INTERNAL WAVES

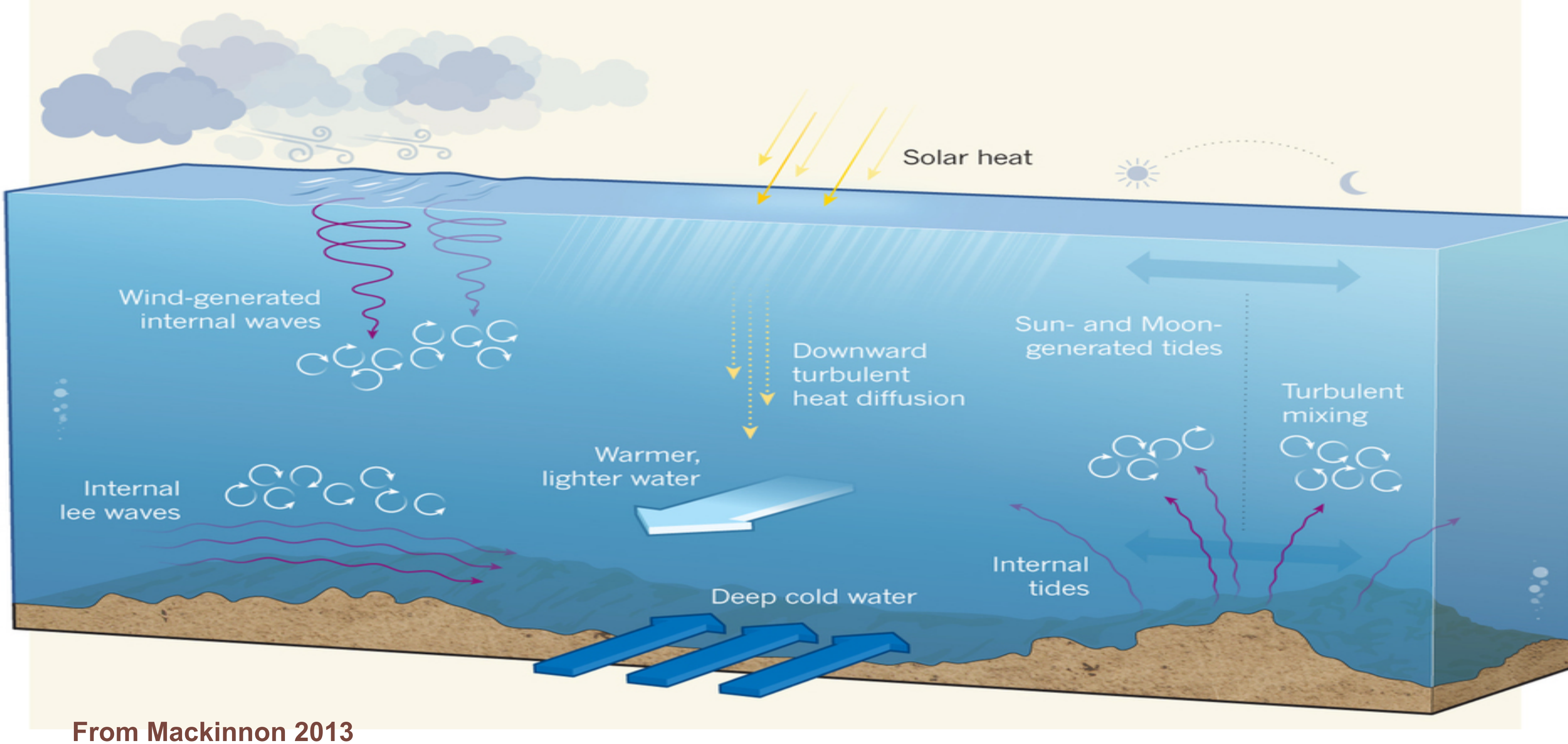
4. PROPAGATION AND DISSIPATION

- **1.4 : Propagation and dissipation of internal waves**

- *Waves in the Ocean*, LeBlond & Misak
- *The excitation, dissipation, and interaction of internal waves in the deep ocean,* Thorpe, 1975
- *Internal waves in the ocean*, Garrett & Munk, 1979
- *Internal Waves and Small-Scale Processes,* Munk, 1981

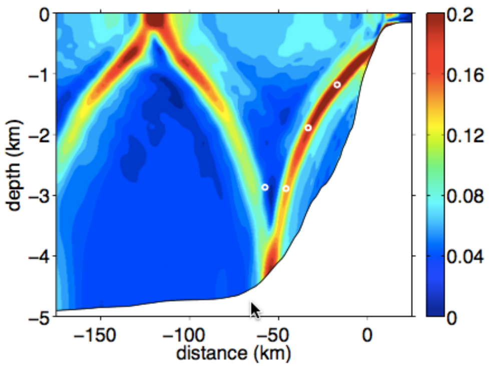
Internal wave generation

- Mechanisms:



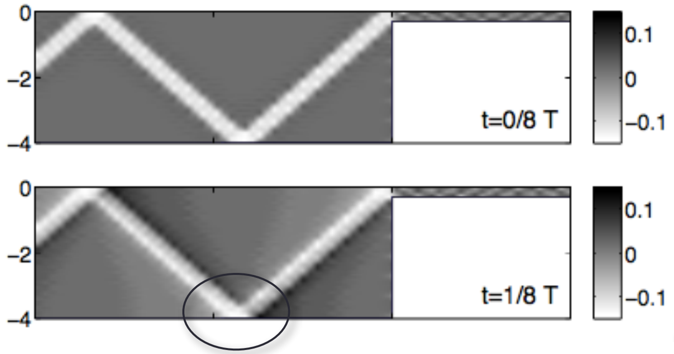
From Mackinnon 2013

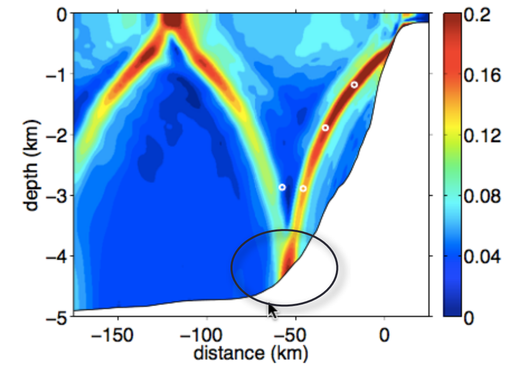
1.4. Propagation and dissipation of waves



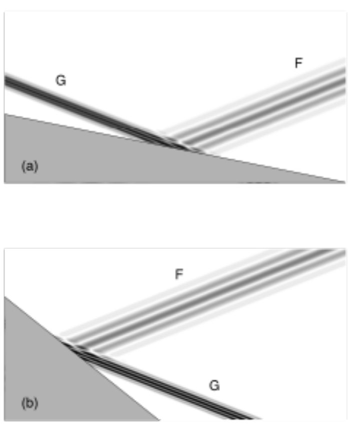
What happens after internal wave generation?

1.4. Propagation and dissipation of waves





What happens after internal wave generation?



A. Wave reflection at the surface and bottom.

In reflection from a rigid boundary, frequency is conserved, and the waves retain their inclination to the horizontal

$$\tan\theta = \pm\left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2}$$

(See LeBlond & Mysak, p 54)

1.4. Propagation and dissipation of waves

- Review of mechanisms *(see Thorpe75.pdf):*

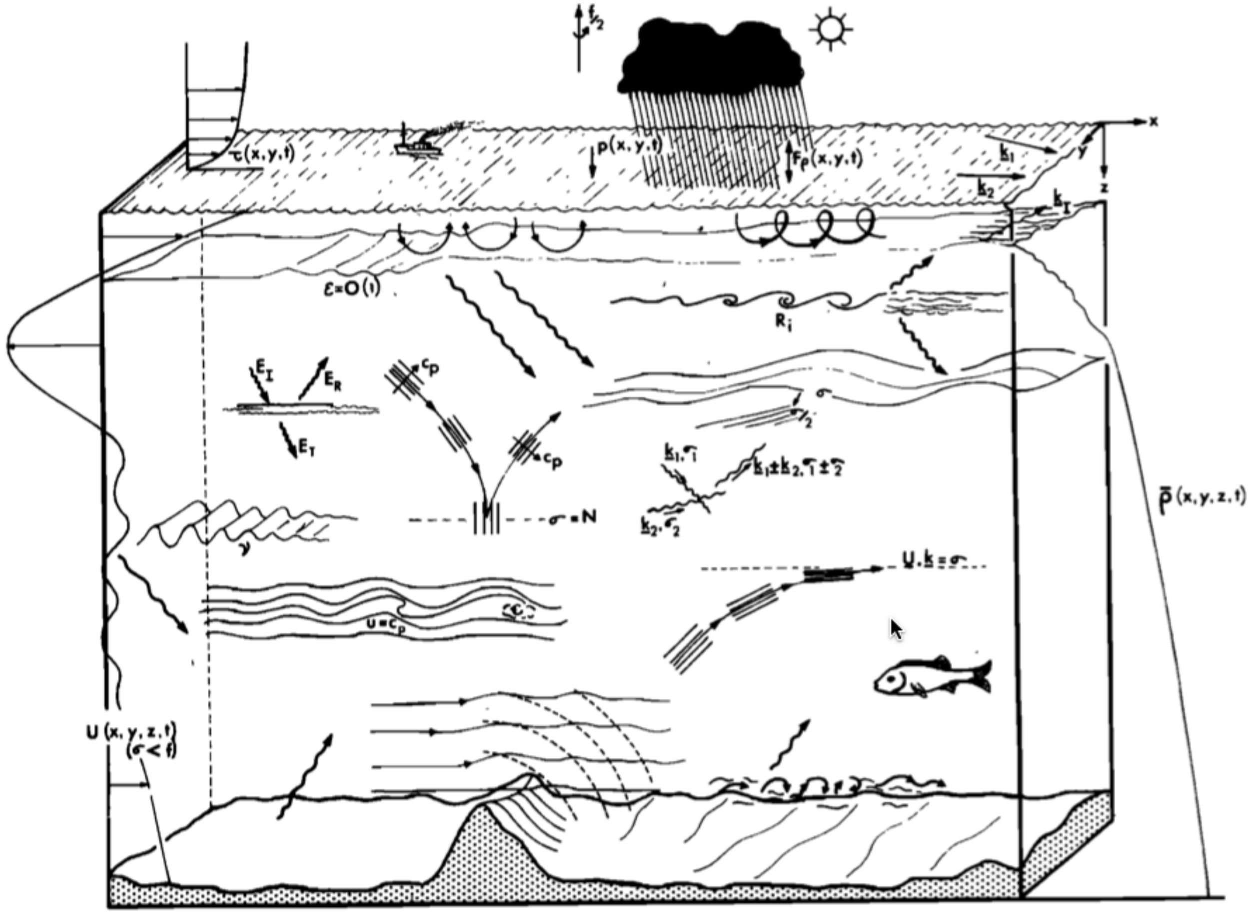


Fig. 5. Physical processes affecting internal waves.

1.4. Propagation and dissipation of waves

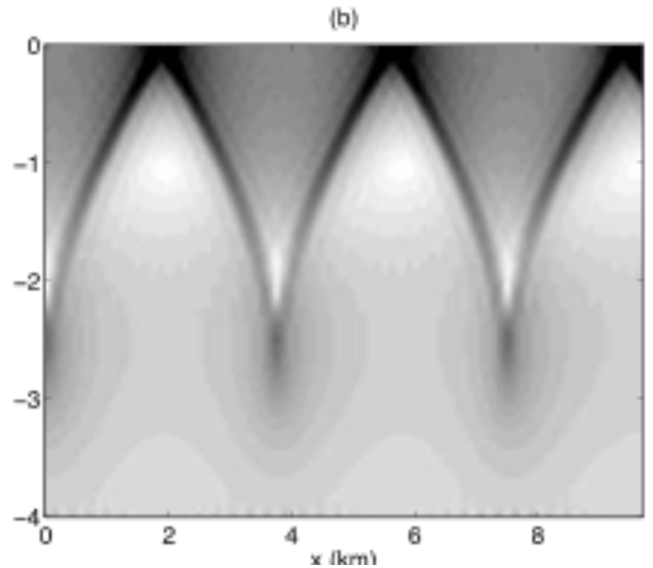
B. Wave reflection where : $f < N < \omega$

Remember the dispersion relation

$$\boxed{\omega^2 = \frac{N^2k^2 + f^2m^2}{k^2 + m^2}}$$

Such that solutions exist only in a range of allowable internal-wave frequencies:

$$(\mathrm{I}) \quad N \leq \omega \leq |f| \quad \text{or} \quad (\mathrm{II}) \quad |f| \leq \omega \leq N$$



If the waves propagate into a region where N falls to a value less than that of the wave frequency, they will be reflected, their group velocity becoming vertical near this surface.

They may be trapped into a waveguide if this happens below and above.

1.4. Propagation and dissipation of waves

C. Wave reflection/refraction by a density jump

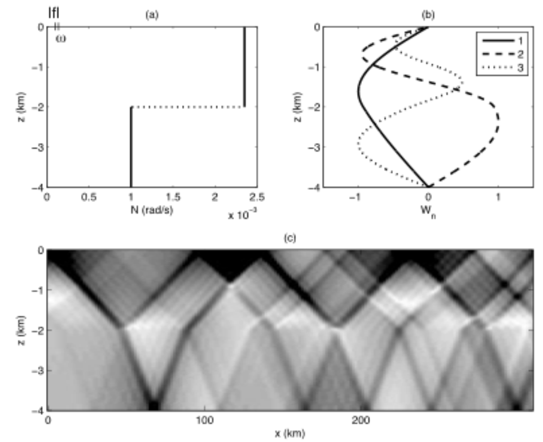
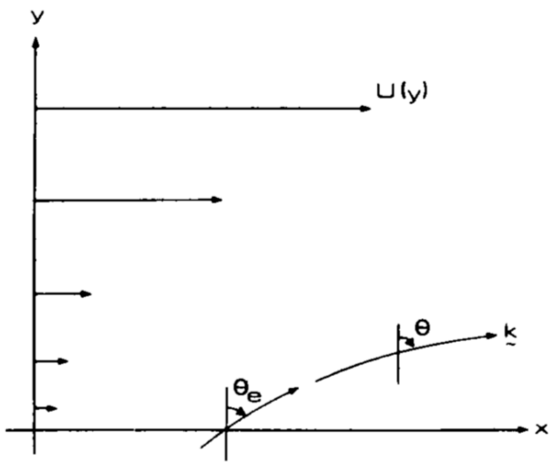


Fig. 5.7: Stratification with two layers of constant N, with $|f| < \omega < N_2 < N_1$ (**a**). Panel **b** shows the first three eigenmodes (5.30), with $C_{1,n}$ chosen such that their amplitudes are one. Modal coefficients are $a_n = 1/n$. The resulting superposition of 20 modes, representing the amplitude of u, is shown in **c**. White denotes zero; black, maximum values.

or by a horizontal shear of mean velocity:



1.4. Propagation and dissipation of waves

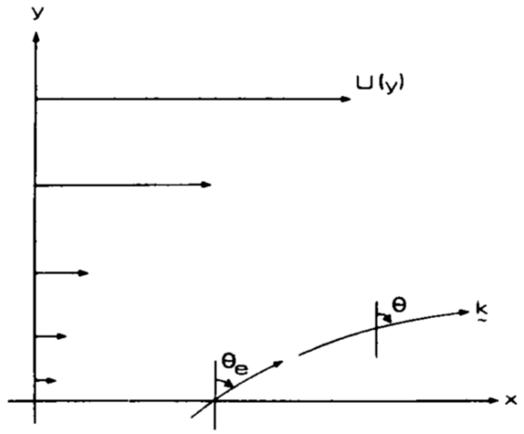
Waves in a moving medium:

- Frequency
  (= frequency observed by a stationary observer)

$$\omega = \vec{k} \cdot (\vec{c_i} + \vec{U})$$

- Intrinsic *[doppler-shifted]* frequency
  (= frequency measured in a frame of reference moving with the fluid)

$$\omega_i = \omega - \vec{k} \cdot \vec{U}$$



1.4. Propagation and dissipation of waves

- Activity:

We consider a long surface gravity wave ($c_i = \sqrt{gH}$, $\vec{k} = [k_x, k_y, 0]$)

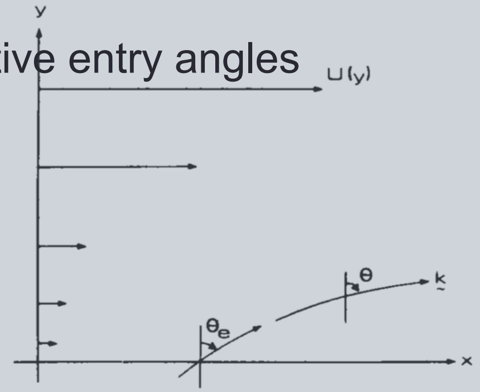
in a domain with constant depth, in a horizontal flow $\vec{U} = [U(y), 0, 0]$.

At the entry point the velocity is $U_e = 0$

1. Write the expression of θ as a function of $U(y), c_i$ and θ_e

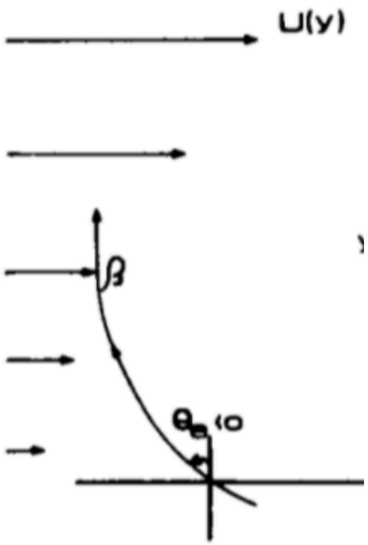
2. Draw (qualitatively) the wave paths for positive and negative entry angles

3. Can the wave be reflected?



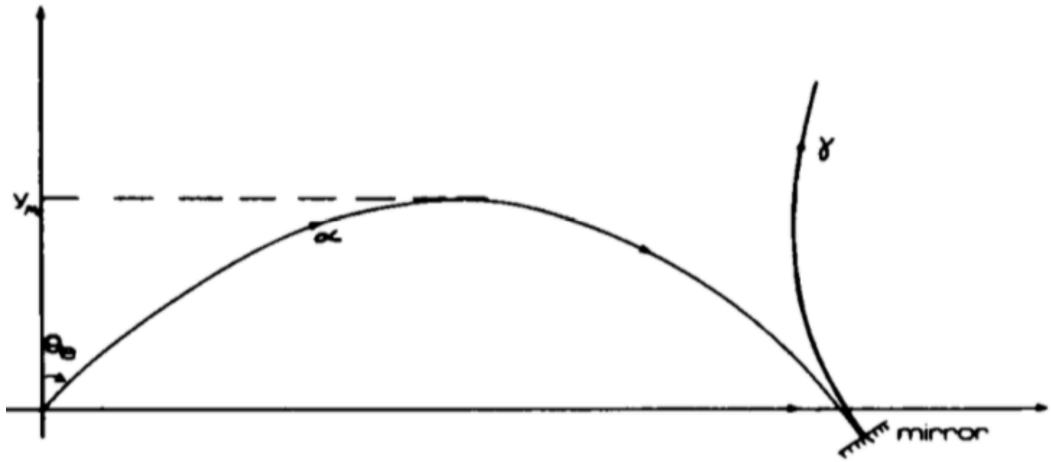
1.4. Propagation and dissipation of waves

- Negative angle: $\theta_e < 0$



The ray is bent towards the normal to the current

- Internal reflection:



1.4. Propagation and dissipation of waves

Waves in a moving medium:

Wave action: $A = \dfrac{E}{\omega_i}$ with intrinsic frequency $\omega_i = \omega - \vec{k} \cdot \vec{U}$

(= frequency measured in a frame of reference moving with the fluid)

And wave energy: $E = \dfrac{1}{2}\rho|u|^2 + \dfrac{1}{2}\rho N^2 \xi^2$

Perturbation velocity u , and vertical particle displacement ξ

Equation for wave action:

With S sources and sinks of wave action due to resonant interactions, wave generation, dissipation, etc.

$$\frac{\partial A}{\partial t} + A\vec{\nabla} \cdot \vec{c_g} + \vec{c_g} \cdot \nabla A = S$$

For a wide class of conservative systems in fluid dynamics changes in wave amplitude along the rays may be computed from conservation of wave action.

