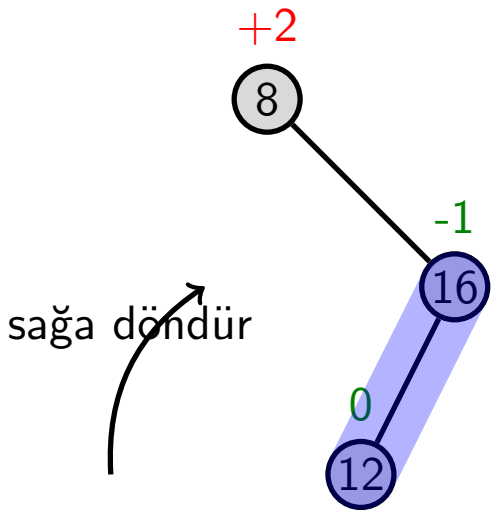
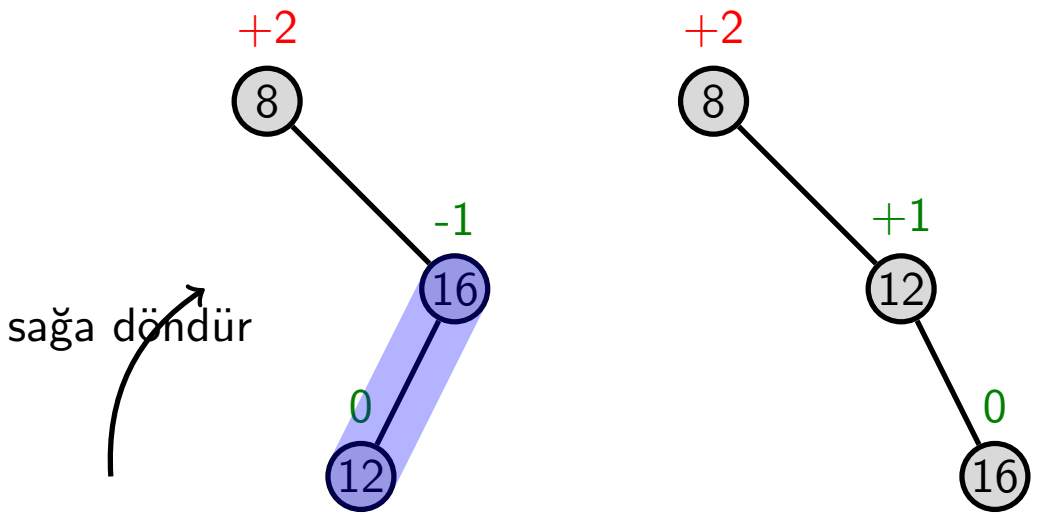


Durum 3 Örnek



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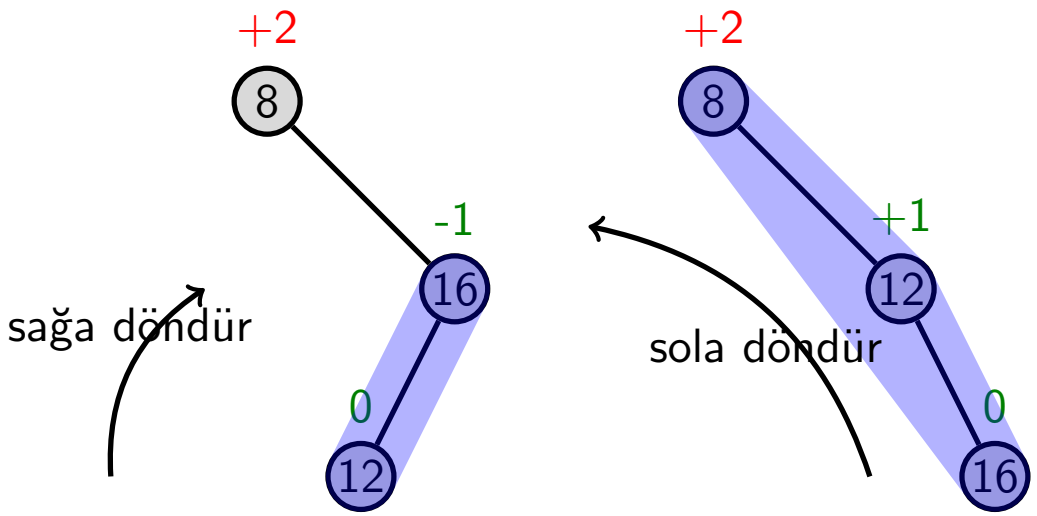
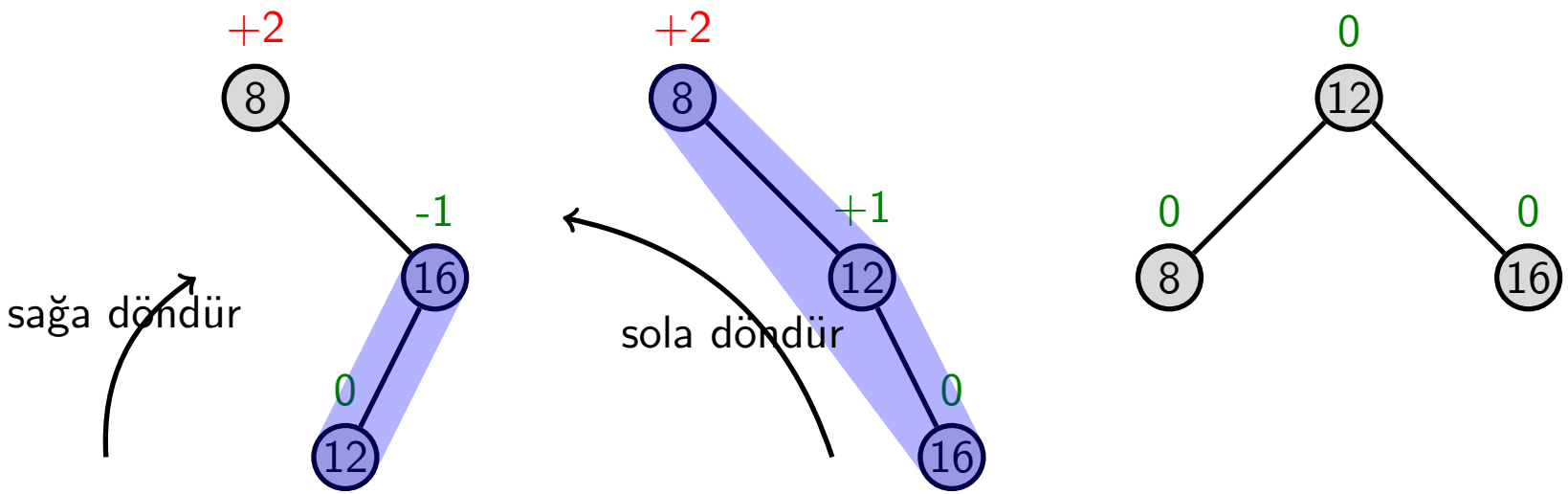


Durum 3 Örnek



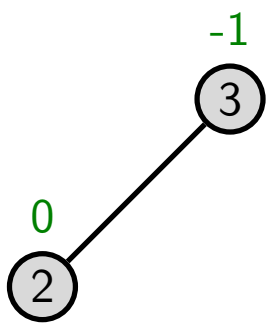
Bir AVL ağacına 3, 2, 1, 4, 5, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9 değerlerini sırasıla ekleyelim.

3 Ekle

Eklenecek: ~~3~~, 2, 1, 4, 5, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9

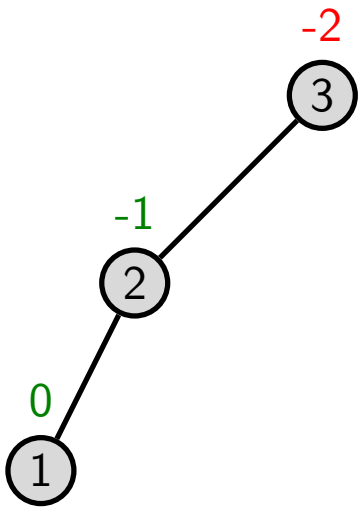
2 Ekle

Eklenecek: ~~3~~, ~~2~~, 1, 4, 5, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9



1 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, 4, 5, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9



1 Ekle

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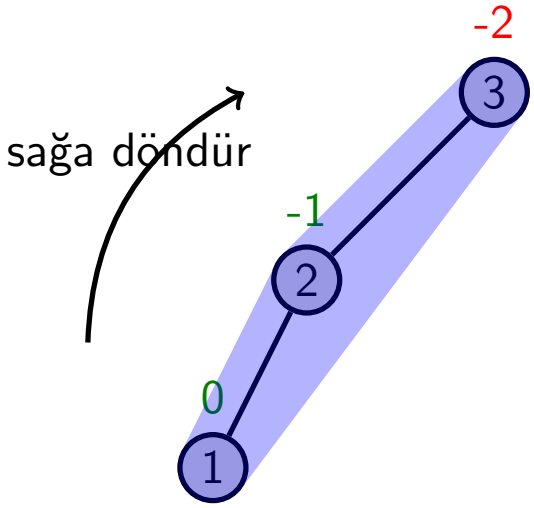

1 Ekle

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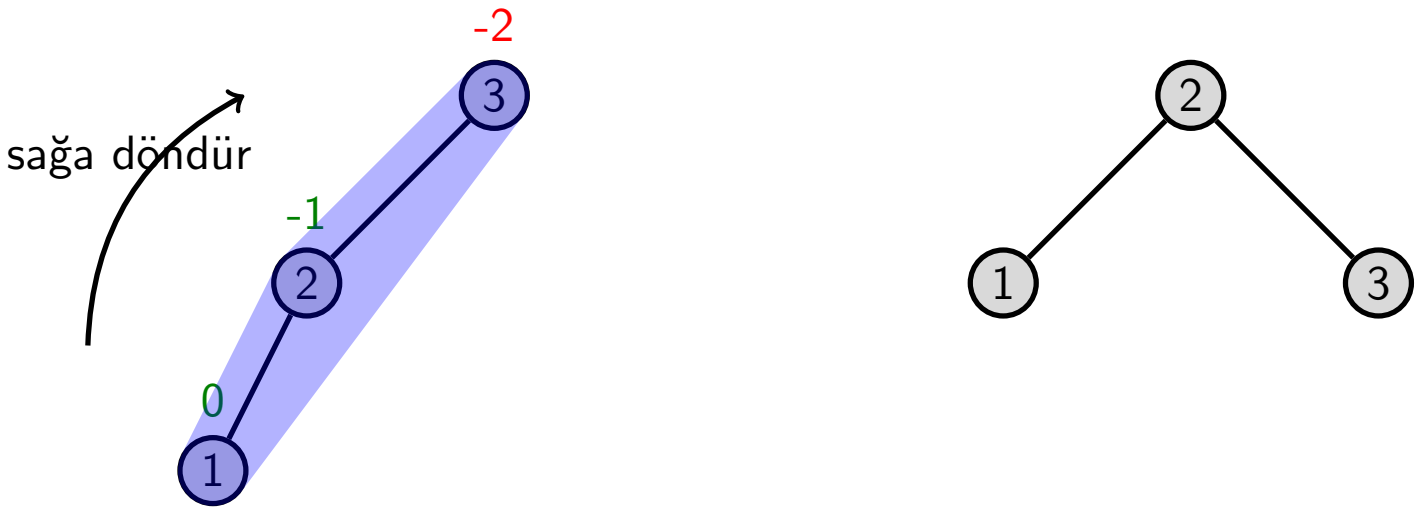

4 Ekle

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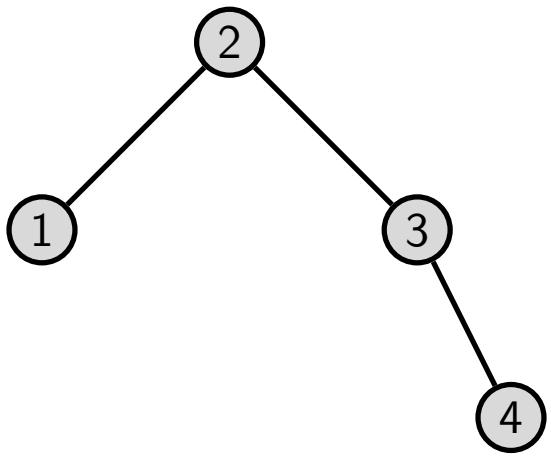

5 Ekle

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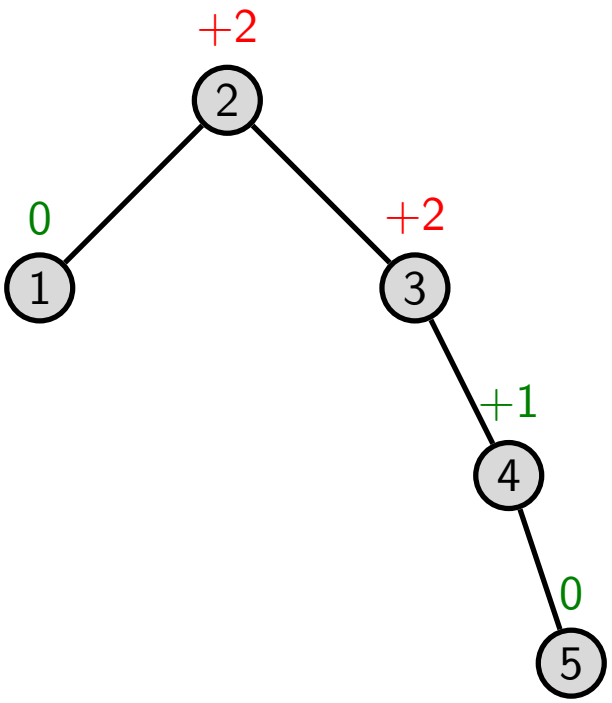
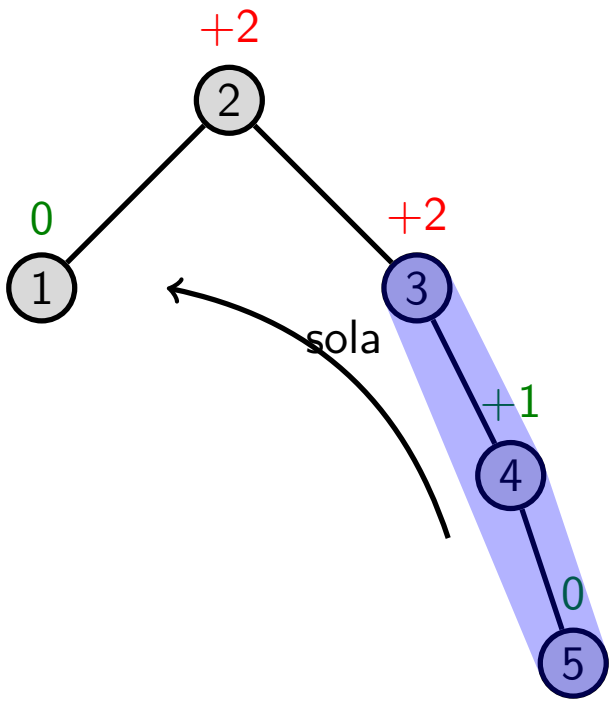


