The Excitation, Dissipation, and Interaction of Internal Waves in the Deep Ocean

S. A. THORPE

Institute of Oceanographic Sciences, Wormley, Godalming, Surrey, England

This paper is a broad review of the theoretical studies of the physical processes affecting the excitation, dissipation, and interaction of internal waves in the deep ocean.

We have very little knowledge of which physical processes are most important in controlling internal gravity waves in the deep ocean. We do not know precisely how or even where internal waves are generated. We do not know how, by the propagation and interaction of waves, the observed spectral shapes [*Garrett and Munk*, 1972*a*] are established. We do not know what processes dominate in the dissipation of internal waves. The complexity of three-dimensional wave propagation in a medium that is itself moving in the mean at speeds comparable to that of the waves, the uncertainty of the processes that dominate generation and dissipation, and the changes that the waves themselves may make to the mean density structure of the medium contrive to make this problem far more difficult than that of surface waves and thereby a much greater challenge to the theoretician. The surface wave problem (as explored for example by *Hasselmann et al.* [1973] Jonswap report) makes, however, a helpful analogy in that it focuses attention on the fundamental difficulty, that of accurately predicting the sources and sinks of energy (or preferably wave action density) in the controlling energy equation of an internal wave with a particular frequency, wave number, and direction. The object of this paper will be to review very briefly the several shreds of information that we have from theoretical studies that tell us about those physical processes that might be important in controlling internal waves with periods less than the local inertial period.

GENERATION: THE SOURCE OF ENERGY

Let us begin with internal wave generation. Imagine a vertical column of the deep ocean some 5 km in depth. Above is the air-sea interface, ruffled by surface waves, responding in a poorly understood way to the stress of the wind and the moving pressure disturbances in the atmosphere boundary layer, heated and cooled by the sun and radiation, evaporation, and precipitation. Below is the deep ocean floor, cracked and rucked by the earth's active mantle and smoothed by erosion and the deposition of sediments. Over this basement flow the transient ocean currents, tidal flows, and 'mesoscale' eddies—a spectrum interacting with this benthic range of roughness scales as well as with the atmosphere above and entering and leaving our column of ocean through its vertical sides. The column has deliberately been chosen to be far from the continental slopes. The generation of internal waves with tidal period at these boundaries is partly understood. Our concern is with the deep ocean beyond the range of these internal tides (although how far one must be is not certain, as will be explained when dissipation is discussed). Our water column is sheared by the transient horizontal currents. Vertical motions are generated by small-scale double-diffusive processes and by convective motions resulting from shear flow instability. Thin turbulent laminas occur, although generally the turbulence is at low Reynolds number. Fish and plankton abound, and long-term chemical reactions are taking place. This is the environment of the internal waves.

Atmospheric Generation

The upper layer of the ocean contains by far the highest density of energy of any part of the water column, and must a priori be regarded as a likely candidate for a source of internal waves. Several physical mechanisms have been discussed in the literature.

Traveling pressure field. Work on this topic dates back to Ekman's observations on 'dead water,' increased drag on a slow-moving craft in strongly stratified water due to the generation of internal waves. The internal wave patterns produced by a moving pressure point on the surface of a two-layer ocean have been described by *Hudimac* [1961] and *Crapper* [1967] [see also *Fedosenko and Cherkesov*, 1971, 1972]. The dispersion relation for these internal waves with frequency much greater than f, the local inertial frequency, is

$$\sigma^2 = \frac{gk(\rho_2 - \rho_1) \tanh kh_1 \tanh kh_2}{\rho_2 \tanh kh_1 + \rho_1 \tanh kh_2} \quad (1)$$

where σ and k are the wave frequency and wave number, respectively, and the fluids are of depth h_i and density ρ_i for $i = 1, 2$ ($i = 2$ is the lower layer). The fractional density difference $\Delta = (\rho_2 - \rho_1)/\rho_2$ is assumed to be small. The phase speed of the waves σ/k is less than $[g\Delta h_1h_2/(h_1 + h_2)]^{1/2}$, and if the speed of the pressure point is less than this critical value, both transverse and divergent wave systems are generated, similar in form to surface waves behind a moving ship (Hudimac, Figure 6). If, however, the speed of the pressure point exceeds this critical value, there are no waves traveling in the direction of the moving pressure that can keep up with it, and the wake consists of a set of divergent waves spreading laterally on either side of the line of motion of the pressure point (e.g., Hudimac, Figure 7). Their amplitude is greatest in a region near the pressure point (Hudimac, Figure 9). *Keller and Munk* [1970] considered the problem for a continuously stratified ocean and found a similar pattern of wave fronts for a three-layer-thermocline ocean. Waves of successively higher modes were found to be more and more concentrated to the line of motion of the pressure point. In general, the pattern is determined by the principle of stationary phase, and *Lighthill* [1967] has shown how this can be done if the dispersion relation is known. Figure 1 shows the shape of one of a set of wake or lee wave surfaces following a body moving horizontally at the surface of fluid of constant Brunt-Väisälä frequency N. (A 'constant N ocean' will hereafter be used to refer to an ocean in which the Brunt-Väisälä frequency N is constant.) When the fluid is rotating, the surface becomes more complex as a result of different propagation characteristics (which we shall discuss later), and the set of waves in the vertical plane through the line of motion of the body is shown in Figure 2. The analogous problem of waves produced by steady stratified flow over an isolated obstacle or ridge is treated in the same way and will be mentioned later.

There are a large number of papers on internal wave generation by a moving pressure field. Although *Polyanskaya* [1969] has shown how a plane pressure front will produce waves if its speed does not exceed the maximum speed of the internal waves and has estimated the amplitude of waves thus generated, the majority of theories depend upon a resonant coupling between the moving field and the internal waves. Resonance is possible for those waves with horizontal wave numbers $\mathbf{k}$ and frequency σ such that

$$\mathbf{k} \cdot \mathbf{u} = \sigma \tag{2}$$

and

$$k_s(\mathbf{u} \cdot \mathbf{k}/uk) = k \tag{3}$$

where $\mathbf{u}$ is the convection velocity of the pressure disturbance that has a wave number k_s in the direction $\mathbf{u}$. Such resonant waves will grow linearly in the absence of dissipation. This theory is a simple extension of that described by *Phillips* [1957] for surface wave generation by moving atmospheric pressures. The direction and speed of the waves is such that they keep in phase with the applied pressure distribution. The speed of the pressure field in the direction normal to the internal wave crests is equal to the internal wave speed, and the wavelength of the disturbance projected in the same direction is equal to the horizontal wavelength of the internal waves. *Keunecke* [1970] has demonstrated in the laboratory that standing internal waves may be produced by surface pressure fluctuations.

Forcing by a variety of different pressure fields has been considered, in particular by *Voyt* [1959], *Cherkesov* [1962, 1965], *Bukatov and Cherkesov* [1970], *Dotsenko and Cherkesov* [1971], and *Bukatov* [1971]. Further references are given by *Krauss* [1966].

In the last few years a more general approach to the problem has been taken, and attention has been centered not on the response to a particular pressure pattern but on the response to a general applied pressure distribution that allows the internal wave field to be expressed as a function of the spectrum of the pressure. *Leonov and Miropolsky* [1973] formulated the problem in this way and presented calculations for the response of both a two-layer ocean and a constant N ocean. The dominant waves are those that satisfy the resonance conditions (2) and (3). For a plane pressure disturbance $P_0 \sin(kx - \sigma t)$ traveling over the surface of an inviscid two-layer ocean with a speed σ/k corresponding to that of an internal wave of length k at the interface, so that

$$\frac{\sigma}{k} = \left(\frac{g\Delta}{k(1 + \coth kh_1)}\right)^{1/2}$$

where h_1 is the thickness of the upper layer and the lower layer is very deep, they find the mean rate of increase of the wave amplitude $\dot{A}$ is

$$\dot{A} = \frac{1}{2}\frac{P_0}{\rho_1}\left(\frac{\Delta}{gh_1}\right)^{1/2}(kh_1 \sinh kh_1 \exp 3kh_1)^{1/2} \tag{4}$$

(see Figure 3). This has a maximum for waves of length about 9 times the upper layer depth. If, for example, $h_1 = 50$ m, $\Delta = 5 \times 10^{-4}$, and $P_0/\rho_1 = 10^3$ cm^2 s^{-2}, corresponding to a pressure fluctuation of 1 mbar and moving at about 37 cm s^{-1} with a wavelength of 450 m, the internal wave amplitude would grow at a rate of $1.3 \cdot 10^{-3}$ cm s^{-1}, a little more than 1 m/d. For the continuously stratified ocean model of depth H and constant Brunt-Väisälä frequency N, the mean growth rate of the amplitude of the nth mode $\dot{A}_n$ is

$$\dot{A}_n = \frac{P_0}{\rho_0}\frac{n\pi k^2H^2N^2}{g(n^2\pi^2 + k^2H^2)^{3/2}(k^2H^2N^2 + f^2n^2)^{1/2}} = G_P \tag{5}$$

$$n = 1$$

where ρ_0 is the density at the surface.

