Objectives:

To create and implement a mathematical model for the control system of Tesla Model S P85 using Simulink.

Mathematical Model Creation:

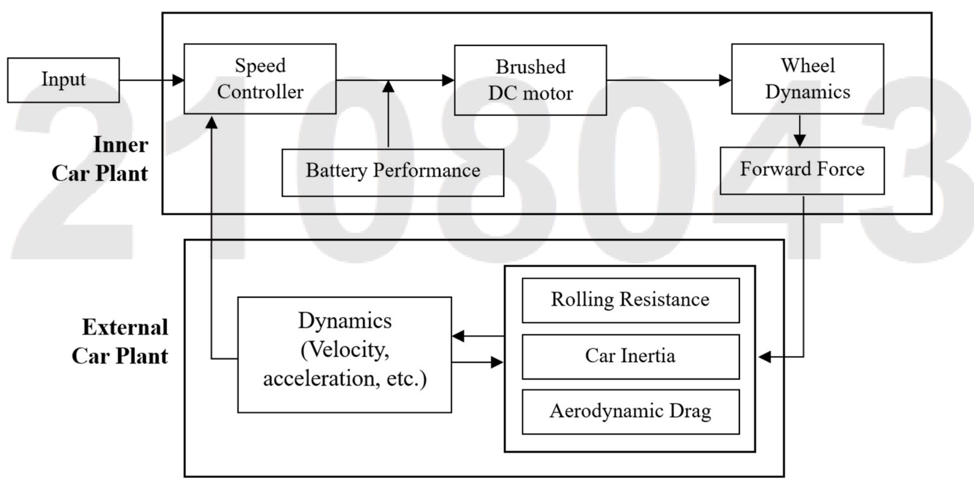
General Characteristics:

1.	Weight (m):	2,108 kg
2.	Power:	460 HP/ 343 KW @ 8600 rpm
3.	Torque(τ):	600 Nm @ 0 rpm
4.	Drag Co-efficient(μ):	0.24
5.	Frontal Area:	2.34 m^2
6.	Rolling Resistance Co-eff on Asphalt:	0.02

Components:

1. Input (0 to 100%)
2. Wheels
3. 85kWh Battery
4. Brushed DC Motor

Model Diagram:



Designing Process:

1. Derive the mathematical expressions for each part of our model.
2. Translate these expressions to a MATLAB/SIMULINK model.
3. Use this model to program our speed controller.
4. Enjoy the results and compare to real world examples.

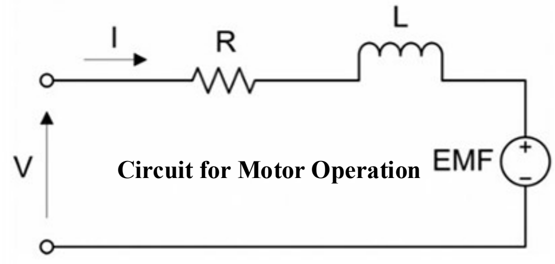
The Battery and Actuator (Input):

The Model S battery has 7,104 18650 Li-ion cells arranged in 16 series-wired modules (each module contains 6 groups of 74 cells wired in parallel), totaling 85 kWh and weighing 540 kg. Each cell offers 3,300 mAh at nominal voltage of 3.6V (11.9 Wh), with a max voltage of 4.2V and a discharge limit of 2.5V.

So, max operating voltage, **V = 3.6*16*6 = 346V** and torque cap will set max current.

The Electro-Motor:

Though Tesla model S uses a 3-phase AC four pole induction motor, here a DC brush motor will be used for model.



