Existence, propagation and collisions of localized dipolar vortices in the rotating shallow water model on the f-plane: numerical simulations with high-resolution finite-volume scheme

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Motivations and method

Recent result by Kizner *et al* (2008): existence of steady propagating modon solutions of full rotating shallow water equations on the f-plane; Method: Newton iterations starting from the known solution for geostrophic modon in quasi-geostrophic equations

Questions: *Is the solution stable? An attractor? (Robustness ?) What are its transport properties? Interactions?*

Method: DNS with high-resolution well-balanced entropy-satisfying finite-volume scheme by Bouchut [2] with following properties:

- Non-dissipative, unless shock formation; dissipation concentrates only in high-gradient zones
- preserves PV (unless dissipation)
- b.c.: periodic, free-slip, sponges.

Geostrophic vs Ageostrophic modon

Geostrophic Modon: steady translation; cyclone - anticyclone symmetry. Solution of QG equation. Not a solution of full RSW equations

Streamfunction of the geostrophic modon:

$$\Psi = U_0H_0L\left[\frac{J_1(\Pi r/L)}{J_1(\Pi)} - r\right]\left(\frac{\Lambda}{\Pi}\right)^2 \sin\theta \qquad r < L$$

$$= U_0H_0L\left[r - \frac{K_1(\Lambda r/L)}{K_1(\Lambda)}\right]\sin\theta \qquad r \geq L$$

$$\Pi = \sqrt{A^2 - \Lambda^2};\ \left(\frac{\mathrm{d}Q}{\mathrm{d}\Psi}\right) \simeq -A^2$$

$$\Lambda = \frac{L}{L_R};\ L_R = \sqrt{g'H_0}/f$$

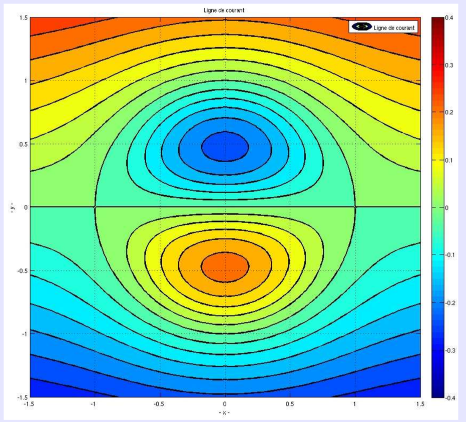

Fig. 1. Streamfunction of the geostrophic modon with $H_0 = 1;\ L = 1;\ U_0 = 0,2$

Ageostrophic Modon by Kizner *et al.* [1].

Steady solution of the full RSW equations:

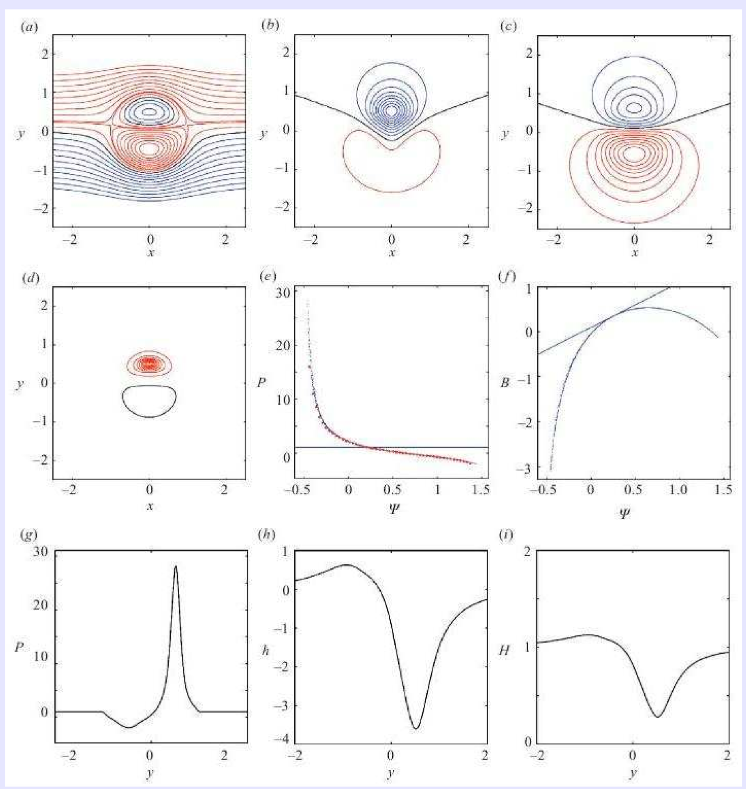

Fig. 2. $\Lambda = 1; R_0 = 0,2$*; (a) Co-moving streamfunction; (b) Layer thickness anomaly; (d) PV; (g) cross-section of PV; (h) cross-section of the thickness anomaly; (i) cross section of thickness*

Existence and attracting character of ageostrophic modon solutions: proof by DNS

Initial value problem with geostrophic modon profile for h and v (Resolution: $\mathrm{d}x = \mathrm{d}y = 0.025L_R$). In a couple of inertial periods, the flow adjusts to the ageostrophic configuration, with a structure similar to Fig. 2, with weak energy loss due to the IGW absorption at the boundary, and a important reorganization of the modon.

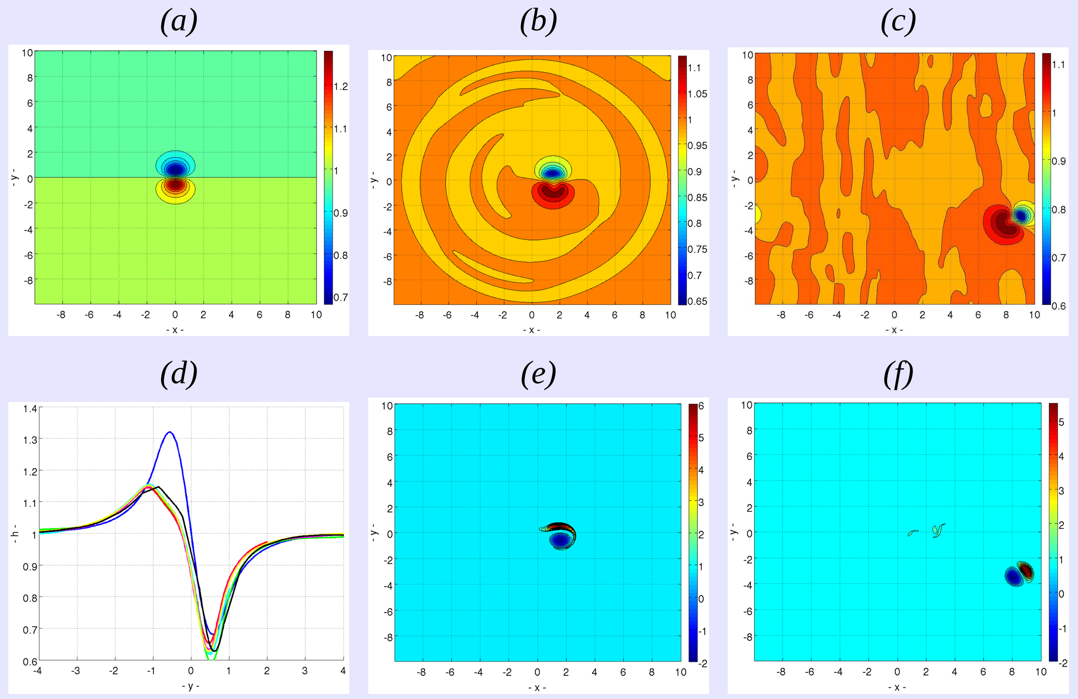

Fig. 3 (a) Thickness h at t=0/f; (b) h(t=9/f); (c) h(t=60/f); (d) Cross-sections through the center of the dipole at t=[0,30,60, 90,114,150,170]/f; (e) PV at t=9/f; (f) PV at t=60/f

Attracting property: a wide range of dipolar initial configurations end up with the same final dipole (not presented).

