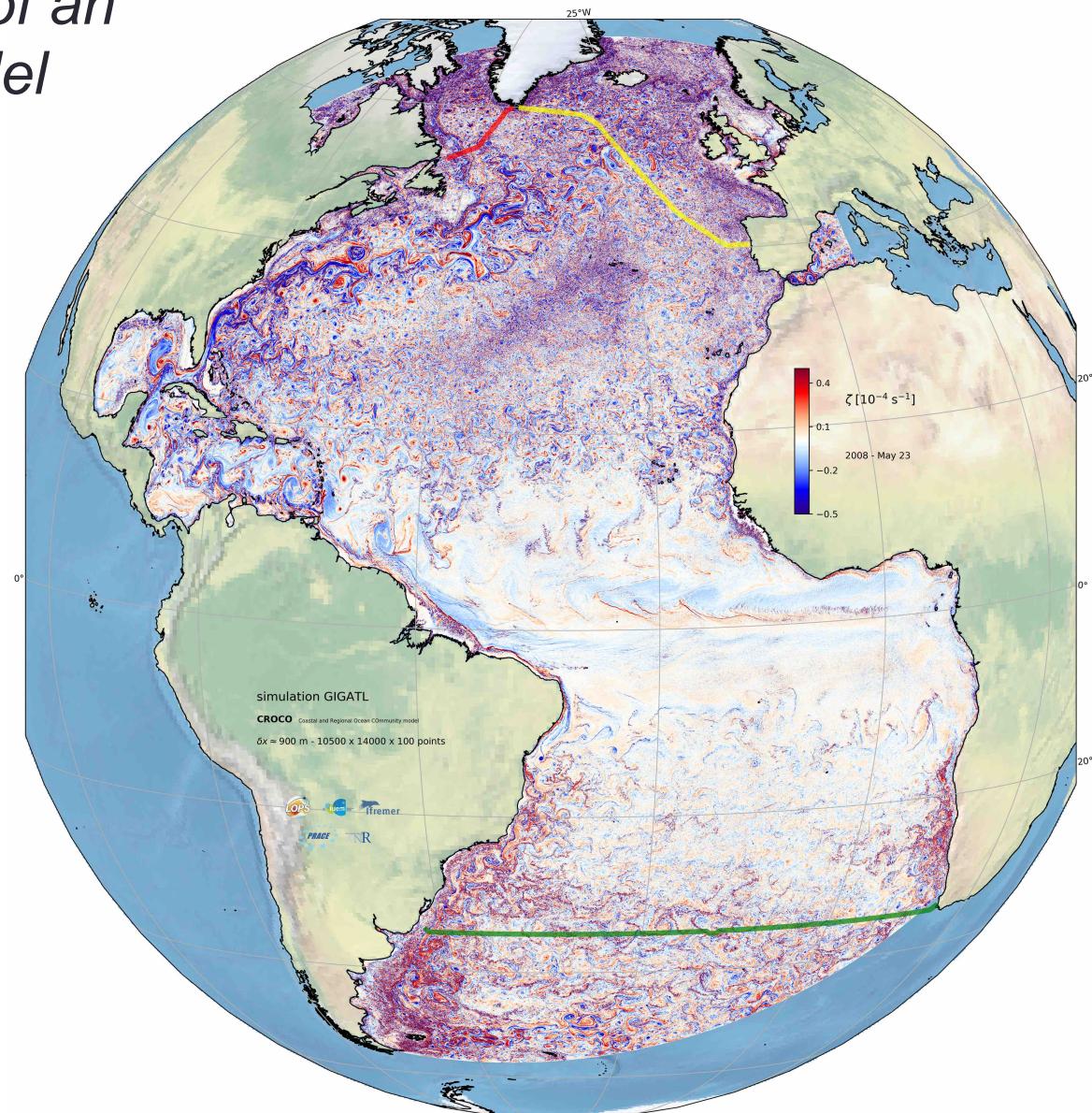


Numerical Modelling

Jonathan GULA
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*the anatomy of an
ocean model*



- **Lesson 1 : [D109]**
 - Introduction
 - Equations of motions
 - *Activity 1 [run an ocean model]*
- **Lesson 2 : [D109]**
 - Subgrid-scale parameterization
 - Dynamics of the ocean gyre
 - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 3 : [D109]**
 - Horizontal Discretization
 - Dynamics of the ocean gyre (suite)
 - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 4 : [D109]**
 - Numerical schemes
 - *Activity 3 [Impacts of numerics]*
- **Lesson 5 : [D109]**
 - Vertical coordinates
 - *Activity 4 [Impact of topography]*
- **Lesson 6 : [D109]**
 - Boundary Forcings
 - Presentation of the model CROCO
- *Activity 5 [Design a realistic simulation]*
- **Lesson 7 : [D109]**
 - Diagnostics and validation
 - *Activity 6 [Analyze a realistic simulation]*
- **Lesson 8 : [D109]**
 - *Project*

Presentations and material
will be available at :

jgula.fr/ModNum/

Useful references

Extensive courses:

- MIT: <https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/>
- Princeton: https://stephengriffies.github.io/assets/pdfs/GFM_lectures.pdf

Overview on ocean modelling and current challenges:

- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling. <http://jgula.fr/ModNum/Griffiesetal00.pdf>
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. http://jgula.fr/ModNum/Griffies_Chapter.pdf
- Fox-Kemper et al, 19, "Challenges and Prospects in Ocean Circulation Models" <http://jgula.fr/ModNum/FoxKemperetal19.pdf>

ROMS/CROCO:

- <https://www.myroms.org/wiki/>
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell. <http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf>

#2 Subgrid-scale parameterization

Incompressible Navier-Stokes Equations:

- Dissipation of energy/momentum in the NS equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

Viscosity

Type of models

Navier
Stokes

- DNS = Direct Numerical Simulation
- LES = Large Eddy Simulation

CFD

Process
studies

PE

- PE = Primitive Equations models

Ocean
Circulation
Models

SW

- SW = Shallow-Water models

SQG

- SQG = Surface Quasi-Geostrophic models

QG

- QG = Quasi-Geostrophic models

Idealized
models

- Etc.

Incompressible Navier-Stokes Equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

Non-linear
terms

Viscosity

- Importance of NL terms and viscosity = Reynolds Number

$$Re = \frac{UL}{\nu}$$

Where U is a typical velocity of the flow and L is a typical length describing the flow.

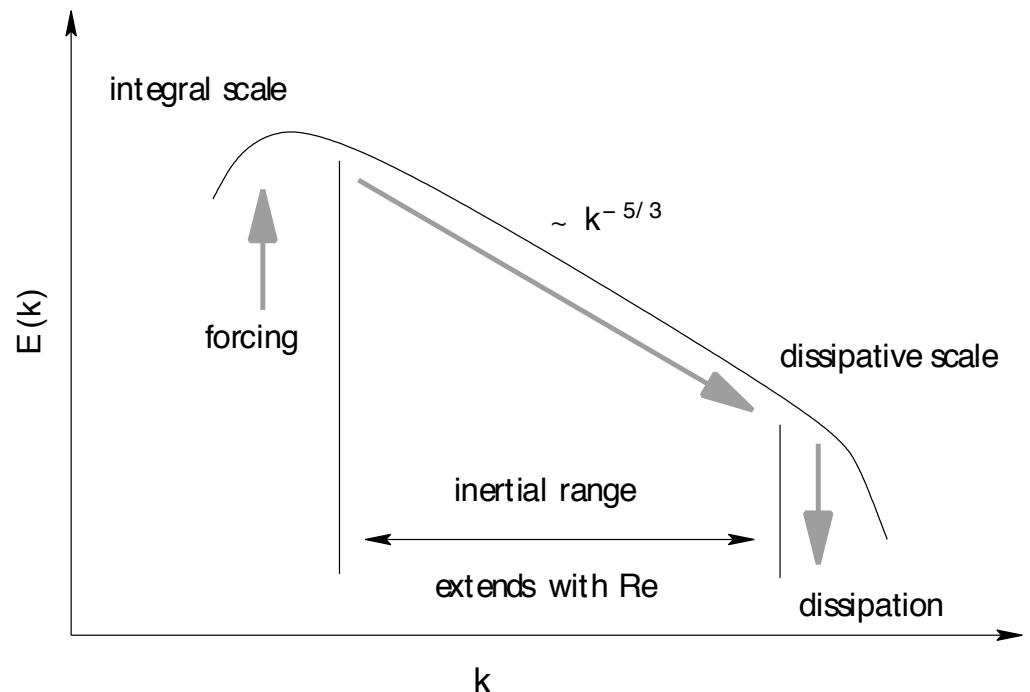
Direct numerical simulation (DNS)

