M2 - Marine Physics **2020–2021**

Internal Waves Evaluation (Homework)

Due date : Dec. 18, 2020

Exercise 1: Dispersion relation

We can write an equation for the evolution of internal waves as:

$$\frac{\partial^2}{\partial t^2}(\nabla^2 w) + f^2\frac{\partial^2 w}{\partial z^2} + N^2\nabla_h^2 w = 0, \tag{1}$$

where w is the vertical velocity, f the Coriolis frequency, and N is the Brunt-Vaisala frequency.

- Which assumptions are necessary to derive this equation?

We restrict ourselves to the vertical plane (x, z), and assume solutions in the form of plane waves: $w(x, z, t) = w_0 e^{\mathrm{i}(\vec{k}\cdot\vec{x}-\omega t)}$, with $\vec{k} = (k_x, 0, k_z)$.

- Compute the dispersion relation $\omega(k_x, k_z)$.

- Simplify the dispersion relation using the angle θ of the direction of propagation of the wave.

- Discuss what happens when $\omega \to f$, and $\omega \to N$.

Exercise 2: Critical layers for internal waves

We now consider a two-dimensional flow ($v = 0, \frac{\partial .}{\partial y} = 0$) of a stably stratified, non rotating, inviscid fluid in which there is a steady mean shear flow $U(z)$ in the x-direction. $p_0(z)$ and $\rho_0(z)$ denote the hydrostatic pressure and density. We consider small perturbations of the basic state:

$$\vec{u} = (u' + U, 0, w'), \quad p = p_0 + p', \quad \rho = \rho_0 + \rho',$$

where the perturbation (primed) quantities are functions of x, z, and t and are assumed to be small compared to the basic state variables.

- Starting from the linearized momentum, density and continuity equations under the Boussineq approximation ($\rho' << \rho_*$ the constant background density), show that the vertical perturbation velocity w' satisfies:

$$\frac{D_0^2}{Dt^2}(w'_{xx} + w'_{zz}) - \frac{D_0}{Dt}(U_{zz}w'_x) + N^2 w'_{xx} = 0 \tag{2}$$

where $\frac{D_0}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x}$, $N^2 = -g\rho_{0z}/\rho_*$, and ρ_* is a constant background density.

We assume solutions in the form of plane waves: $w'(x, y, z, t) = w_0 e^{\mathrm{i}(\vec{k}\cdot\vec{x}-\omega t)}$, with $\vec{k} = (k_x, 0, k_z)$ and a constant background vertical shear such that $U_{zz} = 0$.

- Compute the dispersion relation $\omega(k_x, k_z)$.

- Compute the vertical group velocity c_{gz}.

- What happens when the wave encounter a critical level in the flow, *i.e.* the background flow velocity is equal to the horizontal phase speed of the wave: $U(z_c) = \frac{\omega}{k_x} = c_\phi$?

Exercise 3: Viscous damping of internal waves

We consider a two-dimensional flow ($v = 0, \frac{\partial .}{\partial y} = 0$) without rotation ($f = 0$). We also introduce, on the right hand sides of the momentum and density equations, the linear damping terms $-Ku'$, $-Kw'$, and $-K\rho'$.

- Write the equation for the vertical velocity w'.

- Find the dispersion relation for a solution of the form: $w'(x, y, z, t) = w_0 e^{\mathrm{i}\vec{k}\cdot\vec{x}-\omega t}$, with $\vec{k} = (k_x, 0, k_z)$ and where ω has both real and imaginary parts.

- Explain what is the impact of the damping term.

Exercise 4: Generation and dissipation of Internal waves

- A number of processes related to interaction and dissipation of internal waves are shown figure 1. Identify and briefly describe the processes a,b,c,d,e,f, and g.

