LiNEER SİSTEMLER $\underset{\sim}{AX = B}$

$$A\underline{X} = B$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{\underline{X}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{B}$$

__örnek:__ $A = \begin{bmatrix} -1 & 2 \\ 7 & 5 \\ 3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 \\ 1 & -4 \\ -9 & 7 \end{bmatrix}$

$$2A - 3B = \begin{bmatrix} -2 & 4 \\ 14 & 10 \\ 6 & -8 \end{bmatrix} - \begin{bmatrix} -6 & 9 \\ 3 & -12 \\ -27 & 21 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 11 & 22 \\ 33 & -29 \end{bmatrix}$$

__örnek:__ $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 8 & -6 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 & 1 \\ 3 & 8 & -6 \end{bmatrix} = \begin{bmatrix} 19 & 20 & -16 \\ 7 & 34 & -25 \end{bmatrix}$$

__SORU:__ $\begin{bmatrix} 0.125 & 0.2 & 0.4 \\ 0.375 & 0.5 & 0.6 \\ 0.5 & 0.3 & 0.0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2.3 \\ 4.8 \\ 2.9 \end{bmatrix}$

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

__SORU:__ $A = \begin{bmatrix} 2 & 3 & 8 \\ -4 & 5 & -1 \\ 7 & -6 & 9 \end{bmatrix} \Rightarrow |A| = \det(A) = ?$

$$= 2(45-6) - 3(-36+7) + 8(24-35) = 62 + 3\cdot(29) - 88$$

$$= 78 + 87 - 88 = 78 - 1 = \underline{77}$$

ex: $A\underline{X} = B \Rightarrow A^{-1}A\underline{X} = A^{-1}B$

$$\underline{X} = A^{-1}B$$

$$3x_1 + x_2 = 2$$
$$7x_1 + 4x_2 = 5$$

$$\underbrace{\begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{X}} = \underbrace{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{B}$$

$$A = \begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix} \Rightarrow A^{-1} = \frac{\begin{bmatrix} 4 & -1 \\ -7 & 3 \end{bmatrix}}{\begin{vmatrix} 3 & 1 \\ 7 & 4 \end{vmatrix}} = \frac{\begin{bmatrix} 4 & -1 \\ -7 & 3 \end{bmatrix}}{12-7}$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -1 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} 4/5 & -1/5 \\ -7/5 & 3/5 \end{bmatrix}$$

$$\underline{X} = \frac{1}{5}\begin{bmatrix} 4 & -1 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 8-5 \\ -14+15 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 \\ 1/5 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{2} & \cdots & a_{2n} \\ \vdots & & & \\ \vdots & & & a_{nn} \\ a_{n1} & a_{n2} & \cdots & \end{bmatrix} \Rightarrow AX = B$$

$$AX = B$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$a_3x_3 + \cdots + a_{3n}x_n = b_3$$
$$\vdots \qquad \vdots$$
$$a_{nn}x_n = b_n$$

Back substitution (Geri yerine koyma)

$$x_n = \frac{b_n}{a_{nn}}$$

$$x_{n-1} = \frac{b_{n-1} - a_{n-1n}x_n}{a_{n-1n-1}}$$

$\vdots$

$$x_{n-2} = \frac{b_{n-2}\ a_{n-2n-1}x_{n-1} - a_{n-2n}x_n}{a_{n-2n-2}}$$

Genel durumda $$x_k = \frac{b_k - \sum_{j=k+1}^{n} a_{kj}x_j}{a_{kk}}$$

$a_{kk} \neq 0$ ve $k = n-1, n-2, \ldots, 2, 1.$

Örnek:

$$4x_1 - x_2 + 2x_3 + 3x_4 = 20$$
$$-2x_2 + 7x_3 - 4x_4 = -7$$
$$6x_3 + 5x_4 = 4$$
$$3x_4 = 6$$

$$x_4 = \frac{6}{3} = 2$$

$$x_3 = \frac{4-5(2)}{6} = -1$$

$$x_2 = \frac{-7-7(-1)+4\cdot(2)}{-2} = -4$$

$$x_1 = \frac{20+(-4)-2(-1)-3(2)}{4} = 3$$

% Program Back substitution

% $x_k = \dfrac{b_k - \sum_{j=k+1}^{n} a_{kj} x_j}{a_{kk}}$, $k = n-1, n-2, \ldots 1$

```
function X = backsub(A,B)
%  A   nxn   upper Triangular matrix
%  B   nx1   matrix

n = length(B);
X = zeros(n,1);
X(n) = B(n) / A(n,n);
 for k = n-1: -1:1
 X(k) = (B(k) - A(k,k+1:n)*X(k+1:n)) /A(k,k);
 end
```

<u>örnek</u>

$$\begin{aligned} 2x_1 + 4x_2 - 6x_3 &= -4 \\ x_1 + 5x_2 + 3x_3 &= 10 \\ x_1 + 3x_2 + 2x_3 &= 5 \end{aligned} \Rightarrow \begin{aligned} 2x_1 + 4x_2 - 6x_3 &= -4 \\ 3x_2 + 6x_3 &= 12 \\ 3x_3 &= 3 \end{aligned}$$

örnek: $x_1 + x_2 = 11$
$x_1 - x_2 = 1$ sisteminin çözüm kümesi nedir?