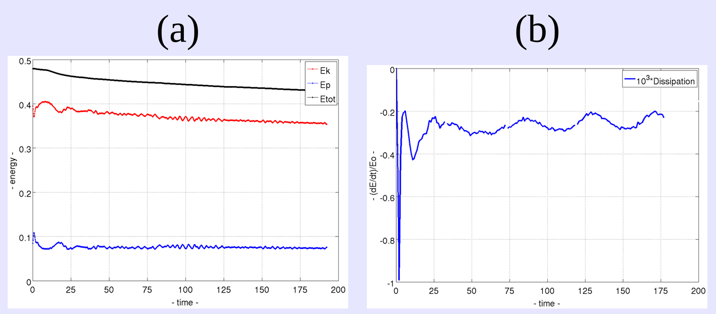

Fig. 4. (a) Energy evolution during the adjustment: Ek, Ep, Etot; (b) Dissipation rate as a function of time

Properties of the ageostrophic modon

Ageostrophic effects: important, destroy the symmetry between cyclone and anticyclone.

- The propagation speed is lower than in the geostrophic case. Probable reason: dissipation induced by strong gradients.
- The modon trajectory is curved (Fig. 3). Stokes theorem ⇒ cyclone propagation speed is proportional to the anticyclone circulation ⇒ Difference in circulations (ageostrophy) curves the trajectory towards the stronger circulation region. Dissipation is more important in the cyclonic region due to higher gradients, and therefore the curvature tends to increase in time.

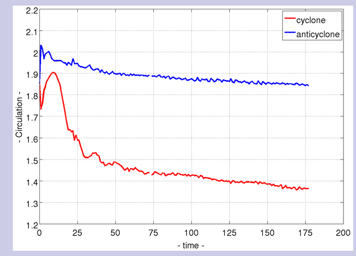

Fig. 5. Evolution of cyclone and anticyclone circulation

Anomalous (ballistic) transport

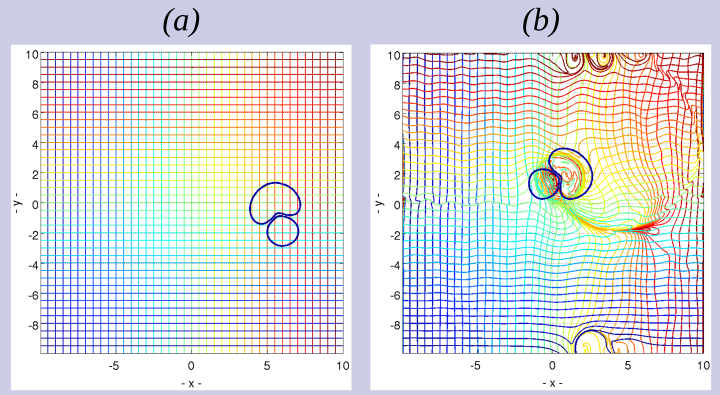

Fig. 6. Contours of a pair of passive tracer fields with superimposed boundary of an ageostrophic modon with $\Lambda = 1;\ R_0 = 0.2$ (blue) at (a) t=0/f, tracer contours vertical and horizontal, resp.; (b) t=40/f, tracer field enrolled and captured by the modon

Collision between a modon and a wall

B.C.: free-slip at the wall.

