Realistic Modelling of the ocean

From Stommel to today’s simulations

From Stommel to today's simulations

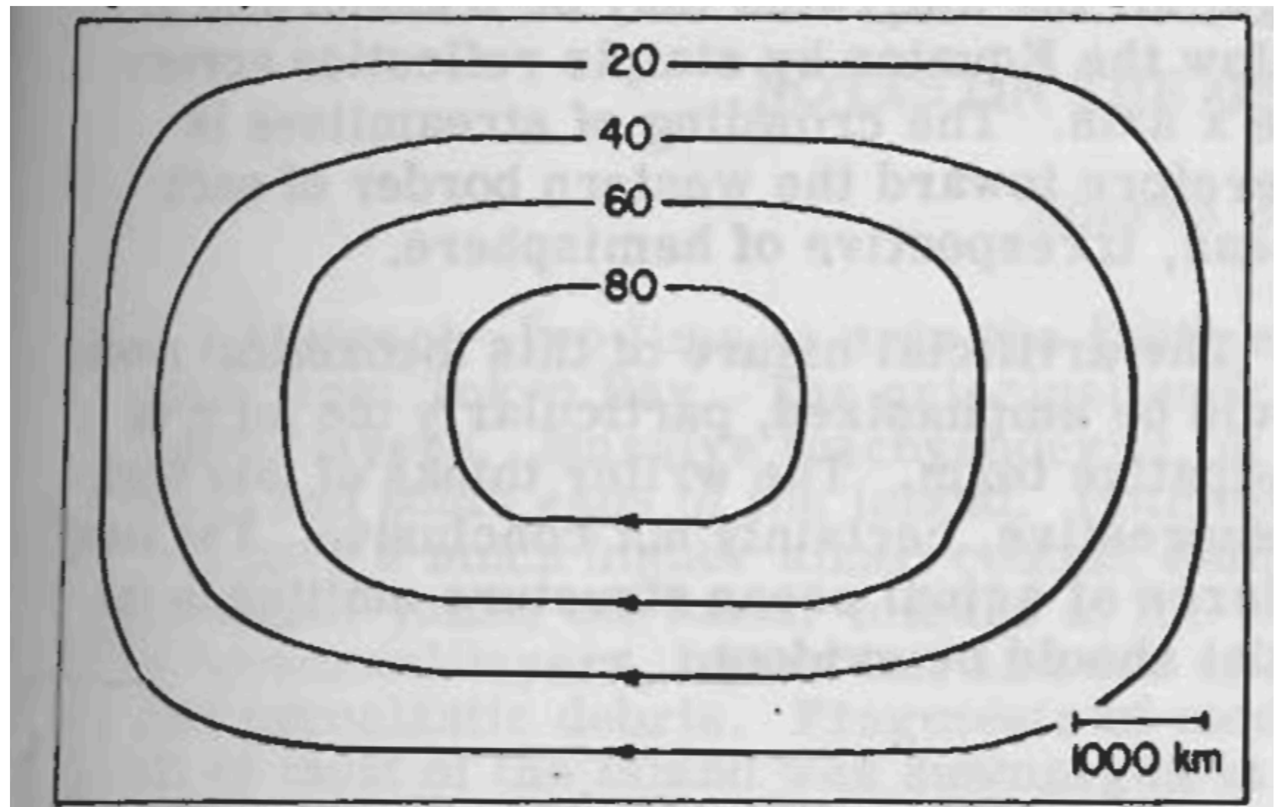


Fig. 2--Streamlines for the case of both the non-rotating and uniformly rotating oceans

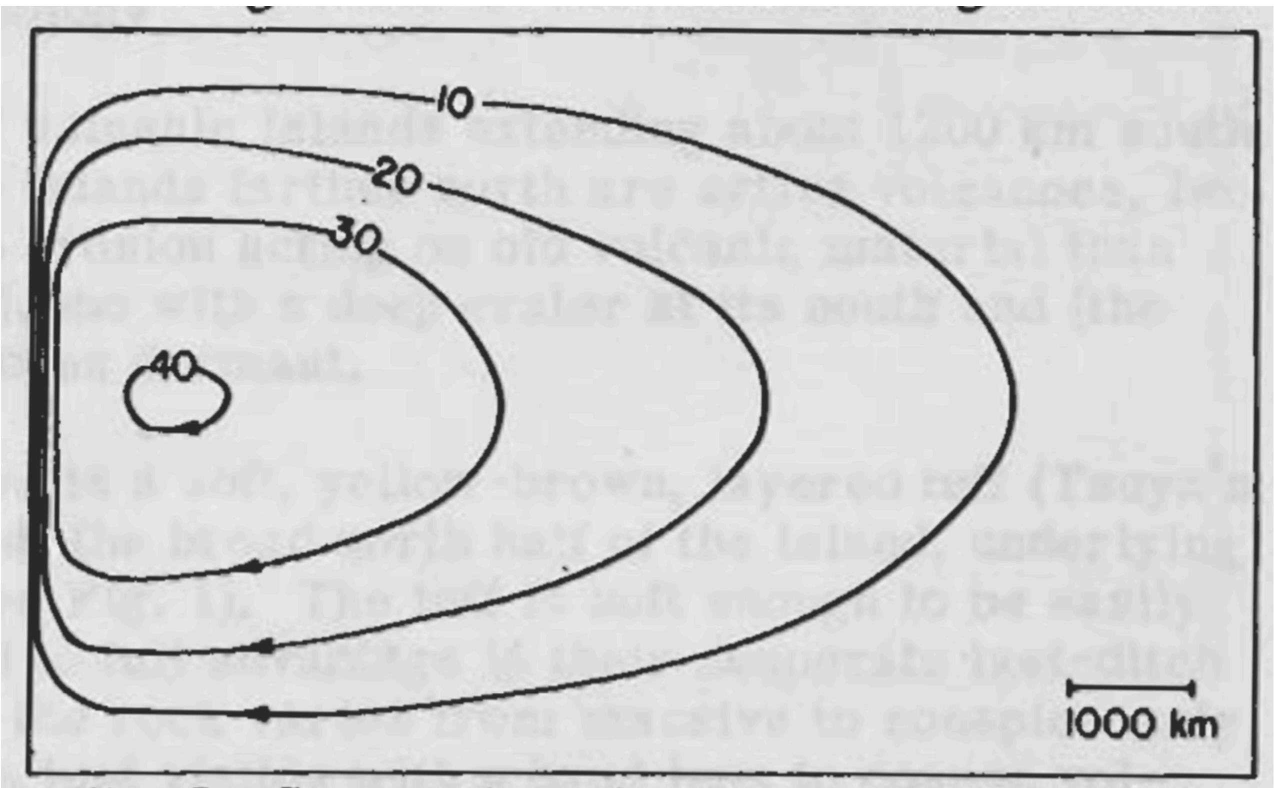
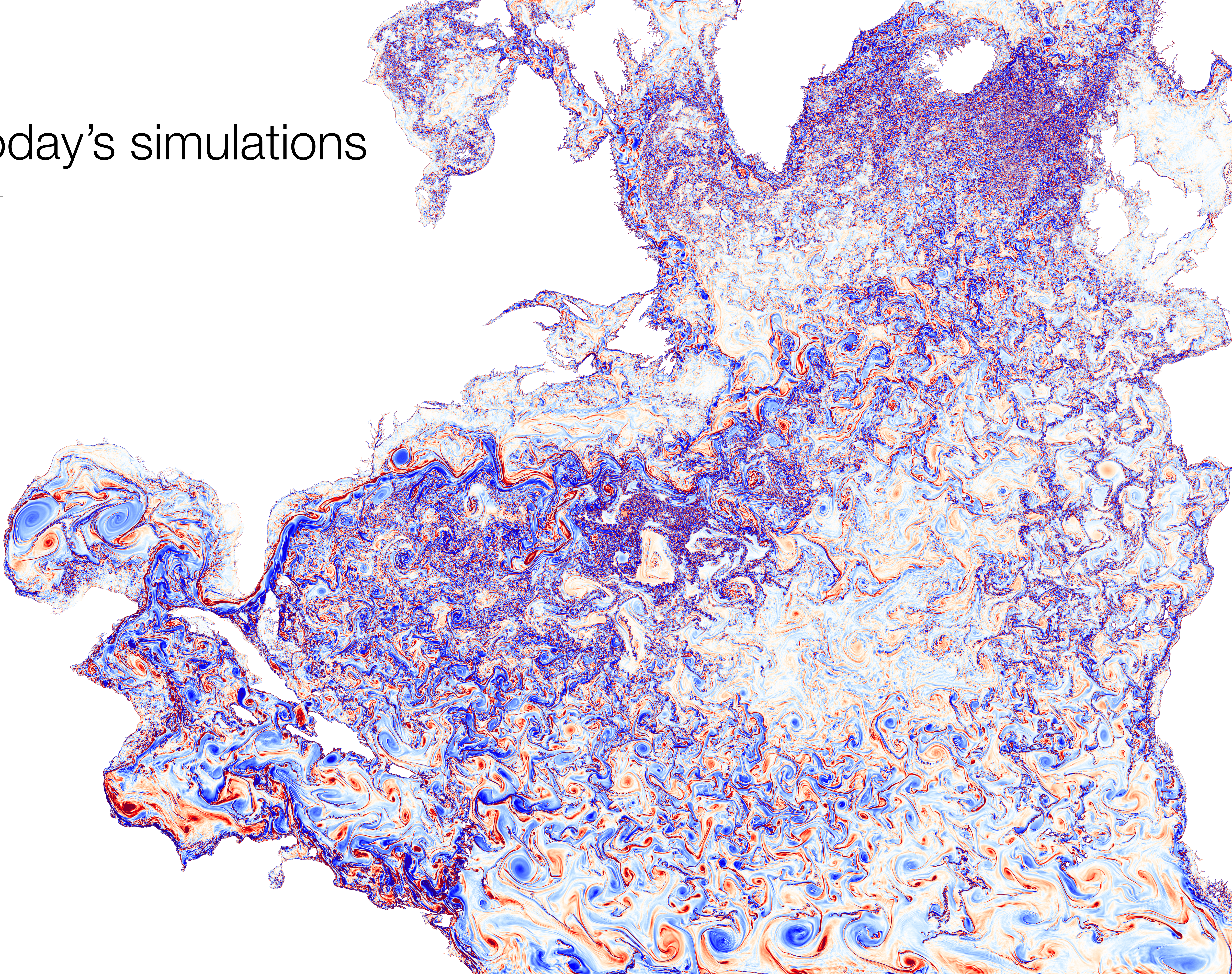


Fig. 5--Streamlines for the case where the Coriolis force is a linear function of latitude



Stommel's gyre

THE WESTWARD INTENSIFICATION OF WIND-DRIVEN OCEAN CURRENTS

Henry Stommel

(Contribution No. 408, Woods Hole Oceanographic Instiution)

Abstract--A study is made of the wind-driven circulation in a homogeneous rectangular ocean under the influence of surface wind stress, linearised bottom friction, horizontal pressure gradients caused by a variable surface height, and Coriolis force.

An intense crowding of streamlines toward the western border of the ocean is discovered to be caused by variation of the Coriolis parameter with latitude. It is suggested that this process is the main reason for the formation of the intense currents (Gulf stream and others) observed in the actual oceans.

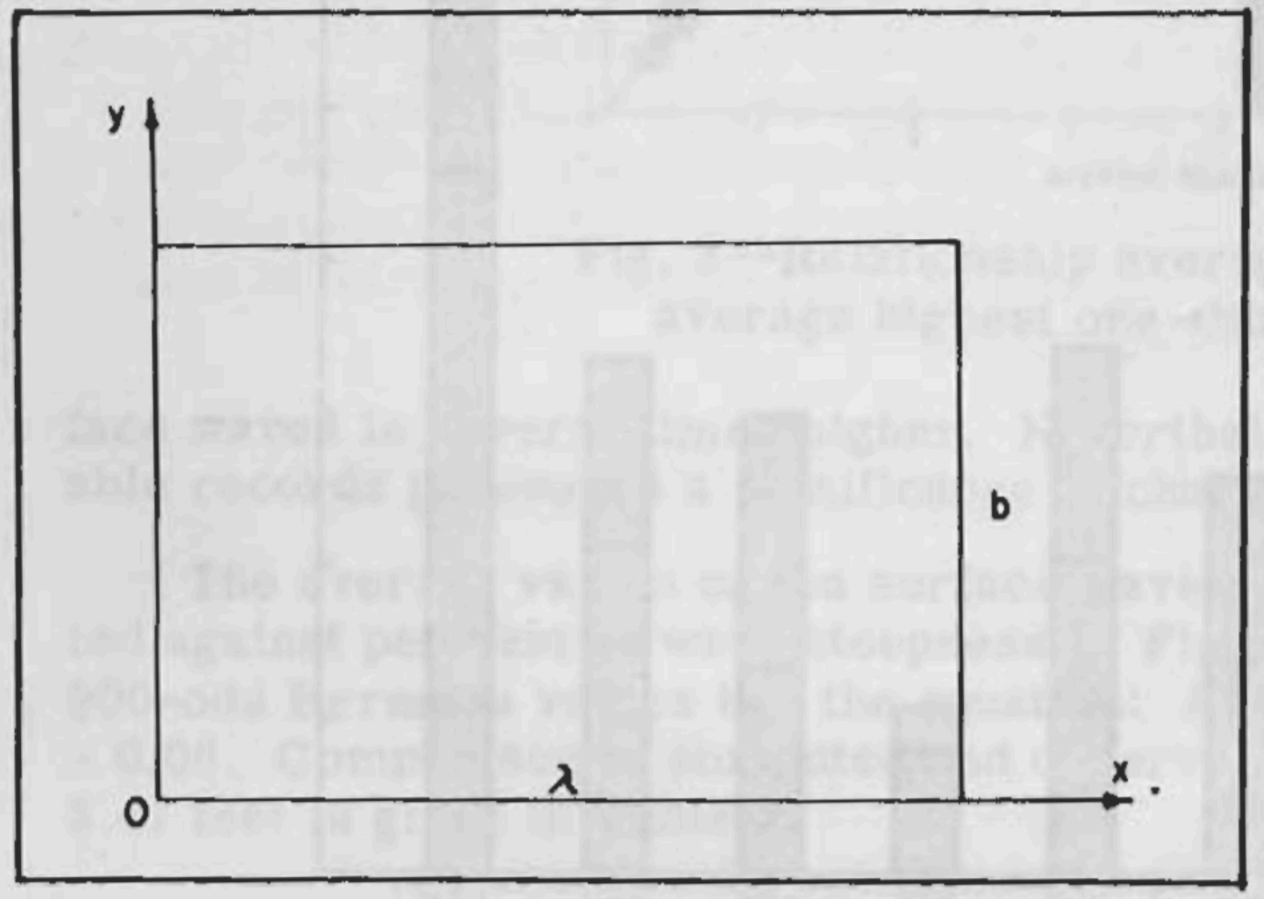


Fig. 1--Ocean basin dimensions and the coordinate system

- Momentum equations:

