CSE 211: Discrete Mathematics **(Due: 17/11/20)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)
p=if it snows tonight, q=i will stay at home.
The statement is $p \Rightarrow q$
Converse:
The converse of statement p $\Rightarrow q$ is $q \Rightarrow p$. Converse is that "If i will stay at home, then it snows tonight".
Contrapositive:
The contrapositive of statement p $\Rightarrow q$ is $q' \Rightarrow p'$.
contrapositive is that" If i will not stay at home, then it does not snow tonight".
Inverse:
The inverse of statement p $\Rightarrow q$ is $p' \Rightarrow q'$.
inverse is that "if it doesnt snow tonight, then i will not stay at home."

(b) I go to the beach whenever it is a sunny summer day.
(Solution)
p = it is a sunny summer day, q = I go to the beach.
The statement is p $\Rightarrow q$.
Converse:
The converse of statement p $\Rightarrow q$ is $q \Rightarrow p$.
converse is that "it is a sunny summer day whenever i go to the beach."
Contrapositive:
The contrapositive of statement p $\Rightarrow q$ is $q' \Rightarrow p'$. Contrapositive is that " it is not a sunny summer day

whenever I do not go to the beach. "
Inverse:
The inverse of statement p $\Rightarrow q$ is $p' \Rightarrow q'$.
Inverse is that " I do not go to the beach whenever it is not a sunny summer day."

(c) If I stay up late, then I sleep until noon.
(Solution)
p = I stay up late, q = I sleep until noon.
The statement is p $\Rightarrow q$.
Converse:
The converse of statement p $\Rightarrow q$ is $q \Rightarrow p$.
The converse is that " If I sleep until noon, then I stay up late."
Contrapositive:
The contrapositive of statement p $\Rightarrow q$ is $q' \Rightarrow p'$.
The contrapositive is that "If I do not sleep until noon, then I do not stay up late."
Inverse:
The inverse of statement p $\Rightarrow q$ is $p' \Rightarrow q'$.
Inverse is that "If I do not stay up late, then I do not sleep until noon."

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
(a) (p $\oplus$ ¬ q)
(Solution)

p	q	q¬	p $\oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b) (p $\iff$ q) $\oplus$ (¬ p $\iff$ ¬ r)
(Solution)

p	q	r	$\neg p$	$\neg r$	p$\leftrightarrow q$	$\neg p \leftrightarrow \neg r$	p$\leftrightarrow q \oplus \neg p \leftrightarrow \neg r$
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
F	T	T	T	F	F	F	F
F	F	T	T	F	T	F	T
T	F	F	F	T	F	F	F
F	F	F	T	T	T	T	F
F	T	F	T	T	F	T	T

(c) (p $\oplus$ q) $\Rightarrow$ (p $\oplus$ ¬ q)
(Solution)

p	q	$\neg q$	p$\oplus q$	p$\oplus\neg q$	p$\oplus q \Rightarrow p \oplus \neg q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Problem 3: Predicates and Quantifiers (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): "x can speak English."
- Q(x): "x knows Python."
- H(x): "x is happy."

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all students at the university.

(a) There is a student at the university who can speak English and who knows Python.
(Solution)
$\exists x(Q(x) \wedge P(x))$

(b) There is a student at the university who can speak English but who doesn't know Python.
(Solution)
$\exists x(P(x) \wedge \neg Q(x))$

(c) Every student at the university either can speak English or knows Python.
(Solution)
$\forall x(P(x) \bigoplus Q(x))$

(d) No student at the university can speak English or knows Python.
(Solution)
$\forall x \neg(P(x) \vee Q(x))$

(e) If there is a student at the university who can speak English and know Python, then she/he is happy.
(Solution)
$\forall x((P(x) \wedge Q(x)) \Rightarrow H(x))$

(f) At least two students are happy.
(Solution)
$\exists x, y(H(x) \wedge H(y) \wedge x \neq y)$

(g) $\neg\forall x(Q(x) \wedge P(x))$
(Solution)
"Not everyone at the university who can speak english and who knows python."