Circuit for Motor Operation

Using KVL,

$$V = I\,(t) * R + L\frac{dI(t)}{dt} + E(t)$$

Putting back EMF, $E(t) = K_E * \omega(t)$ in equation and transforming it to Laplace domain,

$$V(s) = RI(s)\ +\ sLI(s) + K_E\omega(s)$$

So,

$$I(s) = \frac{V(s)\ -\ K_T\omega(s)}{sL\ +\ R}$$

The Torque in an electrical DC motor, $T(t) = K_T * I(t)$ and in Laplace domain

$$T(s) = K_T * I(s)$$

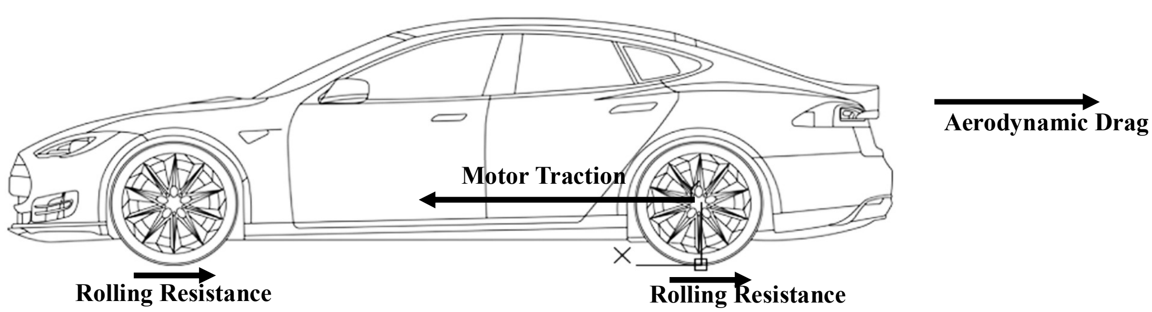
$$T(s) = K_T * \frac{V(s)\ -\ K_T\omega(s)}{sL\ +\ R}$$

Analyzing online data the estimated values are:

R = $5.3*10^{-3}$ Ohm
L = $493*10^{-9}$ $Henrys$
$K_{=}$ 0.12 Vs/rad
K_T = 0.25 Nm/Amp
So, the equation for motor will be

$$\boldsymbol{T(s) = 0.25 * \frac{V(s)\ -\ 0.12\ *\ \omega(s)}{493 * 10^{-9}s\ +\ 5.3 *\ 10^{-3}}}$$

Forces at play:



Motor traction force,$F_f = \frac{T}{L} * G_r$ where gear ratio, G_r = 9.73 and distance from center of rotation, L = 24 cm.
So, $F_f = 40.5T$

Aerodynamic drag, $D = \frac{1}{2}\rho V^2 S C_D$ where, air density ρ = 1.225 kgm^{-3} the frontal area, S = 2.3 m^2 and coefficient of Drag is C_D = 0.24.

So D = $0.3381V^2$

Rolling resistance, $F_r = C_r * m_r * g$, where, for car tires on a dry/asphalt road rolling resistance coefficient, C_r = 0.02, total mass of Model S, m_r = 2108 kg and the gravitational constant is 9.81 m/s^2.

So the Rolling resistance of the car, Fr = 413 N

<u>The Model's Plant Dynamics:</u>

Using Newton's 2nd law

$$0.7 * F_f - F_r - D = ma$$

So,

$$a = \frac{dv}{dt} = \frac{\mathbf{0.7 * 40.5 - 413 - 0.3381}v^2}{\mathbf{2108}}$$

Here,
Motor Traction: F_f = 40.5T
Overall efficiency = 70%
Rolling Resistance: F_r = 413 N
Aerodynamic Drag: $D = 0.3381v^2$
Total mass of Model S: m = 2108 kg

This equation describes the plant dynamics.