The steady state equations of motion, with the inertial terms omitted because they are small, are written in the form

$$0 = f(D + h)v - F\cos(\pi y/b) - Ru - g(D + h)\partial h/\partial x \quad (1)$$

$$0 = -f(D + h)u - Rv - g(D + h)\partial h/\partial y \quad (2)$$

- Barotropic vorticity equation:

$$v(D + h)(\partial f/\partial y) + (F\pi/b)\sin(\pi y/b) + R(\partial v/\partial x - \partial u/\partial y) = 0$$

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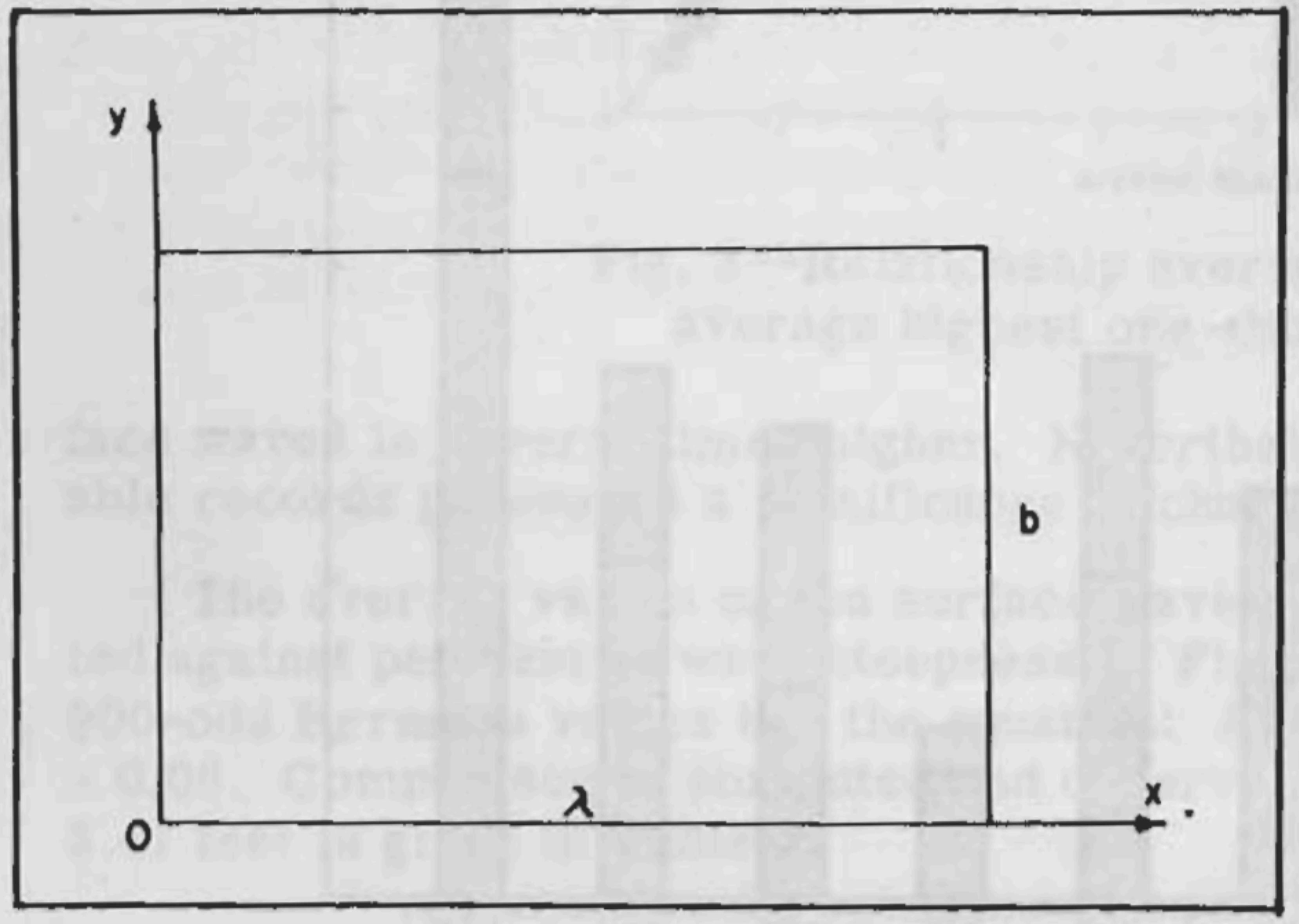

Fig. 1--Ocean basin dimensions and the coordinate system

- Momentum equations:

Coriolis Wind Drag Pressure gradient

The steady state equations of motion, with the inertial terms omitted because they are small, are written in the form

$$0 = f(D + h)v - F\cos(\pi y/b) - Ru - g(D + h)\partial h/\partial x \quad (1)$$

$$0 = -f(D + h)u - Rv - g(D + h)\partial h/\partial y \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

-

Stommel's gyre

- No rotation / or constant rotation ($\dfrac{\partial f}{\partial y} = \beta = 0$)

The equation for the stream function is therefore

$$\psi = \gamma (b/\pi)^2 \sin (\pi y/b)[e^{(x - \lambda)\pi/b} + e^{- x\pi/b} - 1]$$

-

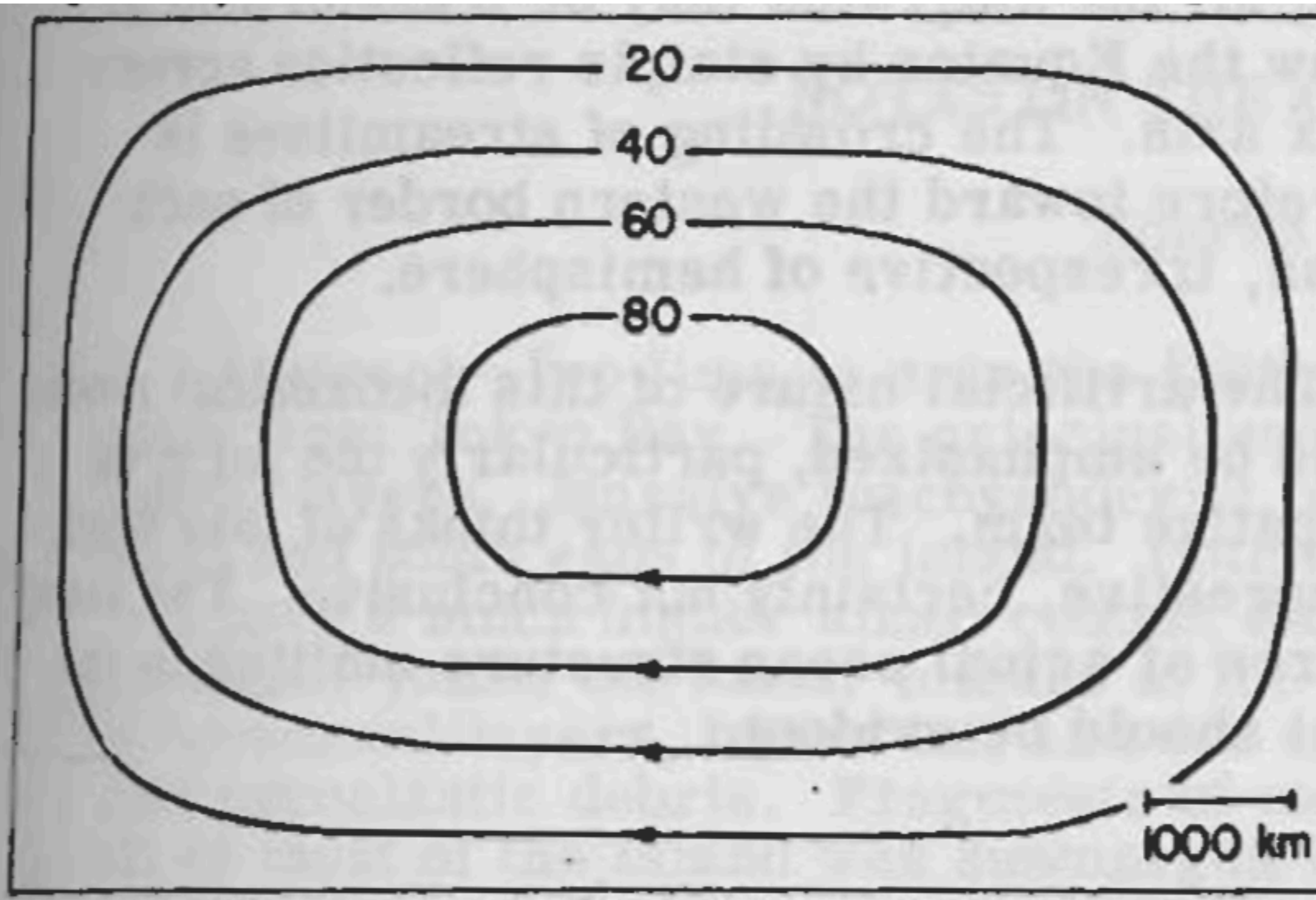

Fig. 2--Streamlines for the case of both the non-rotating and uniformly rotating oceans

Momentum equations

- $$\frac{\partial u}{\partial t} = -u_j \frac{\partial u}{\partial x_j} - w \frac{\partial u}{\partial z} + fv - \frac{P_x}{\rho_0} + \mathcal{V}_u + \mathcal{D}_u + \mathcal{S}_u$$
$$\underbrace{\frac{\partial v}{\partial t}}_{rate} = -\underbrace{u_j \frac{\partial v}{\partial x_j}}_{hadv} - \underbrace{w \frac{\partial v}{\partial z}}_{vadv} - \underbrace{fu}_{cor} - \underbrace{\frac{P_y}{\rho_0}}_{Prsgrd} + \underbrace{\mathcal{V}_v}_{vmix} + \underbrace{\mathcal{D}_v}_{hmix+hdiff} + \underbrace{\mathcal{S}_v}_{nudg}$$

Barotropic vorticity balance

- Barotropic vorticity: $$\Omega = \frac{\partial \overline{v}}{\partial x} - \frac{\partial \overline{u}}{\partial y}$$ with $$\overline{u} = \int_{-h}^{\zeta} u \, dz,$$
- The barotropic vorticity balance equation of the flow is obtained by integrating the momentum equations in the vertical and cross differentiating them:

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = - \underbrace{\vec{\nabla}.(f\vec{\overline{u}})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}} + \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$
$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

$$A_\Sigma = \frac{\partial^2(\overline{vv} - \overline{uu})}{\partial x \partial y} + \frac{\partial^2 \overline{uv}}{\partial x \partial x} - \frac{\partial^2 \overline{uv}}{\partial y \partial y},$$

Stommel's gyre

THE WESTWARD INTENSIFICATION OF WIND-DRIVEN OCEAN CURRENTS

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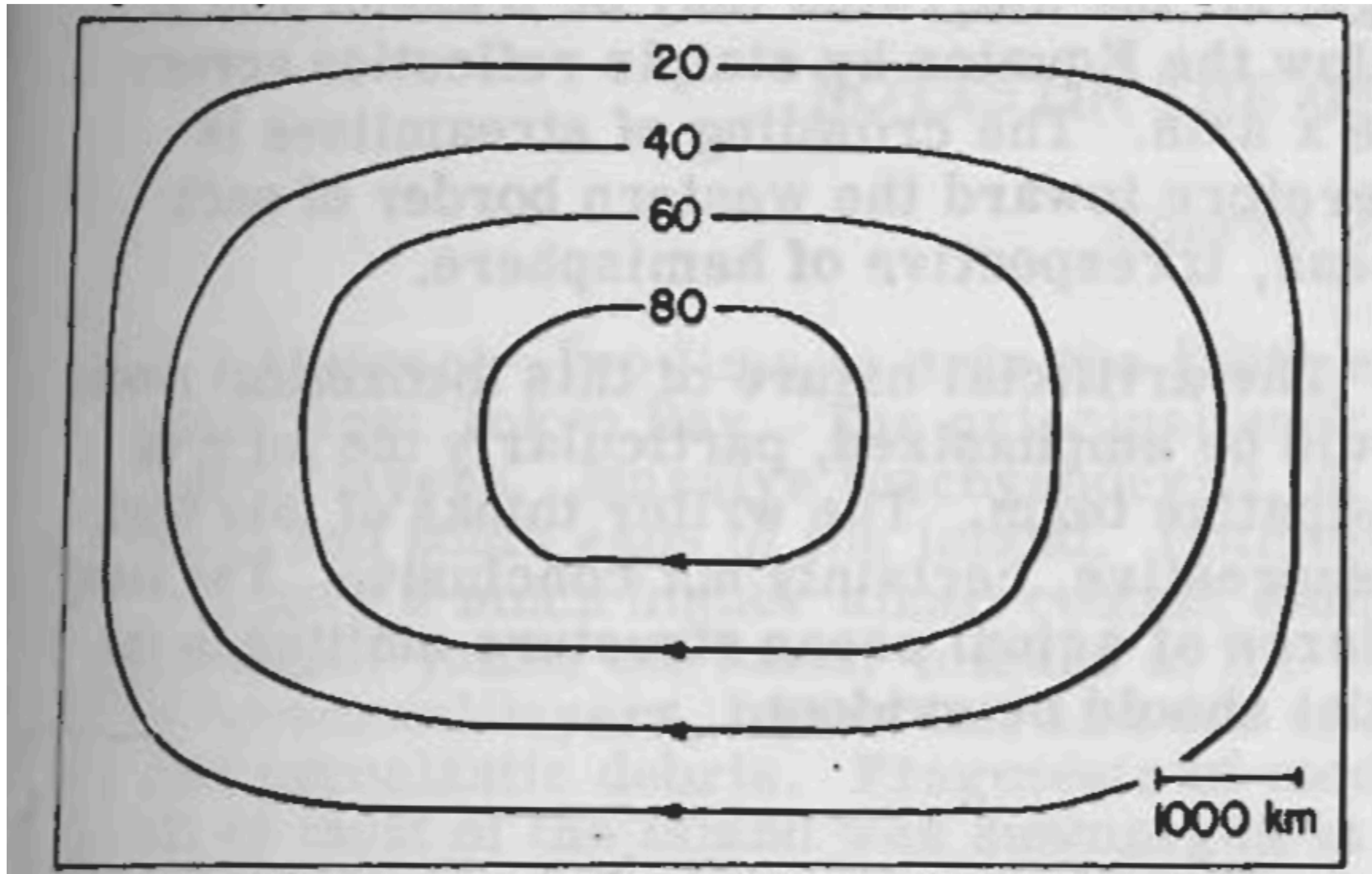