5 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9



5 Ekle

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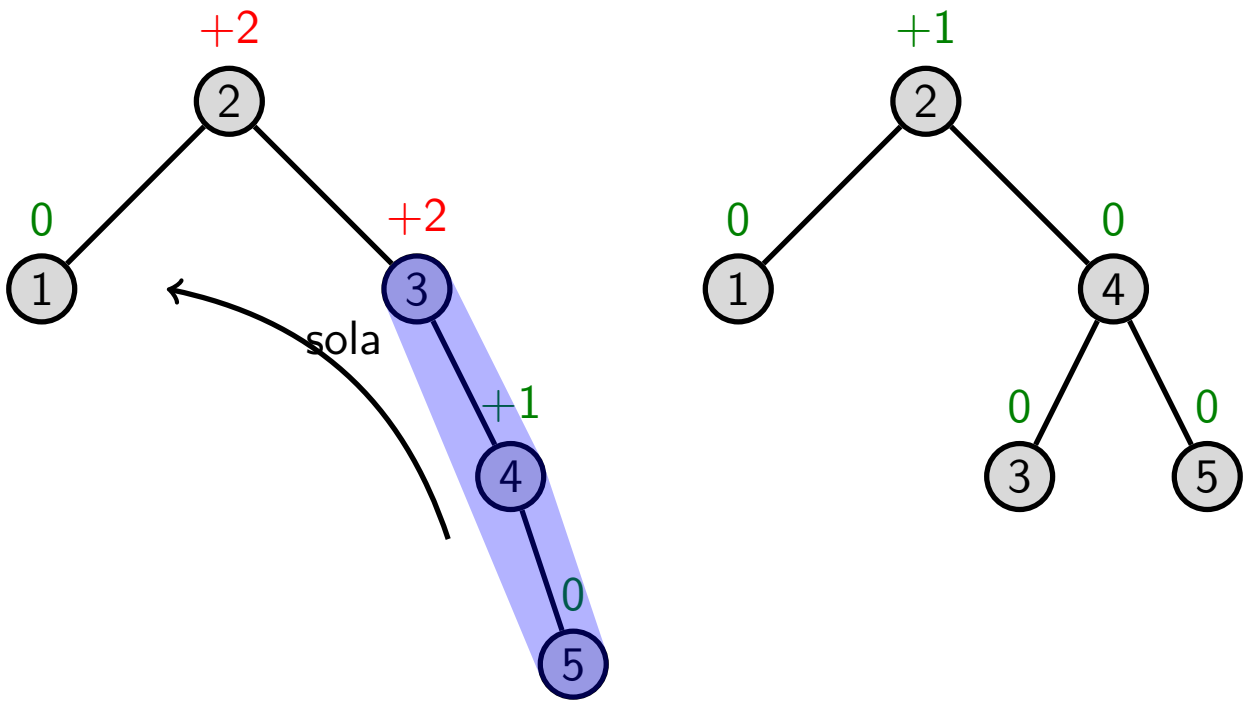


6 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9

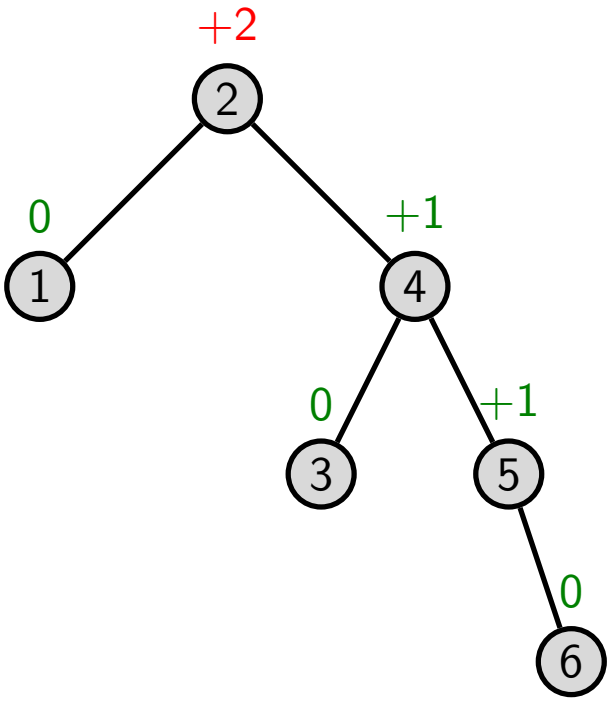


Figure: Sola döndür

6 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9

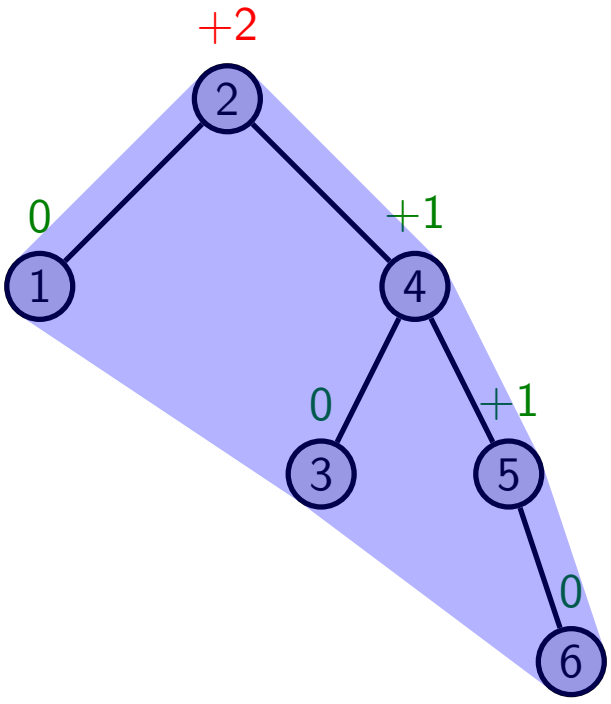


Figure: Sola döndür

6 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9

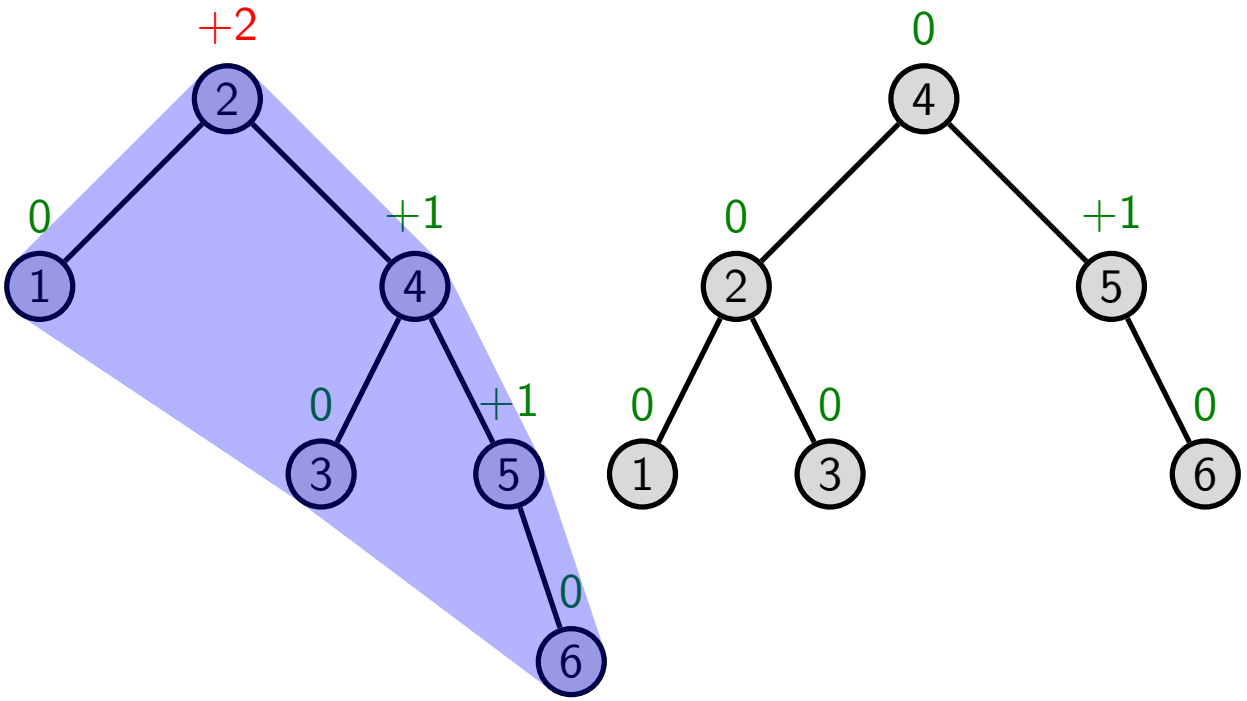


Figure: Sola döndür

7 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, 16, 15, 14, 13, 12, 11, 10, 8, 9

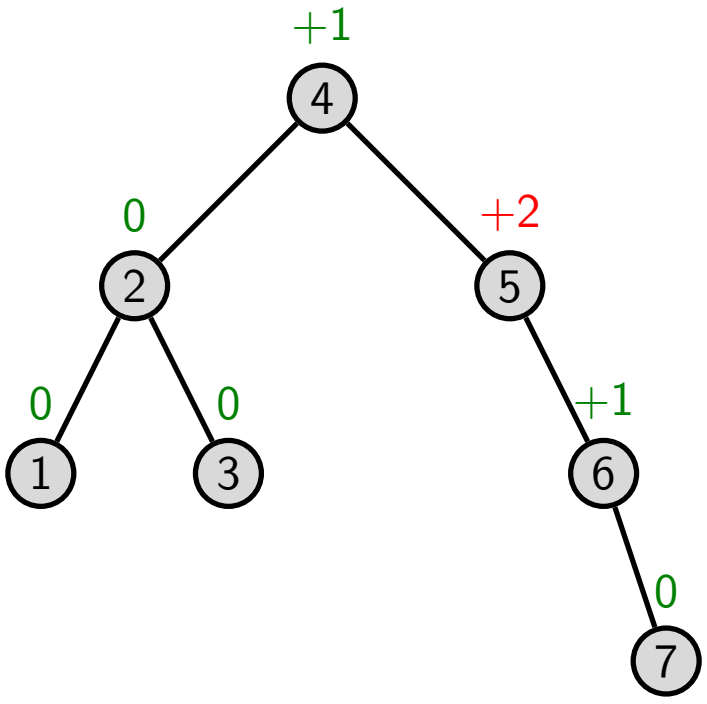


Figure: Sola döndür

7 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, 16, 15, 14, 13, 12, 11, 10, 8, 9

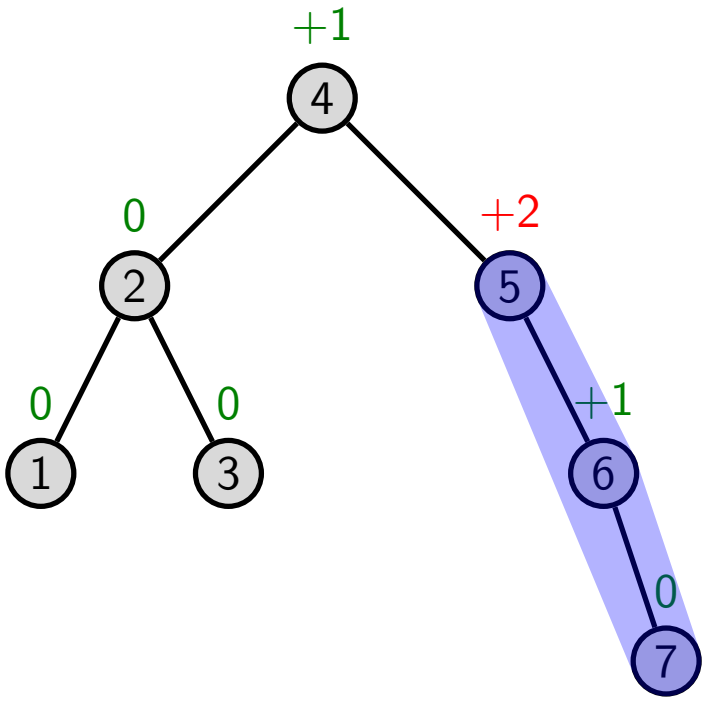


Figure: Sola döndür

7 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, 16, 15, 14, 13, 12, 11, 10, 8, 9

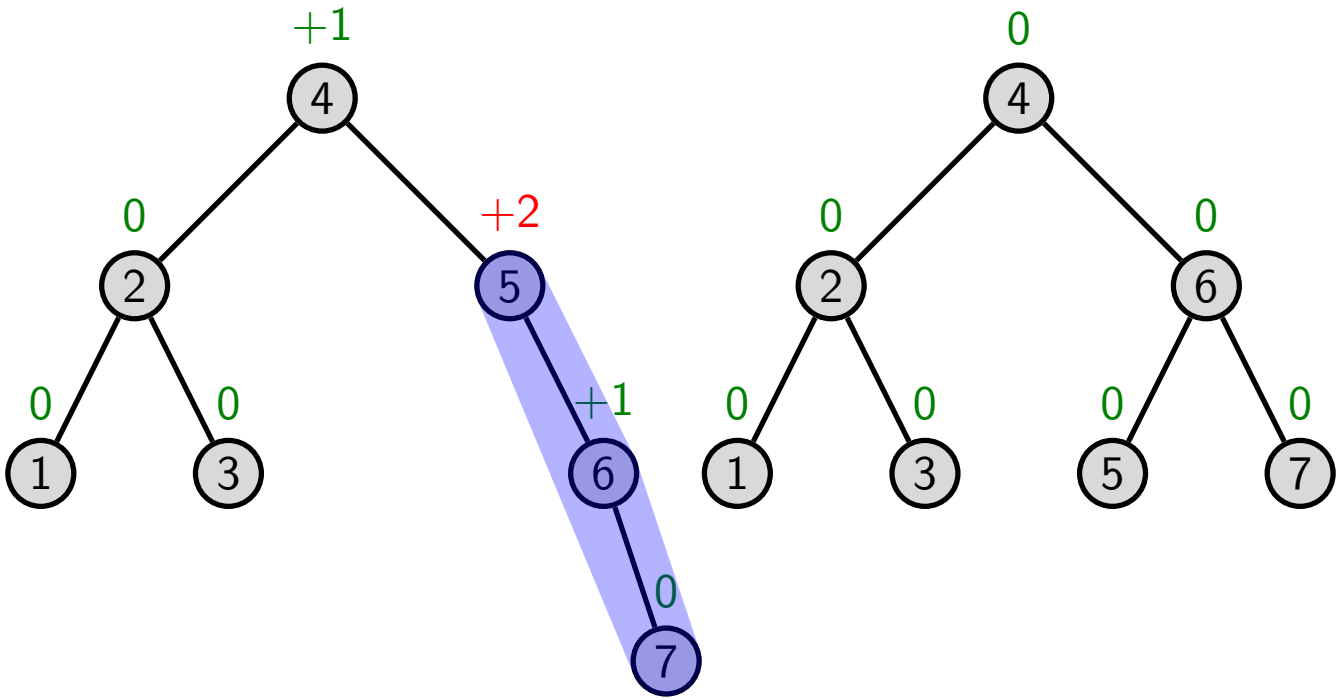
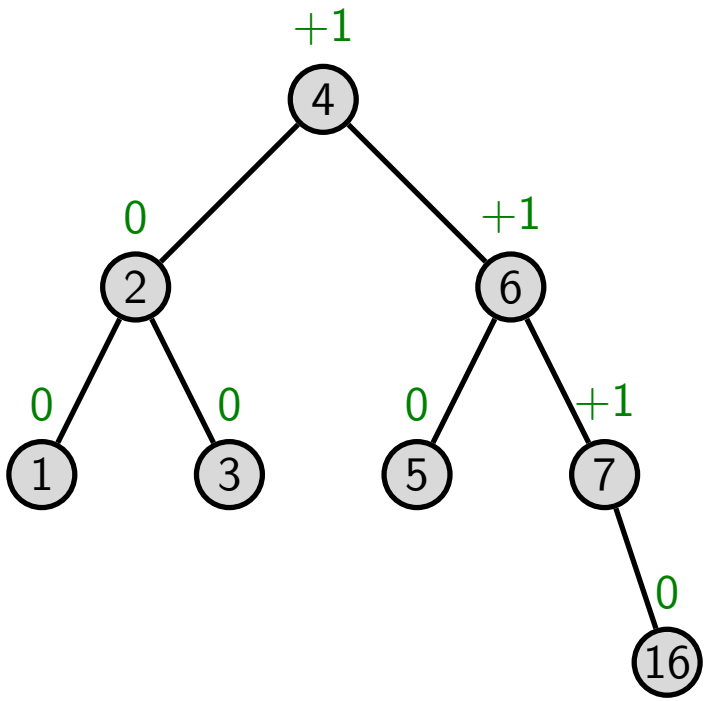


