Lab 3

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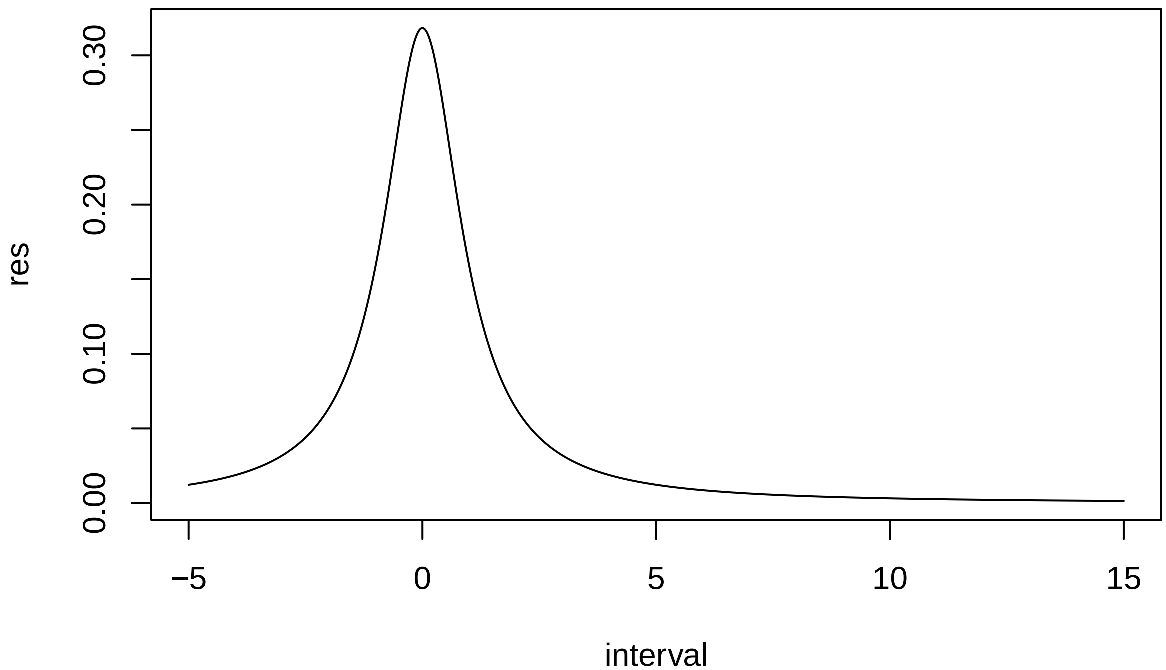
3.1.1