1.4. Propagation and dissipation of waves

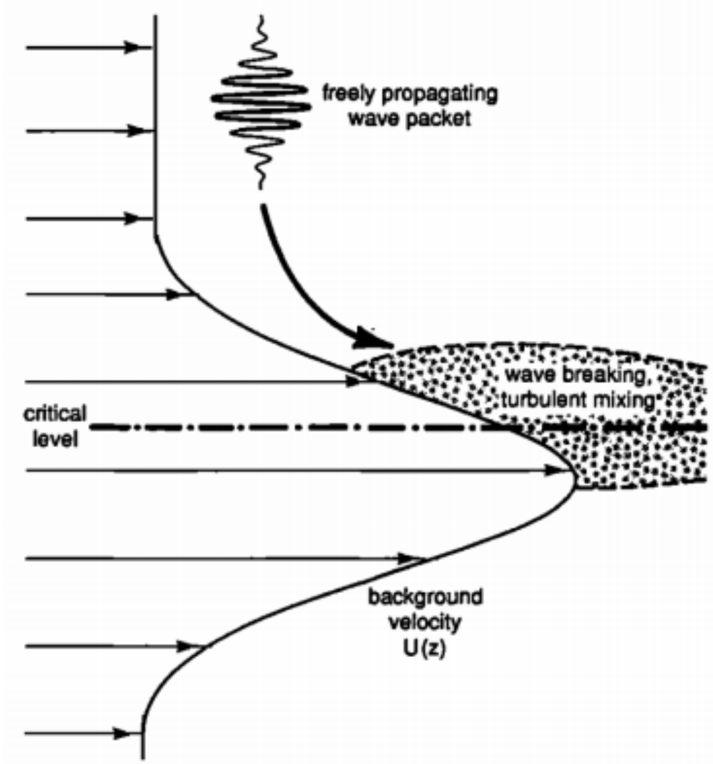
D. Critical layer absorption

A critical layer is a region where the velocity of the flow is equal to the phase speed of the waves **[Bretherton & Garrett, 1968]**

$$\omega = \vec{k} \cdot \vec{U}$$

At such level, energy is transferred to the mean flow and turbulence is generated.

(See LeBlond & Mysak, p 387)



[Winters & D'Asaro, 1989, 1995]

1.4. Propagation and dissipation of waves

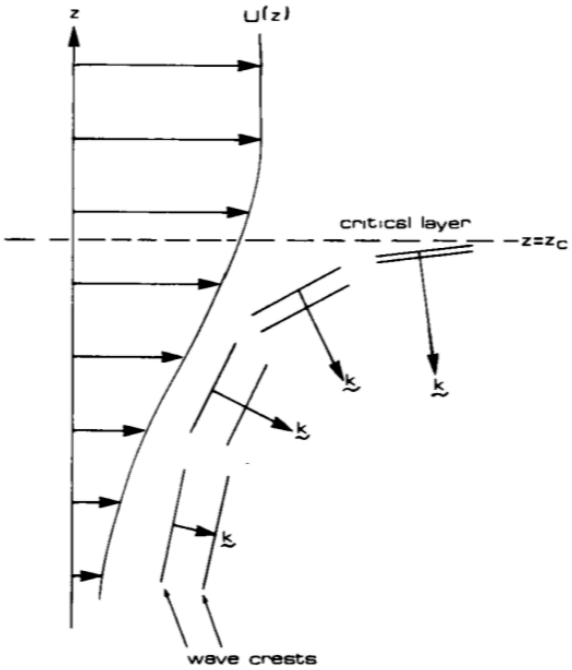
D. Critical layer absorption

- Activity:

We consider an internal wave propagating upward in a vertically sheared flow $\vec{U} = [U(z), 0, 0]$
(with no rotation for simplicity)

1. Write the vertical component of the group speed

2. Explain what happens when the wave reaches a critical layer



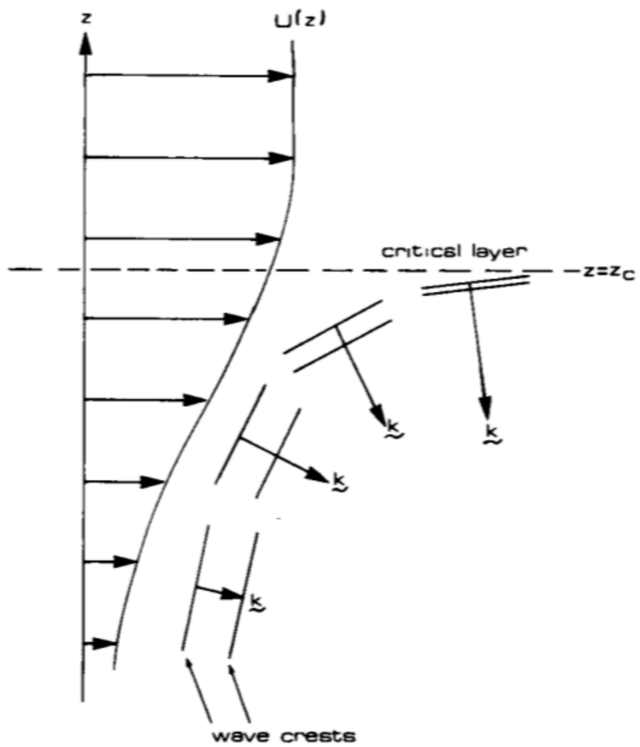
[LeBlond & Misak]

1.4. Propagation and dissipation of waves

D. Critical layer absorption

- For small wave in a presence of a vertical shear: If the background flow increases as a small wave approaches, it is stretched and rotated due to the shear until it's group velocity is nearly horizontal, wavelength increases infinitely, lending the wave's energy to the background.

- For large amplitude waves, the steepness of the wave causes breaking before absorption occurs.

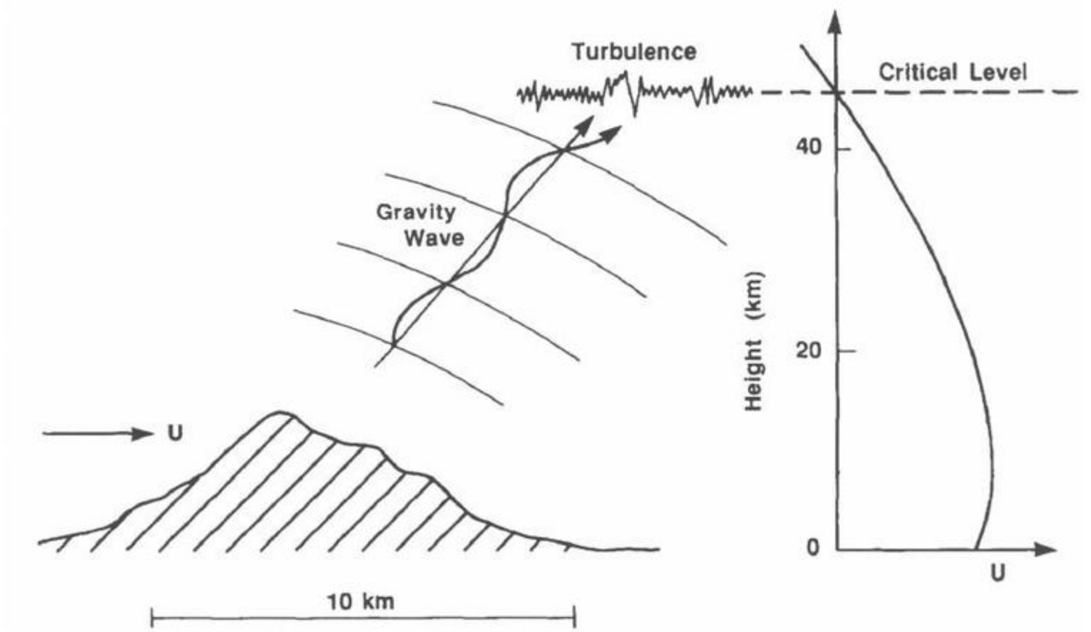


[LeBlond & Misak]

1.4. Propagation and dissipation of waves

D. Critical layer absorption

- Short internal Lee waves have a high probability of encountering critical layers and be absorbed in the lower 1 km above ocean bottom:



1.4. Propagation and dissipation of waves

E. Resonant interactions

A resonant second-order interaction between three internal waves may occur if :

$$\vec{k_1} \pm \vec{k_1} \pm \vec{k_3} = 0$$

$$\omega_1 \pm \omega_2 \pm \omega_3 = 0$$

Under this special circumstance, non-linear terms efficiently transfer energy from one scale to another.

(See LeBlond & Mysak, p 333)

1.4. Propagation and dissipation of waves

E. Resonant interactions

Theory of weak wave-wave interactions

Let's consider a general equation: $$L(u) + M(u, u) = 0$$

With a linear (L) and a quadratic (M) operator. We assume that u can be expanded as a power series of the form:

$$u = \epsilon u_1 + \epsilon^2 u_2 + \dots$$

Where ϵ characterizes the relative size of the nonlinear term.

1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Theory of weak wave-wave interactions</u>

At order ϵ, the equation is $L(u_1) = 0$

We can write it as a plane wave solution

$$u_1 = \mathrm{Re}[\mathrm{A}\exp\{\mathrm{i}(\mathrm{k}\,\mathrm{x} - \omega \mathrm{t})\}]$$

Which satisfies the relation dispersion: $\Delta(\omega, k)$

At order ϵ^2, the equation is $L(u_2) = -M(u_1, u_1)$
and gives the correction to the linear solution.

1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Theory of weak wave-wave interactions</u>

Suppose u_1 is the sum of two plane waves $u_1 = u_a + u_b$

(with in general different wavenumbers and frequencies)

Then a new wave u_c can be generated as the result of the interaction between them following:

$$L(u_c) = -M(u_a + u_b, u_a + u_b)$$

With frequency and wavenumbers equal to the forcing terms, which are all combinations of u_a, u_b and their derivatives, which include terms with frequencies:

$$0, \pm 2\omega_a, \pm 2\omega_b, \pm(\omega_a \pm 2\omega_b)$$

1.4. Propagation and dissipation of waves

E. Resonant interactions

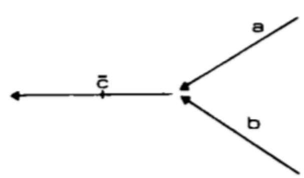
<u>Theory of weak wave-wave interactions</u>

Possible interactions are summarized by:

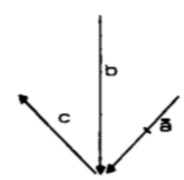
$$\omega_a \pm \omega_b \pm \omega_c = 0$$

$$\vec{k}_a \pm \vec{k}_b \pm \vec{k}_c = 0$$

Feynman diagrams for triplets:



a) $ab \rightarrow \bar{c}$

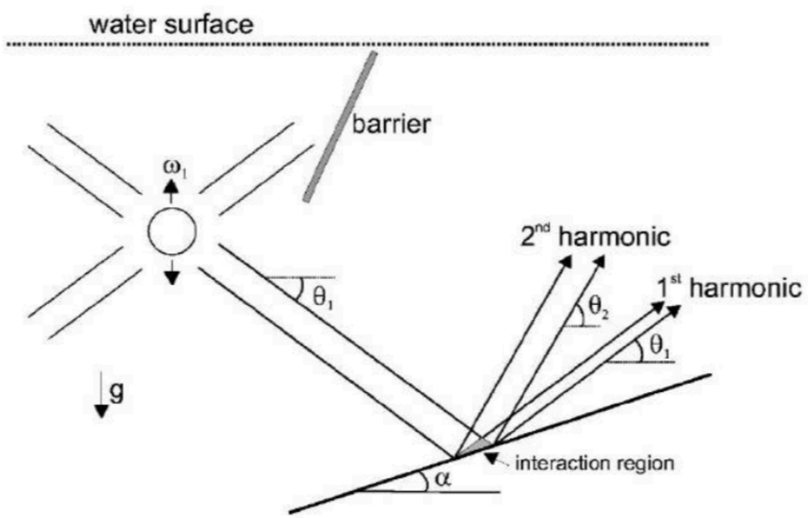


b) $\bar{a}b \rightarrow c$

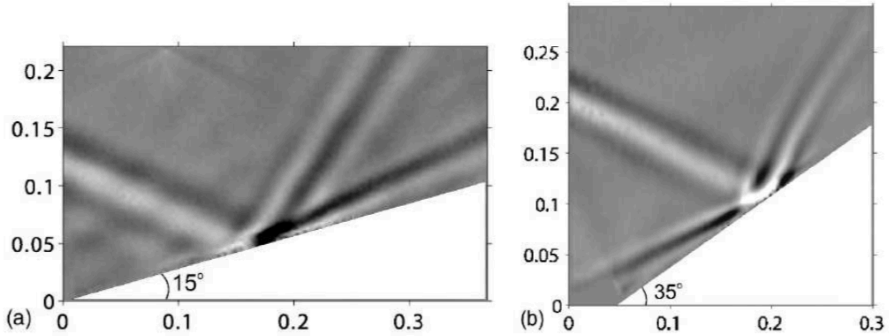
1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Generation of higher harmonics at boundaries (or density gradients)</u>: *(see Gerkema, p 190)*



Results from laboratory experiments:
[Peacock and Tabei, 2005]



1.4. Propagation and dissipation of waves

E. Resonant interactions

Generation of higher harmonics at boundaries (or density gradients): *(see Gerkema, p 190)*

Equations with non-linearities:

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho_*}\frac{\partial p'}{\partial x}$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho_*}\frac{\partial p'}{\partial y}$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho_*}\frac{\partial p'}{\partial z} + b$$

$$u_x + v_y + w_z = 0$$

$$\frac{Db}{Dt} + N^2 w = 0\,.$$

If we assume uniformity in the y-direction, i.e. $\partial/\partial y = 0$, we can introduce a streamfunction, defined via $u = \psi_z$ and $w = -\psi_x$. Eq. 9.4d) is then automatically satisfied. Furthermore, we can combine (9.4a) and (9.4c) to obtain

$$\begin{aligned} \nabla^2\psi_t + J(\nabla^2\psi, \psi) - fv_z + b_x &= 0\,, \\ v_t + J(v, \psi) + f\psi_z &= 0 \\ b_t + J(b, \psi) - N^2\psi_x &= 0\,. \end{aligned}$$

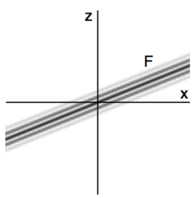
1.4. Propagation and dissipation of waves

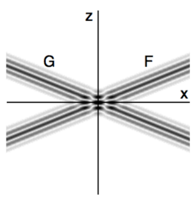
E. Resonant interactions

<u>Generation of higher harmonics at boundaries (or density gradients)</u>: *(see Gerkema, p 190)*

The general form of the solution of the linear equations (with constant N) can be written as an internal wave beam:

$$w = F(\xi_+) + G(\xi_-)\,,$$





$$\begin{aligned} u &= \mu_+^{-1} F(\xi_+) + \mu_-^{-1} G(\xi_-) \\ v &= -i\,\frac{f}{\omega}\left[\mu_+^{-1} F(\xi_+) + \mu_-^{-1} G(\xi_-)\right] \\ b &= -i\,\frac{N^2}{\omega}\left[F(\xi_+) + G(\xi_-)\right]. \end{aligned}$$

$$\mu_\pm = \pm\left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2}$$

$$\xi_\pm = \mu_\pm x - z\,,$$

- Activity:

  1. Check if $w = F(\xi)$ is also solution of the non-linear equations

1.4. Propagation and dissipation of waves

E. Resonant interactions

Generation of higher harmonics at the bottom: *(see Gerkema, p 190)*

$$
\begin{aligned}
J(\nabla^2\psi,\psi) &= -u\nabla^2 w + w\nabla^2 u = -\mu^{-1}F(\mu^2+1)F'' + F\mu^{-1}(\mu^2+1)F'' = 0 \\
J(v,\psi) &= uv_x + wv_z = -i\,\frac{f}{\omega}\mu^{-1}FF' + i\,\frac{f}{\omega}\mu^{-1}FF' = 0 \\
J(b,\psi) &= ub_x + wb_z = -i\,\frac{N^2}{\omega}FF' + i\,\frac{N^2}{\omega}FF' = 0\,.
\end{aligned}
$$

Hence a beam taken in isolation F (with G=0) also fulfills the non-linear equation. The same holds for G (if F=0).

However, wherever F and G occur together (reflection at the bottom), nonlinear effects will, in general, set in.

