Master Physique Marine (M1)

Fluides II

HW2:
The **short** gravity waves
Written answers are due for Wednesday February 13 (O. Arzel mailbox)

Homework n°1

The particle (or Lagrangian) velocity $\mathbf{u_L}$ is the velocity of a particle which was at $\mathbf{x}=\mathbf{x_o}$ at t=0. Its position at later t is then: $\mathbf{x}=\mathbf{x_o}+\int_0^t \mathbf{u_L}(\mathbf{x_o},t')dt'$. The velocity at point $\mathbf{x}$ is $\mathbf{u_L}(\mathbf{x_o},t)$ but also the Eulerian form $\mathbf{u}(\mathbf{x},t)$. Hence:

$$\mathbf{u_L}(\mathbf{x_o},t)=\mathbf{u}(\mathbf{x_o}+\int_0^t \mathbf{u_L}(\mathbf{x_o},t')dt',t)$$

a Taylor expansion for small displacements is:

$$\mathbf{u_L}(\mathbf{x_o},t)=\mathbf{u}(\mathbf{x_o},t)+(\int_0^t \mathbf{u}\ (\mathbf{x_o},t')dt').\nabla_{\mathbf{x_o}}\mathbf{u}(\mathbf{x_o},t)$$

Derive the linear approximation above. The first term dominates if k times displacement <<1. Show that particle motions in a surface wave (short wave limit) are then circles with radius $\eta_0\exp(kz)$ where η_0 is the amplitude of the surface displacement.

optional:

The second term when averaged over a period is called the Stokes drift. It is non zero in the horizontal direction. Compute it.

Homework n°2

Compute the streamlines of a surface gravity wave at a given time. Sketch the solution (in x z space). Add the surface displacement on the figure. Compare with the lagrangian description found in HW n° 1.

Hint: from the surface displacement, compute first u and w.

Homework n°3

You know that a storm has occurred on January 1, 2018 in the North Atlantic. You observe waves on a beach in Brest which have a 12” period. The next day you observe waves that have an 8” period. Estimate the location of the storm (assuming of course that the waves that you observe have been forced by the same storm).

Hint: Use a characteristic x-t diagram.