

# OCEAN WAVES

## INTRODUCTION

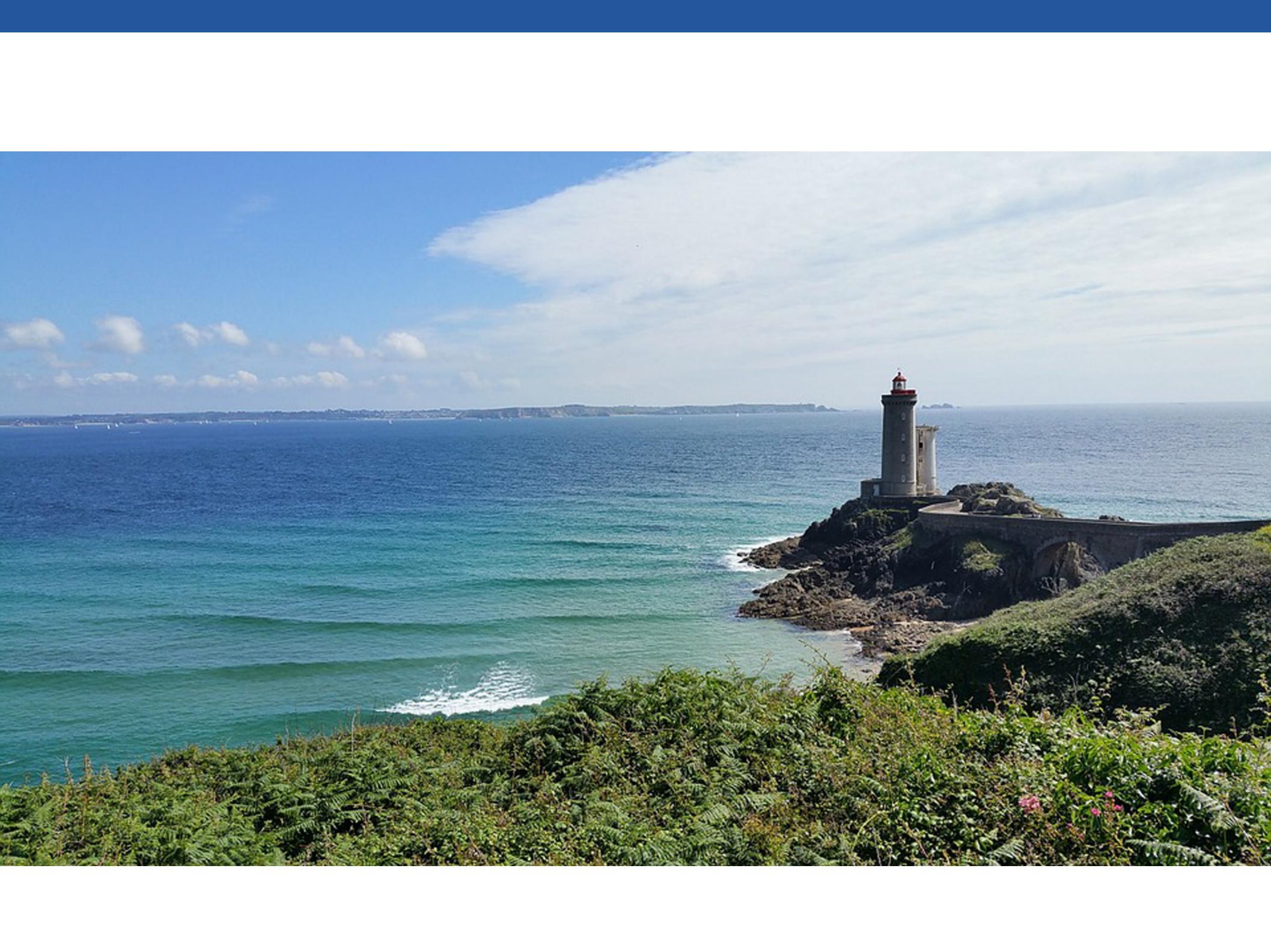
---

Jonathan GULA  
[gula@univ-brest.fr](mailto:gula@univ-brest.fr)

- **Lesson 1 :**
    - Introduction about Ocean waves
    - Surface waves
    - Internal Waves (Introduction)
    - Internal Waves in the 2-layer model
  - **Lesson 2 :**
    - Internal Waves with a continuous stratification
  - **Lesson 3 :**
    - Generation of internal waves
  - **Lesson 4 : [D109]**
    - Dissipation and interaction of internal waves
    - Activities: Numerical simulation of internal waves
  - **Lesson 5 :**
    - Long waves
    - Rossby waves [Rossby, Poincare, etc.]
    - Coastal trapped waves [Kelvin waves, etc.]
    - Equatorial waves [Rossby, Kelvin, Yanai]
- Presentations and material will be available at :
- [jgula.fr/Ondes/](http://jgula.fr/Ondes/)**

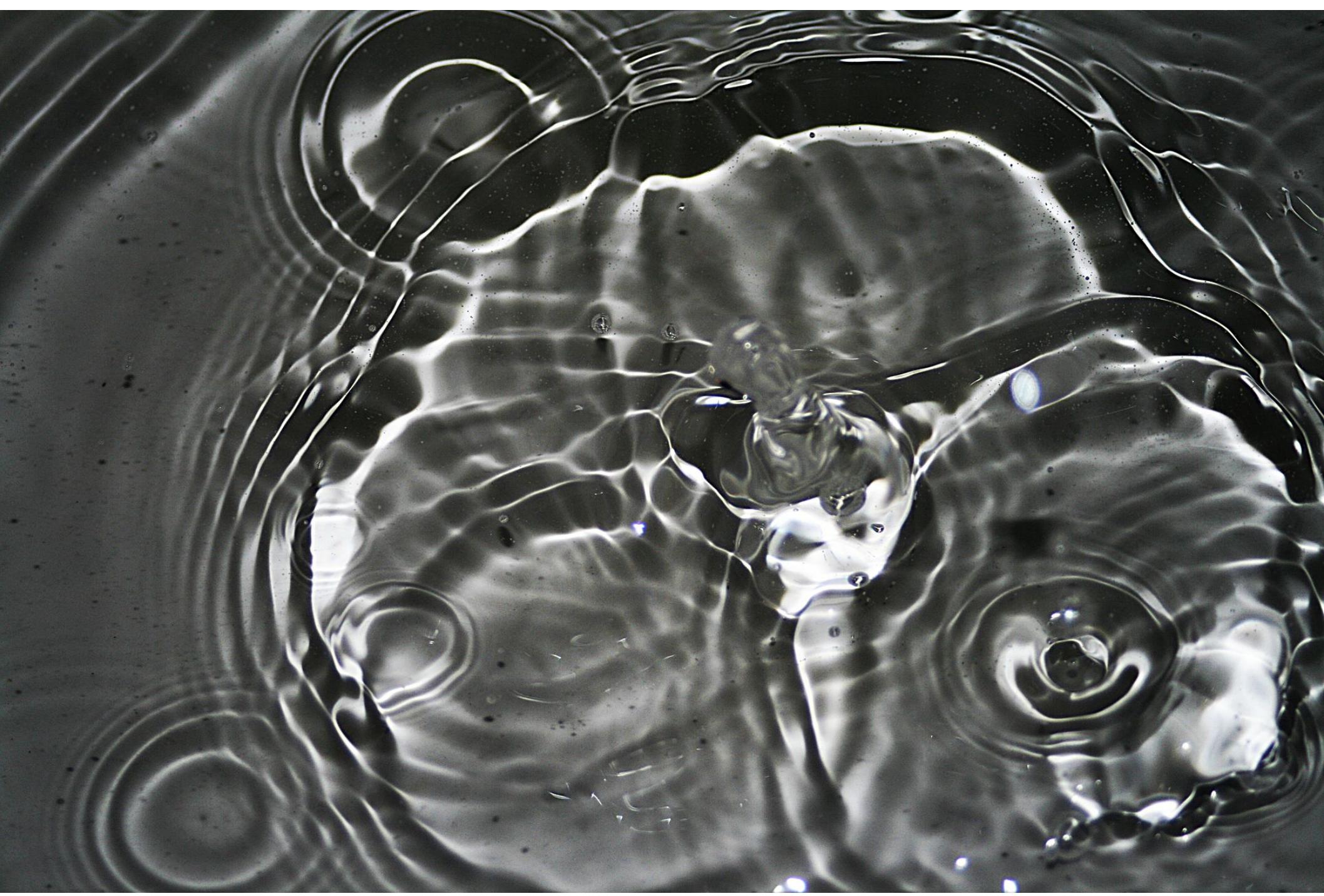
# Useful references

- *The course is largely based on:*
  - Gerkema- Zimmerman (2008). *An introduction to internal waves* <https://www.jgula.fr/Ondes/gerkema.pdf>
- *Other useful references are:*
  - Leblond-Mysak (1977) : *Waves in the ocean*
  - Whitham (1974) : *Linear and nonlinear waves*
  - Gill (1982) : *Atmosphère-Ocean Dynamics*
  - Kundu-Cohen (1987). *Fluid Mechanics. Third edition*
  - Cushman-Roisin. *Introduction to geophysical fluid Dynamics*
- *Some important research articles and reviews are available here:* <https://www.jgula.fr/Ondes/>









