Özyinelemeli algoritma analizi Seans 1

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n diski 1'den 3'e taşı

- $n-1$ diski $1 \to 2$
- 1 diski $1 \to 3$
- $n-1$ diski $2 \to 3$

$$T(n) = 2T(n-1) + 1 \;,\; T(1) = 1$$

$$= 2(2T(n-2)+1)+1 = 4T(n-2) + 2 + 1$$

$$= 4(2T(n-3)+1)+2+1 = 8T(n-3) + 4 + 2 + 1$$

$$= 2^i \cdot T(n-i) + \underbrace{2^0 + 2^1 + 2^2 + \ldots + 2^{(i-1)}}_{2^i - 1}$$

$$= 2^i \cdot T(n-i) + 2^i - 1 \;,\quad n - i = 1$$

$$i = n-1$$

$$= 2^{n-1} \cdot T(1) + 2^{n-1} - 1$$

$$= 2^{n-1} + 2^{n-1} - 1 = \boxed{2^n - 1} \in \Theta(2^n)$$

Counting bits

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$$

$$T(n) = T\left(\frac{n}{2}\right) + 1 \qquad T(1) = 0$$

$$T(n) = T\left(\frac{n}{4}\right) + 1 + 1 = T\left(\frac{n}{4}\right) + 2$$

$$T(n) = T\left(\frac{n}{8}\right) + 1 + 1 + 1 = T\left(\frac{n}{8}\right) + 3$$

$$T(n) = T\left(\frac{n}{2^i}\right) + i \;,\quad \frac{n}{2^i} = 1$$

$$2^i = n$$

$$\log_2 2^i = \log_2 n$$

$$i \cdot \log_2 2 = \log_2 n$$

$$i = \log_2 n$$

$$T(n) = T(1) + \log_2 n$$

$$T(n) = \log_2 n \in \Theta(\log n)$$

$$T(n) = 2T(\sqrt{n}) + 1 \quad , \quad T(2) = 0$$

$$T(n) = 2 \cdot T\left(n^{\frac{1}{2}}\right) + 1$$

$$= 2 \cdot \left(2T\left(n^{\frac{1}{4}}\right) + 1\right) + 1$$

$$= 4\, T\left(n^{\frac{1}{4}}\right) + 2 + 1$$

$$= 4\left(2T\left(n^{\frac{1}{8}}\right) + 1\right) + 2 + 1$$

$$= 8T\left(n^{\frac{1}{8}}\right) + 4 + 2 + 1$$

$$= 2^i \cdot T\left(n^{\frac{1}{2^i}}\right) + \sum_{k=0}^{i-1} 2^i \rightarrow 2^i - 1$$

$$= \underbrace{\log_2 n \cdot T(2)}_{0} + 2^i - 1$$

$$= \log_2 n - 1 \quad \in \Theta(\log n)$$

$$T\left(n^{\frac{1}{2}}\right) = 2T\left(\left(n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right) + 1$$

$$= 2T\left(n^{\frac{1}{4}}\right) + 1$$

$$n^{\frac{1}{2^i}} = 2$$

$$\log_n n^{\frac{1}{2^i}} = \log_n 2$$

$$\frac{1}{2^i} = \log_n 2$$

$$2^i = \frac{1}{\log_n 2} = \log_2 n$$

$$\log_2 2^i = \log_2 \log_2 n$$

$$i = \log_2 \log_2 n$$