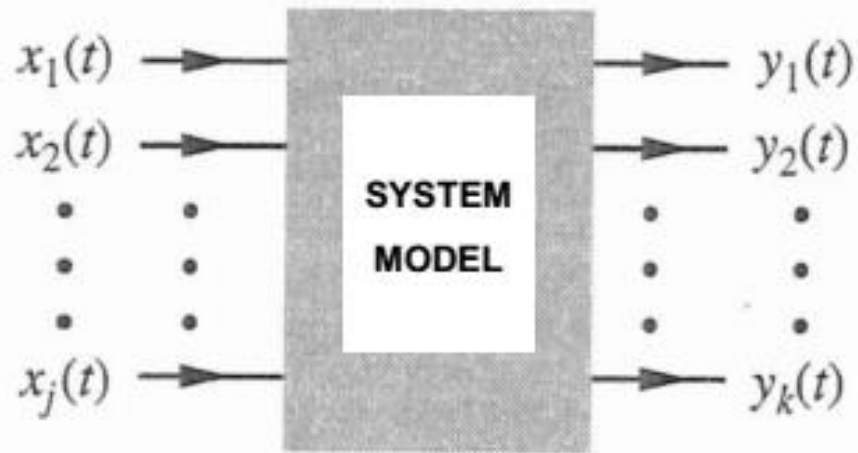


İşaret İşleme

Zaman Domaini Analizi-H3CD1

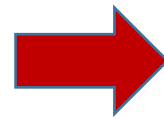
Dr. Meriç Çetin
versiyon041022

Sürekli zamanlı sistemlerde zaman domaini analizi



In general, relationship between $x(t)$ and $y(t)$ in a **linear time-invariant (LTI)** differential system is given by (where all coefficients a_i and b_i are constants):

$$\begin{aligned} \frac{d^N y}{dt^N} + a_1 \frac{d^{N-1} y}{dt^{N-1}} + \cdots + a_{N-1} \frac{dy}{dt} + a_N y(t) \\ = b_{N-M} \frac{d^M x}{dt^M} + b_{N-M+1} \frac{d^{M-1} x}{dt^{M-1}} + \cdots + b_{N-1} \frac{dx}{dt} + b_N x(t) \end{aligned}$$



Use compact notation **D** for **operator d/dt** , i.e. $\frac{dy}{dt} \equiv Dy(t)$ and $\frac{d^2 y}{dt^2} \equiv D^2 y(t)$ etc.

We get:
$$\begin{aligned} (D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y(t) \\ = (b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N) x(t) \end{aligned}$$