Fig. 2--Streamlines for the case of both the non-rotating and uniformly rotating oceans

- Momentum equations:

The steady state equations of motion, with the inertial terms omitted because they are small, are written in the form

$$0 = \underbrace{f(D + h)v}_{\text{Coriolis}} - \underbrace{F\cos(\pi y/b)}_{\text{Wind}} - \underbrace{Ru}_{\text{Drag}} - \underbrace{g(D + h)\partial h/\partial x}_{\text{Pressure gradient}} \qquad (1)$$

$$0 = \underbrace{-f(D + h)u}_{\text{Coriolis}} - \underbrace{Rv}_{\text{Drag}} - \underbrace{g(D + h)\partial h/\partial y}_{\text{Pressure gradient}} \; \ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

- Barotropic vorticity equation:

$$\underbrace{v(D + h)(\partial f/\partial y)}_{\text{Planetary vorticity advection}} + \underbrace{(F\pi/b)\sin(\pi y/b)}_{\text{Wind Curl}} + \underbrace{R(\partial v/\partial x - \partial u/\partial y)}_{\text{Drag Curl}} = 0$$

Stommel's gyre (no beta)

- Forcings and data
  - Bottom topography + Land mask
  - Atmospheric surface boundary forcing
  - Initial oceanic conditions
  - Lateral oceanic boundary conditions

- **Flat bottom**
- **Constant wind** `(sustr(i,j)=-cff1*cos(2.*pi/el*yr(i,j)))`
- **Resting state**
- **Vertical walls**

Stommel's gyre (no beta)

- No rotation / or constant rotation ($\dfrac{\partial f}{\partial y} = \beta = 0$
- **cppdefs.h**

```
# define UV_COR
# define UV_VIS2
# define TS_DIF2

# define ANA_GRID
# define ANA_INITIAL
```

- **croco.in**

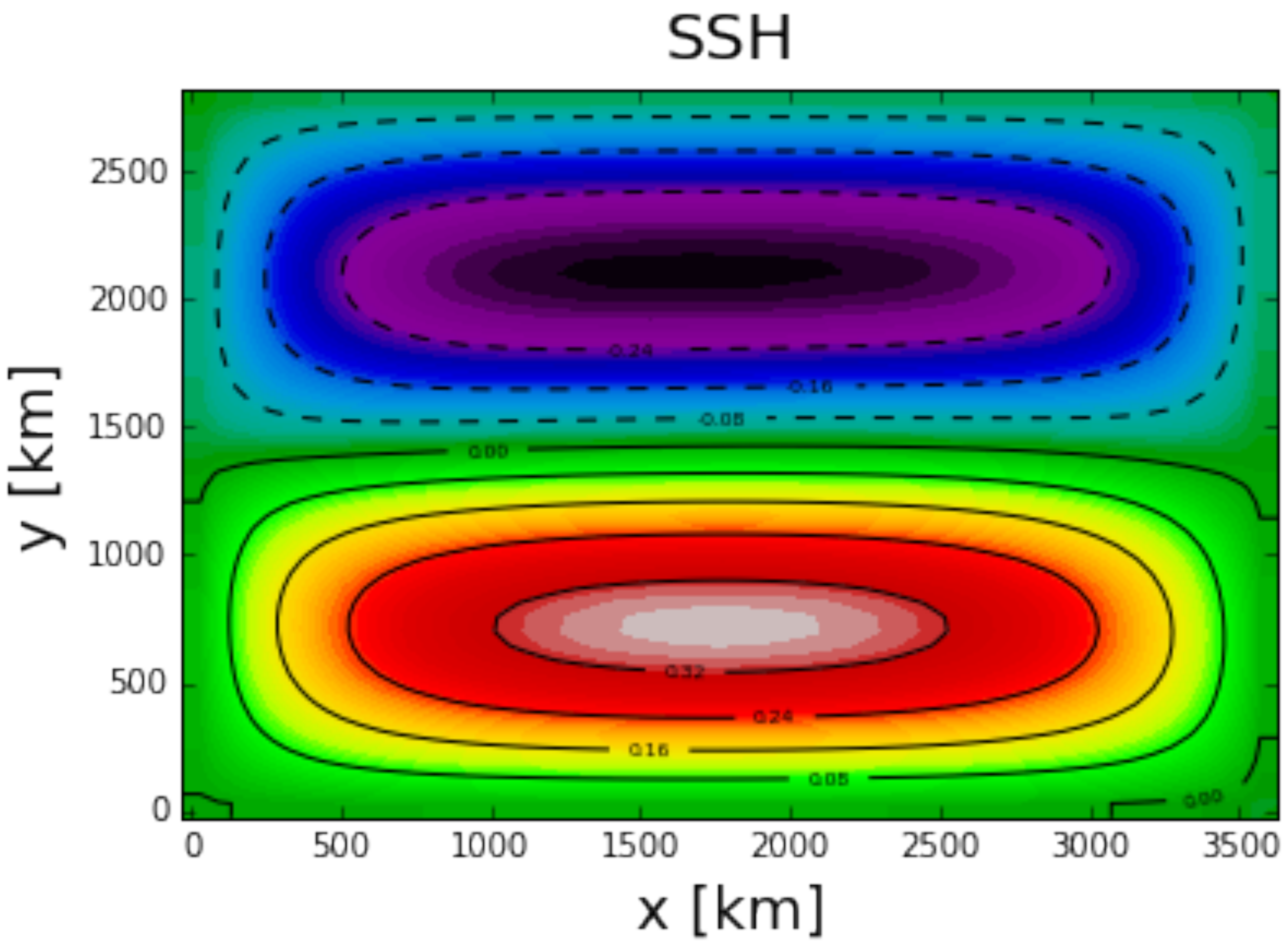
```
bottom_drag:     RDRG(m/s),       RDRG2, Zob [m],  Cdb_min, Cdb_max
                 3.e-4              0.      0.         0.       0.
gamma2:
                 1.
lin_EOS_cff:  R0 [kg/m3], T0 [Celsius], S0 [PSU], TCOEF [1/Celsius], SCOEF [1/PSU]
              30.          0.            0.        0.28                 0.
lateral_visc:   VISC2     [m^2/sec ]
                 1000.  0.
tracer_diff2: TNU2          [m^2/sec]
                 1000. 0.
```

- **ana_grid.F**

```
f0=1.E-4
beta=0.
```

- **param.h**

```
      parameter (LLm0=60,    MMm0=50,    N=10)
```



ROMS simulation after 20 years

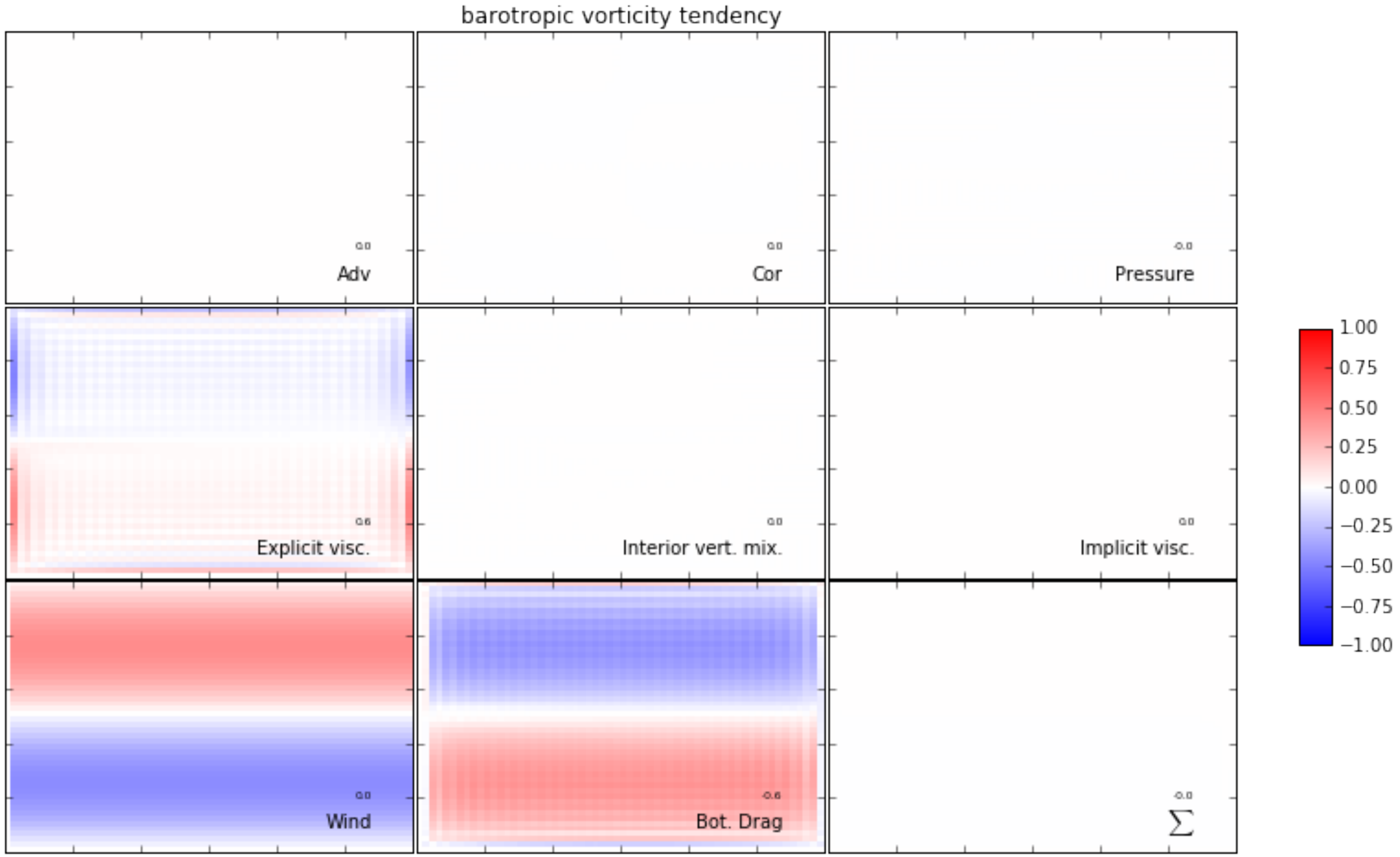
Stommel's gyre (no beta)

$$\underbrace{\cancel{\frac{\partial \Omega}{\partial t}}}_{\text{rate}} = -\underbrace{\cancel{\vec{\nabla}\cdot(f\vec{u})}}_{\text{planet. vort. adv.}} + \underbrace{\cancel{\frac{\mathbf{J}(P_b, h)}{\rho_0}}}_{\text{bot. pres. torque}} \boxed{+\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} \boxed{-\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}}$$
$$+\underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{\cancel{\mathcal{A}_\Sigma}}_{\text{NL advection}}$$

- No rotation / or constant rotation

$$\frac{\partial f}{\partial y} = \beta = 0$$

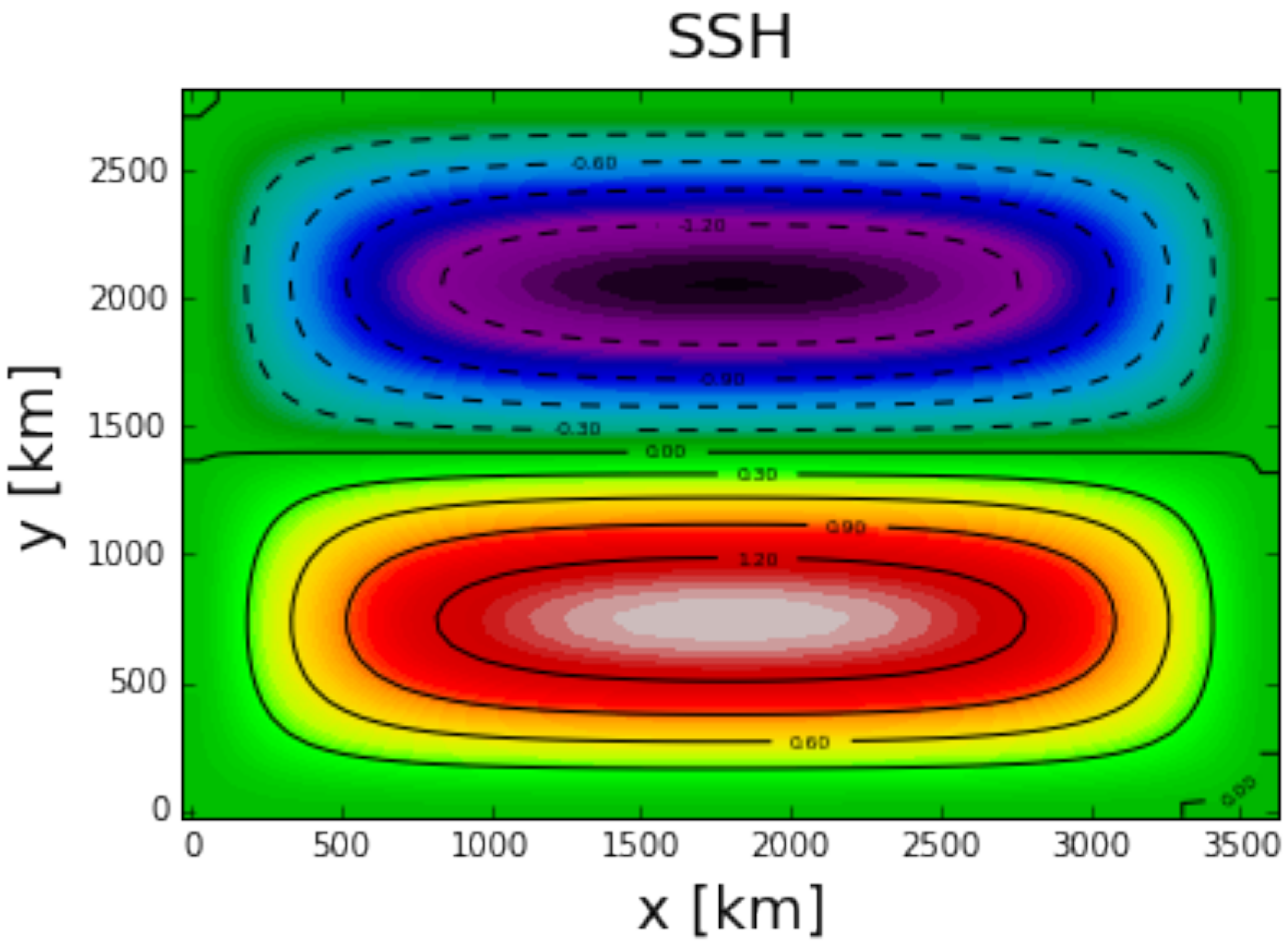
-



Stommel's gyre (no beta)

