

# FLUIDS 2

---

Jonathan GULA  
[gula@univ-brest.fr](mailto:gula@univ-brest.fr)

## II. INSTABILITIES

Jonathan GULA  
[gula@univ-brest.fr](mailto:gula@univ-brest.fr)

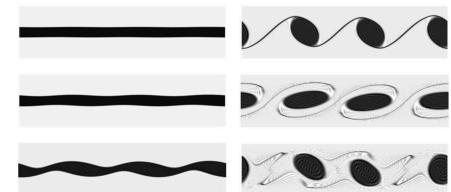
## II. INSTABILITY

### II.1. Concept of stability

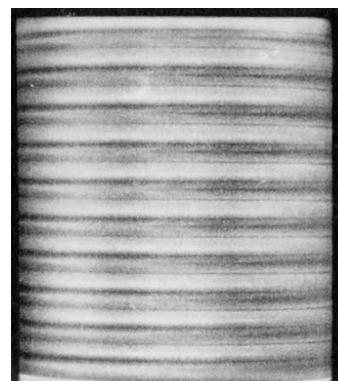
### II.2. Kelvin-Helmholtz Instability



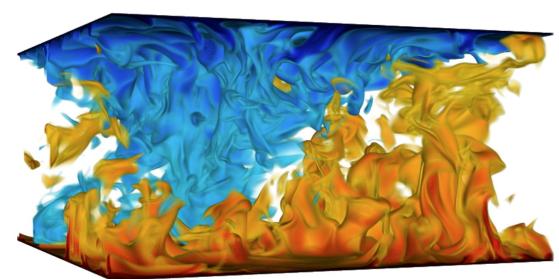
### II.3. Parallel Shear instability



### II.4. Convective instability (Rayleigh–Bénard)

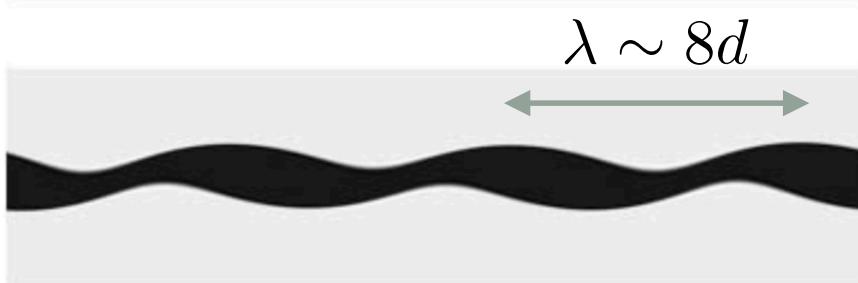
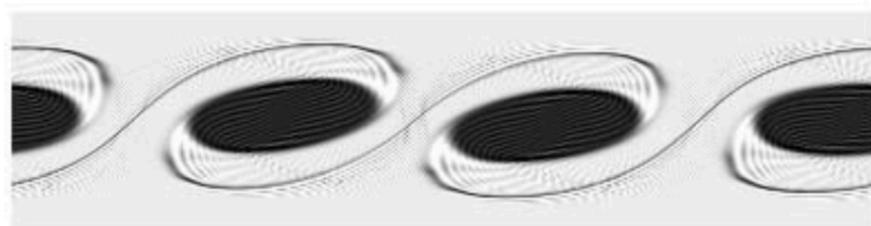


### II.5. Taylor–Couette



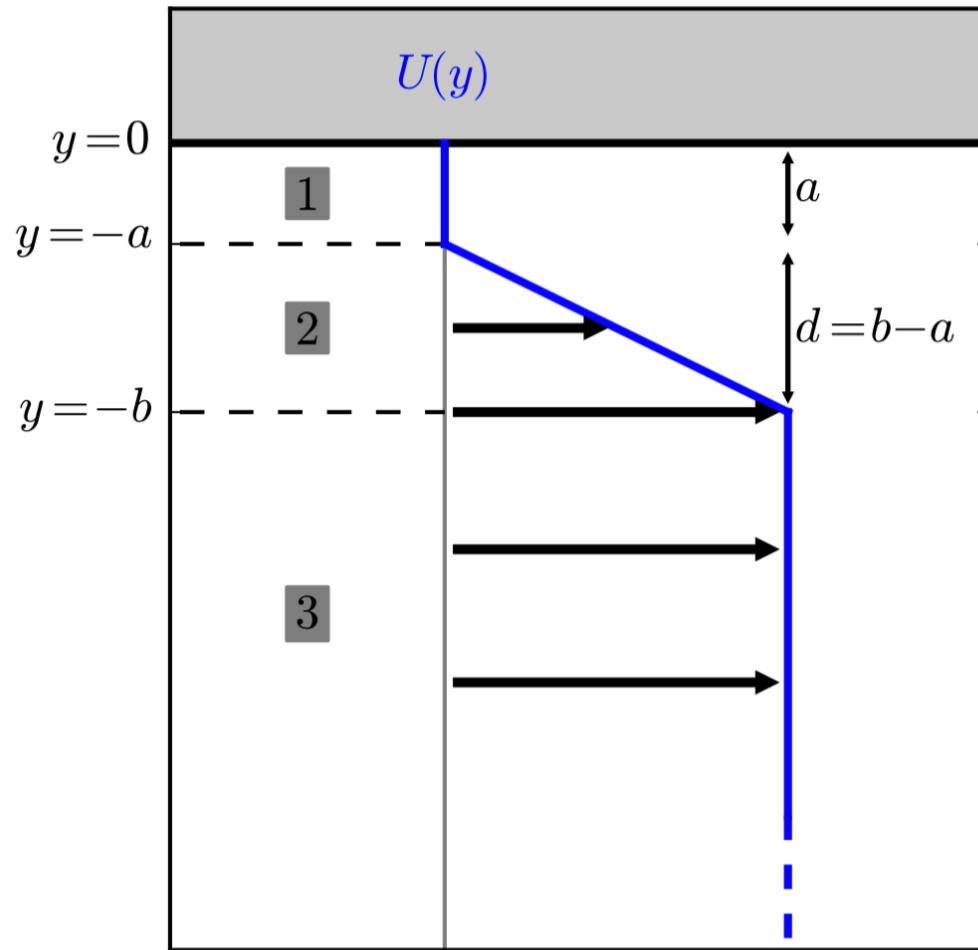
## II.3. Parallel Shear instability

### Rayleigh shear instability



## II.3. Parallel Shear instability

### Rayleigh shear instability

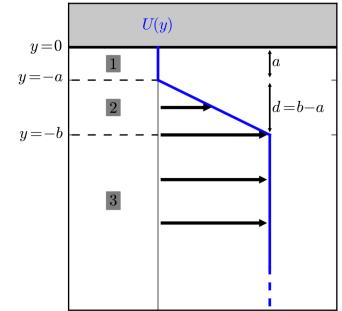


$$\left\{ \begin{array}{ll} U_1 = 0, & -a < y < 0 \\ U_2 = -\frac{U_0}{d}(y + b) + U_0, & -b < y < -a \\ U_3 = U_0, & y < -b \end{array} \right.$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

- Solutions are:

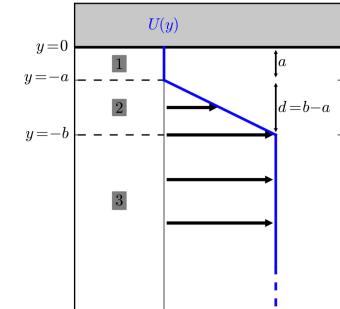


$$\begin{cases} \psi_1 = A e^{-k(y+a)} + B e^{k(y+a)}, & -a < y < 0 \\ \psi_2 = C e^{k(y+a)} + D e^{k(y+b)}, & -b < y < -a \\ \psi_3 = E e^{k(y+b)}, & y < -b \end{cases}$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