1.4. Propagation and dissipation of waves

E. Resonant interactions

Generation of higher harmonics at the bottom: *(see Gerkema, p 190)*

Non-linear equations:

$$\begin{aligned} \nabla^2\psi_t + J(\nabla^2\psi,\psi) - f v_z + b_x &= 0 \\ v_t + J(v,\psi) + f\psi_z &= 0 \\ b_t + J(b,\psi) - N^2\psi_x &= 0\,, \end{aligned}$$

We consider weakly nonlinear waves, monochromatic (with frequency ω) at lowest order, and write the fields in a formal expansion in which ε, a measure of the intensity of the wave, serves as the small parameter:

$$\begin{aligned} \psi &= \epsilon\{\Psi\exp(-i\omega t) + \text{c.c.}\} + \epsilon^2\{\Psi_0 + [\Psi_2\exp(-2i\omega t) + \text{c.c.}]\} + \cdots \\ v &= \epsilon\{V\exp(-i\omega t) + \text{c.c.}\} + \epsilon^2\{V_0 + [V_2\exp(-2i\omega t) + \text{c.c.}]\} + \cdots \\ b &= \epsilon\{\Gamma\exp(-i\omega t) + \text{c.c.}\} + \epsilon^2\{\Gamma_0 + [\Gamma_2\exp(-2i\omega t) + \text{c.c.}]\} + \cdots \end{aligned}$$

1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Generation of higher harmonics at the bottom</u>: *(see Gerkema, p 190)*

Lower order:

$$\begin{aligned} -i\omega\nabla^2\Psi - fV_z + \Gamma_x &= 0 \\ -i\omega V + f\Psi_z &= 0 \\ -i\omega\Gamma - N^2\Psi_x &= 0\,, \end{aligned}$$

Which gives the well-known equation and solution:

$$\begin{aligned} (N^2-\omega^2)\Psi_{xx} - (\omega^2 - f^2)\Psi_{zz} &= 0\,, \\ \Psi &= F(\xi_+) + G(\xi_-)\,, \end{aligned}$$

1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Generation of higher harmonics at the bottom</u>: *(see Gerkema, p 190)*

Order ϵ^2, mean field:

$$\begin{aligned} [J(\nabla^2\Psi,\Psi^*)+\mathrm{c.c.}] - fV_{0,z} + \Gamma_{0,x} &= 0 \\ [J(V,\Psi^*)+\mathrm{c.c.}] + f\Psi_{0,z} &= 0 \\ [J(\Gamma,\Psi^*)+\mathrm{c.c.}] - N^2\Psi_{0,x} &= 0\,. \end{aligned}$$

Which gives :

$$\Psi_0 = \frac{i}{\omega}J(\Psi,\Psi^*)\,.$$

Using previous low order solution:

$$\Psi_0 = \frac{2}{\omega}(\mu_+ - \mu_-)\mathrm{Im}[F'(\xi_+)G'(\xi_-)^*]\,.$$

This expression confirms that no nonlinear contributions arise from an interaction of one plane internal wave (F, say) with itself; only junctions of plane waves, involving both F and G, provide nonlinear terms.

1.4. Propagation and dissipation of waves

E. Resonant interactions

Generation of higher harmonics at the bottom: *(see Gerkema, p 190)*

Order ϵ^2, second harmonic:

$$
\begin{aligned}
-i\omega_2 \nabla^2 \Psi_2 + J(\nabla^2\Psi, \Psi) - f V_{2,z} + \Gamma_{2,x} &= 0 \\
-i\omega_2 V_2 + J(V, \Psi) + f \Psi_{2,z} &= 0 \\
-i\omega_2 \Gamma_2 + J(\Gamma, \Psi) - N^2 \Psi_{2,x} &= 0\,,
\end{aligned}
$$

Which gives : $$(N^2 - \omega_2^2)\Psi_{2,xx} - (\omega_2^2 - f^2)\Psi_{2,zz} = 3i\omega J(\nabla^2\Psi, \Psi)\,.$$

The left-hand side of this equation describes the propagation of free waves at frequency 2ω; the right-hand side, the nonlinear forcing by the lowest-order terms.

The left-hand-side can be written:

$$J(\nabla^2\Psi, \Psi) = -2\mu(1+\mu^2)\left[F'''(\xi_+)G'(\xi_-) - G'''(\xi_-)F'(\xi_+)\right].$$

1.4. Propagation and dissipation of waves

E. Resonant interactions

<u>Generation of higher harmonics at the bottom</u>: *(see Gerkema, p 190)*

We can solve it with

$$F(\xi_+) = \int_0^\infty dk\, a(k)\, e^{ik\xi_+}\, .$$

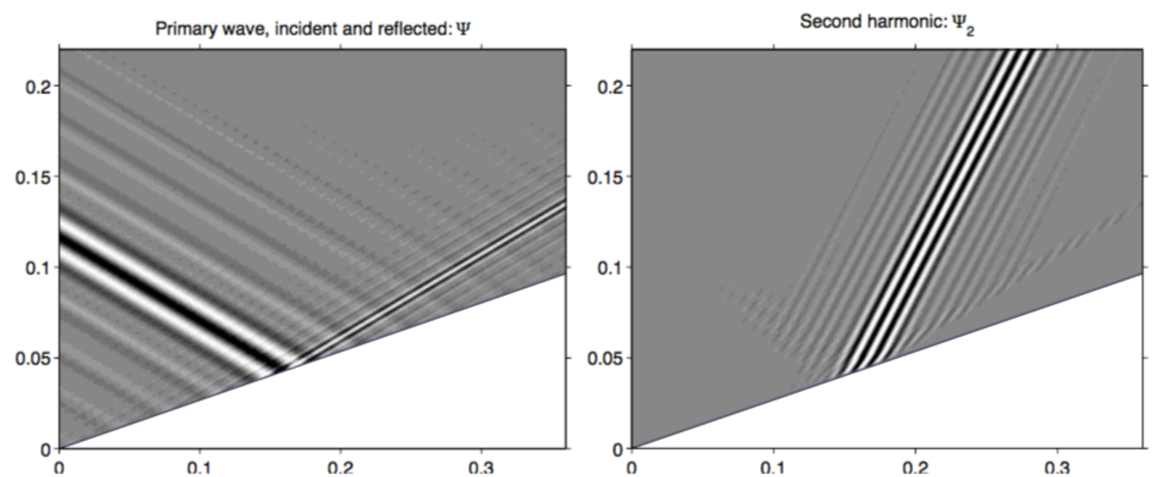


Fig. 9.5: On the left, the primary incident and reflected waves (9.37); the incident wave enters from the left. On the right, the nonlinearly generated second harmonic, (9.41). Parameters as in Figure 9.4a.

1.4. Propagation and dissipation of waves

F. Viscous dissipation of waves

Internal waves are damped by viscosity. But viscosity acts mostly on small scales.

- *Short internal waves* (< 100 m) are strongly damped
- *Long internal waves* (about 200 km) can propagate for distances of 2000 km or more, but will be damped before being able to establish cross-ocean baroclinic standing waves.
- *Very long internal waves* (> 200 km) are damped by bottom friction

1.4. Propagation and dissipation of waves

F. Viscous dissipation of waves

- Activity:

1. Write Boussinesq equations without rotation, and include viscous terms using constant eddy viscosity diffusion coefficients for momentum:

$$K_{Mh} = K_{Mv} = K_M$$

And density:

$$K_{Bh} = K_{Bv} = K_B$$

2. Write an equation for the vertical velocity only

3. Find the dispersion relation for a solution of the form

$$w = w_0 \exp(\omega t + i(k_x x + k_z z))$$

(where imaginary part of ω is the frequency, and its real part the damping rate)

1.4. Propagation and dissipation of waves

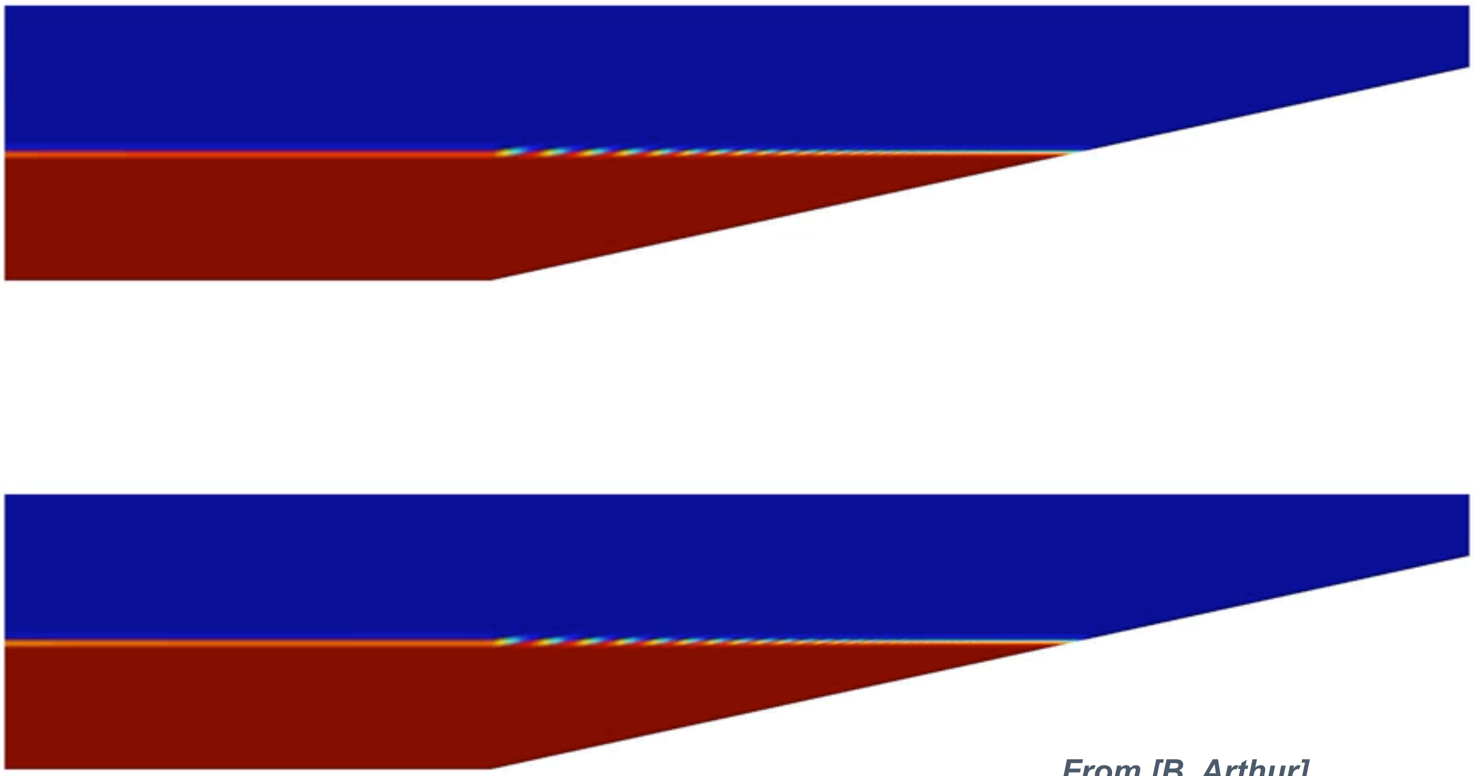
G. Wave breaking

- Wave breaking due to Convective instability: Unstable density gradients may occur where the local horizontal particle speed exceeds the phase speed of the wave.

The crest (or trough) overtakes the rest of the wave, and the wave rolls over not unlike the surf on the sea surface upon approaching a beach.

1.4. Propagation and dissipation of waves

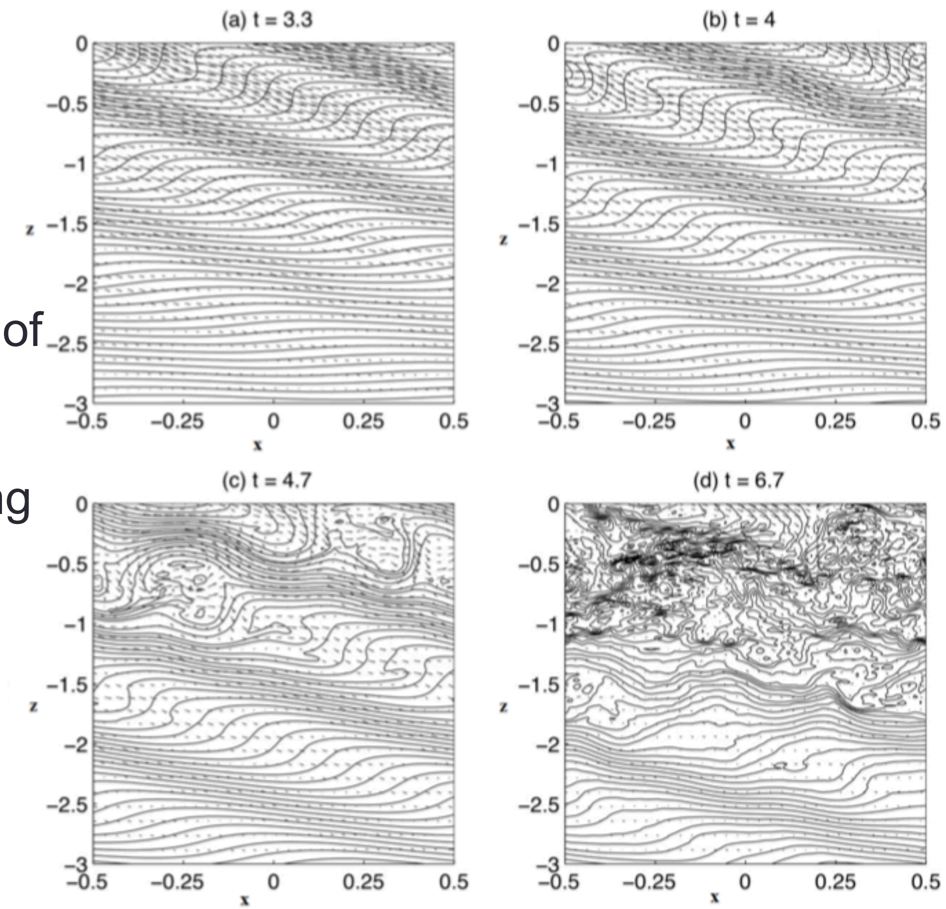
G. Wave breaking



From [B. Arthur]

1.4. Propagation and dissipation of waves

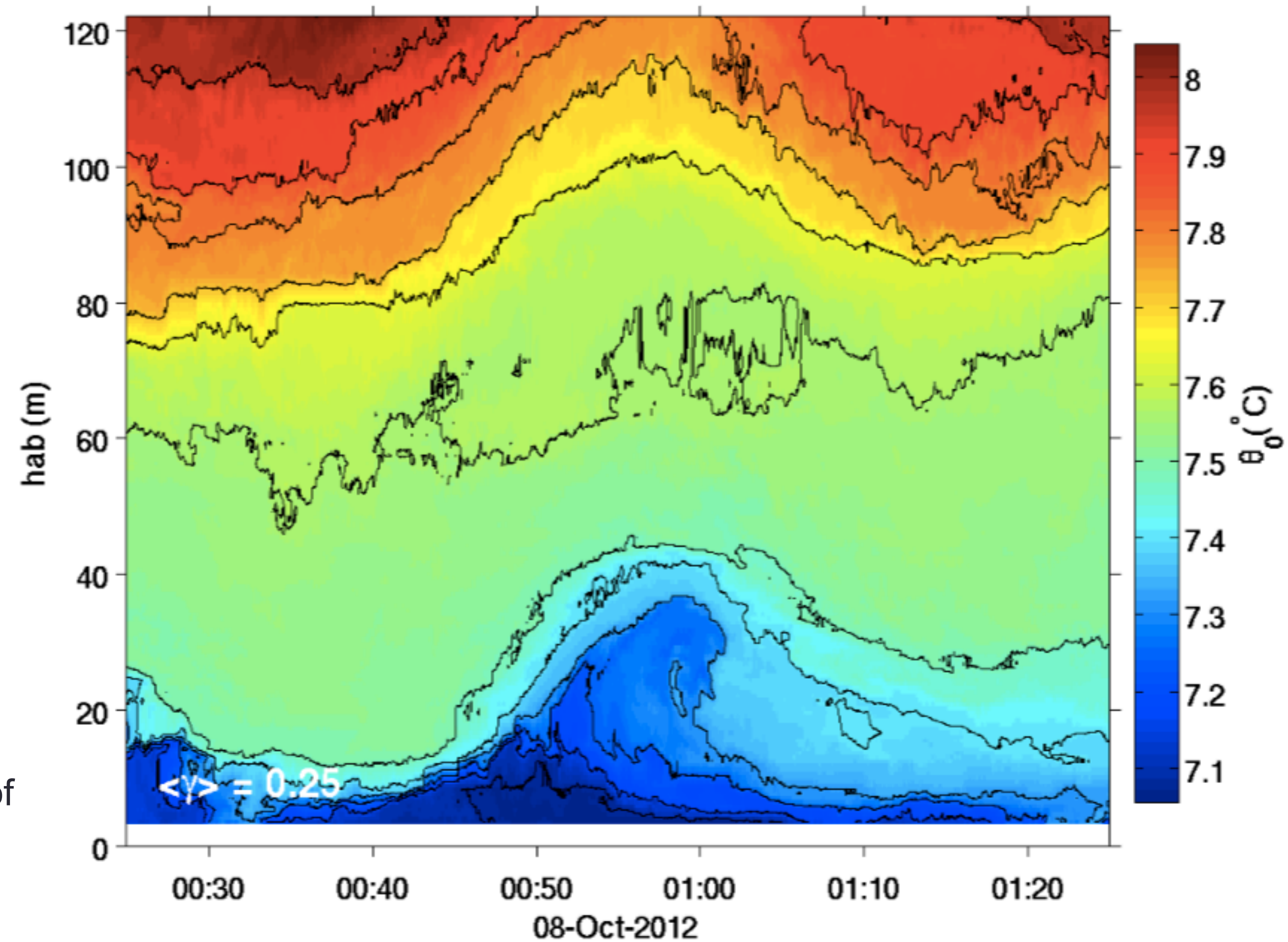
G. Wave breaking



time-series cross-sectional view of a set of isopycnals from relative stability, Figure (a), to relative levels of overturning and breaking in Figures (b-d).
[Liu ,2010]

1.4. Propagation and dissipation of waves

G. Wave breaking

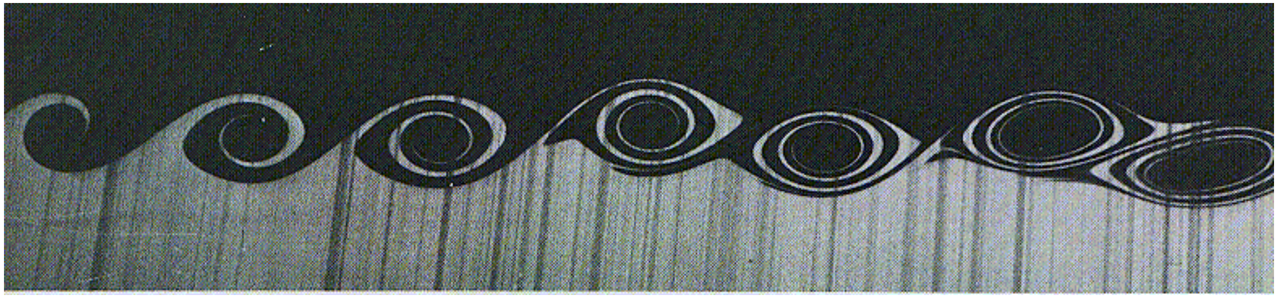


1-hour snapshot of the temperature time/depth series as measured from moored temperature sensors on October-8 2012 on the slopes of Rockall Bank, NE Atlantic (925 m total depth).
[Cyr & Van Haren, 2016]