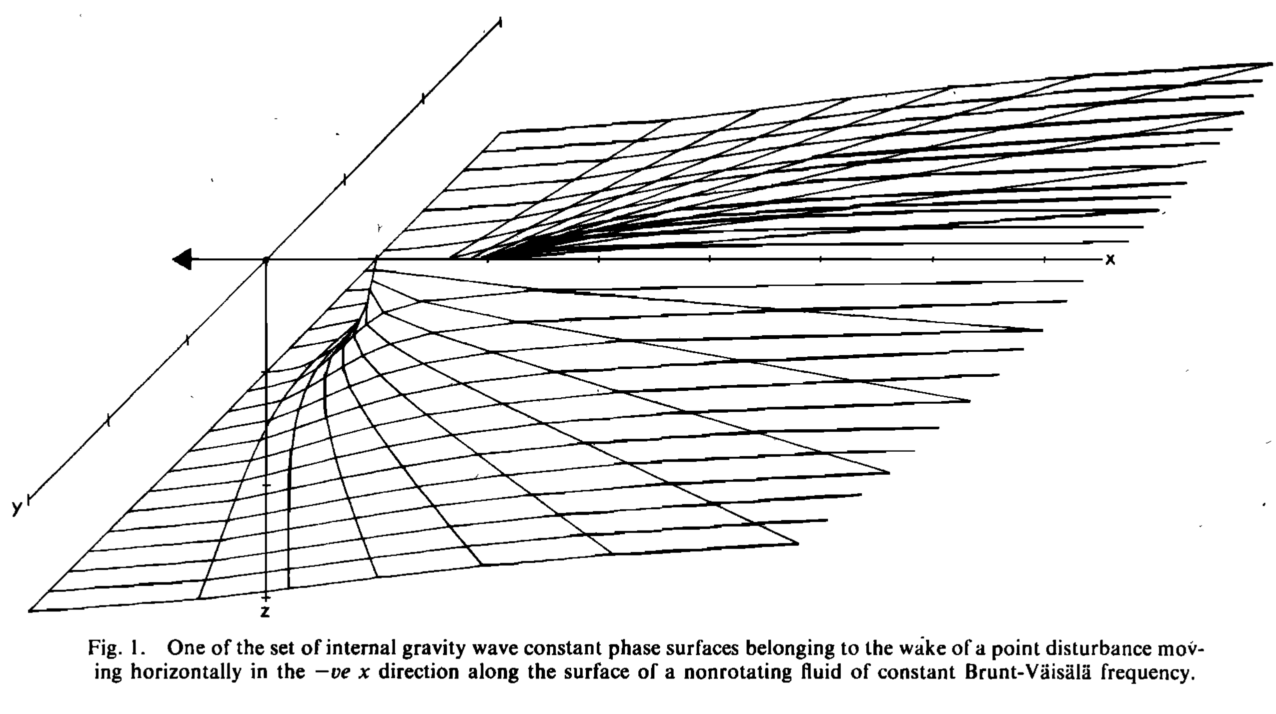

Fig. 1. One of the set of internal gravity wave constant phase surfaces belonging to the wake of a point disturbance moving horizontally in the $-ve$ x direction along the surface of a nonrotating fluid of constant Brunt-Väisälä frequency.

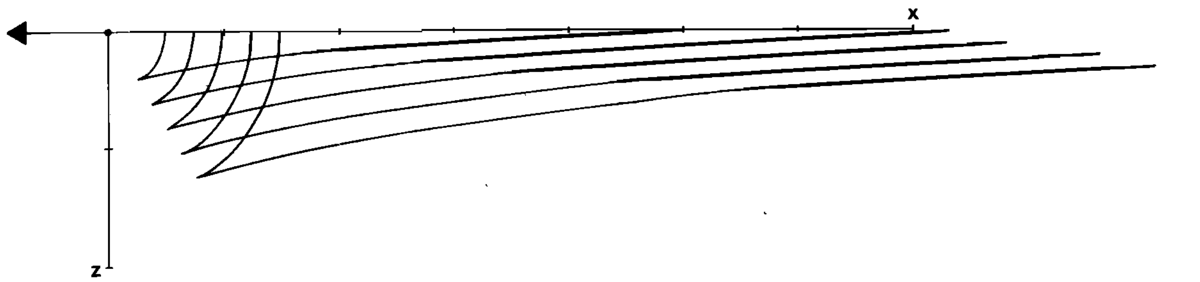

Fig. 2. The constant phase lines in the plane $y = 0$ of the wake behind a point disturbance moving horizontally in the $-ve$ x direction along the surface of a fluid of constant Brunt-Väisälä frequency N rotating about a vertical axis with angular velocity Ω for $\Omega/N = 0.05$.

The first-order mode will usually grow more rapidly than the higher modes.

Krauss [1972*a*] has computed the response functions for a constant N ocean, including variable eddy coefficients of viscosity, for forcing at various frequencies, σ, and wave numbers. He finds that five to ten modes are of importance when $\sigma > f$, but that except at low wave numbers the response to a moving wind stress pattern will usually dominate over that due to pressure variation, supporting an earlier conclusion by *Mork* [1968] [see also *Magaard*, 1971].

The problem thus satisfactorily formulated cannot yet, however, be applied to the ocean, for we have insufficient knowledge of the sea surface pressure spectrum in the range of wave numbers and frequencies (10 m–200 km and 1-min inertial period) of importance to the internal wave spectrum. (It might be possible to extrapolate the present wind tunnel and atmospheric boundary layer observations to lower wave numbers and frequencies by using appropriate scaling.) *Faller* [1966] has suggested that cloud streets may represent an important atmospheric scale for a resonant coupling in the pressure field, since their speed and horizontal scale correspond to those of internal waves.

Traveling stress field. Even less is known about the spectrum of the stress of the wind on the ocean surface than that of the pressure! *Tomczak*'s [1966, 1967] work showed how internal waves with frequencies above the inertial were mainly caused by vertical motion induced by divergent wind fields. This idea has recently been explored in a rather general form by R. H. Käse (unpublished manuscript, 1974), who has examined the response of the interior of a stratified ocean to a vertical velocity field induced at the bottom of the surface boundary layer. The manner in which the velocity field is induced is determined from the generating mechanism. The appropriate vertical velocity for a traveling stress field has been calculated by *Magaard* [1973], but the results of the theories await the appropriate specification of the stress field. As an example, Käse finds that for waves of frequency $2f$ and a stress pattern $\tau/\rho_0 = 1$ cm^2 s^{-2} moving at a speed 1 m s^{-1} the internal wave amplitude will grow at a rate of 2.7 m/d. *Krauss* [1972*a*, *b*] has also discussed this problem and calculated response functions as he did for the pressure field.

The majority of this work has been motivated by the requirement to understand the dynamic response of shallow seas (notably the Baltic), and the response of an essentially deep ocean, one which is so deep that waves traveling vertically with their group velocity do not reach the bottom in a time short in comparison with the duration of the forcing, requires a special study that it has not yet received but that by analogy with the waves formed at the floor of the ocean (see the section on propagation, reflection, critical layers, and bottom topography again) is quite feasible and should produce interesting results.

Traveling buoyancy flux. *Magaard* [1973] has calculated the vertical velocity induced at the lower boundary of a surface layer by the pressure field produced by variable buoyancy flux and applying Käse's theory has estimated the growth rate of the first-mode waves for which there is a resonant response to a moving periodic temperature perturbation $T = T_s \exp i(kx - \sigma t)$ at the surface of an ocean of constant N.

$$\dot{A}_1 = \frac{g\alpha\kappa\pi T_s}{H^2 N^2}(1 - f^2/\sigma^2) = G_B \tag{6}$$

say. Here κ is an exchange coefficient for buoyancy, and α is the coefficient of thermal expansion. Magaard compares G_B with the corresponding growth rate for first-mode waves produced under resonant conditions by a moving wind stress G_w. Their ratio is

$$\frac{G_B}{G_W} = \frac{g\alpha\rho_0\kappa T_s}{c\tau} \tag{7}$$

where $c = \sigma/k$ is the phase speed of the surface stress disturbance τ. Magaard quotes a typical value of $g\alpha\rho_0/c$ of 2×10^{-3} g cm^{-3} s^{-1}, corresponding to $c = 1$ m s^{-1} and $\alpha = 2 \cdot 10^{-4}$ °C^{-1}. Here κT_s may be as large as 10^2 cm^2 s^{-1} °C, and hence the efficiency of the fluctuating buoyancy flux in generating internal waves may be comparable to that of a wind that fluctuates with an amplitude of 3.5 m s^{-1} (corresponding to a stress τ of about 0.2 g cm^{-1} s^{-2}). At these values the growth rate of the first-mode wave amplitude is about 0.2 m/d for an ocean of depth 5 km with $N = 10^{-3}$ s^{-1}. The ratio G_B/G_P, where G_P is the growth rate caused by the pressure fluctuations given by (5), may also be found either directly from (5) and (6) or by comparing the scales of the effective vertical velocities produced by the pressure and buoyancy fields.