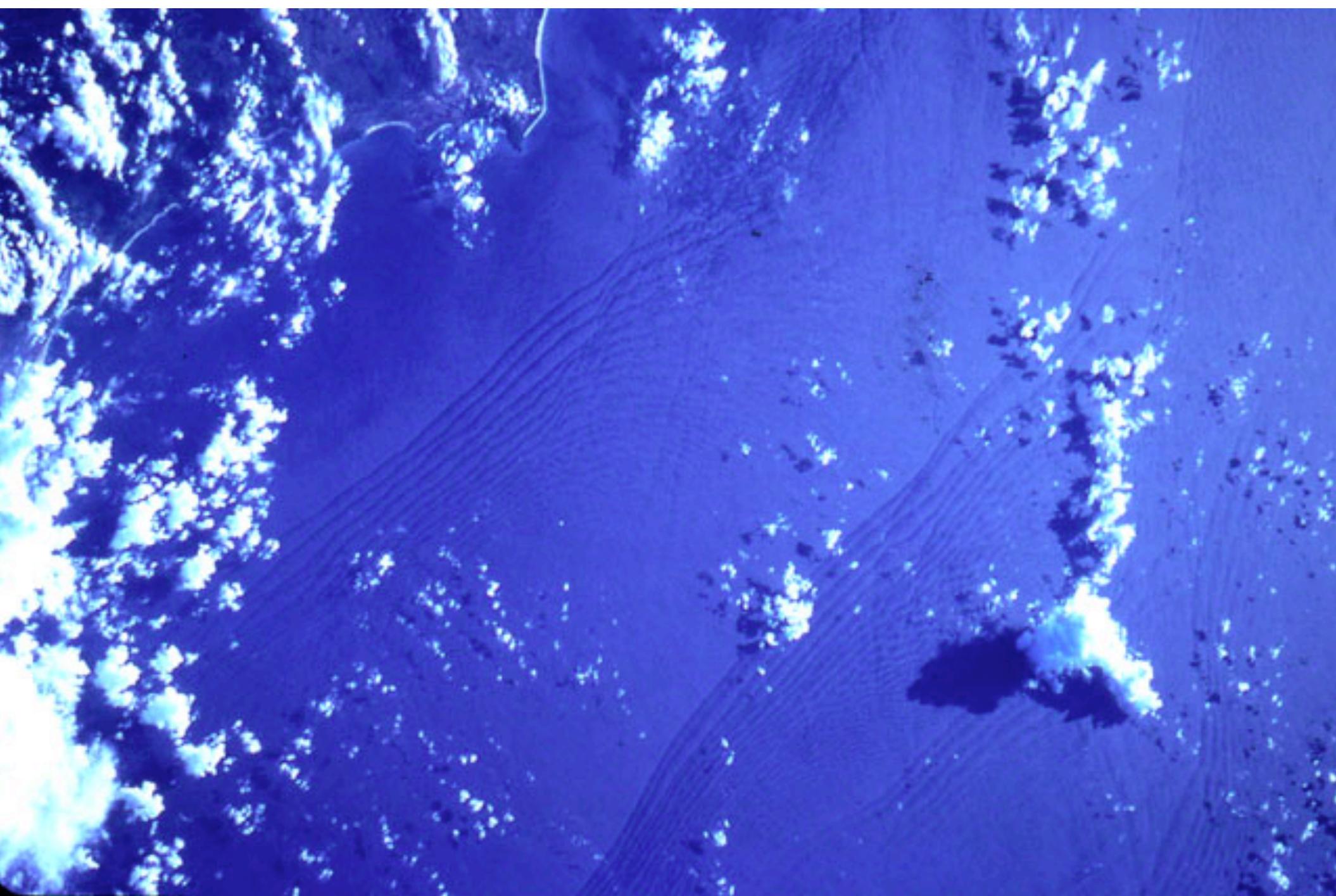


龍泉洞  
Ryusendo Cave  
5 Km

三王岩  
Sannowi Rocks  
20 Km

浄土ヶ浜  
Jodogahama Beach  
4 Km

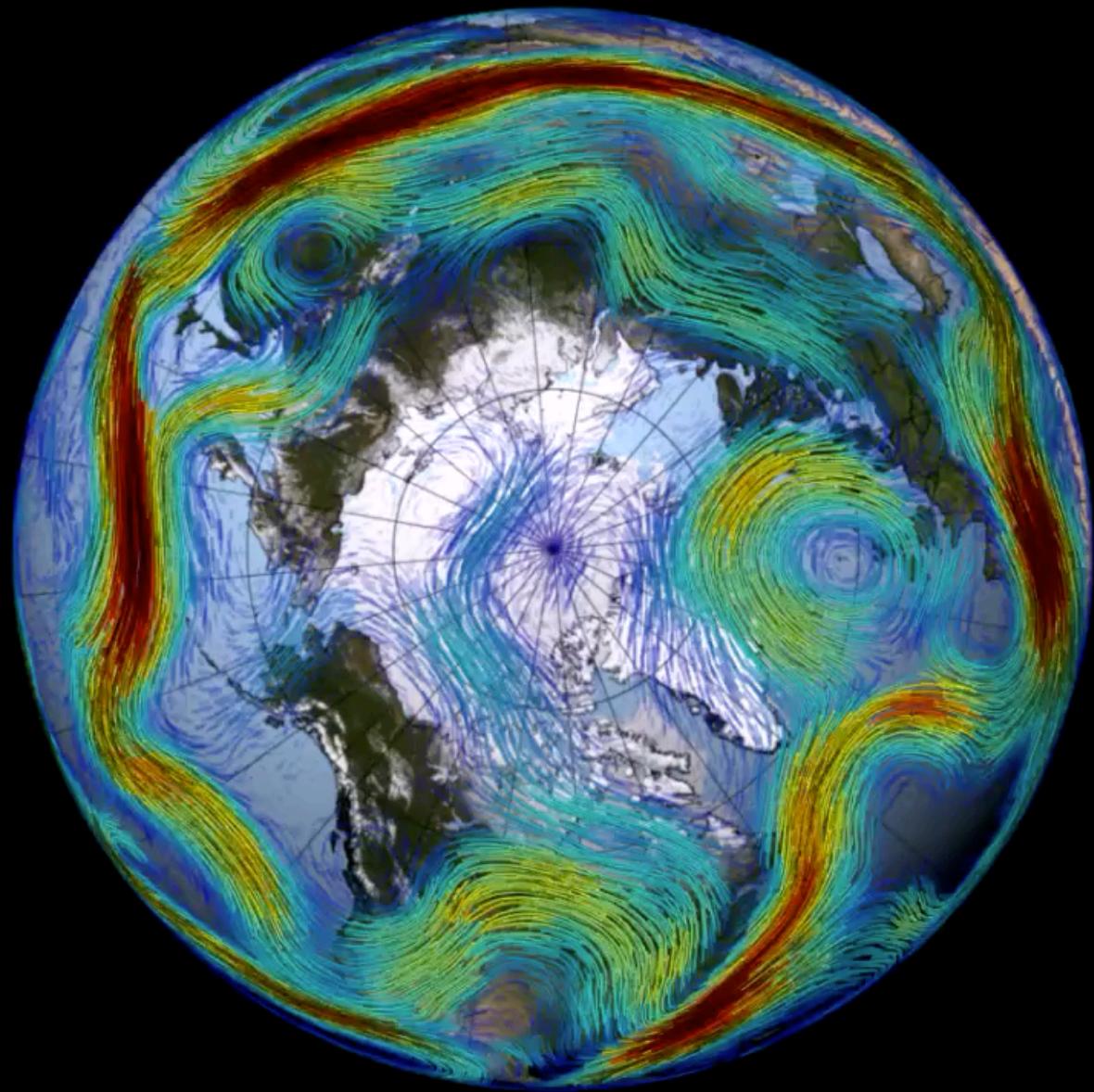
シートピアなあど  
Sealapia Naojo  
1 Km

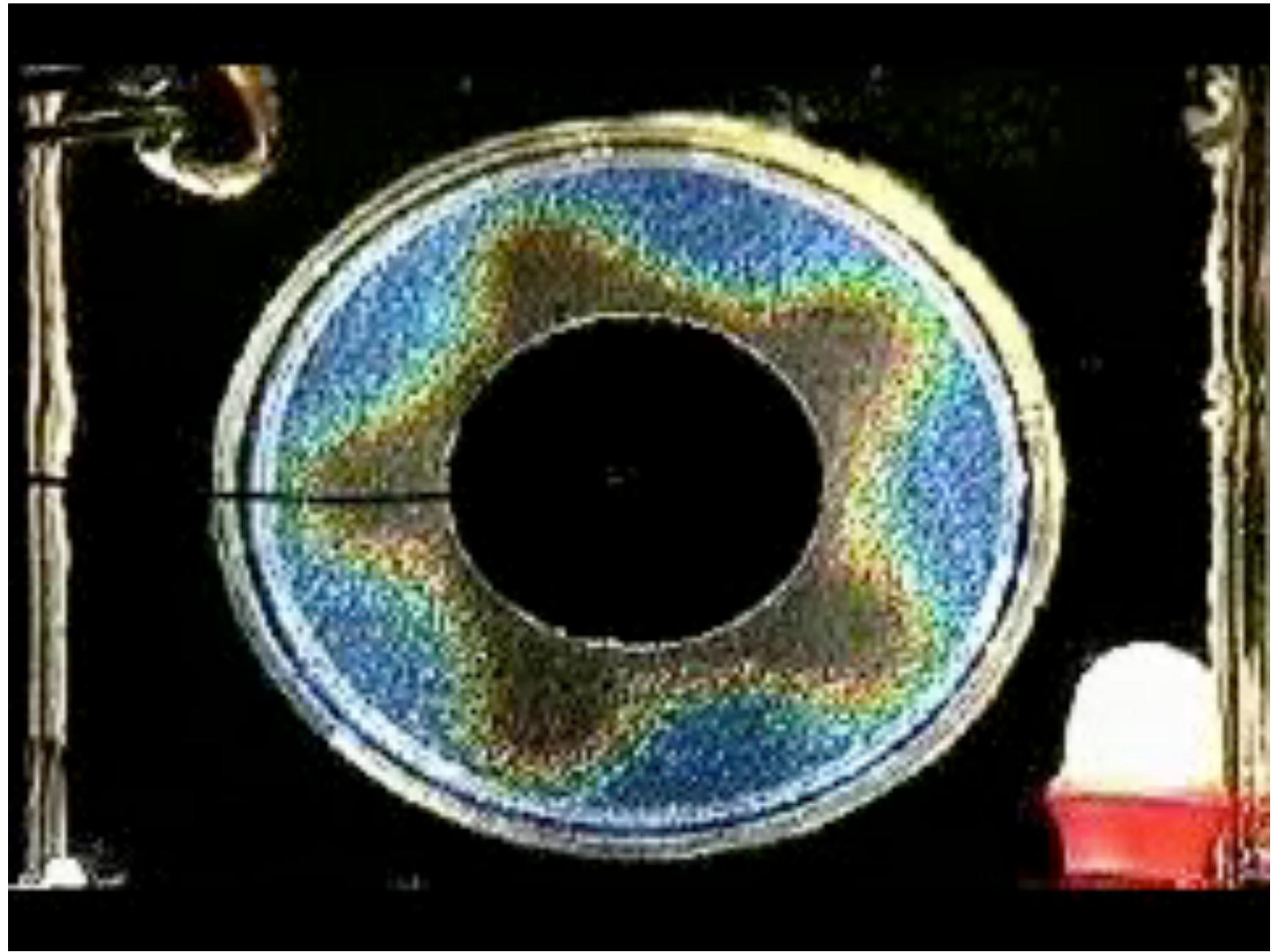


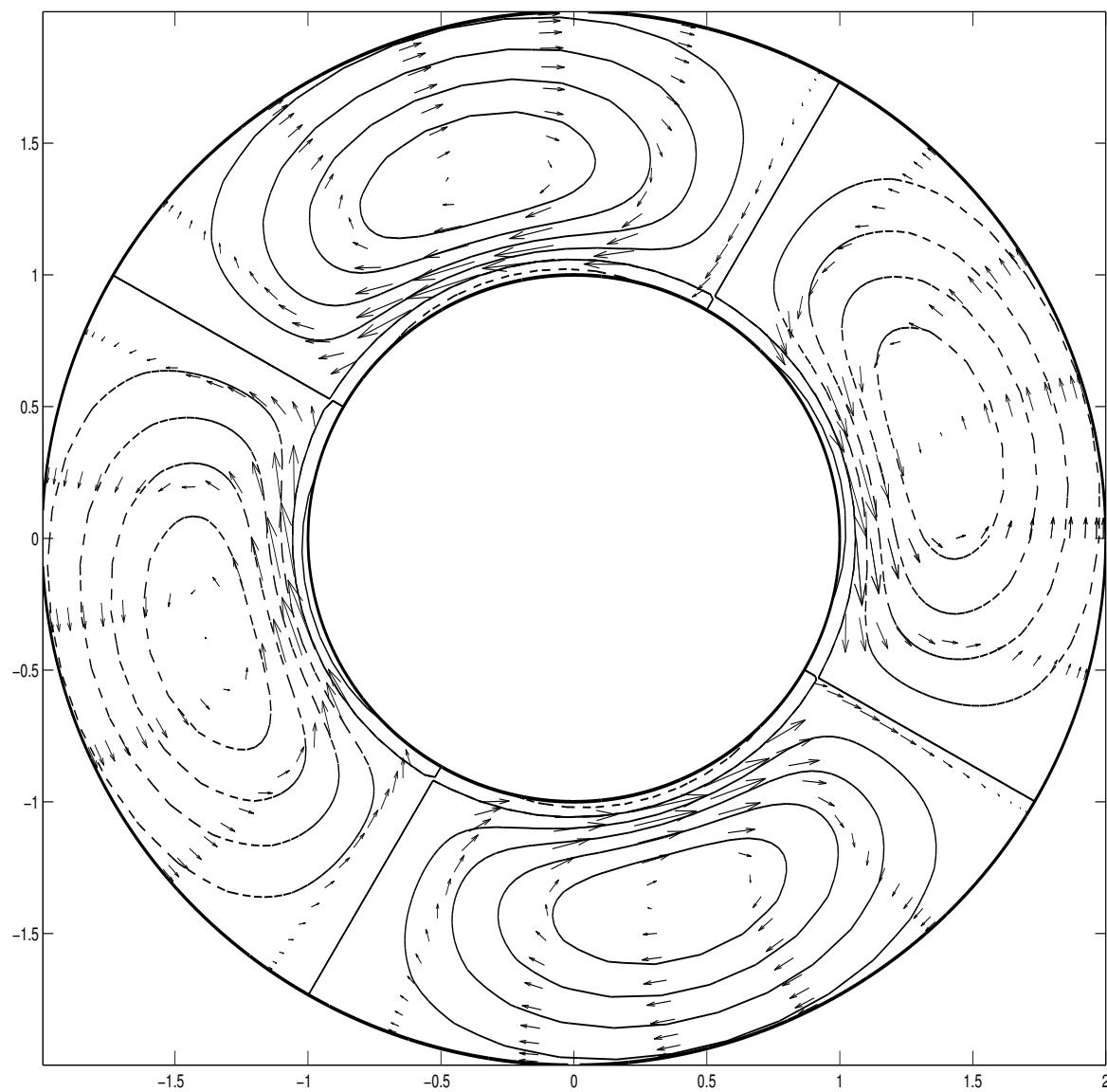




© marit kastberg

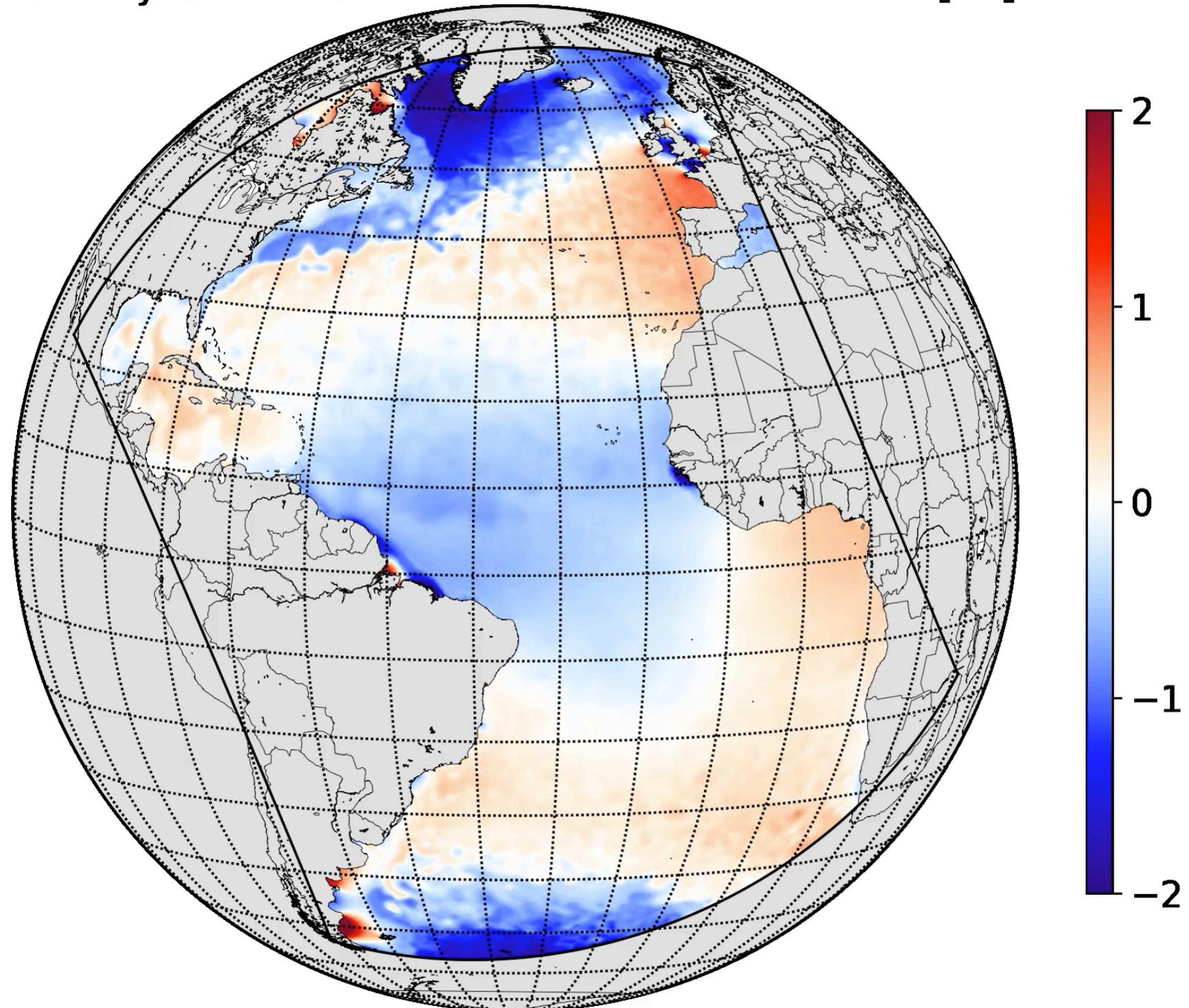


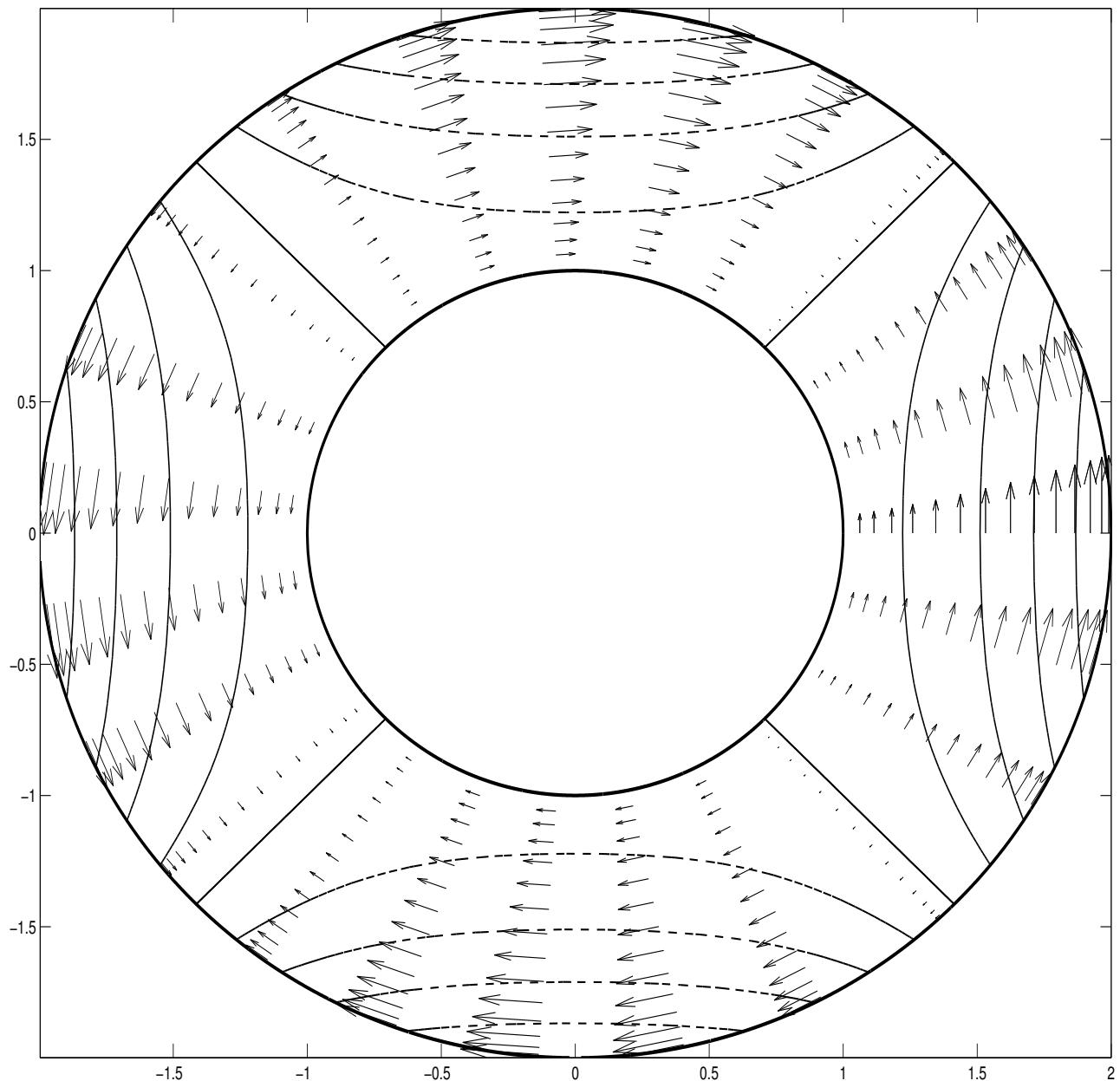


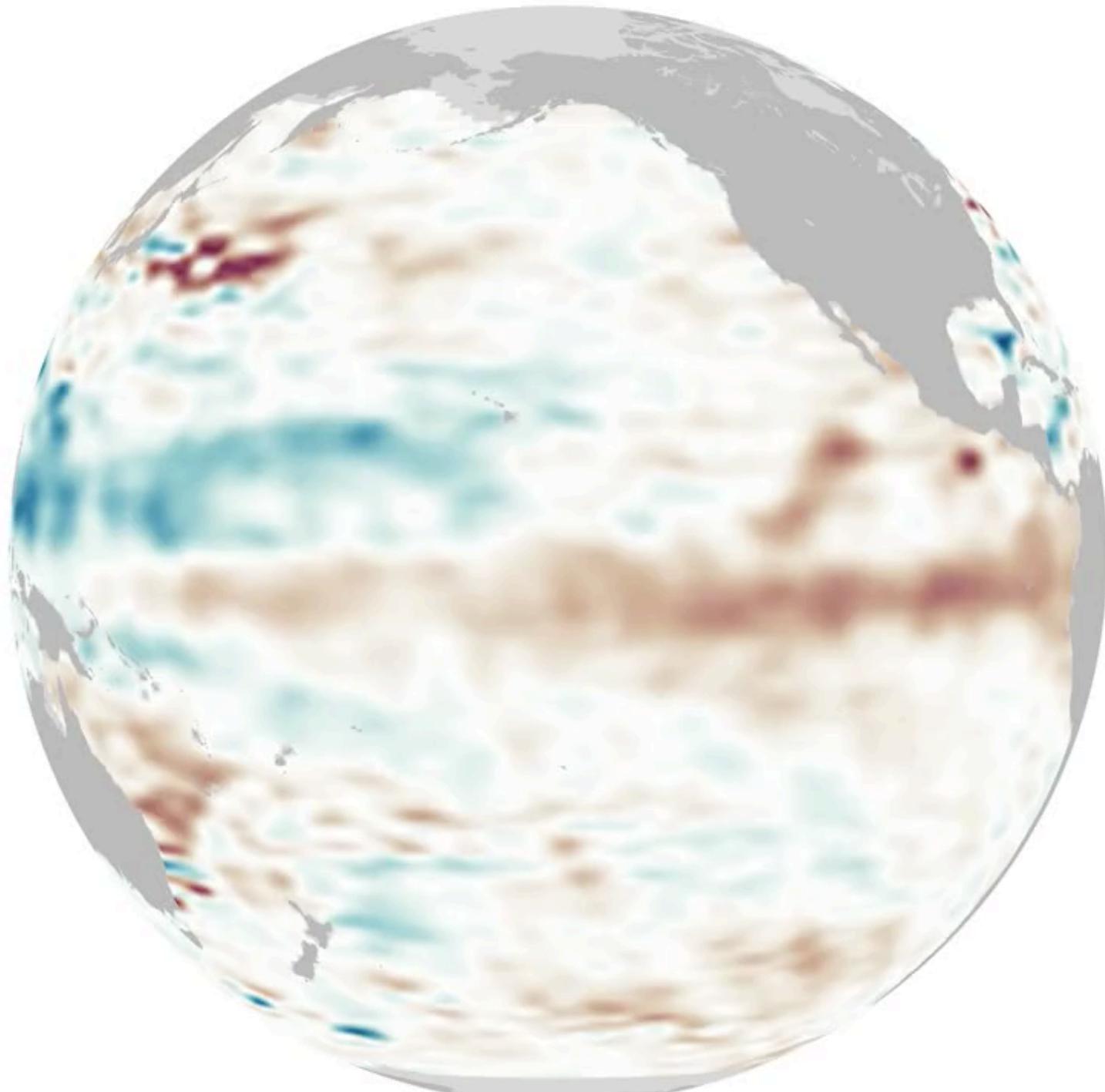


2005 - Jan 15 - 03:00