1.4. Propagation and dissipation of waves

G. Wave breaking

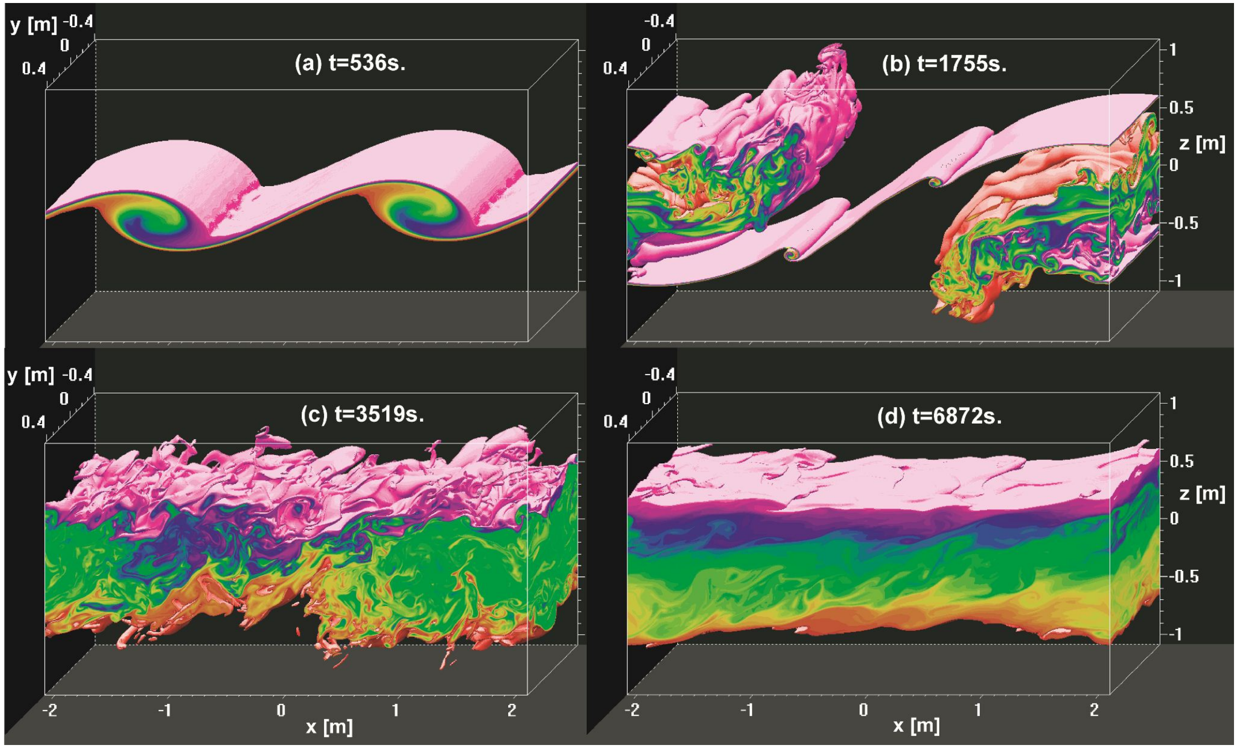
- ***Wave breaking due to shear instability:*** If the shear generated by the wave, added to any preexisting shear, becomes so large that the local Richardson number falls below a critical value (usually 0.25) long enough for a shear instability to grow and produce density gradients or billows.



1.4. Propagation and dissipation of waves

G. Wave breaking

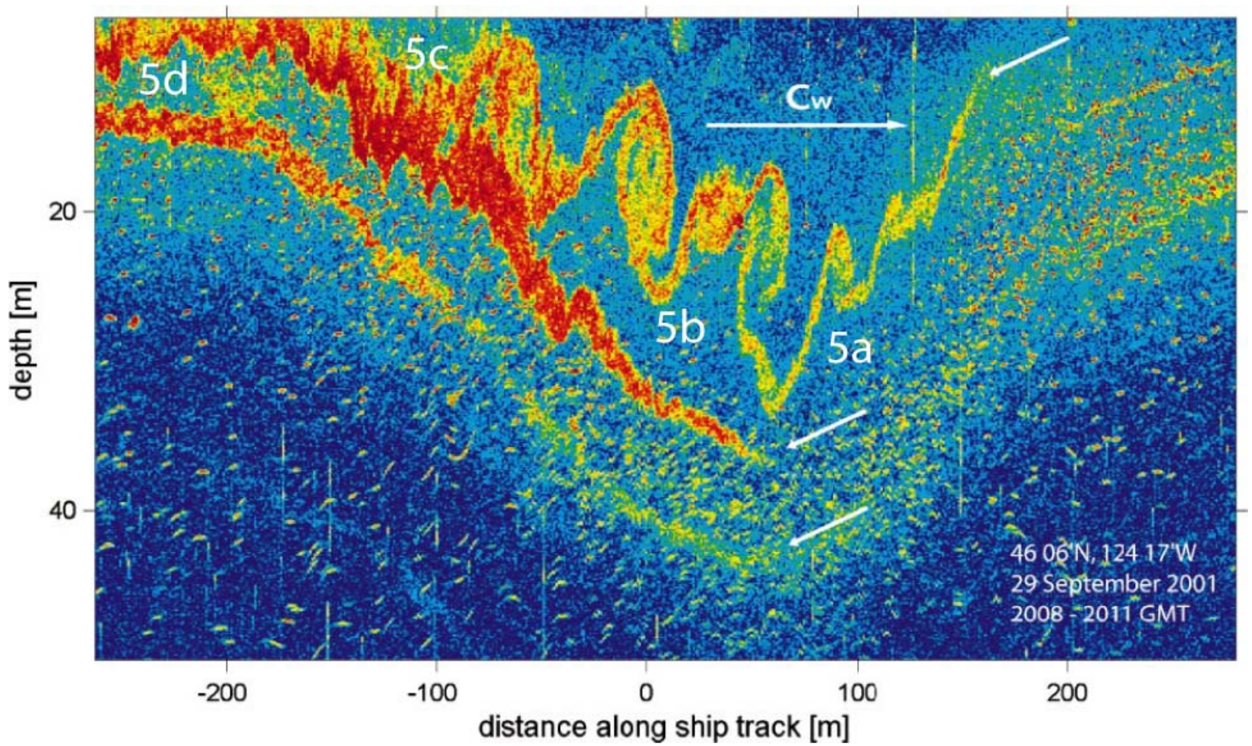
Most of the energy is transferred to turbulent kinetic energy, a fraction is transferred to potential energy (raising the center of gravity of the mixed fluid), and a fraction can be radiated as secondary internal waves (with horizontal scales related to billows scale)



Direct numerical simulation mixing across an interface in the ocean. Colors indicate intermediate values of salinity found in the interfacial layer ***[W. Smyth]***

1.4. Propagation and dissipation of waves

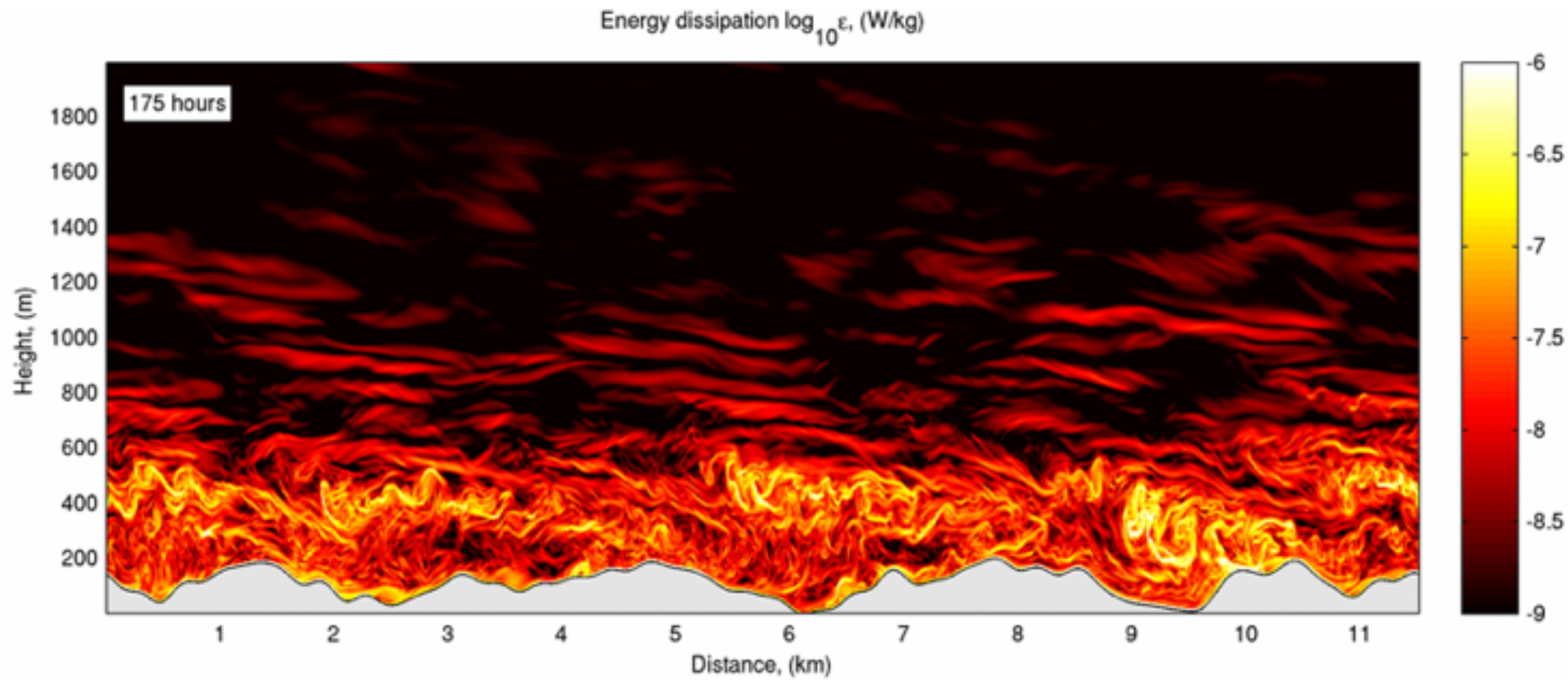
G. Wave breaking



Acoustical snapshot of a nonlinear internal gravity wave approaching the Oregon coast. Near the surface the current flows from left to right, the direction of wave propagation, while the underlying fluid flows left to compensate. The resulting current shear generates the Kelvin-Helmlholtz billows and turbulence. ***[Moum et al, 2003]***

1.4. Propagation and dissipation of waves

G. Wave breaking



Breaking Lee waves: Energy dissipation in log10(W/kg) from a high-resolution numerical simulation of lee waves forced by the prescribed mean flow with parameters typical for the Drake Passage region of the Southern Ocean **[From N. Nikurashin]**

1.4. Propagation and dissipation of waves

- Theory of linear internal waves is applicable only if their amplitude is small, but internal-wave amplitudes can be quite large

- When do internal-wave dynamics become nonlinear?

  If particles displacements are comparable to the wavelength

- Ex: $$u = U_0 \cos(k_x x + k_z z - \omega t)$$

- Particle displacement: U_0/ω

- Horizontal wavelength: $\dfrac{2\pi}{k_x}$

$$\longrightarrow \quad U \ll \frac{2\pi\, N}{\sqrt{k_x^2 + k_z^2}} \leq \frac{2\pi\, N}{k_z}$$

- Or equivalently Froude Number $$Fr = \frac{U}{NH} \ll 1$$

1.4. Propagation and dissipation of waves

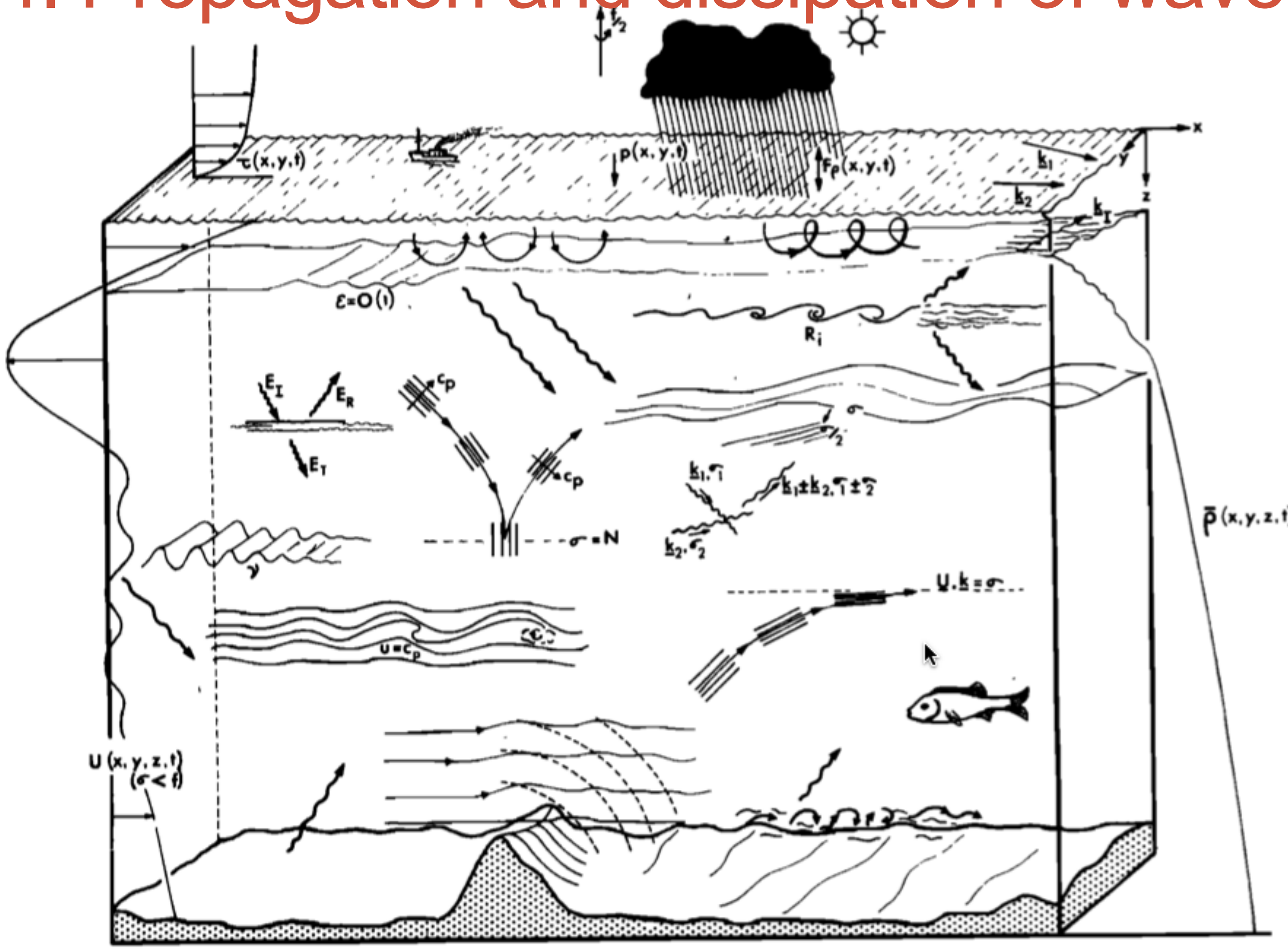


Fig. 5. Physical processes affecting internal waves. [Thorpe, 1975]

1.4. Propagation and dissipation of waves

- Review of mechanisms *(see Thorpe75.pdf):*

- Reflection of waves

$$\omega = N$$

- Critical layer absorption

$$\vec{u}.\vec{k} = \omega$$

- Resonant interactions between triads

$$\vec{k_1} \pm \vec{k_1} \pm \vec{k_3} = 0$$

$$\omega_1 \pm \omega_2 \pm \omega_3 = 0$$

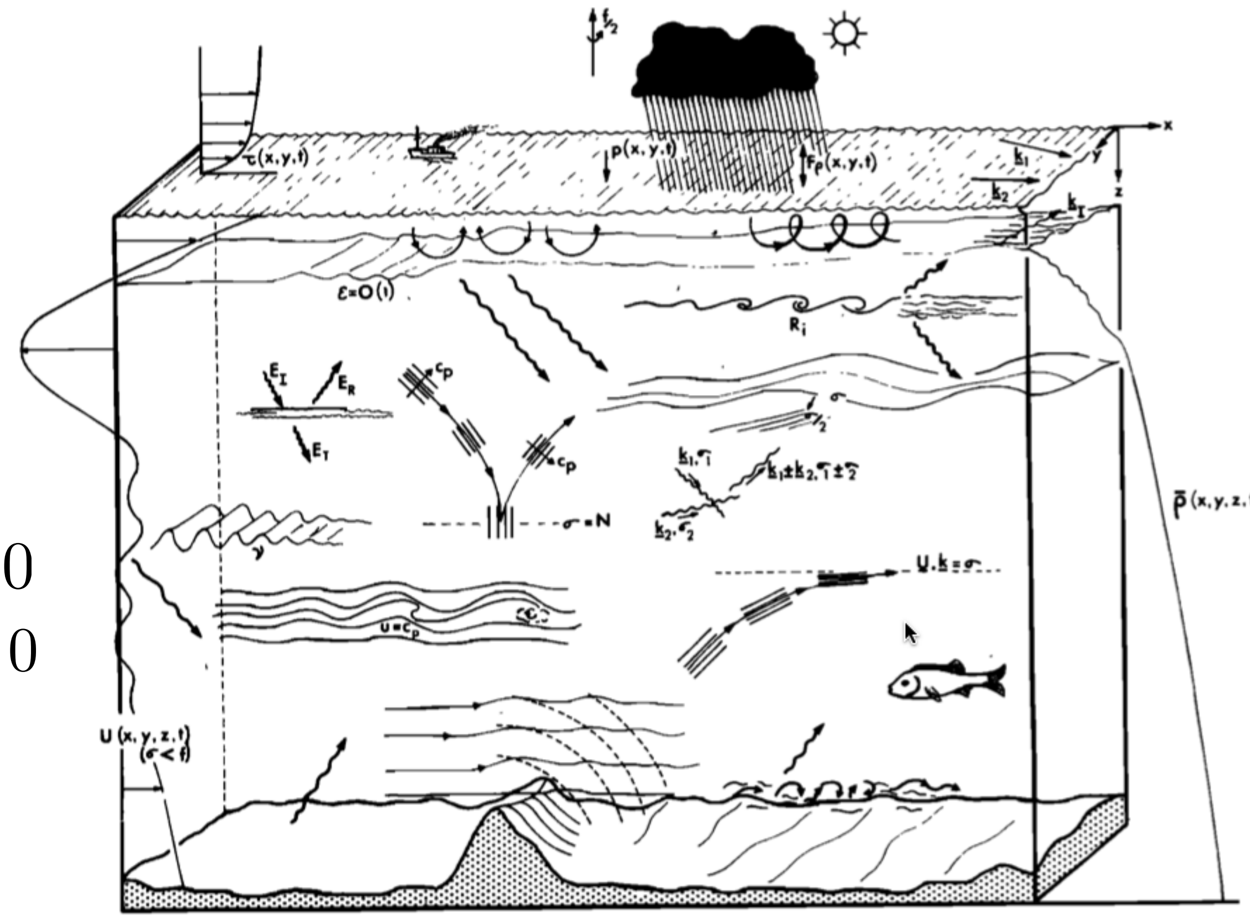


Fig. 5. Physical processes affecting internal waves.