DNS resolves the entire range of turbulent length scales down to the smallest dissipative scales (Kolmogorov scale):

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4} \approx \left(\frac{\nu^3 L}{U^3} \right)^{1/4} = Re^{-3/4} L$$

where ν is the kinematic viscosity

And ϵ the rate of kinetic energy dissipation



Direct numerical simulation (DNS)

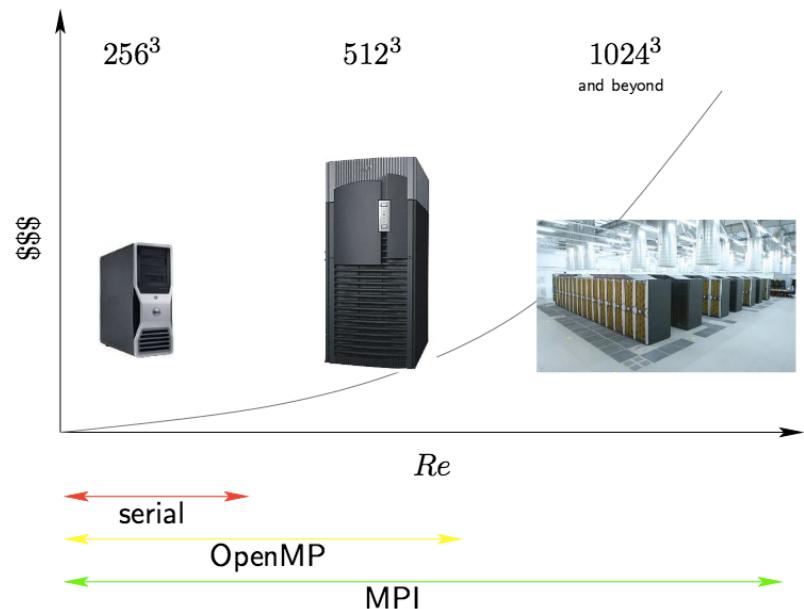
The number of floating-point operations required to complete the simulation is proportional to the number of mesh points:

$$N_x = \frac{L}{\eta} = Re^{3/4}$$

$$\frac{T}{\Delta t} = \frac{TU}{\eta} = \frac{TU}{L} Re^{3/4}$$

and the number of time steps:

It is extremely expensive as the computational cost scales as Re^3



Type of models

Navier
Stokes

- DNS = Direct Numerical Simulation
- LES = Large Eddy Simulation

PE

- PE = Primitive Equations models

SW

- SW = Shallow-Water models

SQG

- SQG = Surface Quasi-Geostrophic models

QG

- QG = Quasi-Geostrophic models

- Etc.

CFD

Process
studies

Ocean
Circulation
Models

Idealized
models

Reynolds averaging

To reduce the computational cost, one need to reduce the range of time- and length-scales that are being solved for.

The idea is based on separation of mean and turbulent component:

$$u = \bar{u} + u'$$

Where

$$\bar{u} = \frac{1}{T} \int_0^T u \, dt \text{ or } \bar{u} = \frac{1}{X} \int_0^X u \, dx$$

With by definition

$$\overline{u'} = 0$$

Reynolds averaging

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

- Activity:

Adapt the momentum equation:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + f \mathbf{k} \times u_i + \frac{\rho}{\rho_0} g \mathbf{k} = -\frac{1}{\rho_0} \frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}$$

For the mean velocity: $\frac{\partial \bar{u}_i}{\partial t} = ?$

Reynolds averaging

So we resolve only the equations for the mean variables:

$$\frac{\partial \bar{u}_i}{\partial t} + \overline{u_j} \frac{\partial u_i}{\partial x_j} + f \mathbf{k} \times \bar{u}_i + \frac{\bar{\rho}}{\rho_0} g \mathbf{k} = - \frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j}$$



$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + f \mathbf{k} \times \bar{u}_i + \frac{\bar{\rho}}{\rho_0} g \mathbf{k} = - \frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} - \frac{\partial \bar{u}'_i \bar{u}'_j}{\partial x_j}$$

Advection for the
averaged flow

Reynolds stress
= effect of subgrid-scale turbulence

Turbulence closure

The Closure Problem :

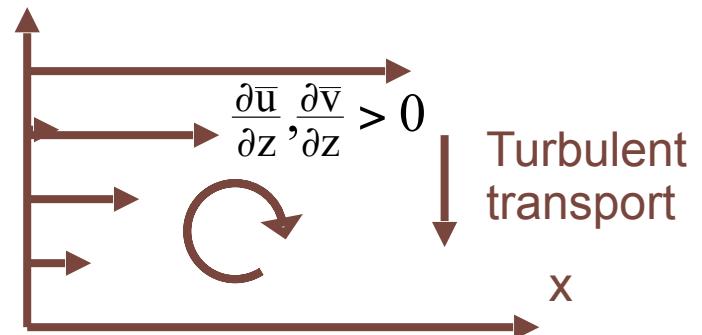
- The number of unknowns is larger than the number of equations
- When we start deriving equations for unknowns, more variables appear.

Progn. Eq. for:	Moment	Equation	No. Eqs.	No. Unknowns
$\overline{U_i}$	First	$\frac{\partial \overline{U_i}}{\partial t} = \dots - \frac{\partial u'_i u'_j}{\partial x_j}$	3	6
$\overline{u'_i u'_j}$	First	$\frac{\partial \overline{u'_i u'_j}}{\partial t} = \dots - \frac{\partial u'_k u'_i u'_j}{\partial x_k}$	6	10
$\overline{u'_i u'_j u'_k}$	First	$\frac{\partial \overline{u'_i u'_j u'_k}}{\partial t} = \dots - \frac{\partial u'_k u'_i u'_j u'_m}{\partial x_m}$	10	15

Turbulence closure

- In PE models the equations are closed by parameterizing the Reynolds stresses as:

$$\overline{u'w'} = -K_M v \frac{\partial u}{\partial z}$$
$$\overline{v'w'} = -K_M v \frac{\partial v}{\partial z}$$



Turbulence closure

In CROCO:

$$\mathcal{F}_u = \frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial u}{\partial z} \right) + \nabla_h (K_{Mh} \cdot \nabla_h u)$$

$$\mathcal{F}_v = \frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial v}{\partial z} \right) + \nabla_h (K_{Mh} \cdot \nabla_h v)$$

$$\mathcal{S}_T = \frac{\partial}{\partial z} \left(K_{Tv} \frac{\partial T}{\partial z} \right) + \nabla_h (K_{Th} \cdot \nabla_h T)$$

$$\mathcal{S}_S = \frac{\partial}{\partial z} \left(K_{Sv} \frac{\partial S}{\partial z} \right) + \nabla_h (K_{Sh} \cdot \nabla_h S)$$

Vertical mixing

Horizontal diffusion

Turbulence closure

Vertical Mixing:

$$K_{Mv}, K_{Tv}, K_{Sv}$$

- Local Turbulence closure: GLS, k-kl (MY2.5), k- ϵ , κ - ω , etc. [e.g. *Warner et al, 2005, Ocean Modelling*]
- Non local K-profile parameterization (KPP) [*Large et al, 1994, Rev. of Geophysics*]

Horizontal diffusion:

- Explicit diffusion

$$K_{Mh}, K_{Th}, K_{Sh}$$

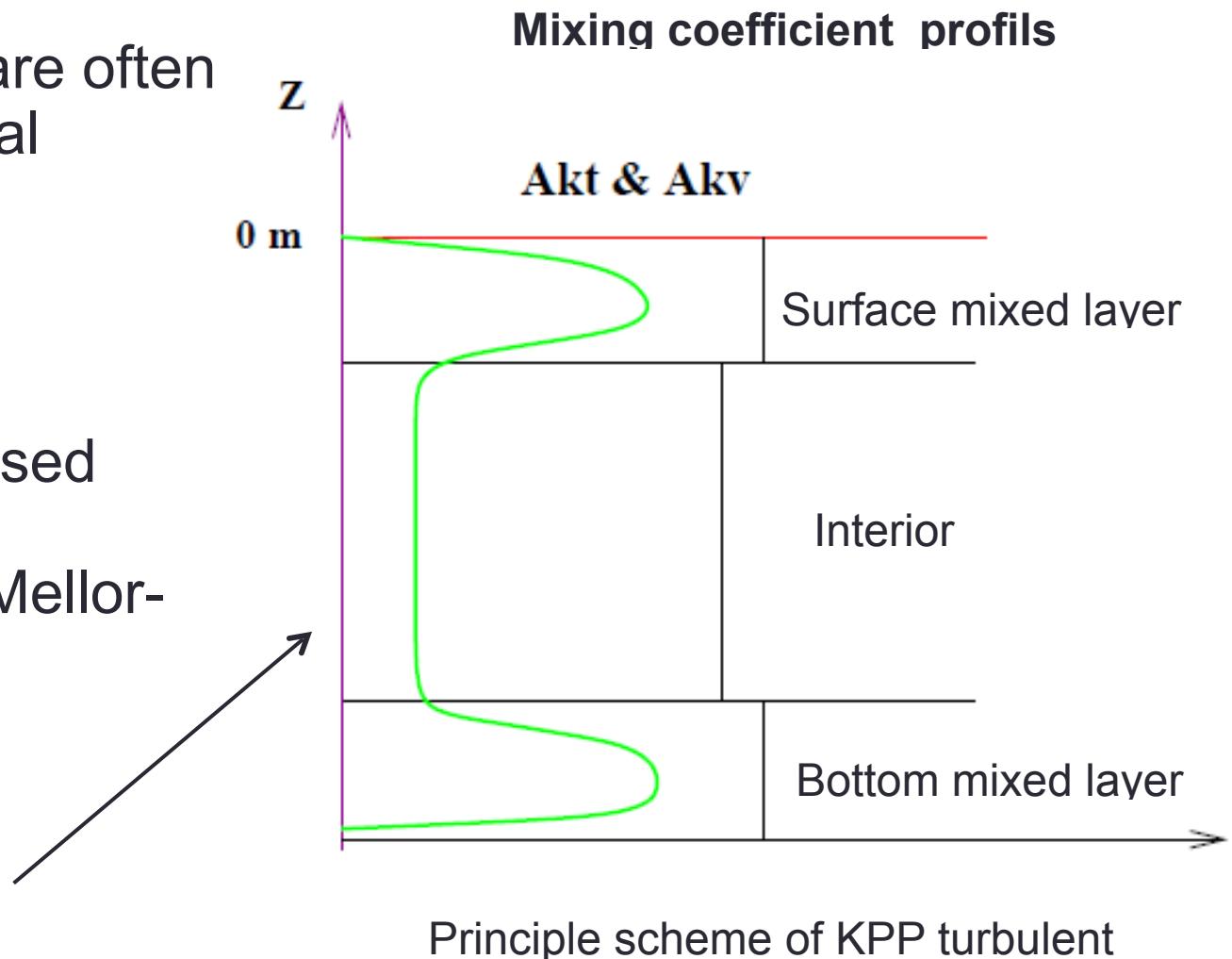
- Implicit (comes from the advective scheme)

Non local K-profile parameterization

- Mixed layer schemes are often based on one-dimensional « column physics »

- Boundary layer parameterizations are based either on:

- Turbulent closure (Mellor-Yamada, TKE)
- **K profile (KPP)**



Turbulence closure

In CROCO:

$$\mathcal{F}_u = \frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial u}{\partial z} \right) + \nabla_h (K_{Mh} \cdot \nabla_h u)$$

$$\mathcal{F}_v = \frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial v}{\partial z} \right) + \nabla_h (K_{Mh} \cdot \nabla_h v)$$

$$\mathcal{S}_T = \frac{\partial}{\partial z} \left(K_{Tv} \frac{\partial T}{\partial z} \right) + \nabla_h (K_{Th} \cdot \nabla_h T)$$

$$\mathcal{S}_S = \frac{\partial}{\partial z} \left(K_{Sv} \frac{\partial S}{\partial z} \right) + \nabla_h (K_{Sh} \cdot \nabla_h S)$$

Vertical mixing

Horizontal diffusion

Boundary conditions

- Boundary conditions become:

top ($\sigma = 0$)

$$\begin{aligned}\left(\frac{K_m}{H_z} \right) \frac{\partial u}{\partial \sigma} &= \tau_s^x(x, y, t) \\ \left(\frac{K_m}{H_z} \right) \frac{\partial v}{\partial \sigma} &= \tau_s^y(x, y, t) \\ \left(\frac{K_C}{H_z} \right) \frac{\partial C}{\partial \sigma} &= \frac{Q_C}{\rho_o c_P} \\ \Omega &= 0\end{aligned}$$

and bottom ($\sigma = -1$)

$$\begin{aligned}\left(\frac{K_m}{H_z} \right) \frac{\partial u}{\partial \sigma} &= \tau_b^x(x, y, t) \\ \left(\frac{K_m}{H_z} \right) \frac{\partial v}{\partial \sigma} &= \tau_b^y(x, y, t) \\ \left(\frac{K_C}{H_z} \right) \frac{\partial C}{\partial s} &= 0 \\ \Omega &= 0.\end{aligned}$$

Boundary conditions

- Boundary conditions become:

top ($\sigma = 0$)	$\left(\frac{K_m}{H_z} \right) \frac{\partial u}{\partial \sigma} = \tau_s^x(x, y, t)$ $\left(\frac{K_m}{H_z} \right) \frac{\partial v}{\partial \sigma} = \tau_s^y(x, y, t)$ $\left(\frac{K_C}{H_z} \right) \frac{\partial C}{\partial \sigma} = \frac{Q_C}{\rho_o c_P}$ $\Omega = 0$	Wind stress
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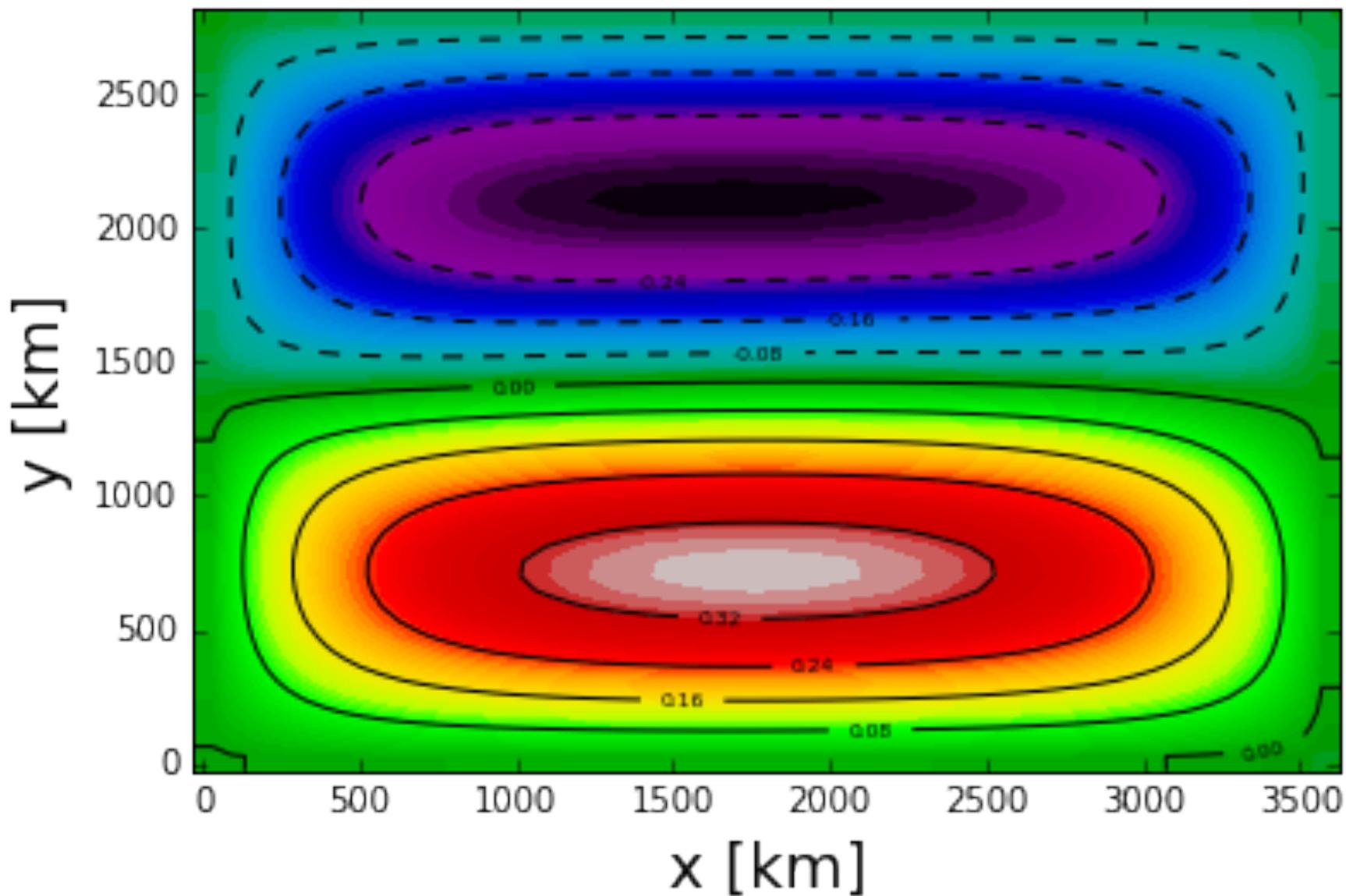
and bottom ($\sigma = -1$)	$\left(\frac{K_m}{H_z} \right) \frac{\partial u}{\partial \sigma} = \tau_b^x(x, y, t)$ $\left(\frac{K_m}{H_z} \right) \frac{\partial v}{\partial \sigma} = \tau_b^y(x, y, t)$ $\left(\frac{K_C}{H_z} \right) \frac{\partial C}{\partial s} = 0$ $\Omega = 0.$	Bottom friction
------------------------------	--	-----------------