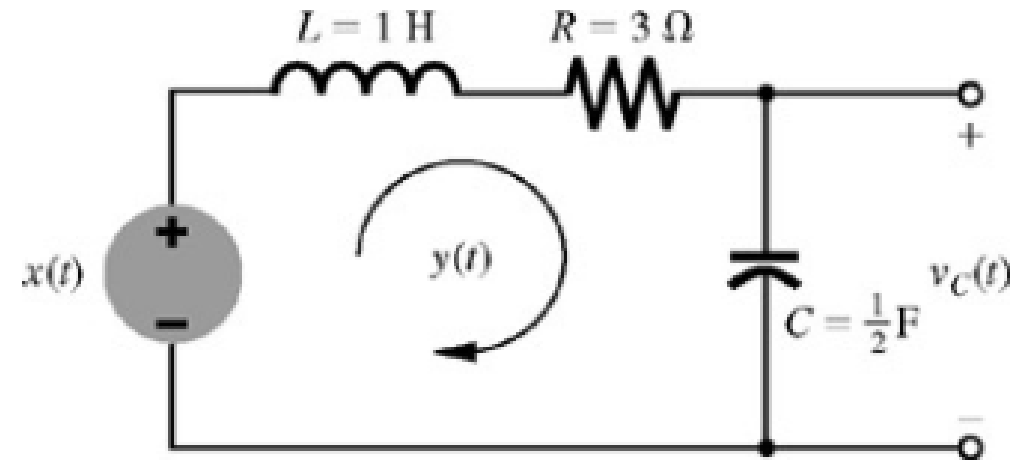
or

$$Q(D)y(t) = P(D)x(t)$$

$$Q(D) = D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N$$

$$P(D) = b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N$$

Önceki ders yaptığımız son örneği hatırlayalım



Application of Kirchhoff's voltage law around the loop yields

$$v_L(t) + v_R(t) + v_C(t) = x(t) \quad (1.47)$$

By using the voltage-current laws of each element (inductor, resistor, and capacitor), we can express this equation as

$$\frac{dy}{dt} + 3y(t) + 2 \int_{-\infty}^t y(\tau) d\tau = x(t) \quad (1.48)$$

Differentiating both sides of this equation, we obtain

$$\frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y(t) = \frac{dx}{dt} \quad (1.49)$$

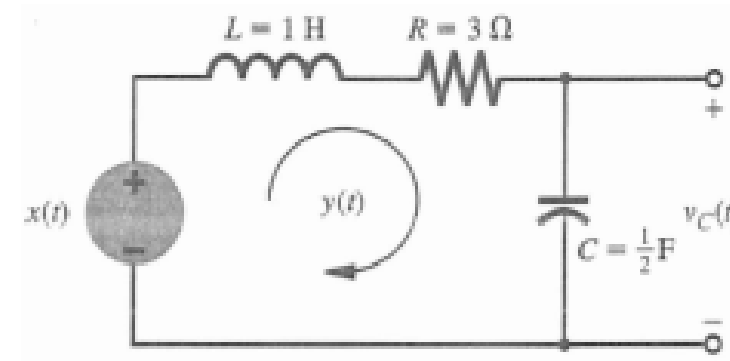
This differential equation is the input-output relationship between the output $y(t)$ and the input $x(t)$.

Önceki ders yaptığımız son örneği hatırlayalım

Let us consider this example again:

The system equation is:

$$\frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y(t) = \frac{dx}{dt}$$



This can be re-written as:

$$\underbrace{(D^2 + 3D + 2)}_{Q(D)} y(t) = \underbrace{D}_{P(D)} x(t)$$

For this system, $N = 2$, $M = 1$, $a_1 = 3$, $a_2 = 2$, $b_1 = 1$, $b_2 = 0$.

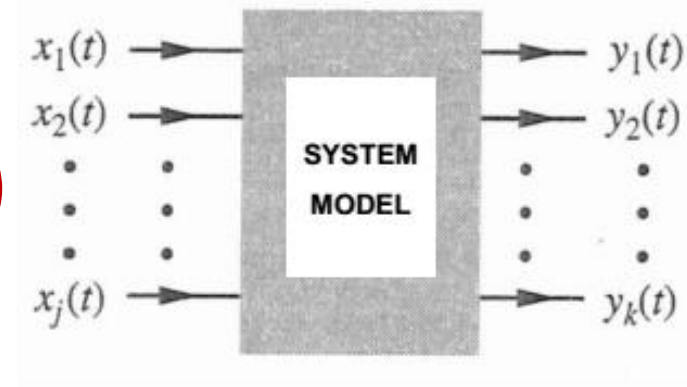
Also

$$\int_{-\infty}^t y(\tau) d\tau \equiv \frac{1}{D} y(t)$$
$$\frac{d}{dt} \left[\int_{-\infty}^t y(\tau) d\tau \right] = y(t)$$

For practical systems, $M \leq N$. It can be shown that if $M > N$, a LTI differential system acts as an $(M - N)$ th-order **differentiator**.

A differentiator is an unstable system because **bounded input** (e.g. a step input) results in an **unbounded output** (a Dirac impulse $\delta(t)$).