- With matching conditions



1. Continuity of pressure across interfaces (namely, at  $y = -a$  and  $y = -b$ ) :

$$\Delta \left[ (U - c) \frac{\partial \psi}{\partial y} - \psi \frac{\partial U}{\partial y} \right] = 0,$$

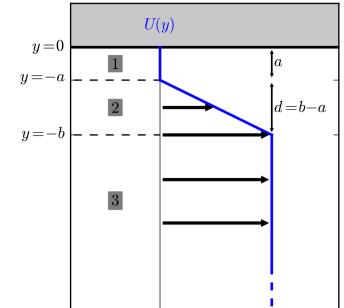
2. Continuity of normal velocity across interfaces :

$$\Delta \left[ \frac{\psi}{U - c} \right] = 0,$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

- With matching conditions



$$A(kc) + B(-kc) + C \left( kc - \frac{U_0}{d} \right) + D \left[ \left( kc - \frac{U_0}{d} \right) e^{-kd} \right] = 0$$

$$A + B - C - D e^{-kd} = 0$$

$$C \left[ \left( (U_0 - c)k + \frac{U_0}{d} \right) e^{-k(b-a)} \right] + D \left( (U_0 - c)k + \frac{U_0}{d} \right) + E (- (U_0 - c)k) = 0$$

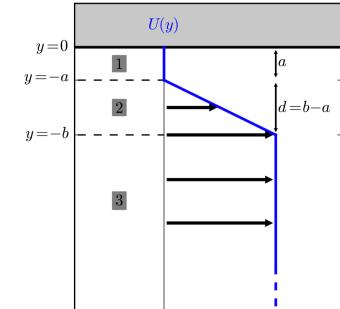
$$C e^{-kd} + D - E = 0$$

$$A e^{-ka} + B e^{-ka} = 0$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

- With matching conditions



This set of 5 homogeneous equations (5-6) may be written in the form of a matrix equation :

$$\mathbf{M} (A \ B \ C \ D \ E)^T = 0 \quad (7)$$

with :

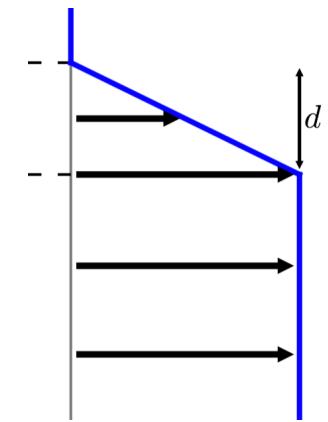
$$\mathbf{M} = \begin{pmatrix} e^{ka} & e^{-ka} & 0 & 0 & 0 \\ kc & -kc & kc - \frac{U_0}{d} & \left(kc - \frac{U_0}{d}\right)e^{-kd} & 0 \\ 1 & 1 & -1 & -e^{-kd} & 0 \\ 0 & 0 & \left((U_0 - c)k + \frac{U_0}{d}\right)e^{-kd} & (U_0 - c)k + \frac{U_0}{d} & -(U_0 - c)k \\ 0 & 0 & e^{-kd} & 1 & -1 \end{pmatrix} \quad (8)$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

The determinant of the matrix must be zero for non-trivial solutions

(Assuming the wall is far away):



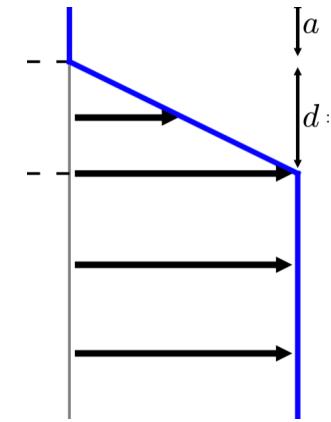
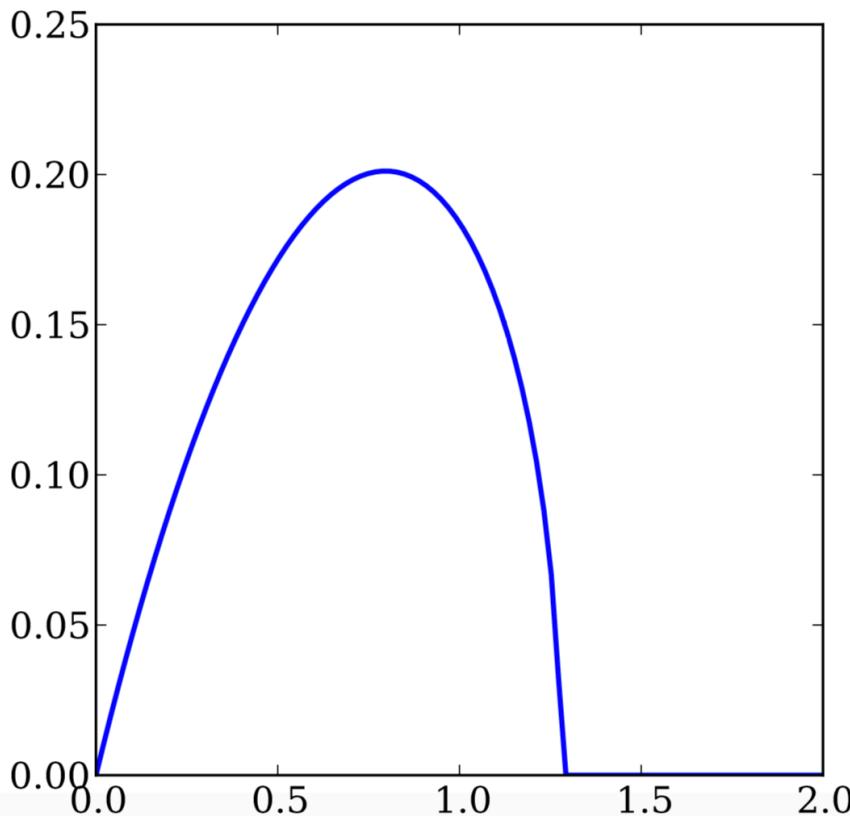
$$\det(\mathbf{M}') = \frac{4e^{2dk}}{d^2} \left[ d^2 k^2 c^2 + \frac{U_0^2}{4} (e^{-2dk} - (1 - dk)^2) \right]$$

$$\det(\mathbf{M}') = 0 \iff c^2 = \frac{U_0^2}{4d^2 k^2} [(1 - dk)^2 - e^{-2dk}]$$

## II.3. Parallel Shear instability

### Rayleigh shear instability

Growth rate as a function of wavenumber:

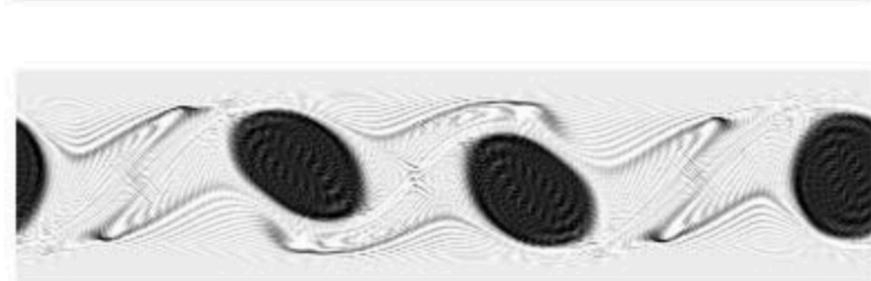
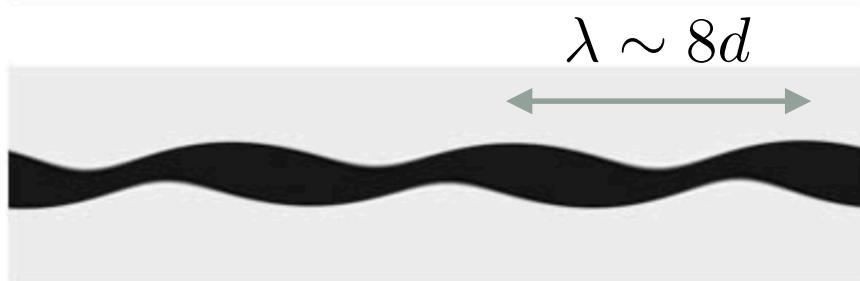
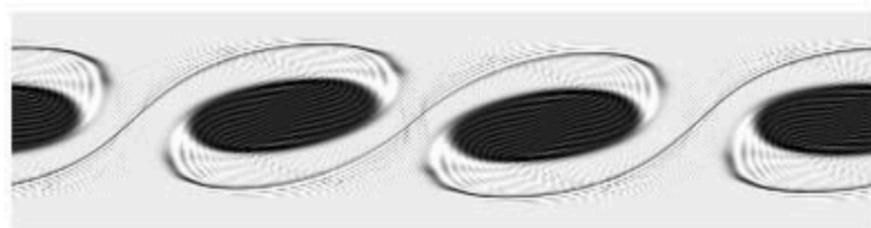