Bottom Boundary conditions

1. Linear friction, with
r friction velocities [m/s] $\rightarrow (\tau_b^x, \tau_b^y) = -r (u_b, v_b)$
2. Quadratic friction, controled by a constant drag coefficient **Cd** $\rightarrow (\tau_b^x, \tau_b^y) = C_d \sqrt{u_b^2 + v_b^2} (u_b, v_b)$
3. Quadratic friction coefficient, using variable **Cd** (Von Karman log. layer)
$$\rightarrow \begin{cases} (\tau_b^x, \tau_b^y) = C_d \sqrt{u_b^2 + v_b^2} (u_b, v_b) \\ C_d = \left(\frac{\kappa}{\log[\Delta z_b/z_r]} \right)^2 \text{ si } C_d^{min} < C_d < C_d^{max} \\ C_d = C_d^{min} \text{ ou } C_d^{max} \\ \kappa = 0.41 \\ z_r = \text{Roughness Length} \\ \Delta z_b = \text{thickness of the first bottom level} \end{cases}$$

Activity 2 - Run an idealized ocean basin II

SSH



Stommel's gyre

THE WESTWARD INTENSIFICATION OF WIND-DRIVEN OCEAN CURRENTS

Henry Stommel

(Contribution No. 408, Woods Hole Oceanographic Institution)

Abstract--A study is made of the wind-driven circulation in a homogeneous rectangular ocean under the influence of surface wind stress, linearised bottom friction, horizontal pressure gradients caused by a variable surface height, and Coriolis force.

An intense crowding of streamlines toward the western border of the ocean is discovered to be caused by variation of the Coriolis parameter with latitude. It is suggested that this process is the main reason for the formation of the intense currents (Gulf stream and others) observed in the actual oceans.

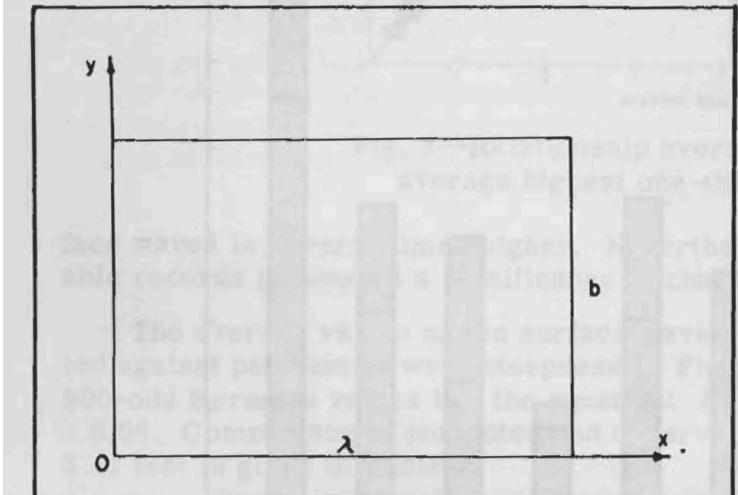


Fig. 1--Ocean basin dimensions and the coordinate system

- Momentum equations:

The steady state equations of motion, with the inertial terms omitted because they are small, are written in the form

$$0 = f(D + h)v - F \cos(\pi y/b) - Ru - g(D + h)\partial h / \partial x \quad (1)$$

- Barotropic vorticity equation:

$$v(D + h)(\partial f / \partial y) + (F \pi / b) \sin(\pi y / b) + R(\partial v / \partial x - \partial u / \partial y) = 0$$

Stommel's gyre

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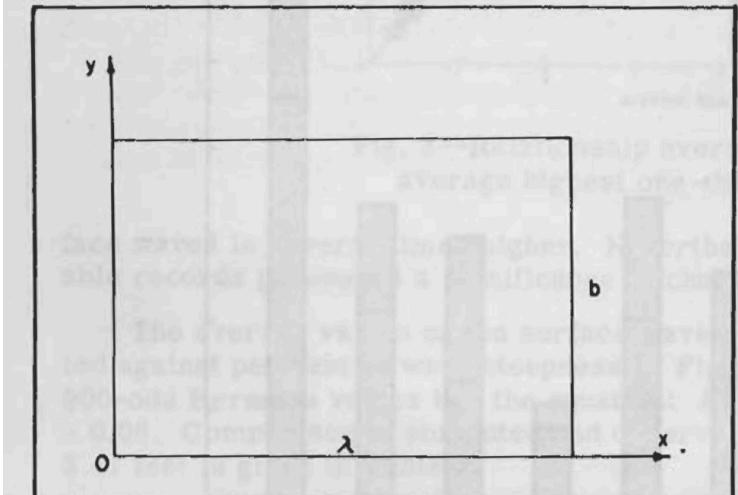


Fig. 1--Ocean basin dimensions and the coordinate system

- Momentum equations:

Coriolis

Wind

Drag

The steady state equations of motion, with the inertial terms omitted because they are small, are written in the form

$$0 = f(D + h)v - F \cos(\pi y/b) - Ru - g(D + h)\partial h / \partial x \quad (1)$$

- Barotropic vorticity equation:

Planetary vorticity advection

Wind Curl

Drag Curl

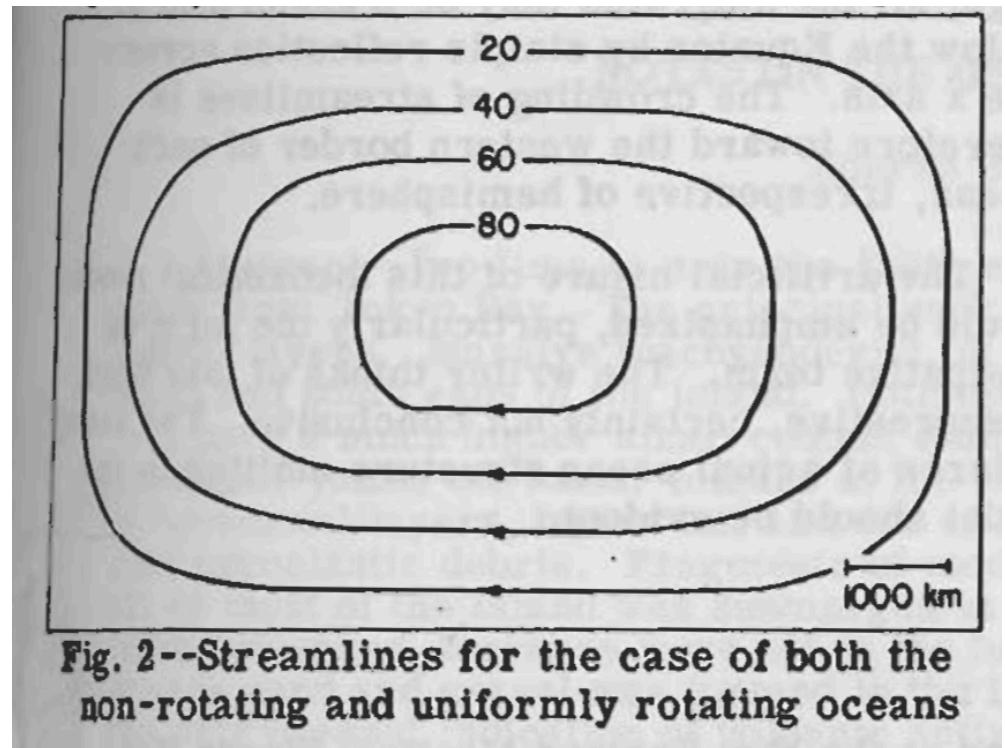
$$v(D + h)(\partial f / \partial y) + (F \pi / b) \sin(\pi y / b) + R(\partial v / \partial x - \partial u / \partial y) = 0$$