Figure: Sola döndür

16 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, 15, 14, 13, 12, 11, 10, 8, 9



15 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, 14, 13, 12, 11, 10, 8, 9

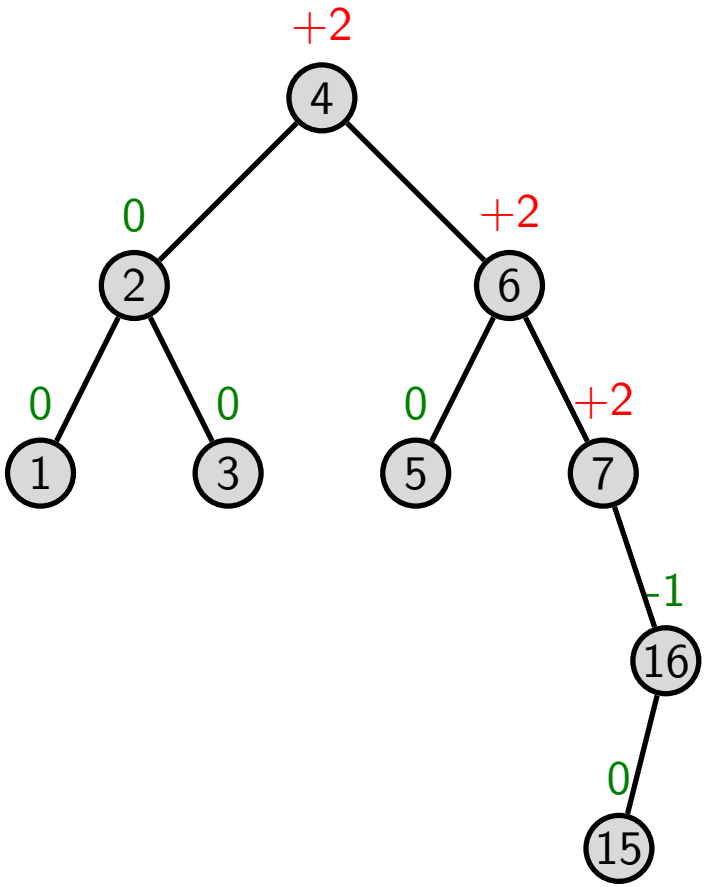


Figure: Mavi sağa, sonra kırmızı sola

15 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, 14, 13, 12, 11, 10, 8, 9

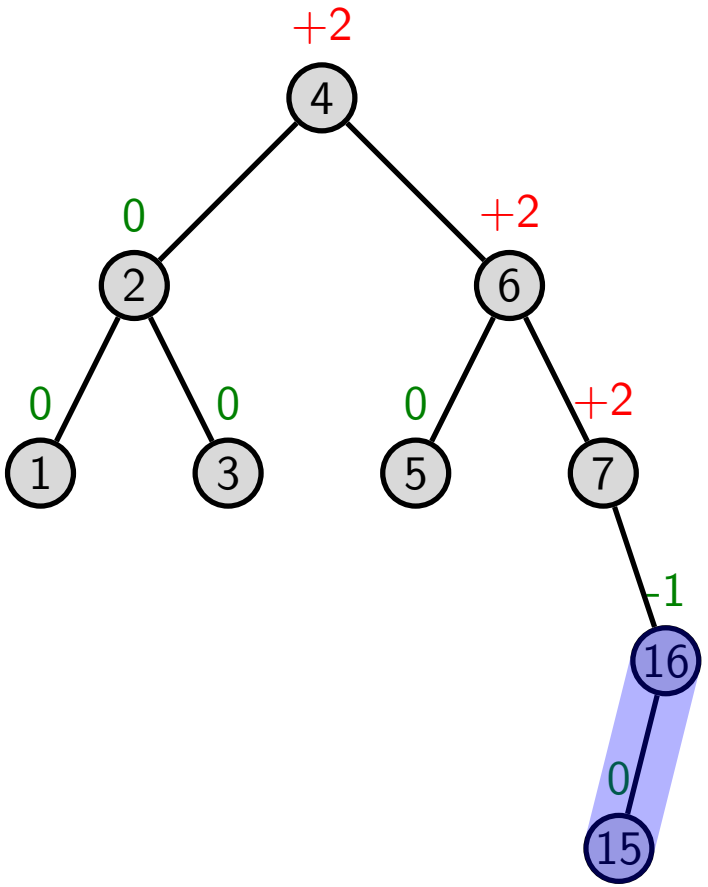


Figure: Mavi sağa, sonra kırmızı sola

15 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, 14, 13, 12, 11, 10, 8, 9

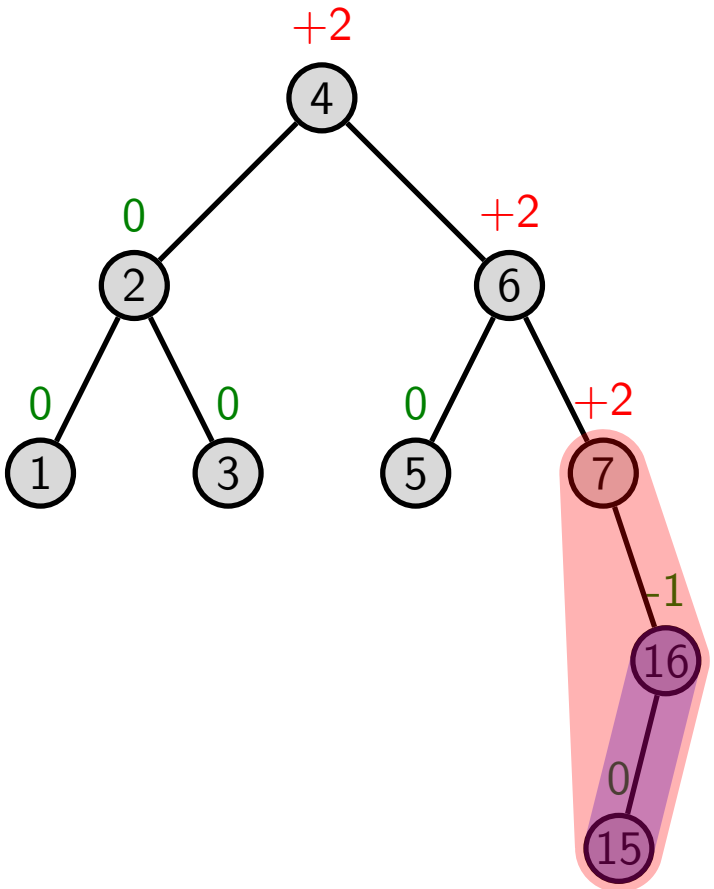
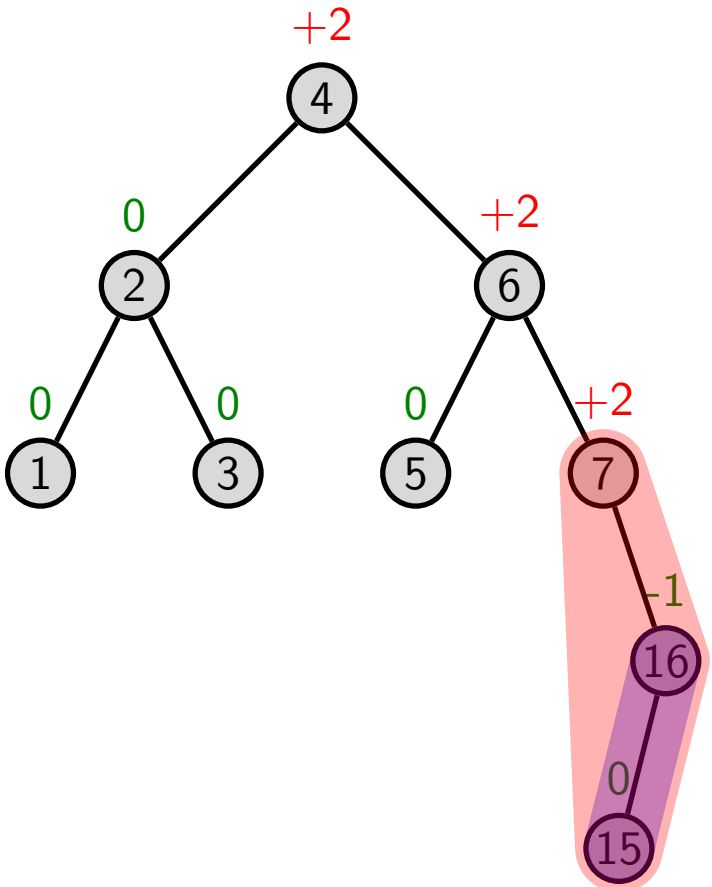


Figure: Mavi sağa, sonra kırmızı sola

15 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, 14, 13, 12, 11, 10, 8, 9



