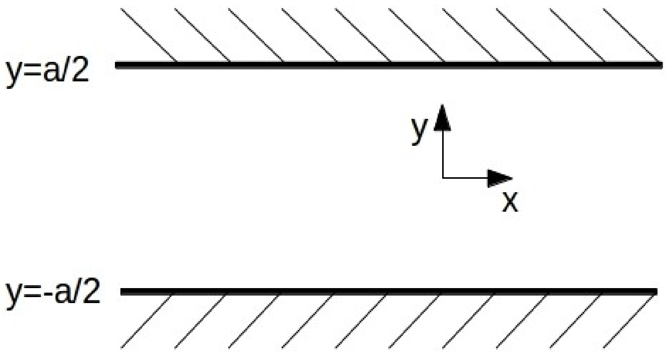
Master Physique Marine (M1)

HW1

Linear non dispersive waves

Homework n°1

We seek to compute the solution of a long gravity wave in an infinite channel as shown below.



1/ Recall the hypotheses of the problem and determine the wave equation satisfied by η

2/ Seek a solution of the form $\eta(x, y, t)=f(y)\exp[i(kx-\omega t)]$. (consider the no normal flow boundary condition at $y=\pm a/2$)

3/ Determine the dispersion relationship between ω and k . Find the associated wavelength for a semi-diurnal tide of period 12 hours. Are the hypotheses of the shallow water theory satisfied ?

4/ Show that there exists a cut-off frequency beyond which the propagation of long gravity waves in the channel is not possible. What is this cut-off frequency for the English channel (a=200 km, H=50 m). Conclude.

Homework n°2

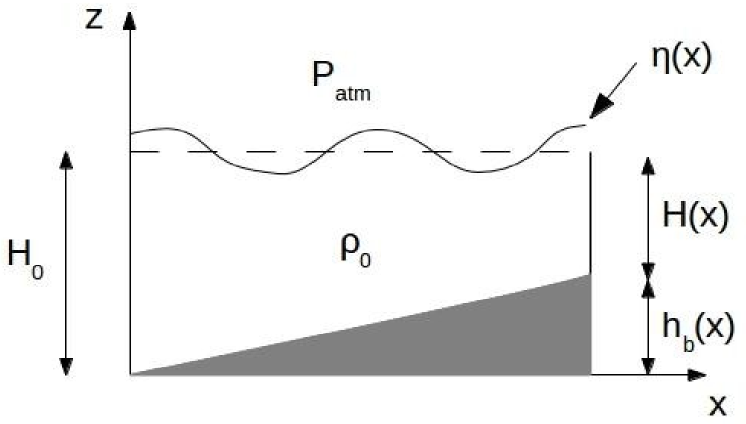
We consider the study of long gravity waves in an homogenous 1D estuary system (see figure below) whose depth and width depends only on x, namely H=H(x) and s=s(x), respectively. Bottom friction is assumed to be quadratic with velocity and inversely proportional to water depth ($-r u|u|/h$, where r is a dimensionless drag coefficient).

1/ Show that the momentum balance and mass conservation equations are given by

$$\frac{\partial F}{\partial t}+\frac{1}{s}\frac{\partial}{\partial x}\left[s\frac{F^2}{h}+\frac{1}{2}gh^2 s\right]=\frac{gh^2}{2s}\frac{\partial s}{\partial x}+gh\frac{\partial H}{\partial x}-r\frac{F|F|}{h^2}$$

$$\frac{\partial hs}{\partial t}+\frac{\partial uhs}{\partial x}=0$$

where $F=hu$ is the mass flux per unit length in the transverse direction, and $h=H+\eta$ is the total water depth.

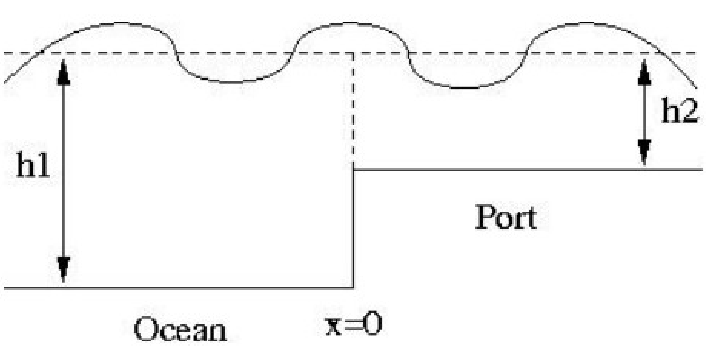


2/ Write the equation satisfied by salinity S, including the turbulent dissipation term. Can you provide a scale for the turbulent diffusivity K for a typical estuary system (hint: scale K in terms of a velocity scale U and a length scale L) ?

3/ Which boundary conditions would you use on both the mouth and source sides of the river for S, h and u ?

<u>Homework n°3</u>

To protect a harbour from the continuous inflow of sand and sediments from the open ocean, the idea is proposed to build a bank at the harbour entrance (represented in the following figure by an abrupt change in the depth of the sea). The objective of this exercise is to evaluate the impact of such a modification of the sea floor on the amplitude of long-gravity waves entering the harbour.



The vertical displacement η of the air-sea interface is a combination of an incident wave, a reflected wave, and a transmitted wave. In the following expressions we suppose that the wavenumbers k_1 and k_2 are different but that the frequency is conserved during transmission ($k_1 c_1 = k_2 c_2$).

$$\text{for } x<0: \eta = a \exp[ik_1(x-c_1 t)] + Ra \exp[-ik_1(x+c_1 t)]$$
$$\text{for } x>0: \eta = Ta \exp[ik_2(x-c_2 t)]$$

Use the continuity of pressure and mass flux at $x=0$ to determine both the reflexion R and transmission T coefficients as a function of h_1 and h_2 . Is the amplitude of the transmitted wave larger of smaller than that of the incident one ? Conclude.