**CSE 211: Discrete Mathematics** **(Due: 07/12/20)**

# Homework #2

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*Assistant: Gizem Süngü*

**Course Policy**: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

**Problem 1: Relations** (15 points)

Draw the Hasse diagram for the "greater than or equal to" relation on {0, 1, 2, 3, 4, 5}. Show each step to build the diagram. In order to draw the diagram, you can choose one of the following options:

- You can use an online drawing tool such as https://app.diagrams.net/.
- You can draw the diagram by hand, take a picture of it to put on Latex as a figure.
- Word Office, Libre Office are also options to use them as drawing tools.

***(Solution)***
( {0, 1, 2, 3, 4, 5} ,$\geq$)

for figure 1:
R = (5,5),(5,4),(5,3),(5,2),(5,1),(5,0),(4,4),(4,3),(4,2),(4,1),(4,0),(3,3),(3,2),(3,1),(3,0),(2,2),(2,1),(2,0),(1,1),(1,0),(0,0)

R is reflexive if for all x A, xRx if we remove reflexitivity:

for figure 2:
R = (5,4),(5,3),(5,2),(5,1),(5,0),(4,3),(4,2),(4,1),(4,0),(3,2),(3,1),(3,0),(2,1),(2,0),(1,0)

R is transitive if for all x,y, z A, if xRy and yRz, then xRz. if we remove transitivity:

for figure 3:
R = (5,4),(4,3),(3,2),(2,1),(1,0)

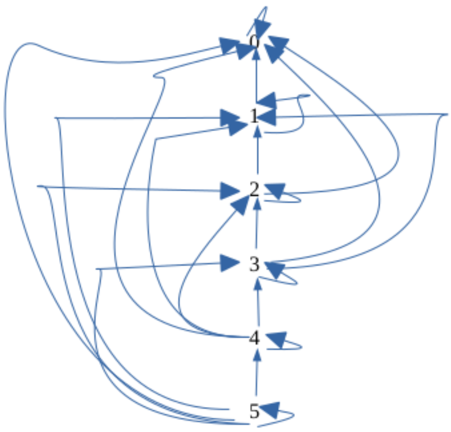


Figure 1: (a)

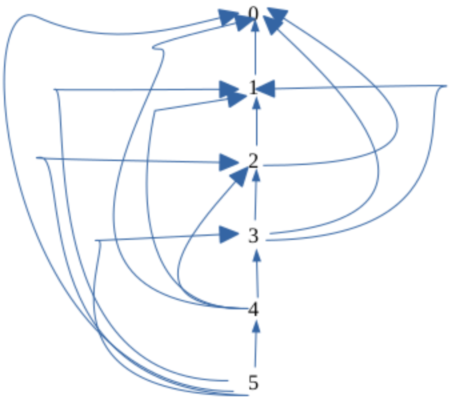


Figure 2: (b)


Figure 3: (c)

**Problem 2: Relations** (15 points)

Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆).

Our set is :
( {1},{1,2}),({1},{1,3,4}),({1},{1,4}),({2},{1,2}),({2},{2,4}),({2},{2,3,4}),({4},{3,4}),({4},{2,4}),({4},{2,3,4}),({4},{1,3,4})
,({4},{1,4}),({1,4},{1,3,4}),({2,4},{2,3,4}),({3,4},{1,3,4}),({3,4},{2,3,4}),({1},{1}),({2},{2}), ({4},{4}),({1,2},{1,2}),
({1,4},{1,4}),({2,4},{2,4}),({3,4},{3,4}),({1,3,4},{1,3,4}),({2,3,4},{2,3,4}),

R is reflexive if for all x A, xRx and if we remove reflexive elements from our set:

( {1},{1,2}),({1},{1,3,4}),({1},{1,4}),({2},{1,2}),({2},{2,4}),({2},{2,3,4}),({4},{3,4}),({4},{2,4}),({4},{2,3,4}),({4},{1,3,4})
,({4},{1,4}),({1,4},{1,3,4}),({2,4},{2,3,4}),({3,4},{1,3,4}),({3,4},{2,3,4})

R is transitive if for all x,y, z A, if xRy and yRz, then xRz.
and if we remove transitive elements from our set:

( {1},{1,2}),({1},{1,4}),({2},{1,2}),({2},{2,4}),({4},{3,4}),({4},{2,4})
,({4},{1,4}),({1,4},{1,3,4}),({2,4},{2,3,4}),({3,4},{1,3,4}),({3,4},{2,3,4} )

Let's draw hasse diagrams.