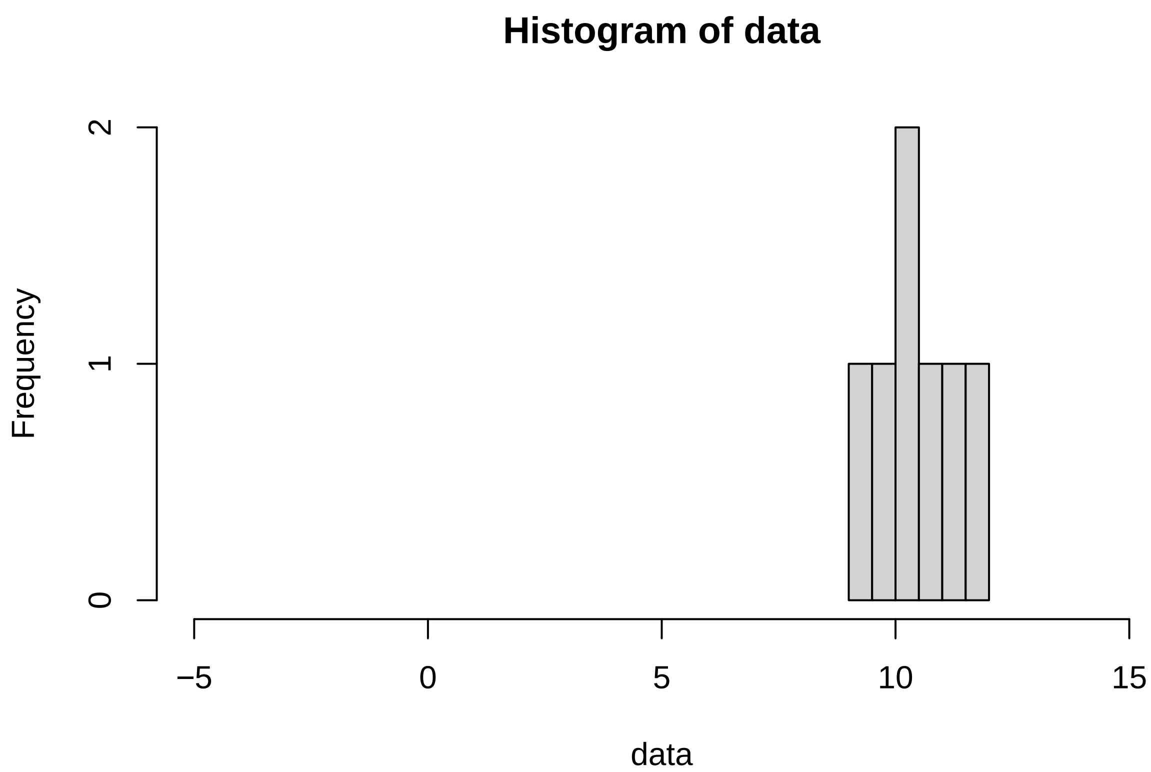
1)

```
interval <- seq(-5, 15, 0.01)
res <- dt(x = interval, df = 1)
plot(interval, res, type="l")
```



2)

```
data <- c(11.3710, 9.4353, 10.3631, 10.6329, 10.4043, 9.8939, 11.5115)
hist(data, xlim = c(-5, 15))
```