$$d k \sim 0.8$$

$$\lambda \sim 8d$$

## II.3. Parallel Shear instability

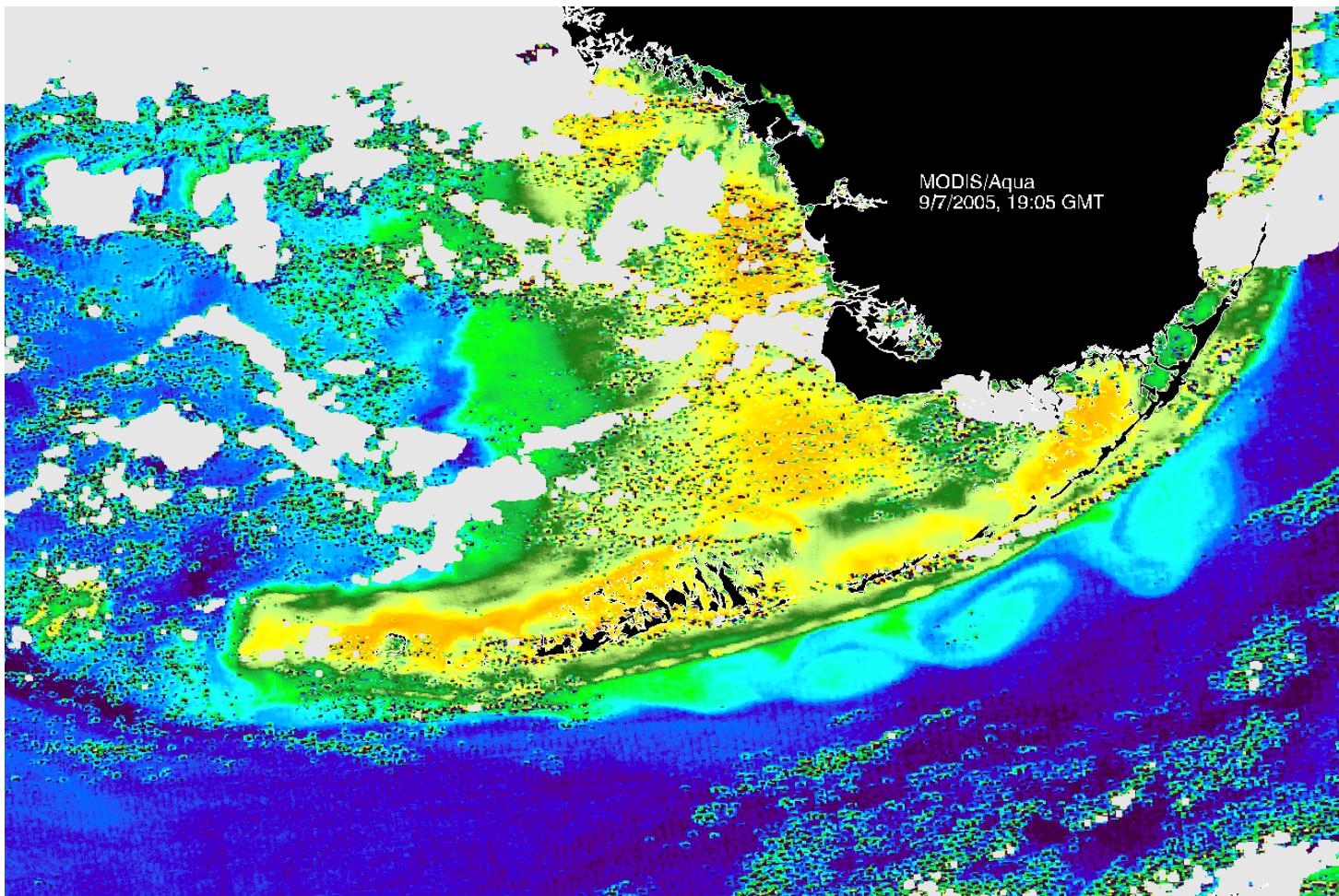
### Rayleigh shear instability



## II.3. Parallel Shear instability

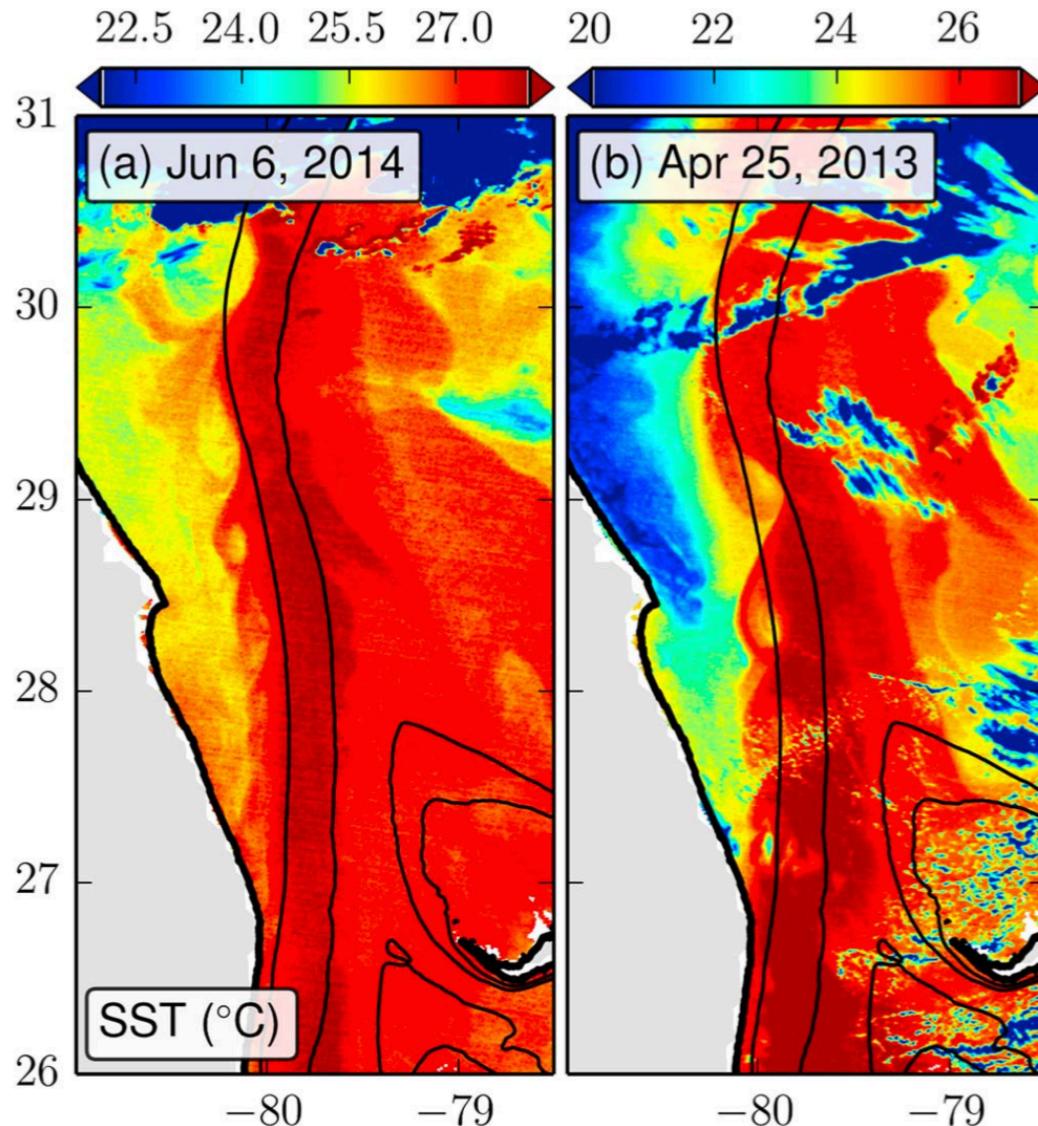
### Realistic flows

Satellite Chlorophyl  
observations along the  
Florida Keys



## II.3. Parallel Shear instability

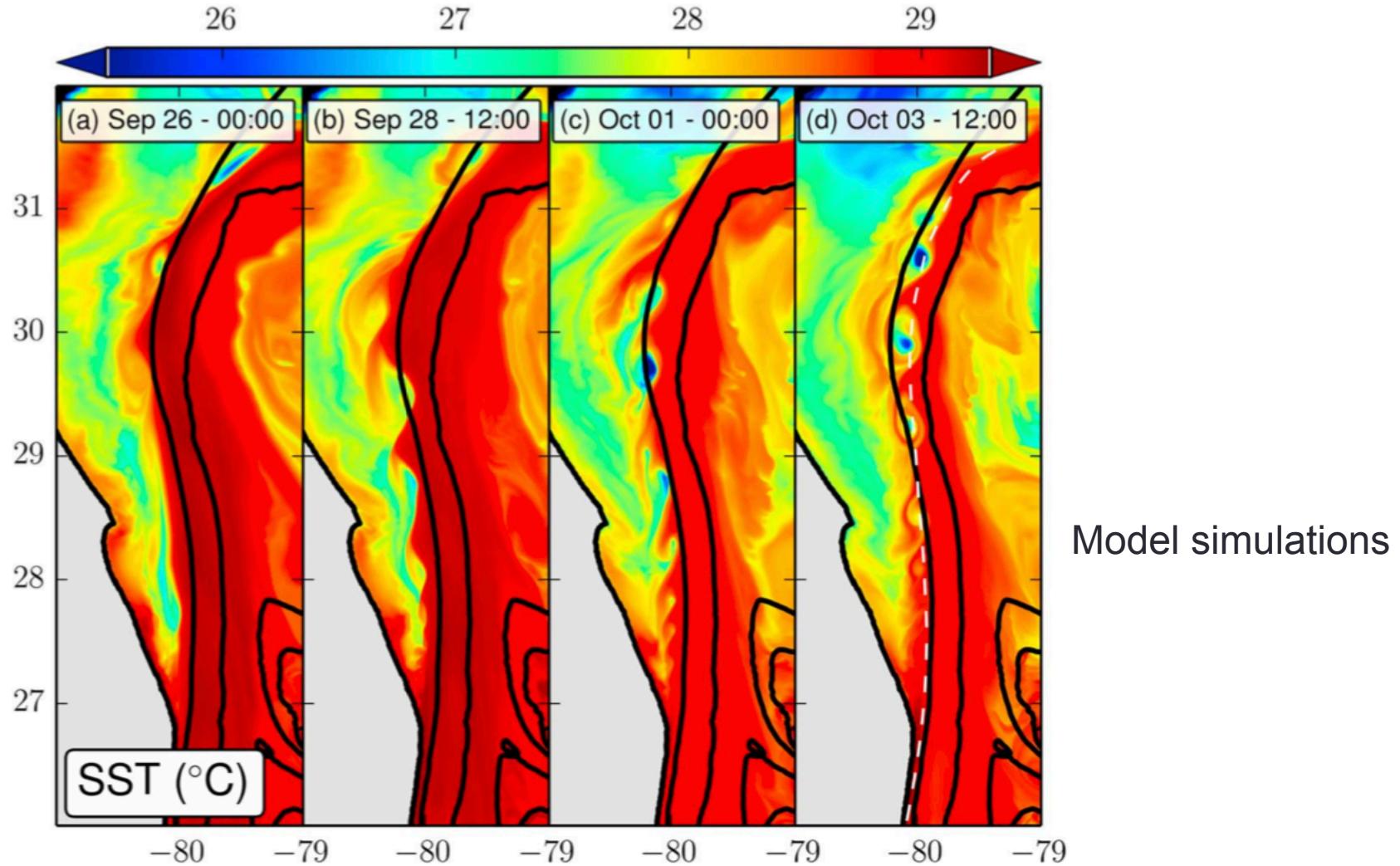