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

çözüm: 1

$$\begin{aligned} x_1 + x_2 &= 11 \\ + \quad x_1 - x_2 &= 1 \\ \hline 2x_1 &= 12 \end{aligned}$$

Ç.K $= \{6, 5\}$

$\boxed{x_1 = 6} \Rightarrow x_1 + x_2 = 11 \Rightarrow 6 + x_2 = 11$
$x_2 = 5$

2

$$\begin{aligned} x_1 + x_2 &= 11 \\ x_1 - x_2 &= 1 \end{aligned} \Rightarrow R_2 \to R_1 + R_2 \quad \begin{aligned} x_1 + x_2 &= 11 \\ 2x_1 + 0x_2 &= 12 \end{aligned}$$

$$\begin{aligned} x_1 + x_2 &= 11 \\ 2x_1 \quad &= 12 \end{aligned} \Rightarrow \begin{aligned} x_1 &= 6 \\ x_2 &= 5 \end{aligned}$$

YADA

$$\begin{aligned} x_1 + x_2 &= 11 \\ x_1 - x_2 &= 1 \end{aligned} \quad R_2 \to R_1 - R_2 \quad \begin{aligned} x_1 + x_2 &= 11 \\ 2x_2 &= 10 \end{aligned}$$

$$R_2 \to \frac{1}{2} R_2 \quad \begin{aligned} x_1 + x_2 &= 11 \\ x_2 &= 5 \end{aligned} \Rightarrow \begin{aligned} x_1 + x_2 &= 11 \\ x_1 + 5 &= 11 \\ x_1 &= 6 \end{aligned}$$

3

$$\begin{aligned} x_1 + x_2 &= 11 \\ x_1 - x_2 &= 1 \end{aligned} \Rightarrow \underline{A}\,\underline{X} = B$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow A^{-1} = \frac{\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}}{-1-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$A \cdot A^{-1} \stackrel{?}{=} I \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{1}{2} & \frac{1}{2}+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A\underline{X} = B \Rightarrow \underline{X} = A^{-1} \cdot B \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$ ②

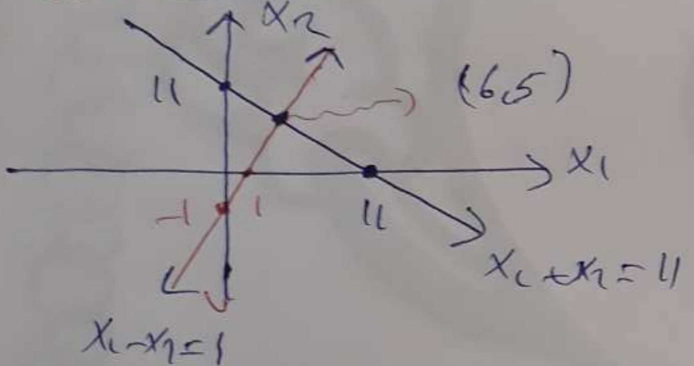
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 11 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{11}{2} + \frac{1}{2} \\ \frac{11}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{12}{2} \\ \frac{10}{2} \end{bmatrix}$$

$$\Rightarrow \underline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\underline{\underline{x_1 = 5}}$ $\underline{\underline{x_2 = 6}}$

④ Grafik metodu



⑤ Kramer Yöntemi

$$x_1 + x_2 = 11$$
$$x_1 - x_2 = 1$$

$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} 11 & 1 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-11-1}{-1-1} = \frac{-12}{-2} = \underline{\underline{6}}$$

$$x_2 = \frac{\begin{vmatrix} 1 & 11 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{1-11}{-1-1} = \frac{-10}{-2} = \underline{\underline{5}}$$

⑥ Yerine koyma metodu

$$x_1 + x_2 = 11$$
$$x_1 - x_2 = 1$$

$$\Rightarrow \quad x_1 = 1 + x_2 \; , \quad x_1 + x_2 = 11$$
$$1 + x_2 + x_2 = 11$$
$$2x_2 = 11 - 1$$
$$2x_2 = 5 \cdot 2$$
$$x_2 = 5$$

$\underline{x_1 = 6}$