Newton Yöntemi (Non-lineer sistemi)

sistem newton ①

Doğrusal olmayan

$$f_1(x,y)=0$$
$$f_2(x,y)=0$$

sistemi verilsin

sistemin çözümü için (x_0,y_0) başlangıç değerler olsun. Sistemin çözümü $\underline{a}=(a,b)$ olsun.

$$\underline{f}(x,y)=\begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix}$$

Jakobi matrisi $J(x,y)$, $\underline{x}_0=(x_0,y_0)$

$$\underline{f}(\underline{x}_0)+J(\underline{x}_0)(\underline{a}-\underline{x}_0)$$

$$\underline{a}=\underline{x}_0-J(\underline{x}_0)^{-1}\cdot\underline{f}(\underline{x}_0):=\underline{x}_1$$

$n\geq 0$ için

$$\boxed{\underline{x}_{n+1}=\underline{x}_n-J(\underline{x}_n)^{-1}\cdot\underline{f}(\underline{x}_n)}$$

Newton yineleme formülü elde edilir.

Eğer $\Delta\underline{x}_n=\underline{a}-\underline{x}_n$ alınırsa

$$-J(\underline{x}_n)\,\Delta\underline{x}_n=\underline{f}(\underline{x}_n)$$ mümkündür,

$$\underline{x}_{n+1}=\underline{x}_n+\Delta\underline{x}_n$$ mümkündür.

Örnek:

Sistem Newton (2)

$$f_1(x,y) = x^2 + y^2 - 2x - 2y + 1$$

$$f_2(x,y) = x + y - 2xy$$

non-lineer sistemini $(x_0, y_0) = (0.5, 2)$ başlangıç noktasını kullanırsak

$$f(x,y) = \begin{pmatrix} x^2 + y^2 - 2x - 2y + 1 \\ x + y - 2xy \end{pmatrix}$$

$$J(x,y) = \begin{pmatrix} 2x-2 & 2y-2 \\ 1-2y & 1-2x \end{pmatrix}$$ olduğundan

$$f(0.5, 2) = \begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix}, \quad J(0.5, 2) = \begin{pmatrix} -1 & 2 \\ -3 & 0 \end{pmatrix}$$

bulunur

$$-J(\underline{x}_n) \cdot \Delta \underline{x}_n = \underline{f}(\underline{x}_n)$$

$$-\begin{pmatrix} -1 & 2 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_0 \\ \Delta y_0 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_0 \\ \Delta y_0 \end{pmatrix} = -\begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix}$$

$$\begin{pmatrix} \Delta x_0 \\ \Delta y_0 \end{pmatrix} = \begin{pmatrix} 0.166666 \\ -0.041666 \end{pmatrix}$$

İlk iterasyonda

$$x_1 = x_0 + \Delta x_0 = \begin{pmatrix} 0.666667 \\ 1.958333 \end{pmatrix}$$ elde edilir

n	x_n	y_n
1	0.6666667	1.95833
2	0.6729391	1.945117
3	0.6730072	1.945027
4	0.6730072	1.945027

15 Ekim 2021

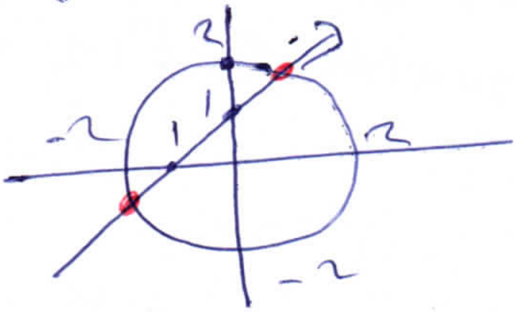
Newton sistem

3

Örnek $x - y + 1 = 0$

$x^2 + y^2 - 4 = 0$

$\underline{F}(\underline{x}) = 0$ Newton ile $(x_0, y_0) = (0.8, 1.8)$



$\underline{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\underline{F}(\underline{x}) = \begin{bmatrix} x - y + 1 \\ x^2 + y^2 - 4 \end{bmatrix}$