Stommel's gyre

- No rotation / or constant rotation ($\frac{\partial f}{\partial y} = \beta = 0$)

The equation for the stream function is therefore

$$\psi = \gamma(b/\pi)^2 \sin(\pi y/b) [e^{(x-\lambda)\pi/b} + e^{-x\pi/b} - 1]$$



Planetary vorticity
and rotation

Wind
Curl

Drag
Curl

$$v(Dv/Dy - \partial f / \partial y) + (F\pi/b) \sin(\pi y/b) + R(\partial v / \partial x - \partial u / \partial y) = 0$$

Stommel's gyre

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

- Formation of a western boundary

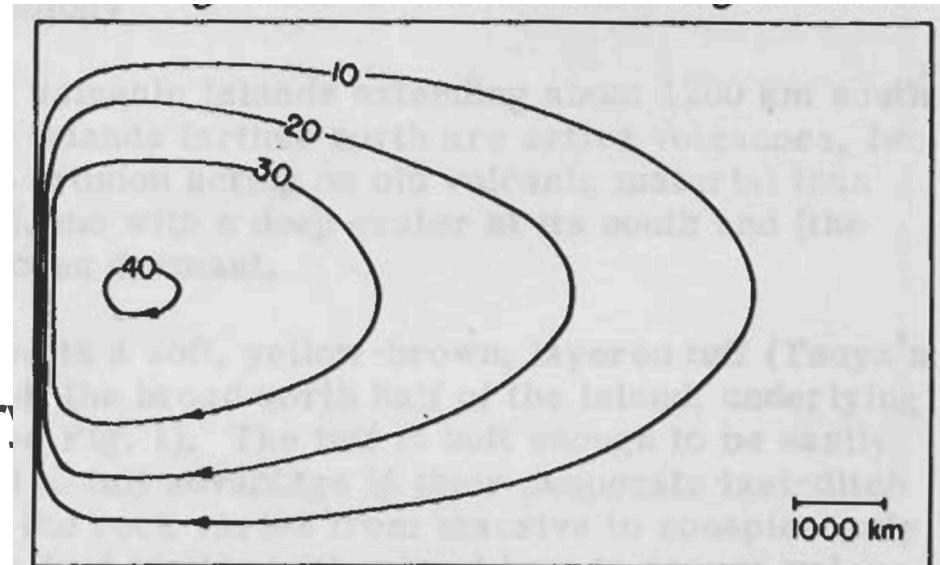
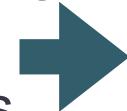


Fig. 5 --Streamlines for the case where the Coriolis force is a linear function of latitude

Stommel's gyre (no beta)

- Forcings and data
 - Bottom topography + Land mask
 - Atmospheric surface boundary forcing
 - Initial oceanic conditions
 - Lateral oceanic boundary conditions
- Flat bottom
- Constant wind ($sustr(i,j) = -cff1 * \cos(2. * \pi / el * yr(i,j))$)
- Resting state
- Vertical walls



Stommel's gyre (no beta)

- No rotation / or constant rotation (.)

$$\frac{\partial f}{\partial y} = \beta = 0$$

- cppdefs.h**

```
# define UV_COR
# define UV_VIS2
# define TS_DIF2
```

```
# define ANA_GRID
# define ANA_INITIAL
```

- croco.in**

```
bottom_drag: RDRG(m/s), RDRG2, Zob [m], Cdb_min, Cdb_max
            3.e-4          0.    0.    0.    0.

gamma2:
            1.

linEOS_cff: R0 [kg/m3], T0 [Celsius], S0 [PSU], TCOEF [1/Celsius], SCOEY [1/PSU]
            30.      0.        0.     0.28      0.

lateral_visc: VIS2 [m^2/sec]
            1000.   0.

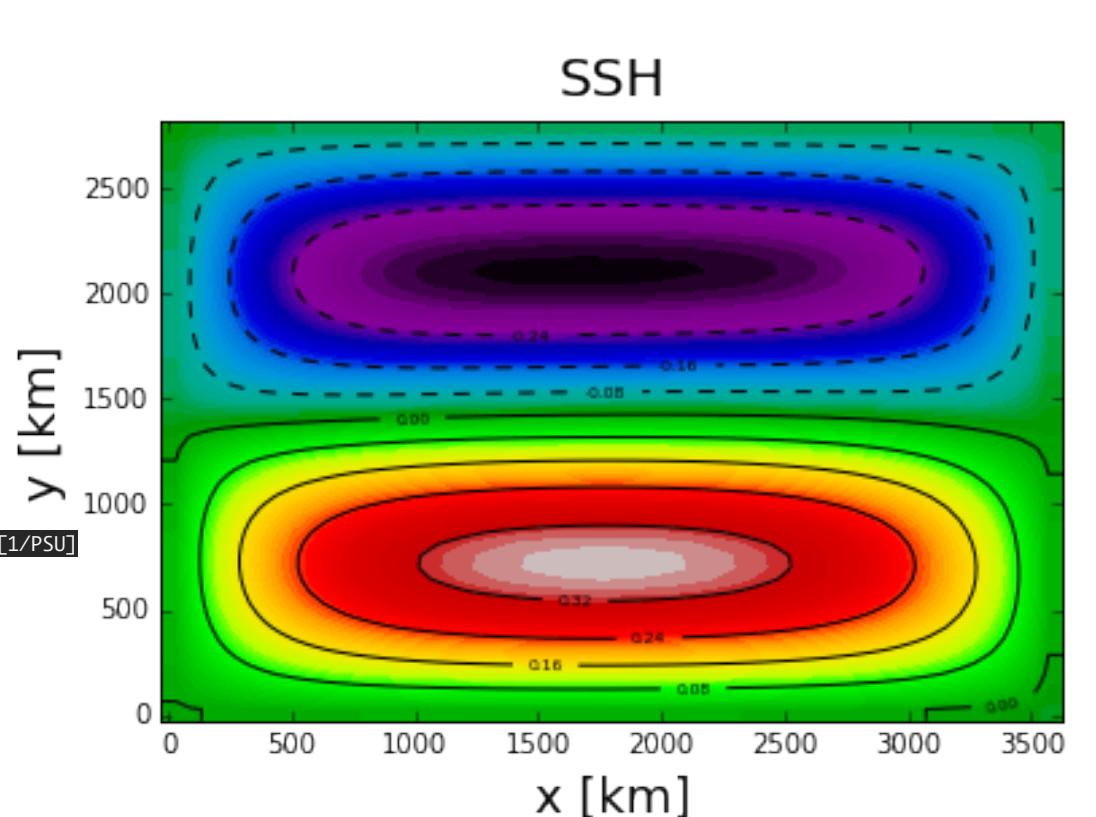
tracer_diff2: TNU2 [m^2/sec]
            1000.   0.
```

- ana_grid.F**

```
f0=1.E-4
beta=0.
```

- param.h**

```
parameter (LLm0=60, MMm0=50, N=10)
```



CROCO simulation after 20 years

Barotropic vorticity balance

- Barotropic vorticity: $\Omega = \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y}$ with $\bar{u} = \int_{-h}^{\zeta} u \, dz,$
- The barotropic vorticity balance equation of the flow is obtained by integrating the momentum equations in the vertical and cross differentiating them:
 - [cf: https://www.jgula.fr/ModNum/vort_balance.pdf]