$$\frac{G_B}{G_P} = \frac{\alpha\kappa T_s g^{1/2} H^{1/2} \pi^2}{\Delta^{3/2}(P_0/\rho_0)} f(kH) \tag{8}$$

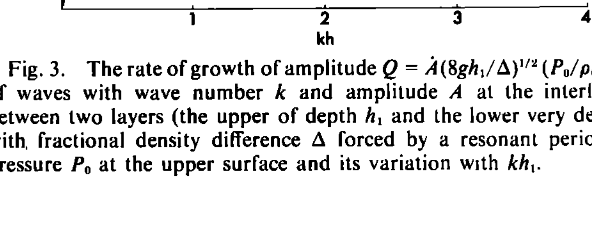

Fig. 3. The rate of growth of amplitude $Q = \dot{A}(8gh_1/\Delta)^{1/2}(P_0/\rho_0)^{-1}$ of waves with wave number k and amplitude A at the interface between two layers (the upper of depth h_1 and the lower very deep) with fractional density difference Δ forced by a resonant periodic pressure P_0 at the upper surface and its variation with kh_1.

where $f(x) = (1 + x)^{3/2}/x$. This has a minimum at $x = 2$, where $f(x) = 2.6$. The ratio will usually be large supporting a conclusion that of the three mechanisms so far considered the wind stress is likely to be capable of causing the most rapidly growing internal waves. Whether it is more appropriate to describe the forcing function in either mechanism as a number of discrete moving regions or in spectral terms must depend upon the climatological situation and perhaps upon the scales that are under review. However, there is the need to reexamine the way in which energy can be transferred to the deep ocean by internal waves of relatively high vertical wave number.

The interaction of a pair of surface waves. At first sight it appears highly unlikely that surface waves, with their relatively high frequencies and wave numbers and rapid phase speeds, will interact in a coherent way so as to generate internal waves. When, however, we recall that microseisms are generated by the seemingly improbable process of a second-order interaction of a pair of surface waves moving in opposite directions [*Longuet-Higgins*, 1950], we may be persuaded at least to consider the possibility. Hasselmann was the first to remark that a second-order coupling between a pair of surface waves and an internal wave was possible when

$$\mathbf{k}_1 - \mathbf{k}_2 = \mathbf{k} \qquad \sigma_1 - \sigma_2 = \sigma \tag{9}$$

where $(\mathbf{k}_i, \sigma_i)$, $i = 1, 2$ are the surface wave numbers and frequencies and $\mathbf{k}$, σ are the horizontal wave number and frequency of the internal wave. These conditions may be satisfied by a pair of surface waves with almost equal wave numbers and frequencies, and the locus of surface wave numbers that can take part in the interaction is given by *Phillips* [1966, Figure 5.6].

The calculation of the interaction coefficients is tedious but has been carried out for a two-layer fluid [*Thorpe*, 1966; *Nesterov*, 1972] and for a three-layer fluid, the central layer only being stratified with exponential gradient [*Brekhovskikh* et al., 1972]. An earlier calculation by Thorpe [1966] for a fluid of exponential density gradient was found to be incorrect, since making the Boussinesq approximation was apparently not valid. The coefficients are reported to have been calculated by *Kenyon* [1968] but are not given explicitly. The amplitude A_n of an internal wave of mode n is given by an expression

$$\dot{A}_n = C_n a_1 a_2 \tag{10}$$

where a_1 and a_2 are the surface wave amplitudes, which vary very slowly, and C_n is an interaction coefficient.

For a two-layer ocean the interaction coefficient is

$$C = (k_1\sigma/2)(1 + \cos\theta)(e^{-(k_1+k_2)h_1} - e^{-kh_1}) \tag{11}$$

where θ is the angle between the surface waves. Brekhovskikh et al. plot the maximum rate of increase of the amplitude of the first-mode internal wave of their three-layer model against θ for various layer thicknesses. These are compared in Figure 4 with the corresponding rates of growth for the two-layer model, in which h_1 is taken to be equal to the mean depth of the thermocline in the model of Brekhovskikh et al. The numerical values of the growth rates are similar in the two models, although they are smaller for the two-layer model. The angle between the surface waves that gives the most rapid response is small, and there is good agreement between the two models. If $\dot{A}_1 = 0.3a_1a_2k_2N$, typical of growth rates shown in Figure 4, we take $a_1 = 1$ m, $a_2k_2 = 0.1$, and $N = 10^{-3}\text{s}^{-1}$, and then $\dot{A} = 3 \cdot 10^{-3}$ cm s^{-1}, or 2.6 m/d. This rate is at least comparable with that of the processes considered earlier. Higher modes are found to have slower rates of growth. The small angle between the surface wave numbers and their almost equal magnitudes imply (from (9)) that the fastest growing internal wave is almost at right angles to the surface wave direction. The directional spectrum of surface waves has typically a form like $\cos^{2s}(\theta/2)$, where s may be as large as 14 for long swell waves (and is usually much smaller for shorter waves) and thus narrow, but the spectrum is not so narrow as to exclude waves that will interact to give the largest growth rates. The growth rate of the first mode in a constant N ocean is given by Brekhovskikh et al. as

$$\dot{A}_1 = \frac{a_1k_1a_2N\pi k_1^3H^3(k/k_1)^{1/2}(4 - k^2/k_1^2)^2}{2(\pi^2 + 4k_1^2H^2)(\pi^2 + k^2H^2)^{3/2}} \tag{12}$$

If we take $a_1k_1 = 0.1$, $k_1H = 0.1$, $k/k_1 = 0.2$, and $N = 10^{-3}\,\text{s}^{-1}$ and $a_2 = 1$ m, we find $\dot{A}_1 = 3$ cm/d!

Joyce [1974] has made laboratory experiments on the generation of internal waves by interacting surface standing waves and has shown that their growth rates are fairly well predicted by the second-order theory. However, *Kenyon* [1968], using a constant N model and oceanographic data to evaluate the energy transfer rates, concluded that the time for surface wave–internal wave interaction was much larger than the time for mutual surface wave interactions (typically 1 day) and that energy would be redistributed within the internal wave field by resonant internal wave interaction more rapidly than it would be supplied from surface-internal wave interaction. This does not of course mean that the interaction is insignificant in the energy balance of internal waves. We need to know how rapidly the internal wave field may be affected by other factors before any such conclusion can be reached. The interaction coefficients for the shallow thermocline model are an order of magnitude larger than those for a deep constant N ocean mainly because of the improved fit of the vertical eigenfunctions, and Kenyon's conclusions should be reconsidered. A preliminary calculation, however, suggests that his first result may yet hold true. Kenyon's is one of the few studies in which the modification to internal wave spectra by the generating field has been considered, being possible because surface wave spectra, the input function, is known in this case. This situation highlights the requirement for more data with which the alternative theories can be tested.

Ekman layer instability and parametric instability. Passing reference is made here to two mechanisms that appear potentially viable as generators of internal waves in the ocean. Their application has so far been to the atmosphere. Instability of a stratified Ekman boundary layer and the generation of internal waves has been described by *Kaylor and Faller* [1972]. Instability in the ocean boundary layer may produce waves in a similar way. The problem may be regarded in broader terms as that of the way in which the surface boundary (mixing) layer is coupled to the deep ocean. Such a turbulent boundary layer has also been considered as a source of internal waves [*Townsend*, 1968]. In this case, however, it is convenient to analyze the internal waves in terms of groups or wave packets, a technique which recommends itself in view of the remarks made earlier about the development of normal modes. Townsend finds that the dominant waves are those that satisfy a resonance condition like (2) and (3). Linden (private communication, 1974) has observed internal waves generated by turbulence in his grid experiments.