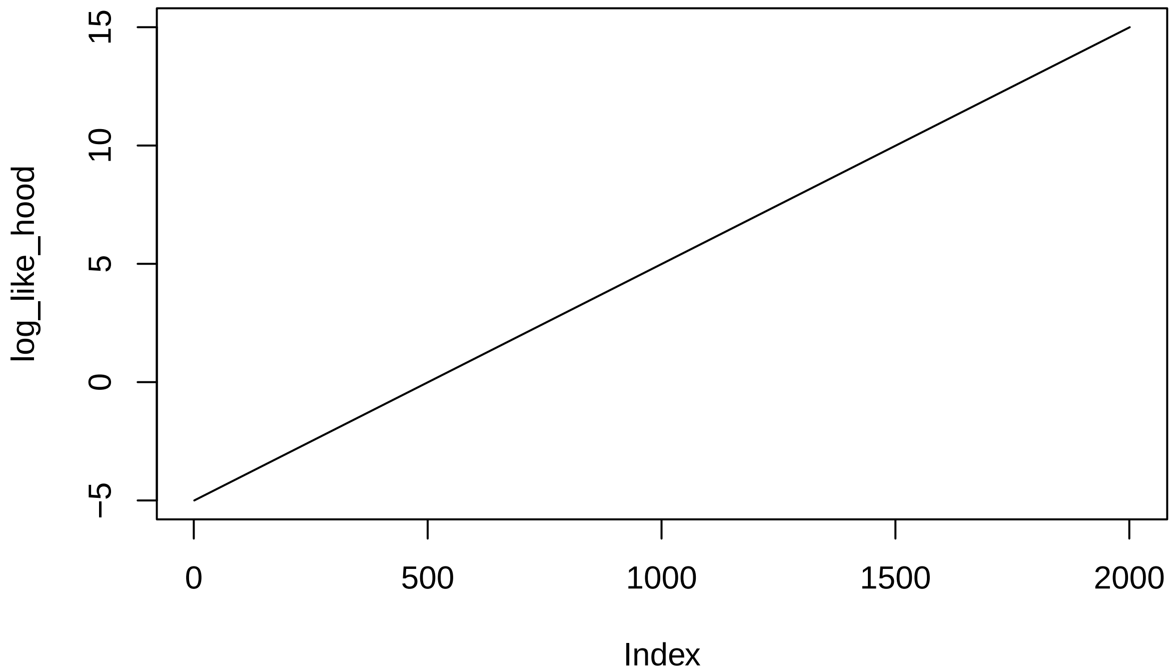


3)

```r
normal_log_likelihood <- function(mu, data) {
    return((-length(data)/2) * log(2*pi) - (1/2) * sum((data - mu)^2))
}
llik <- normal_log_likelihood(5, data)
print(round(llik, 1))
```

```
## [1] -114.6
```

```r
log_like_hood <- c()
for (mu in interval) {
    log_like <- c(log_like_hood, normal_log_likelihood(mu, data))
}
plot(interval, log_like_hood, type="l")
```



4)

Proof for proportional posterior for my

$$p(\theta \mid y) \propto p(y \mid \theta) * p(\theta)$$

Because it is the my parameter we want, we changes the theta to my instead.

$$p(\mu \mid y) \propto p(y \mid \mu) * p(\mu)$$

The likelihood function:

$$p(y \mid \mu) = (2\pi\sigma^2)^{\frac{-n}{2}} * \exp\left(\frac{-1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \mu)^2\right)$$

Sigmaˆ2 = 1 gives:

$$p(y \mid \mu) = (2\pi)^{\frac{-n}{2}} * \exp\left(\frac{-1}{2}\sum_{i=1}^{n}(y_i - \mu)^2\right)$$

We can shorten the terms that doesn't contain my.

$$p(y \mid \mu) = \exp\left(\frac{-1}{2}\sum_{i=1}^{n}(y_i - \mu)^2\right)$$

Prior function:

$$p(\mu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1 + \frac{\mu^2}{\nu}\right)^{-\frac{\nu+1}{2}},$$

v = 1 gives:

$$p(\mu) = \frac{\Gamma(1)}{\sqrt{\pi}\,\Gamma(\frac{1}{2})\,(1+\mu^2)} = \frac{1}{\sqrt{\pi}\,\Gamma(\frac{1}{2})\,(1+\mu^2)}$$

Then the posterior will be:

$$p(\mu \mid y) = (\exp(\frac{-1}{2}\sum_{i=1}^{n}(y_i-\mu)^2))\,(\frac{1}{\sqrt{\pi}\,\Gamma(\frac{1}{2})\,(1+\mu^2)}) =$$

$$= \frac{\exp(\frac{-1}{2}\sum_{i=1}^{n}(y_i-\mu)^2)}{\sqrt{\pi}\,\Gamma(\frac{1}{2})\,(1+\mu^2)} = \frac{1}{\sqrt{\pi}\,\Gamma(\frac{1}{2})} * \frac{\exp(\frac{-1}{2}\sum_{i=1}^{n}(y_i-\mu)^2)}{1+\mu^2} = c * \frac{\exp(\frac{-1}{2}\sum_{i=1}^{n}(y_i-\mu)^2)}{1+\mu^2}$$