R = { {1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4} }
HASSE DIAGRAM:

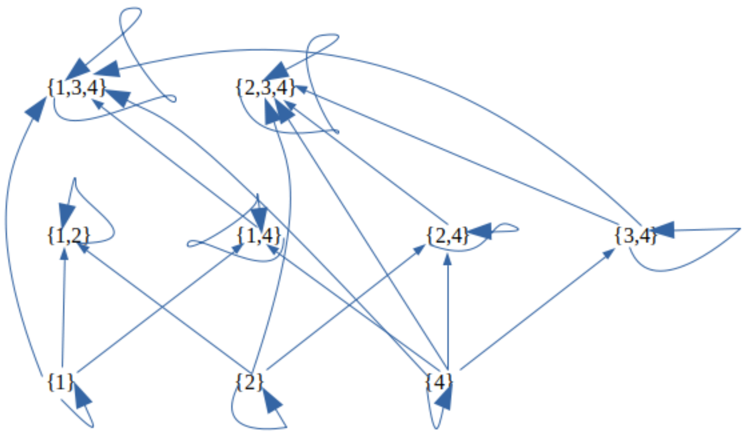


Figure 4: (a)

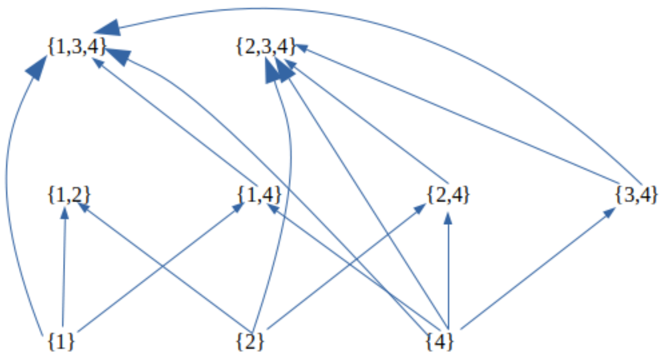


Figure 5: (b)

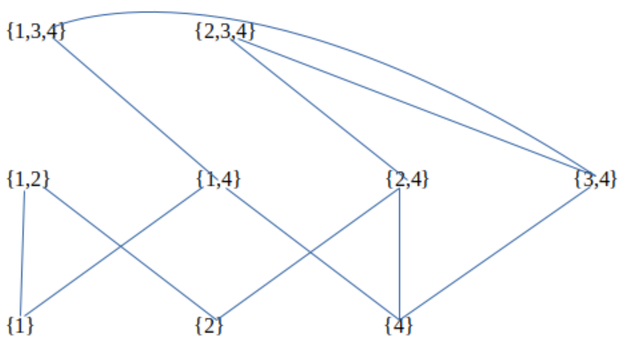


Figure 6: (c)

**(a)** Find the maximal elements.
***(Solution)***
An element R in S is called a maximal element in S if there exist no $x \in S$ such that $R \prec x$.
so, maximal elements = {1,2},{1,3,4},{2,3,4}
These elements does not have any elements above itself in hasse diagram.

**(b)** Find the minimal elements.
***(Solution)***
An element R in S is called a minimal element in S if there exist no $y \in S$ such that $y \prec R$.
so, minimal elements = {1},{2},{4}.
These elements does not have any elements down itself in hasse diagram.

**(c)** Is there a greatest element?
***(Solution)***
An element $a$ is the greatest element of $S$ if, for any element $x \in S$, we have $x \prec a$.
no, it is not. Because if there is only one maximal elements, then there is greatest element.
But there is three maximal elements.