Parametric instability in the atmosphere due to diurnal forcing has been proposed as a source of waves with frequencies that are multiples of ½ per day by *Orlanski* [1973], and diurnal or other regular period forcing in the upper layers of the

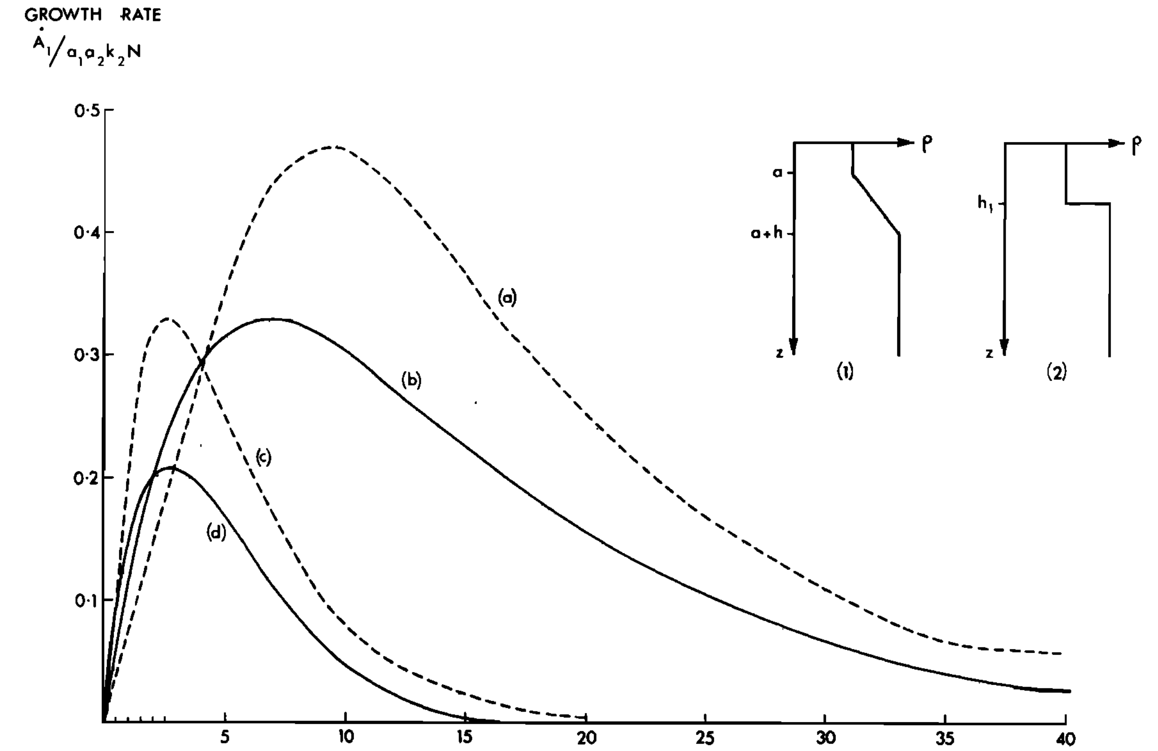

Fig. 4. Comparison of growth rates of internal waves generated by resonant interaction of a pair of surface waves in a fluid with continuous density as shown in inset 1 or a step as shown in inset 2. $N = (g\Delta/h)^{1/2}$ and $h_1 = a + h/2$. Curves *a* and *b* correspond to inset 1 and 2, respectively, with $k_1a = 1$ and $h = 10a$ and the same fractional density difference Δ. Curves *c* and *d* are the similar curves for $k_1a = 10$ and $h = a$. Smaller values of h_1 for the model in inset 2 give growth rates that correspond more closely to the curves of the model in inset 1.

ocean may be sufficient to force internal waves. We shall return later to the problem of parametric instability within the depth of ocean and to turbulent boundary layer generation.

Generation Due to Bottom Topography

The flow of a stratified ocean over a rough bottom topography has many features in common with that of air flow over mountains, which has been extensively studied by meteorologists. An example of the pattern of lee waves produced by uniform flow of a constant N fluid over a two-dimensional ellipsoidal ridge is given by *Huppert and Miles* [1969, Figure 4]. The semicircular pattern of waves of constant phase in this figure is of course exactly that seen in the plane $y = 0$ in the pattern behind a moving pressure point shown in Figure 1. When the speed is not uniform with height, small perturbations to the flow may be analyzed in terms of Fourier components of vertical velocity that satisfies Scorer's equation [*Scorer*, 1949]. This equation has been solved in a number of particular cases to successfully model the pattern and amplitude of lee waves that can be generated by an obstacle on the ground.

The thickness δ of the benthic boundary layer is about u_*/f (where $u_* = (\tau/\rho_0)^{1/2}$, τ being the stress) and is probably about 0.2 cm s^{-1} for a flat bottom [*Wimbush and Munk*, 1970]. Hence δ is about 20 m. The most energetic eddies are likely to have a comparable scale, and so the horizontal scale of the internal waves generated by the turbulence [*Townsend*, 1968] will be of the same order and quite small. However, in regions of very rough topography, where flow separation may occur and enhance the turbulence, longer waves may be generated.

The flow of an oscillatory current over rough topography is a much more important problem in the deep ocean than is found in the atmosphere because of the presence of the dominant signal of the barotropic tides. *Cox and Sandstrom* [1962] formulated the problem in terms of an arbitrary bottom topography and solved it in terms of normal modes. The conversion of tidal energy from the barotropic to the baroclinic tides is significant, amounting perhaps to as much as one sixth of the energy of the barotropic tides [*Munk*, 1966]. The problem has recently been reconsidered by *Bell* [1975], and I shall refer to his results later. Tidal flow over an isolated ridge [*Baines*, 1973] may also be an important source of internal tidal waves in those areas where such features exist.

The very different problem of internal tsunamis generated by earthquakes has been examined by *Cherkesov* [1968], but these are probably not significant in the total energy balance.

Internal Generation

Very little is known about the physics that controls the cascade of energy from large scales to internal wave scales or even whether such a cascade does occur. It is known that internal waves can interact to transfer energy to lower frequencies, but the direction of the net transfer is unknown. It is likely that internal waves may be generated in the process of decay of large-scale circulations and mesoscale eddies by breaking or baroclinic instability, but no measurements or estimates are available. The gap that often separates internal gravity waves from waves with periods of 4 or 5 days in the frequency spectra of horizontal kinetic energy suggests that no significant homogeneous local cascade of energy occurs between low-frequency motions and internal waves [*Rhines*, 1973]. The transfer of energy between different internal waves by resonant interaction and as a result of internal wave breaking will be described later.

PROPAGATION OF INTERNAL WAVES

Propagation, reflection, critical layers, and bottom topography again. Internal wave modes and waves on an interface between two layers (when there exists only one internal mode) have been referred to already. Various analytical solutions for the eigenfunctions are known, and for more realistic density profiles they can be calculated numerically [*Fjeldstad*, 1933]. Their general properties have been summarized by *Yih* [1966].

We are here concerned with the propagation of a group or packet of internal waves that for generality we may suppose has been generated deep within the depths of the ocean. We assume that variations in N are small over one wavelength and that the Richardson number is sufficiently large, so that a WKB approximation may be made. The frequency of the waves σ relative to the fluid must lie between N and f and is given locally by

$$\sigma^2 = \frac{N^2(l^2 + m^2) + f^2n^2}{l^2 + m^2 + n^2} \tag{13}$$

where $\mathbf{k} = (l, m, n)$ is the wave number. The group velocity

$$\mathbf{c}_g = \left(\frac{\partial \sigma}{\partial l}, \frac{\partial \sigma}{\partial m}, \frac{\partial \sigma}{\partial n}\right)$$

is at right angles to $\mathbf{k}$ (that is $\mathbf{k}\cdot\mathbf{c}_g = 0$), so that as the group propagates the lines of constant phase move at right angles and toward the horizontal ($N > f$ is assumed). The inclination of the direction of propagation, the group velocity, to the horizontal α is given by

$$\tan \alpha = [(\sigma^2 - f^2)/(f^2 - \sigma^2)]^{1/2} \tag{14}$$

so that near-inertial waves propagate almost horizontally and high-frequency waves propagate vertically. In reflection from a rigid boundary (maintaining frequency) the waves retain their inclination to the horizontal. If the waves propagate into a region where N falls to a value less than that of the wave frequency, they will be reflected, their group velocity becoming vertical near the surface $N = \sigma$. The waves may be trapped in a wave guide if N falls below σ both above and below the group. Laboratory experiments by *Mowbray and Rarity* [1967] and *Stevenson* [1968] have demonstrated these properties, as well as the patterns of waves behind moving bodies.