Exercise 5: Summary of research articles

Choose two research articles available on this page: `https://www.jgula.fr/Ondes/` and summarise them (in french or english) using your own words. Each summary should be about one or two pages long and should:

- State the question of the research and explain why it's important.
- State the hypotheses that were tested (if applicable)
- Explain the methods that were used
- State the different results and their interpretation
- State what the key implications were

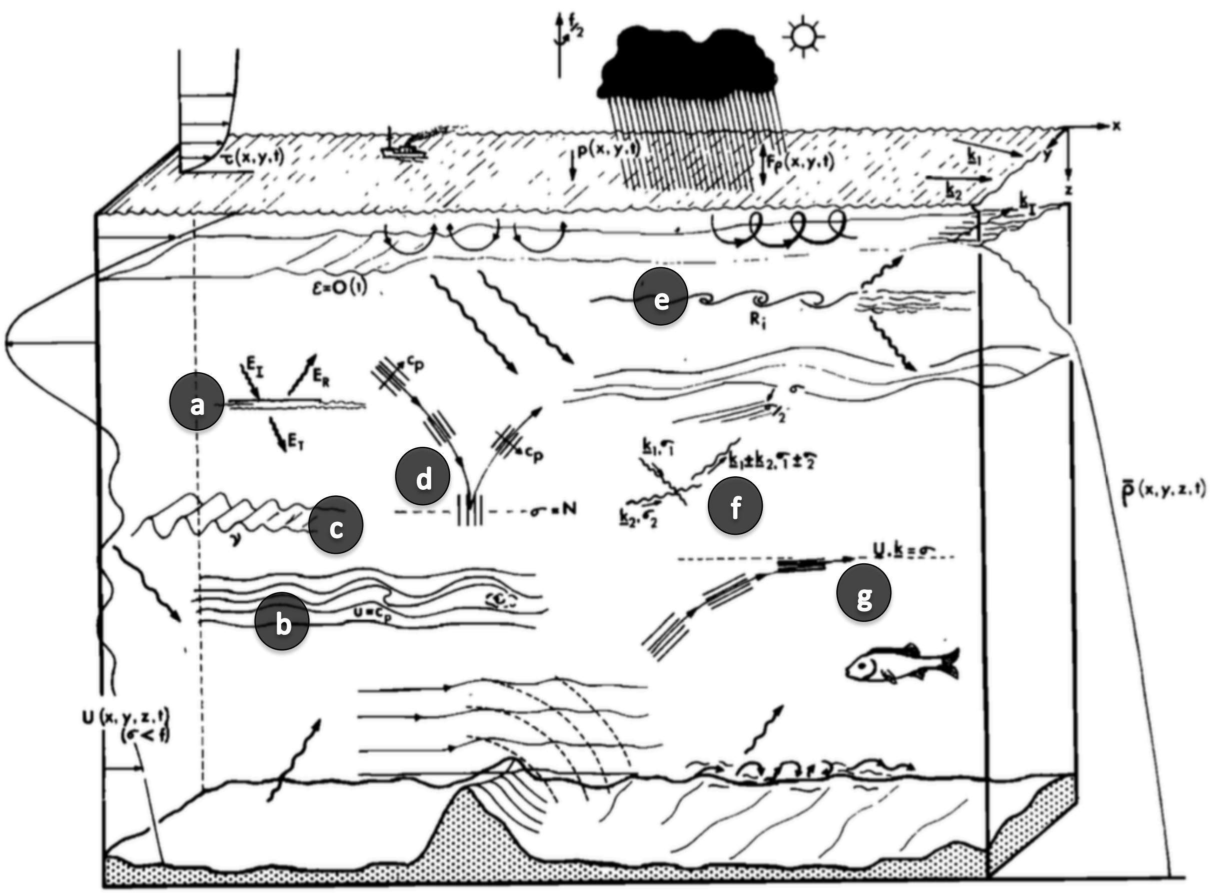

Figure 1: Pictural representation of some physical processes involved in the generation, interaction and dissipation of internal waves (From Thorpe, 1975).