- No rotation / or constant rotation ($\dfrac{\partial f}{\partial y} = \beta = 0$)
- **No-slip lateral boundaries**
- **croco.in**

```
gamma2:
                    -1.
```



Stommel's gyre (no beta)

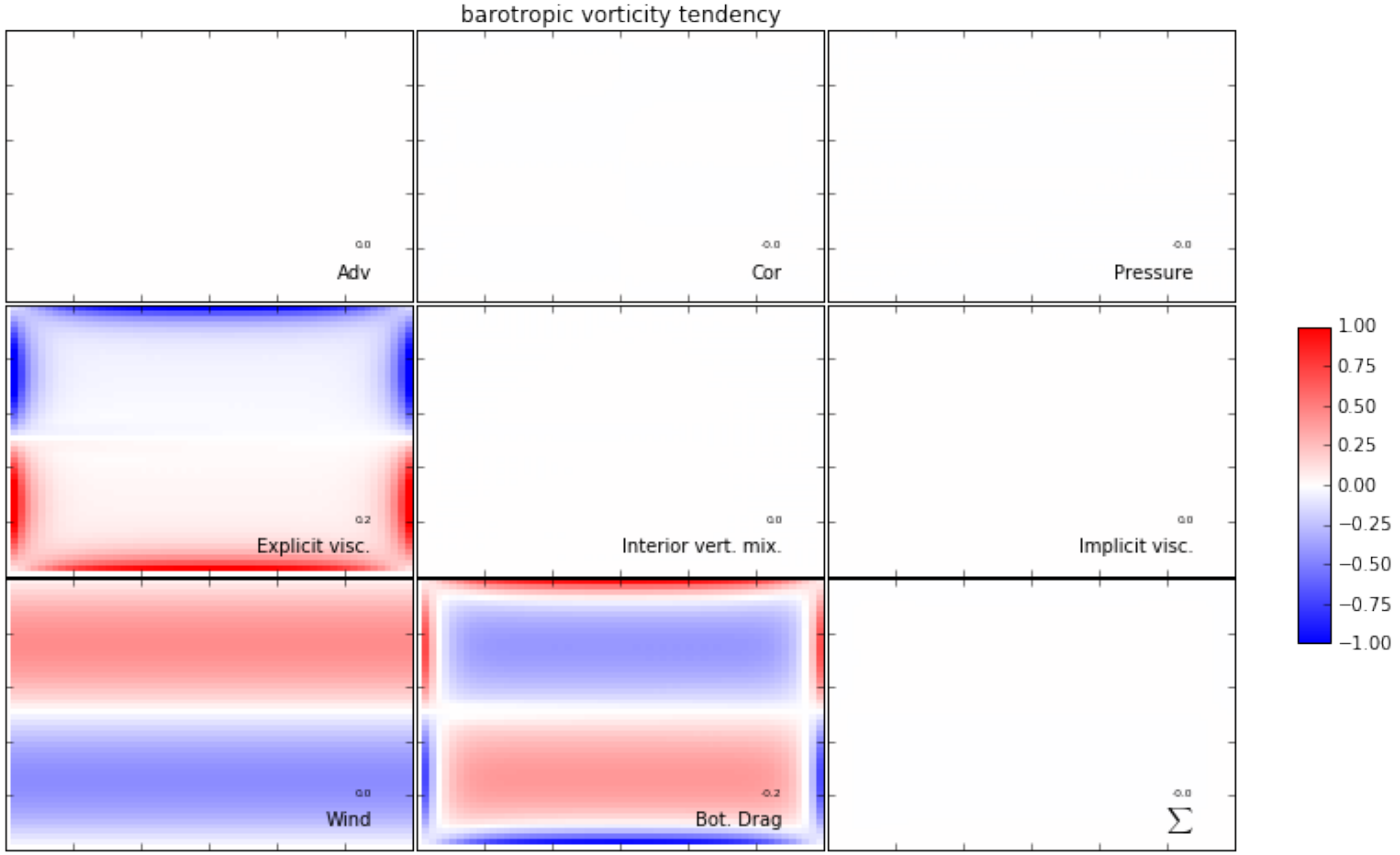
$$\underbrace{\cancel{\frac{\partial \Omega}{\partial t}}}_{\text{rate}} = -\underbrace{\cancel{\vec{\nabla} \cdot (f\vec{u})}}_{\text{planet. vort. adv.}} + \underbrace{\cancel{\frac{\mathbf{J}(P_b, h)}{\rho_0}}}_{\text{bot. pres. torque}} + \boxed{\underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \boxed{\underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}}$$

$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{\cancel{\mathcal{A}}}_{\text{NL advection}}$$

- No rotation / or constant rotation

$$\frac{\partial f}{\partial y} = \beta = 0$$

-



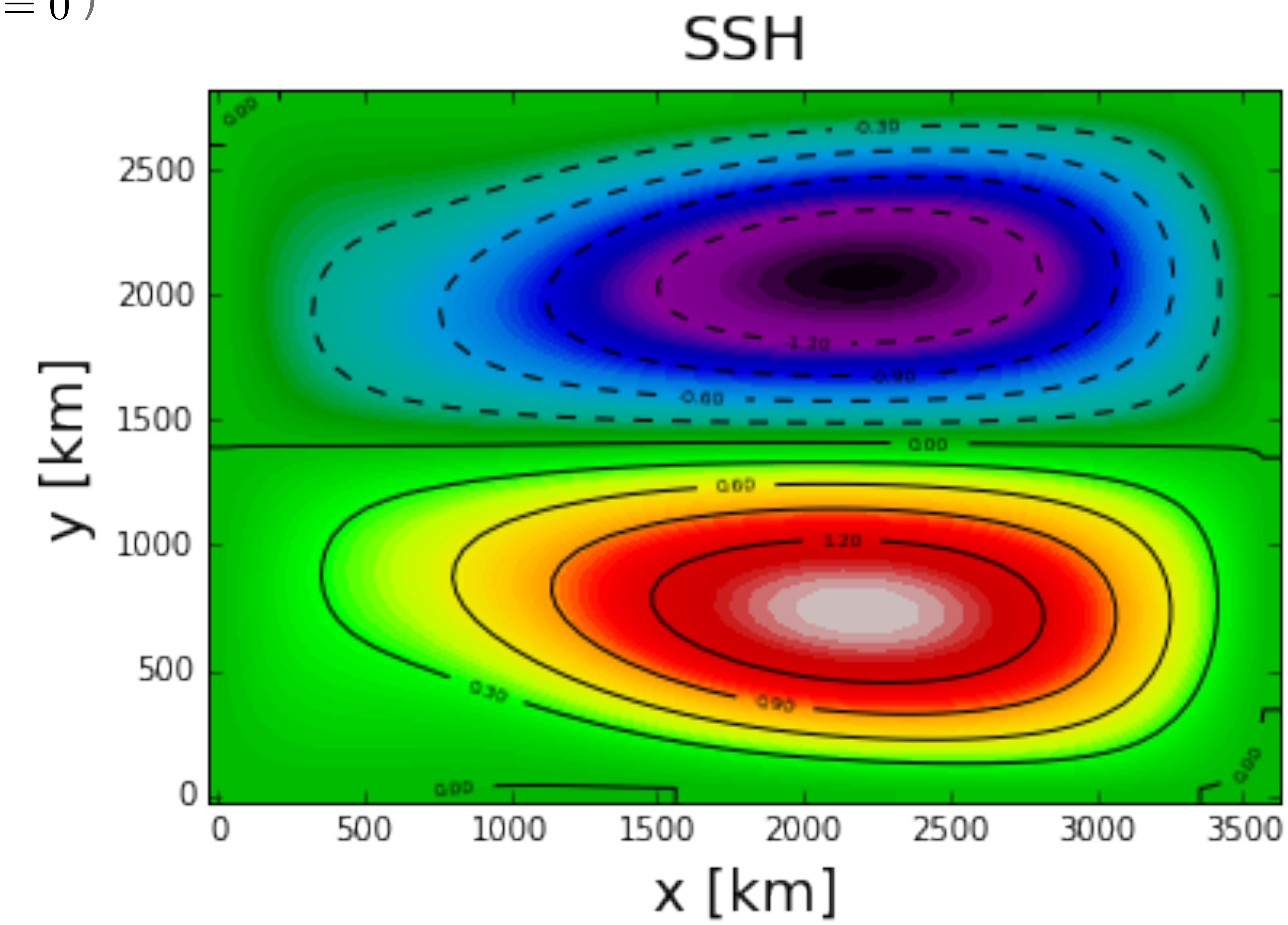
Stommel's gyre (no beta)

- No rotation / or constant rotation ($\frac{\partial f}{\partial y} = \beta = 0$)

- **Add some non-linear terms**

- **cppdefs.h**

```
# define UV_ADV
```



Stommel's gyre (no beta)

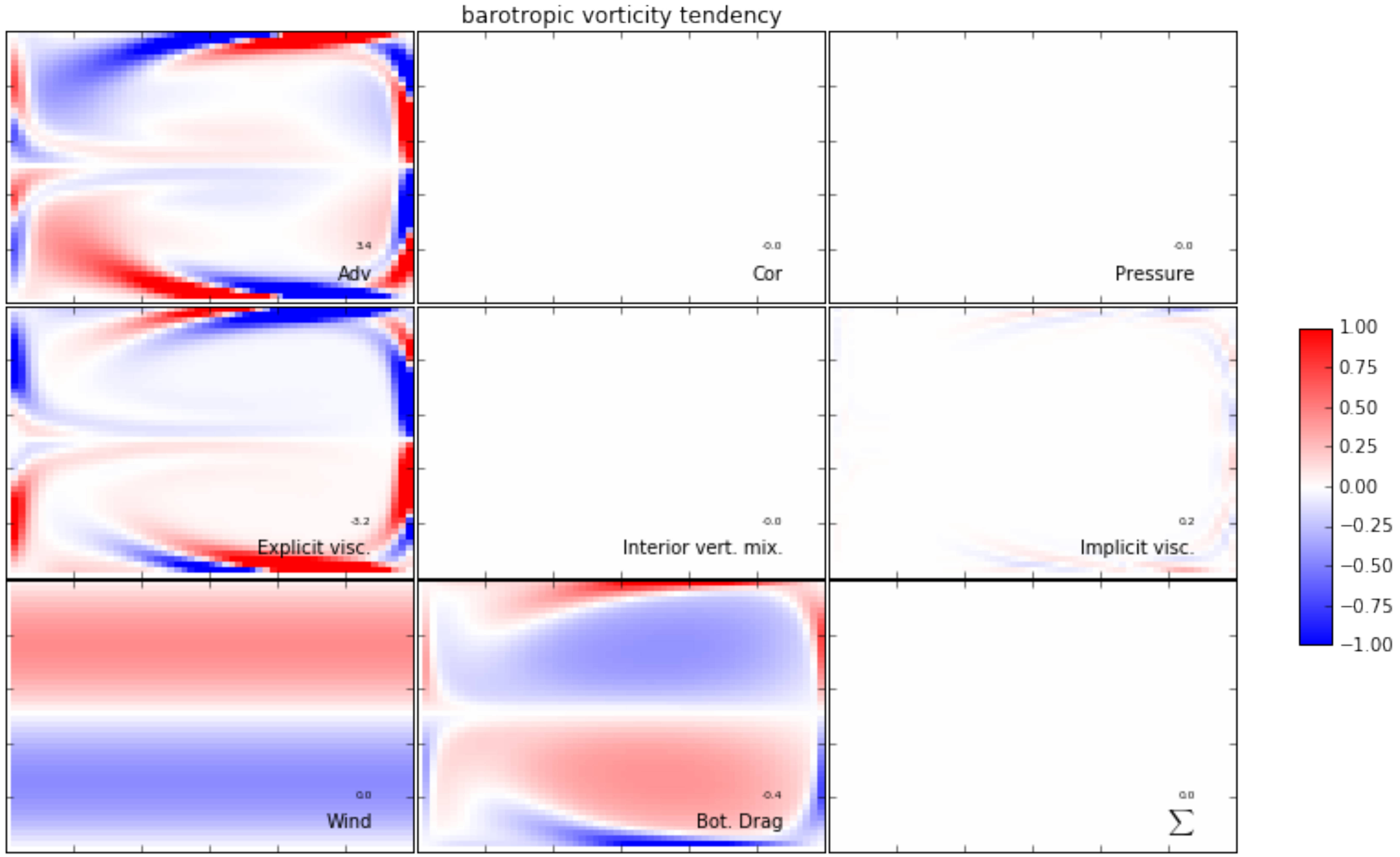
$$\underbrace{\cancel{\frac{\partial \Omega}{\partial t}}}_{\text{rate}} = -\underbrace{\cancel{\vec{\nabla}\cdot(f\vec{u})}}_{\text{planet. vort. adv.}} + \underbrace{\cancel{\frac{J(P_b,h)}{\rho_0}}}_{\text{bot. pres. torque}} + \boxed{\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \boxed{\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}}$$

$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

- No rotation / or constant rotation

$$\frac{\partial f}{\partial y} = \beta = 0$$

-



Stommel's gyre (with beta)

- With latitudinal variation of Coriolis ($\dfrac{\partial f}{\partial y} = \beta \neq 0$)
-

Stommel's gyre (with beta)