### Realistic flows



Satellite SST observations  
in the Florida Strait

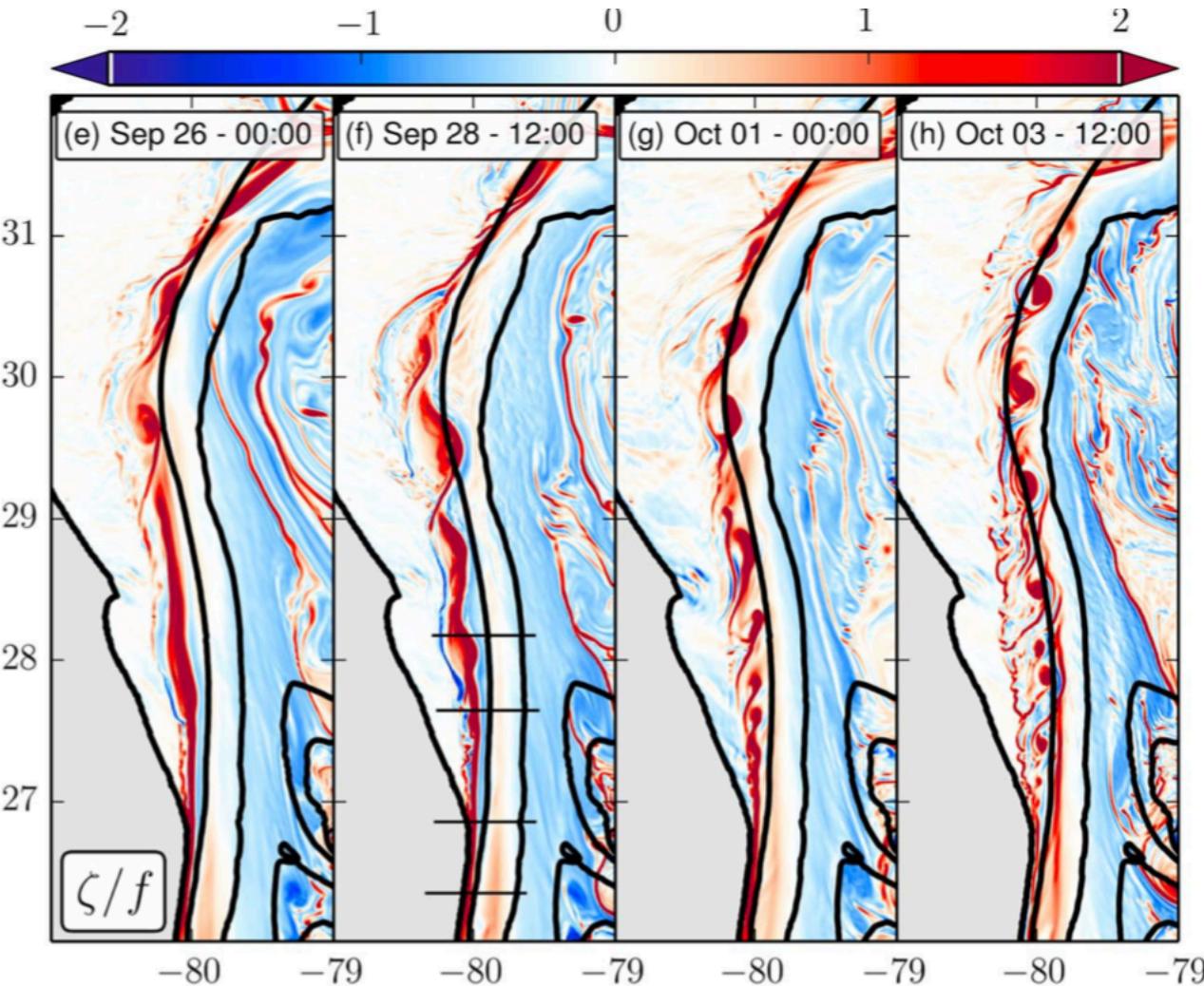
## II.3. Parallel Shear instability

### Realistic flows



## II.3. Parallel Shear instability

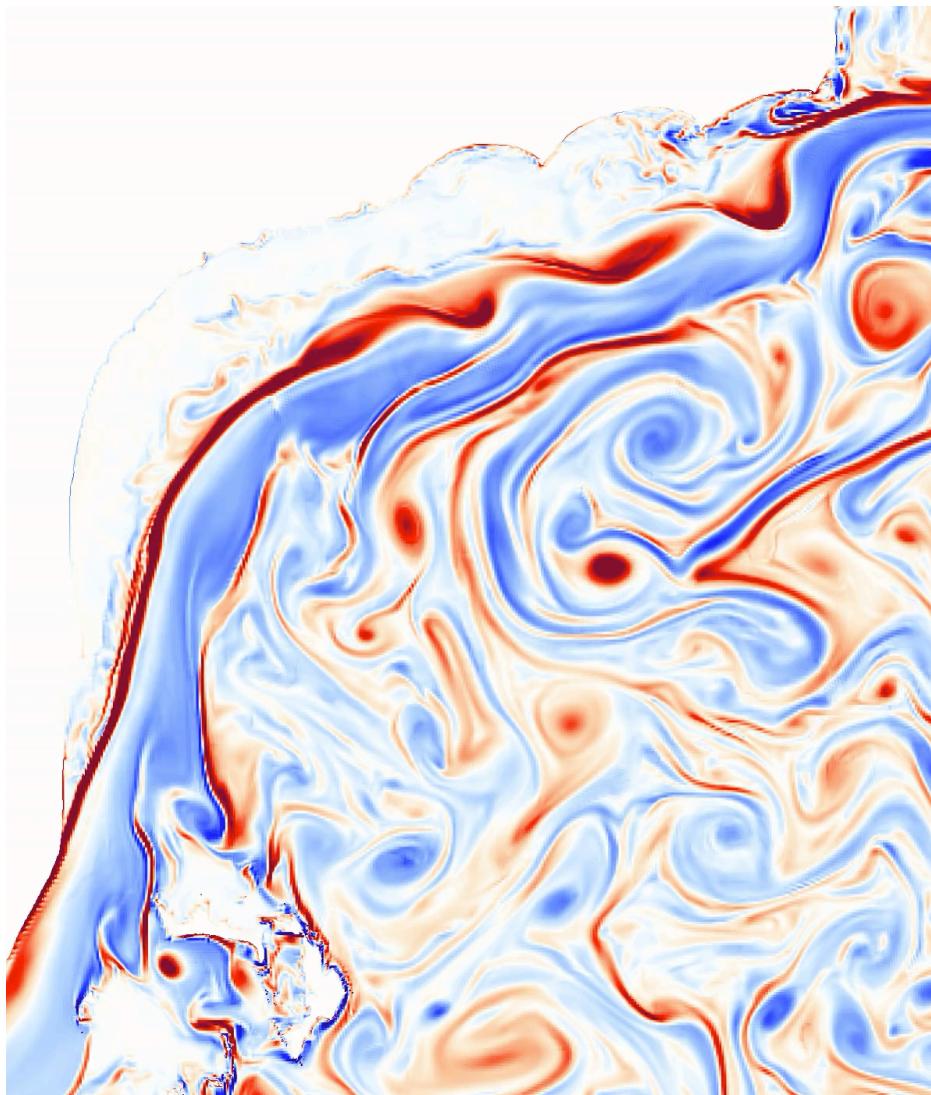
### Realistic flows



Model simulations

## II.3. Parallel Shear instability

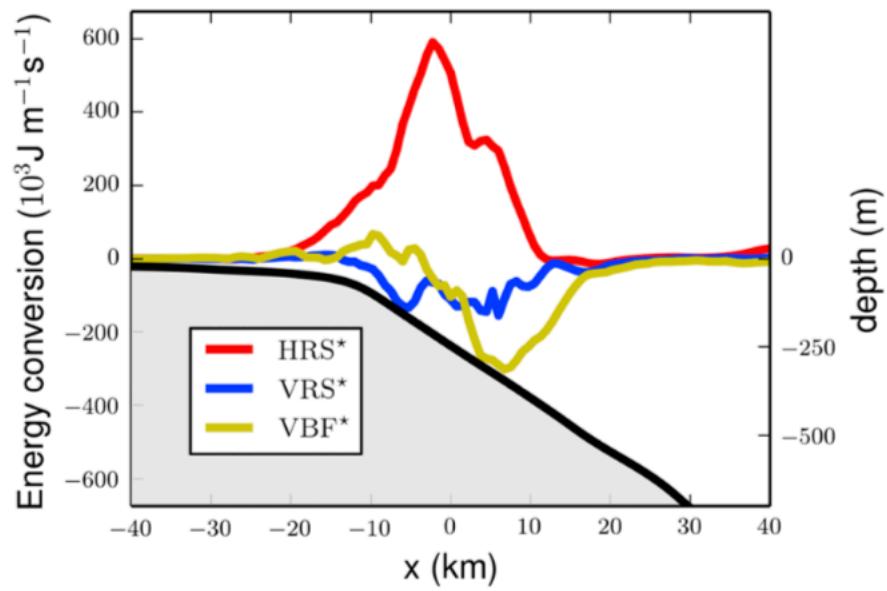
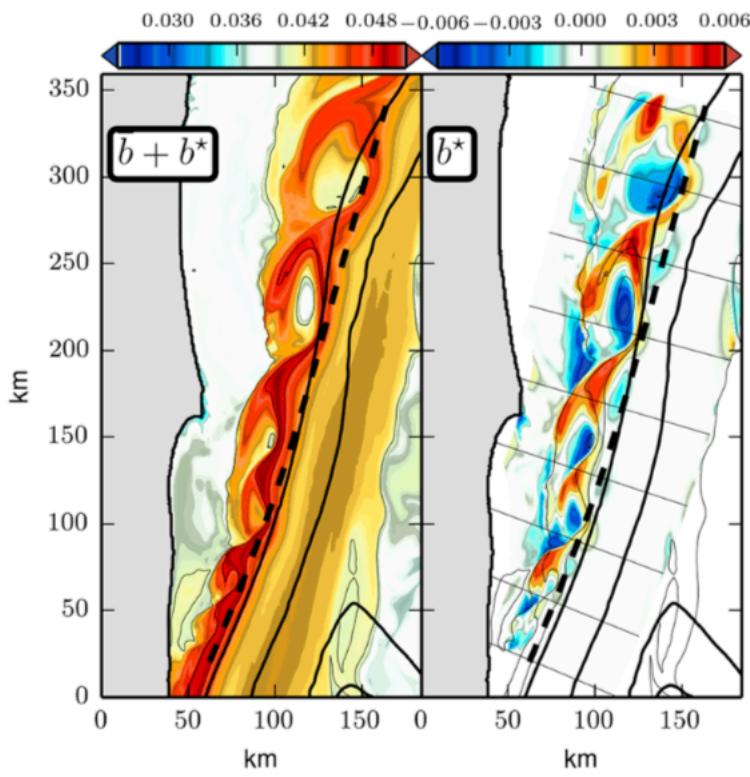
### Realistic flows