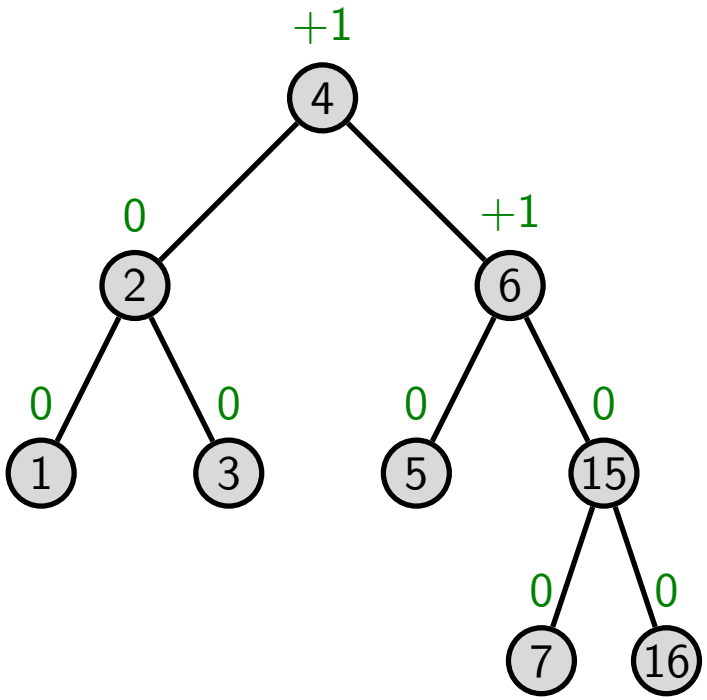


Figure: Mavi sağa, sonra kırmızı sola

14 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, 13, 12, 11, 10, 8, 9

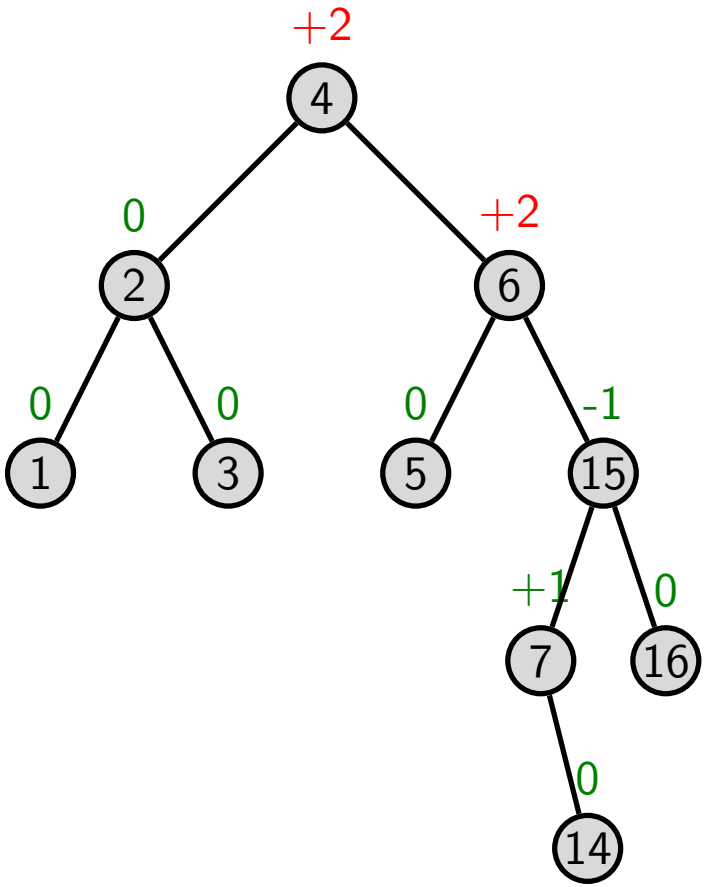


Figure: Mavi sağa, sonra kırmızı sola

14 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, 13, 12, 11, 10, 8, 9

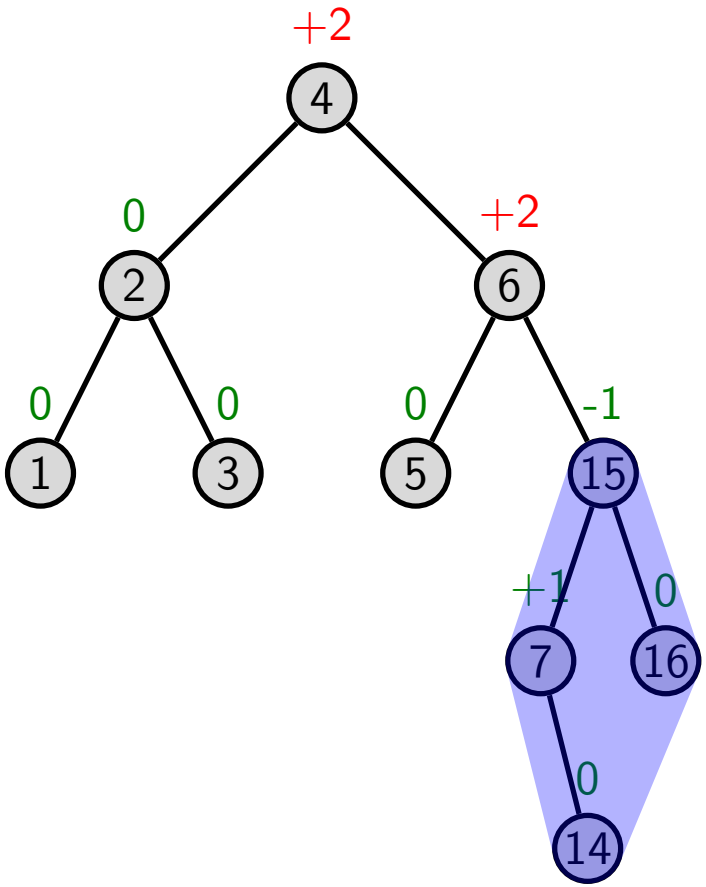


Figure: Mavi sağa, sonra kırmızı sola

14 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, 13, 12, 11, 10, 8, 9

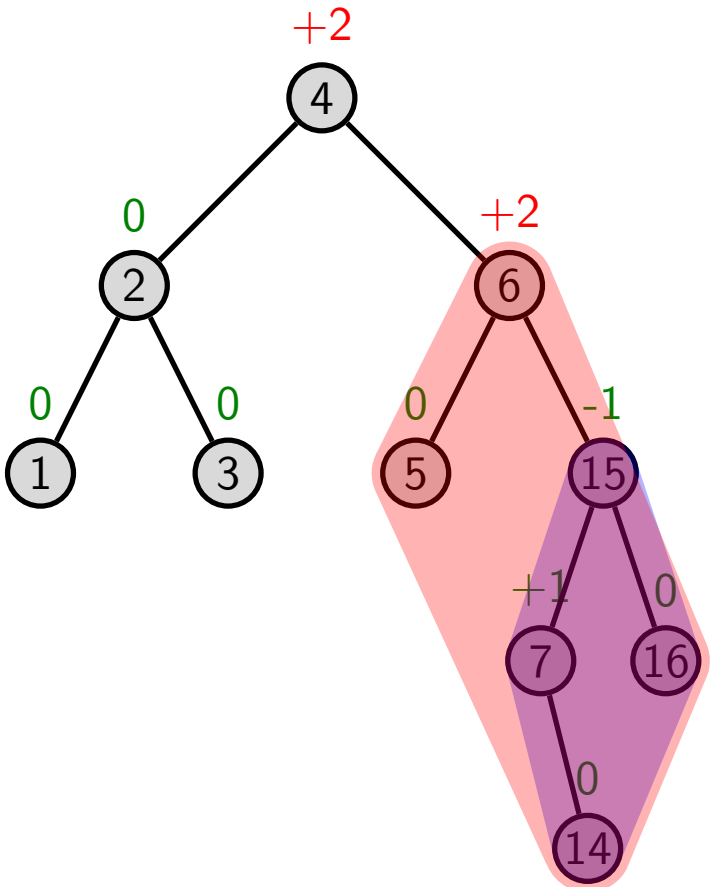


Figure: Mavi sağa, sonra kırmızı sola

14 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, 13, 12, 11, 10, 8, 9

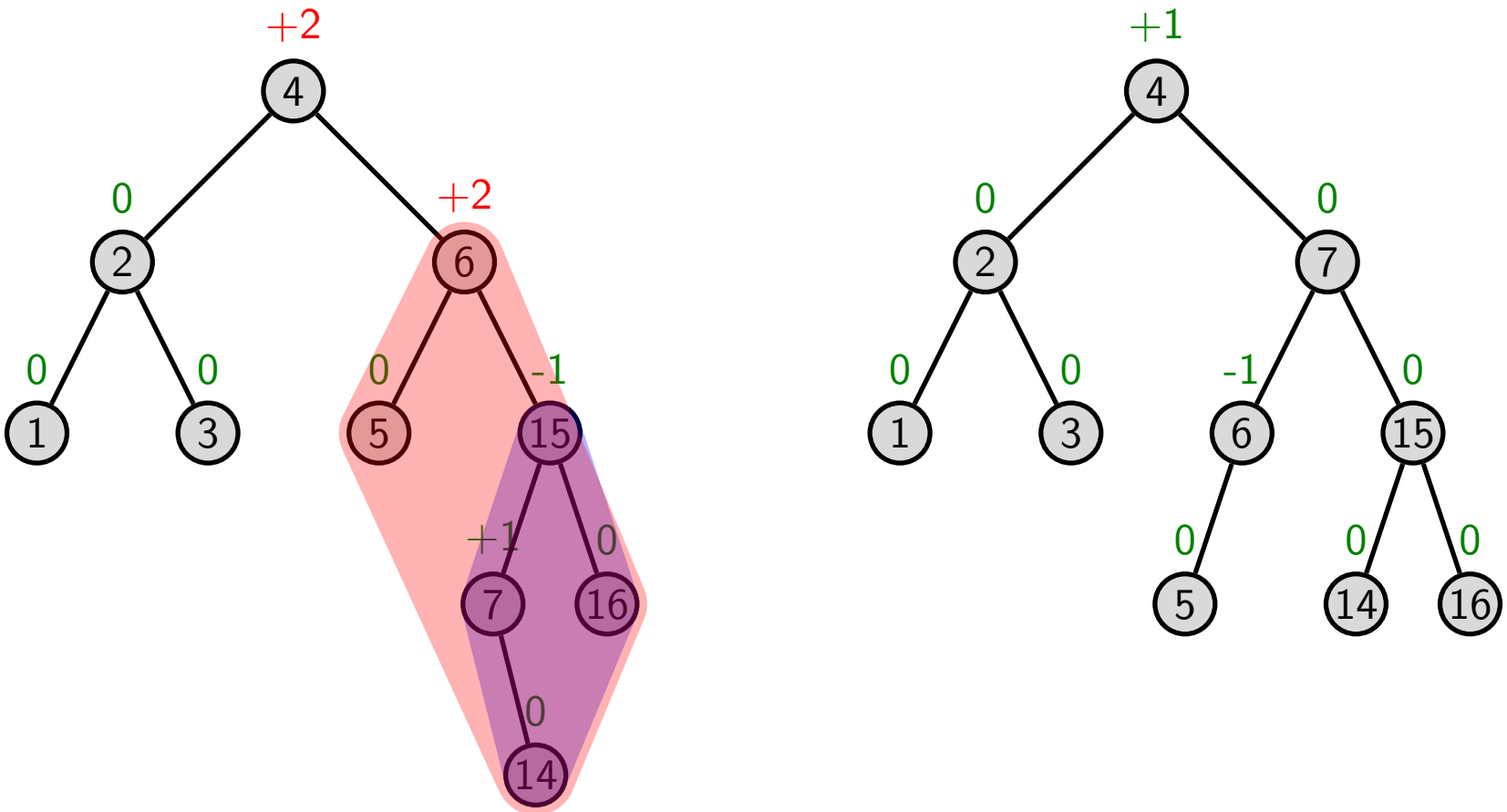


Figure: Mavi sağa, sonra kırmızı sola

13 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, 12, 11, 10, 8, 9

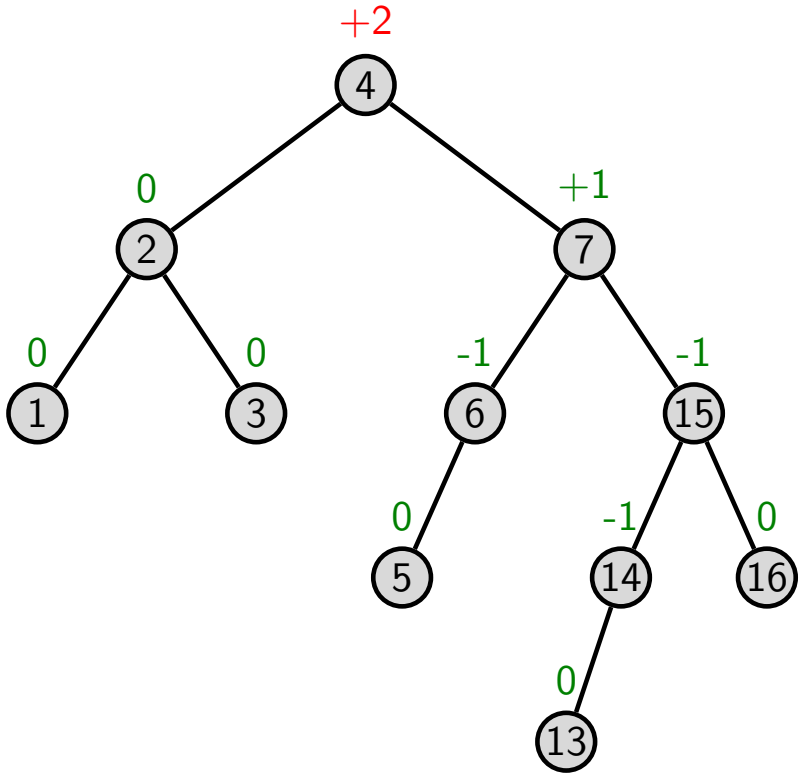


Figure: Tüm ağacı sola döndür

13 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, 12, 11, 10, 8, 9

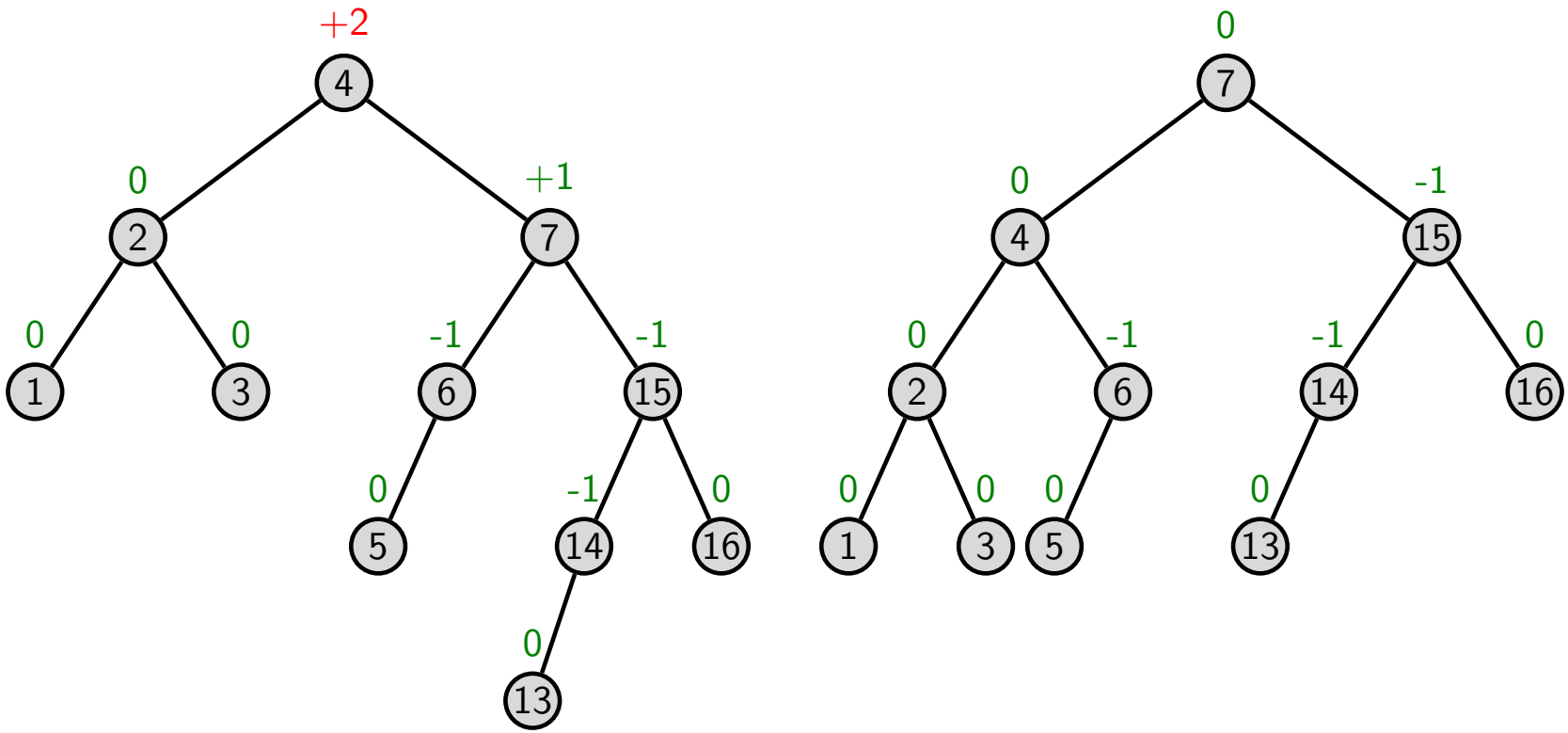


