Laplace Transform

Diagram

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Definition: The tools that makes it possible to represent arbitrary input f(t) in terms of exponential component.

Forward Laplace Transform

f(t) ----------L---------> F(s) where s = + jw (complex)

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Inverse Laplace Transform

f(t) <-------L-1-------- F(s)

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Bilateral Laplace Transform Pair

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Example: Find Laplace Transform of:

a) (t) 🡪 delta, impulse function

b) u(t) 🡪 unit step function

c) cos(w0 . t) . u(t) 🡪 cosine function

Solution:

(a)

Text, letter

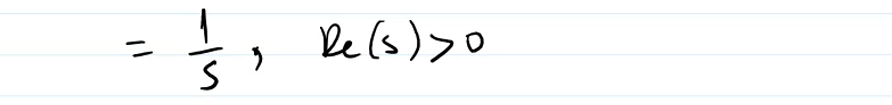
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Delta function has only non-zero value at t=0, rest is 0

(b)

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(c)

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Shortcut: Table 4.1 in your textbook, Laplace Transform pairs. You have this in exam, don’t memorize.

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Inverse Laplace Transform

Example: Partial Fraction Expansion (PFE) method

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*Side notes:*

A picture containing diagram

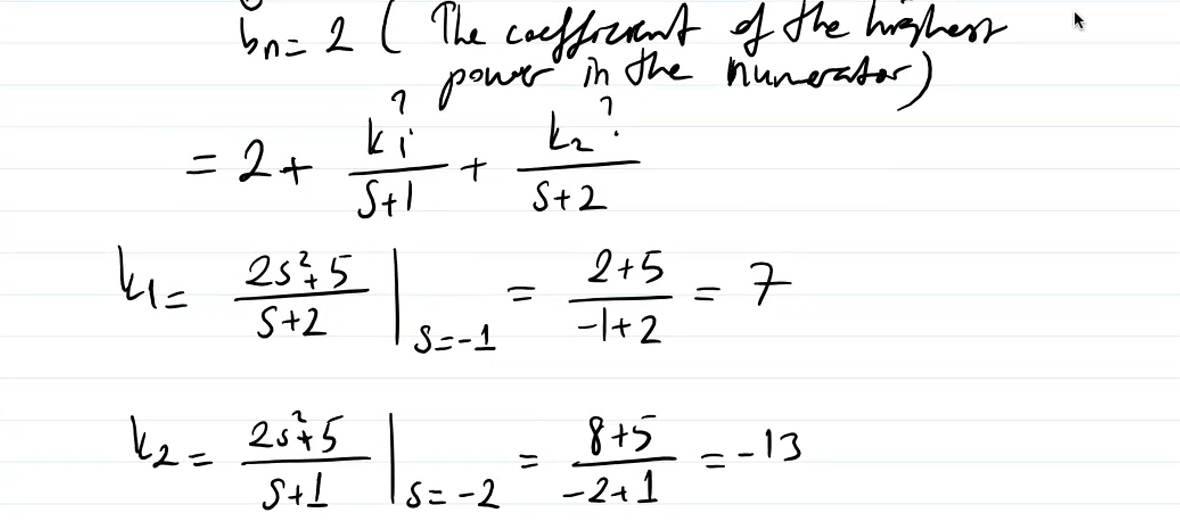
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Example:

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Inverse of 1 is (t).

Some Properties of Laplace Transform

*Time-Shifting:*

if f(t) ⬄ F(s), then for t0 >= 0,

f(t – t0) ⬄ F(s) . e-s . t0

Example:

Text, letter

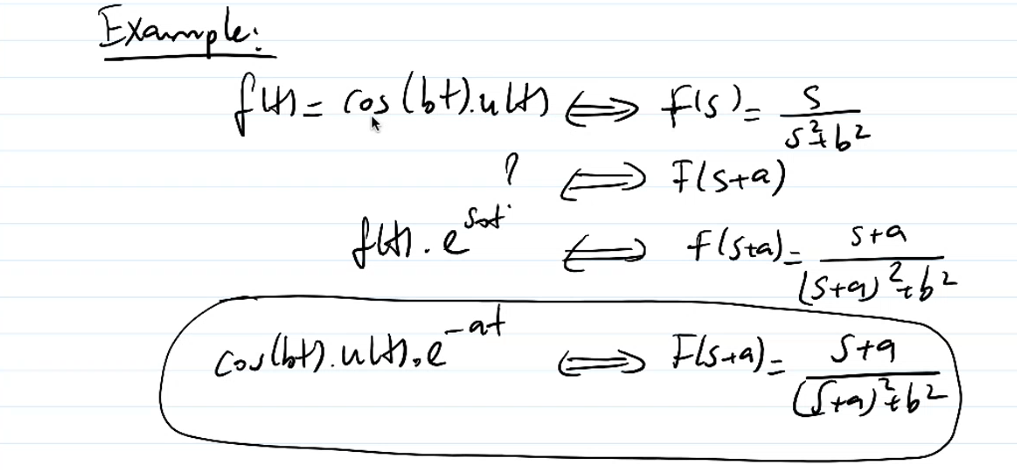
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*Frequency-Shifting:*

If f(t) ⬄ F(s), then

f(t) . es0 . t ⬄ F(s – s0)