Model simulations

## II.3. Parallel Shear instability

### Realistic flows

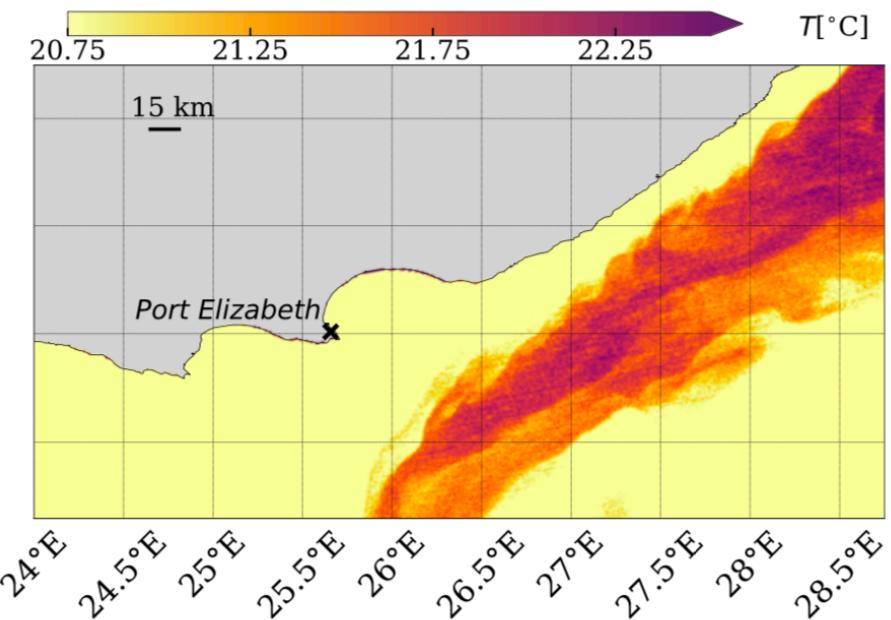
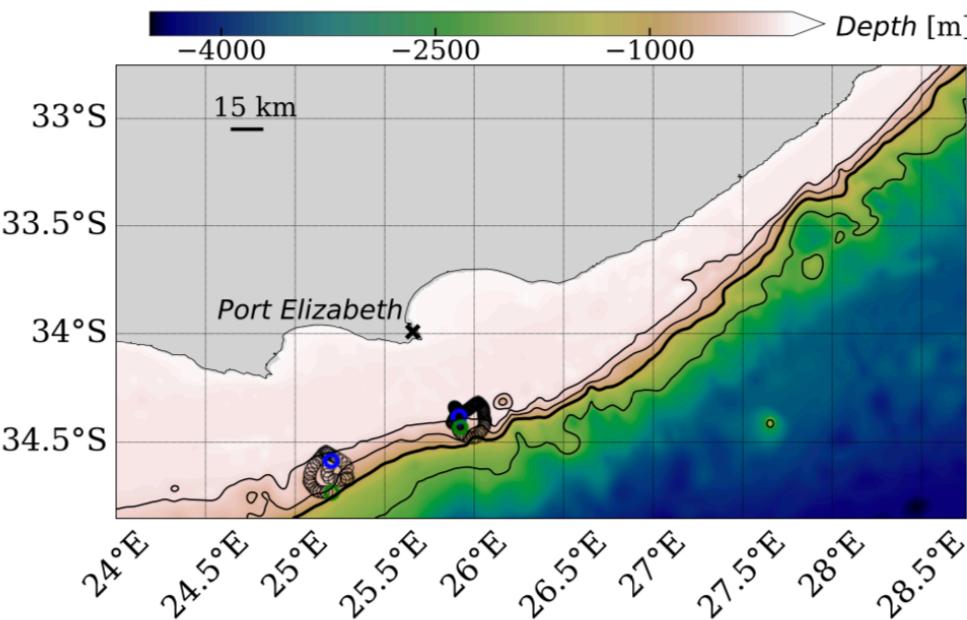


$$\text{HRS} = -\langle \mathbf{u}' v' \rangle \frac{\partial \langle \mathbf{u} \rangle}{\partial y} - \langle \mathbf{u}' u' \rangle \frac{\partial \langle \mathbf{u} \rangle}{\partial x}$$