Figure: Tüm ağacı sola döndür

12 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, 11, 10, 8, 9

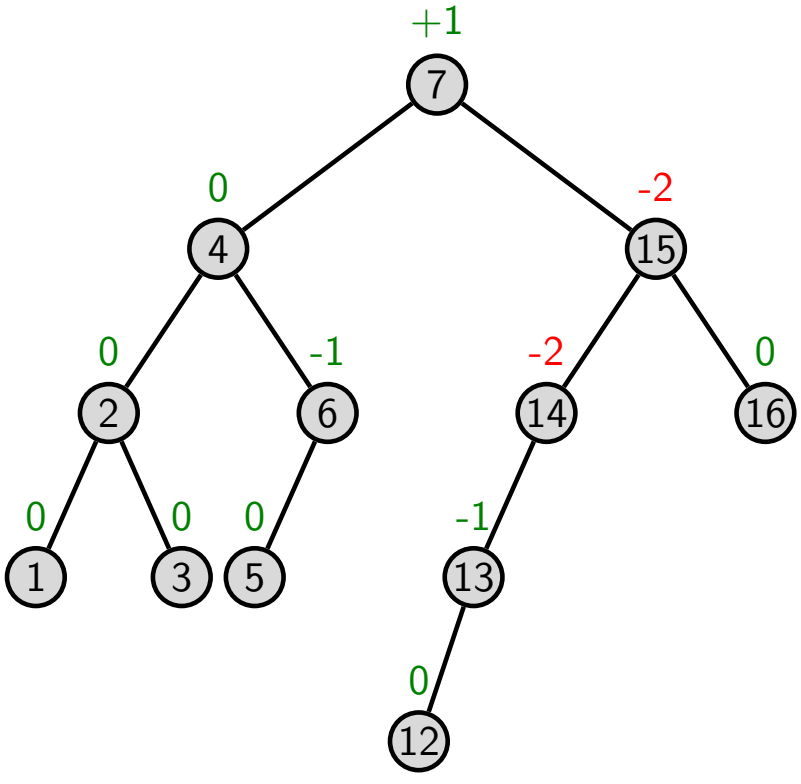


Figure: Sağa döndür

12 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, 11, 10, 8, 9

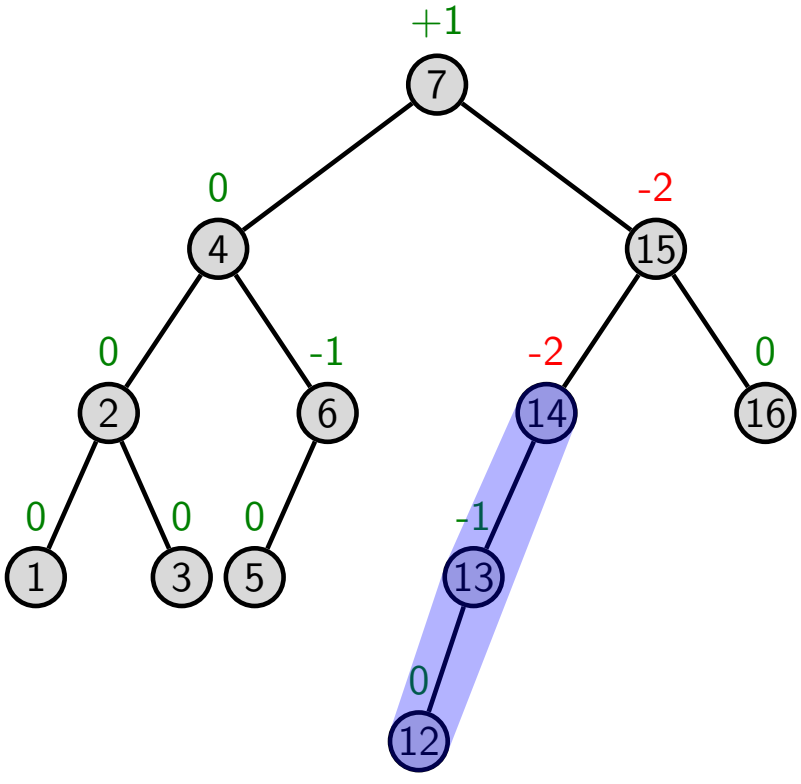


Figure: Sağa döndür

12 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, 11, 10, 8, 9

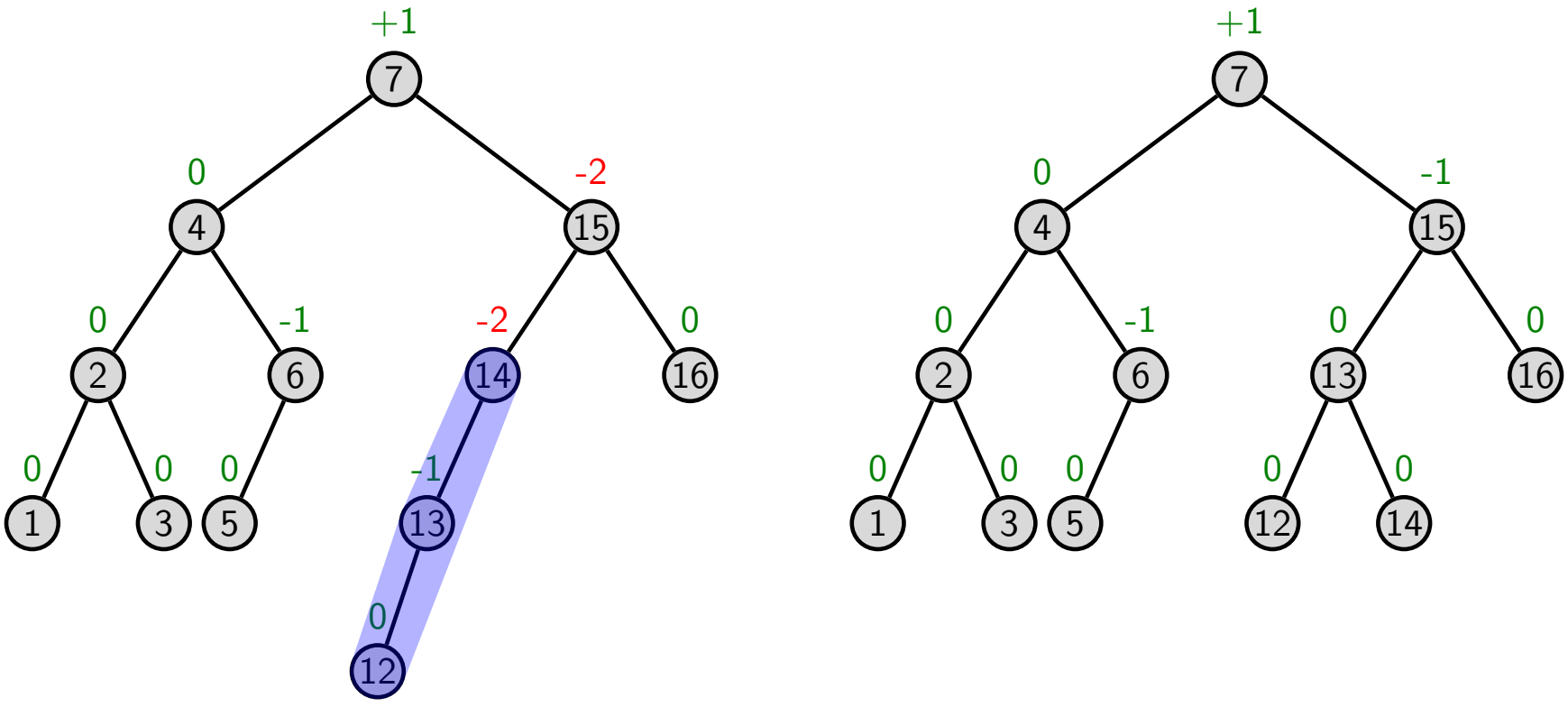


Figure: Sağa döndür

11 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, 10, 8, 9

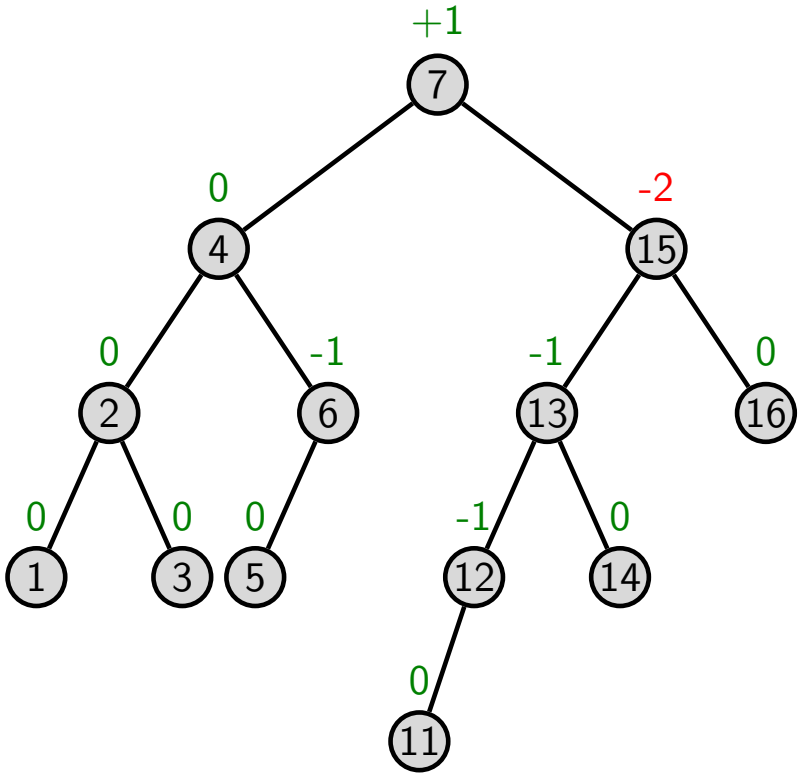


Figure: Sağa döndür

11 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, 10, 8, 9

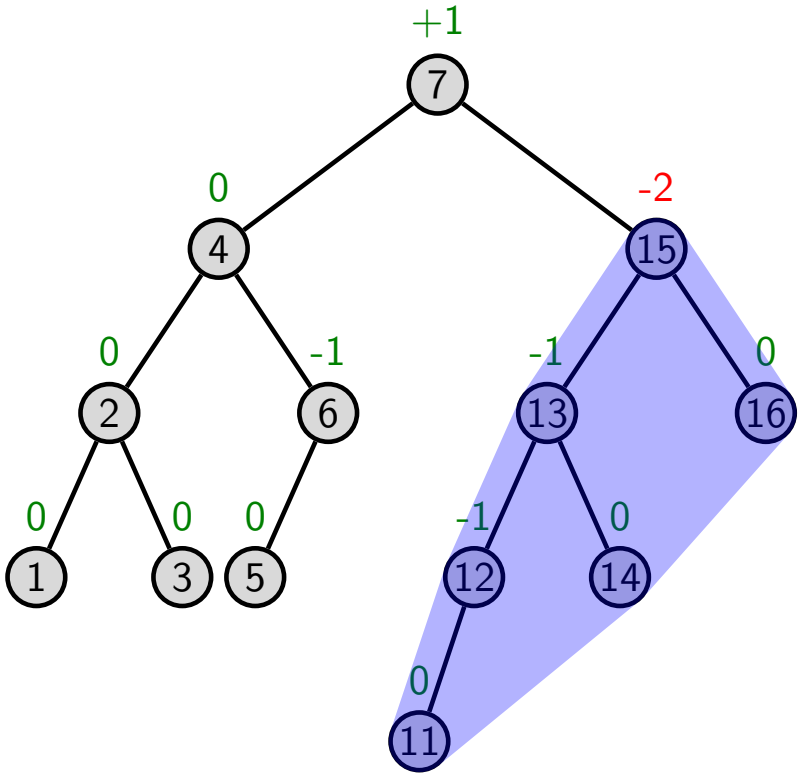


Figure: Sağa döndür

11 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, 10, 8, 9

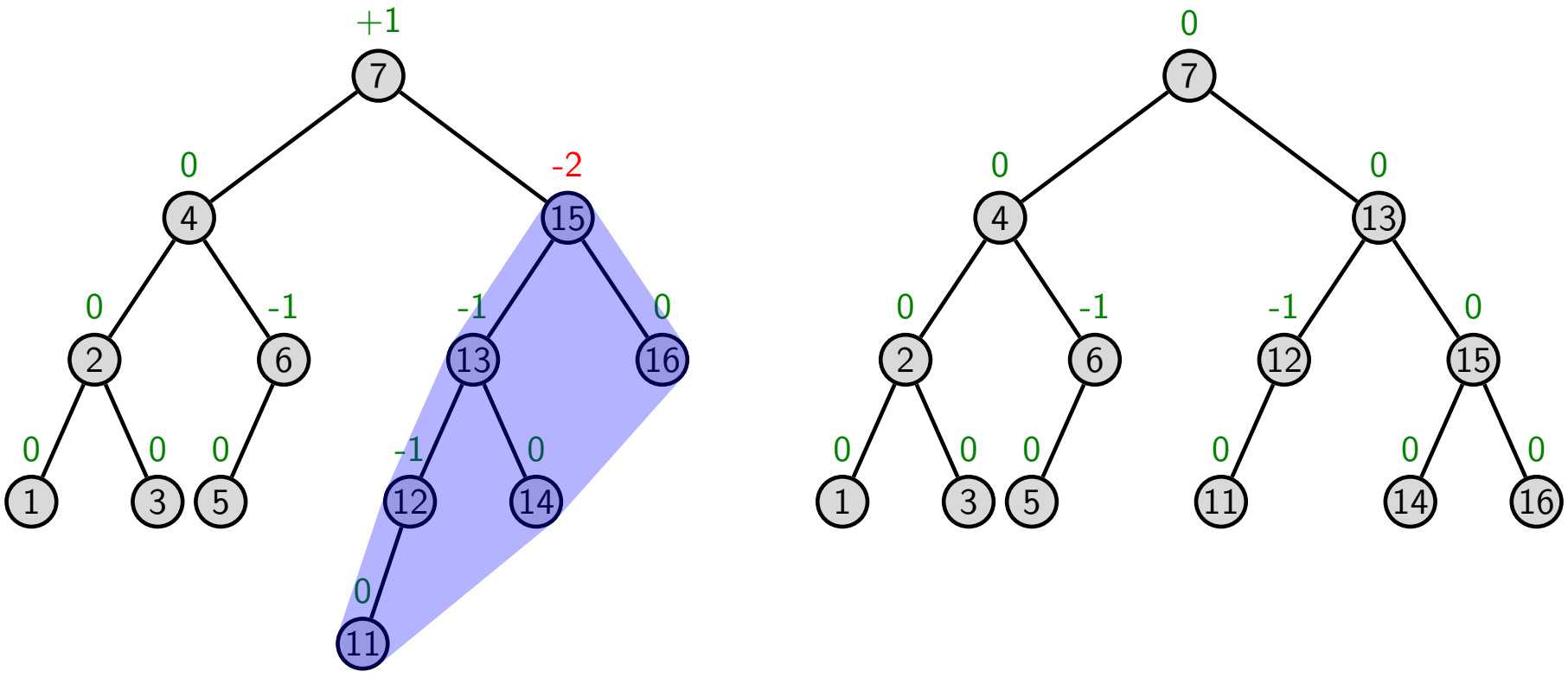


Figure: Sağa döndür

10 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, 8, 9

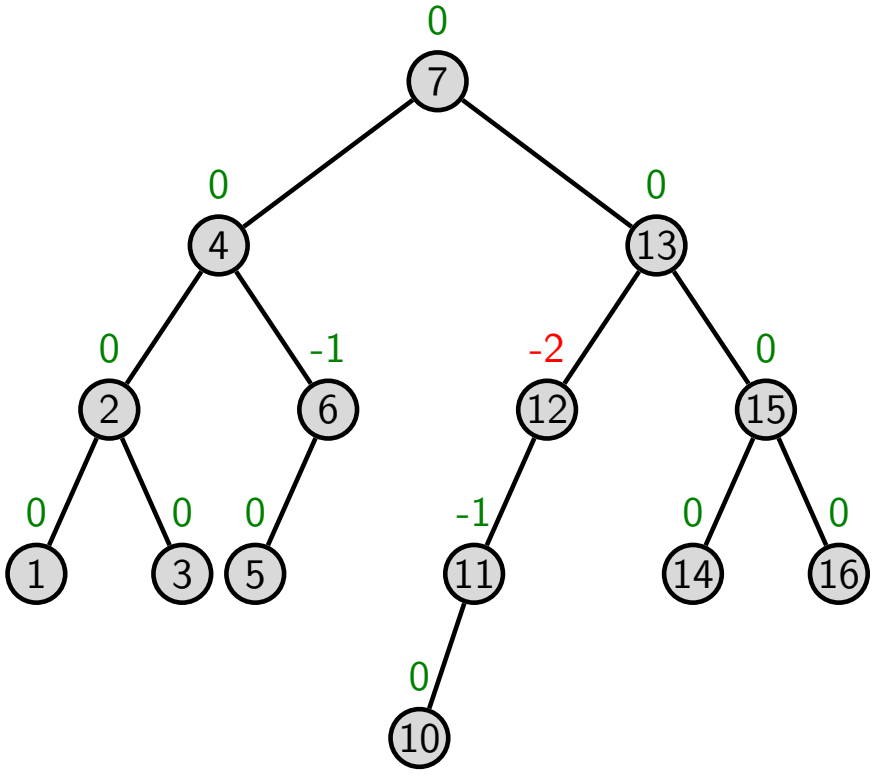


Figure: Sağa döndür