Problem 4: Mathematical Induction (21 points)

Prove that 3 + 3 . 5 + 3 . 5^2 + . . . + 3 . 5^n $=\frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.
(Solution)

Basis step = apply n=1 on the equation;
3+3.5 = $\frac{3(5^2-1)}{4}$
18 = 18
inductive step = apply n=k on the equation and accept that the equation is true for n=k;
$3 + 3.5 + 3.5^2 + . . . + 3.5^k = \frac{3(5^{k+1}-1}{4} = a$

apply n=k+1 on the equation and prove that the equation is true based on the equation of n=k.
$3 + 3.5 + 3.5^2 + \ . \ . \ . + 3.5^k + 3.5^{k+1} = \frac{3(5^{k+2}-1}{4}$
$\frac{3(5^{k+1}-1}{4} + 3.5^{k+1} = \frac{3(5^{k+2}-1}{4}$

$3.5^{k+1} = \frac{3(5^{k+2}-1}{4}$-$\frac{3(5^{k+1}-1}{4}$

$3.5^{k+1} = \frac{3.5^{k+1}(5-1)}{4}$

$3.5^{k+1} = \frac{3.5^{k+1}(4)}{4}$

$3.5^{k+1} = 3.5^{k+1}$

Problem 5: Mathematical Induction (20 points)

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer.
(Solution)
Basis step = apply n=1 on the equation;
1^2-1 = 0
0 % 8 = 0
inductive step = apply n=k on the equation and accept that the equation is true for n=k and k is odd positive integer.;
k^2-1 % 8 = 0
apply n=k+2 on the equation and prove that the equation is true based on the equation of n=k. the reason we use k + 2 is because it is an odd number, it must increase by 2 by 2.
$(k + 2)^2$-1
$(k + 2)^2$-1 = k^2 + 4k + 4 -1
k^2-1 + (4k+4)
We have already assumed that k^2-1 is divided by 8.
since the smallest positive odd number is 1, (4k + 4) is divided by 8 exactly. 4.1 + 4 = 8
Whatever we substitute for k, (4k + 4) will be a multiple of 8, so all values of k (4k + 4) are divided by 8.

Problem 6: Sets (8 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of $x^2 - 6$x + 8 = 0}
When we factor the equation, we get the following equation.
(x-2)(x-4) = 0
x-2=0 , x-4=0
x=2 or x=4
t : 2,4

(b) {y : y is a real number in the closed interval [2, 3]}
y : 2,2.1,2.3,...3

(c) {4, 2, 5, 4}
y : 2,4,5

(d) {4, 5, 7, 2} - {5, 7}
u : 2,4
(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}
the rectangle has 4 sides and any number between 11 and 99 has 2 digits thus, the sets is that
k : 2,4

(Solution)
The sets a, d and e are equal sets because they have the same number of elements and have the same elements.

Problem Bonus: Logic in Algorithms (20 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

circuit.png

Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.
(a) Write the sentence that "result" output has.
(Solution)

result $= (p \land q).\neg s + (p \lor \neg q).s$
If s = 0 the result is that "It is sunny and the flowers are blooming."
If s = 1 the result is that "It is sunny or the flowers are not blooming."
then the result is that s=1, so "It is sunny or the flowers are not blooming."

(b) Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).
(Solution)

$include < iostream >$ // including library for function cout
using namespace std;
int main()
//putting the sentence "it is sunny" into the string p
//putting the sentence "the flowers are blooming." into the string q
string p="It is sunny", q="The flowers are blooming.";

// putting the result from 'and' gate into the firstGate string
//$p \land q$
string firstGate = "It is sunny and the flowers are blooming.";
// putting the result from 'or' gate into the firstGate string
//$p \lor \neg q$
string secondGate = "It is sunny or the flowers are not blooming.";

//according to result $= (p \land q).\neg s + (p \lor \neg q).s$ formula
// if s is zero, then first statement is executed.
if(s == 0)

cout << firstGate
newline //according to result $= (p \land q).\neg s + (p \lor \neg q).s$ formula
//if s is one, then first statement is executed.
else
cout << secondGate;
return 0;

[1] https://www.geeksforgeeks.org/multiplexers-in-digital-logic/