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

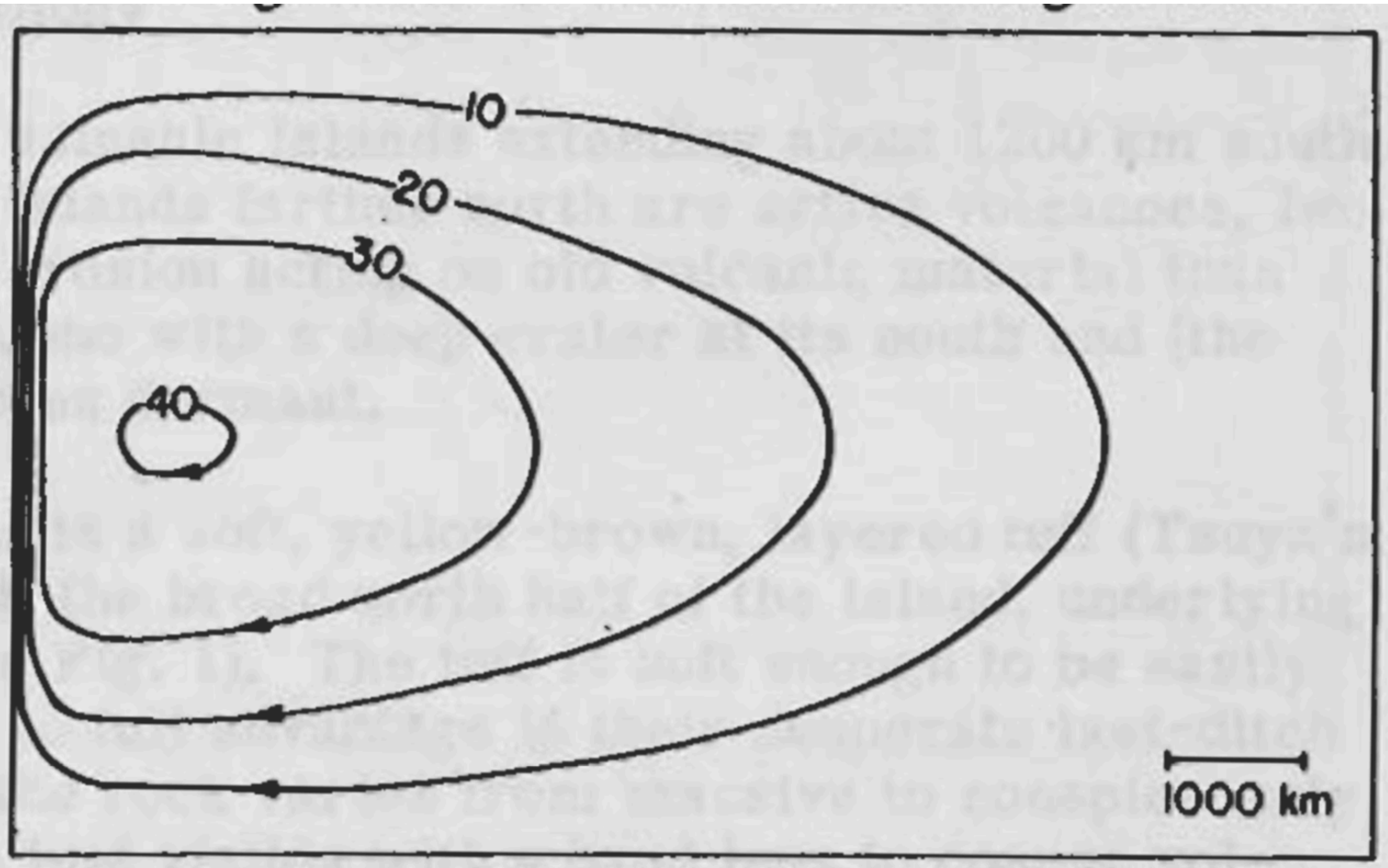
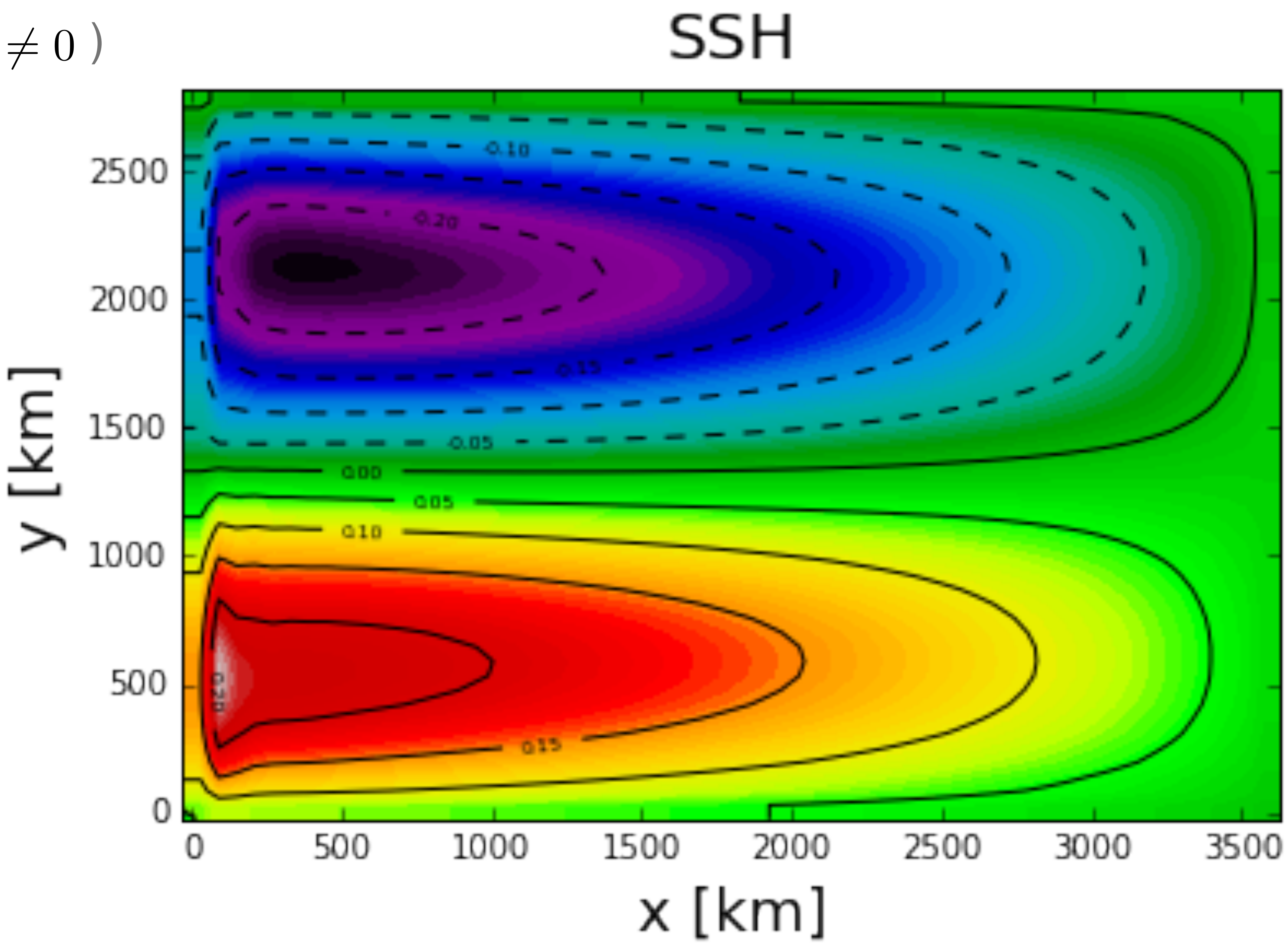


Fig. 5--Streamlines for the case where the Coriolis force is a linear function of latitude

- **Formation of a western boundary current !**

Stommel's gyre (with beta)

- With latitudinal variation of Coriolis ($\frac{\partial f}{\partial y} = \beta \neq 0$)



- **ana_grid.F**

```
f0=1.E-4
beta=2.E-11
```

Stommel's gyre (with beta)

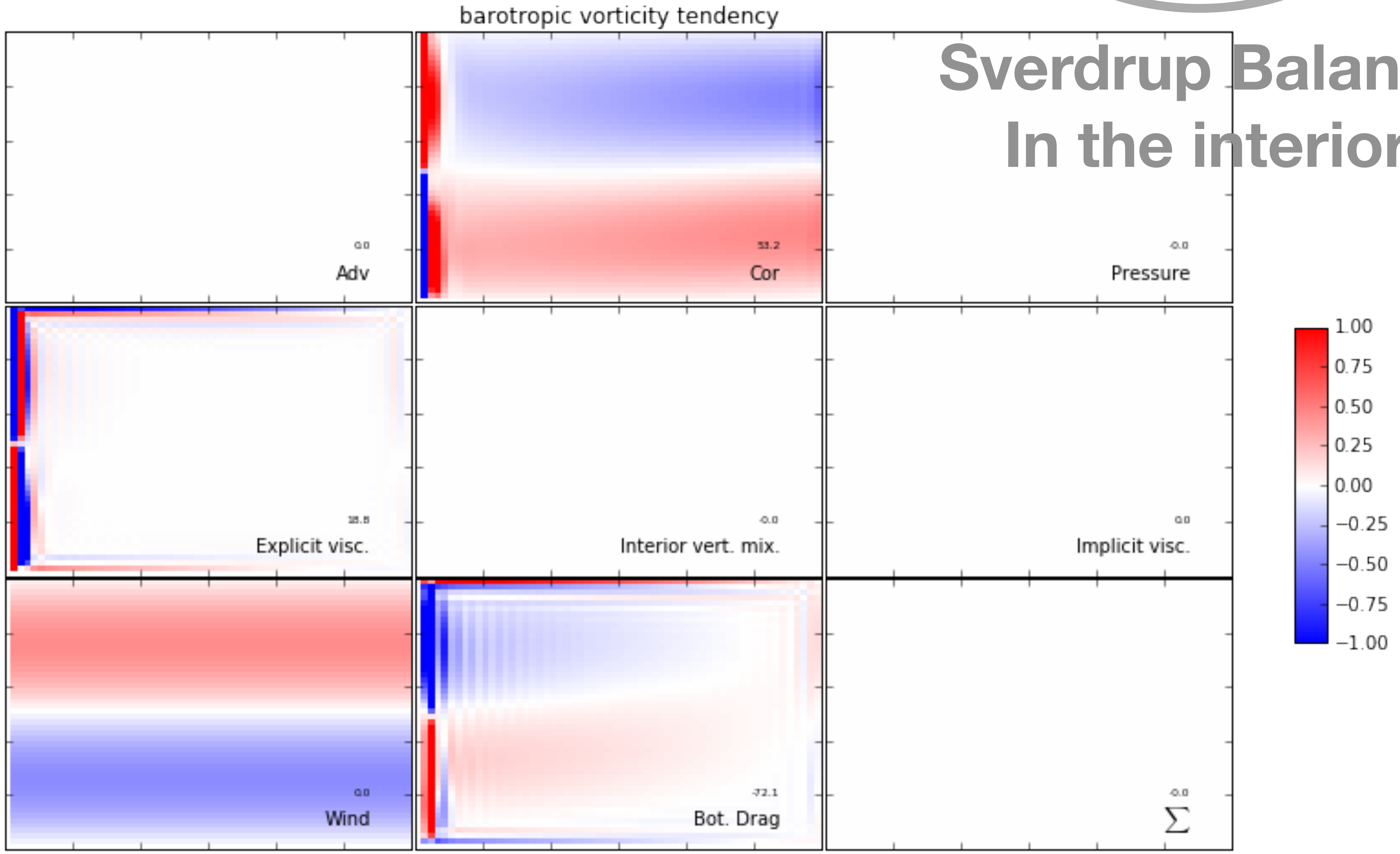
$$\cancel{\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}}} = \boxed{\underbrace{-\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \cancel{\underbrace{\mathbf{J}(P_b, h)}_{\text{bot. pres. torque}}} \boxed{\underbrace{+\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} \underbrace{-\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$

$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \cancel{\underbrace{\ }_{\text{NL advection}}}$$

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

-



Sverdrup Balance
In the interior

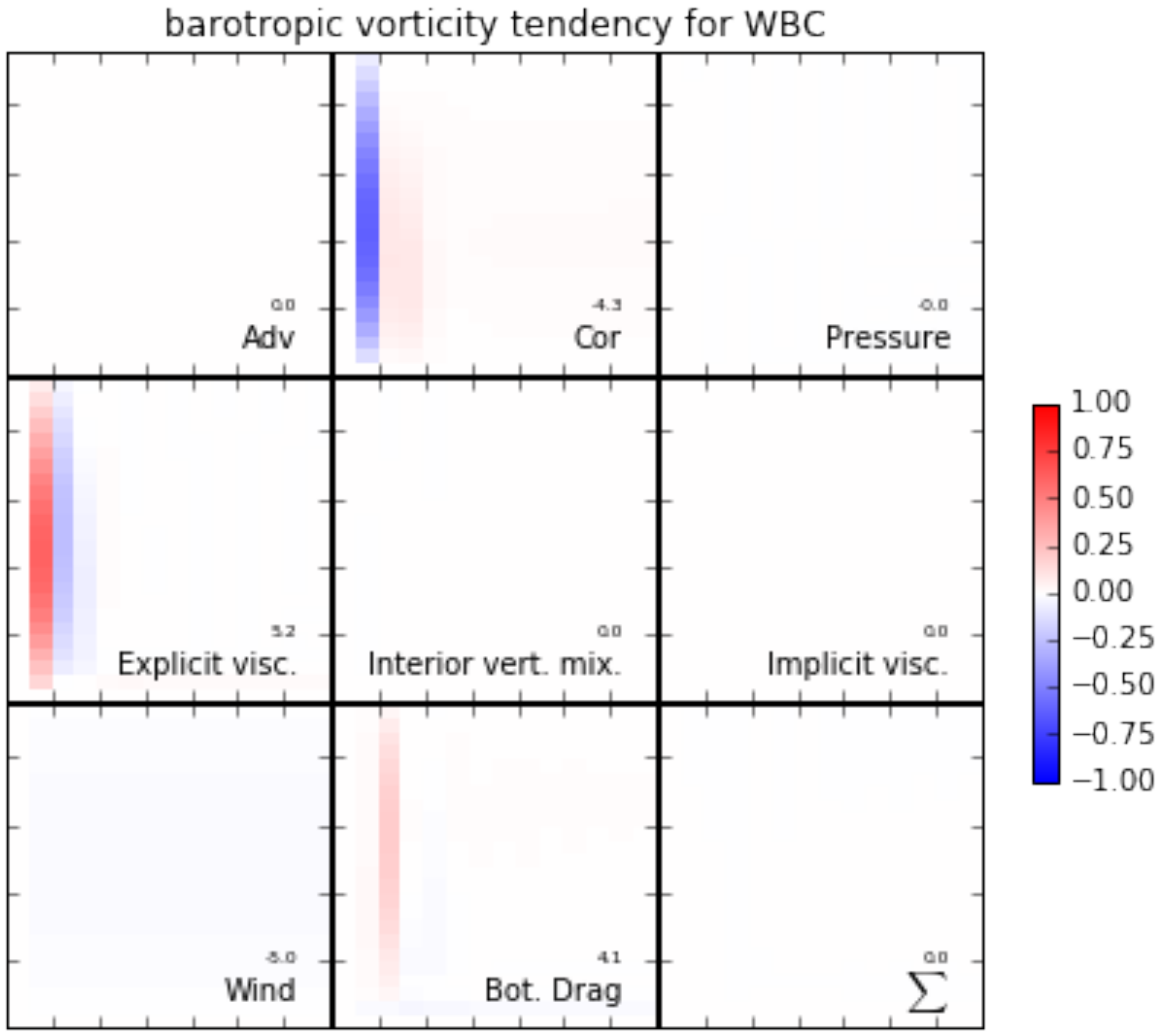
Stommel's gyre (with beta)