1.4. Propagation and dissipation of waves

- Review of mechanisms *(see Thorpe75.pdf):*
- Viscous attenuation
- Attenuation due to turbulent layers
- Shear instability (Ri<1/4)
- Wave breaking

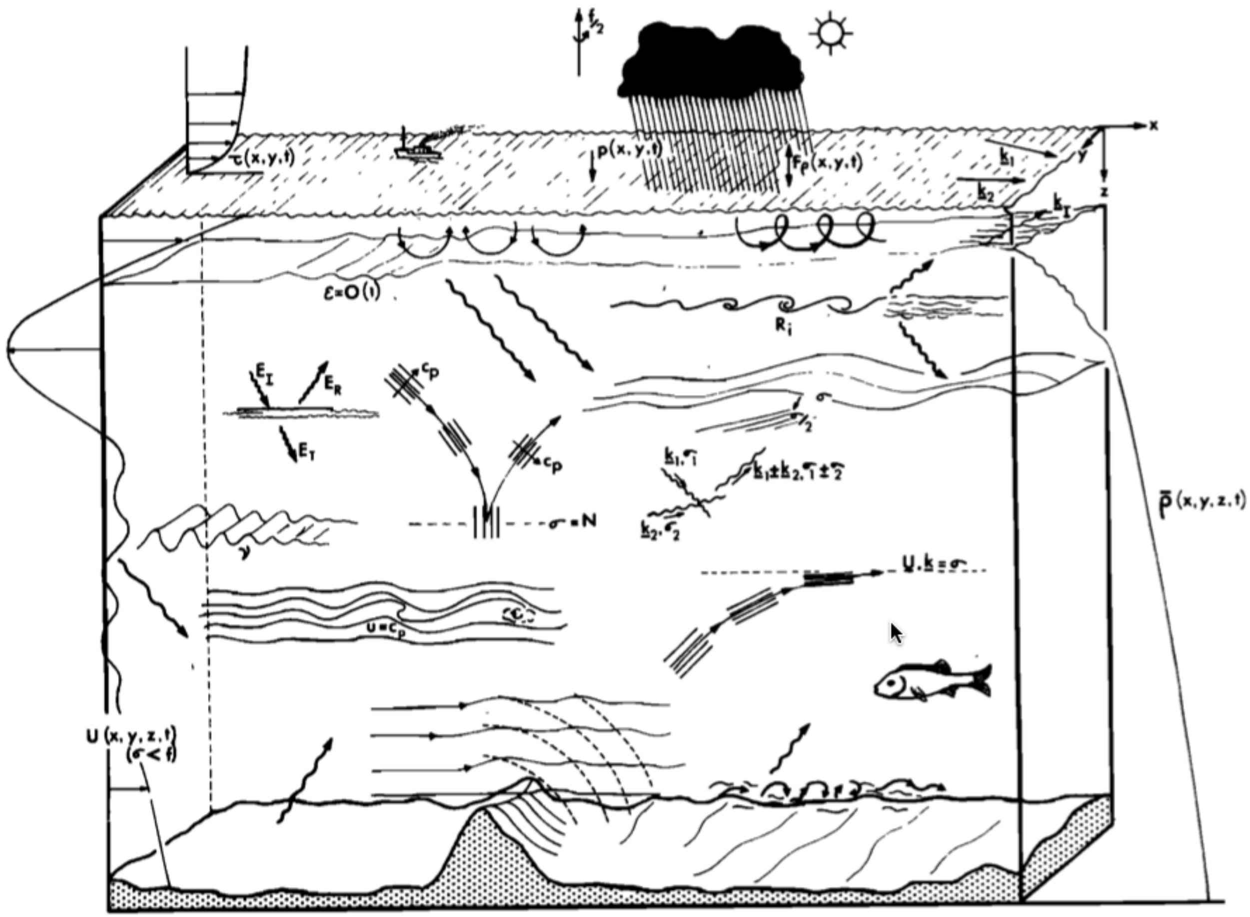


Fig. 5. Physical processes affecting internal waves.

1.5. Wave energy spectra

In nature, It is impossible to isolate triplets or quartets of interacting waves. There is a continuous spectra of waves spanning several order of magnitudes of wavenumbers and frequencies. The interactions take place in a continuous fashion and resonant and forced energy transfers act continuously over wavenumber and frequency domain.

Redistribution of wave action between all possible interacting multiplets. Wave action is not conserved for individual multiplets, but becomes a conserved quantity of the whole spectrum.

1.5. Wave energy spectra

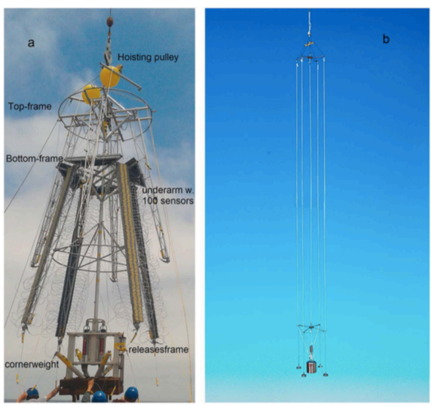
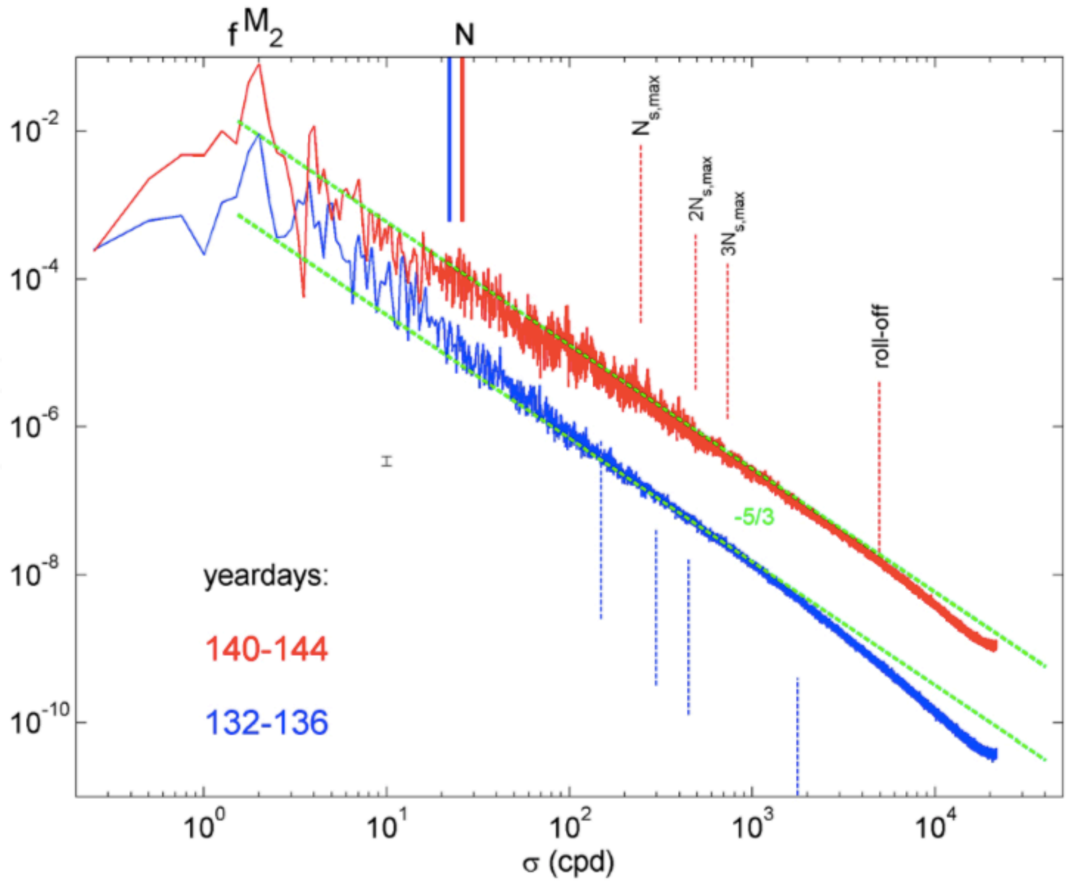


Figure 1. (a) Compacted 3-D mooring array of temperature sensors just before deployment in the ocean. (b) Model of the unfolded array to scale.

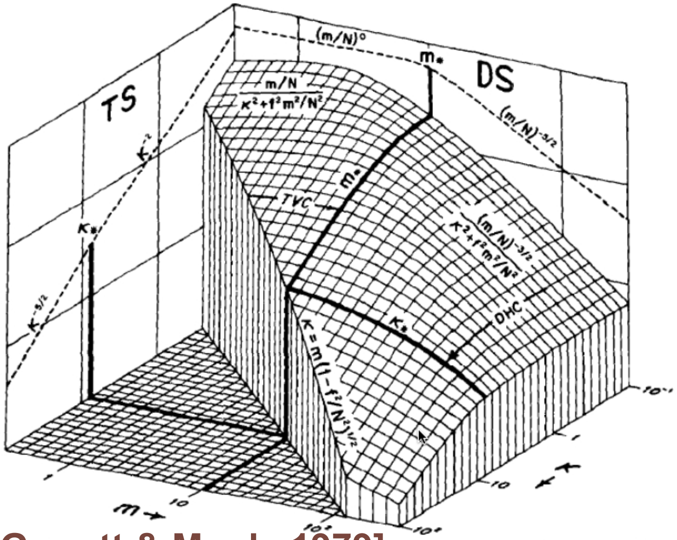


Observed Temperature power spectra. Besides inertial (f) and semidiurnal lunar tidal (M2) frequencies several buoyancy frequencies are indicated including 4 day large-scale mean N and the maximum small-scale Ns,max. **[Van Haren et al., 2016]**

1.5. Wave energy spectra

- The ***Garrett–Munk spectrum*** *is* an empirical spectrum for internal-wave energy based on observations, simple dimensional considerations, and elementary physics. It has been shown to conform to a large number of observations

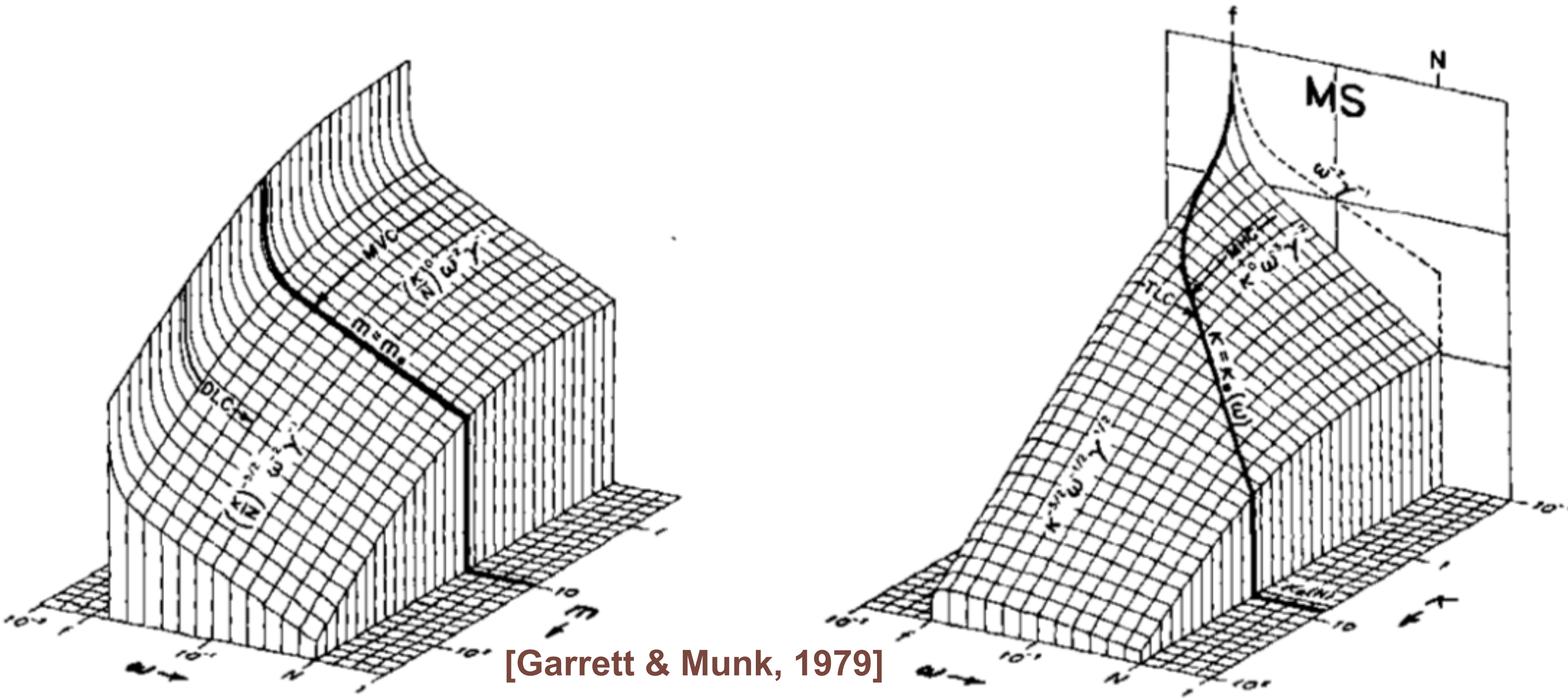
- It gives the spectral energy density anywhere in the ocean *(dimensionless constant E setting the overall energy level)*



[Garrett & Munk, 1979]

$$E(k,m) = \frac{3fN\mathrm{E}\,m\,m_*^{3/2}}{\pi(m+m_*)^{5/2}\,(N^2k^2+f^2m^2)},$$

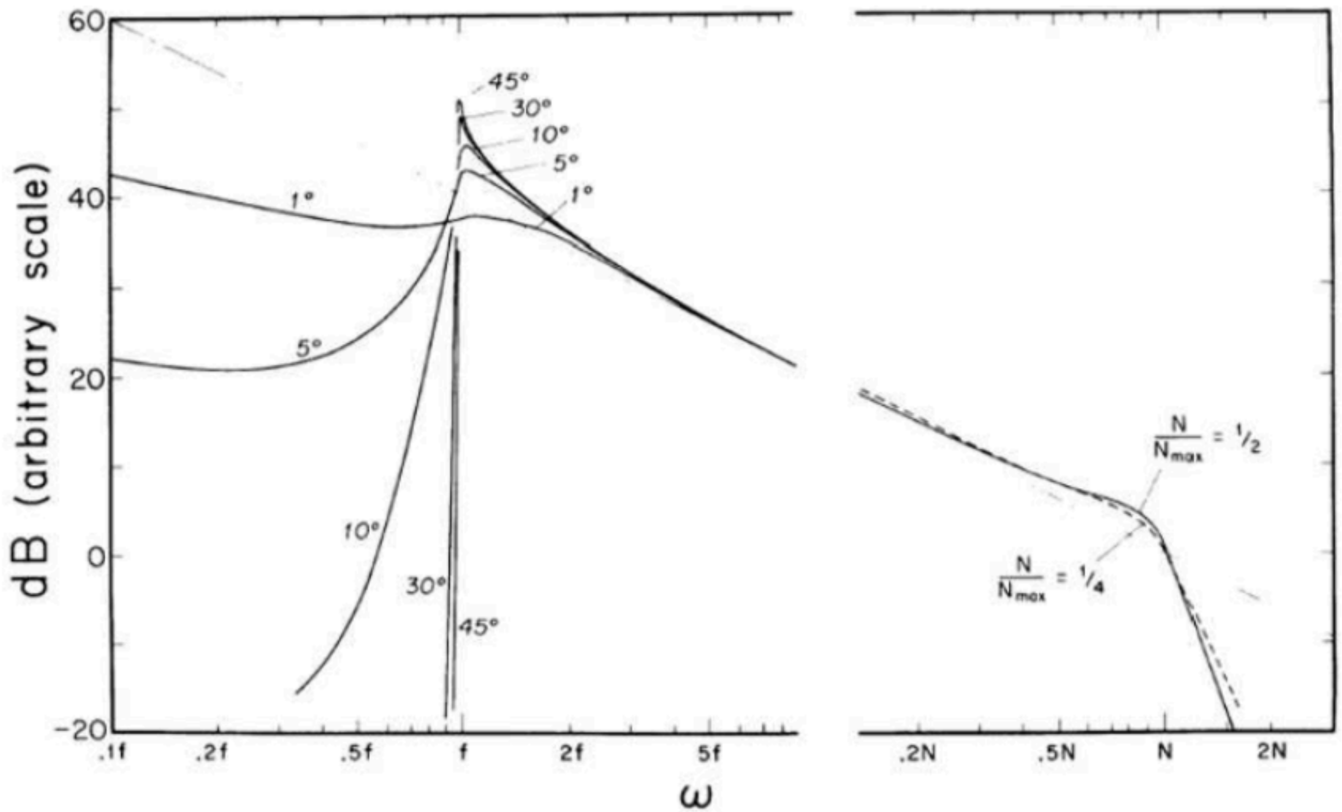
1.5. Wave energy spectra



[Garrett & Munk, 1979]

As a function of frequency and/or wavenumbers.

