IE 400

Term Project

Mert Özerdem

21300835

IP model:

Parameters:

$\mathbf{dc_{ij}}$: destination(travel) cost of point i to point j
$\mathbf{dc_{ij} = dc_{ji}}$, for all i,j = {1,2,..., N}
$\mathbf{sc_i}$: study cost of student i to finish homework

Decision Variables:

$\mathbf{x_{ij}}$ = { x_{ij} = 1 , if road i to j is used, x_{ij}=0 , otherwise}
$\mathbf{u_i}$ = auxiliary variables to eliminate subtours, for i= 1,2,...,N

Model:

$$\textbf{minimum} \sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{dc_{ij}x_{ij}} + \sum_{i=1}^{N} \mathbf{sc_{ij}}$$

subject to:

$$\sum_{i=1,i\neq j}^{N} \mathbf{x_{ij}}, \ \mathbf{j = 1,2,..,N}$$,This is done to ensure that we arrive to point once

$$\sum_{j=1,i\neq j}^{N} \mathbf{x_{ij}}, \ \mathbf{i = 1,2,..,N}$$,This is done to ensure that we leave the point once

$\mathbf{u_i - u_j + (N-1)x_{ij} \leq N - 2, \ i \neq j, \ i,j=2,...,N,}$ To eliminate subtours

$\mathbf{x_{ij} \in \{0,1\}, \ i,j = 1,..,N}$

$\mathbf{0 \leq u_i \leq n - 1, \ for \ 2 \leq i \leq N}$

$\mathbf{u_i \in Z}$ **, for i = 2,...,N**

DP Model:

Parameters:

$\mathbf{dc_{ij}}$: destination(travel) cost of point i to point j
$\mathbf{dc_{ij} = dc_{ji}}$, for all i,j = {1,2,..., N}
$\mathbf{sc_i}$: study cost of student i to finish homework

A = {Professor, Student1, …, StudentN}, set of nodes in the problem

Decision Variables:

$\mathbf{x_i}$ = decision at stage i, for i = 1,...,N

Model:

Functional Equation:

$\mathbf{f_k(i,A)}$=minimum destination cost to travel to destination point while in stage k, with initial point i, and from set of A points. Plus all study costs of visited points.

Optimal Value:

$$\mathbf{f_N(1)}$$

Recursion using principle of optimality:

$$\mathbf{f_k(i,A) = minimum\{dc_{ij} + f_{k+1}(x_i , A - \{x_i\}) + sc_i\}}$$

Random Travel Times

Due to the nature of the project, NxN matrices grow very fast and it is possible to fit or fit properly to the document. So I could not include travel time matrices but I provided them with my submission file. You can locate each travel time matrices and homework completion time matrix form the path {PathOfTheFolder}/Özerdem_IE400_project/Data. Inside this folder every matrix is available. Additionally, I specified the addresses of the files as I used them for any of my solutions.

Running Time Comparison Graph

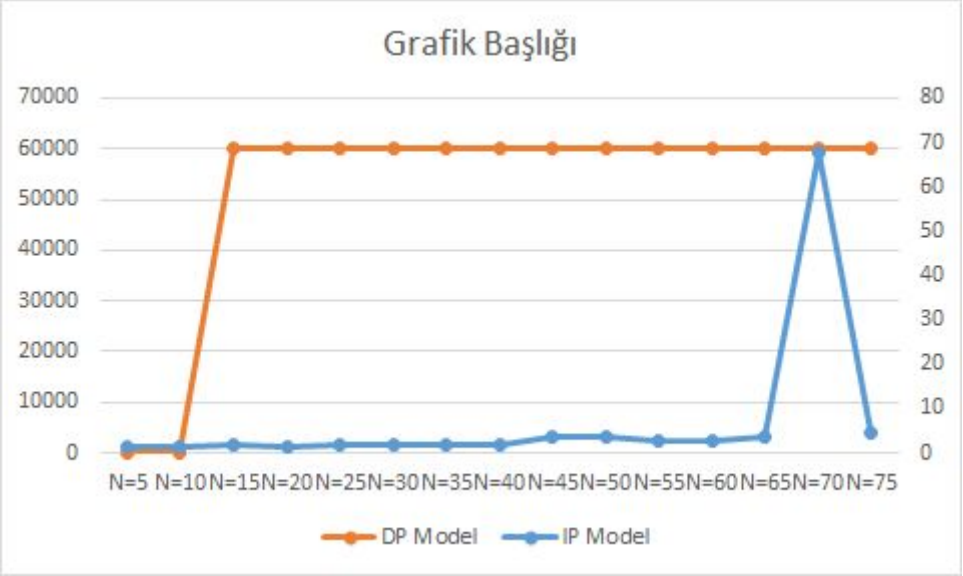

figure 1: Computation Time Comperison Graph

DP model's computation time is $O(2^n n^2)$ so I estimated the values of DP computation times because after N=15 computation time exceeds 1 hour. So to be able to skech the graph I fixed it to value of 60000 after N=10.