SSH [m]







Sea Surface Height Anomaly (mm)

-220

0

220

Date

Dec

Jan

Feb

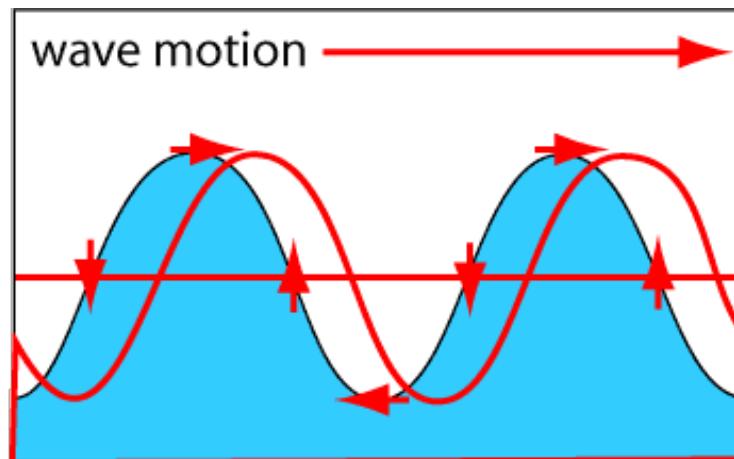
# Ocean Waves

## Definition of a wave:

- *A wave is a recognizable signal that is transferred from one part of the medium to another with a recognizable velocity of propagation. The signal may be any feature of the disturbance, such as a maximum or an abrupt change in some quantity, provided that it can be clearly recognized and its location at any time can be determined. [Whitham: « Linear and nonlinear waves »]*