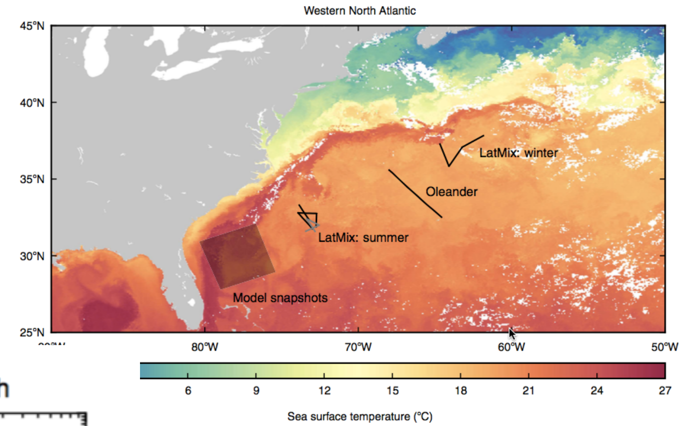
1.5. Wave energy spectra

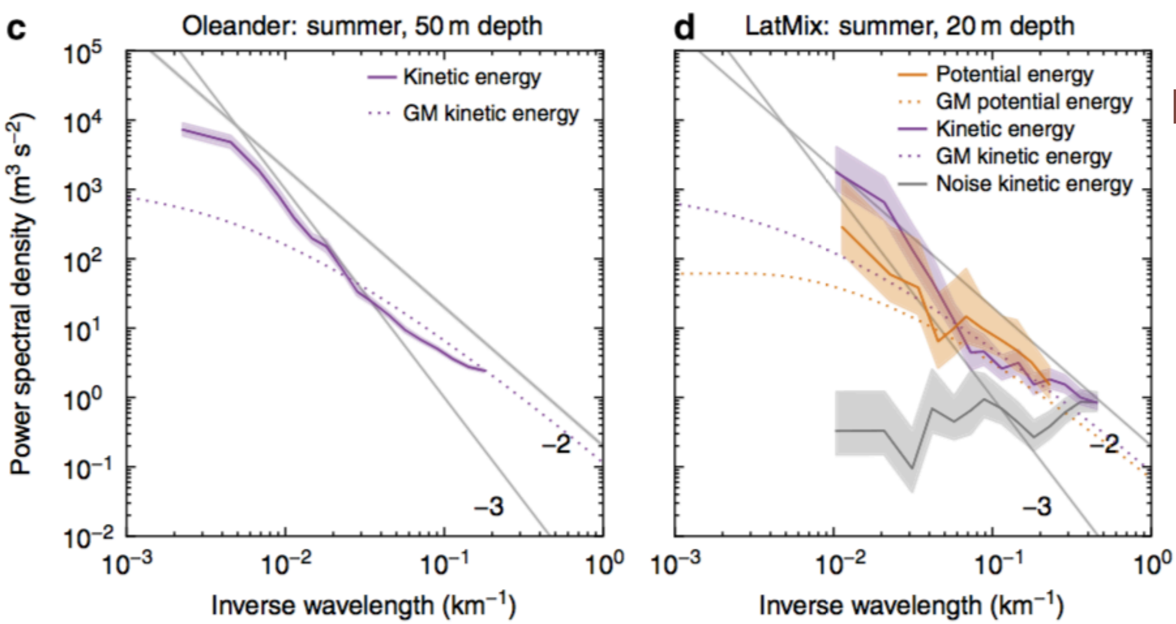


The Garrett-Munk spectrum, at different latitudes, showing a peak at the inertial frequency (f), and a steady fall-off for higher frequencies. **[Munk, 1981].**

1.5. Wave energy spectra

- Ex: Interpretation of observation

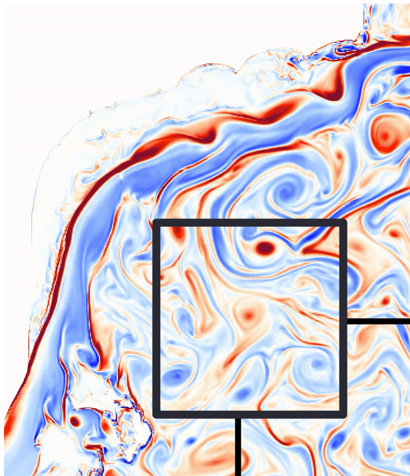




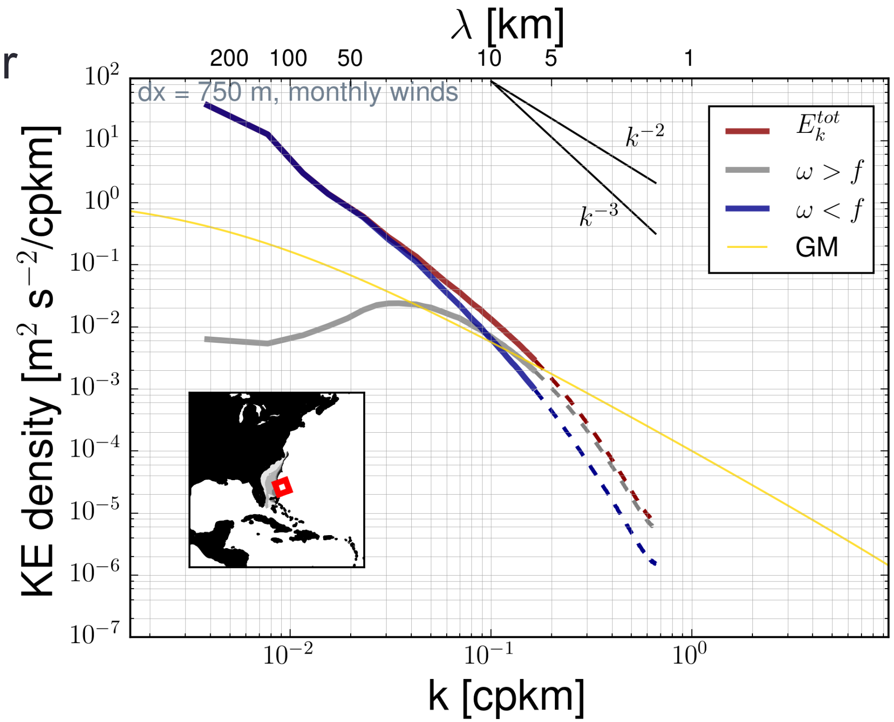
[Callies et al, 2015]

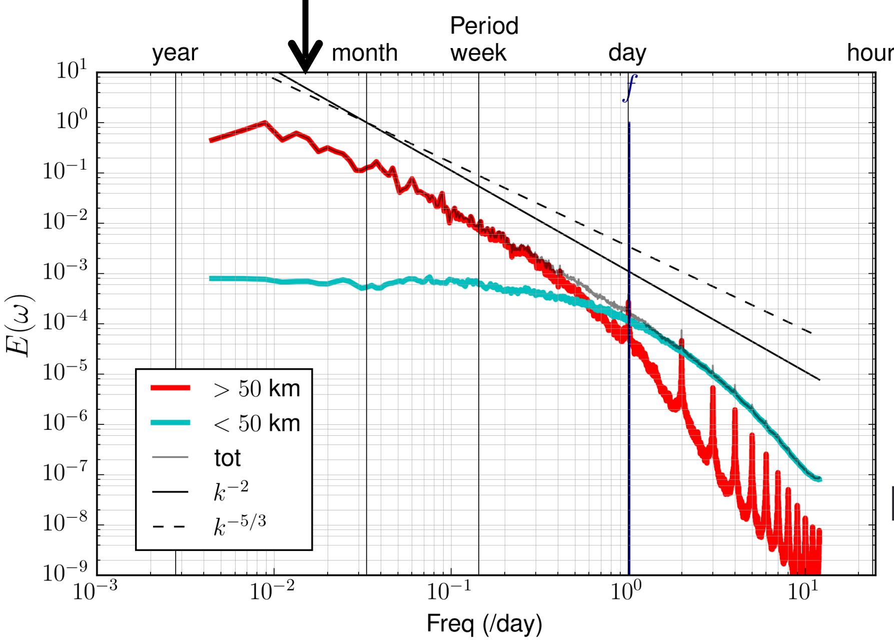
(c) Kinetic energy spectrum at 50 m depth for the Oleander summer data. (d) Potential and kinetic energy spectra at 20 m depth for the LatMix summer experiment. Also shown are the GM model spectra for internal waves in the seasonal thermocline and reference lines with slopes 2 and 3.

1.5. Wave energy spectra



Wavenumber spectra



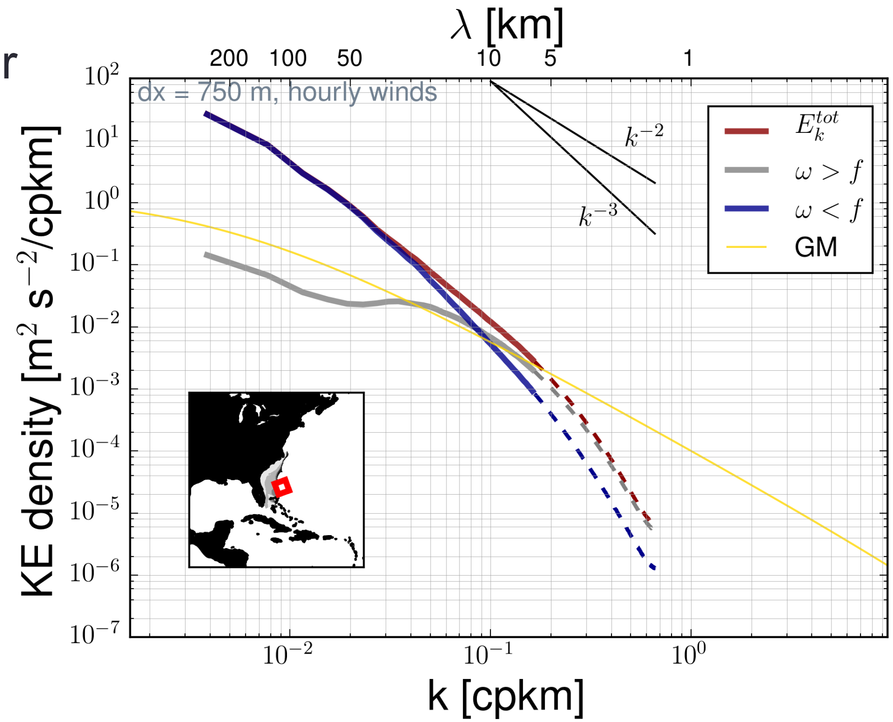


Monthly wind forcings / No tides

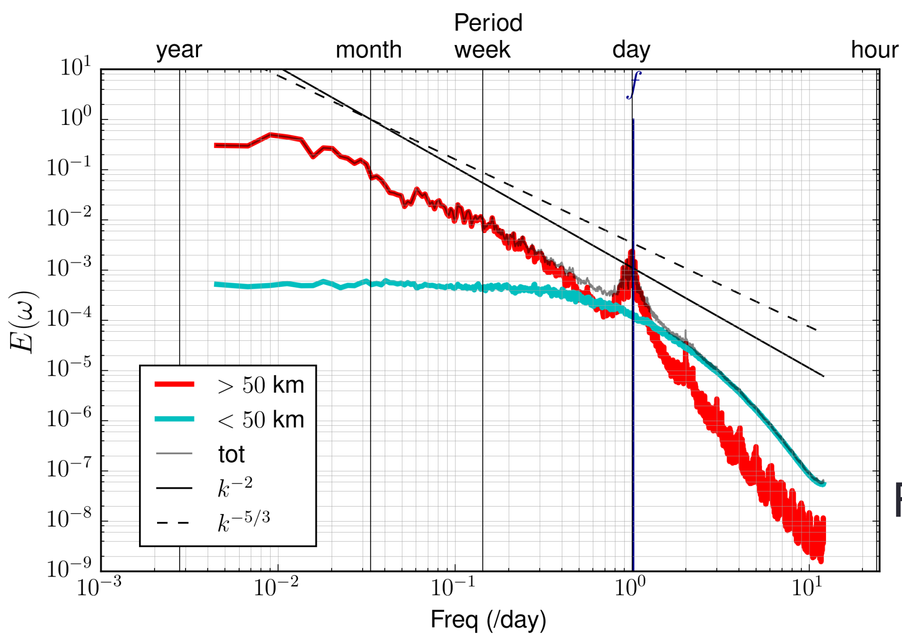
Frequency spectra

1.5. Wave energy spectra

Wavenumber spectra



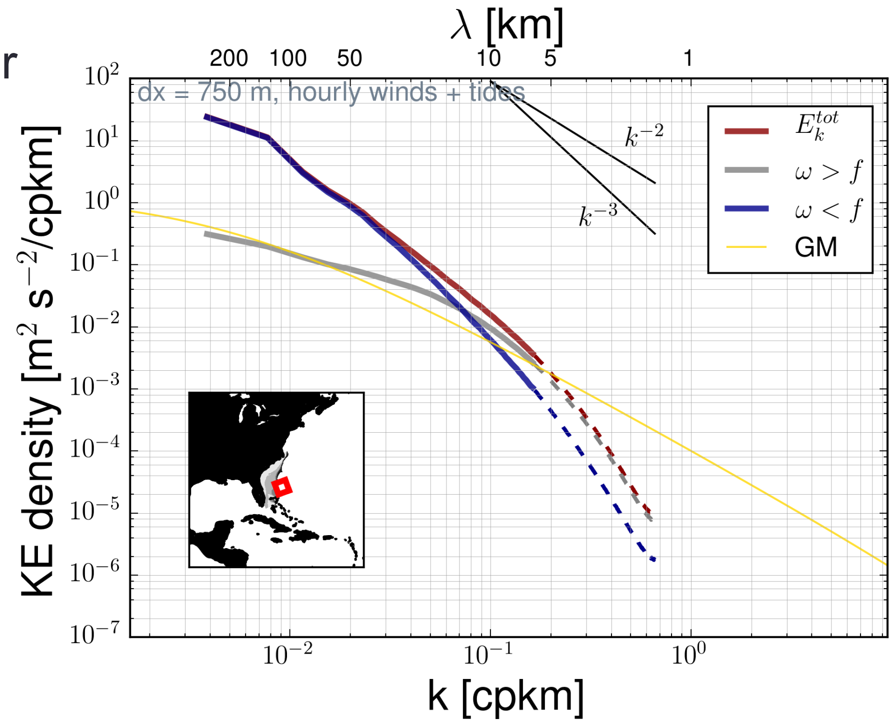
Hourly wind forcings / No tides

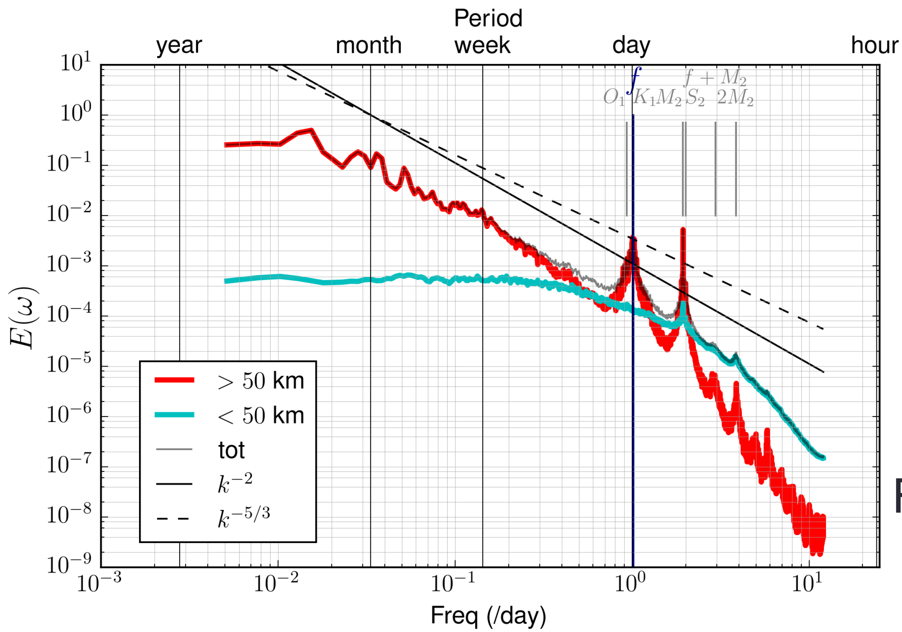


Frequency spectra

1.5. Wave energy spectra

Wavenumber spectra





Hourly wind forcings / Tides

Frequency spectra

Activity: Numerical simulation of Internal waves

- Download and unzip Fluid2d:
  http://stockage.univ-brest.fr/~gula/Ondes/fluid2d.tar.gz

- Install
  - module load anaconda3
  - tar -xf fluid2d.tar.gz
  - cd fluid2d
  - make
  - bash
  - source activate.sh

Activity: Numerical simulation of Internal waves

Run the Interfacial wave case:

- cd nhom/Interfacial
- python interfacialwave.py

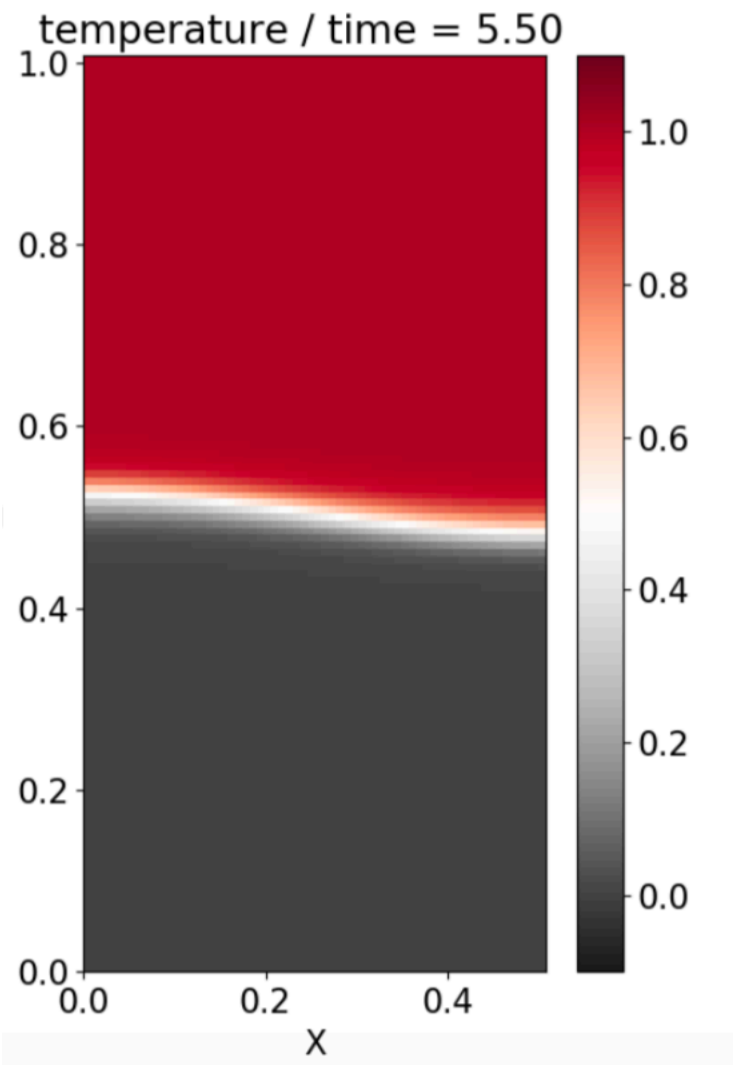
Either

- 1) a sine wave in a tank (stationary wave)
  or
  2) a localized perturbation on the left (propagating wave)

In 2) look at how the Stokes drift deforms the tracer field

In 1) play with hydroepsilon (not smaller than 0.2), look at the structure of the velocity field