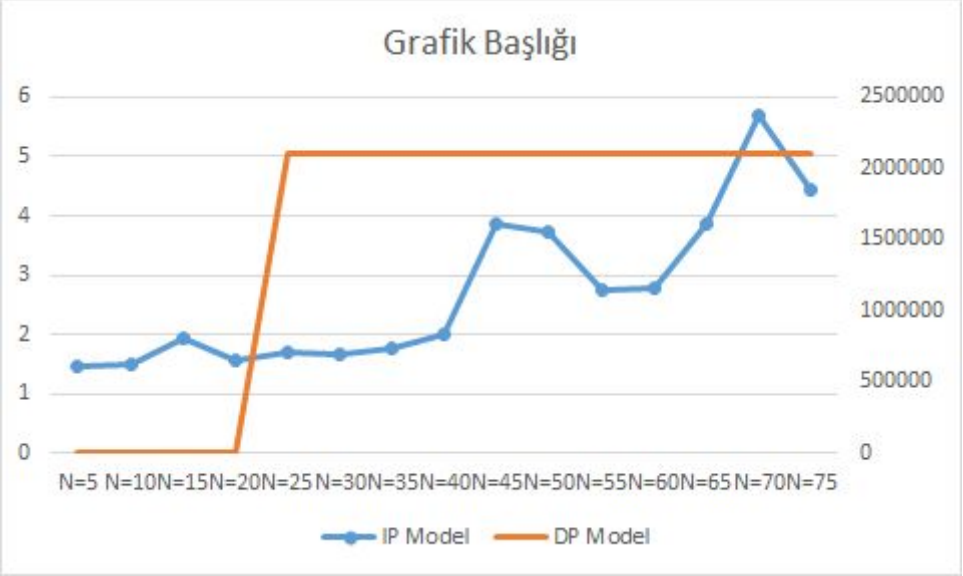

figure 2: Computation Time Comperison Graph with smoothing

In figure 2 I smoothened the value that happens in the IP model at N=70 where computation time is 67 seconds, which is a strange thing. And I equate it to the average value of 5.7. Also for DP programming, I estimated time of computation by using computationTime=$2^n n^2$ formula from starting N=10, because after 10

students computation time exceeds 1 hour. But after N=20 computation time growth exceeded normal amounts so I fixed it as I did in the previous graph.
[Go Data/Time_Comperison for excel data]

Algorithm Comparison

From the above graph, we can see that the DP model's computation time grows exponentially while the IP model's computation time grows exponentially but with lesser coefficients. This happens because the computation time of the Travelling Salesman Problem(TSP) with DP solution is O(2nn2). Because of this, DP's computation time grows exponentially very fast and after N=10 it takes above 1 hour to compute. For this reason, solving TSP problems via DP is not a viable option. Instead, IP solution should be used with computation time is much much cheaper.

Solutions to Different Sets of Problems

I solved different sets of problems by using the codes generated based on models that I specified earlier. For the IP model, I used IBM Cplex, and for DP model I used Python 3.7.

Note that optimal paths beginning and ending do not matter since it is a circle but for displaying purposes I chose to use end and start notations.

Computation times are in seconds and Nan means computation time is more than one hour.

Student homework complation times are taken from "StudentWorkTimes" excel file. [Go Data/StudentWorkTimes for excel data]

N = 5 Students

IP Model Computation Time : 01:47
DP Model Computation Time: 0.000997304916381836
Optimal path: end-1-2-4-6-3-5-1-start
Optimal Solution: 3140.25 units of time
[Go Data/StudentN5 for excel data]

N = 10 Students

IP Model Computation Time : 01:48
DP Model Computation Time: 10.266908168792725
Optimal path: end-1-8-2-5-10-7-9-4-3-11-6-1-start
Optimal Solution: 5904.086956522 units of time

[Go Data/StudentN10 for excel data]

N = 15 Students

IP Model Computation Time : 01:92
DP Model Computation Time: Nan
Optimal path: end-1-2-7-4-3-6-14-8-12-13-16-5-11-9-10-15-1-start
Optimal Solution: 9080.125 units of time
[Go Data/StudentN15 for excel data]

N = 20 Students

IP Model Computation Time : 01:57
DP Model Computation Time: Nan
Optimal path: end-1-12-15-5-9-11-17-7-20-3-2-13-21-14-16-6-18-19-4-10-8-1-start
Optimal Solution: 12098.033898305 units of time
[Go Data/StudentN20 for excel data]

N = 25 Students

IP Model Computation Time : 01:68
DP Model Computation Time: Nan
Optimal path:
end-1-25-13-18-8-6-10-12-26-21-15-2-11-22-23-17-14-24-5-3-19-4-20-9-7-16-1-start
Optimal Solution: 14990.45 units of time
[Go Data/StudentN25 for excel data]

