2. For the following functions prove whether $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$ or $f(n) \in \theta(g(n))$ by using limit approach.

   (a) $f(n) = 99n$ and $g(n) = n$

   (b) $f(n) = \sum_{x=1}^{n} x$ and $g(n) = 4n + logn$

Solution:

$$\boxed{\begin{array}{l} f(n) \in O(g(n)) \longrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \\ f(n) \in \Omega(g(n)) \longrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \\ f(n) \in \Theta(g(n)) \longrightarrow 0 < \lim_{n\to\infty} \frac{f(n)}{g(n)} = c < \infty \end{array}}$$

(a) $f(n) = 99n$ and $g(n) = n$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{99n}{n} = \lim_{n\to\infty} 99 = 99$

$0 < c = 99 < \infty$

Therefore $f(n) \in \Theta(g(n))$.

(b) $f(n) = \sum_{x=1}^{n} x$ and $g(n) = 4n + logn$

$\rightarrow \quad f(n) = \sum_{x=1}^{n} = \frac{n*(n+1)}{2}$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n*(n+1)}{2}}{4*n+logn} = \frac{n*(n+1)}{8*n+2logn} = \frac{\infty}{\infty}$

$\xrightarrow{L'hospital} \quad \lim_{n\to\infty} \frac{2*n+1}{8+\frac{2}{n*ln2}} = \frac{(2*n+1)*(n*ln2)}{8*n*ln2+2} = \frac{2*n^2*ln2+n*ln2}{8*n*ln2+2} = \frac{\infty}{\infty}$

$\xrightarrow{L'hospital} \quad \lim_{n\to\infty} \frac{4*n+ln2}{8*ln2} = \infty$

Therefore $f(n) \in \Omega(g(n))$.