Mathematical Model Implementation:

Using MATLAB Simulink the following inner car plant was created which is an open loop system

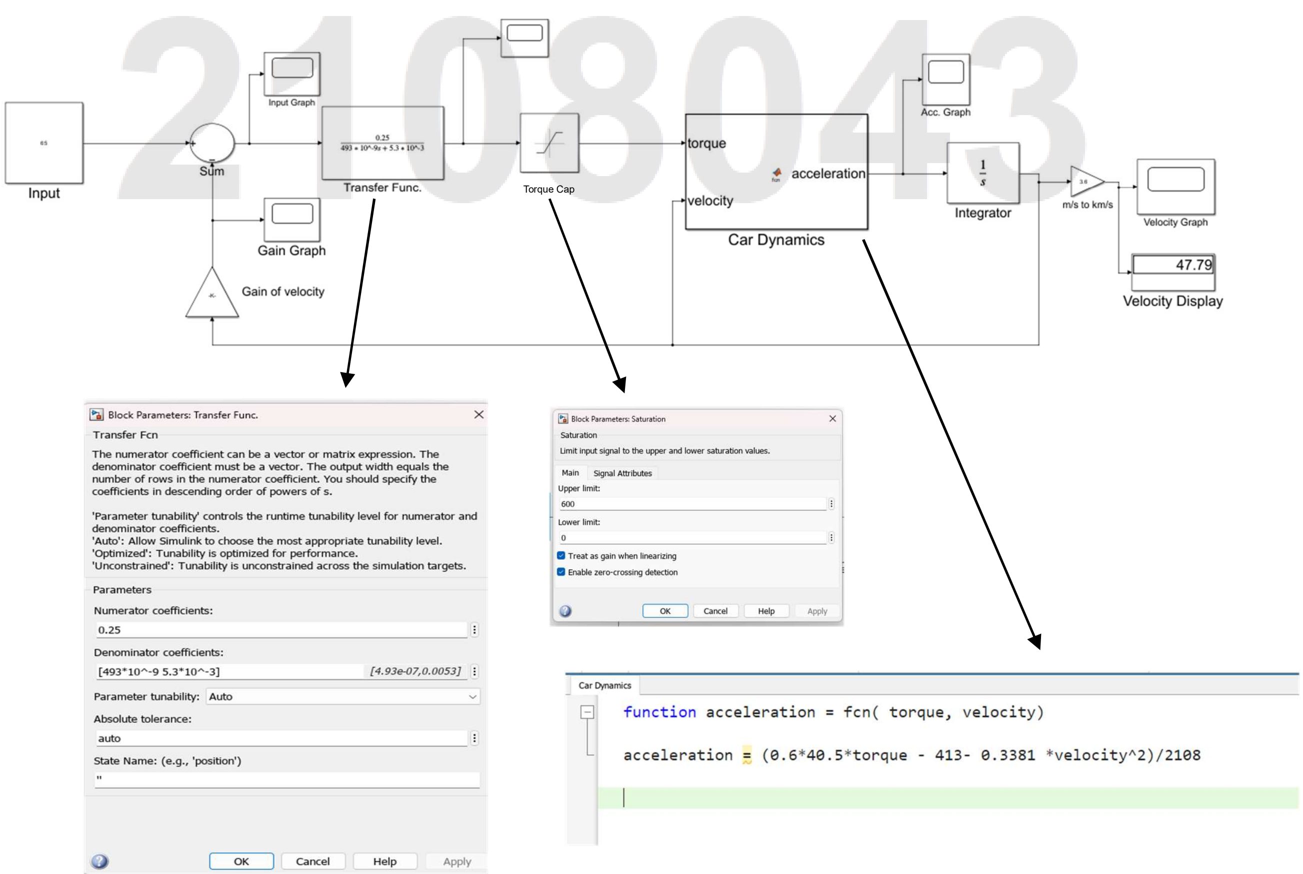
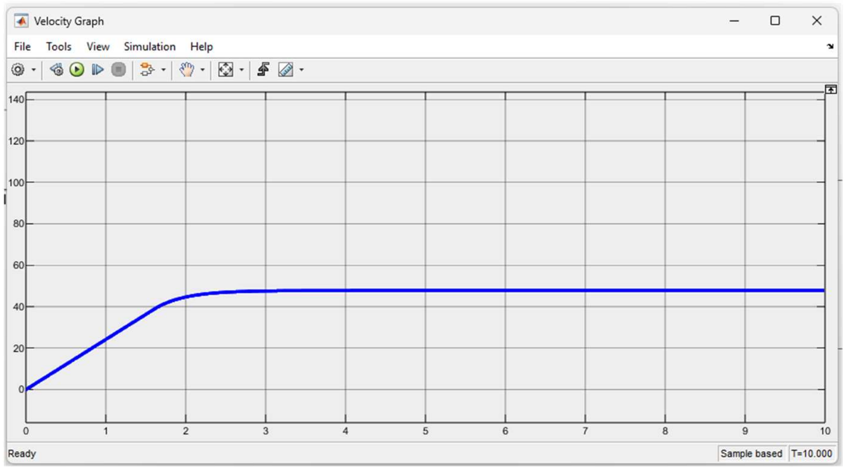


