

ADDIS ABABA UNIVERSITY
 MATHEMATICS DEPARTEMENT
 Introduction to Combinatorics and Graph Theory (Math2231)
 Comprehensive Assignment

1. How many 5-letter code words are possible from the first 6 letters of the alphabet if
 - a. No letter is repeated?
 - b. Letters are repeated?
 - c. Adjacent letters must be different?
2. How many 3-digit numbers can be formed with the digits 2, 4, 6, 8, 9
 - a. If each digit is used at most once?
 - b. How many of them these numbers are smaller than 500?
3. An examination consists of 13 questions. Find the number of different ways a student can select the questions to be answered if:
 - a. He is required to answer any 10 questions?
 - b. He must answer the 4th and the 9th questions and any 8 other from the remaining?
 - c. He must answer 5 of the first 7 questions and 5 of the last 6 questions?
4. Determine the number of positive integers n , where $1 \leq n \leq 3000$, that are
 - a. Divisible by 2 or 3.
 - b. Divisible by none of 2 or 3,
 - c. Divisible by none of 2, 3 or 7.
5. Show that for $n = 3, 4, 5, \dots$

$$\binom{n}{2} + \binom{n}{3} = \binom{n+1}{3}$$

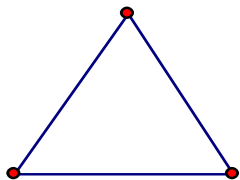
6. A bowl contains 6 red balls and 4 blue balls. In how many ways can 3 balls be selected so that both red and blue balls are obtained?
7. Find the coefficient of x^4 in the expansion of $(x - x^{-2})^{10}$.
8. Show the sequence $a_n = 3n$ are solution to the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$.
9. Find a_n for $n = 1, 2, 3, 4$. Then solve the recurrence relation: $a_n = 2a_{n-1} + a_{n-2} - a_{n-3}$, with $a_0 = a_1 = 1$ and $a_2 = 2$
10. Find a cubic polynomial whose roots are 5, -1 and 3. Then find a LHRRWCC with this characteristic polynomial.
11. Solve the following recurrence relations
 - a) $a_n = 4a_{n-1} - 4a_{n-2}$, $n \geq 2$, with $a_0 = 6, a_1 = 8$
 - b) $a_n - 2a_{n-1} + 2a_{n-2} - a_{n-3} = 0$, $n \geq 3$, given that $a_0 = a_1 = a_2 = 1$
 - c) $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$, $n \geq 3$, given that $a_0 = 7, a_1 = -4, a_2 = 8$.
 - d) $a_n = 3a_{n-1} - 2n^2 + 6n - 3$, with $a_0 = 1$.
12. Given that $a_0 = 0, a_1 = 1, a_2 = 4, a_3 = 12$ satisfy the recurrence relation $a_n + c_1a_{n-1} + c_2a_{n-2} = 0$. Determine a_n .
13. Solve the following linear inhomogeneous recurrence relations
 - a) $a_n = 3a_{n-1} - 2a_{n-2} + 2$, with $a_0 = a_1 = 1$.
 - b) $a_n = 3a_{n-1} - n^2 - 3$, with $a_0 = 1$.

Chinese Proverb: I HEAR, AND I FORGET I SEE, AND I REMEMBER I DO, AND I UNDERSTAND

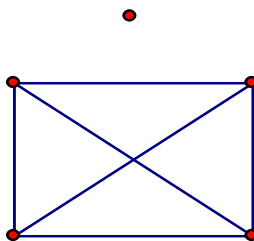
Mathematics is a “do” field. In order to learn it, you must “do” it. Good luck and don’t give up!

- c) $a_n = a_{n-1} - n$, with $a_1 = 2$.
 d) $a_n = 9a_{n-1} - 56n + 63$, with $a_0 = 2$.
 e) $a_n = 3a_{n-1} - 2n^2 + 6n - 3$, with $a_0 = 1$.
 f) $a_n = 2a_{n-1} + 3^{n-1}$, with $a_0 = 2$.
14. Find the general solution to the following recurrence relations:
 a) $a_n = 4a_{n-1} - 7a_{n-2} + (n+2)2^n$
 b) $a_n = a_{n-1} + 6a_{n-2} - (4+2n)3^n$.
15. Someone deposits Birr 10,000 in a savings account at a bank yielding 5% per year with interest compounded annually. Find a recurrence relation and how much money will be in the account after 30 years?
16. A complete graph is one in which there is an edge connecting every vertex to every other vertex. For what values of n does complete graph with n vertices have an Euler circuit?
17. Draw K_6 . How many edges does its complement have?
18. Draw a 3-regular graph having more than 4 vertices.
19. Find the chromatic number of each graph below.

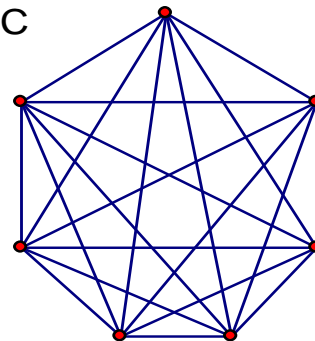
A



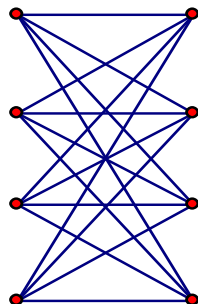
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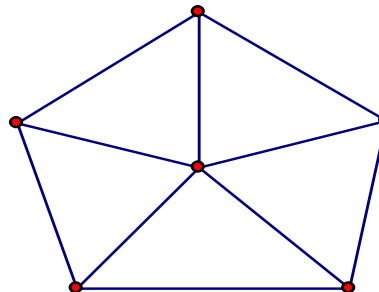
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D



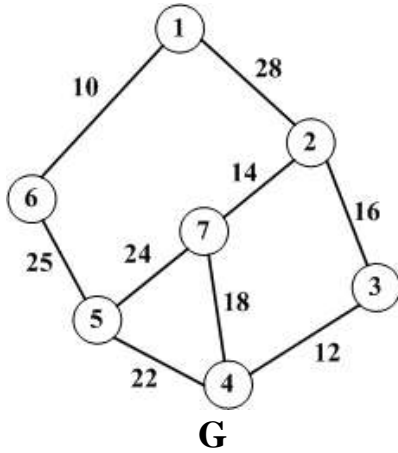
E



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20. Consider the following weighted graph G . Then Build (draw) a minimum spanning tree.
What is the minimum weight of the tree?



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