Momentum equations

$$\frac{\partial u}{\partial t} = -u_j \frac{\partial u}{\partial x_j} - w \frac{\partial u}{\partial z} + fv - \frac{P_x}{\rho_0} + \mathcal{V}_u + \mathcal{D}_u + \mathcal{S}_u$$
$$\underbrace{\frac{\partial v}{\partial t}}_{rate} = -u_j \underbrace{\frac{\partial v}{\partial x_j}}_{hadv} - w \underbrace{\frac{\partial v}{\partial z}}_{vadv} - \underbrace{fu}_{cor} - \underbrace{\frac{P_y}{\rho_0}}_{Prsgrd} + \underbrace{\mathcal{V}_v}_{vmix} + \underbrace{\mathcal{D}_v}_{hmix+hdiff} + \underbrace{\mathcal{S}_v}_{nudg}$$

Barotropic vorticity balance

- Barotropic vorticity: $\Omega = \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y}$ with $\bar{u} = \int_{-h}^{\zeta} u \, dz,$
- The barotropic vorticity balance equation of the flow is obtained by integrating the momentum equations in the vertical and cross differentiating them:
 - [cf: https://www.jgula.fr/ModNum/vort_balance.pdf]

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = - \underbrace{\vec{\nabla} \cdot (\vec{f} \vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}} + \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{wind}}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{bot}}}{\rho_0}}_{\text{bot. drag curl}} \\
 + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz. diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

$$A_\Sigma = \frac{\partial^2(\bar{v}\bar{v} - \bar{u}\bar{u})}{\partial x \partial y} + \frac{\partial^2 \bar{u}\bar{v}}{\partial x \partial x} - \frac{\partial^2 \bar{u}\bar{v}}{\partial y \partial y},$$

Stommel's gyre (no beta)

- No rotation / or constant rotation ($\frac{\partial f}{\partial y} = \beta = 0$)

•

$$\cancel{- \text{rate}} = - \cancel{\text{Dvert. vort. adv. } f(\vec{u})} + \cancel{\text{bot. press. torque } \rho_0 \nabla h} + \boxed{\cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}} - \boxed{\cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}}$$

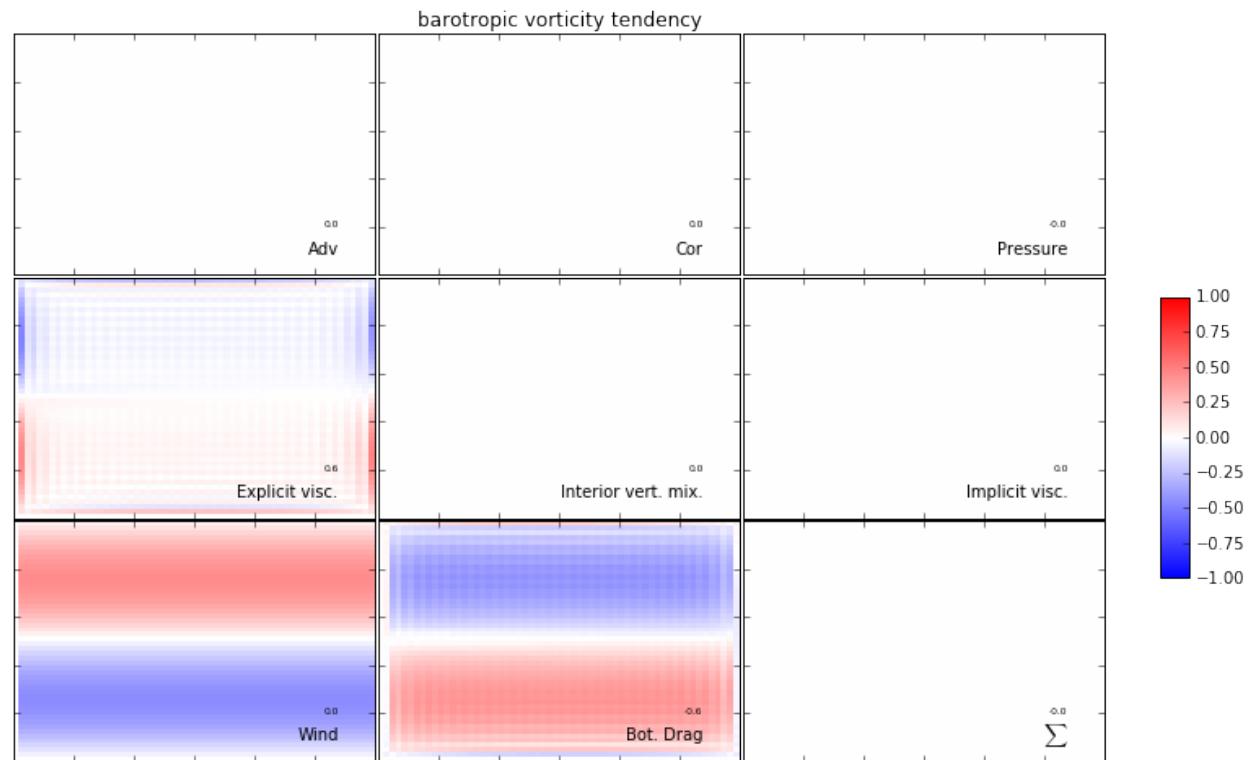
The equation shows the balance of forces in Stommel's gyre. The left side is the rate of change of the flow field. The right side consists of several terms, each crossed out with a large blue X:

- The first term, $- \text{rate}$, is crossed out.
- The second term, $- \text{Dvert. vort. adv. } f(\vec{u})$, is crossed out.
- The third term, $+ \text{bot. press. torque } \rho_0 \nabla h$, is crossed out.
- The fourth term, $+ \cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}$, is enclosed in a yellow box and labeled "wind curl".
- The fifth term, $- \cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}$, is enclosed in a blue box and labeled "bot. drag curl".

Stommel's gyre (no beta)

- No rotation / or constant rotation

$$\begin{aligned}
\cancel{\frac{\partial \vec{u}}{\partial t}} = & - \cancel{\vec{\nabla} \times (\vec{u})} + \cancel{\frac{\mathbf{J} \times \vec{P}(\lambda)}{A}} \\
& + \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} \quad \boxed{\underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}} \\
& + \cancel{\mathcal{D}_\Sigma} - \cancel{\vec{\nabla} \times (\vec{u})} \\
& \text{horiz. diffusion.} \quad \text{NL advection}
\end{aligned}$$



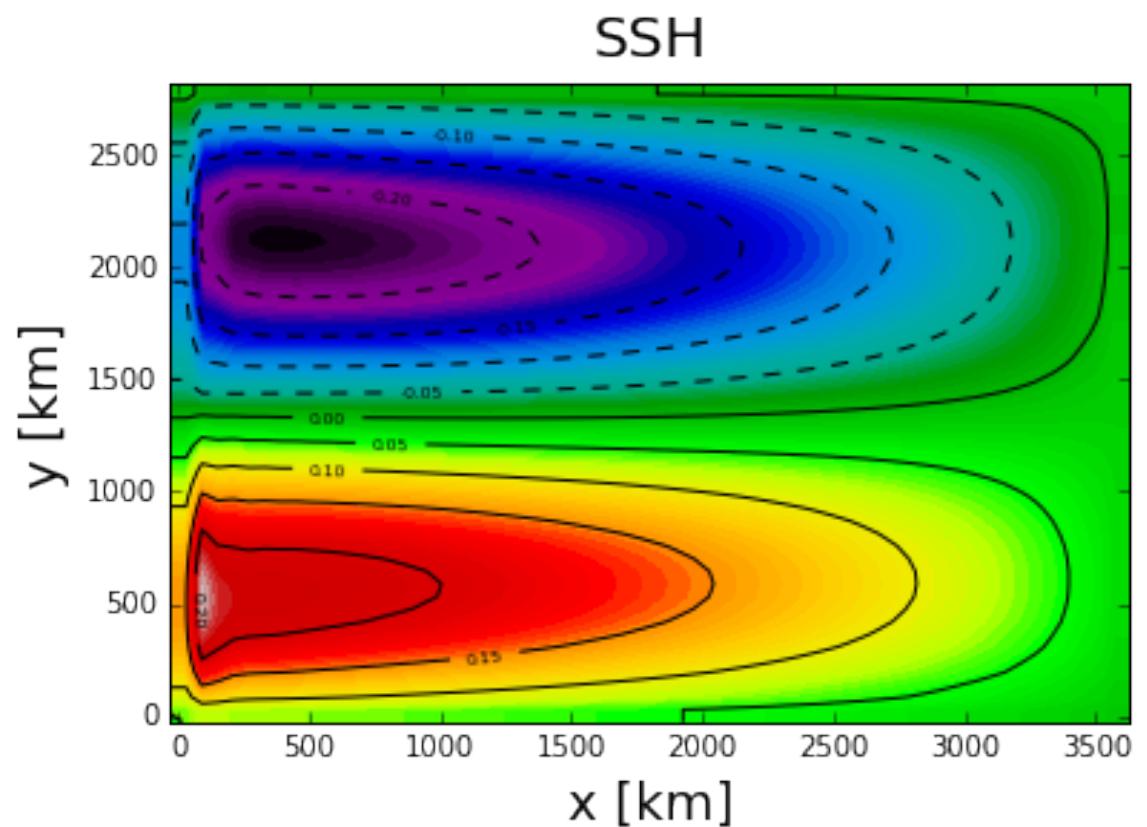
Stommel's gyre (with beta)