-

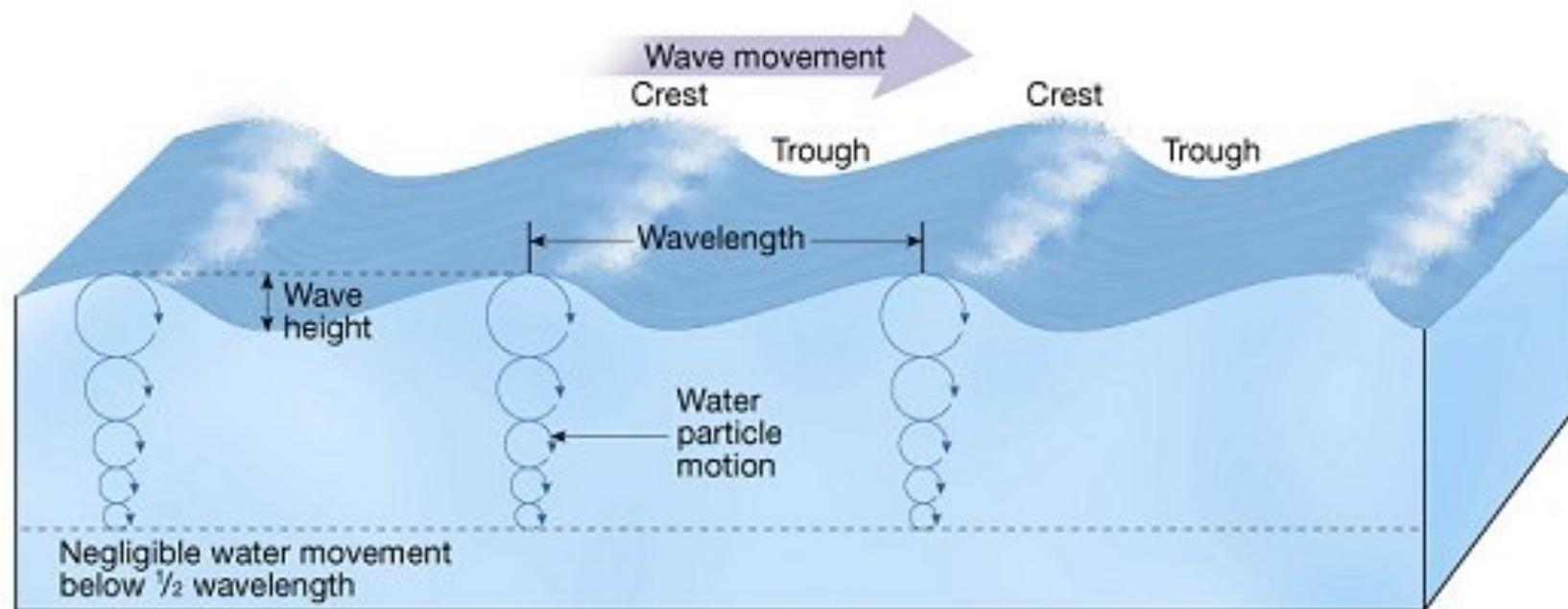
# Ocean Waves

- A wave results when fluid is displaced from a position of equilibrium.
- The restoration of the fluid to equilibrium will produce a movement of the fluid back and forth, called a wave orbit.



# Ocean Waves

- Waves propagates energy but do not transport water



# Ocean Waves

The different type of waves can be classified on the basis of:

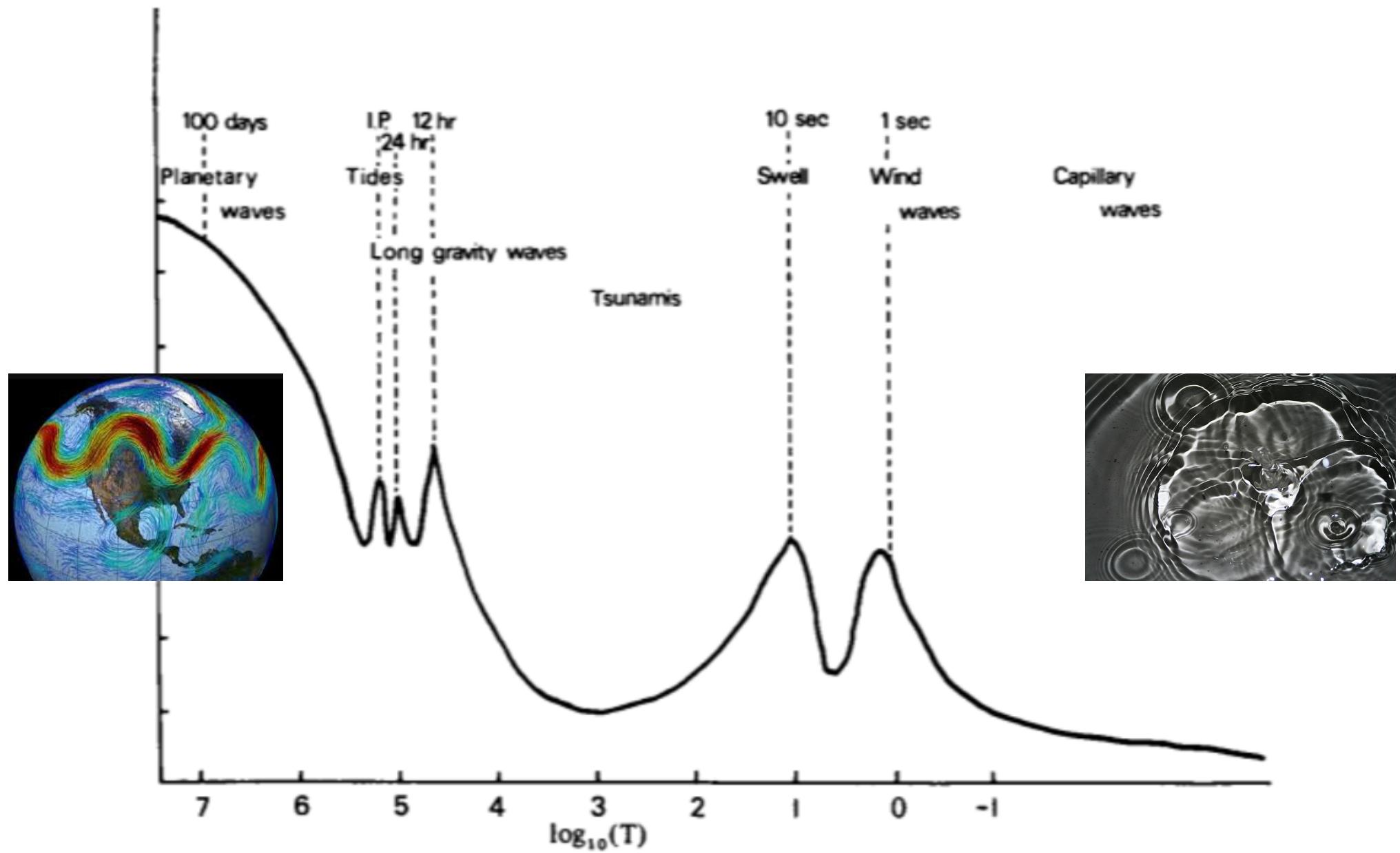
- Restoring force
- Disturbing force
- Wavelength
- Free wave Vs forced wave

# Ocean Waves

# Ocean Waves

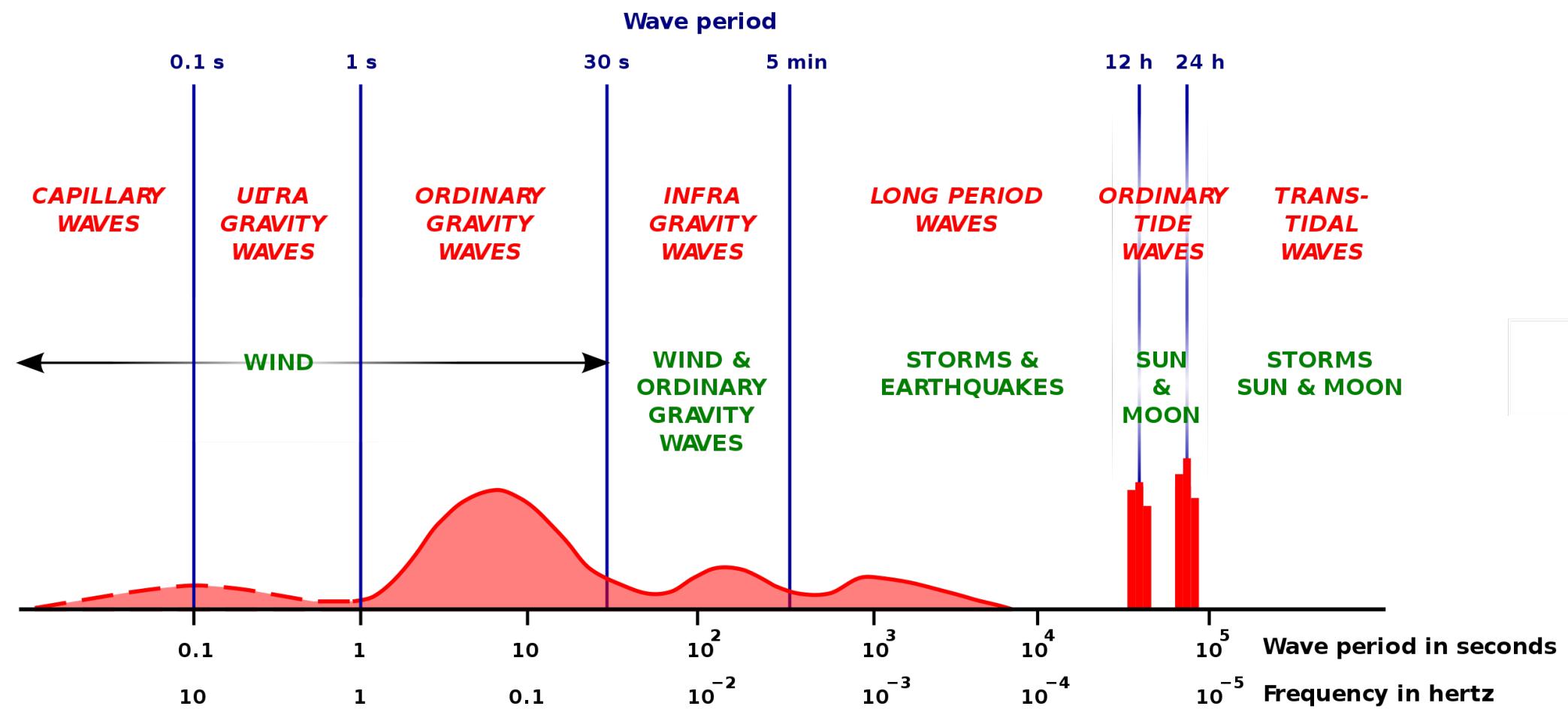
	<b>Disturbing Force</b>	<b>Restoring Force</b>	<b>Wavelength</b>	<b>Periods</b>
<b>Acoustic waves</b>		Pressure (compressibility)	1 mm - 10 km	< 1 s
<b>Capillary waves</b>		Surface tension	1 mm - 1 cm	< 1 s
<b>Surface gravity waves</b>	Wind, boat, earthquake, etc.	Gravity	1 cm - 100 km	1 s - 1 day
<b>Internal waves</b>	Tides, Wind, Topography, etc.	Gravity (stratification) + Coriolis	1 m - 100 km	1 s - 1 day
<b>Rossby waves</b>		PV (variation of Coriolis with latitude)	100 km - 1000 km	Days - Months
<b>Kelvin wave</b>		Pressure gradient + Coriolis	10 km - 100 km	Days - Months
<b>Equatorial Waves</b>		Pressure gradient + Coriolis	10 km - 1000 km	Days - Years

# Ocean Waves

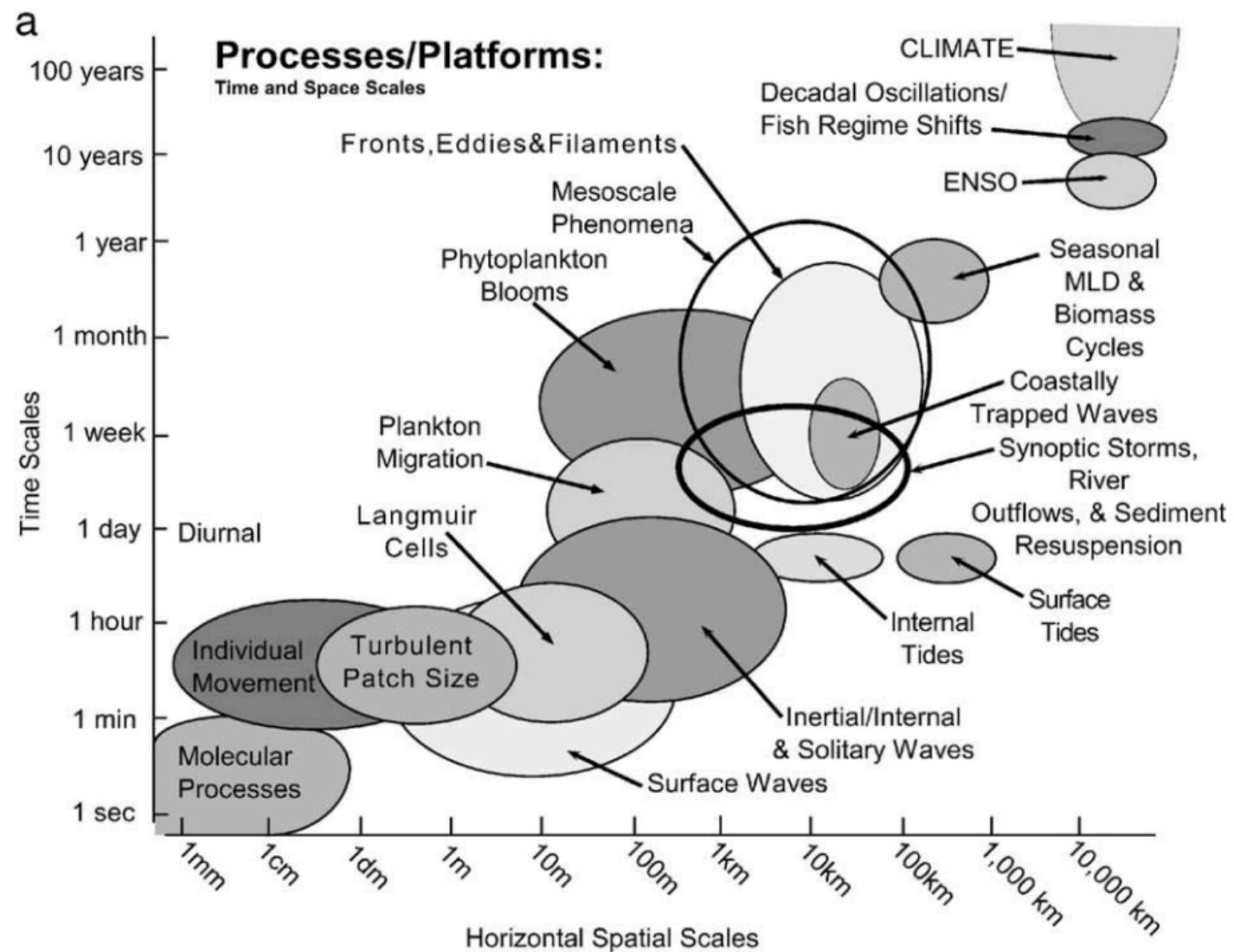


Schematic energy spectrum of ocean variability [Leblond & Mysak]