Sıfır Giriş Cevabı (SGC)-(Zero input response)



Remember that for a Linear System

Total response = zero-input response + zero-state response

In this lecture, we will focus on a linear system's **zero-input response**, $y_0(t)$, which is the solution of the system equation when input $x(t) = 0$.

$$(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y(t) = (b_{N-M} D^M + b_{N-M+1} D^{M-1} + \cdots + b_{N-1} D + b_N) x(t)$$

$$\Rightarrow Q(D)y(t) = P(D)x(t)$$

$$\Rightarrow Q(D)y_0(t) = 0$$

$$\Rightarrow (D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N) y_0(t) = 0$$

SGC için genel çözüm-1

From maths course on differential equations, we may solve the equation:

$$(D^N + a_1 D^{N-1} + \dots + a_{N-1} D + a_N) y_0(t) = 0 \quad \dots\dots\dots (3.1)$$

by letting $y_0(t) = ce^{\lambda t}$, where **c and λ are constants**

Then:

$$Dy_0(t) = \frac{dy_0}{dt} = c\lambda e^{\lambda t}$$

$$D^2 y_0(t) = \frac{d^2 y_0}{dt^2} = c\lambda^2 e^{\lambda t}$$

$$\vdots$$

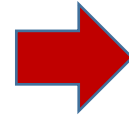
$$D^N y_0(t) = \frac{d^N y_0}{dt^N} = c\lambda^N e^{\lambda t}$$

} Substitute into (3.1)

SGC için genel çözüm-2

We get:

$$c(\lambda^N + a_1\lambda^{N-1} + \dots + a_{N-1}\lambda + a_N)e^{\lambda t} = 0$$



$$\lambda^N + a_1\lambda^{N-1} + \dots + a_{N-1}\lambda + a_N = 0$$

This is identical to the polynomial $Q(D)$ **with λ replacing D** , i.e.

$$Q(\lambda) = 0$$

We can now express $Q(\lambda)$ in **factorized form**:

$$Q(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_N) = 0 \quad \text{..... (3.2)}$$

Therefore **λ has N solutions**: $\lambda_1, \lambda_2, \dots, \lambda_N$, assuming that all λ_i are distinct.

Characteristic Polynomial of a system

$Q(\lambda)$ is called the **characteristic polynomial** of the system

$Q(\lambda) = 0$ is the **characteristic equation** of the system

The **roots** to the characteristic equation $Q(\lambda) = 0$, i.e. $\lambda_1, \lambda_2, \dots, \lambda_N$, are extremely important.

They are called by different names:

- Characteristic values
- **Eigenvalues**
- **Natural frequencies**

The exponentials $e^{\lambda_i t}$ ($i = 1, 2, \dots, n$) are the **characteristic modes** (also known as **natural modes**) of the system

Characteristics modes determine the system's behaviour

NOT

- **Sistemin karakteristik polinomuna ait kökler (λ) 3 farklı durumda olabilir.**
 - Katsız (tekrar etmeyen) kök
 - Katlı (tekrar eden) kök
 - Kompleks kök
- **Her 3 durum için sistemin sıfır giriş cevabı farklı genel çözümlerle bulunur.**

SGC için genel çözüm-3

Therefore, equation (3.1): $(D^N + a_1 D^{N-1} + \dots + a_{N-1} D + a_N) y_0(t) = 0$

has **N possible solutions**: $c_1 e^{\lambda_1 t}, c_2 e^{\lambda_2 t}, \dots, c_N e^{\lambda_N t}$

where c_1, c_2, \dots, c_N are arbitrary constants.

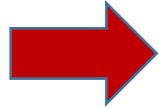
It can be shown that the **general solution** is the sum of all these terms:

$$\Rightarrow y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \dots + c_N e^{\lambda_N t} \quad \Leftarrow \text{Katsız kök olduğunda}$$

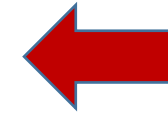
In order to determine the N arbitrary constants, we need to have **N constraints** (i.e. initial or boundary or auxiliary conditions).

