



$$T(n) = 2T(n-1) + 1$$

$$T(1) = 1$$

$$= 2 \cdot (2T(n-2) + 1) + 1$$

$$T(n-1) = 2 \cdot T(n-2) + 1$$

$$= 4T(n-2) + 2 + 1$$

$$T(n-2) = 2T(n-3) + 1$$

$$= 4(2T(n-3) + 1) + 2 + 1$$

$$= 8T(n-3) + 4 + 2 + 1$$

$$= 2^i \cdot T(n-i) + \underbrace{1 + 2 + \dots + 2^{i-1}}$$

$$\sum_{k=0}^{i-1} 2^k = 2^i - 1$$

$$= 2^i \cdot T(\underbrace{n-i}_1) + 2^i - 1, \quad n-i=1, \quad i=n-1$$

$$= 2^{n-1} \cdot T(1) + 2^{n-1} - 1 = 2^{n-1} + 2^{n-1} - 1 = 2^n - 1$$

$$2^n - 1 \in \Theta(2^n)$$

$$T(n) = T\left(\frac{n}{2}\right) + 1, \quad T(1) = 0$$

$$= T\left(\frac{n}{2}\right) + 1 + 1$$

$$= T\left(\frac{n}{8}\right) + 1 + 1 + 1$$

$$= T\left(\frac{n}{2^i}\right) + i$$

$$T(n) = T(1) + \log_2 n$$

$$T(n) = \log_2 n \in \Theta(\log n)$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + 1$$

$$\frac{n}{2^i} = 1, \quad 2^i = n$$

$$\log_2 2^i = \log_2 n$$

$$i \cdot \log_2 2 = \log_2 n$$

$$i = \log_2 n$$

$$T(n) = 2T(\sqrt{n}) + 1, \quad T(2) = 0$$

$$T(n) = 2T(n^{\frac{1}{2}}) + 1$$

$$= 2(2T(n^{\frac{1}{4}}) + 1) + 1$$

$$= 4T(n^{\frac{1}{4}}) + 2 + 1$$

$$= 4(2T(n^{\frac{1}{8}}) + 1) + 2 + 1$$

$$= 8T(n^{\frac{1}{8}}) + 4 + 2 + 1$$

$$T(n^{\frac{1}{2}}) = 2T((n^{\frac{1}{2}})^{\frac{1}{2}})$$

$$+ 1$$

$$= 2T(n^{\frac{1}{4}}) + 1$$

$$T(n^{\frac{1}{4}}) = 2T(n^{\frac{1}{8}}) + 1$$

$$\sqrt{\frac{1}{n^{\frac{1}{2^i}}}} = 2$$

$$= 2^i T(n^{2^i}) + \underbrace{1 + 2^1 + \dots + 2^{i-1}}_{2^i - 1}$$

$$= 2^i \cdot T\left(n^{\frac{1}{2^i}}\right) + 2^i - 1$$

$$= \cancel{\log_2 n \cdot T(2)} + \log_2 n - 1$$

$$T(n) = \log_2 n - 1 \in \Theta(\log n)$$

$$\log_n n^{\frac{1}{2^i}} = \log_n 2$$

$$\frac{1}{2^i} = \log_n 2$$

$$2^i = \frac{1}{\log_n 2}$$

$$2^i = \log_2 n$$

$$\log_2 2^i = \log_2 \log_2 n$$

$$i = \log_2 \log_2 n$$