Figure: Inner Car Plant without controller and feedback plant

The output velocity graph is



Let's add a PID controller and take velocity gain as feedback of the controller,

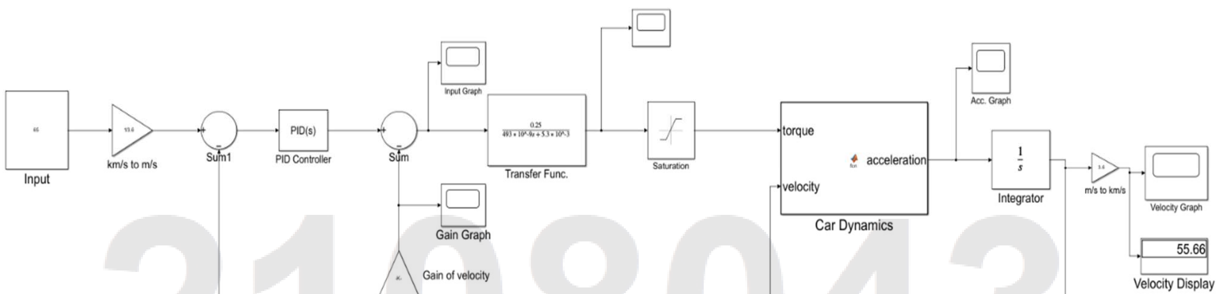
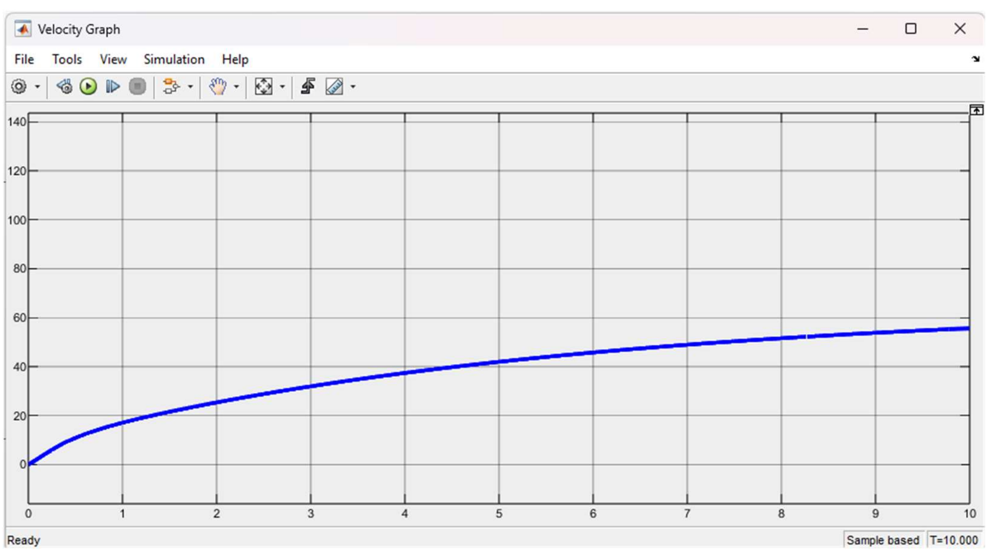
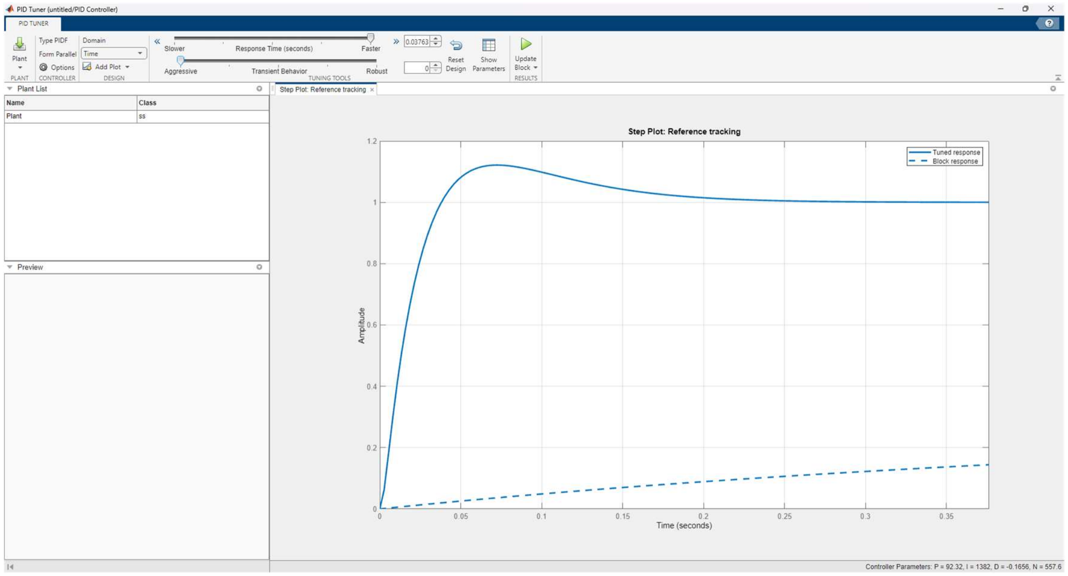