In 1) increase the amplitude until you trigger KHI and wave breaking

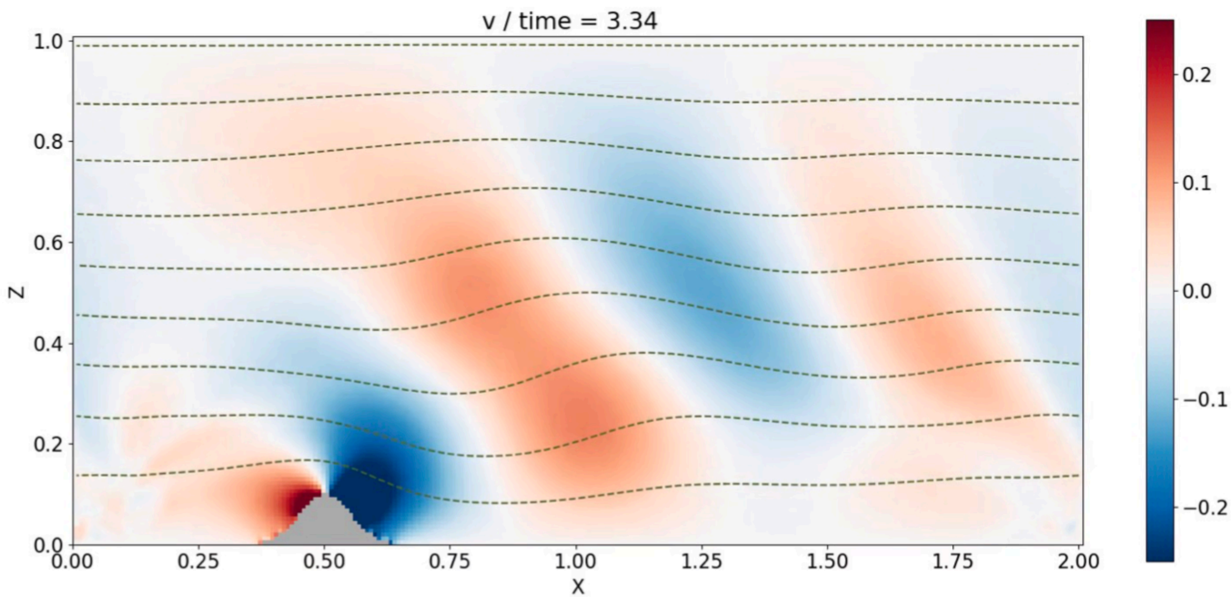


Activity: Numerical simulation of Internal waves

Run the Lee wave case:

- cd nhom/Leewave
- python leewave.py

- A flow past a seamount
- See the wave setting up
- Increase N and observ

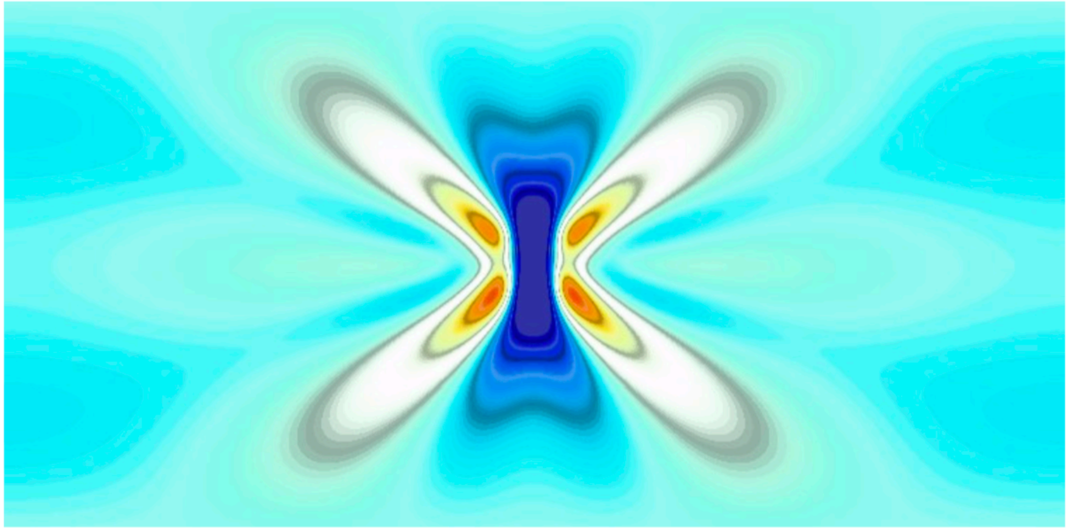


Activity: Numerical simulation of Internal waves

Run the Internal wave case:

- cd nhom/Internal_IVP
- python internalwave.py

- This is an initial value problem: gaussian perturbation of b’
- Play with the wave amplitude
- Play with the hydroepsilon (0.2 is the minimum)
- Play with the size of the initial perturbation

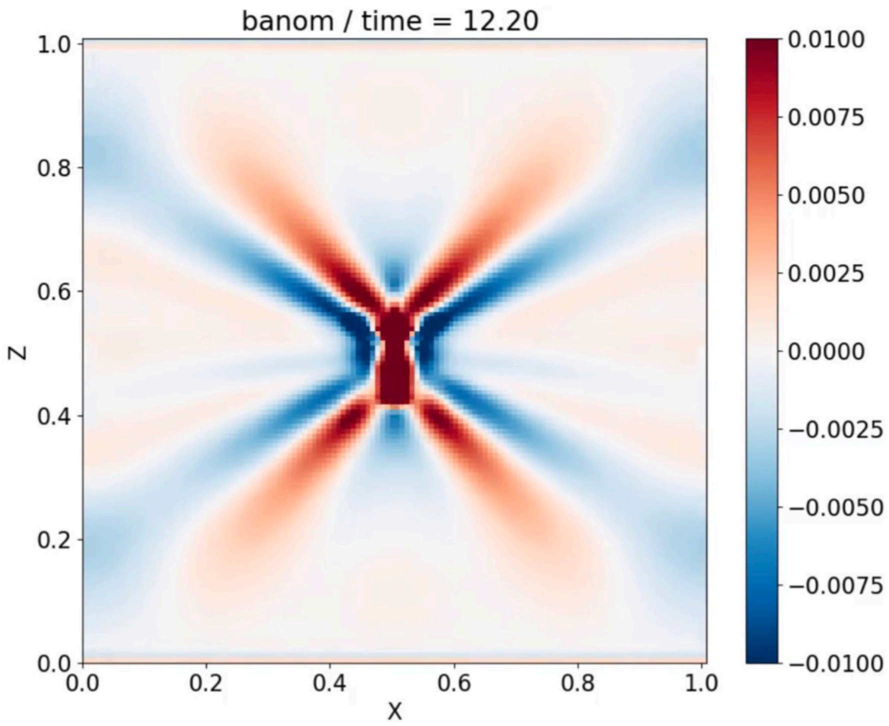


Activity: Numerical simulation of Internal waves

Run the Internal wave case:

- cd nhom/Internal_forced
- Python forcedinternal.py

- This is a localized periodic forcing
- Play with the forcing frequency (in forcedinternal.py)
- Generate evanescent waves



Activity: Numerical simulation of Internal waves

Run the KHI case:

- cd nhom/KHI
- Python kelvin_helmholtz.py

- Look at the initial velocity profile
- Check the condition for instability S2/N2 < 0.25 (S=shear)
- Look at the exponential growth (on v or banom)
- Check the evolution of Kinetic, Potential and total Energy. Check brms (the r.m.s. of buoyancy)
- How to make the rolls smaller?
- Look at the vorticity at a late time, you can observe the inverse cascade of K.

Activity: Numerical simulation of Internal waves

- Run the internal wave case:
  - cd nhom/Internal_forced
  - gedit forcing.py
  - python forcedinternal.py

- Run the internal wave case:
  - cd nhom/Internal_forced
  - gedit forcing.py
  - python forcedinternal.py

A few open questions concerning internal waves (among many others)

a) How to compute surface velocities from SSH in presence of internal waves?
b) Extraction of energy from the mesoscale by internal waves
c) How to observe internal waves?

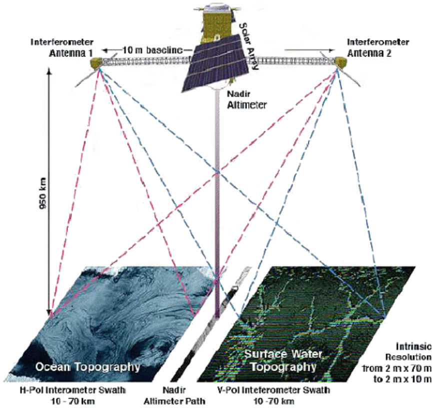
a. Diagnosing surface currents from satellite SSH measurements

- SSH observations allow to retrieve surface currents assuming the geostrophic balance:

$$\mathbf{u}_g = \frac{g}{f}\,\hat{\mathbf{z}} \times \nabla_h \eta$$

- This framework has served us very well for present altimetry measurements of large-mesoscale eddies with wavelength > 100-200 km.

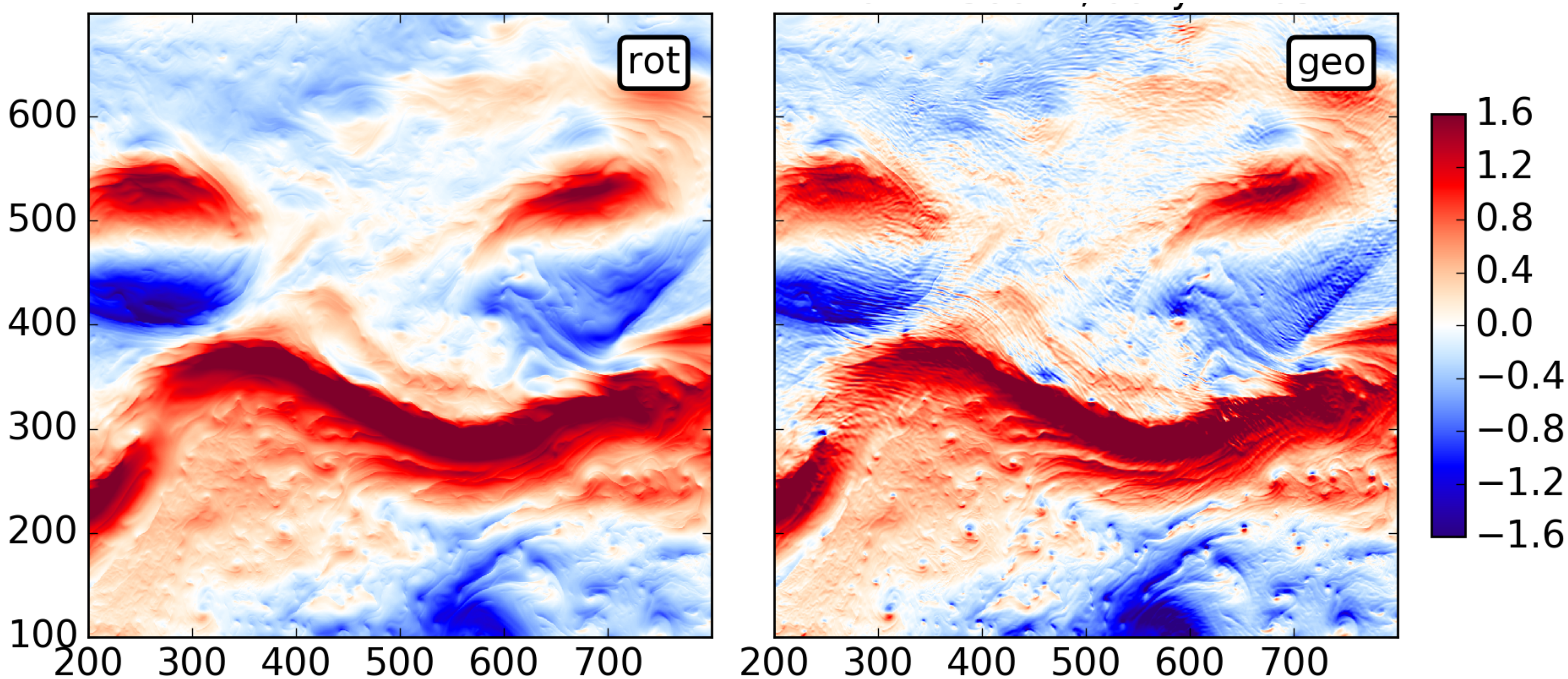
- With **SWOT** (launch 2021) we will see much smaller scales in SSH (~10 km) but we get into submesoscale and internal waves territory and we may need more general surface layer balances.



a. Diagnosing surface currents from satellite SSH measurements

Unbalanced signal in the SSH interferes with the geostrophic velocity computation.

Snapshot from a 500 m resolution simulation over the Gulf Stream



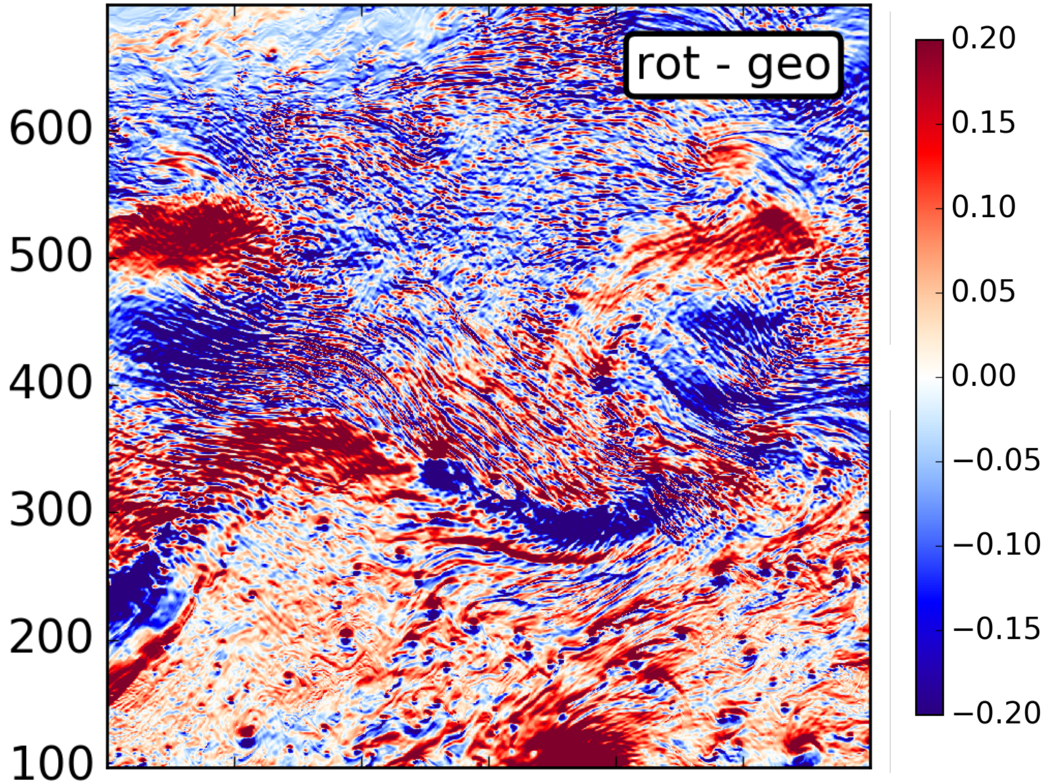
"True" Model velocity (rotational part only)

Geostrophic velocity

$$\vec{u}_g = \frac{g}{f}\vec{k} \times \vec{\nabla}_h \eta$$

a. Diagnosing surface currents from satellite SSH measurements

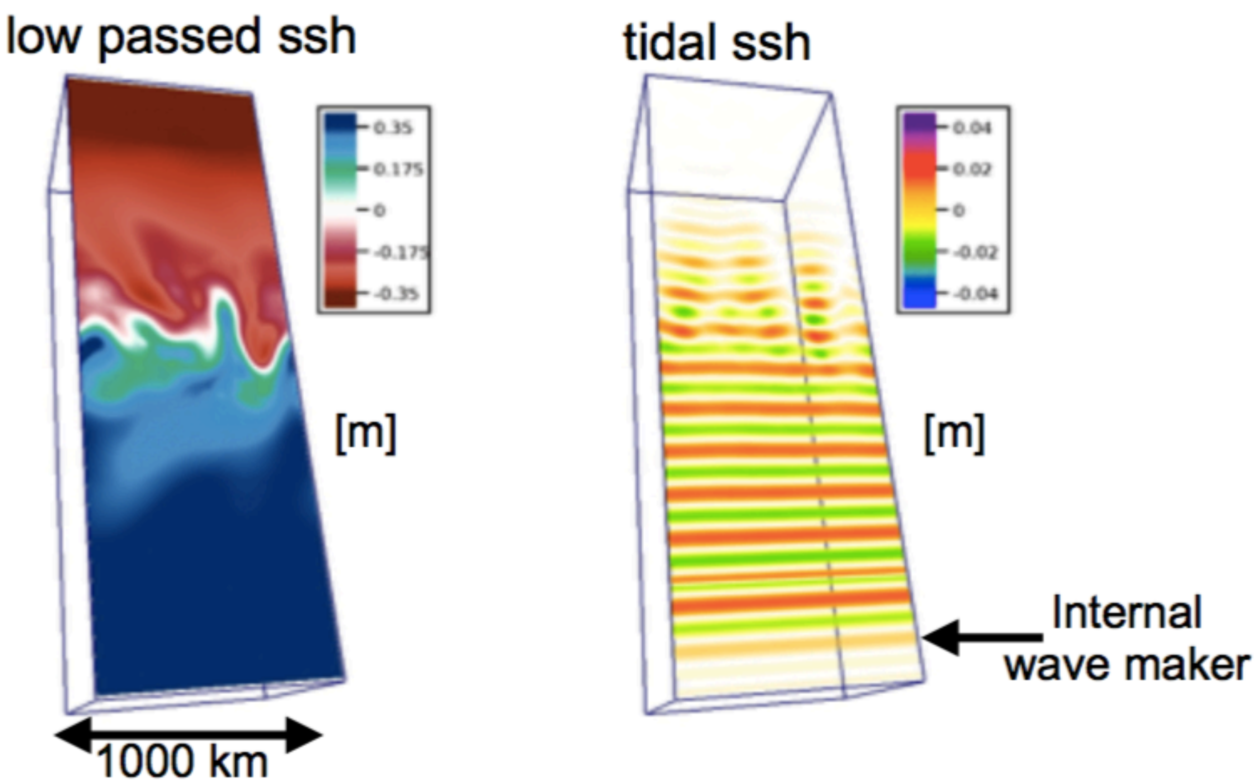
Unbalanced signal in the SSH interferes with the geostrophic velocity computation.