10 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, 8, 9

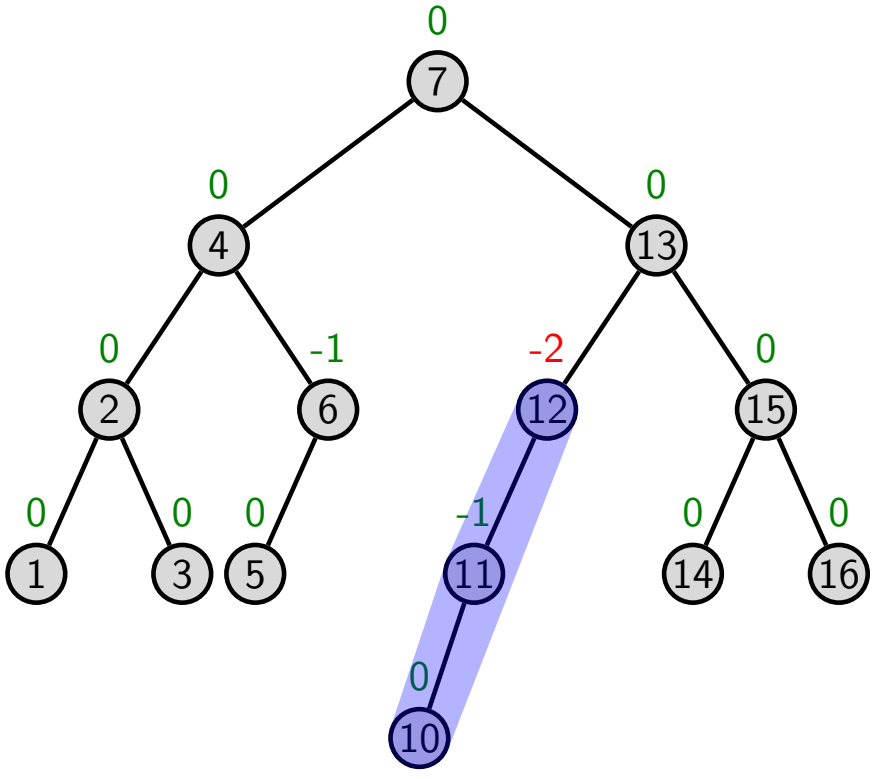


Figure: Sağa döndür

10 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, 8, 9

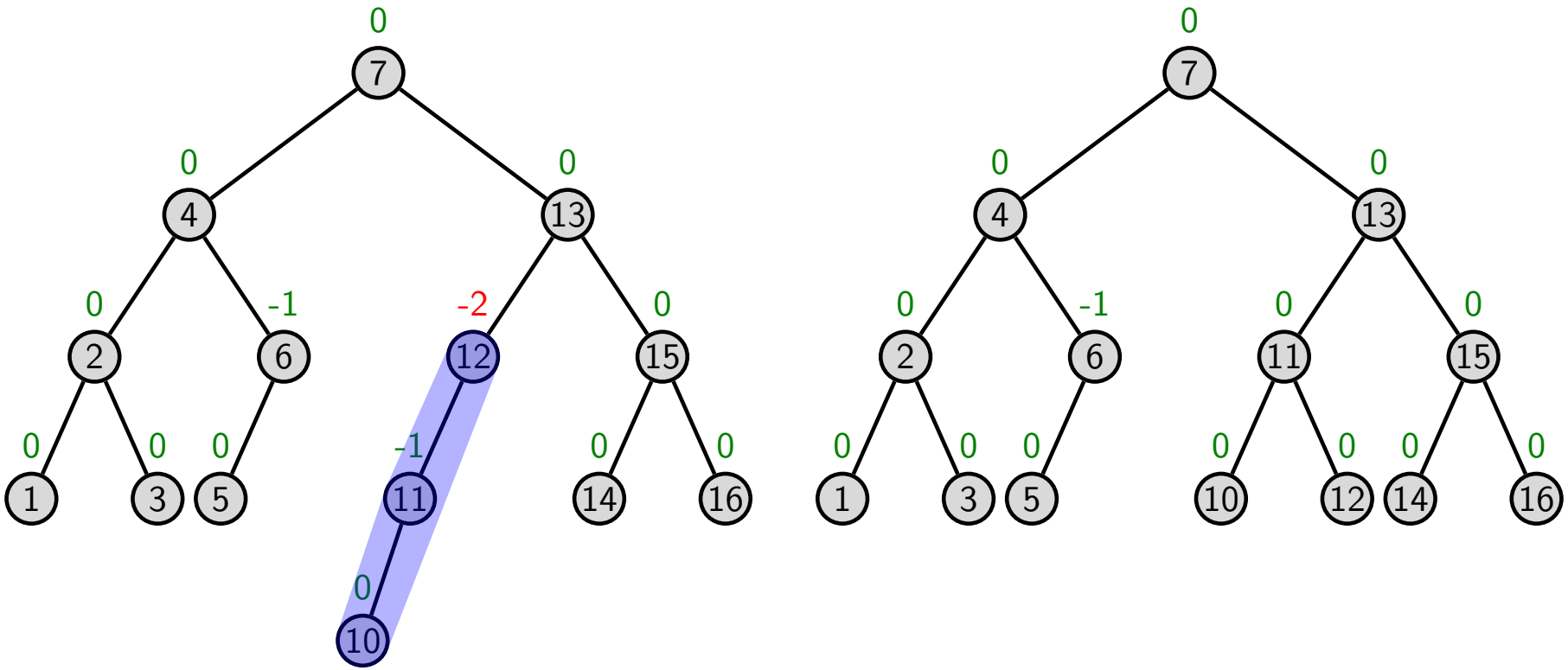


Figure: Sağa döndür

8 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, ~~8~~, 9

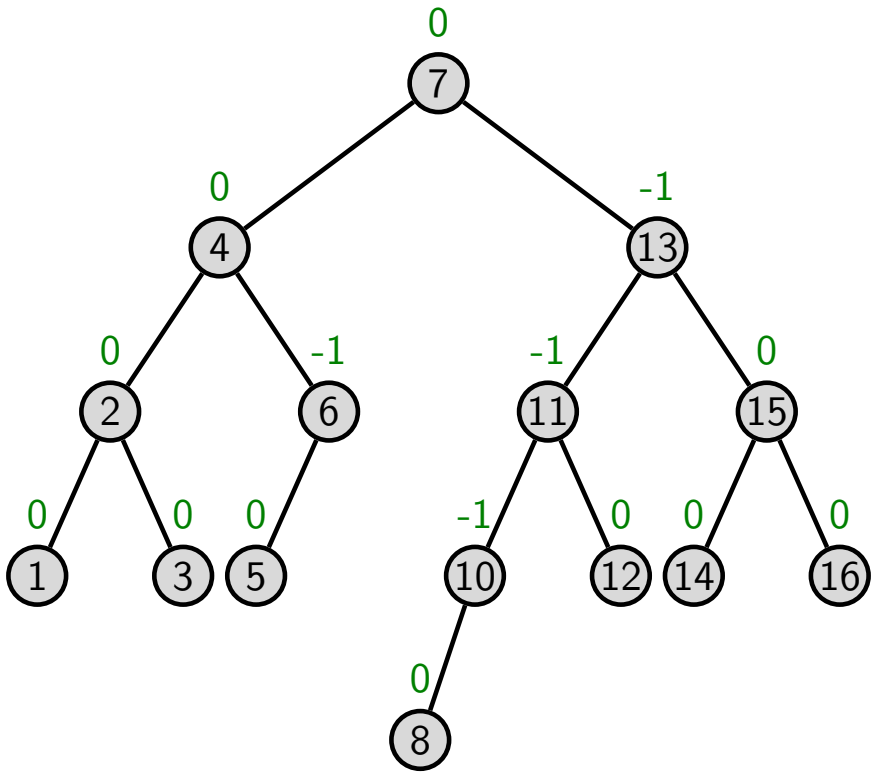


Figure: Sağa döndür

9 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, ~~8~~, ~~9~~

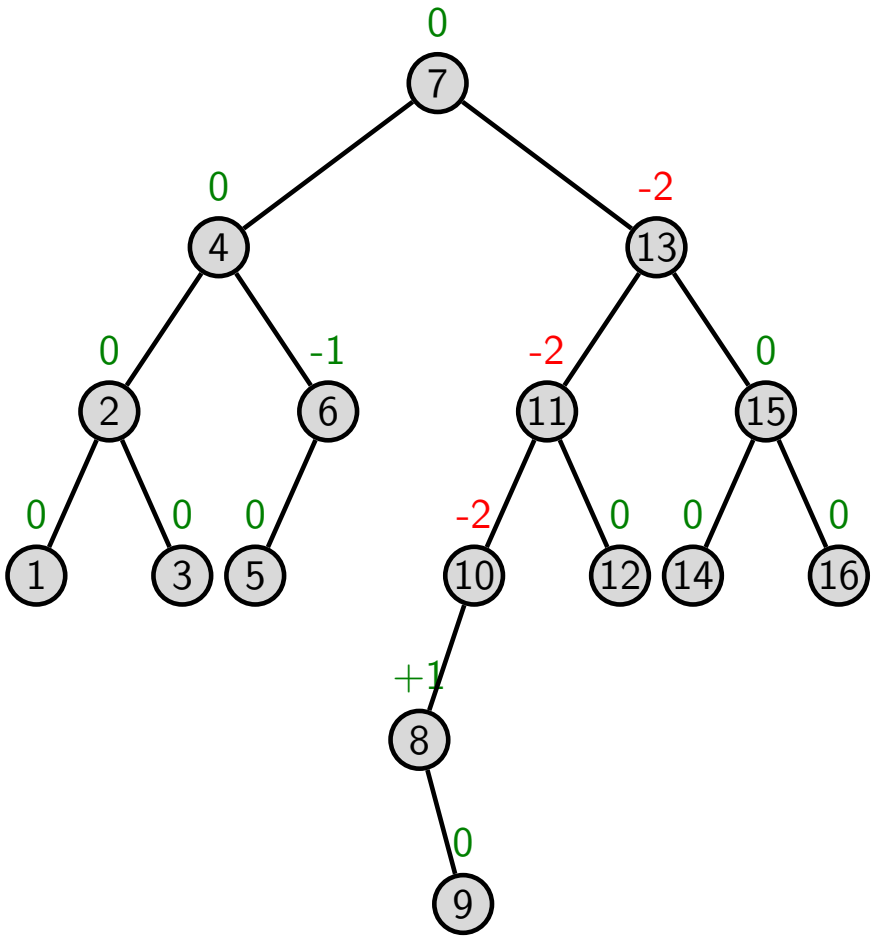


Figure: Kırmızı sola, mavi sağa

9 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, ~~8~~, ~~9~~

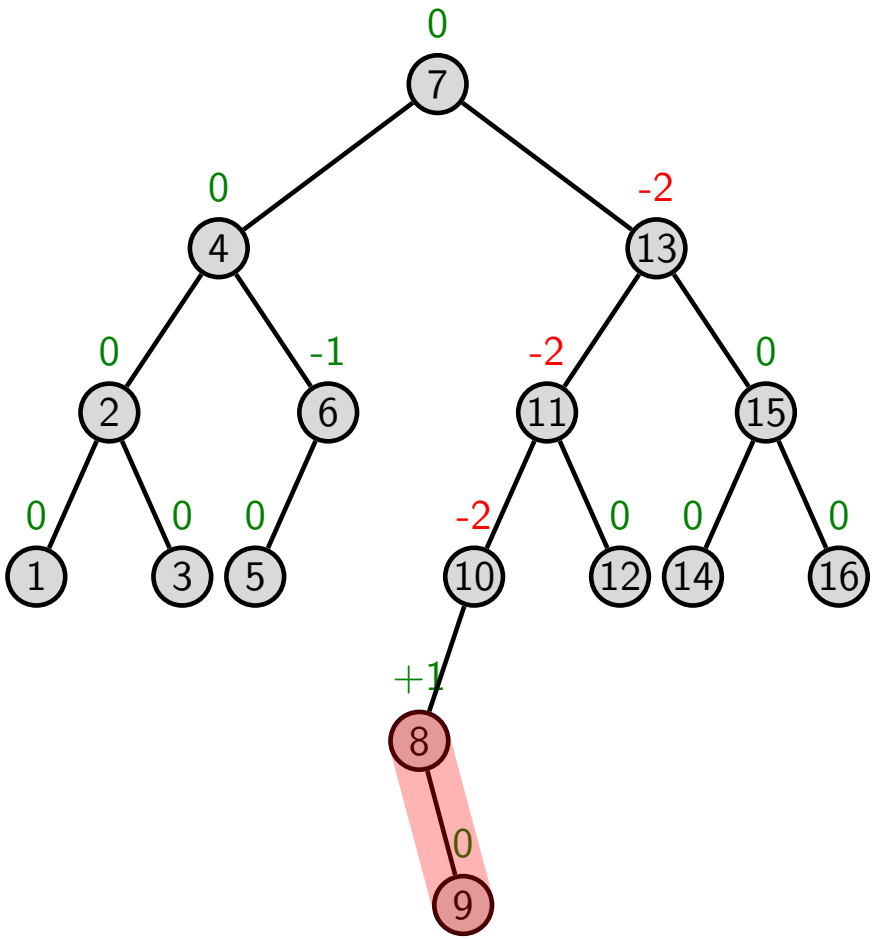


Figure: Kırmızı sola, mavi sağa

9 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, ~~8~~, ~~9~~

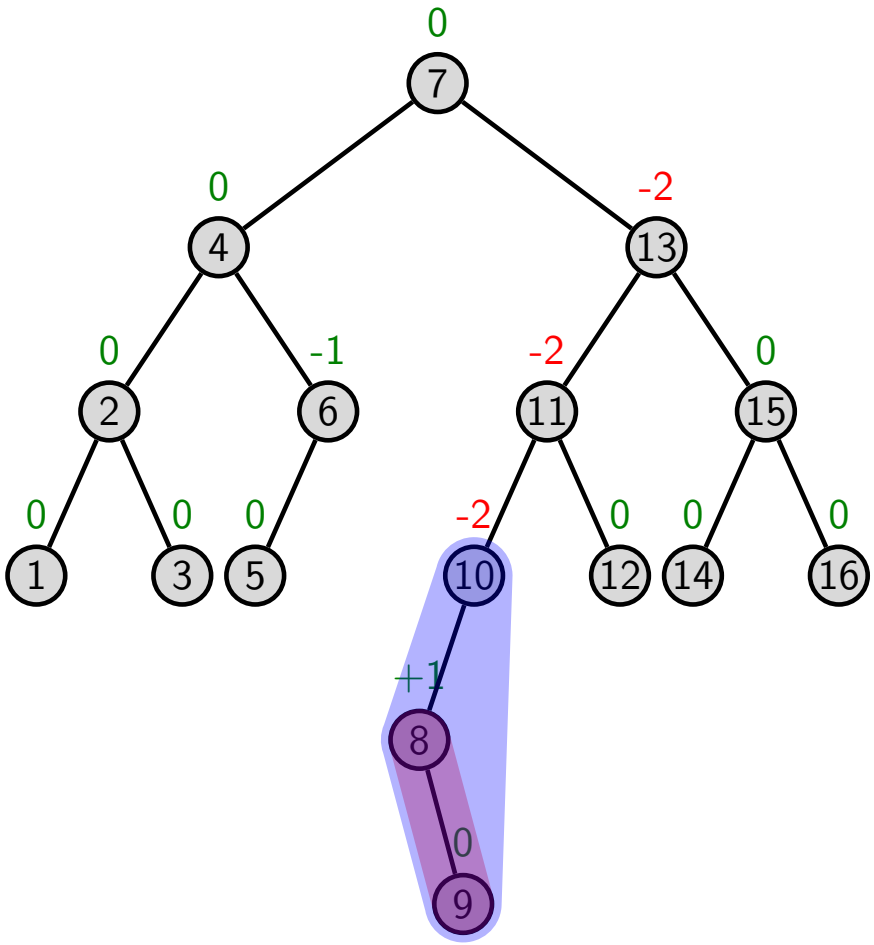


Figure: Kırmızı sola, mavi sağa

9 Ekle

Eklenecek: ~~3~~, ~~2~~, ~~1~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~16~~, ~~15~~, ~~14~~, ~~13~~, ~~12~~, ~~11~~, ~~10~~, ~~8~~, ~~9~~