- With latitudinal variation of Coriolis ($\frac{\partial f}{\partial y} = \beta \neq 0$)

- ana_grid.F**

$f_0=1.E-4$

$\beta=2.E-11$



Stommel's gyre (no beta)

- No rotation / or constant rotation ($\frac{\partial f}{\partial y} = \beta = 0$)

•

$$\cancel{\text{rate}} = - \underbrace{\vec{\nabla} \cdot (f \vec{u})}_{\text{planet. vort. adv.}} + \cancel{\frac{\partial u}{\partial z}(z, h)} + \cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}} + \cancel{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}$$

+

$$\cancel{\mathcal{D}_\Sigma} - \cancel{A_\Sigma}$$

horiz. diffusion. NI advection

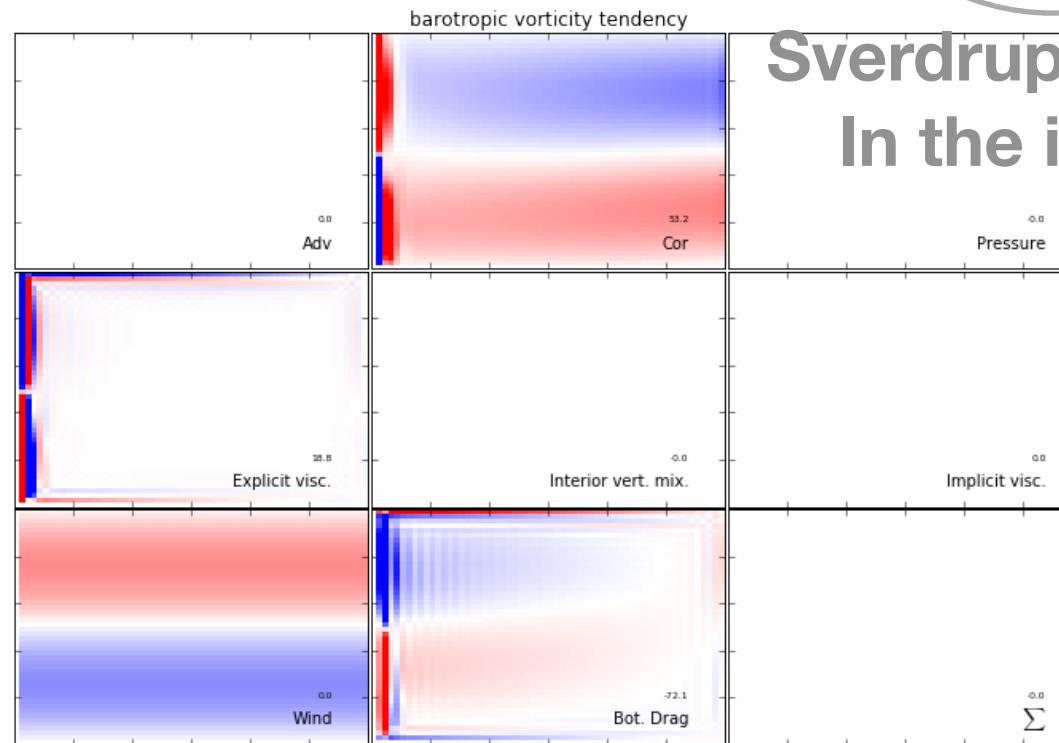
Wind curl
Bottom drag curl

Stommel's gyre (with beta)

- With latitudinal variation of Coriolis

$$\cancel{\frac{\partial \vec{u}}{\partial t} = -\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\vec{J}(\vec{P}, h)}{R}}_{\text{bot. pres. torque}} + \underbrace{k.\vec{\nabla} \times \frac{\vec{\tau}^{\text{wind}}}{\rho_0}}_{\text{wind curl}} - \underbrace{k.\vec{\nabla} \times \frac{\vec{\tau}^{\text{bot}}}{\rho_0}}_{\text{bot. drag curl}}}$$

+ $\cancel{D_\Sigma}$ horiz. diffusion. $\cancel{N.L. \text{ adv.}}$



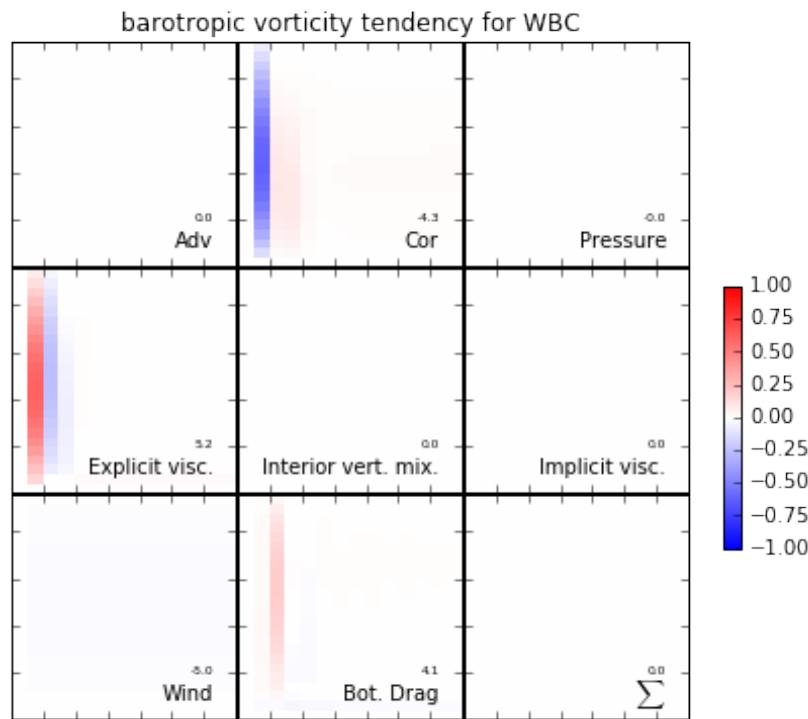
**Sverdrup Balance
In the interior**

Stommel's gyre (with beta)

- With latitudinal variation of Coriolis

$$\frac{\partial \Omega}{\partial t} = - \underbrace{\vec{\nabla} \cdot (\vec{f} \vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}(\mathbf{R}_b, \vec{u})}{k \cdot \vec{\nabla}}}_{\text{bot. pres. torque}} + \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{wind}}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{bot}}}{\rho_0}}_{\text{bot. drag curl}}$$

~~rate~~ ~~NL advection~~



Zoom over the
western boundary
current

Munk' gyre

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ON THE WIND-DRIVEN OCEAN CIRCULATION

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(Manuscript received 24 September 1949)

$$P = \int_{-h}^{z_0} p \, dz, \quad \mathbf{M}_H = \int_{-h}^{z_0} \rho \mathbf{v}_H \, dz, \quad (2a, b)$$

designate the integrated pressure and mass transport.

$$\nabla P + f \mathbf{k} \times \mathbf{M} - \boldsymbol{\tau} - A \nabla^2 \mathbf{M} = 0. \quad \mathbf{M} = \mathbf{k} \times \nabla \psi,$$

$$(A \nabla^4 - \beta \partial / \partial x) \psi = - \operatorname{curl}_z \boldsymbol{\tau},$$

For boundary conditions we choose

$$\psi_{\text{bdry}} = 0, \quad (\partial \psi / \partial \nu)_{\text{bdry}} = 0, \quad (7a, b)$$

In Ekman's and Stommel's model the ocean is assumed homogeneous, a case in which the currents extend to the very bottom. Not only is this in contrast with observations, according to which the bulk of the water transport in the main ocean currents takes place in the upper thousand meters, but it also leads to mathematical complications which rendered Ekman's

analysis very difficult, and forced Stommel to resort to a rather arbitrary frictional force along the bottom.