**(d)** Find all upper bounds of {{2}, {4}}.
***(Solution)***
Upper Bound: Consider B be a subset of a partially ordered set A. An element x A is called
an upper bound of B if y x for every y B.
When we follow common arrows of {2} and {4} toward up in hasse diagram, then we will reach upper bounds.
Upper bounds = {2,4}, {2,3,4}

**(e)** Find the least upper bound of {{2}, {4}}, if it exists.
***(Solution)***
Least upper bound is {2,4}, because {2,4} lower than {2,3,4} in hasse diagram.

**(f)** Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
***(Solution)***
Lower Bound: Consider B be a subset of a partially ordered set A. An element z A is called
a lower bound of B if z x for every x B.
When we follow common arrows of {{1, 3, 4} and {2, 3, 4}} toward down in hasse diagram, then we will reach lower bounds.
Lower bounds = $\{4\}$, $\{3, 4\}$

**(h)** Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.
***(Solution)***
Greatest lower bound is $\{3, 4\}$ because $\{3, 4\}$ higher than $\{4\}$ in hasse diagram.

**Problem 3: Relations** (70 points)

Remember that a relation $R$ on a set $A$ can have the properties reflexive, symmetric, anti-symmetric and transitive.

- **Reflexive:** R is reflexive if (a, a) $\in R$, $\forall$ a $\in$ A.
- **Symmetric:** R is symmetric if (b, a) $\in$ R whenever (a, b) $\in$ R, $\forall$ a, b $\in$ A.
- **Anti-symmetric:** R is antisymmetric if $\forall$ a, b $\in$ A, (a, b) $\in$ R and (b, a) $\in$ R implies that a = b.
- **Transitive:** R is transitive if $\forall$ a, b, c $\in$ A, (a, b) $\in$ R and (b, c) $\in$ R implies that (a, c) $\in$ R.

For the details about the properties, please check the 4th lecture slide on Moodle.

As we solved the problem 3 in PS4 document - which is available on Moodle - in the problem session, we can determine any given relation if it is reflexive, symmetric, anti-symmetric, and transitive.

Write an algorithm to determine if a given relation $R$ is reflexive, symmetric, anti-symmetric, and transitive. Your code should meet the following requirements, standards and tasks.

- Read the relations in the text file "input.txt".
- Let $R$ be a relation on a set $A$ where $\exists a, b \in A, (a, b) \in R$. Each relation $R$ is represented with the lines in the file:
  1. The first line says how many relations in $R$.
  2. The second line gives the elements of the set $A$.
  3. The following lines give each relation in $R$.
- After determining each relation in input.txt whether it is reflexive, symmetric, anti-symmetric and transitive with your algorithm, write its result to the file which is called "output.txt" with the following format.
- output.txt:
  1. Start a new line with "n" which indicates a new relation.
  2. The set of $R$
  3. Reflexive: Yes or No, explain the reason if No (e.g. "(a, a) is not found").
  4. Symmetric: Yes or No, explain the reason if No (e.g. "(b, a) is not found whereas (a, b) is found.")
  5. Antisymmetric: Yes or No, explain the reason if No (e.g. "(b, a) and (a, b) are found.")
  6. Transitive: Yes or No, explain the reason if No (e.g. "(a, c) is not found whereas (a, b) and (b, c) are found.")
- An example of the output format is given in "exampleoutput.txt". The file has the result of the first relation in "input.txt".
- When explaining why a property does not exist in the relation, one reason is enough to explain if there are more. For example, in "exampleoutput.txt", the relation is not symmetric because (b, a) and (e, a) are not found. Detecting one of them is enough to explain the reason.
- **Bonus (20 points):** If you can explain why a property exists in the relation, it brings you bonus of 20 points.
- Your code is responsible to provide exception and error handling. The input file may be given with a wrong information, then your code must be prepared to detect them. For instance, "The element b of the relation (1, b) is not found in the set A = $\{1, 2, 3, 4\}$.".
- You can implement your algorithm in Python, Java, C or C++.
- **Important:** Put comments almost for each line of your code to describe what the line is going to do.
- You should put your source code file (file name is problem1.{$.c$, $.java$, $.py$, $.cpp$}) and output.txt into your homework zip file (check Course Policy).