## II.3. Parallel Shear instability

### Realistic flows

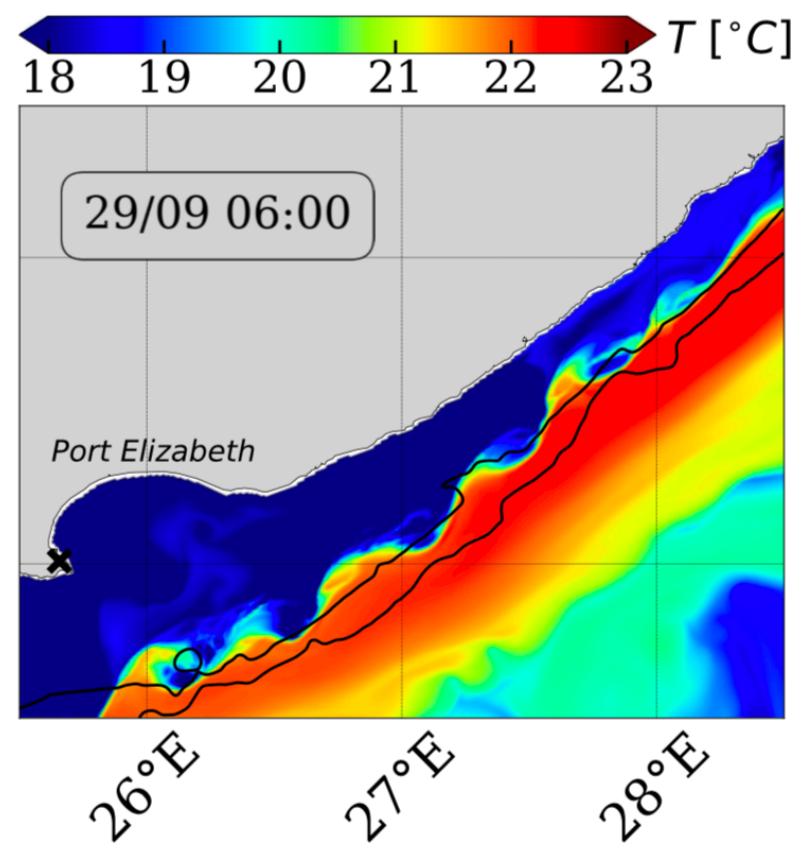
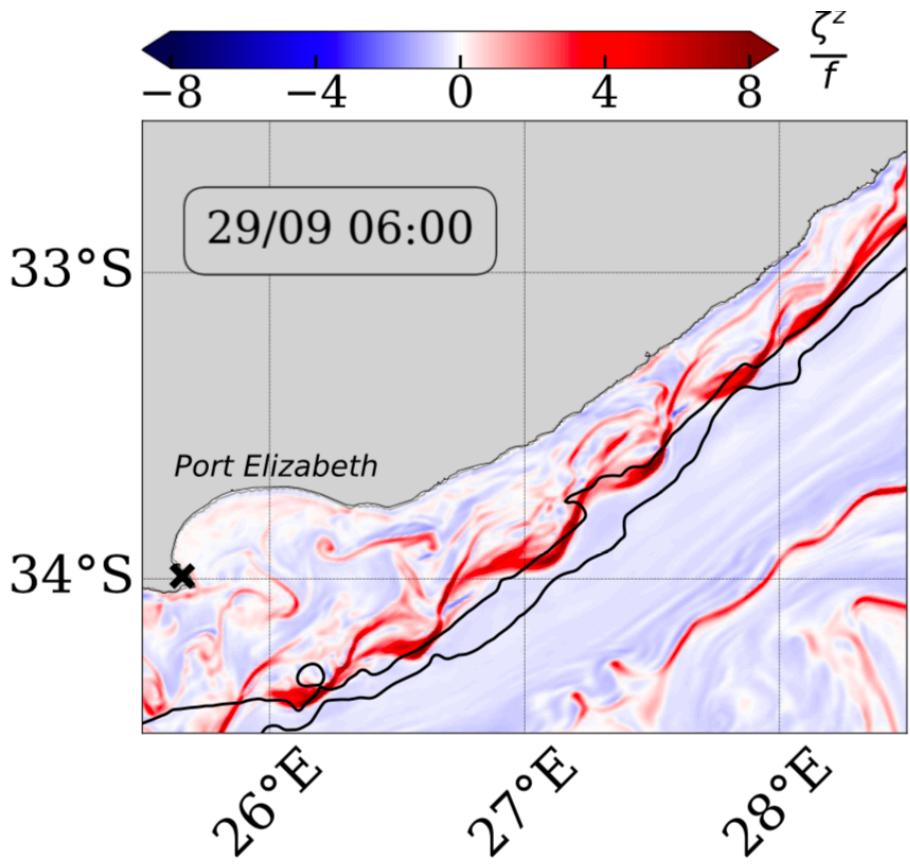
Satellite SST observations  
in the Agulhas Current



## II.3. Parallel Shear instability

### Realistic flows

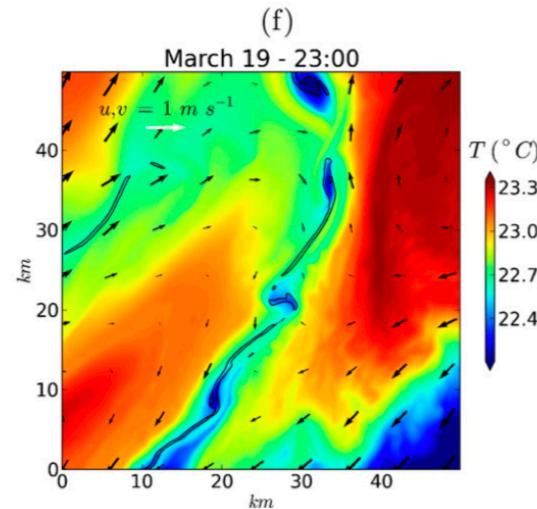
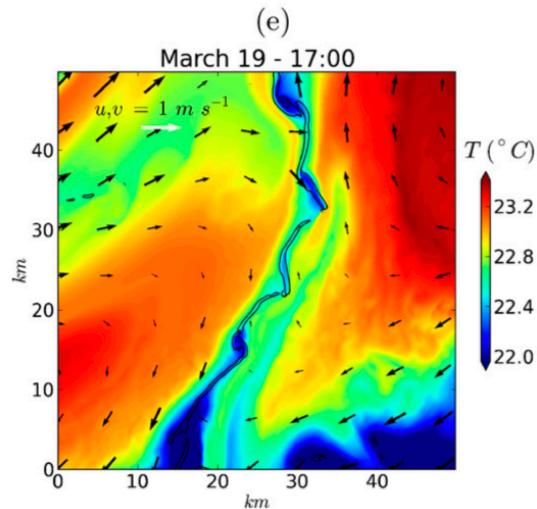
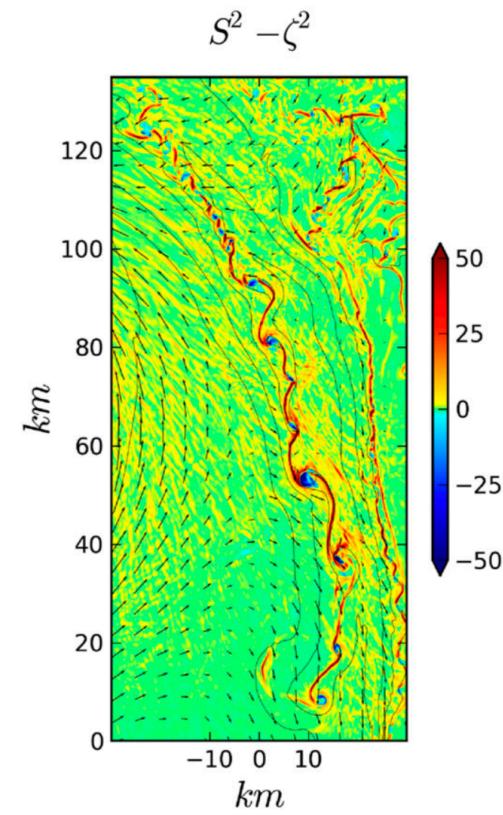
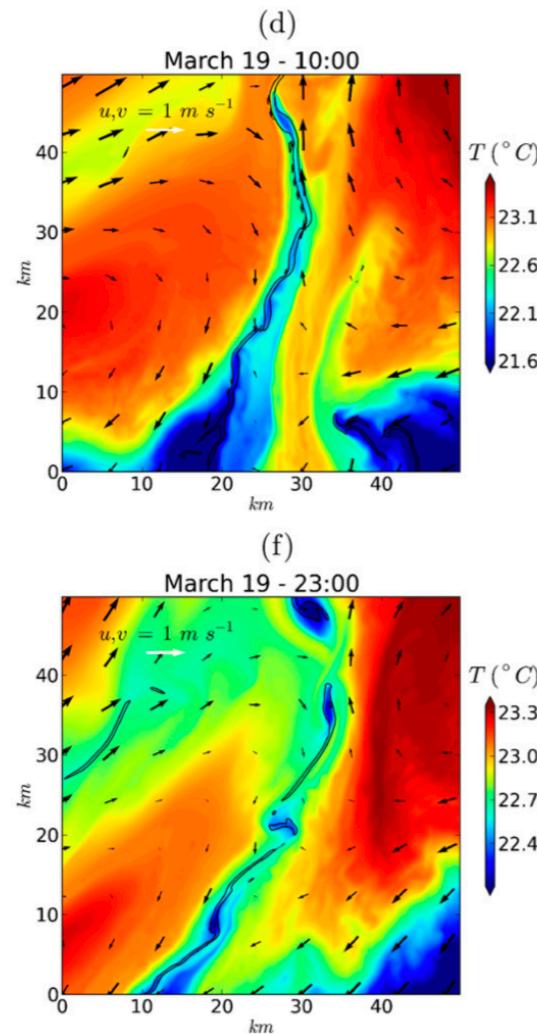
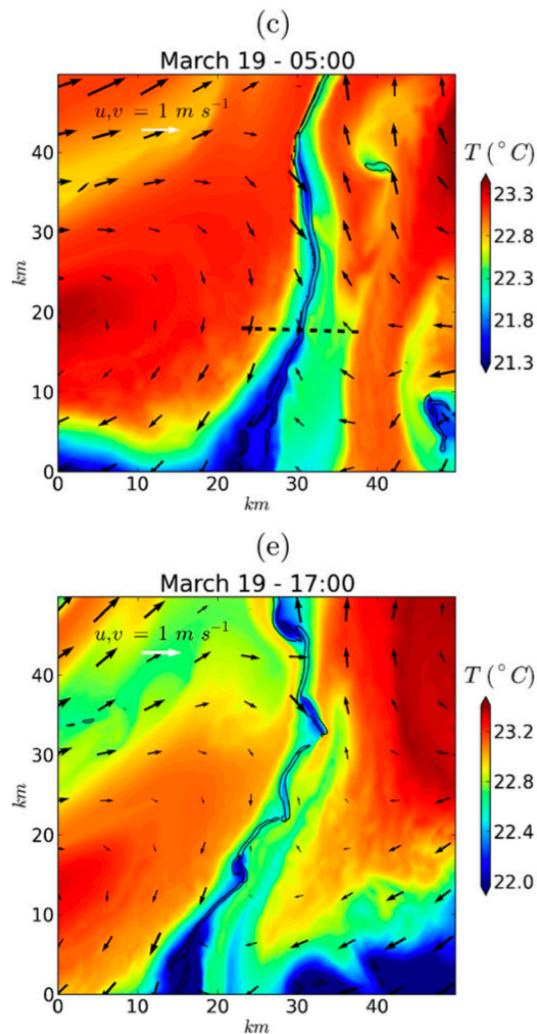
Simulation of the  
Agulhas Current



