

Özyineleme İlişkileri

Şevket Umut Çakır

Pamukkale Üniversitesi

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Master Teoremi

- $T(n)$, aşağıdaki ilişkiyi sağlayan, azalmayan bir fonksiyon olsun
 - ▶ $T(n) = aT(\frac{n}{b}) + f(n)$, $n = b^k$, $k = 1, 2, \dots$
 - ▶ $T(1) = c$
- $a \geq 1$, $b \geq 2$, $c > 0$, Eğer $f(n) \in \Theta(n^d)$ ise, $d \geq 0$

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$$T(n) \in \begin{cases} \Theta(n^d) & \text{eğer } a < b^d \\ \Theta(n^d \log n) & \text{eğer } a = b^d \\ \Theta(n^{\log_b a}) & \text{eğer } a > b^d \end{cases}$$

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Örnek



$$T(n) = T\left(\frac{n}{2}\right) + c$$

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Örnek



$$T(n) = T\left(\frac{n}{2}\right) + c$$

- $a = 1, b = 2, f(n) = c$
- $f(n) \in \Theta(n^0) = \Theta(1), d = 0$
- $a = b^d$, durum 2
- $T(n) \in \Theta(n^0 \log n) = \Theta(\log n)$

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Örnek



$$T(n) = 2T\left(\frac{n}{2}\right) + cn$$

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Örnek



$$T(n) = 2T\left(\frac{n}{2}\right) + cn$$

- $a = 2, b = 2, f(n) = cn$
- $f(n) \in \Theta(n^1) = \Theta(n), d = 1$
- $a = b^d$, durum 2
- $T(n) \in \Theta(n^1 \log n) = \Theta(n \log n)$

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Örnek



$$T(n) = 7T\left(\frac{n}{2}\right) + 18n^2$$

Master Teoremi

Örnek



$$T(n) = 7T\left(\frac{n}{2}\right) + 18n^2$$

- $a = 7, b = 2, f(n) = 18n^2$
- $f(n) \in \Theta(n^2) = \Theta(n^2), d = 2$
- $a > b^d$ durum 3
- $T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_2 7})$
- $\log_2 7 \approx 2.81$

Master Teoremi

Örnek



$$T(n) = 9T\left(\frac{n}{3}\right) + n$$

Master Teoremi

Örnek



$$T(n) = 9T\left(\frac{n}{3}\right) + n$$

- $a = 9, b = 3, f(n) = 18n^2$
- $f(n) \in \Theta(n), d = 1$
- $a > b^d$ durum 3
- $T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_3 9}) = \Theta(n^2)$

Master Teoremi I

Örnek Tam Çözüm

$$T(n) = 9T\left(\frac{n}{3}\right) + n \quad (1)$$

$$T(n) = 9\left(9T\left(\frac{n}{3^2}\right) + \frac{n}{3}\right) + n \quad (2)$$

$$T(n) = 9^2T\left(\frac{n}{3^2}\right) + 3n + n \quad (3)$$

$$T(n) = 9^2\left(9T\left(\frac{n}{3^3}\right) + \frac{n}{3^2}\right) + 3n + n \quad (4)$$

$$T(n) = 9^3T\left(\frac{n}{3^3}\right) + 3^2n + 3n + n \quad (5)$$

$$T(n) = 9^mT\left(\frac{n}{3^m}\right) + n(3^{m-1} + \dots + 1) \quad (6)$$

- $T(1)$ 'i elde etmek için $\frac{n}{3^m} = 1$, $m = \log_3 n$

Master Teoremi II

Örnek Tam Çözüm

$$T(n) = 9^{\log_3 n} T(1) + n \frac{3^m - 1}{3 - 1} \quad (7)$$

$$T(n) = n^2 T(1) + n \frac{n - 1}{2} \quad (8)$$

$$T(n) \in \Theta(n^2) \quad (9)$$