The propagation of waves in moving media requires special care. The internal wave packet retains not the wave energy E (the integral of $\frac{1}{2}\rho|\mathbf{u}|^2 + \frac{1}{2}\rho N^2\xi^2$ over the volume of the group, where ξ is the vertical particle displacement and $\mathbf{u}$ the perturbation velocity), but instead there is conservation of wave action.

$$\mathbf{A} = E/\sigma - \mathbf{U}\cdot\mathbf{k}$$

where $\sigma - \mathbf{U}\cdot\mathbf{k}$ is the intrinsic frequency of the waves, the frequency measured in a frame of reference moving with the fluid. (The fluid speed is $\mathbf{U}$.) The energy equation [*Bretherton and Garrett*, 1968; *Garrett*, 1968] becomes

$$\frac{\partial \mathbf{A}}{\partial t} + \mathbf{c}_g\cdot\nabla\mathbf{A} + \mathbf{A}\nabla\cdot\mathbf{c}_g = 0$$

This may be generalized to the form

$$\frac{\partial \mathbf{A}}{\partial t} + \mathbf{c}_g\cdot\nabla\mathbf{A} + \mathbf{A}\nabla\cdot\mathbf{c}_g = S \tag{15}$$

where $\mathbf{A}$ represents the wave action of a wave of given frequency and horizontal wave number moving through a moving medium in which there are sinks and sources of energy of waves with the same frequency and wave number. The S would include energy exchange through resonant interactions, wave generation, or dissipation and might be considered as a sum of contributions for these various sources and sinks. The momentum carried by the waves is independent of height and is not transferred to the fluid unless the waves break or encounter a 'critical layer' [*Booker and Bretherton*, 1967] at which their intrinsic frequency falls to zero or equivalently where their horizontal speed becomes equal to the speed of the fluid. If the Richardson number is large, the waves are attenuated at a critical layer, the group velocity becoming more and more nearly horizontal as the waves approach the layer. In a steady state their Reynolds stress is reduced by a factor $\exp -2\pi(\frac{1}{4} - Ri)$ on the other side of the layer. In practice it is supposed (although it is not convincingly demonstrated experimentally) that waves will break down at the critical layer, transferring their momentum to the mean flow and forming turbulent regions. (This behavior will in turn modify the mean flow and change the depth of the critical layer.)

Bretherton [1969] has calculated the momentum transferred to the atmosphere by lee wave drag over the hills of North Wales, and a similar calculation for a more complicated oceanic flow including a tidal component has been made by *Bell* [1975], taking a rough topography of abyssal hills typical of much of the Pacific Ocean floor. The topography is specified by its wave number spectrum, which for scales smaller than 10 km is found to obey a k^{-3} law and is flat at larger scales. Near the bottom the mean flow is topographically steered by the larger roughness scales, and this results in a flow decreasing in height and of direction that varies over scales of some tens of kilometers. The short internal lee waves generated by this mean flow have a high probability of encountering critical layers in the lower 1 km of the ocean and are assumed to be absorbed. The energy extracted from the mean flow and dissipated by internal waves in this way is estimated to be of about 1 erg cm^{-2} s^{-1}. (*Faller* [1966] estimated that about 3 ergs cm^{-2} s^{-1} is transferred from the atmosphere to the mean ocean currents, and the dissipation thus appears to be a significant amount.) The internal waves of tidal frequency can, however, propagate upward through the water column but with a wave Richardson number decreasing as N^{-2}. It is estimated that this will fall well below unity in the main thermocline, where N is large, and that there the internal tidal waves will break (see the section on internal wave breaking) losing energy to turbulent kinetic energy or to the mean potential energy of the ocean. The internal tidal waves have a total residence time of about 12 days. Wave dissipation at some level in the ocean makes a description in terms of normal modes (Cox and Sandstrom) improper, although since the conversion of energy from the barotropic tides to the baroclinic is a process controlled by the bottom boundary condition, their estimate of energy transfer is not affected and indeed it is supported by Bell's conclusions.

Resonant interactions. A resonant second-order interaction between three internal waves of wave numbers $\mathbf{k}_i$ and frequencies σ_i, $i = 1, 2, 3$, may occur if

$$\mathbf{k}_1 \pm \mathbf{k}_2 \pm \mathbf{k}_3 = 0 \qquad \sigma_1 \pm \sigma_2 \pm \sigma_3 = 0 \tag{16}$$

Here $\mathbf{k}_i$ is a horizontal wave number. If the Brunt-Väisälä frequency is constant or we consider an interaction of groups or packets of internal waves in a medium in which N varies slowly so that the WKB approximation is valid, then (16) can be regarded as a condition on the three-dimensional wave numbers. In other circumstances a matching condition between the internal wave eigenfunctions must be satisfied. (If the product of one pair of eigenfunctions is expressed as a sum of the (complete) set of possible eigenfunctions, the coefficient of the eigenfunction belonging to the third wave in this sum must be nonzero. The same matching was necessary for the interaction of a pair of surface waves with an internal wave, and it was because this coefficient was small in the case of a first-mode internal wave in a constant N ocean that the growth of the internal wave was slow, see (12).)

It is now well established that resonant interactions can lead to a significant modification of an internal wave. The laboratory experiments of *Davis and Acrivos* [1967] with a thin interface and those in a constant N fluid by *Martin et al.* [1972] for progressive internal waves and by *McEwan* [1971] and *McEwan et al.* [1972] for standing internal waves have convincingly demonstrated that the interactions occur and at small times grow in accordance with the predictions and linear theory and that they can rapidly lead to the decay of single wave mode. The experiments have, however, shown that it is very difficult to predict exactly what triads (or higher order sets) of waves may take part in the decay of a single wave mode or which waves may result from the interaction of a pair of forced waves. Martin et al. point out that although a single mode will be unstable in a triad interaction to generate a pair of waves of lower frequency in accordance with a theorem of *Hasselmann* [1967], four-wave (quadrad) interaction may lead to the development of higher-frequency waves.

A variety of more complex interactions can take place. Internal waves may be trapped as a result of their nonlinear interactions by periodic variations of density gradient or of weak horizontal steady current. This trapping has been described by *Phillips* [1968] and occurs when the vertical component of the wave number is half that of the density gradient or current variation. Side band instabilities of the kind that cause the classical Stokes surface wave to become unstable are possible in the ocean thermocline but are probably of less importance than second-order resonant interactions [*Thorpe,* 1974]. Interactions between a pair of internal waves and a single surface wave can occur [*Smith,* 1972] and may play a part in the overall energy balance. In particular regions, internal waves can disturb the pattern of surface waves by modifying the surface currents [*Gargett and Hughes,* 1972; *Cavanie,* 1972]. A parametric interaction with exponential growth rates similar to that described by *Orlanski* [1973] may occur between large-scale internal waves and waves of small scale and of half the frequency of the larger-scale wave, and this behavior has been examined and demonstrated in a laboratory experiment by A. D. McEwan and R. M. Robinson (unpublished manuscript, 1974). The finest scales of instability are favored in the lowest-frequency modes and should be most common when N is a maximum. Basing their calculations on the *Garrett and Munk* [1972*a*] spectrum of internal waves, they show that this parametric instability could occur in the ocean at scales down to that of the finest observed microstructure. In *McEwan*'s [1971] laboratory experiments a flow breakdown occurs that leads to the appearance of sharp density discontinuities or 'traumata,' and McEwan and Robinson suggest that these are the result of parametric instability in the flow. They infer that the layered structures found in the ocean may similarly result from parametric instability. Possibly *Lazier*'s [1973] observations of internal waves, which distorted the thermocline structure of a lake, may be due to the same phenomenon.

Kenyon [1968] examined the energy transfer rates for low-order modes in a constant N ocean basing his calculations on real temperature data and an assumption that the spectra were horizontally isotropic. The interactions were disappointingly found to occur very much more slowly than surface wave interactions for waves of comparable slope. However, in view of the likely importance of internal wave interactions on the source terms in the equation of wave action density, this problem deserves careful reconsideration. In particular, it now seems important to examine interactions between higher-order modes. *Müller and Olbers* [1975] have been working on this problem and so too has McComas at Johns Hopkins. We can hope for some progress before long.

Finite Amplitude Effects

The effects of finite amplitude will be mentioned later when wave breaking is discussed. Here we note that in a nonrotating two-layer ocean (or one in which the density increases as tanh az, where z is the depth and a is an inverse length scale that characterizes the shape of the profile) the parameter that determines whether the wave shape differs significantly from a sine wave is

$$\epsilon = A\lambda/h_1^2 \quad \text{or} \quad A\lambda a^2 \tag{17}$$

where A is the wave amplitude and λ the wavelength, and the ocean depth is supposed large in comparison with λ ($h_2 > \lambda$ [*Thorpe,* 1968]). When $\epsilon = O(1)$, the wave shape will be modified by harmonics of the linear sine wave solution (for example, if $\lambda = 1$ km and $h_1 = 50$ m, $\epsilon = 1$ when $A = 2.5$ m). If ϵ is much greater than $O(1)$ then other nonlinear phenomena may occur, for example, the development of internal surges. *Miropol'skii* [1973] has suggested that the non-Gaussian probability density histograms of temperatures in the seasonal thermocline may be partly the result of nonlinear effects. Second-order 'Stokes drift' currents are generated by internal waves and may sometimes be significant [e.g., *Wunsch,* 1973]. A complete discussion of nonlinear effects is beyond the scope of this paper.