Figure: Inner Car Plant without external car plan (controller and feedback)

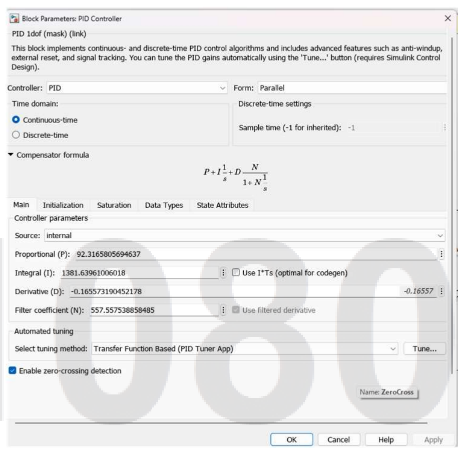
The output velocity graph is



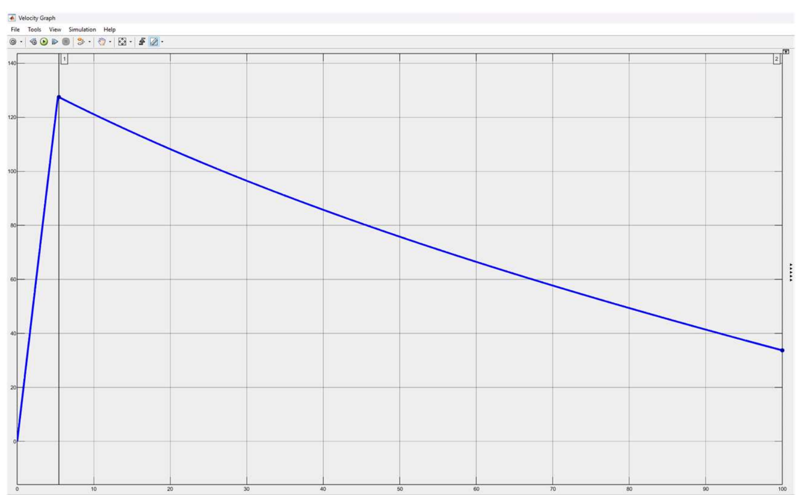
It took 10 sec to reach 55.66 km/h also it doesn't reached the target 65 km/h. For getting an optimal graph lets auto tune PID controller PID tuner app



After setting “Response time” to faster and “Transient behavior” to aggressive, following values of K_p, K_d and K_i are found.