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

-

$$\cancel{\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}}} = \boxed{-\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \cancel{\underbrace{\frac{\mathbf{J}(P_b, H)}{\rho_0}}_{\text{bot. pres. torque}}} + \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \boxed{\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}} + \boxed{\underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}}} - \cancel{\underbrace{\mathcal{A}_\Sigma}_{\text{NL advection}}}$$



Zoom over the western boundary current

Gyre with beta and lateral drag

- With latitudinal variation of Coriolis ($\dfrac{\partial f}{\partial y} = \beta \neq 0$)

- **Weaker drag and no-slip lateral**

- **cppdefs.h**

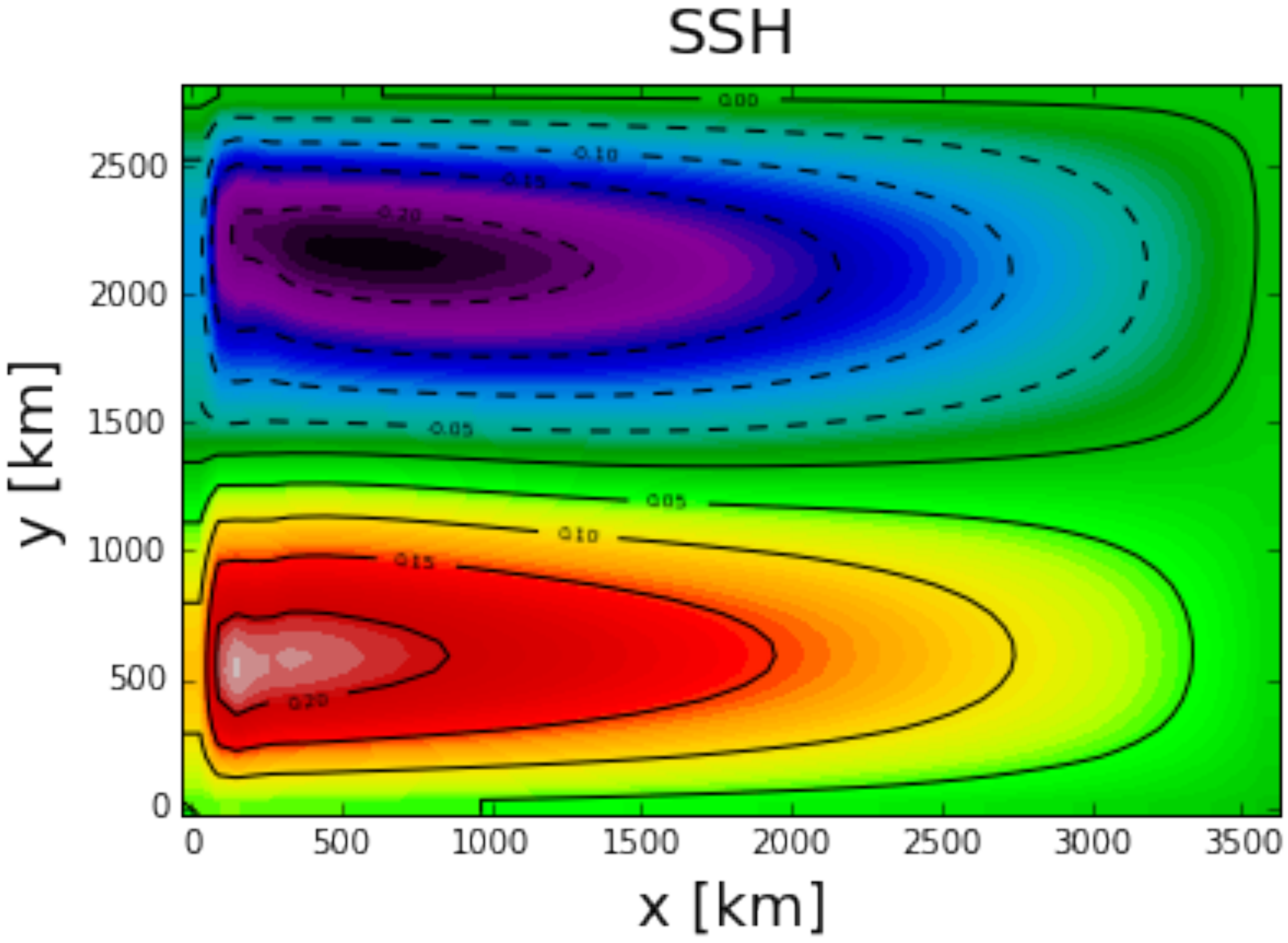
```
# define UV_ADV
```

- **croco.in**

```
bottom_drag:     RDRG(m/s),      RDRG2, Zob [m],  Cdb_min, Cdb_max
                 3.e-4            0.      0.         0.      0.
```

- **croco.in**

```
gamma2:
                 -1.
```



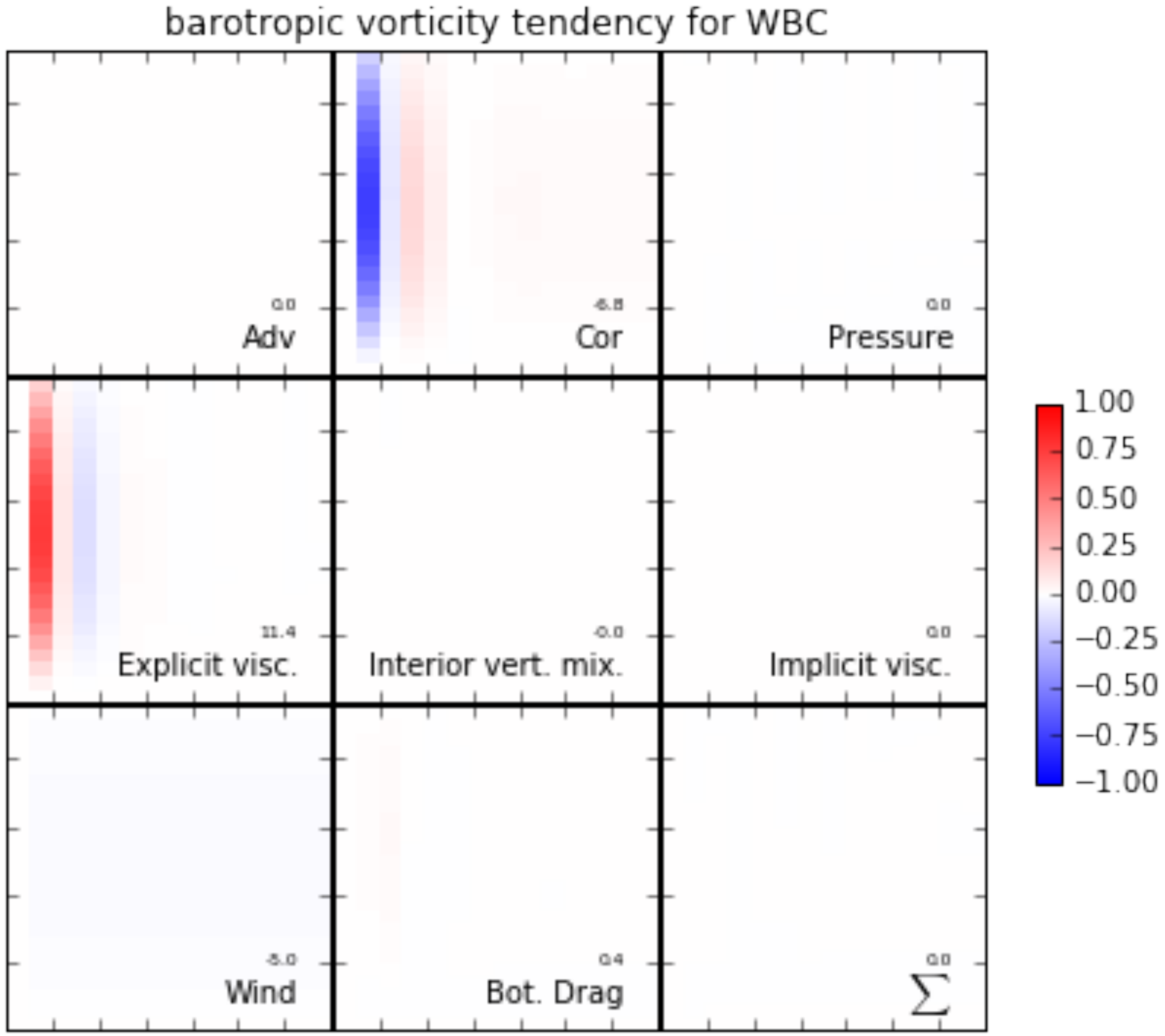
Gyre with beta and lateral drag

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{-\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\cancel{\mathbf{J}(P, h)}}_{\text{bot. pres. torque}} + \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \boxed{\underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

-



Zoom over the western boundary current

Munk' gyre

In Ekman's and Stommel's model the ocean is assumed homogeneous, a case in which the currents extend to the very bottom. Not only is this in contrast with observations, according to which the bulk of the water transport in the main ocean currents takes place in the upper thousand meters, but it also leads to mathematical complications which rendered Ekman's analysis very difficult, and forced Stommel to resort to a rather arbitrary frictional force along the bottom.

To avoid these difficulties, we retain Sverdrup's integrated mass transport as the dependent variable. This device makes it possible to examine the more general case of a baroclinic ocean without having to specify the nature of the vertical distributions of density and current. In recognition of the evidence that currents essentially vanish at great depths, we shall depend on lateral friction for the dissipative forces. From Stommel we retain the rectangular boundaries, although we extend the basin to both sides of the equator and deal with the *observed* wind distribution rather than a simple sinusoidal distribution.

JOURNAL OF METEOROLOGY

ON THE WIND-DRIVEN OCEAN CIRCULATION

By Walter H. Munk

Institute of Geophysics and Scripps Institution of Oceanography, University of California[1]

(Manuscript received 24 September 1949)

$$P = \int_{-h}^{z_0} p\, dz, \qquad \mathbf{M}_H = \int_{-h}^{z_0} \rho\, \mathbf{v}_H\, dz, \qquad (2a, b)$$

designate the integrated pressure and mass transport.

$$\nabla P + f\mathbf{k} \times \mathbf{M} - \boldsymbol{\tau} - A\,\nabla^2 \mathbf{M} = 0. \qquad \mathbf{M} = \mathbf{k} \times \nabla\psi,$$

$$\boxed{(A\nabla^4 - \beta\, \partial/\partial x)\psi = -\operatorname{curl}_z \boldsymbol{\tau},}$$

For boundary conditions we choose

$$\psi_{\mathrm{bdry}} = 0, \qquad (\partial\psi/\partial\nu)_{\mathrm{bdry}} = 0, \qquad (7a, b)$$

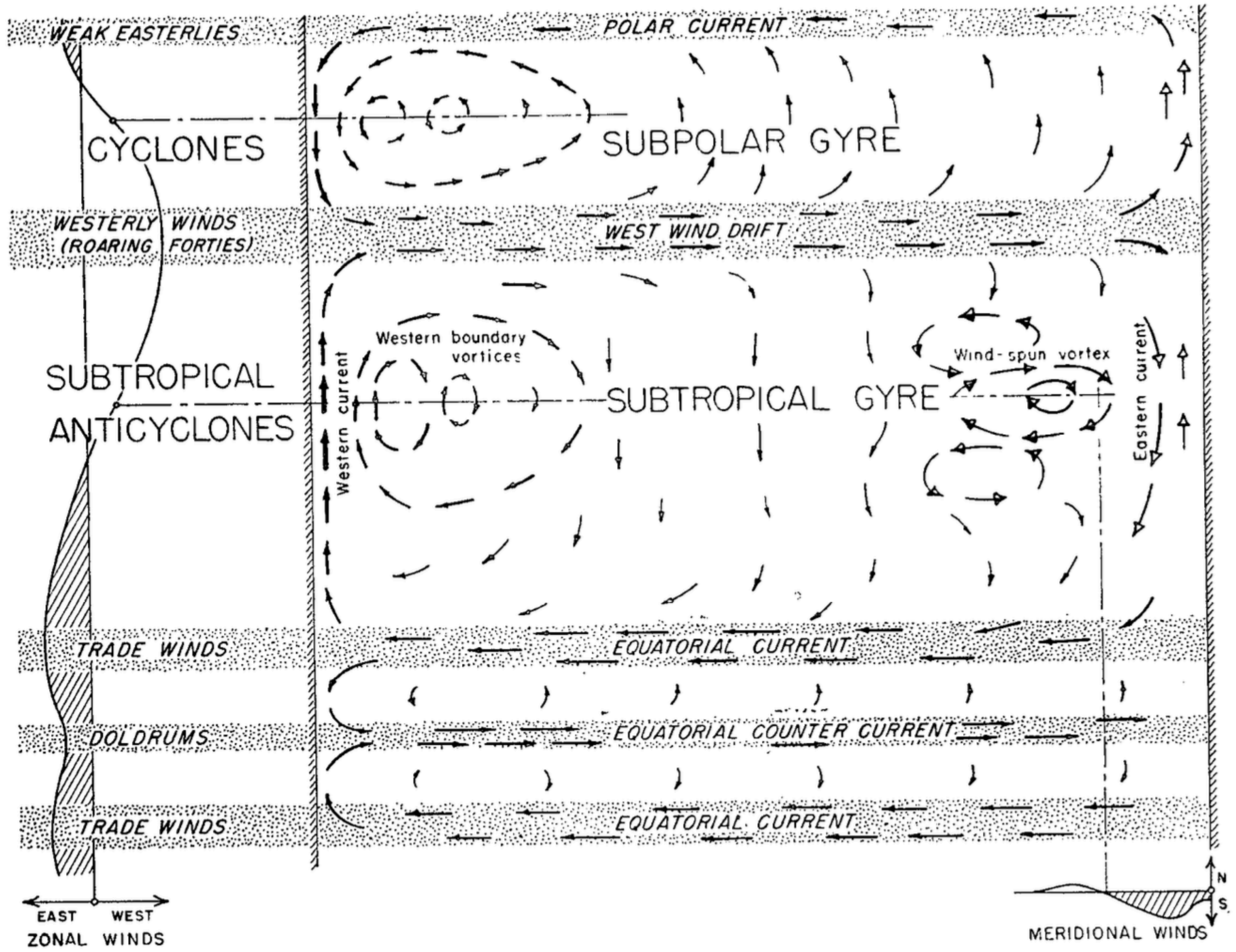


FIG. 8. Schematic presentation of circulation in a rectangular ocean resulting from zonal winds (filled arrowheads), meridional winds (open arrowheads), or both (half-filled arrowheads). The width of the arrows is an indication of the strength of the currents. The nomenclature applies to either hemisphere, but in the Southern Hemisphere the subpolar gyre is replaced largely by the Antarctic Circumpolar Current (west wind drift) flowing around the world. Geographic names of the currents in various oceans are summarized in table 3.