INTERNAL WAVE DISSIPATION: THE ENERGY SINKS

We have already referred to internal wave attenuation by critical layer absorption (section on propagation, reflection, critical layers, and bottom topography again), and here we consider other causes.

Viscous attenuation. *LeBlond* [1966] examined the problem of the attenuation of internal waves in a constant N fluid, making estimates of the horizontal and vertical viscous eddy coefficients. He concluded that internal waves of horizontal scale less than 100 m were strongly attenuated and would decay in a few cycles, but that internal tides that have a length of about 200 km will propagate for 2000 km or more, although standing oscillations in ocean basins are not possible. The deep ocean must be very far from land to avoid the effects of internal tides propagating from continental slopes!

Attenuation due to turbulent layers and microstructure. Parts of the ocean are known to be horizontally layered, and horizontally elongated turbulent patches occur, perhaps resulting from internal wave breaking, shear instability of the mean or transient flow, or double-diffusive effects [*Turner,* 1973]. These will tend to distort and modify waves that having a vertical component of motion, pass through them. The statistical problem has been examined by *Miropol'skii* [1972] and by *McGorman and Mysak* [1973]. The phase speed of internal waves is found to be decreased by the presence of fine structure. Recently, Mysak has studied the transmission of wave energy through a finite depth layer of fine microstructure and has shown how the high wave numbers may be attenuated or reflected.

Internal wave breaking. As yet very little is known about internal wave breaking. We know from the experiments of *Davis and Acrivos* [1967] that when waves of different modes are present on a diffuse interface, overturning may occur owing to the distortion of one mode in the velocity field of the other. For a single wave mode, *Orlanski and Bryan* [1969] demonstrated that unstable density gradients may occur where the local horizontal particle speed exceeds the phase speed of the wave, and *Orlanski* [1972] and *Orlanski and Ross* [1973] have demonstrated in laboratory and computer experiments what happens in standing waves when this (or an equivalent) condition is satisfied. The condition may be interpreted as a condition on the wave amplitude (or wave slope). For waves that are larger than a critical amplitude the particle velocity exceeds the phase velocity, and unstable gradients occur. These unstable gradients lead to convective instability in a small volume of fluid. This is not a complete breakdown of the wave, although energy will be extracted and transferred into turbulent kinetic energy and into increasing the mean potential energy of the fluid. A similar breakdown in atmospheric lee waves leads to rotor formation and possibly rotors may also be produced in the ocean. No estimates have yet been made of the total energy that may be dissipated through these processes in the ocean.

Shear instability.. Internal waves may cause local overturning and turbulence and lose energy by shear instability. For this condition to happen the shear generated by the waves, added to any preexisting shear, must become so large that the local Richardson number falls below a certain critical value (which is ¼ for many flows) and remains below for a time long enough for shear instability to grow and produce unstable density gradients or billows. *Woods* [1968] has observed this happening in the Mediterranean thermocline. Billows are observed to grow near the crests of internal waves, that is, where their shear is greatest. A description of the instability at an interface between two homogeneous layers (Kelvin-Helmholtz instability) in a laboratory experiment has been given by *Thorpe* [1973*a*]. Most of the kinetic energy lost by the flow is transferred to turbulent kinetic energy. A small fraction, typically 10%, is transferred to potential energy in raising the center of gravity of the mixed fluid. The effect of the instability is to increase the thickness of the interface to cause a rapid diffusion. The diffusion of momentum is more significant than that of density, at least in the high Prandtl number laboratory experiments, and the final Richardson number in the layer when turbulence has subsided is near to ⅓. *Thorpe* [1973*b*] has estimated that as much as 16% of the kinetic energy dissipated may be radiated as internal waves if the layers above and below the interface are stratified with values typical of the seasonal thermocline. The horizontal scale of the waves corresponds to the dominant horizontal scales in the turbulence, typically the billow scale, about 7 times the interface thickness.

There is a second form of instability that is very common in laboratory experiments on stratified flows and that may lead to the maintenance of layers in the ocean. This may occur when the vertical scale of the velocity variation is much greater than that of the density [*Hazel*, 1972] and leads to a traveling growing disturbance. It is probable that long waves on a thin interface are more prone to this instability than to Kelvin-Helmholtz [*Thorpe*, 1968, 1973*b*]. *Phillips* [1966, p. 187] suggested that the criterion $Ri = 1/4$ might be used to define a saturated internal wave spectrum, one in which there was incipient breaking, and deduced that for long waves of the first mode in the thermocline the displacement spectrum must depend on k^{-2} (where k is the internal wave number). *Garrett and Munk* [1972*b*] have examined the net buoyancy flux that can be produced by shear instability in their spectrum model, but although this can be interpreted in terms of an eddy diffusivity, its numerical evaluation is very uncertain.

Frankignoul [1972] has shown that convective instability rather than a shear instability may be expected for internal waves of large frequency. However, *Garrett and Munk* [1972*b*] found that in the ocean, shear instability will be more important because much of the internal wave energy is in the small near-inertial frequencies. Garrett and Munk estimate that about 5 ergs cm^{-2} s^{-1} of energy may be dissipated from internal waves in the ocean by shear instability, with the result being about 9 days to dissipate the total internal wave energy of approximately $4 \cdot 10^6$ ergs cm^{-2}.

CONCLUSION

The discussion is summarized in Figure 5, which shows in diagrammatic form the various physical processes that have been mentioned. (As a challenge, the reader is left to decipher the symbols himself.)

In planning this paper, I had intended to treat waves in the thermocline (especially the seasonal thermocline) separately from those in deeper layers, particularly as the former appear to respond more readily to atmospheric forcing. This distinction was soon lost, although I hope that it has been clear which situation I had in mind at any particular time. The two-layer model is of course more appropriate to the thermocline, and the constant N model to the deep ocean.

The basic problem remains that of identifying and quantifying the sources and sinks of energy in the spectrum of internal waves. There are areas in which a concentrated theoretical effort might produce important results. The first is that of resonant interactions between internal waves and waves with mean currents to discover what importance this has on the energy balance (15). This is a problem that *Müller and Olbers* [1975] have studied. There is then the whole problem of wave generation by the atmosphere. This problem might sensibly be reformulated in terms of wave packets generated near the surface, and the problem would then be one of properly coupling the oceanic surface boundary layer (the 'mixed' layer or Ekman spiral) through the seasonal thermocline to the interior. The important practical problems of determining the spectrum of pressure and, more important, stress needs attention, and better understanding of the dynamics of the boundary layer, its response to wind, and the process of coupling of air to boundary layer and of boundary layer to interior in the wave number-frequency range of internal waves deserves much more experimental study. There is need to relate the internal wave spectra to external parameters of the system, for example, to an overall Richardson number. A further useful field of investigation is that [see *Miropol'skii*, 1973] of describing internal wave properties in terms of probability theory, extending the available results from surface waves (relating, for example, to the probability of waves of maximum height and to the zero crossing frequency), and relating this to a description of waves in the spectral form.

The source-sink function S in the action density balance (15) may be thought of as a sum of contributions. This is a helpful concept because it may allow us to consider and assess the importance of various contributions separately, and also to divide the ocean into regions in which the contributions are of different relative importance and dominated by different physical phenomena. A further useful subdivision is in frequency, and it is important to identify each physical process with the internal wave frequency range in which it is impor-