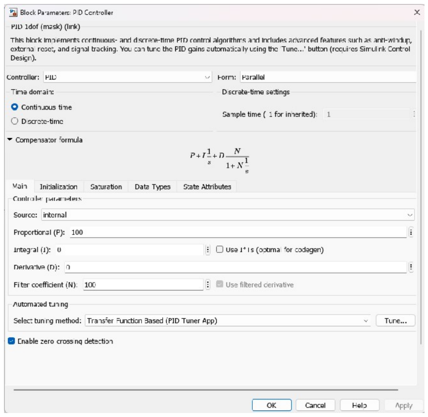


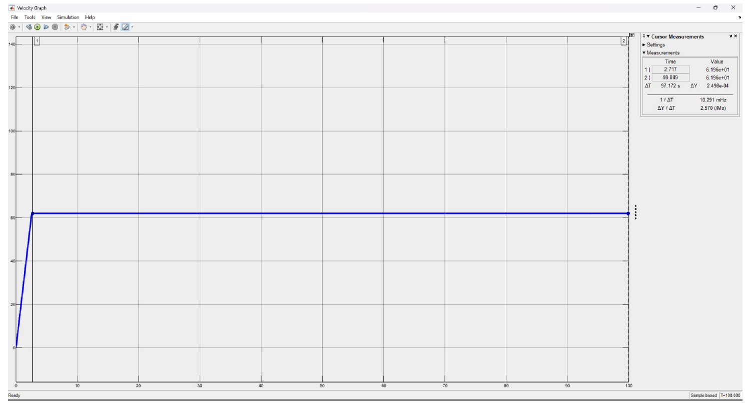
Now the output velocity graph



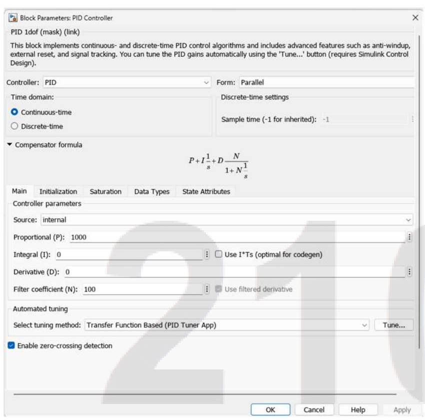
Here, it can be seen the velocity reached at a peak value of 127km/s in 5.453 seconds and the gradually decreased to 33.71 km/h after 100 seconds. Which means the auto configuration of PID of this system did not follow the requirement. Peak value of 127 Km/s in 5.453 seconds is impossible in real life also it has a fixed max value of 8600 rpm which will not allow the system to reach that peak value.

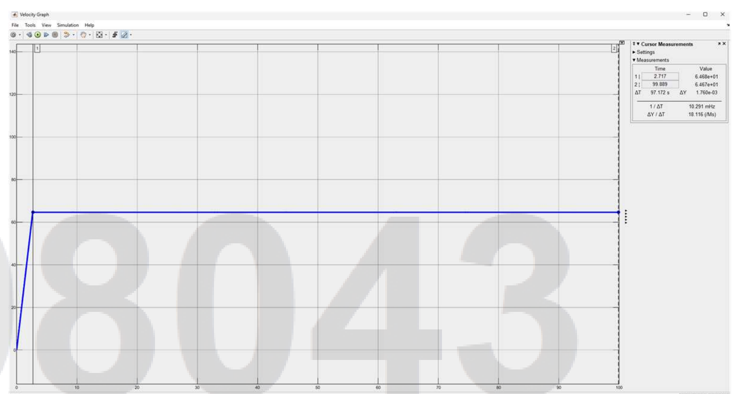
Lets set the PID manually.
After setting (K_p , K_i , K_d) = (100, 0,0) the following results are found, which ouputs 61.7 Km/s .



