Barotropic Vorticity Diagnostics for CROCO

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1 Barotropic vorticity equation

1.1 Continuous equation

The full barotropic vorticity balance equation of the flow is obtained by integrating the momentum equations in the vertical and cross differentiating them:

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = - \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}(P_b,h)}{\rho_0}}_{\text{bot. pres. torque}} + \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}} \tag{1}$$

where the barotropic vorticity is defined as the vorticity of the vertically integrated velocities[1]

$$\Omega = \frac{\partial \overline{v}}{\partial x} - \frac{\partial \overline{u}}{\partial y}$$

with (u,v) the (x,y) components of the horizontal flow, and the overbar denotes a vertically integrated quantity,

$$\overline{u} = \int_{-h}^{\zeta} u\, dz,$$

where $\zeta(x,y,t)$ is the free-surface height and $h(x,y) > 0$ the depth of the resting topography. $H(i,j,t) = \int_{-h}^{\zeta} dz = \zeta(i,j,t) + h(i,j)$ is the total depth of the water column. Finally, the curl of non-linear advection terms can be written as

$$A_\Sigma = \frac{\partial^2(\overline{vv} - \overline{uu})}{\partial x \partial y} + \frac{\partial^2 \overline{uv}}{\partial x \partial x} - \frac{\partial^2 \overline{uv}}{\partial y \partial y},$$

and $\mathcal{D}_\Sigma$ is the term due to the horizontal diffusion in the model implicitly part of the advective scheme, plus eventually some explicit diffusion.

Examples of barotropic vorticity budget with ROMS/CROCO using these diagnostics and interpretations can be found for example in **?**, **?**, **?**.

[1]Note that the barotropic vorticity is not identical to the vertically integrated vorticity. The curl and the vertical integration can be interchanged at the expense of introducing terms due to the horizontal variations of the limits of the integral. The difference $\Omega - \overline{\zeta} = \vec{u_s}\times\vec{\nabla}\zeta + \vec{u_b}\times\vec{\nabla}h$, where $\vec{u_s}$ and $\vec{u_b}$ are the horizontal velocities at the surface and bottom, respectively, can be non-negligible at places where we have both significant bottom currents and large topography slopes.

1.2 online diagnostics

Variables included in the croco_diags_vrt.nc files are:

- vrt_rate = rate of change of barotropic vorticity [*step3d_uv2.F*]
- vrt_xadv = contribution of advection + implicit dissipation along xi-axis+ grid curvature terms (see CURVGRID) [*rhs3d.F*]
- vrt_yadv = contribution of advection + implicit dissipation along eta-axis (see [*rhs3d.F*]
- vrt_hdiff = implicit dissipation along xi- and eta- axis [*rhs3d.F*] [already included in vrt_xadv+ vrt_yadv]
- vrt_cor = planetary vorticity advection [*rhs3d.F*]
- vrt_prsgrd = bottom Pressure torque [*prsgrd.F*]
- vrt_hmix = contribution of Horizontal diffusion (explicit) [*uv3dmix4_GP.F, uv3dmix_GP.F, uv3dmix_spg.F, uv3dmix4_S.F, uv3dmix_S.F*]
- vrt_vmix = contribution of Vertical mixing = vrt_Wind + vrt_Drag [*step3d_uv2.F*]
- vrt_nudg = contribution of Nudging, restoring, boundary conditions, etc. [*step3d_uv2.F*]
- vrt_wind = Wind stress curl [*step3d_uv2.F*] [already included in vrt_vmix]
- vrt_drag = Bottom drag curl [*step3d_uv2.F*] [already included in vrt_vmix]

All variables are 2D on the horizontal psi-grid. The following pointwise budget is closed:

vrt_rate = vrt_xadv + vrt_yadv + vrt_prsgrd + vrt_cor + vrt_vmix + vrt_hmix + vrt_nudg

Spatial discretization: Momentum terms are first computed and vertically averaged on their native u- and v-grids, then vorticity terms are computed on the psi-grid:

1.3 offline diagnostics

Variables computed offline are:

- vortadv_sol2 = contribution of advection + implicit dissipation along horizontal axis
- vortadv_mix = implicit dissipation only [computed by substracting 4th order centered advection from upstream advection]
- vortplantot_sol2 = planetary vorticity advection + grid curvature terms
- bpt = vrt_Prsgrd = bottom Pressure torque
- rotwind = vrt_Wind = Wind stress curl
- rotbot = vrt_Drag = Bottom drag curl

All variables are 2D on horizontal psi-grid. The sum of the following terms should be close to zero when time averaged:

vortadv_sol2 + vortplantot_sol2 + bpt + rotwind + rotbot = 0

[The horizontal mixing terms in the sponge layer and/or if an explicit dissipation is added are not included yet but should be very soon]