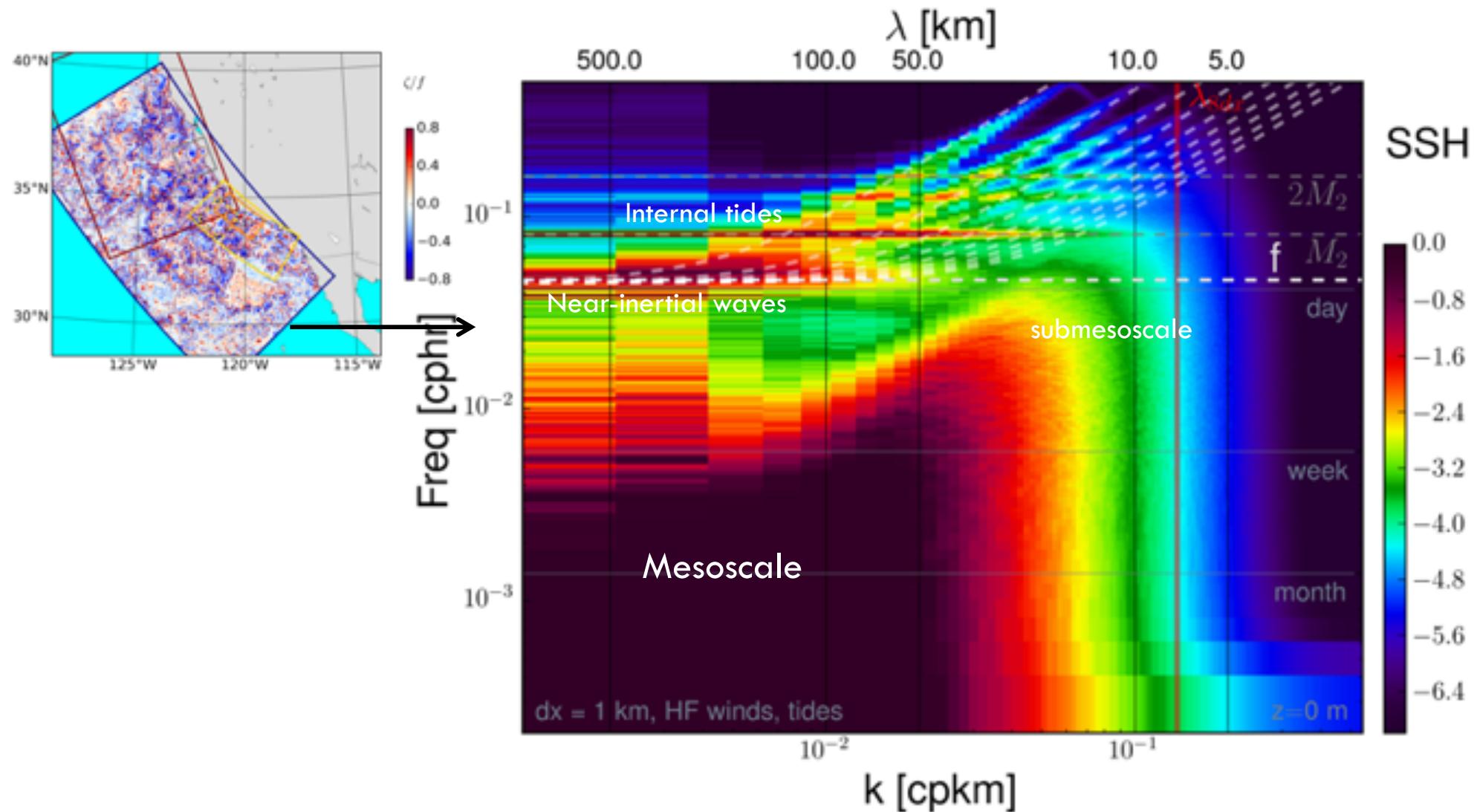
# Surface gravity Waves



# Ocean Waves



# Ocean Waves



Azimuthally-averaged 2D frequency-wavenumber spectra for SSH in California Current

- Mathematically two main classes of waves:

**Hyperbolic waves and dispersive waves**

# Hyperbolic waves

1. **Hyperbolic waves** are formulated in terms of hyperbolic partial differential equations, for example:

$$\eta_t + c\nabla\eta = 0$$

$$\eta_{tt} - c^2\nabla^2\eta = 0$$

# Hyperbolic waves

1. **Hyperbolic waves** are formulated in terms of hyperbolic partial differential equations, for example:

$$\eta_t + c\nabla\eta = 0$$

$$\eta_{tt} - c^2\nabla^2\eta = 0$$

- With general solutions in the form:

$$\eta = f(x - ct)$$

$$\eta = f(x - ct) + g(x + ct)$$

- Very frequent in acoustics, elasticity, electromagnetism, etc.

# Hyperbolic waves

## 1. Examples of Hyperbolic waves

- Flood wave, tidal bores



- Shock wave



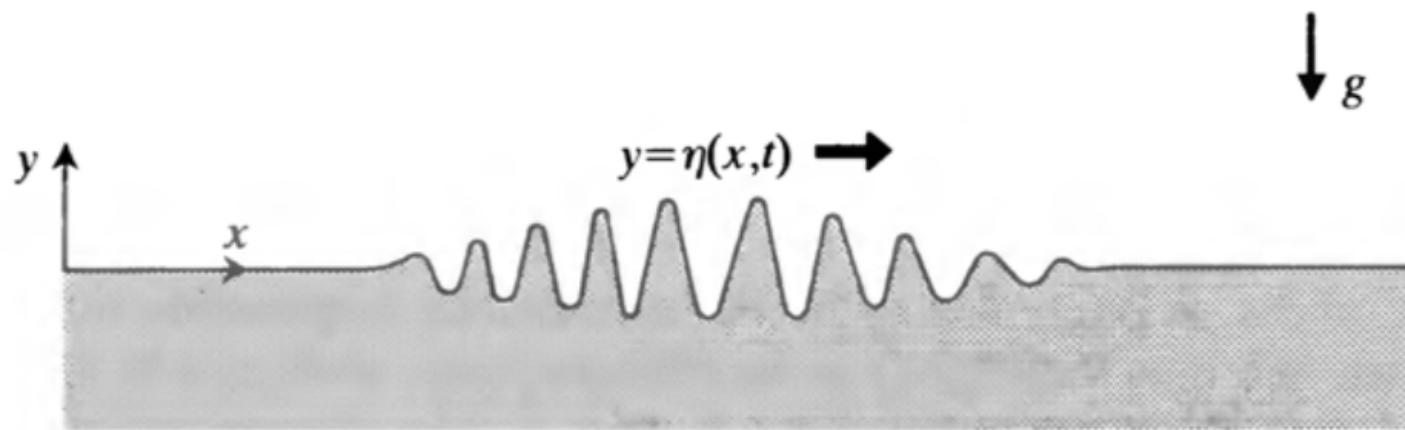
# Dispersive waves

2. **Dispersive waves** come from a variety of partial differential equations, they are characterized principally by their dispersion relation:

$$\omega = f(k)$$

Connecting the frequency and the wave number.

They are visualized as a group of waves where the different Fourier components propagate at different speeds