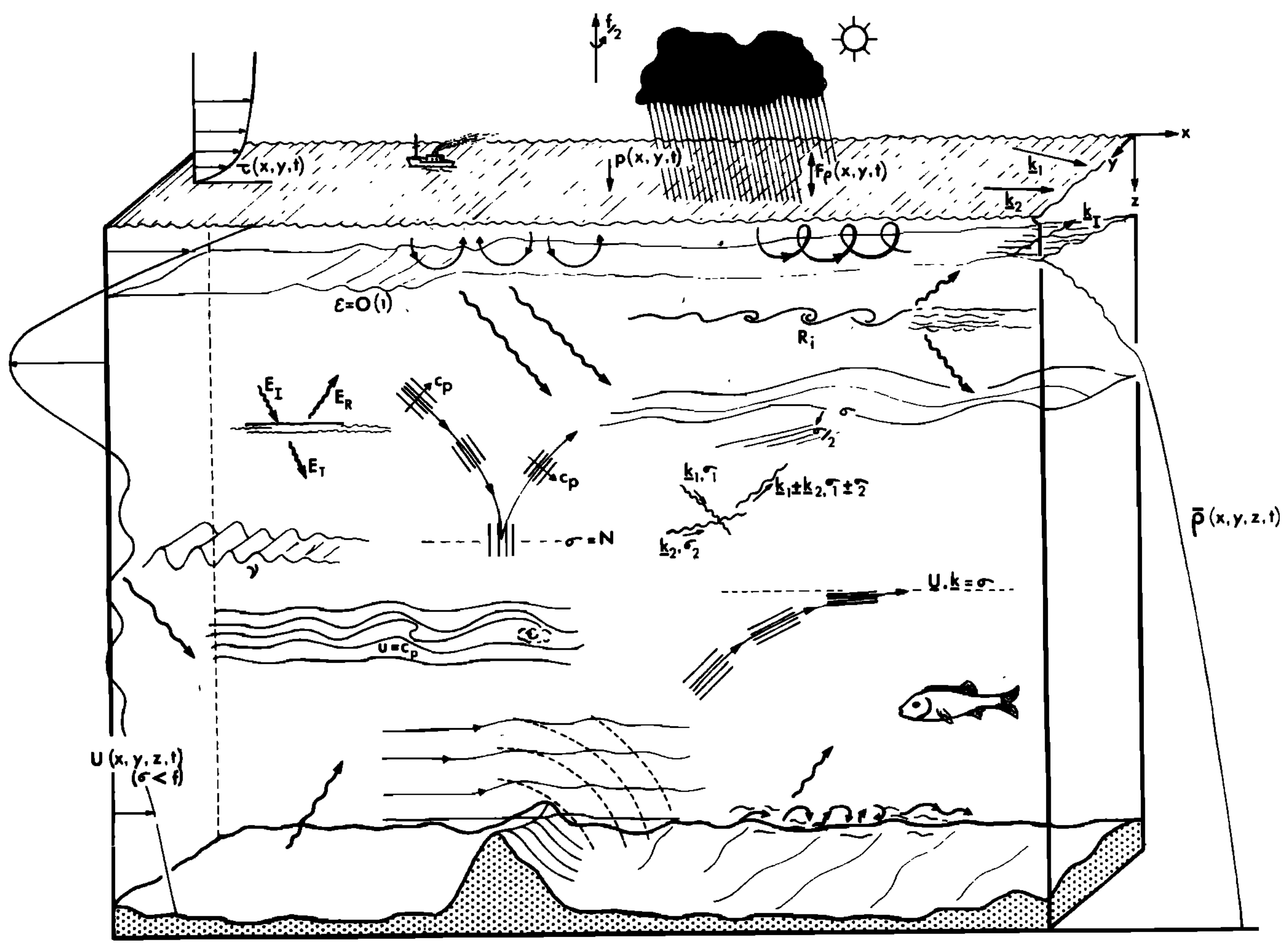

Fig. 5. Physical processes affecting internal waves.

tant. If, for example, we are considering a part of the ocean that includes the seasonal thermocline, we should consider the local input from the atmosphere in the source terms (as well as resonant interactions and the dissipation processes, although the latter may be more important at high frequencies) but not an input from bottom topography. In the interior of the ocean the local generation terms may possibly be insignificant, and the source-sink term S dominated by resonant interactions and dissipation. A difficult but important problem is that of formulating the various source-sink contributions in terms of parameters of the physical processes that can be observed by experiments in the ocean and recognizing the limitations of existing (or possible) measuring instruments. Experiments are needed (some are already planned or being made) that can measure the relative importance of the source-sink contributions in different parts of the ocean and identify the various physical processes that influence them most.

We have briefly described the known physical processes by which internal waves are generated and by which they propagate and decay. There may be others yet unknown. It is not beyond our wit to decide on their relative importance, although their complexity is such that probably most rapid progress can be made by the inverse process of inference from the results of experiments and observations of the waves themselves instead of through the direct approach, which has been followed here, of considering the various mechanisms separately and of trying to establish their relative importance from a calculation of the external forces acting on a part of the deep ocean. Many exciting and significant discoveries in this important but formidable field of research are yet to be made.

Acknowledgment. The talk at the AGU meeting on April 9, 1974, was a summary based on this paper.

REFERENCES

Baines, P. G., The generation of internal tides by flat-bump topography, *Deep Sea Res., 20,* 179–205, 1973.

Bell, T. H., Topographically generated internal waves in the open ocean, *J. Geophys. Res., 80,* this issue, 1975.

Booker, J. R., and F. P. Bretherton, Critical layer for internal gravity waves in a shear flow, *J. Fluid Mech., 27,* 513–539, 1967.

Brekhovskikh, L. M., V. V. Goncharov, V. M. Kurtepov, and K. A. Nangol'nykh, Resonant excitation of internal waves by nonlinear interaction of surface waves, *Izv. Akad. Nauk. SSSR Fiz. Atmos. Okeana, 8,* 192–203, 1972. (*Izv. Acad. Sci. USSR Atmos. Oceanic Phys., 8,* Engl. Transl., 112–117, 1972.)

Bretherton, F. P., Momentum transport by gravity waves, *Quart. J. Roy. Meteorol. Soc., 95*(404), 213–243, 1969.

Bretherton, F. P., and C. J. R. Garrett, Wavetrains in inhomogeneous moving media, *Proc. Roy. Soc., Ser. A, 302,* 529–554, 1968.

Bukatov, A., On internal waves in a continuously stratified ocean, *Morsk. Gidrofiz. Issled., 56*(6), 26–36, 1971.

Bukatov, A. Ye., and L. F. Cherkesov, Internal waves generated by periodic surface disturbances, *Izv. Akad. Nauk SSSR Fiz. Atmos. Okeana, 7,* 649–657, 1971. (*Izv. Acad. Sci. USSR Atmos. Oceanic Phys., 7,* Engl. Transl., 432–443, 1971.)

Cavanie, A. G., Observations de fronts internes dans le Détroit de Gibraltar pendant la campagne océanographique otan 1970 et interpretation des results par un modèle mathématique, *Mem. Soc. Roy. Sci. Liege, 2*(6), 27–41, 1972.

Cherkesov, L. V., On the development of waves on a free surface and a surface of division of two fluids under the action of traveling pressures (in Russian), *Prikl. Mat. Mekh.*, *26*, 559-563, 1962.

Cherkesov, L. V., Waves in a non-homogeneous liquid from periodic surface disturbances, *Izv. Akad. Nauk SSSR Fiz. Atmos. Okeana*, *1*, 517-526, 1965. (*Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *1*, Engl. Transl., 303-307, 1965.)

Cherkesov, L. V., On the problem of tsunamis in a sea with continuously varying density, *Izv. Akad. Nauk SSSR Fiz. Atmos. Okeana*, *4*, 1101-1109, 1968. (*Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *4*, Engl. Transl., 632-635, 1968.)

Cox, C. S., and H. Sandstrom, Coupling of internal and surface waves in water of variable depth, *J. Oceanogr. Soc. Jap.*, anniv. vol. 20, 499-513, 1962.

Crapper, G. D., Ship waves in a stratified ocean, *J. Fluid Mech.*, *29*(4), 667-672, 1967.

Davis, R. E., and A. Acrivos, The stability of oscillatory internal waves, *J. Fluid Mech.*, *30*, 723-736, 1967.

Dotsenko, S. F., and L. V. Cherkesov, Generation of internal waves by a travelling field of pressure, *Morsk. Gidrofiz. Issled.*, *56*(6), 5-14, 1971.

Faller, A. J., Sources of energy for the ocean circulation and a theory of the mixed layer, in *Proceedings of the 5th U.S. National Congress on Applied Mechanics*, pp. 651-672, American Society of American Engineers, New York, 1966.

Fedosenko, V. S., and L. V. Cherkesov, Development of internal waves, *Morsk. Gidrofiz. Issled.*, *52*(2), 9-61, 1971.

Fedosenko, V. S., and L. V. Cherkesov, Unsteady waves in a non-uniform fluid of finite depth, *Morsk. Gidrofiz. Issled.*, *58*(2), 11-26, 1972.

Fjeldstad, J. E., Interne Wellen, *Geofys. Publ.*, *10*(6), 1-35, 1933.

Frankignoul, C. J., Stability of finite amplitude internal waves in a shear flow, *Geophys. Fluid Dyn.*, *4*(2), 91-99, 1972.

Gargett, A. E., and B. A. Hughes, On the interaction of surface and internal waves, *J. Fluid Mech.*, *52*, 179-191, 1972.

Garrett, C. J. R., On the interaction between internal gravity waves and shear flow, *J. Fluid Mech.*, *34*, 711-720, 1968.

Garrett, C. J. R., and W. H. Munk, Space-time scales of internal waves, *Geophys. Fluid Dyn.*, *3*(3), 225-264, 1972*a*.

Garrett, C. J. R., and W. H. Munk, Oceanic mixing by breaking internal waves, *Deep Sea Res.*, *19*(12), 823-832, 1972*b*.

Hasselmann, K., A criterion for nonlinear wave stability, *J. Fluid Mech.*, *30*, 737-739, 1967.

Hasselmann, K., et al., Measurements of wind-wave growth and swell decay during the joint North Sea Wave Project (Jonswap), *Ergänzungsh. Deut. Hydrogr. Z., Reihe A*, *8*, 12, 7-95, 1973.

