CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

Instructor: Dr. Zafeirakis Zafeirakopoulos *Name:* *Student Id:*
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)

if $a_n = -2^{n+1}$,then what is a_{n-1} ?
$a_{n-1} = -2^{n-1+1} = -2^n$
if we plug the a_{n-1} into $a_n = 3a_{n-1} + 2^n$;
$a_n = 3$ x $-2^n + 2^n$
$a_n = 2^n$ (-3 + 1) $= 2^n$ x (-2)
$a_n = -2^{n+1}$

(b) Find the solution with $a_0 = 1$.
(Solution)
$a_n = 3a_{n-1} + 2^n$ and $a_0 = 1$.
non-homogeneous linear recurrences have two parts. One of these homo part, other is particular part. And this is represented as $a_n = a_n^h + a_n^p$
particular solution is same as in part a. That is $a_n^p = -2^{n+1}$
a_n - $3a_{n-1} = 0$
if we divide by a_{n-1}, then the characteristhic equation is found. That is ;
r - 3 = 0, r = 3.
a_n^h = A x 3^n
a_n = A x $3^n + -2^{n+1}$
for $a_0 = 1$
A x 3^0 -2^{0+1}
A-2 = 1 , A = 3
The equation is : $3^{n+1} - 2^{n+1}$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
non-homogenous recurrence releation has two parts: homogenous part and particular part.

homogeneous part is written on one side and particular part on the other side of the equality.
f(n) - 4f(n-1) + 4f(n-2) = n^2

$f(n)^{(g)} = f(n)^{(h)} + f(n)^{(p)}$

finding characteristhic equation with homogenous part :

$f(n)^{(h)}$ = f(n) - 4f(n-1) + 4f(n-2)

if we divide each term by f(n-2) ;
characteristic equation ;
r^2 -4r + 4 = 0
$= (r-2)(r-2) = 0 => r = 2$

f(n)$^{(h)}$ = p . 2^n + q.n.2^n

solving particular part :

$f(n)^{(p)}$ = A.n^2 + B.n + C

Putting f(n) into the original equation ;

A.n^2 + B.n + C = 4[A.$(n-1)^2$ + B.(n-1) + C] -4[A.$(n-2)^2$ + B.(n-2) + C] + n^2

A.n^2 + B.n + C = 4.[A.(n^2 -2.n +1) + B.n -B + C] -4.[A.(n^2 -4.n + 4) + B.n -2.B + C] + n^2

A.n^2 + B.n + C = 4An^2 -8An + 4A +4Bn -4B + 4C - 4An^2 + 16An -16A - 4Bn + 8B -4C + n^2

An^2 + B.n + C = n^2 + 8An -12A + 4B

variables of the same degree must be equal to the same coefficient

A = 1, B = 8, C = -12.1 + 4.8 = 20

$f(n)^{(p)} = n^2$ + 8n + 20

f(n) = p.2^n + q.n.2^n + n^2 + 8n + 20

f(0) = 2 , f(1) = 5

f(0) = p + 20 = 2
p = -18

f(1) = 2.(-18) + 2.q + 1 + 8 + 20 = 5

q = 6

f(n) = -18.2^n + 6.n.2^n + n^2 + 8n + 20

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)

a_n - $2a_{n-1}$ + $2a_{n-2} = 0$
characteristhic equation :
r^2 - 2r + 2 = 0
delta = b^2 -4ac = $(-2)^2$ -4*1*2 = -4

first root = (-b + $\sqrt{delta}$)/(2*a) = (-(-2) + 2i)/2 = 1+i
second root = (-b - $\sqrt{delta}$)/(2*a) = (-(-2) - 2i)/2 = 1-i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)

a_n = A $(1+i)^n$ + B $(1-i)^n$
a_0 = 1 = A + B (first equation)
a_1 = 2 = A x (1+i) + B x (1-i)
A * i - B * i = 1
A-B = 1/i (second equation)
if we find A and B by using first and second equation,
A = (i+1) / 2i
B = (i-1) / 2i

a_n = ((i+1) / 2i) x $(1+i)^n$ + ((i-1) / 2i) x $(1-i)^n$