To avoid these difficulties, we retain Sverdrup's integrated mass transport as the dependent variable. This device makes it possible to examine the more general case of a baroclinic ocean without having to specify the nature of the vertical distributions of density and current. In recognition of the evidence that currents essentially vanish at great depths, we shall depend on lateral friction for the dissipative forces. From Stommel we retain the rectangular boundaries, although we extend the basin to both sides of the equator and deal with the *observed* wind distribution rather than a simple sinusoidal distribution.

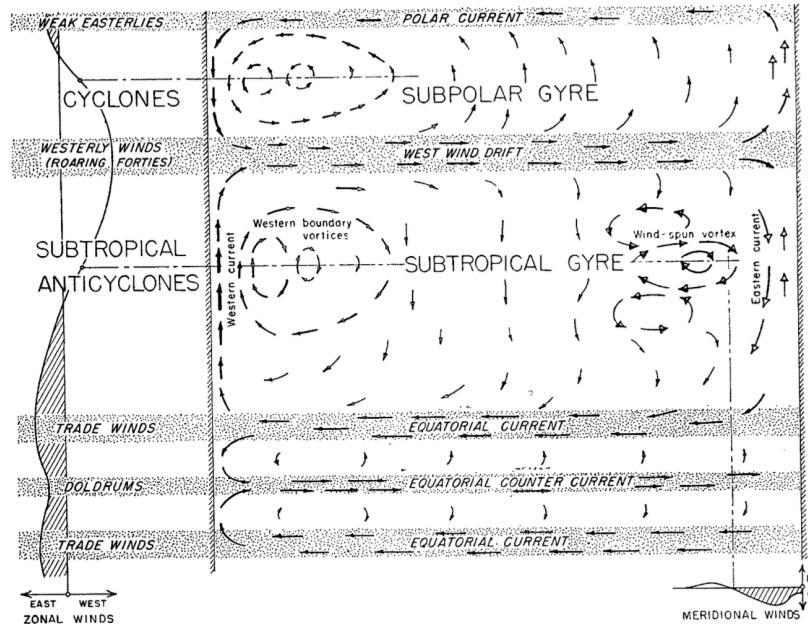


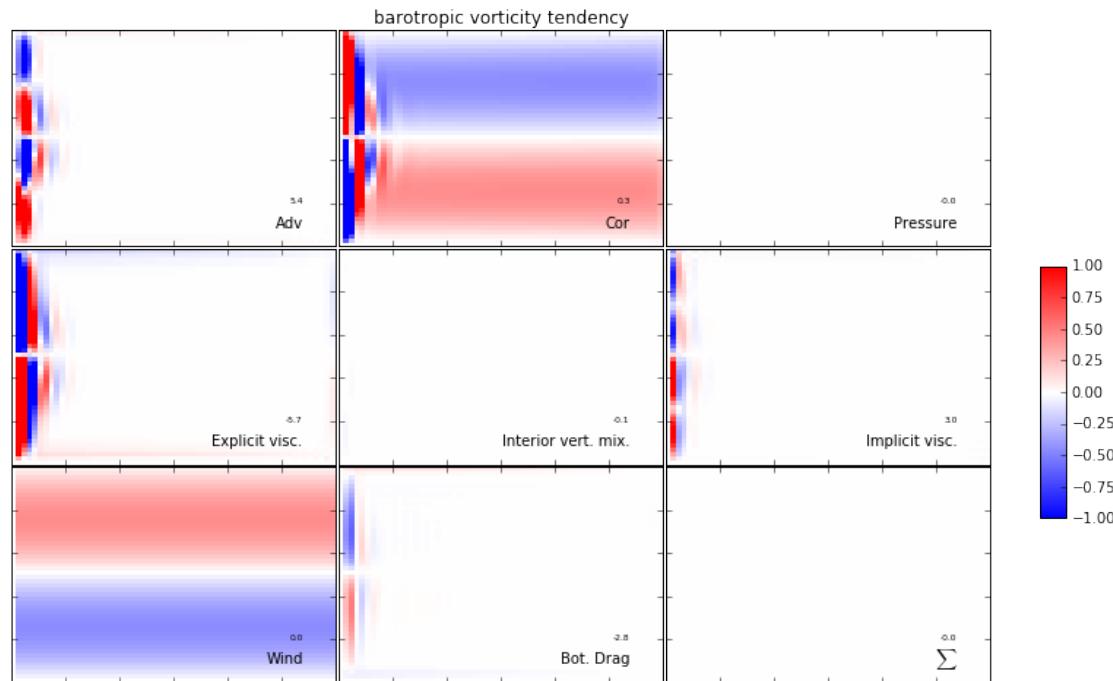
FIG. 8. Schematic presentation of circulation in a rectangular ocean resulting from zonal winds (filled arrowheads), meridional winds (open arrowheads), or both (half-filled arrowheads). The width of the arrows is an indication of the strength of the currents. The nomenclature applies to either hemisphere, but in the Southern Hemisphere the subpolar gyre is replaced largely by the Antarctic Circumpolar Current (west wind drift) flowing around the world. Geographic names of the currents in various oceans are summarized in table 3.

Gyre with beta and lateral drag

- With latitudinal variation of Coriolis

$$\cancel{\frac{\partial \vec{u}}{\partial t}} = - \underbrace{\vec{\nabla} \cdot (f \vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\vec{J} \times P(h)}{R}}_{\text{bot. pres. torque}} + \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{wind}}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{\text{bot}}}{\rho_0}}_{\text{bot. drag curl}}$$

$\cancel{\frac{\partial \vec{u}}{\partial t}}$
 rate
 horiz. diffusion.
 NL advection

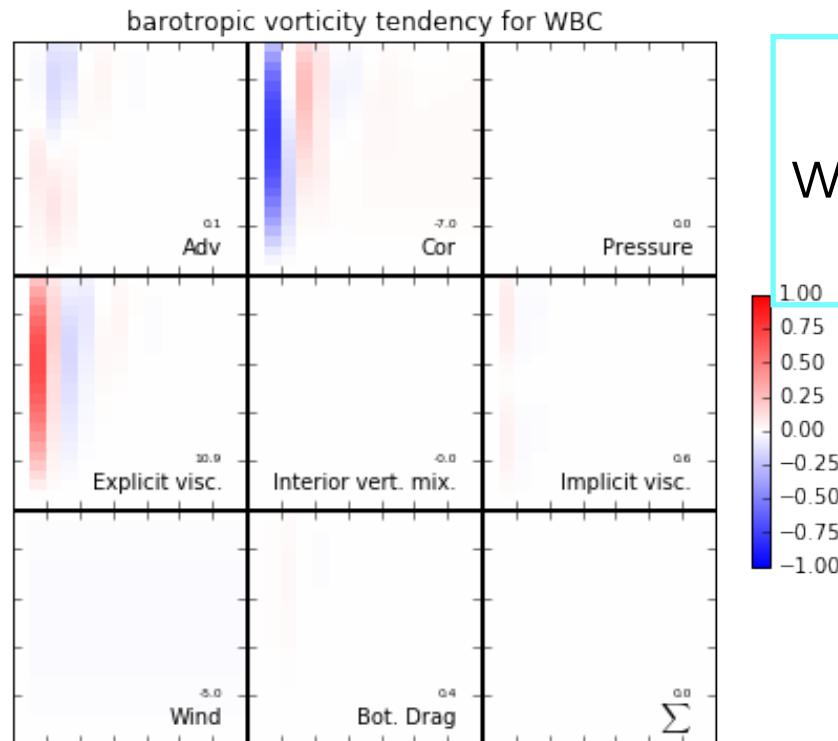


Gyre with beta and lateral drag

- With latitudinal variation of Coriolis

$$\frac{\partial \Omega}{\partial t} = \underbrace{- \nabla \cdot (f \vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}^P \times h}{k \cdot \nabla}}_{\text{bot. pres. torque}} + \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k} \cdot \vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$

\mathcal{D}_Σ horiz. diffusion. A_Σ NL advection



Zoom over the western boundary current