## II.3. Parallel Shear instability

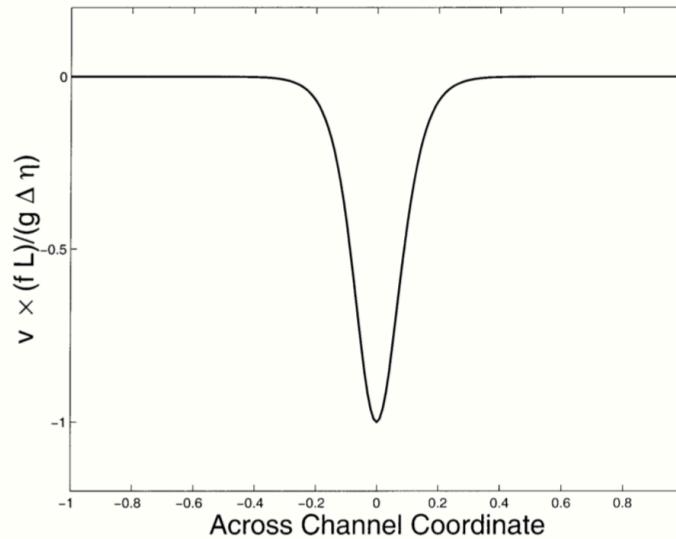
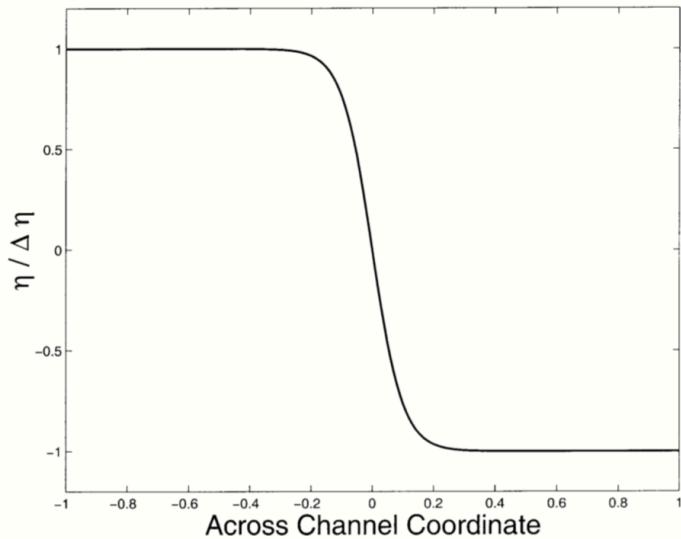
### Realistic flows

Instability of a surface vorticity filament



## II.3. Parallel Shear instability

### The Bickley Jet



$$\bar{\eta} = -\Delta \eta \tanh\left(\frac{x}{L}\right)$$

$$\bar{u} = 0$$

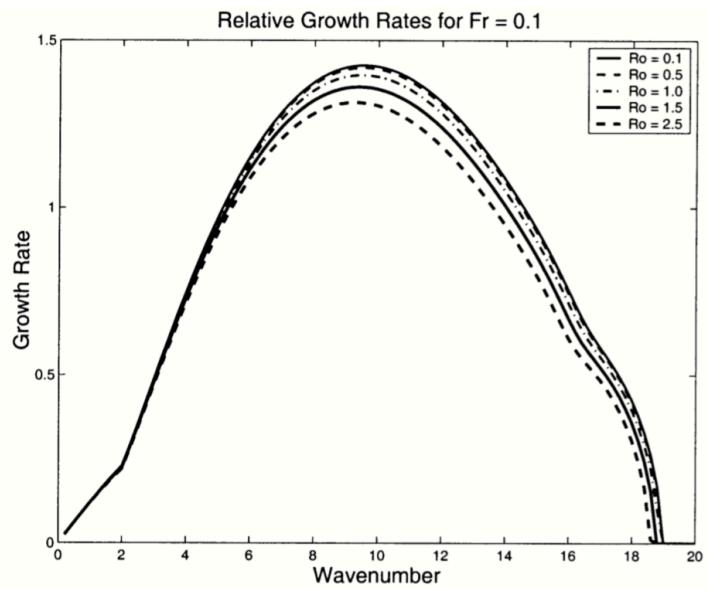
$$\bar{v} = -\frac{g' \Delta \eta}{f L} \operatorname{sech}^2\left(\frac{x}{L}\right).$$

See Poulin et Flierl, JFM, 2003

## II.3. Parallel Shear instability

### The Bickley Jet

$$\begin{bmatrix} \bar{v} & \left( \frac{dH}{dx} + H \frac{d}{dx} \right) & H \\ -\frac{g'}{k^2} \frac{d}{dx} & \bar{v} & \frac{f}{k^2} \\ g' & \left( \frac{d\bar{v}}{dx} + f \right) & \bar{v} \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{u} \\ \hat{v} \end{bmatrix} = c \begin{bmatrix} \hat{\eta} \\ \hat{u} \\ \hat{v} \end{bmatrix}$$



## II.3. Parallel Shear instability

### The Bickley Jet