# Dispersive waves

2. Dispersive waves:

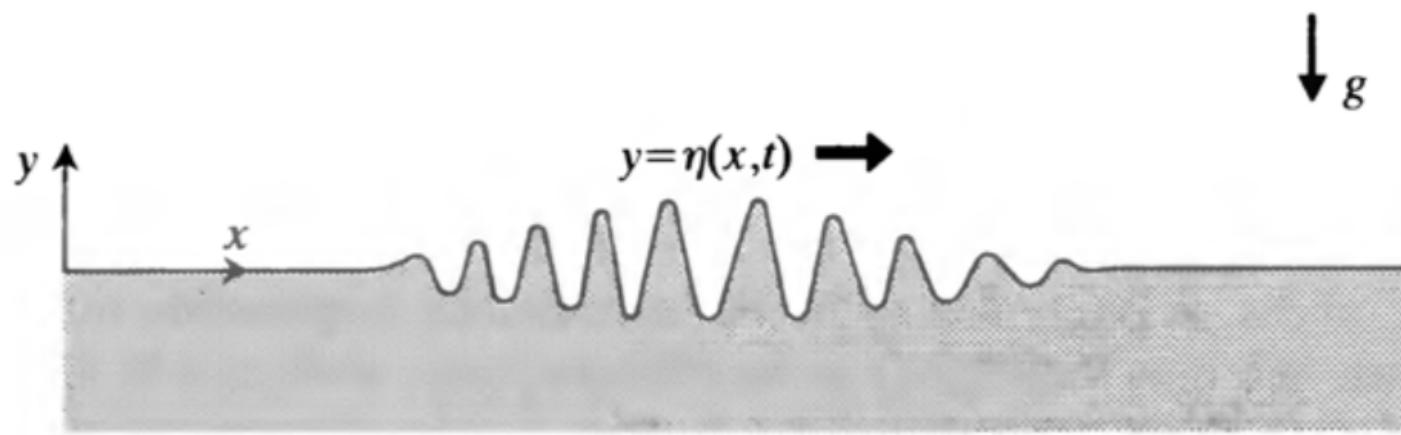
$$\omega = f(k)$$

The wave speed is

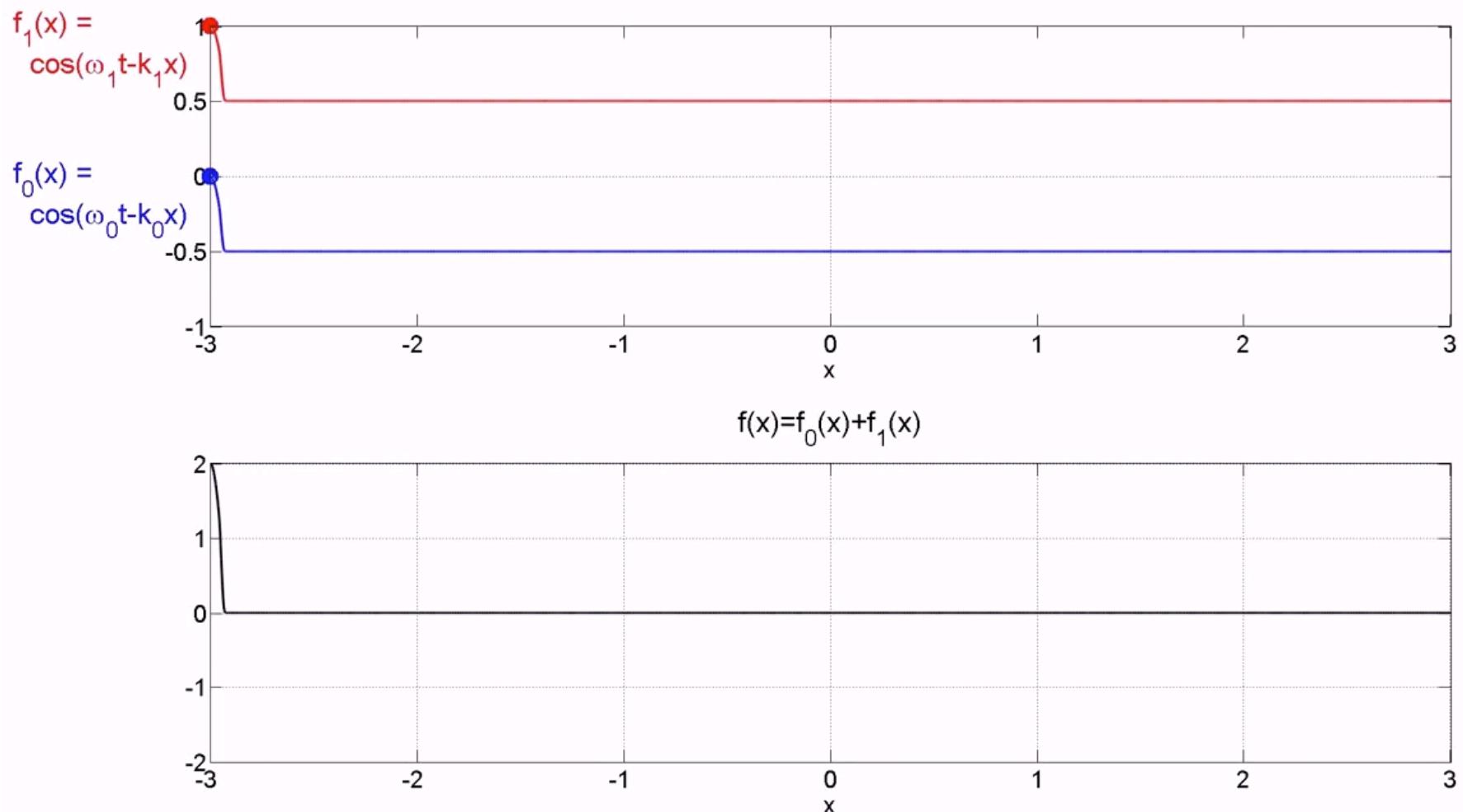
$$c = \frac{\omega}{k}$$

Energy propagates with the group velocity:

$$c_g = \frac{\partial \omega}{\partial k}$$



# Dispersive waves

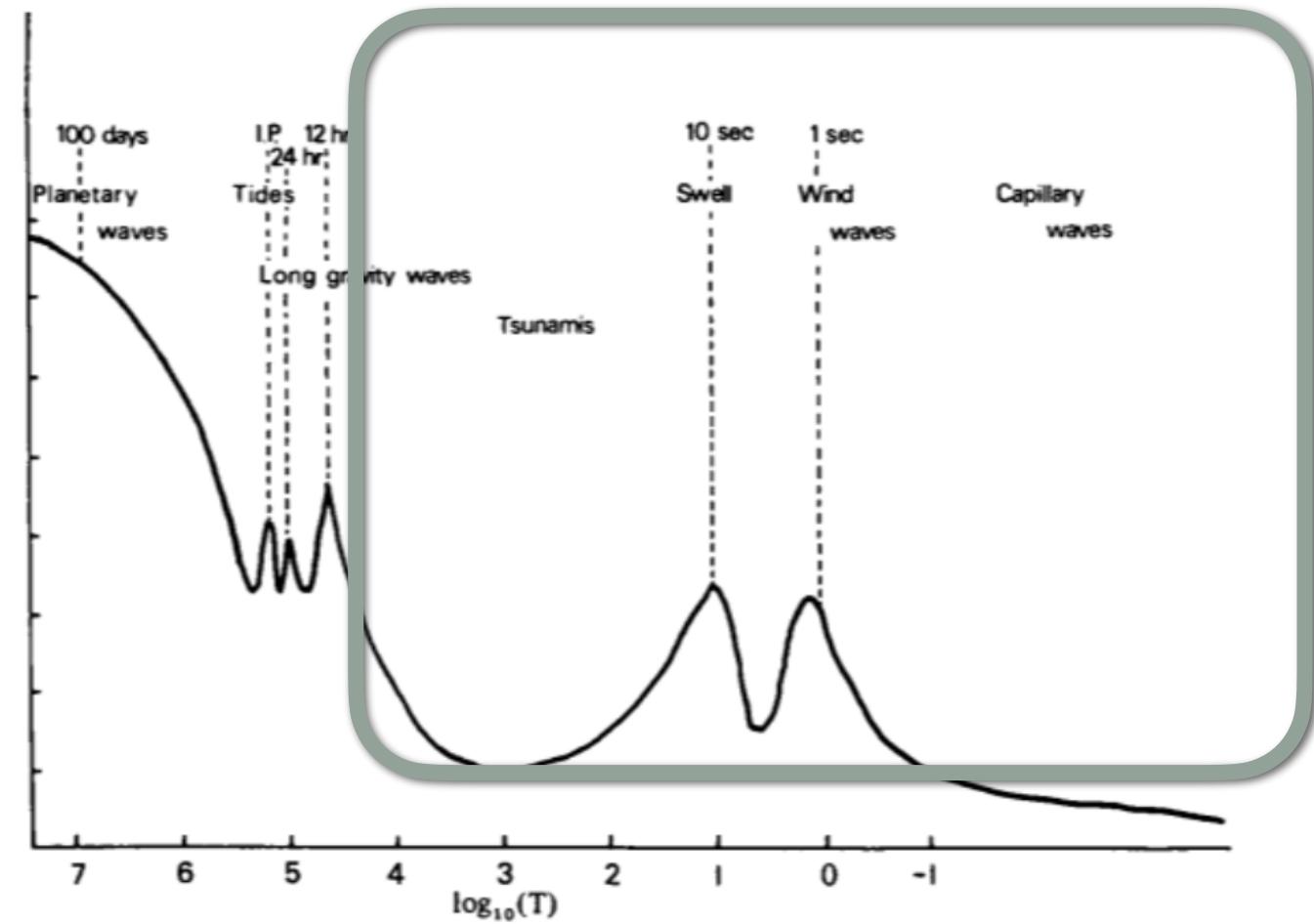


Source: [https://www.youtube.com/watch?v=uii9clp\\_DSg](https://www.youtube.com/watch?v=uii9clp_DSg)

# OCEAN WAVES

1. Surface waves [very brief reminder]
2. Internal waves
3. Long waves
  - a. Rossby waves [Rossby, Poincare, etc.]
  - b. Coastal trapped waves [Kelvin waves, etc.]
  - c. Equatorial waves [Rossby, Kelvin, Yanai]

# 1. Surface waves



Schematic energy spectrum of ocean variability [Leblond & Mysak]

# 1. Surface waves

Solving the linearized incompressible Euler equations:

Incompressible Euler equations (*convective or Lagrangian form*)

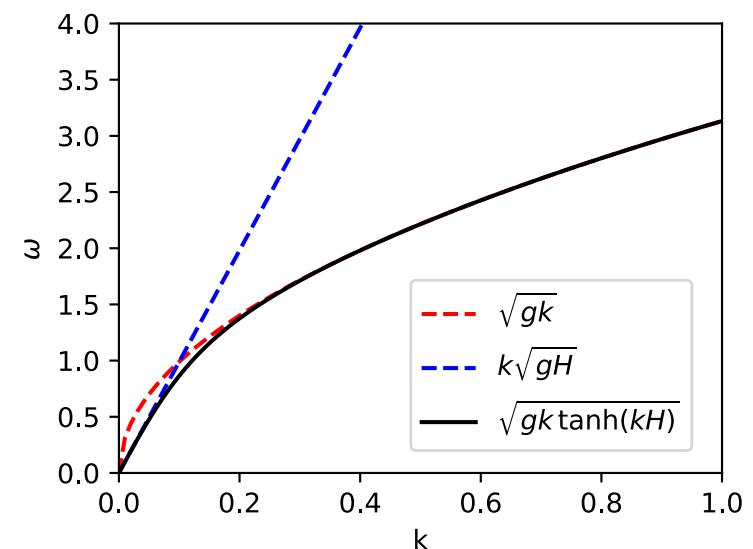
$$\begin{cases} \frac{D\rho}{Dt} = 0 \\ \frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

$$\begin{aligned} u_t &= -p_x \\ v_t &= -p_y \\ w_t &= -p_z - g \\ 0 &= u_x + v_y + w_z, \end{aligned}$$

Looking for a wave solution:

$$\Phi = F(z) e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

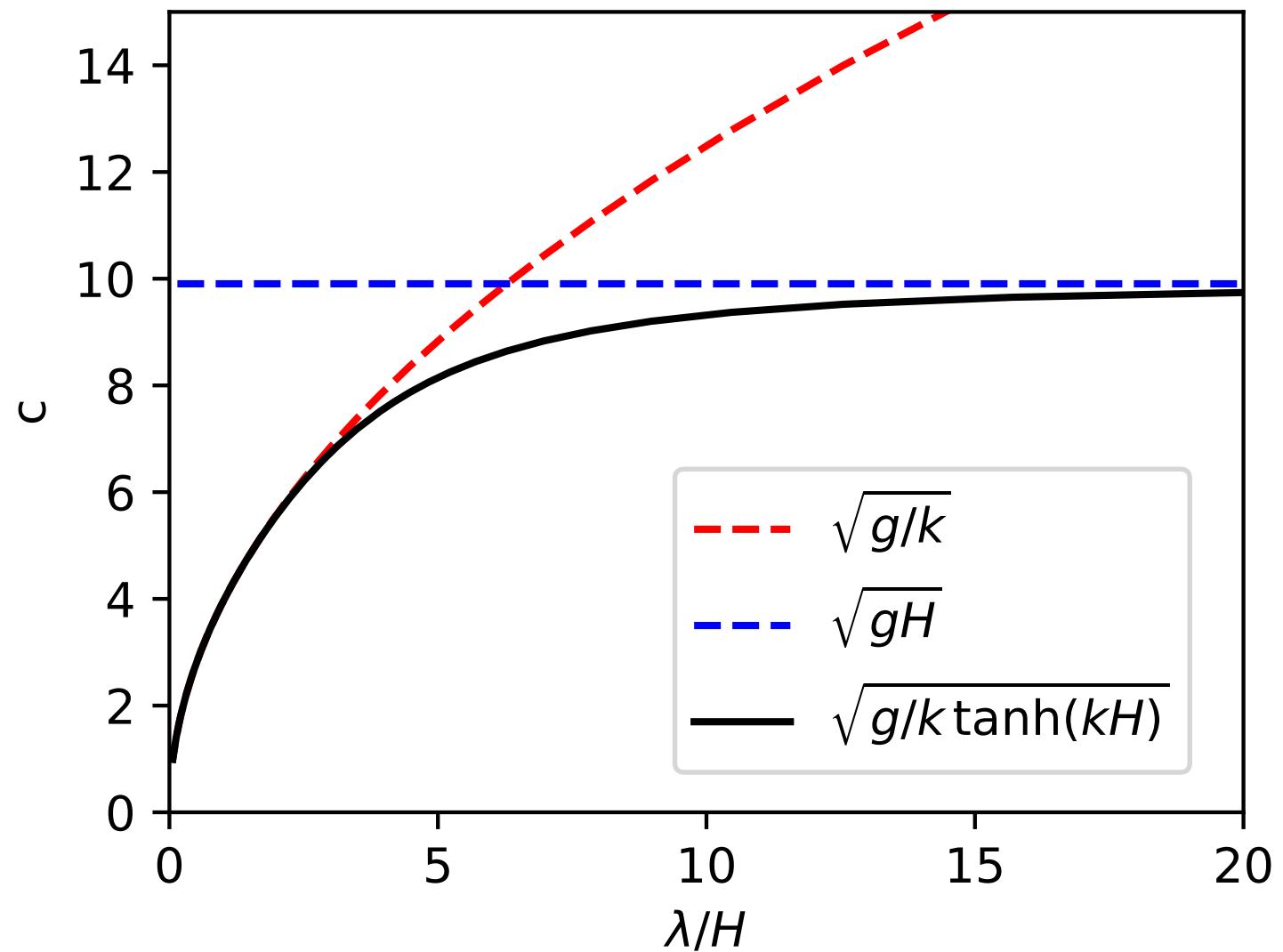
We get the dispersion relation :