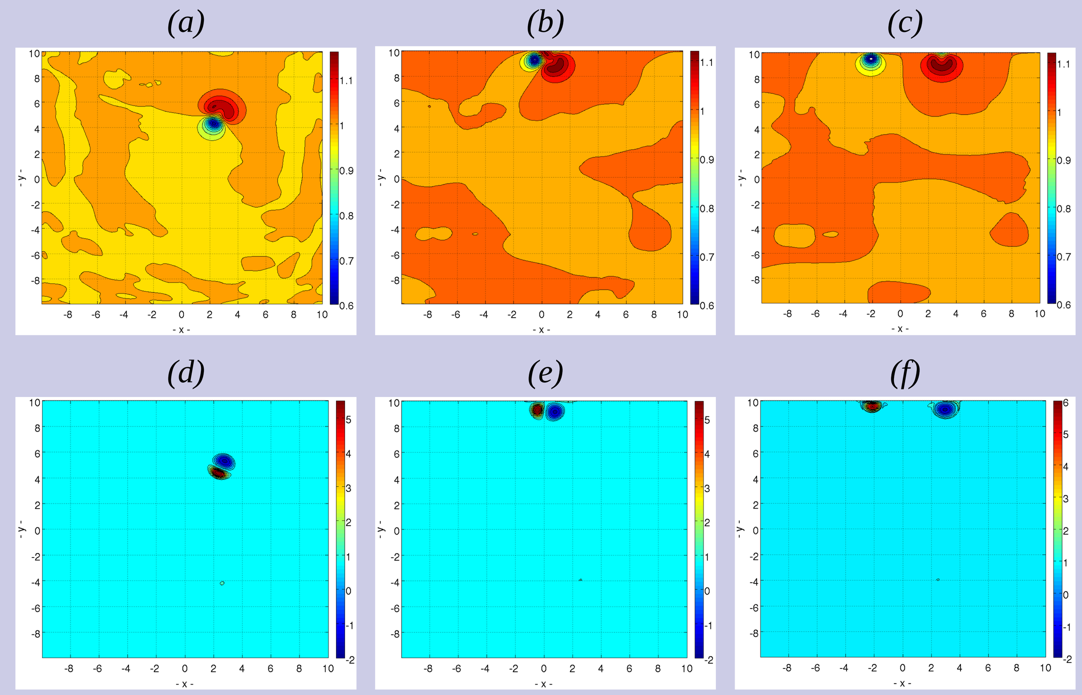

Fig. 7. Upper panel: thickness h at (a) t=20/f; (b) t=54/f; (c) t=70/f; Lower panel: PV at (d) t=20/f; (e) t=54/f; (f) t=70/f

Modon strikes the wall and is destroyed, each component aggregated with a corresponding mirror vortex and moving along the wall in opposite directions: similar to the DNS with quasigeostrophic model [3].

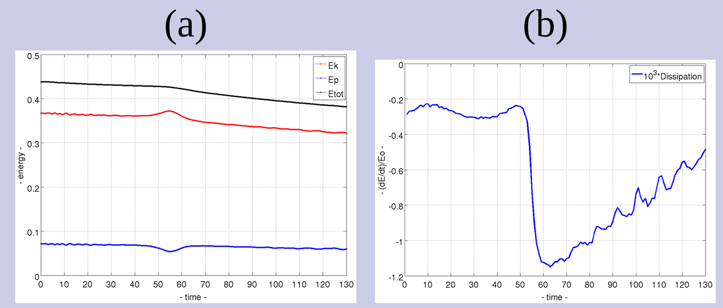

Fig. 8. (a) Energy evolution during the collision: Ek, Ep, Etot; (b) Dissipation rate as a function of time

Frontal collision between two modons

By adjusting initial conditions we managed to organize a **frontal** collision between two ageostrophic modons. They undergo **elastic (!)** scattering with **exchange of cyclonic partners.**