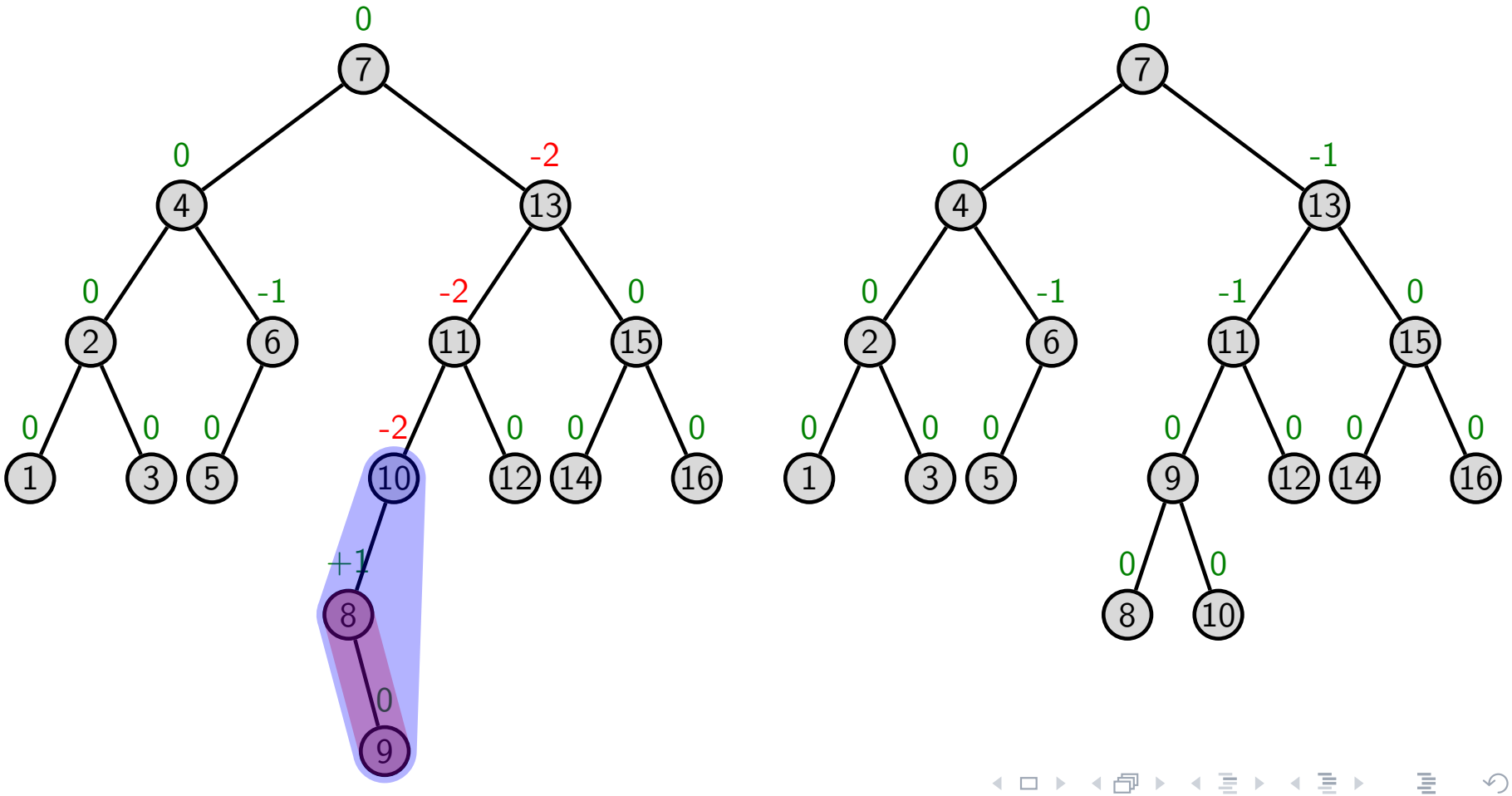


Figure: Kırmızı sola, mavi sağa

Soru

13, 21, 45, 8, 10, 65, 7, 4, 16, 46, 15 değerlerini sırasıyla AVL ağacına ekleyin.

Soru

13, 21, 45, 8, 10, 65, 7, 4, 16, 46, 15 değerlerini sırasıyla AVL ağacına ekleyin.

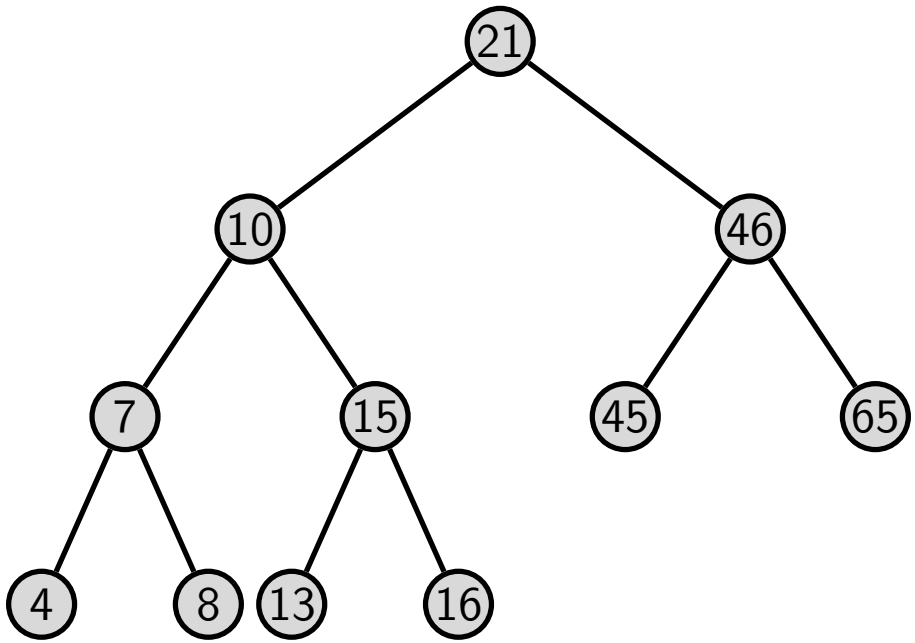


Figure: Değerler eklenince ortaya çıkan AVL Ağacı

Silme İşlemi

- Silinecek düğüm w olsun
- w düğümünü ikili arama ağacındaki gibi sil
- w'dan köke doğru ilk dengesiz z düğümünü bul
- y, z'nin yüksek çocuğu olsun
- x, y'nin yüksek çocuğu olsun
- 4 farklı durum ortaya çıkmaktadır