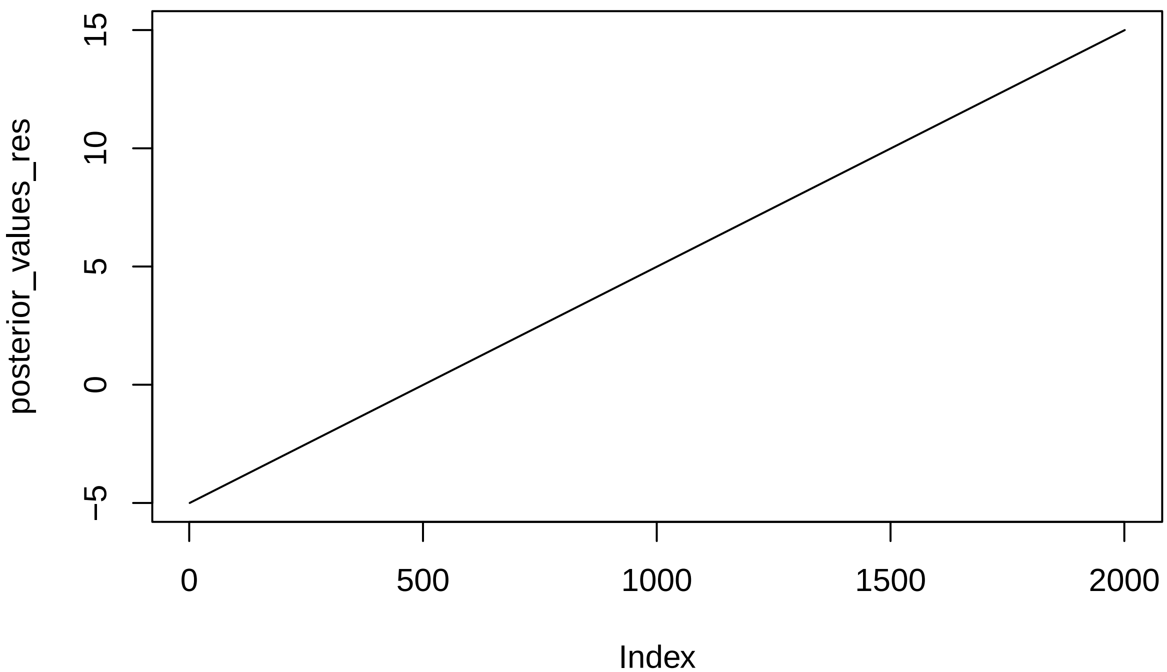
Since we want the proportienal posterior, we could shorten the constant c.

$$p(\mu \mid y) = \frac{\exp(\frac{-1}{2}\sum_{i=1}^{n}(y_i-\mu)^2)}{1+\mu^2}$$

5)

```
posterior <- function(mu, data) {
    return(exp((-1/2)*sum((data - mu)^2)) / (1 + mu^2))
}

posterior_values_res <- c()
for (mu in interval) {
    posterior_values <- c(posterior_values_res, posterior(mu, data))
}
plot(interval, posterior_values_res, type="l")
```



3.2.1

1)

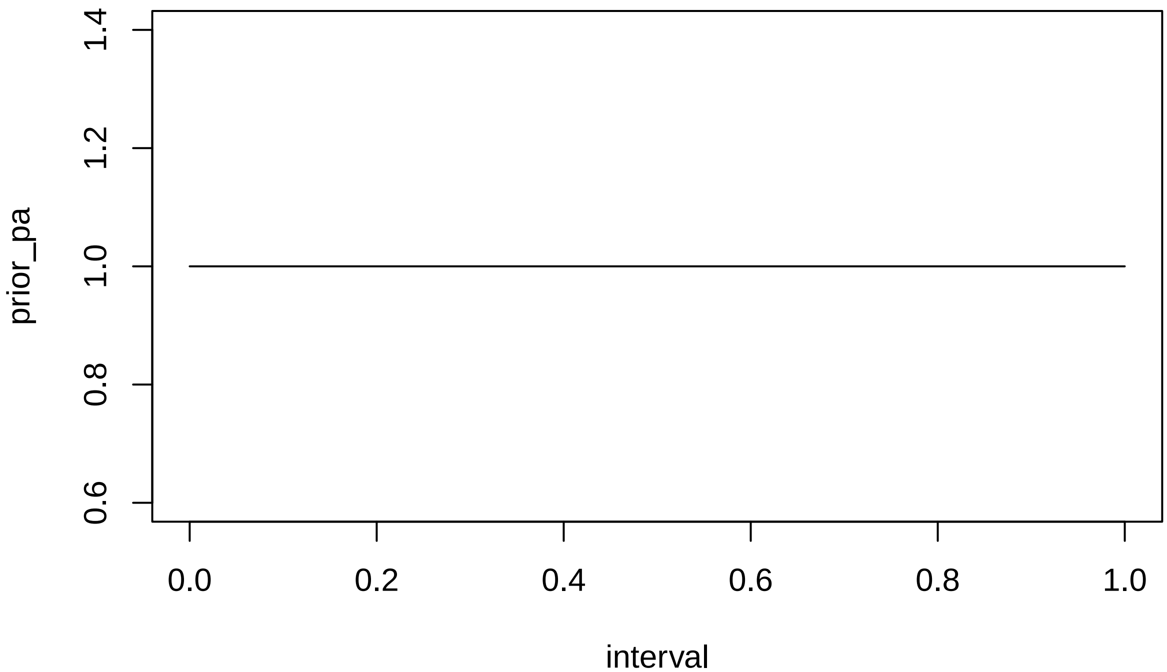
Since we don't have any data:

Proir for Pa: Beta (1, 1)

Proir for Pb: Beta (1, 1)

Eftersom de är samma så plottar vi bara ena Since they are the same we only plot one of them. :)

```
alpha = 1
beta = 1
interval <- seq(0, 1, 0.01)
prior_pa <- dbeta(x = interval, shape1 = alpha, shape2 = beta)
plot(interval, prior_pa, type="l")
```



2) Here is the expected value for beta.

$$\mathbb{E}(x) = \frac{\alpha}{\alpha + \beta}$$

So for product A:

E(x) = (8+1) / (8+1 + 13-8+1) = 9 / 15 = 0.6

For product B:

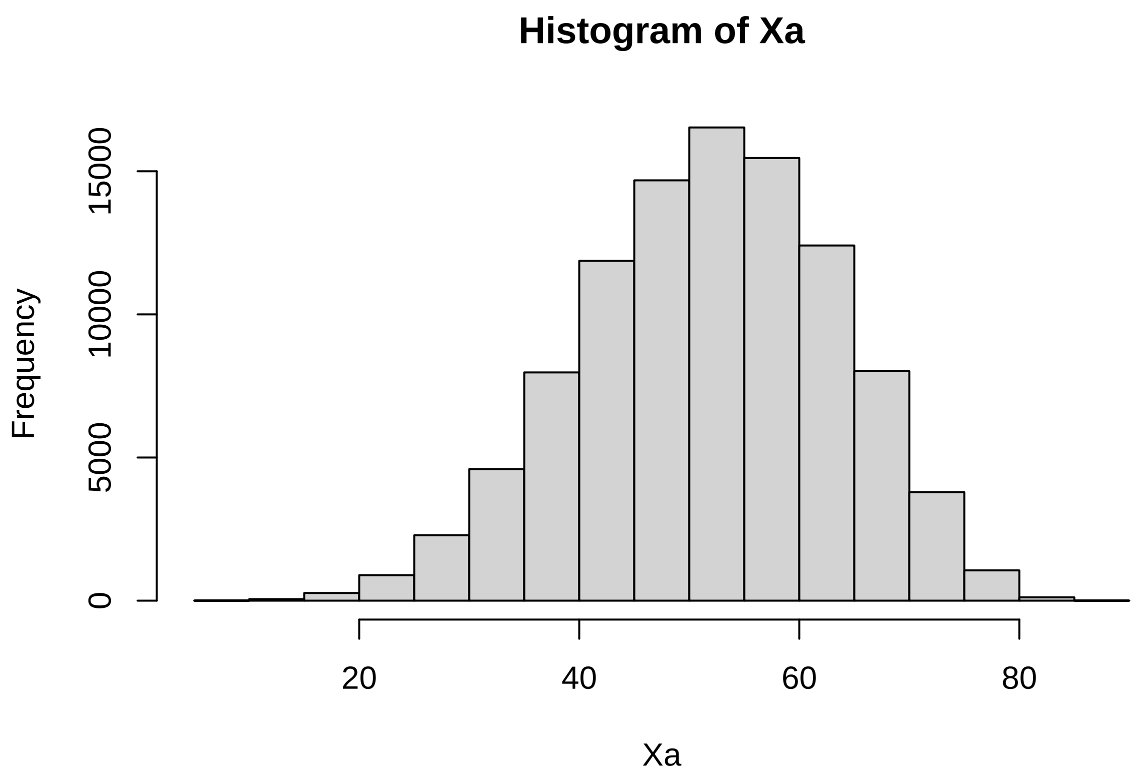
E(x) = (2+1) / (2+1 + 3-2+1) = 3 / 5 = 0.6

We can see that the product has the same expected value based on the formula.