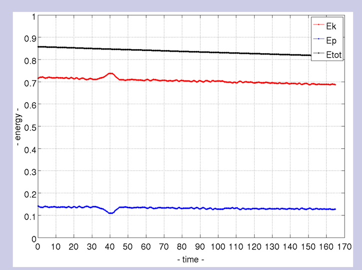

Fig. 9. Energy evolution during the collision: Ek, Ep, Etot

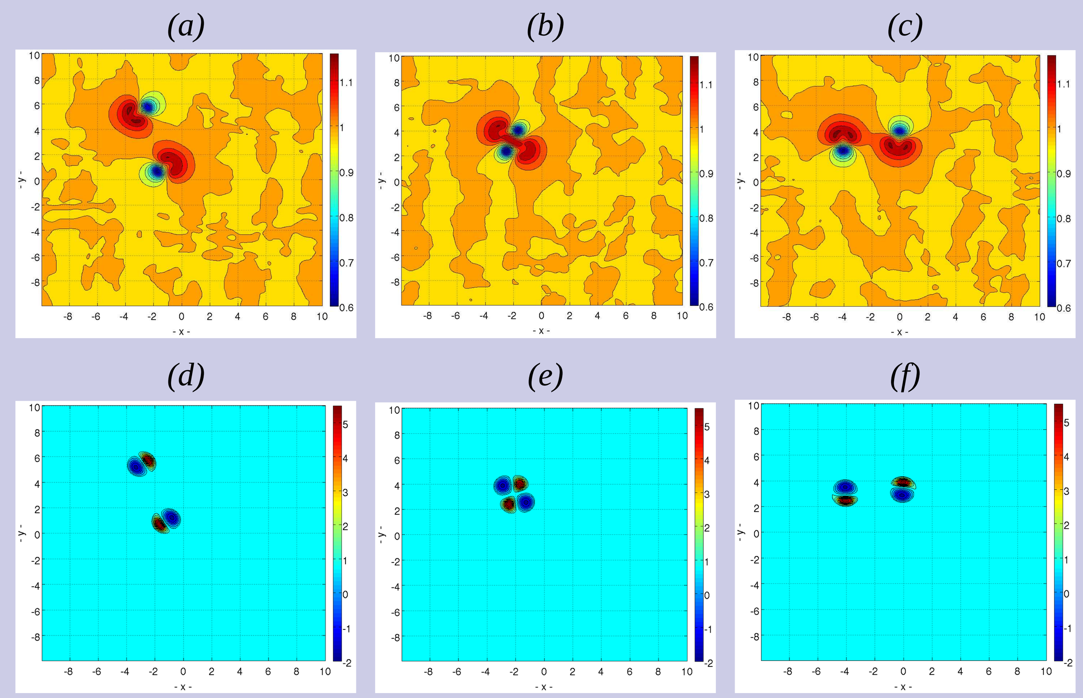

Fig. 10. Upper panel: thickness h at (a) t=28/f; (b) t=39/f; (c) t=50/f; Lower panel: PV at (d) t=28/f; (e) t=39/f; (f) t=50/f

Conclusions and discussion

Predictions of **[1] confirmed** in what concerns the structure of ageostrophic modons, but **trajectories are essentially curved**. Modons are presumably **attracting**. Capture and **transport mass**. Robust: scattering is **elastic** in spite of **exchange of partners**.

Curvature of the trajectory: solution of [1] not exact, or not stable with respect to trajectory bending, or (small) dissipation in the numerical model is responsible?

References

[1] **Kizner Z., Reznik G. , Fridman B., Khvoles R. and McWilliams J.** *Shallow-water modons on the f-plane*, J. Fluid Mech., v. 603, p. 305-329 (2008). [3] **Bouchut F.** *Efficient numerical finite-volume schemes for shallow-water models*, in *Nonlinear dynamics of Rotating Shallow Water: Methods and Advances*, V. Zeitlin, ed. Elsevier, p.189 - 264, (2007). [3] **Carnevale G. F., Velasco O. U. and Orlandi P.** *Invicid dipole-vortex rebound from a wall or a coast*, J. Fluid Mech., v. 351, p. 75-103 (1997).