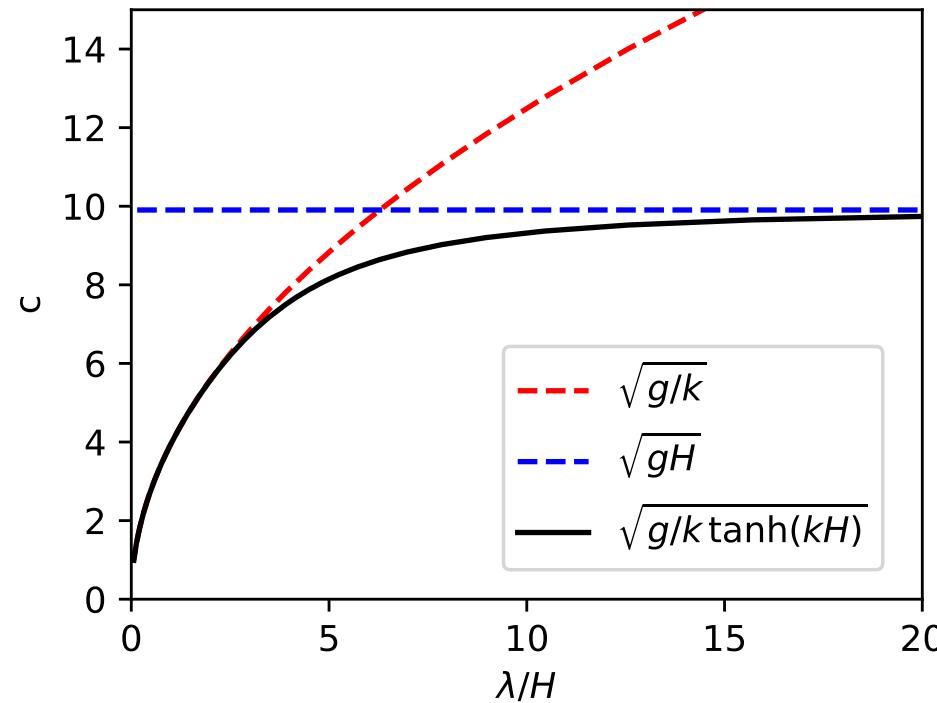
# 1. Surface waves

$$c = \frac{\omega}{k}$$

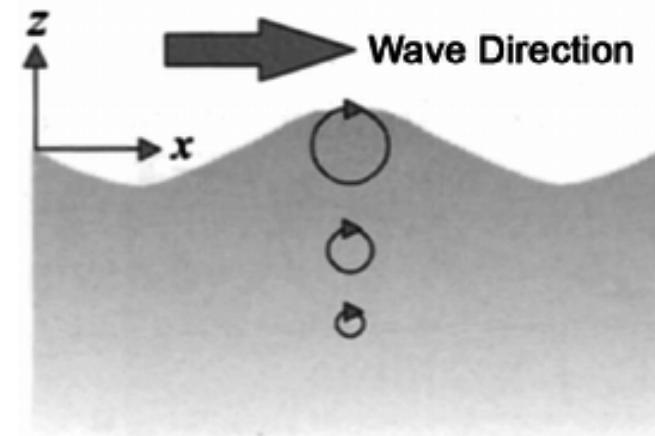
Phase speed:



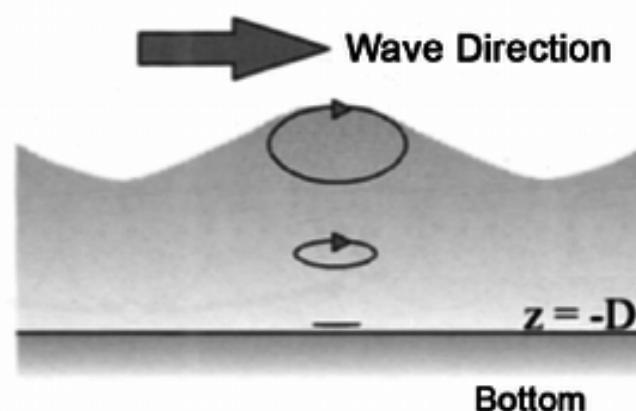
# 1. Surface waves



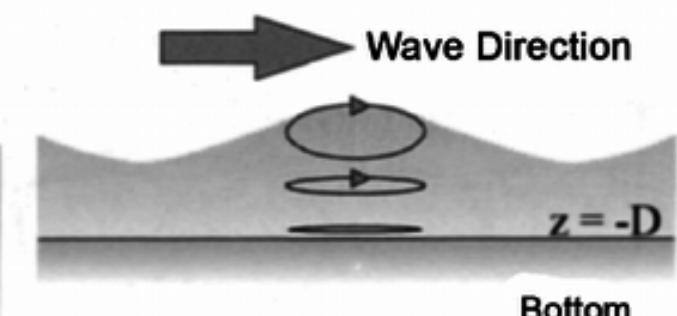
Deep Water



Intermediate Depth



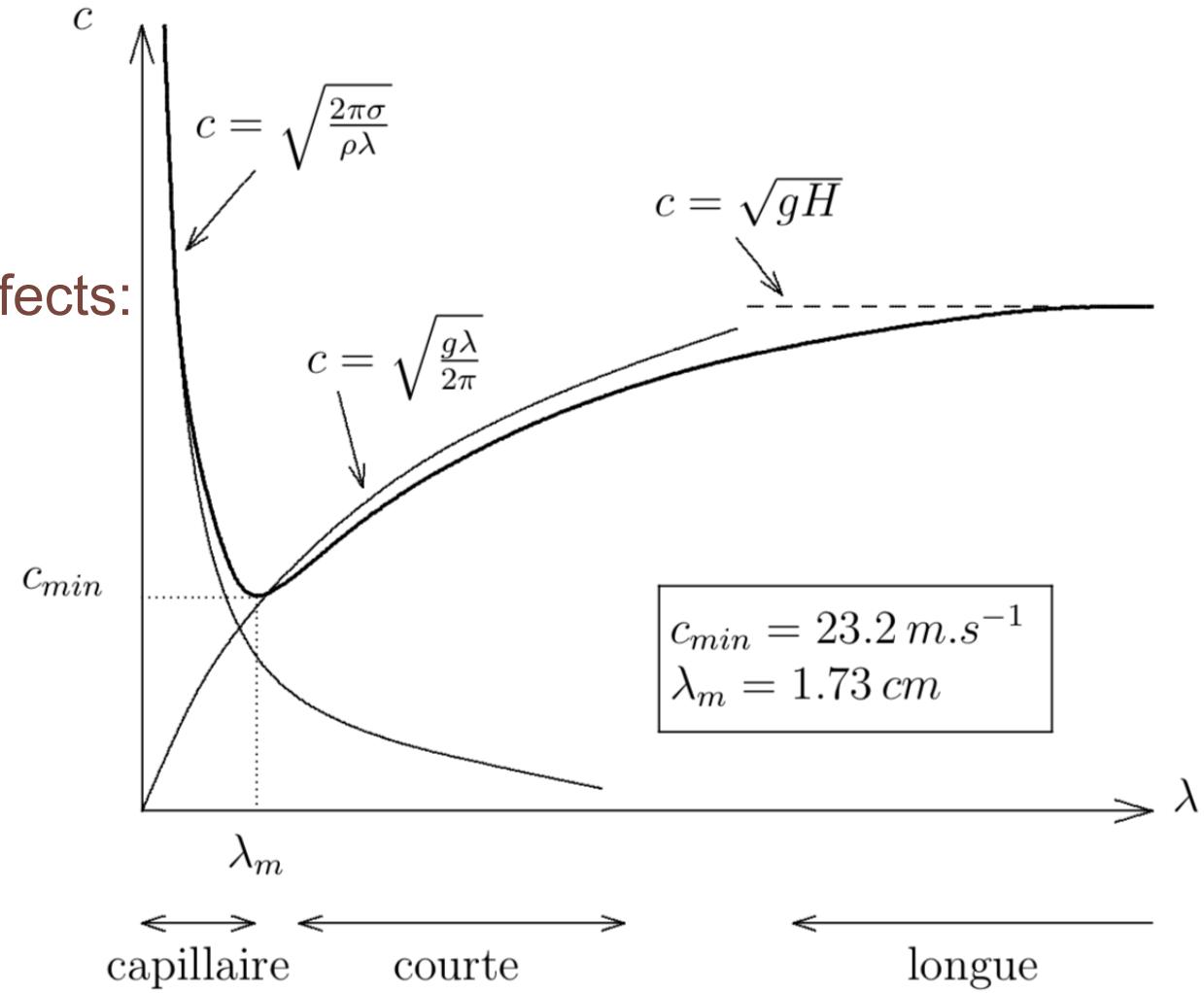
Very Shallow Water



# 1. Surface waves



With surface tension effects:



# OCEAN WAVES

1. Surface waves
2. Internal waves
3. Long waves
  - a. Rossby waves [Rossby, Poincare, etc.]
  - b. Coastal trapped waves [Kelvin waves, etc.]
  - c. Equatorial waves [Rossby, Kelvin, Yanai]