Sol'un solu durumu

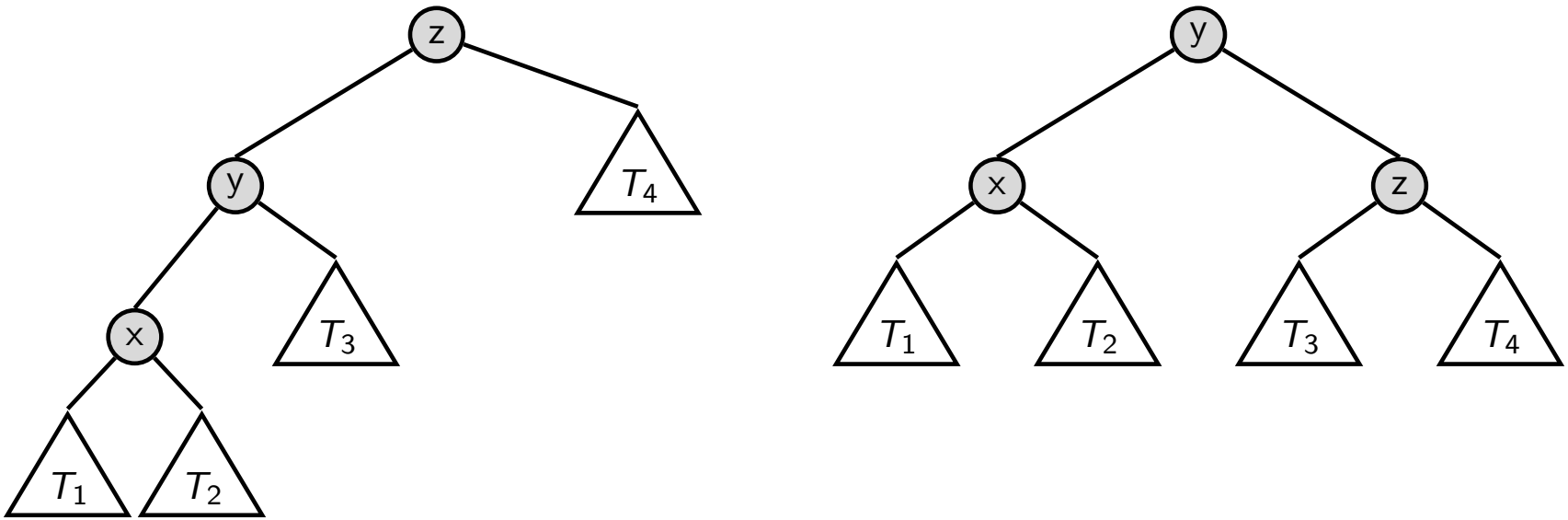


Figure: Sağa döndürme

Sol'un sağı durumu

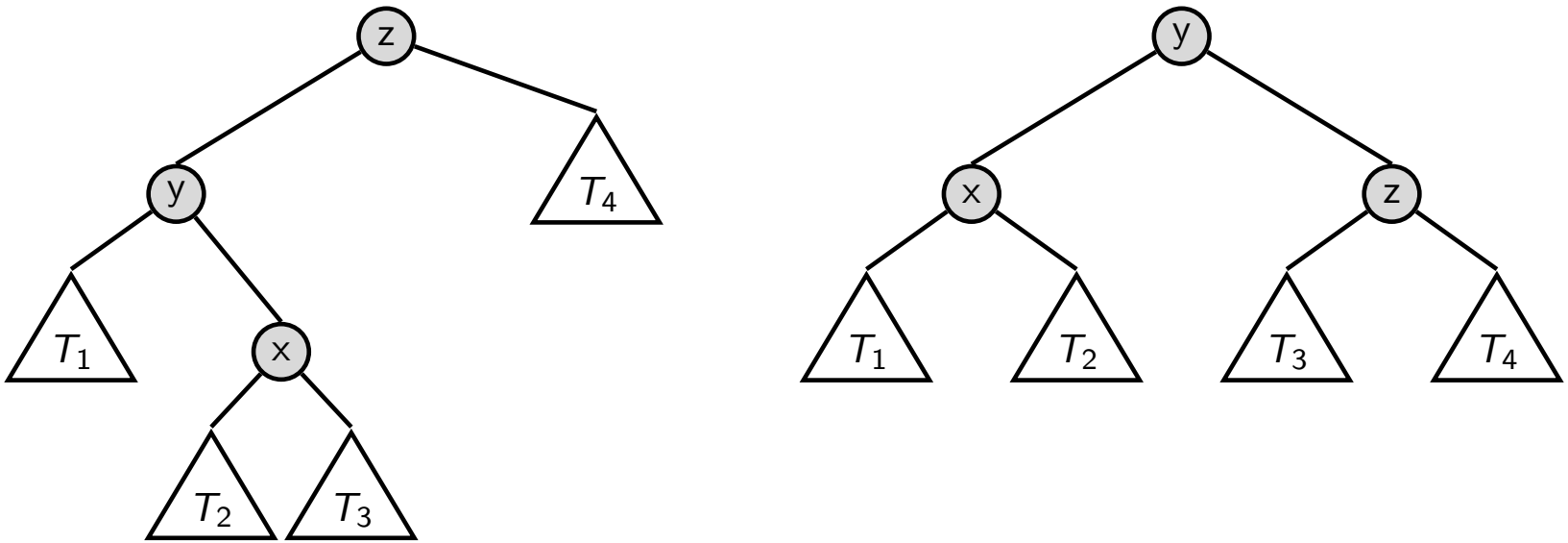


Figure: Önce sola sonra sağa döndürme

Sağ'ın sağı durumu

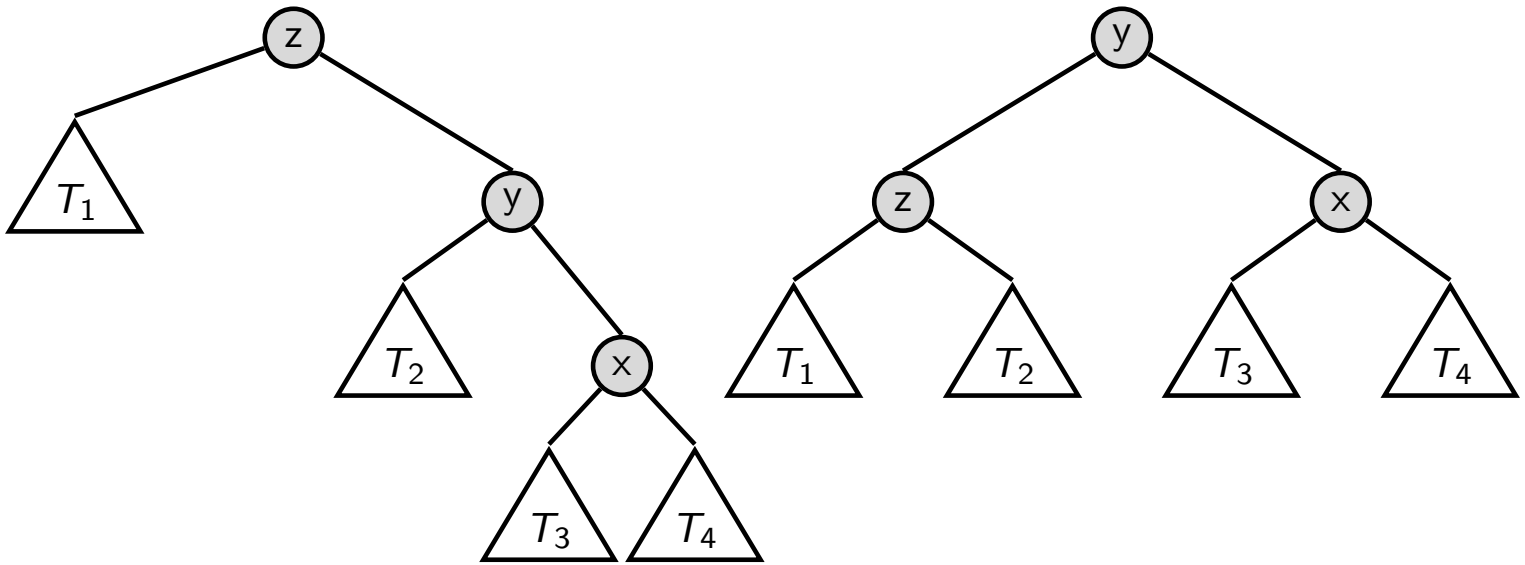


Figure: Sola döndürme

Sağ'ın solu durumu

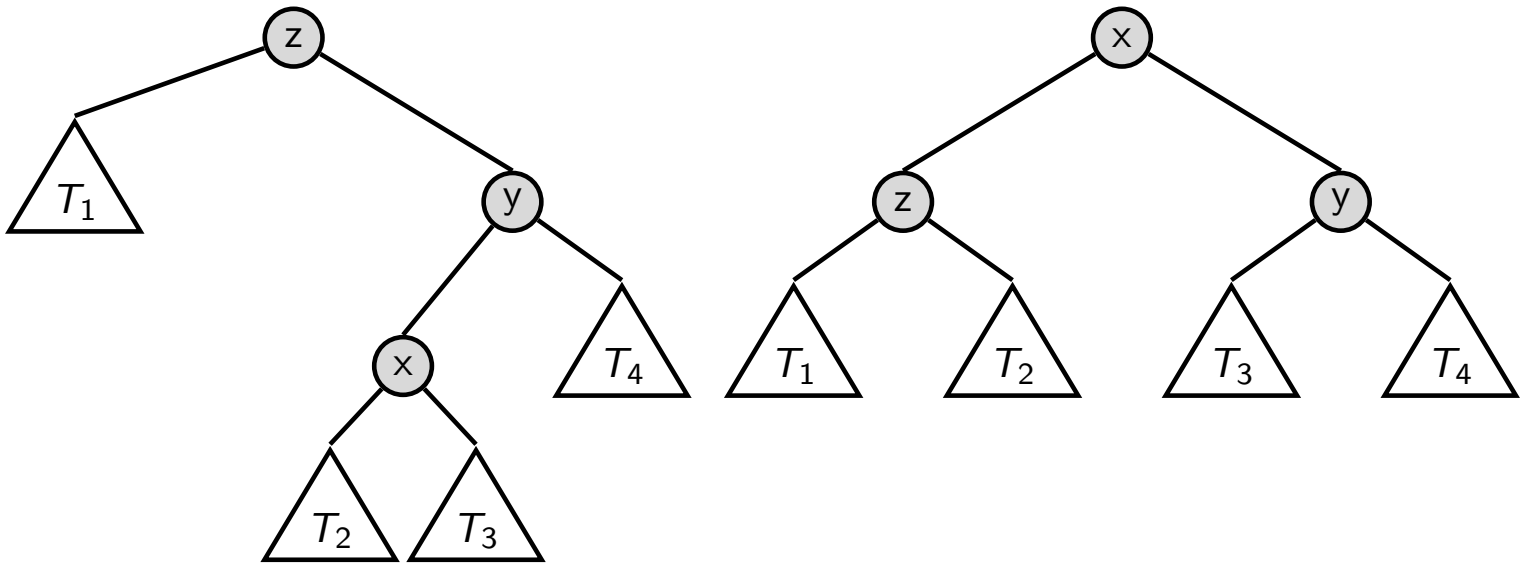
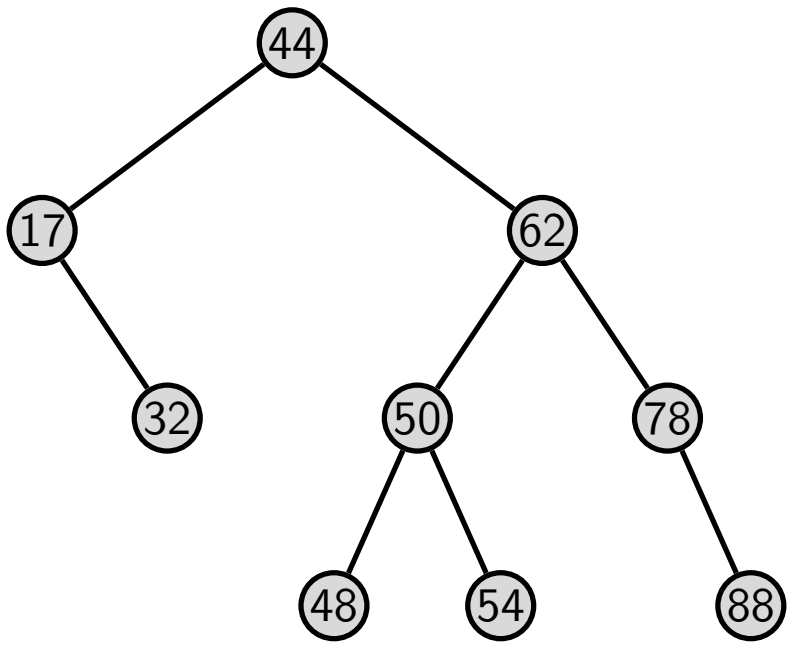


Figure: Önce sağa sonra sola döndürme

Örnek: 32'nin silinmesi



Örnek: 32'nin silinmesi

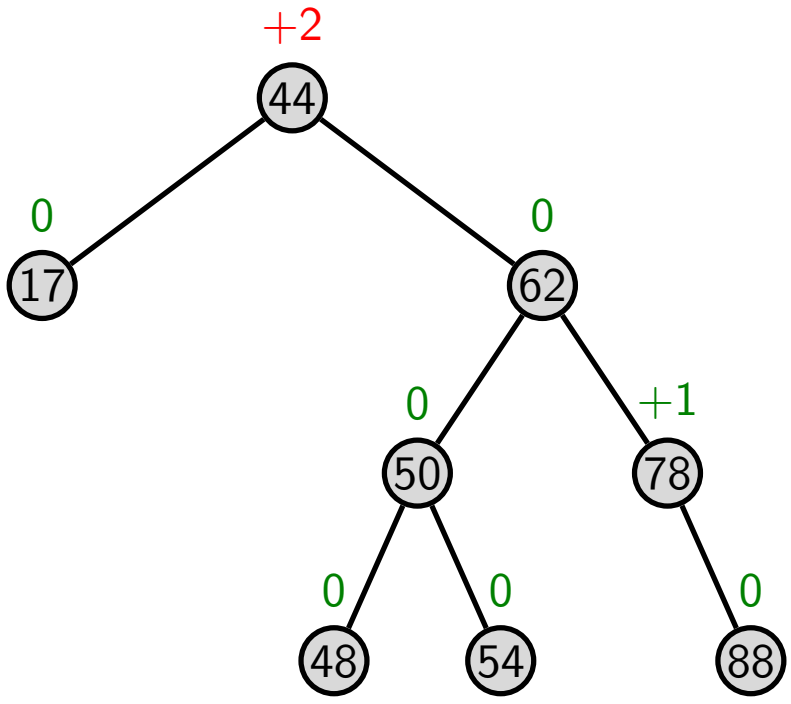


Figure: Sola döndürme

Örnek: 32'nin silinmesi

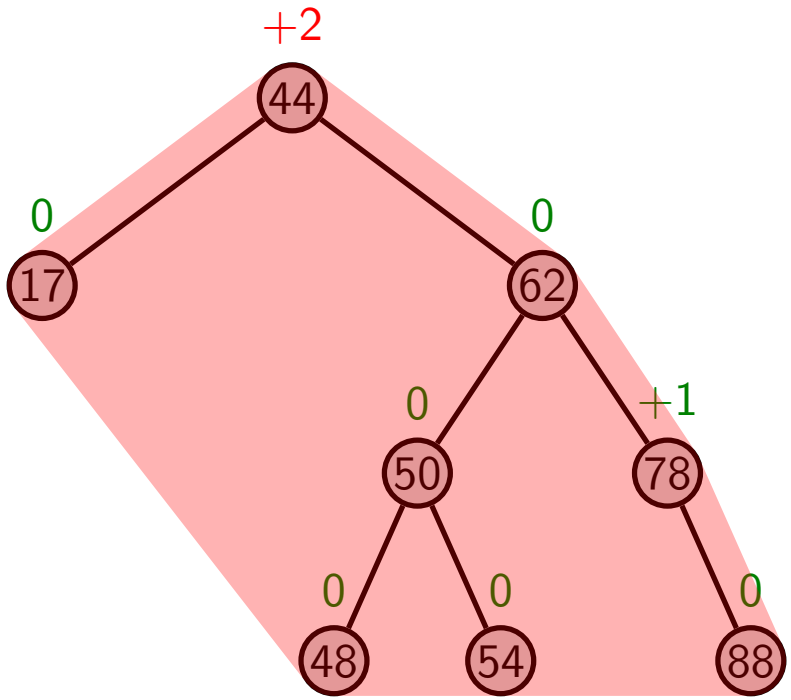


Figure: Sola döndürme

Örnek: 32'nin silinmesi

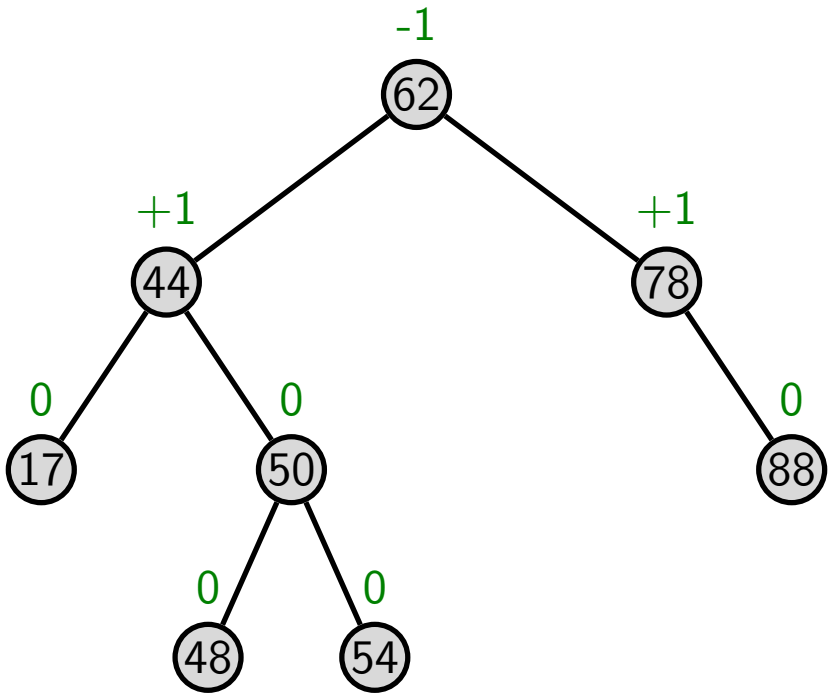
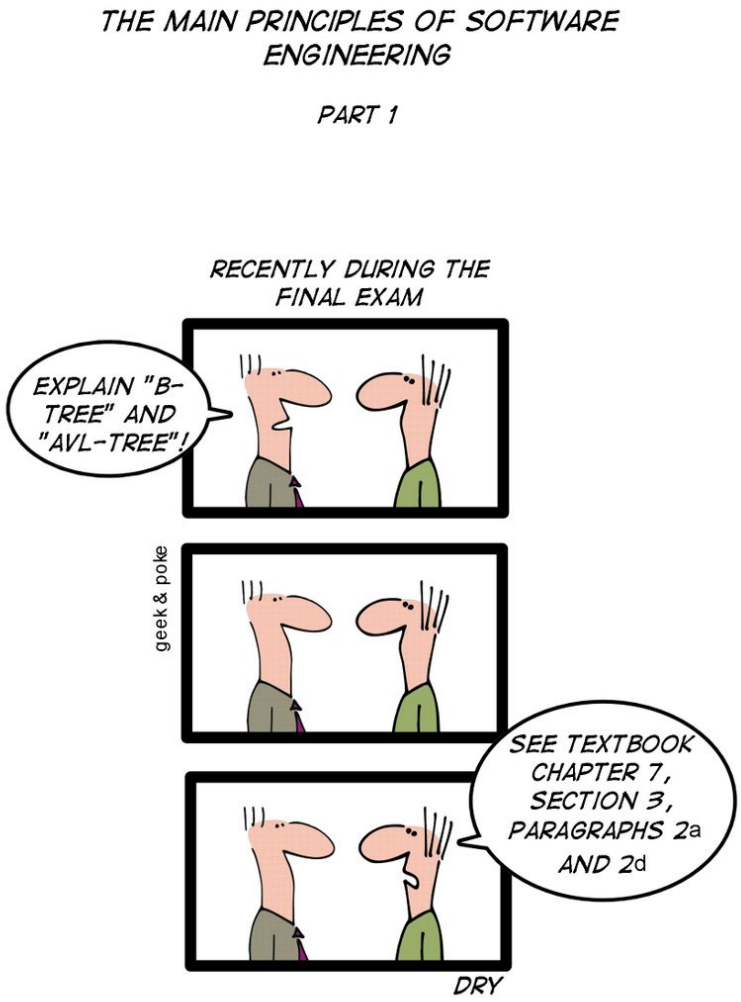


Figure: 32 silindikten sonra ağaç dengelendi

Görselleştirme

- `https://www.cs.usfca.edu/~galles/visualization/AVLtree.html`
- `https://visualgo.net/bn/bst`


THE MAIN PRINCIPLES OF SOFTWARE ENGINEERING
PART 1
RECENTLY DURING THE FINAL EXAM
EXPLAIN "B-TREE" AND "AVL-TREE"!
SEE TEXTBOOK CHAPTER 7, SECTION 3, PARAGRAPHS 2a AND 2d
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