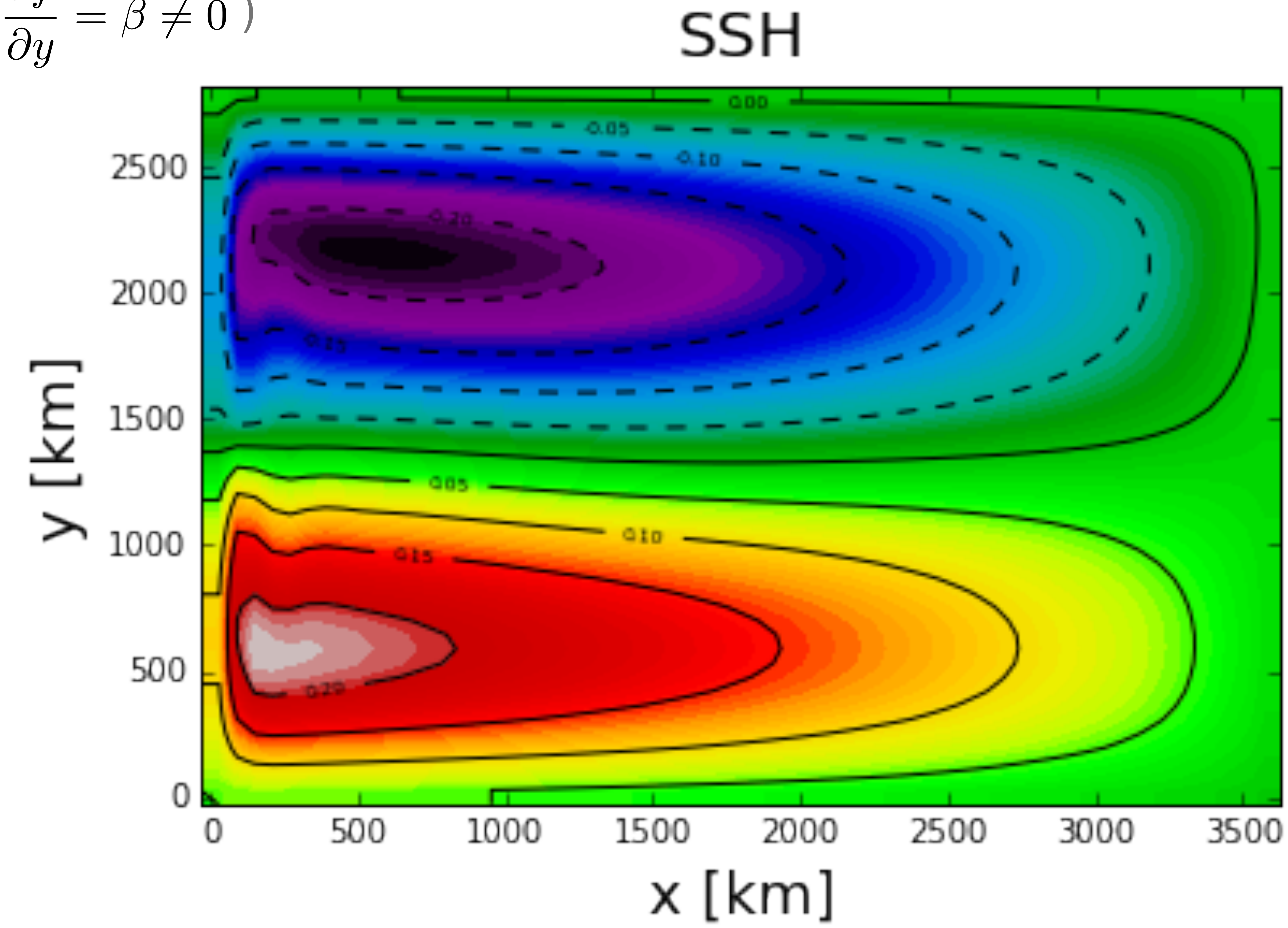
Gyre with beta and lateral drag

- With latitudinal variation of Coriolis ($\dfrac{\partial f}{\partial y} = \beta \neq 0$)

- **Add some non-linear terms**

- **cppdefs.h**

```
# define UV_ADV
```



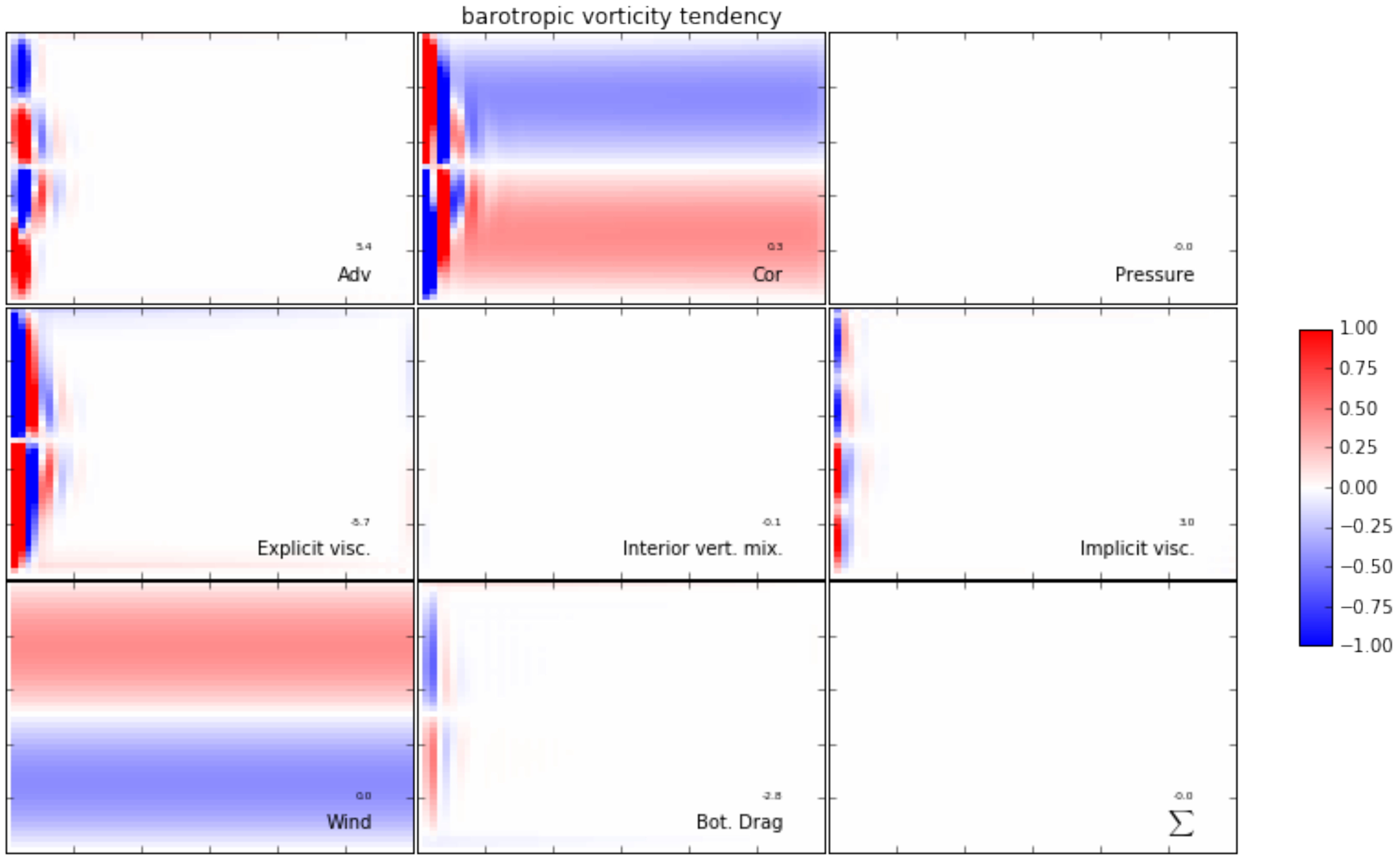
Gyre with beta and lateral drag

$$\cancel{\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}}} = \boxed{- \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \cancel{\underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}}} \boxed{+ \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

-



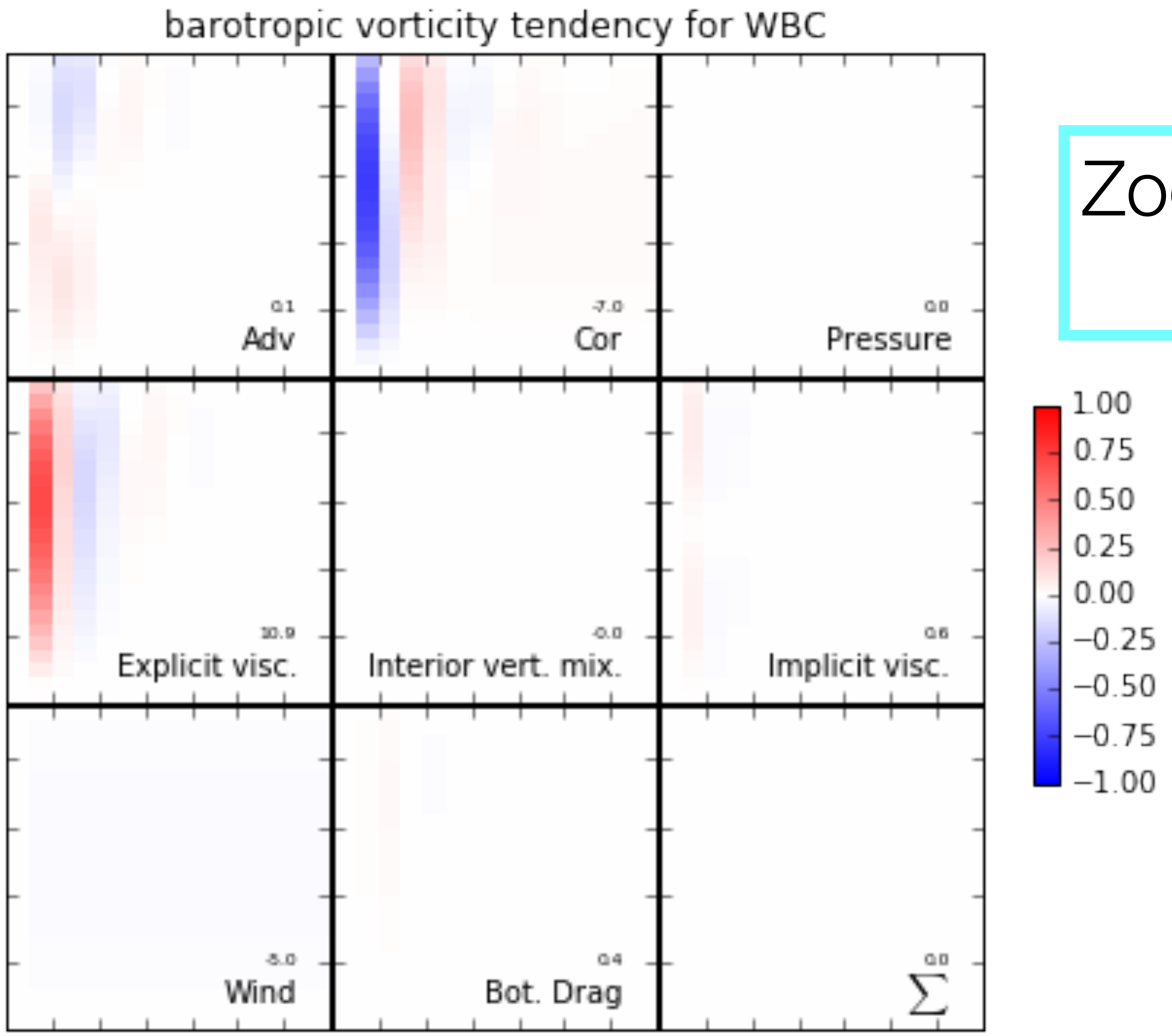
Gyre with beta and lateral drag

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{- \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \cancel{\underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}}} + \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$
$$+ \boxed{\underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

- With latitudinal variation of Coriolis

$$\frac{\partial f}{\partial y} = \beta \neq 0$$

-



Zoom over the western boundary current

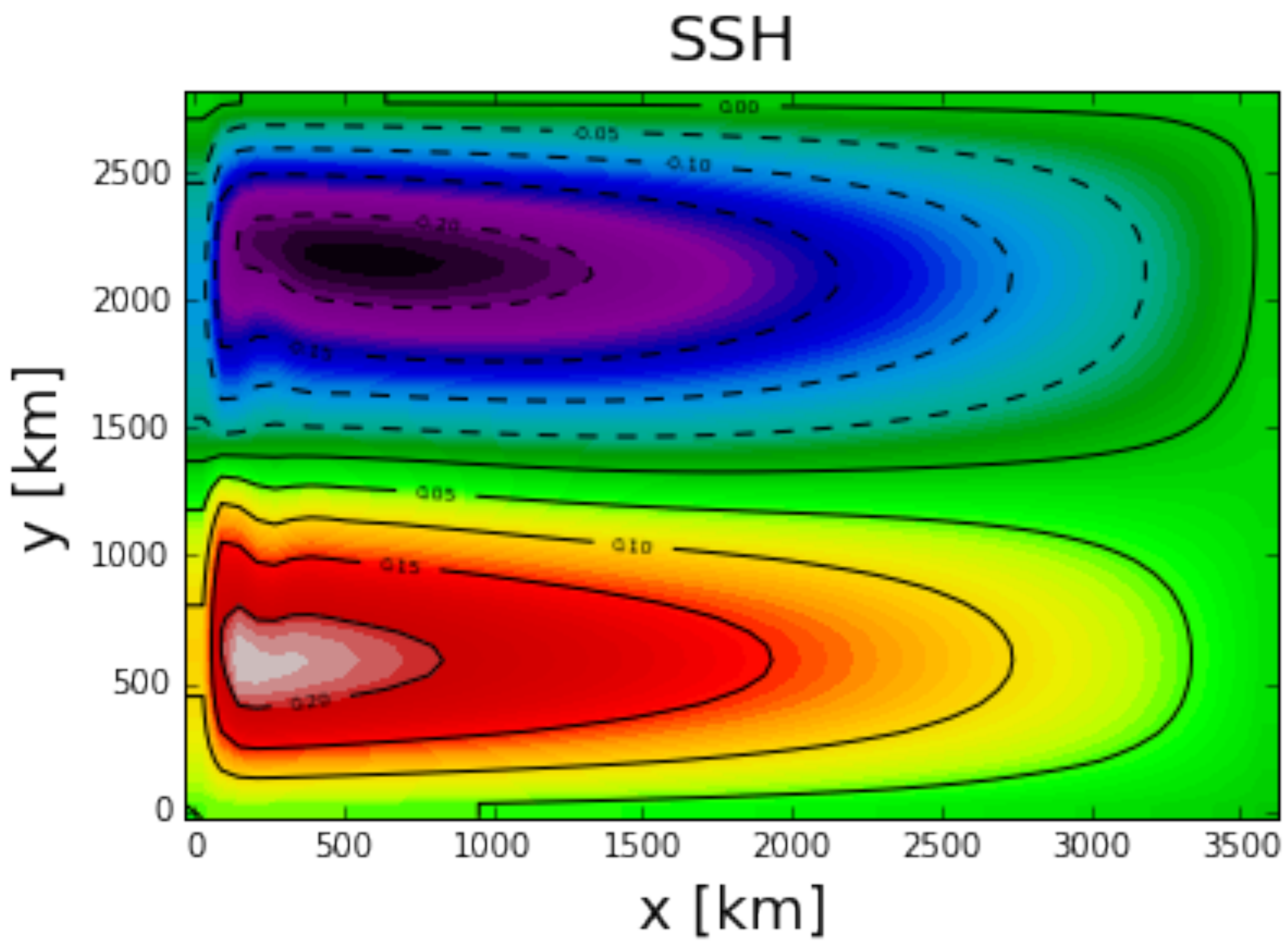
Toward a more turbulent gyre

- **param.h**

```
parameter (LLm0=60,    MMm0=50,    N=10)
```

- **Croco.in**

```
lateral_visc:   VISC2     [m^2/sec ]
                 1000.   0.
tracer_diff2: TNU2          [m^2/sec]
                 1000.  0.
```