Hazel, P., Numerical studies of the stability of inviscid stratified shear flows, *J. Fluid Mech.*, *51*, 39-61, 1972.

Hudimac, A. A., Ship waves in a stratified ocean, *J. Fluid Mech.*, *11*, 229-243, 1961.

Huppert, H. E., and J. W. Miles, Lee waves in a stratified flow, 3, Semi-elliptical obstacle, *J. Fluid Mech.*, *35*, 481-496, 1969.

Joyce, T. M., Nonlinear interactions among standing surface and internal gravity waves, *J. Fluid Mech.*, *63*, 801-825, 1974.

Kaylor, R., and A. J. Faller, Instability of the stratified Ekman boundary layer and generation of internal waves, *J. Atmos. Sci.*, *29*(3), 497-509, 1972.

Keller, J. B., and W. H. Munk, Internal wave wakes of a body moving in a stratified fluid, *Phys. Fluids*, *13*, 1425-1431, 1970.

Kenyon, K., Wave-wave interactions of surface and internal waves, *J. Mar. Res.*, *26*(3), 208-231, 1968.

Keunecke, K.-H., Stehende interne wellen in rechteckigen becken, *Deut. Hydrogr. Z.*, *23*, 61-79, 1970.

Krauss, W., *Methoden und Ergebnisse der Theoretischen Ozeanographie*, vol. 2, *Interne Wellen*, p. 234, Gebruder Borntreger, Berlin, 1966.

Krauss, W., On the response of a stratified ocean to wind and air pressure, *Deut. Hydrogr. Z.*, *25*, 49-61, 1972*a*.

Krauss, W., Wind-generated internal waves and inertial-period motions, *Deut. Hydrogr. Z.*, *25*, 241-250, 1972*b*.

Lazier, J. R. N., Temporal changes in some fresh water temperature structures, *J. Phys. Oceanogr.*, *3*, 226-229, 1973.

Leblond, P. H., On the damping of internal gravity waves in a continuously stratified ocean, *J. Fluid Mech.*, *25*, 121-142, 1966.

Leonov, A. I., and Yi. Z. Miropolsky, Resonant excitation of internal gravity waves in the ocean by atmospheric-pressure fluctuations, *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, Engl. Transl., *9*, 480-485, 1973.

Lighthill, M. J., On waves generated in dispersive systems by travelling forcing effects, with applications to the dynamics of rotating fluids, *J. Fluid Mech.*, *27*, 725-752, 1967.

Longuet-Higgins, M. S., A theory on the origin of microseisms, *Phil. Trans. Roy. Soc. London Ser. A.*, *243*, 1-35, 1950.

Magaard, L., Zur Berechnung von luftdruck-und windbedingten Bewegungen eines stetig geschichteten seitlich unbergrenzten Meeres, *Deut. Hydrogr. Z.*, *24*, 145-158, 1971.

Magaard, L., On the generation of internal gravity waves by a fluctuating buoyancy flux at the sea surface, *Geophys. Fluid Dyn.*, *5*(2), 101-111, 1973.

Martin, S., W. F. Simmons, and C. I. Wunsch, The excitation of resonant triads by single internal waves, *J. Fluid Mech.*, *53*, 17-44, 1972.

McEwan, A. E., Degeneration of resonantly-excited standing internal gravity waves, *J. Fluid Mech.*, *50*, 431-448, 1971.

McEwan, A. D., D. W. Mander, and R. K. Smith, Forced resonant second-order interaction between damped internal waves, *J. Fluid Mech.*, *55*, 589-608, 1972.

McGorman, R. E., and L. A. Mysak, Internal waves in a randomly stratified fluid, *Geophys. Fluid Dyn.*, *5*(2), 243-266, 1973.

Miropol'skii, Yu. Z., The effect of the microstructure of the density field in the sea on the propagation of internal gravity waves, *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *8*, Engl. Transl., 515-517, 1972.

Miropol'skii, Yu. Z., Probability distribution of certain characteristics of internal waves in the ocean, *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *9*, Engl. Transl., 226-230, 1973.

Mork, M., The response of a stratified sea to atmospheric forces, report, 41 pp., Univ. of Bergen Geofys. Inst., Bergen, Norway, 1968.

Mowbray, E. E., and B. S. H. Rarity, A theoretical and experimental investigation of the phase configuration of internal waves of small amplitude in a density stratified fluid, *J. Fluid Mech.*, *28*, 1-16, 1967.

Müller, P., and D. J. Olbers, On the dynamics of internal waves in the deep ocean, *J. Geophys. Res.*, *80*, in press, 1975.

Munk, W. H., Abyssal recipes, *Deep Sea Res.*, *13*, 707-730, 1966.

Nesterov, S. V., Resonant interaction of surface and internal waves, *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *8*, 252-254, 1972.

Orlanski, I., On the breaking of standing internal gravity waves, *J. Fluid Mech.*, *54*, 577-598, 1972.

Orlanski, I., Trapeze instability as a source of internal gravity waves, I, *J. Atmos. Sci.*, *30*, 1007-1016, 1973.

Orlanski, I., and K. Bryan, Formation of the thermocline step structure by large-amplitude internal gravity waves, *J. Geophys. Res.*, *74*(28), 6975-6983, 1969.

Orlanski, I., and B. R. Ross, Numerical simulation of the generation and breaking of internal gravity waves, *J. Geophys. Res.*, *78*(36), 8808-8826, 1973.

Phillips, O. M., On the generation of waves by turbulent wind, *J Fluid Mech.*, *2*, 417-445, 1957.

Phillips, O. M., *The Dynamics of the Upper Ocean*, pp. 172-182, Cambridge University Press, London, 1966.

Phillips, O. M., The interaction trapping of internal gravity waves, *J. Fluid Mech.*, *34*, 407-416, 1968.

Polyanskaya, V. A., The generation of subsurface waves by a plane pressure front moving at constant velocity, *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, *5*, Engl. Transl., 608-611, 1969.

Rhines, P., Observations of the energy-containing oceanic eddies, and theoretical models of waves and turbulence, *Boundary Layer Meteorol.*, *4*, 345-360, 1973.

Scorer, R. S., Theory of waves in the lee of mountains, *Quart. J. Roy. Meteorol. Soc.*, *75*, 41-56, 1949.

Smith, R., An instability of internal gravity waves, *J. Fluid Mech.*, *52*, 393-399, 1972.

Stevenson, T. N., Some two-dimensional internal waves in a stratified fluid, *J. Fluid Mech.*, *33*, 715-720, 1968.

Thorpe, S. A., On wave interactions in a stratified fluid, *J. Fluid Mech.*, *24*, 737-751, 1966.

Thorpe, S. A., On the shape of progressive internal waves, *Phil. Trans. Roy. Soc. London, Ser. A*, *263*, 563-614, 1968.

Thorpe, S. A., Turbulence in stratified fluids: A review of laboratory experiments, *Boundary Layer Meteorol.*, *5*, 95-119, 1973*a*.

Thorpe, S. A., Experiments on instability and turbulence in a stably stratified shear flow, *J. Fluid Mech.*, *61*, 731-751, 1973*b*.

Thorpe, S. A., Near-resonant forcing in a shallow two-layer fluid: A model for the internal surge in Loch Ness?, *J. Fluid Mech.*, *63*, 509, 1974.

Tomczak, M., Jr., Winderzeugte interne Wellen, insbesondere im

Periodenbereich oberhalb der Trägheitsperiode, *Deut. Hydrogr. Z.*, *19*, 1-21, 1966.
Tomczak, M., Jr., Über den Einfluss fluktuierender Windfelder auf ein stetig geschichtetes Meer, *Deut. Hydrogr. Z.*, *20*, 101-129, 1967.
Townsend, A. A., Excitation of internal waves in a stably-stratified atmosphere with considerable wind shear, *J. Fluid Mech.*, *32*, 145-171, 1968.
Turner, J. S., *Buoyancy Effects in Fluids*, pp. 251 ff., Cambridge University Press, London, 1973.
Voyt, S. S., Waves developing at the interface of two fluids induced by a travelling and periodic pressure system, *Trans. Mar. Hydrophys. Inst. Acad. Sci. USSR*, *17*, 117, 1959.
Wimbush, M., and W. Munk, *The Sea*, vol. 4, edited by A. E. Maxwell, pp. 731-758, Interscience, New York, 1970.
Woods, J. D., Wave induced shear instability in the summer thermocline, *J. Fluid Mech.*, *32*, 791-800, 1968.
Wunsch, C., On the mean drift in large lakes, *Limnol. Oceanogr.*, *18*, 793-795, 1973.
Yih, C.-S., *Dynamics of Non-Homogeneous Fluids*, pp. 15 ff., MacMillan, New York, 1966.

(Received June 4, 1974;
accepted October 10, 1974.)