Example:



*Time-Differentiation:*

If f(t) ⬄ F(s), then

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*Time-Integration:*

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*Scaling:*

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*Time-Convolution and Frequency-Convolution:*

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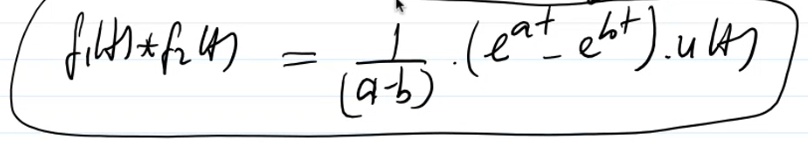
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Example:

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Zero-State Response: The transfer function of LTIC system

Consider nth order LTIC system with:

Q(D) . y(t) = P(D) . f(t)

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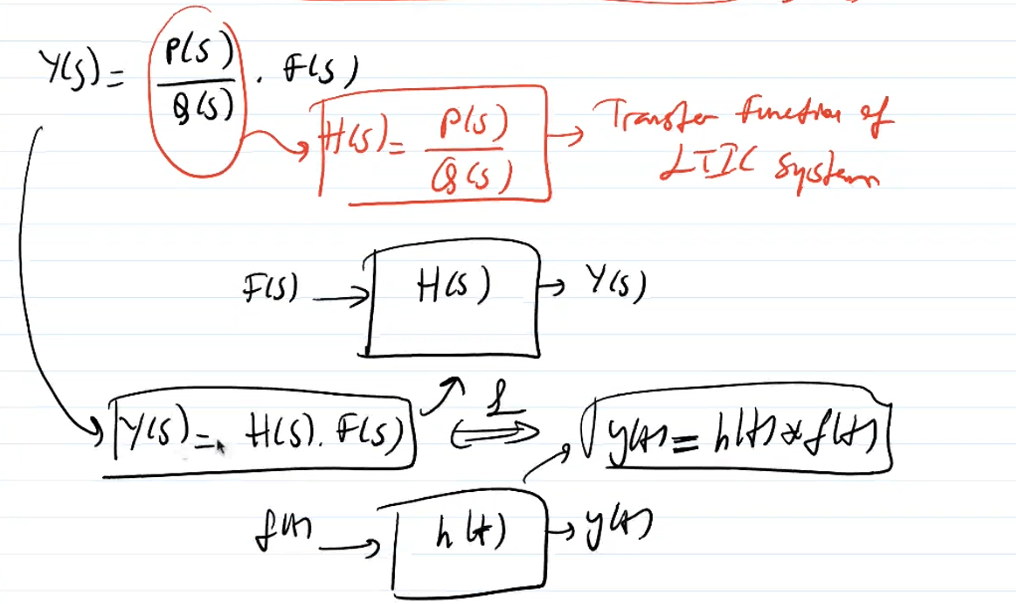
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Example:

Find the response y(t) of the following LTIC system,

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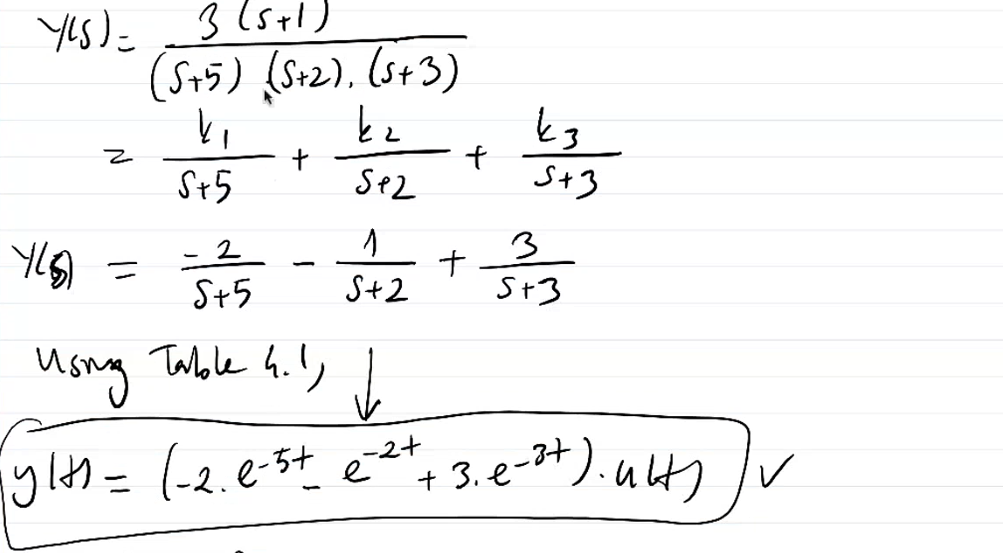
Solution:

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Block Diagrams

Large systems may consist of a very large number of components (subsystems)

Analyzing such large systems all at once could be very complex

In such case, it is convenient to represent a system by suitably interconnected subsystems, each of which can be analyzed easily

Subsytems may be interconnected using 3 interconnection types:

* cascade
* parallel
* feedback

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*Cascade:*

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*Parallel:*

Graphical user interface, text, application

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*Feedback:*

A picture containing graphical user interface

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E(s) --> Error signal