N = 30 Students

IP Model Computation Time : 01:65
DP Model Computation Time: Nan
Optimal path:
end-1-30-2-17-26-23-29-21-15-19-22-4-3-25-14-10-31-7-28-11-16-9-13-6-27-5-20-18-12-24-8-1-start
Optimal Solution: 18159.343137255 units of time
[Go Data/StudentN30 for excel data]

N = 35 Students

IP Model Computation Time : 01:78
DP Model Computation Time: Nan

Optimal path:
end-1-21-6-17-24-10-25-26-5-20-27-33-7-19-28-32-13-31-30-36-8-2-34-22-16-14-29
-9-3-18-12-15-4-23-35-11-1-start
Optimal Solution: 20951.459677419 units of time
[Go Data/StudentN35 for excel data]

N = 40 Students

IP Model Computation Time : 02:00
DP Model Computation Time: Nan
Optimal path:
end-1-29-9-24-13-33-39-32-12-23-40-20-3-7-25-31-2-28-15-35-30-6-34-10-8-19-17-
14-5-37-26-16-18-38-11-27-21-36-22-41-4-1-start
Optimal Solution: 23999.013605442 units of time
[Go Data/StudentN40 for excel data]

N = 45 Students

IP Model Computation Time : 03:85
DP Model Computation Time: Nan
Optimal path:
end-1-29-30-7-5-43-20-18-31-40-45-8-21-3-11-17-35-16-12-9-4-33-15-37-6-19-27-4
4-42-23-38-39-24-2-26-14-34-36-32-22-25-28-13-10-41-46-1-start
Optimal Solution: 27047.076023392 units of time
[Go Data/StudentN45 for excel data]

N = 50 Students

IP Model Computation Time : 03:73
DP Model Computation Time: Nan
Optimal path:
end-1-22-51-34-2-31-18-44-26-30-37-39-47-48-23-32-17-13-10-25-42-50-6-21-20-35
-27-7-19-45-16-4-41-9-29-14-33-24-38-43-46-3-49-28-5-40-11-36-8-15-12-1-start
Optimal Solution: 30099.215384615 units of time
[Go Data/StudentN50 for excel data]

N = 55 Students

IP Model Computation Time : 02:76
DP Model Computation Time: Nan
Optimal path:
end-1-5-54-9-47-8-7-25-45-12-28-49-6-18-21-40-31-13-2-10-44-23-52-16-55-22-50-

14-26-24-29-51-3-15-33-37-39-42-41-30-38-19-35-27-36-11-4-48-46-43-20-56-53-17-34-32-1-start
Optimal Solution: 33021.440909091 units of time
[Go Data/StudentN55 for excel data]

N = 60 Students

IP Model Computation Time : 02:77
DP Model Computation Time: Nan
Optimal path:
end-1-57-29-48-49-9-18-24-6-15-53-32-28-3-23-25-59-22-33-13-14-30-26-27-12-43-35-60-47-31-40-38-61-52-7-44-5-50-2-45-55-19-17-42-8-20-56-21-46-58-54-11-39-4-41-36-51-37-16-10-34-1-start
Optimal Solution: 35927.289795918 units of time
[Go Data/StudentN60 for excel data]

N = 65 Students

IP Model Computation Time : 03:85
DP Model Computation Time: Nan
Optimal path:
end-1-11-54-36-3-46-6-21-23-24-60-12-49-19-61-53-57-55-45-22-51-40-5-38-17-59-64-32-39-65-28-26-48-42-43-37-9-33-62-13-34-31-44-52-58-4-63-14-7-20-35-18-50-47-66-8-15-25-2-16-56-41-10-29-27-30-1-start
Optimal Solution: 38806.933579336 units of time
[Go Data/StudentN65 for excel data]

N = 70 Students

IP Model Computation Time : 01:07:70
DP Model Computation Time: Nan
Optimal path:
end-1-16-9-8-19-34-13-22-54-42-53-7-5-17-49-2-11-68-39-26-52-51-63-43-4-35-25-6-37-70-71-10-27-38-12-18-61-3-29-14-59-21-69-23-57-50-40-15-48-32-64-30-65-45-33-60-67-31-46-66-58-20-44-55-56-36-62-24-41-47-28-1-start
Optimal Solution: 41761.279461279 units of time
[Go Data/StudentN70 for excel data]

N = 75 Students

IP Model Computation Time : 04:45

DP Model Computation Time: Nan
Optimal path:
end-1-69-71-64-16-76-54-5-75-27-60-30-70-65-51-44-7-36-4-15-26-48-52-68-72-32-40-21-45-8-10-63-47-12-39-35-46-18-11-33-38-58-6-37-61-57-2-22-73-74-41-20-67-17-28-14-55-34-24-56-43-49-42-3-23-25-13-50-59-29-19-31-66-53-9-62-1-start
Optimal Solution: 44739.786377709 units of time
[Go Data/StudentN75 for excel data]