Özetle;

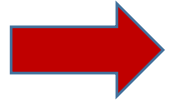
- Sistemin karakteristik polinomuna ait kökler (λ_i) **Katsız (tekrar etmeyen) kökler** olduğunda sistemin sıfır giriş cevabına ait genel çözüm;



$$y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \dots + c_N e^{\lambda_N t}$$



Çalışma Sorusu

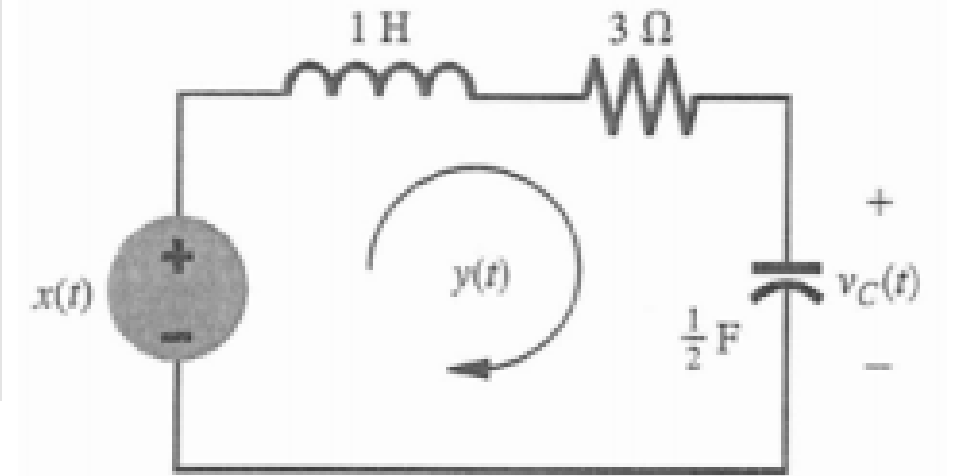


Find $y_0(t)$, the zero-input component of the response, for a LTI system described by the following differential equation:

$$(D^2 + 3D + 2)y(t) = Dx(t)$$

when the initial conditions are

$$y_0(0) = 0, \quad \dot{y}_0(0) = -5.$$



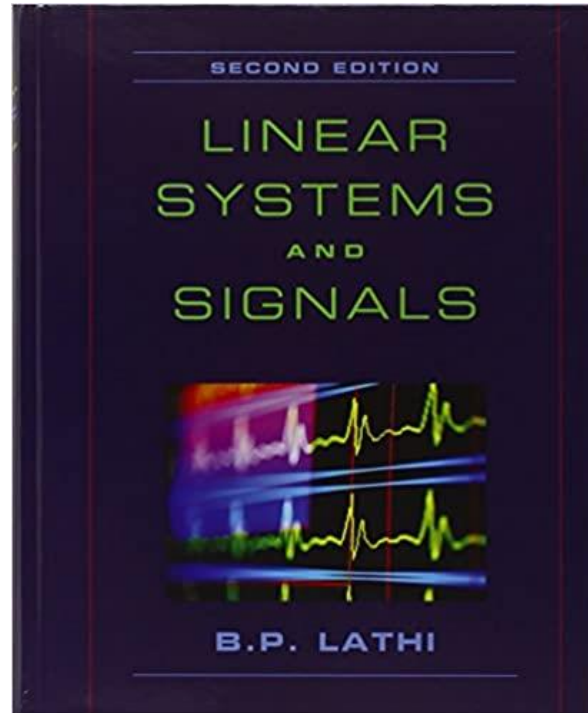
- Yukarıdaki sistemin sıfır giriş cevabını bulunuz.

Bu ders notu için faydalanılan kaynaklar

Lecture 3

Time-domain analysis: Zero-input Response (Lathi 2.1-2.2)

Peter Cheung
Department of Electrical & Electronic Engineering
Imperial College London



EEEN343 Sinyaller ve Sistemler

Ders Notları

Prof. Dr. Serdar İplikçi
Pamukkale Üniversitesi
Mühendislik Fakültesi
Elektrik-Elektronik Mühendisliği