$y = x + 1$

$x^2 + y^2 = 4$

$J = \begin{bmatrix} 1 & -1 \\ 2x & 2y \end{bmatrix}$

$\underline{F}(\underline{x}_0) = \begin{bmatrix} 0.8 - 1.8 + 1 \\ (0.8)^2 + (1.8)^2 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.12 \end{bmatrix}$

$J = \begin{bmatrix} 1 & -1 \\ 1.6 & 3.6 \end{bmatrix}$

$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - J_n^{-1} F[\underline{x}_n]$

$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 1.8 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1.6 & 3.6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -0.12 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.0230769 \\ 0.0230769 \end{bmatrix} = \begin{bmatrix} 0.8230769 \\ 1.8230769 \end{bmatrix}$

$J(\underline{x}_1) = \begin{bmatrix} 1 & -1 \\ 1.6461538 & 3.6461538 \end{bmatrix}$

$F(\underline{x}_1) = \begin{bmatrix} 0 \\ 0.0010651 \end{bmatrix}$

$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.8230769 \\ 1.8230769 \end{bmatrix} - J(\underline{x}_1)^{-1} F(\underline{x}_1)$

$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.8228752 \\ 1.8228752 \end{bmatrix}$

$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.8228756 \\ 1.8228756 \end{bmatrix}$

Exact:

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\sqrt{7} - 1) \\ \frac{1}{2}(\sqrt{7} + 1) \end{bmatrix}$

$$\underline{X}_{n+1} = \underline{X}_n - J^{-1}(\underline{X}_n) \cdot \underline{F}(\underline{X})$$

$$F(\underline{X}) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix}$$

$$\underline{X} = [x_1, x_2, \ldots, x_n]^T$$

$$J(\underline{X}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\underline{x}) & \frac{\partial f_1}{\partial x_2}(\underline{x}) & \cdots & \frac{\partial f_1}{\partial x_n}(\underline{x}) \\ \frac{\partial f_2}{\partial x_1}(\underline{x}) & \frac{\partial f_2}{\partial x_2}(\underline{x}) & \cdots & \frac{\partial f_2}{\partial x_n}(\underline{x}) \\ \vdots & & & \frac{\partial f_n}{\partial x_n}(\underline{x}) \\ \frac{\partial f_n}{\partial x_1}(\underline{x}) & \frac{\partial f_n}{\partial x_n}(\underline{x}) & \cdots & \end{bmatrix}$$

$\underline{X}^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})$ başlangıç değer

$J(\underline{x}^{(0)}), \quad \underline{F}(\underline{x}^{(0)})$

$y^{(0)}$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$J(x^{(0)}) \cdot y^{(0)} = -\underline{F}(\underline{x}^{(0)})$$

$$x^{(k)} = \underline{x}^{(k-1)} - J^{-1}(\underline{x}^{(k-1)}) \cdot \underline{F}(\underline{x}^{(k-1)})$$

örnek

$$3x_1 - \cos(x_2 \cdot x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0, \quad \underline{x}^0 = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.1 \end{bmatrix}$$

$$x + e^{-x} + y^3 = 0$$
$$x^2 + 2xy - y^2 + \tan x = 0$$

15 Ekim 2021

sistem

örnek Newton

$$F(\underline{x}) = 0$$

$$J = \begin{bmatrix} 1 - e^{-x} & 3y^2 \\ 2x + 2y + 1 + \tan^2 x & 2x - 2y \end{bmatrix} \Rightarrow T$$

$$\underline{X}_{n+1} = \underline{X}_n - J^{-1} F(\underline{X}_n), \quad \underline{X_0} = (3, -1.5)$$

tol = 1e-14

$x = 3.13244796038883$

$y = -1.46992840120343$

$$\underline{X}_{n+1} = \underline{X}_n - J_n^{-1} \cdot F(\underline{X}_n)$$

$(x_0, y_0) = (1.98, 1.02)$

örnek

$$f_1(x, y) = 0 = x + xy - 4$$
$$f_2(x, y) = 0 = x + y - 3$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 + y & x \\ 1 & 1 \end{bmatrix}$$