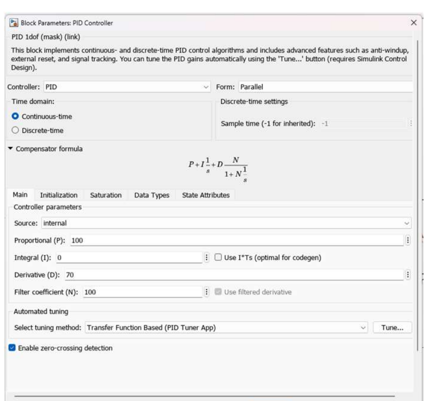


Now set (K_p , K_i , K_d) = (1000, 0,0) , which ouputs 64.7 Km/s which is almost equal to 65km/s , but the settling time is to low.

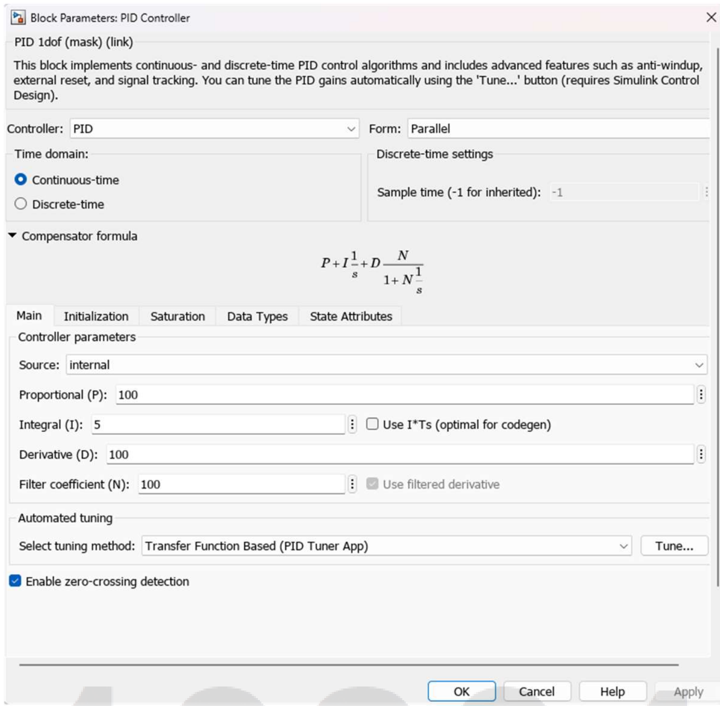




Now set (K_p , K_i , K_d) = (100, 0,70) , which ouputs 61.7 Km/s(resulting in Steady state error) and increases the settling time.



Now set (K_p , K_i , K_d) = (100, 5, 100) , which outputs 65.3 Km/s (desired output) with an optimal settling time and overshoot .



This is the final configuration for PID controller of the system

Discussion:

1. The Tesla Model S P85 was modeled in Simulink using a simplified version, where a brushed DC motor was implemented to simulate vehicle dynamics.

2. A PID controller was set up, which highlighted how small adjustments could significantly impact the vehicle's speed control. Observing these changes provided valuable insights into controller responsiveness.

3. Each model parameter's effect on performance was examined, and the variations in behavior were analyzed, deepening the understanding of how parameter tuning influences system output.

4. Although the model functioned effectively, it was noted that future iterations could benefit from implementing a more complex motor model and conducting finer tuning of the controller for improved accuracy. This modeling experience has strengthened the understanding of vehicle dynamics and control design.