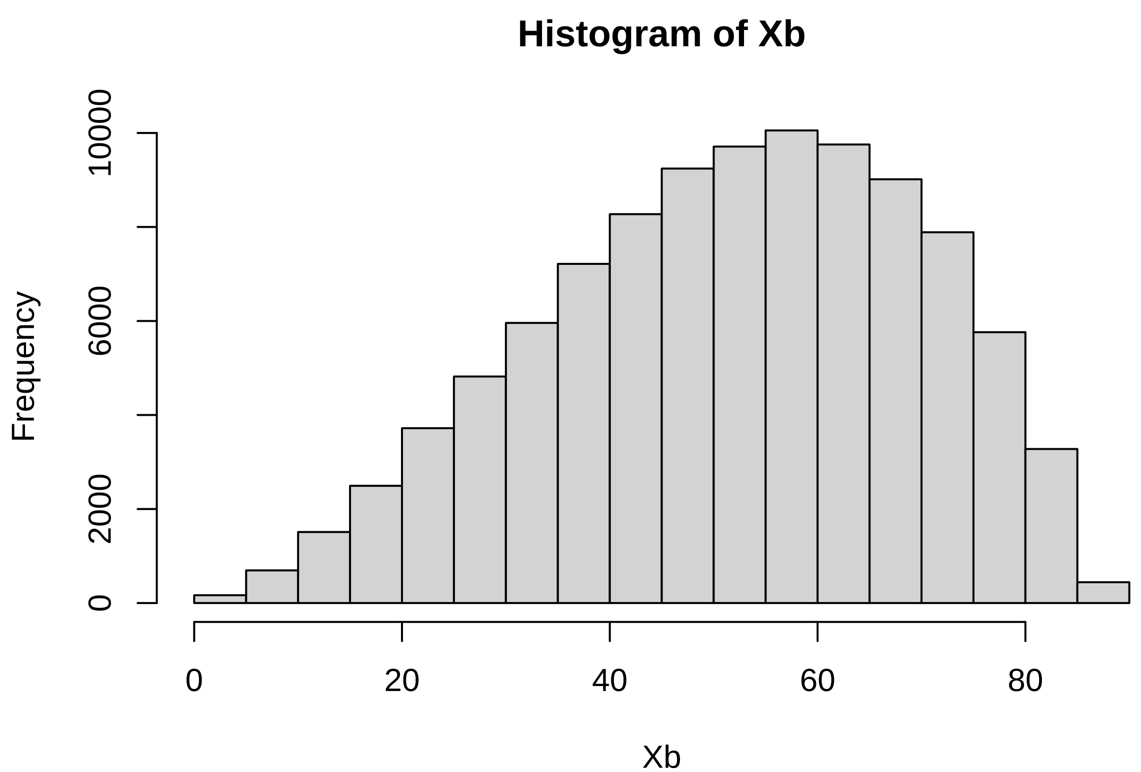
3)

```
n <- 100000
pA <- rbeta(n = n, shape1 = 8+1, shape2 = 13-8+1)
pB <- rbeta(n = n, shape1 = 2+1, shape2 = 3-2+1)
Xa <- rbinom(n = n, size = 87, prob = pA)
Xb <- rbinom(n = n, size = 87, prob = pB)

hist(Xa)
```




```
hist(Xb)
```



a)

```
print(sum(Xa > 40) / n)
```

```
## [1] 0.83936
```

```r
print(sum(Xb > 40) / n)
```

```
## [1] 0.73421
```

b)

```r
print(mean(Xa))
```

```
## [1] 52.28773
```

```r
print(mean(Xb))
```

```
## [1] 52.22945
```