$$n = 0 \Rightarrow \quad J_{(0)} = \begin{bmatrix} 1 + y_0 & x_0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 + 1.02 & 1.98 \\ 1 & 1 \end{bmatrix}$$

$$J_{(0)} = \begin{bmatrix} 2.02 & 1.98 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} 1 + y_0 & x_0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_0 + x_0 y_0 - 4 \\ x_0 + y_0 - 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1.98 \\ 1.02 \end{bmatrix} - \begin{bmatrix} 2.02 & 1.98 \\ 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -4 \cdot 10^{-4} \\ 0 \end{bmatrix} = \begin{bmatrix} 1.9900 \\ 1.0100 \end{bmatrix}$$

$\{x+\bar{e}^{x}+y^3=0,\ x^2+2xy-y^2+\tan x=0\}$

15 Ekim 2021
örnek
newton 2

$\Downarrow$

$(x_0, y_0) = (1.98, 1.02)$
initial values

n	x_n	y_n
0	1.9800	1.0200
1	1.9900	1.0100
2	1.9950	1.0050
3	1.9975	1.0025
4	1.9987	1.0013
5	1.9994	1.0006
6	1.9997	1.0003
7	1.9998	1.0002
8	1.9999	1.0001
9	2.0000	1.000

$\{x+\bar{e}^{x}+y^3=0,\ x^2+2xy-y^2+\tan x=0\}$

Aynı örnek
$(x_0, y_0) = (2.1, 0.9)$ için

n	x_n	y_n
0	2.1000	0.9000
1	2.0500	0.9500
2	2.0250	0.9745
3	2.0125	0.9875
4	2.0062	0.9938
5	2.0031	0.9969
6	2.0016	0.9984
7	2.0008	0.9992
8	2.0004	0.9996
9	2.0002	0.9998
10	2.0001	0.9999
11	2.0000	1.0000

örnek: Aşağıdaki non-lineer sistemi Newton ile $X_0 = (0.1, 0.1, -0.1)$ başlangıç ile çözünüz

$$3x_1 - \cos(x_2 \cdot x_3) - 0.5 = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0$$

$$e^{-x_1 \cdot x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

çözüm: $$F(\underline{x}) = \begin{bmatrix} 3x_1 - \cos(x_2 \cdot x_3) - 0.5 \\ x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 \\ e^{-x_1 \cdot x_2} + 20x_3 + \frac{10\pi - 3}{3} \end{bmatrix}$$

$$J(\underline{x}) = \begin{bmatrix} 3 & x_3 \sin(x_2 x_3) & x_2 \sin(x_2 x_3) \\ 2x_1 & -162(x_2 + 0.1) & \cos x_3 \\ -x_2 e^{-x_1 \cdot x_2} & -x_1 \cdot e^{-x_1 \cdot x_2} & 20 \end{bmatrix}$$

$$F(\underline{x}_0) = \begin{bmatrix} -1.19995 \\ -2.269833417 \\ 8.462025346 \end{bmatrix}$$

$$J(\underline{x}_0) = \begin{bmatrix} 3 & 0.000999983 & -0.000999983 \\ 0.2 & -32.4 & 0.99500465 \\ -0.099004984 & -0.099004983 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{bmatrix} - J^{-1}(\underline{x}_0) \cdot F(\underline{x}_0)$$

$$= \begin{bmatrix} 0.1 \\ 0.1 \\ -0.1 \end{bmatrix} - J^{-1} \cdot F$$

$$= \begin{bmatrix} 0.1 \\ 0.1 \\ -0.1 \end{bmatrix} + \begin{bmatrix} 0.40003702 \\ -0.08053314 \\ -0.42152047 \end{bmatrix} = \begin{bmatrix} 0.50003702 \\ 0.01946686 \\ -0.52152047 \end{bmatrix}$$