Difference between true and diagnosed geostrophic velocity

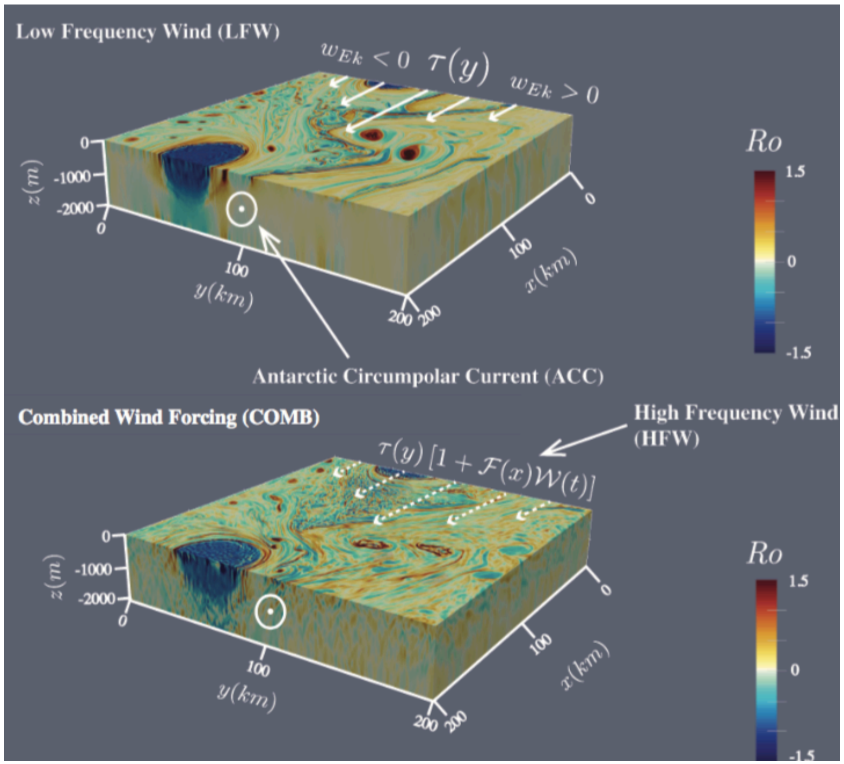
a. Diagnosing surface currents from satellite SSH measurements

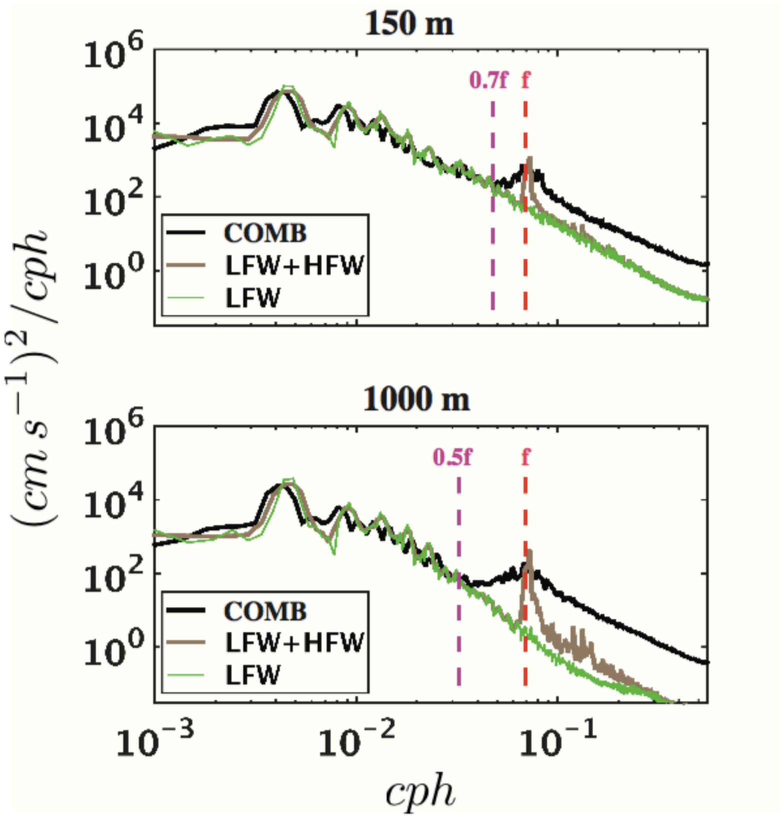
- Emergence of internal tide incoherence in presence of eddy turbulence. In strongly turbulent situations, the internal tide signature on sea level forms complex interference patterns with large amplifications of the initial internal wave **[Ponte & Klein, 2015]**



b. Extraction of energy from the mesoscale by internal waves

Temporal-scale analysis of energy exchanges among low (mesoscale), intermediate (submesoscale), and high (IW) frequency bands shows a corresponding increase in kinetic (Ek) and available potential (APE) energy transfers from mesoscales to submesoscales (stimulated imbalance) and mesoscales to IWs (direct extraction). **[Barkan et al, 2017]**





c. Observation of internal waves using Lagrangian floats

nature COMMUNICATIONS

ARTICLE

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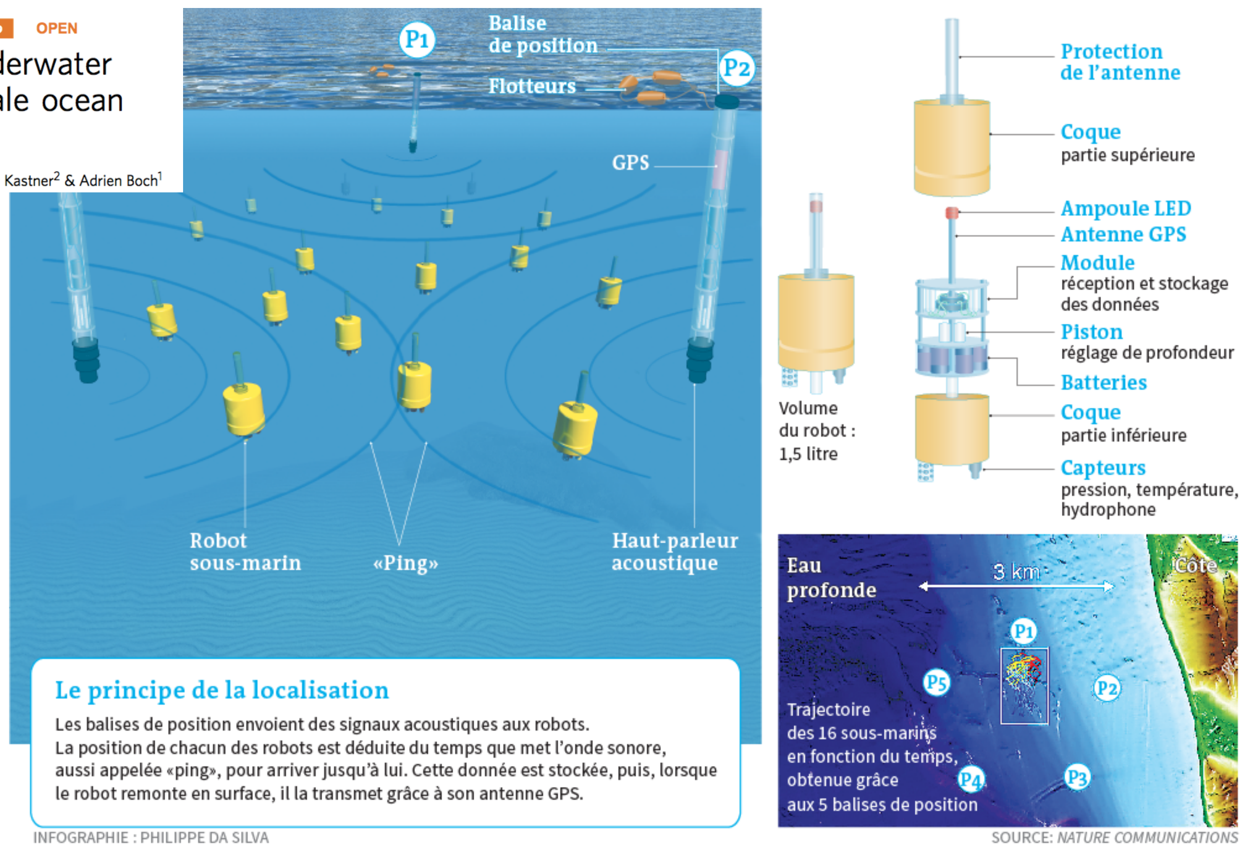
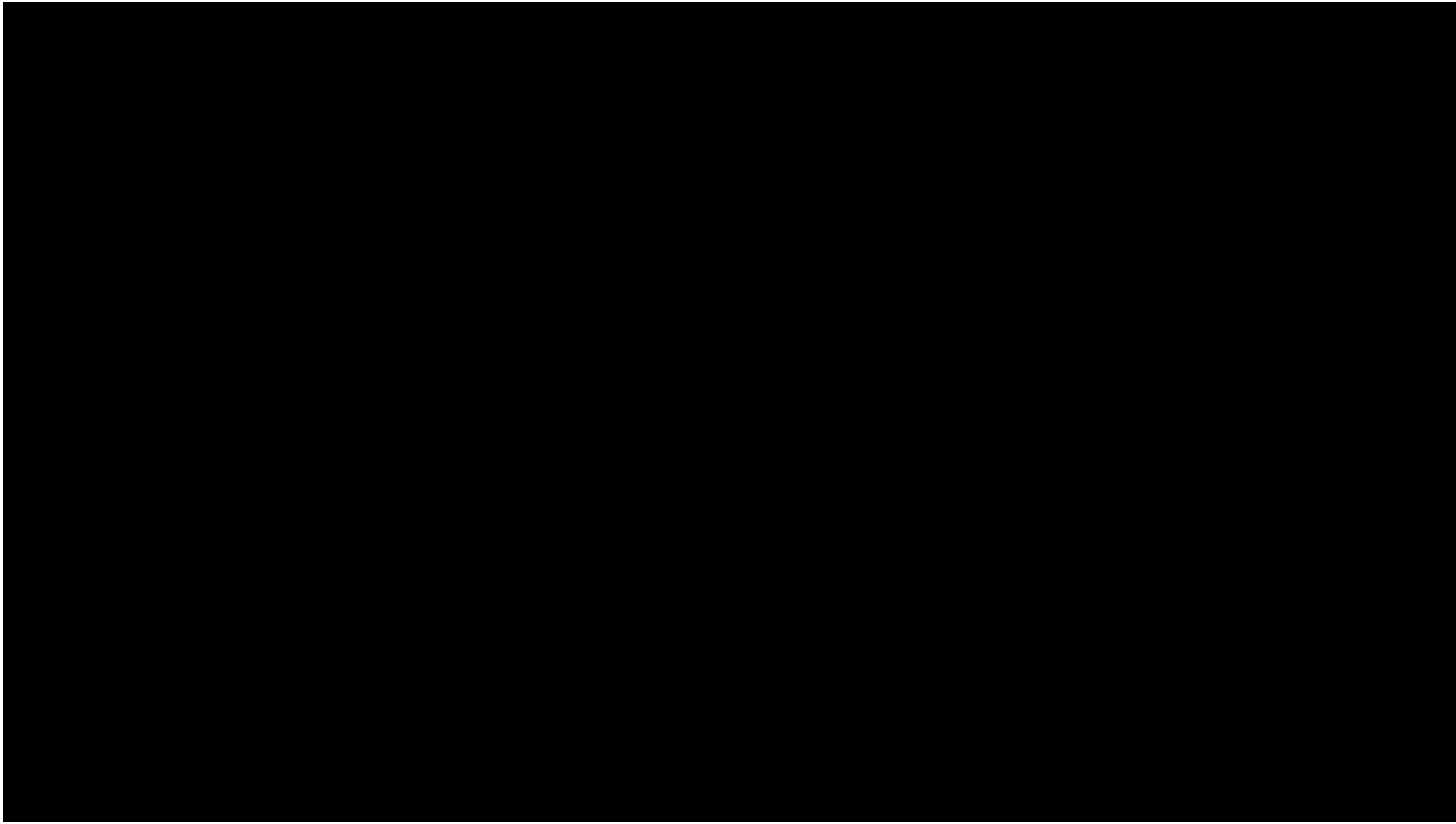
A swarm of autonomous miniature underwater robot drifters for exploring submesoscale ocean dynamics

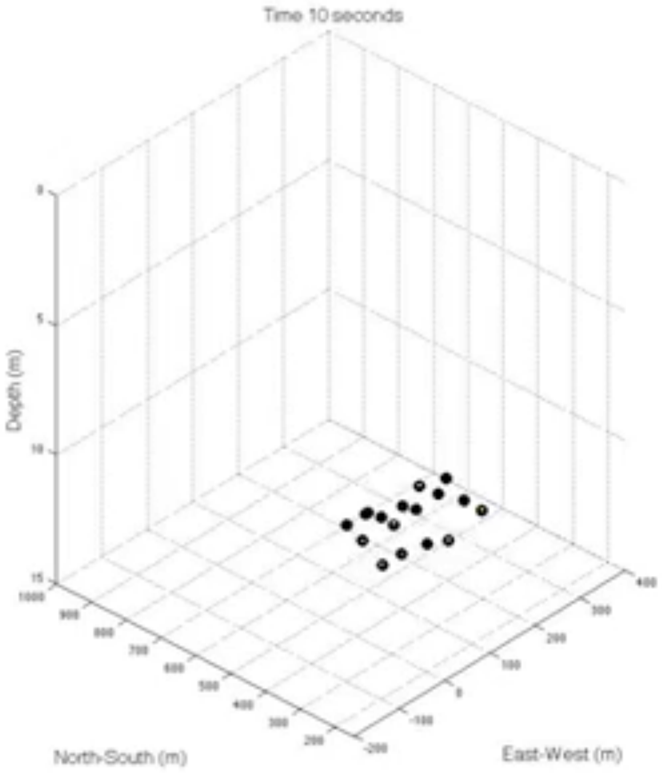
Jules S. Jaffe[1], Peter J.S. Franks[1], Paul L.D. Roberts[1], Diba Mirza[2], Curt Schurgers[3], Ryan Kastner[2] & Adrien Boch[1]

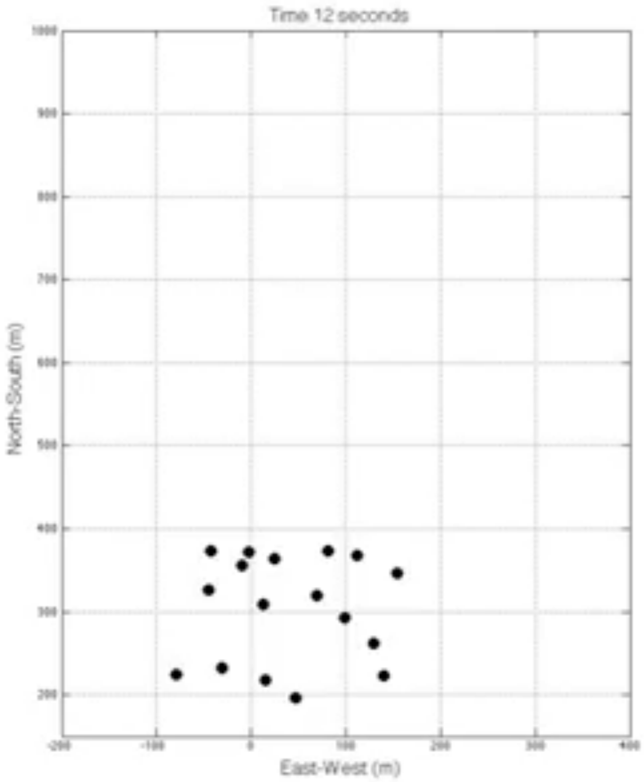


c. Observation of internal waves using Lagrangian floats



c. Observation of internal waves using Lagrangian floats



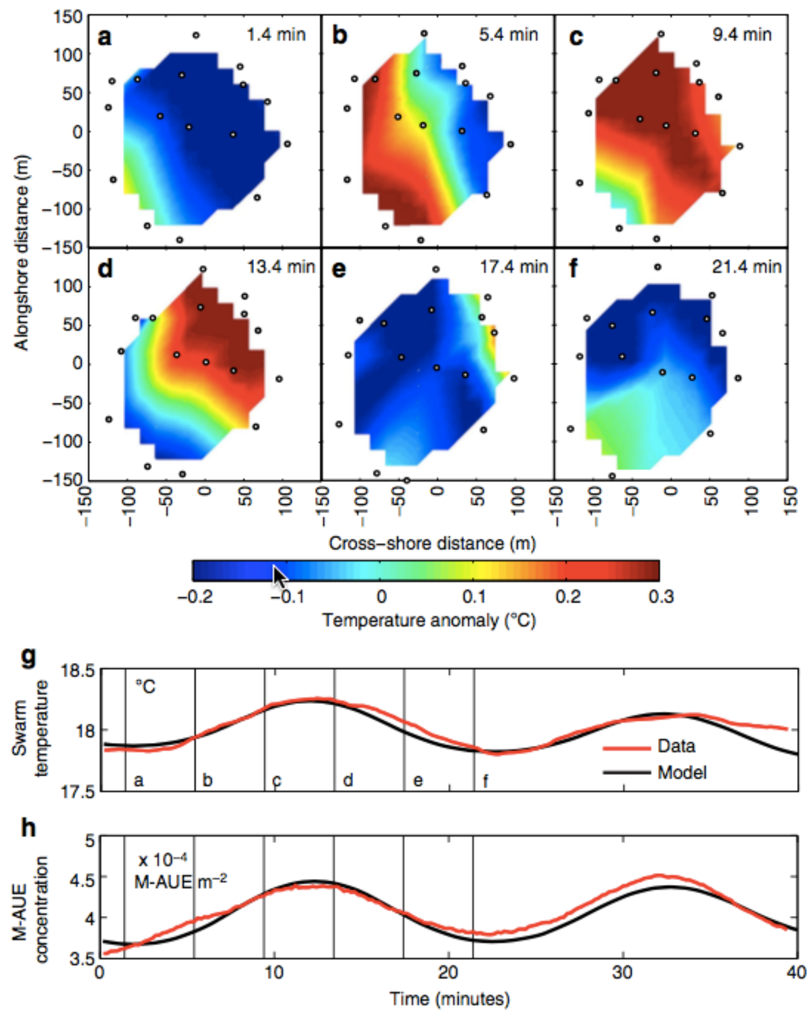


c. Observation of internal waves using Lagrangian floats

Depth-holding objects in the upper water column should accumulate over the troughs of internal waves (that is, in warm water) and disperse over the wave crests (cold water).

Changes in concentration of the M-AUEs were calculated from changes in the area of the swarm: smaller areas indicate higher concentrations (Fig. 5).

As predicted by theory, the swarm concentrated over wave troughs and dispersed over wave crests.



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Presentations and articles will be available at http://stockage.univ-brest.fr/~gula/Ondes/