Toward a more turbulent gyre

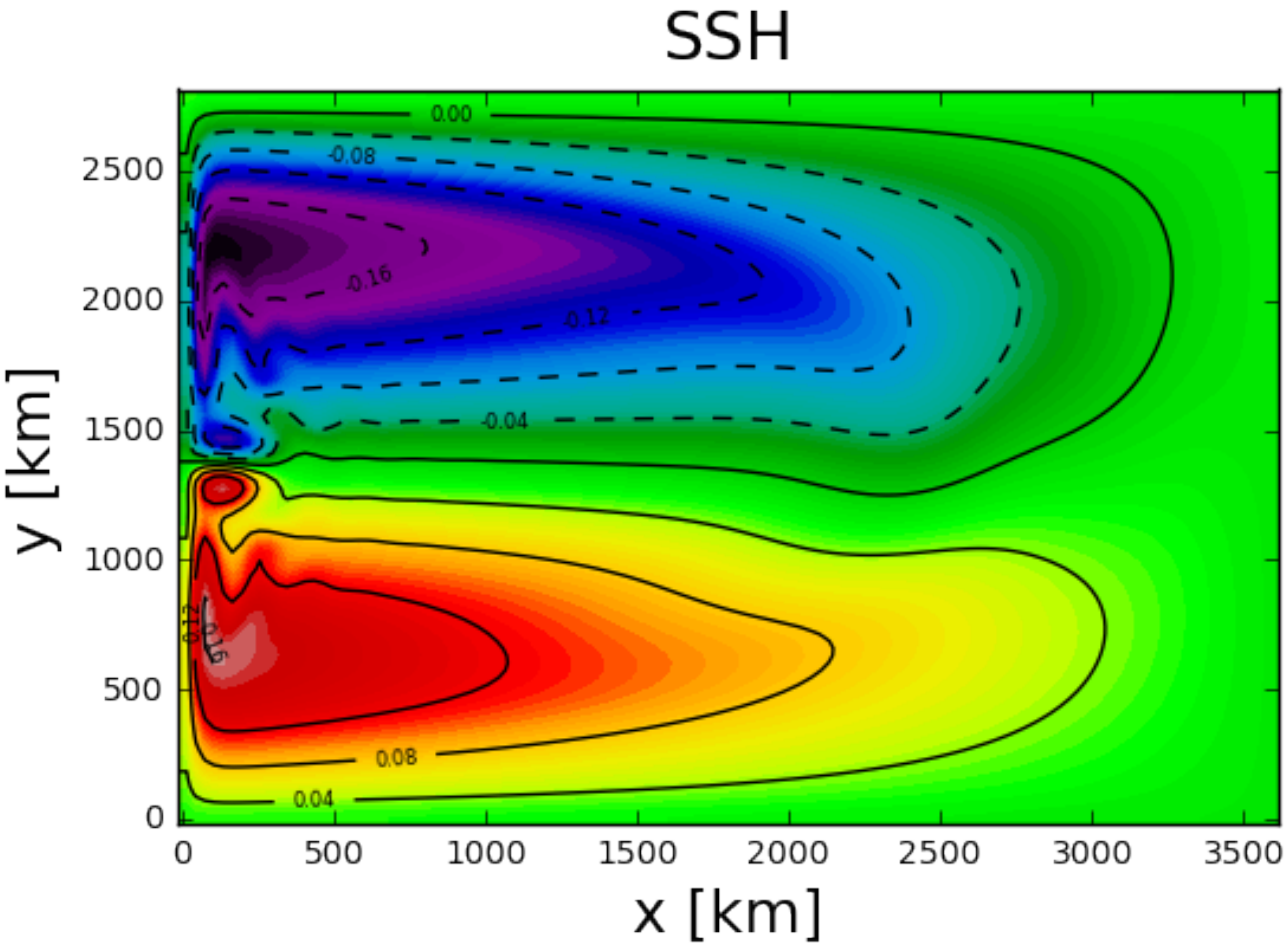
- more resolution and less viscosity:

- **param.h**

```
parameter (LLm0=120,   MMm0=100,   N=20)
```

- **Croco.in**

```
lateral_visc:   VISC2    [m^2/sec ]
                 100.  0.
tracer_diff2: TNU2          [m^2/sec]
                 100. 0.
```



Toward a more turbulent gyre

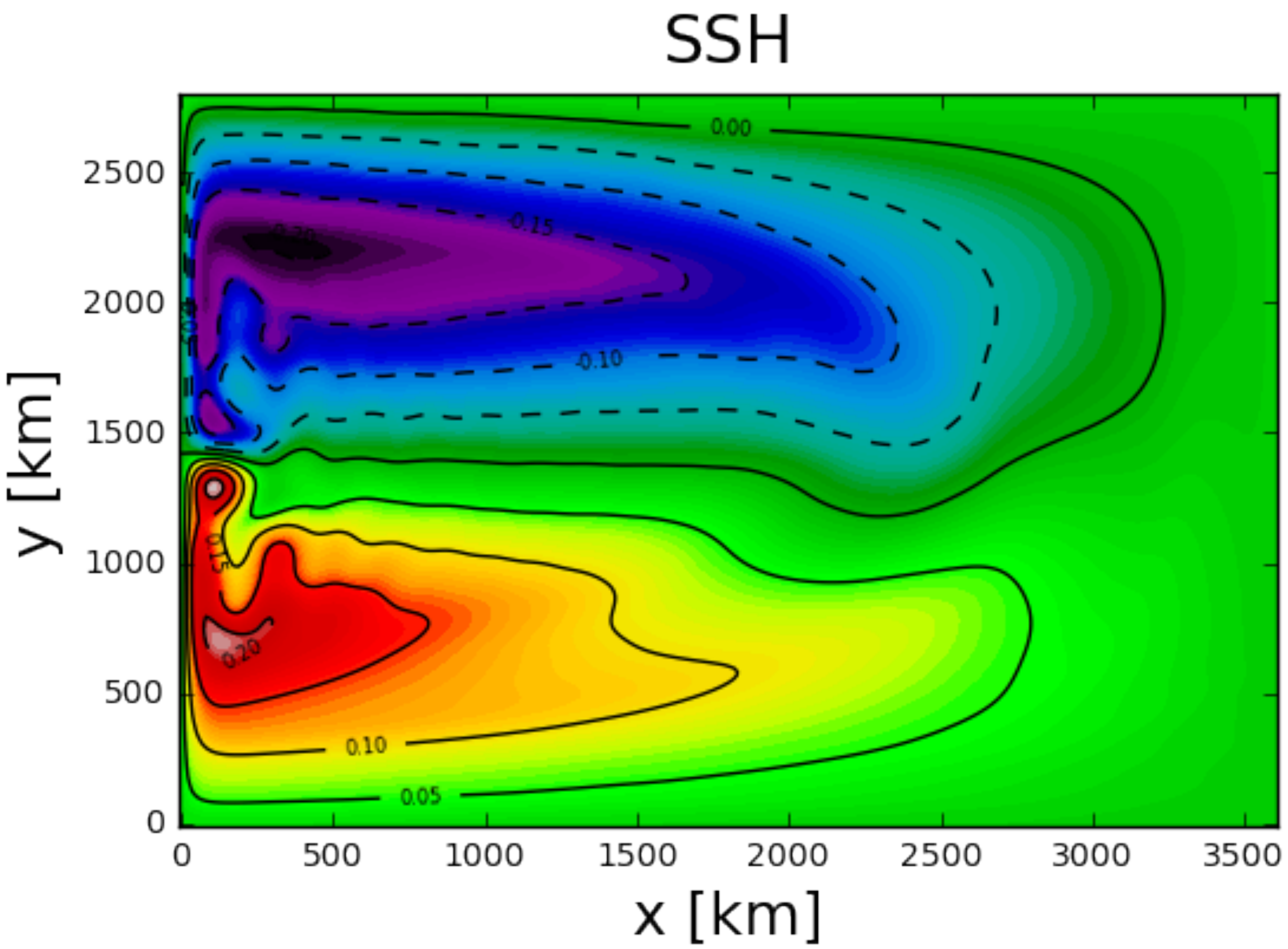
- more resolution and less viscosity:

- **param.h**

```
parameter (LLm0=240,   MMm0=200,   N=20)
```

- **Croco.in**

```
lateral_visc:   VISC2     [m^2/sec ]
                 10.   0.
tracer_diff2: TNU2          [m^2/sec]
                 10. 0.
```



Toward a more turbulent gyre

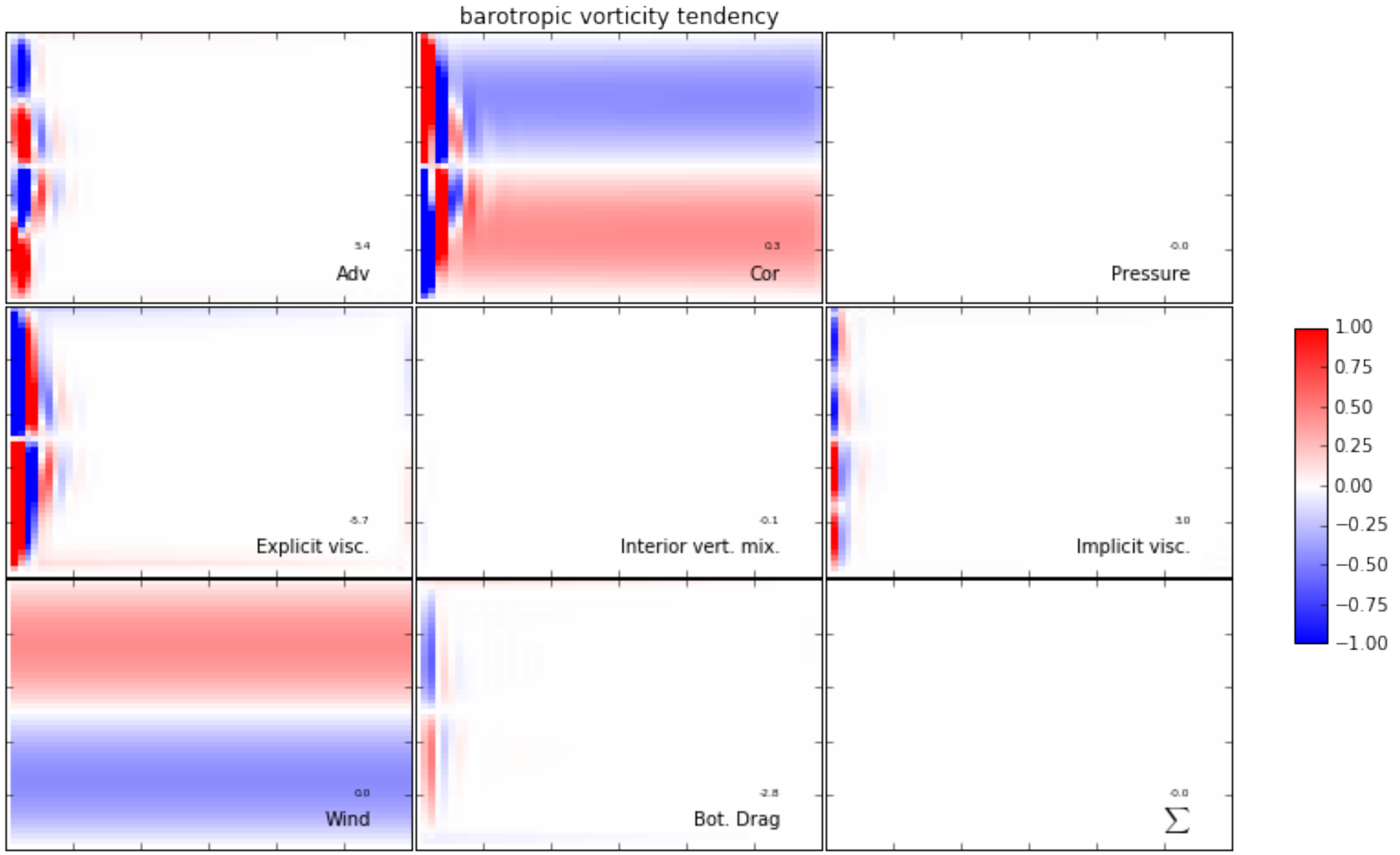
$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{-\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\cancel{\frac{J(P_b,h)}{\rho_0}}}_{\text{bot. pres. torque}} \boxed{+\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

- **param.h**

```
parameter (LLm0=60,    MMm0=50,    N=10)
```

- **Croco.in**

```
lateral_visc:   VISC2     [m^2/sec ]
                  1000.  0.
tracer_diff2: TNU2          [m^2/sec]
                  1000. 0.
```



Toward a more turbulent gyre

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{-\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\cancel{\mathbf{J}(P_b,h)}}_{\text{bot. pres. torque}} \boxed{+\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

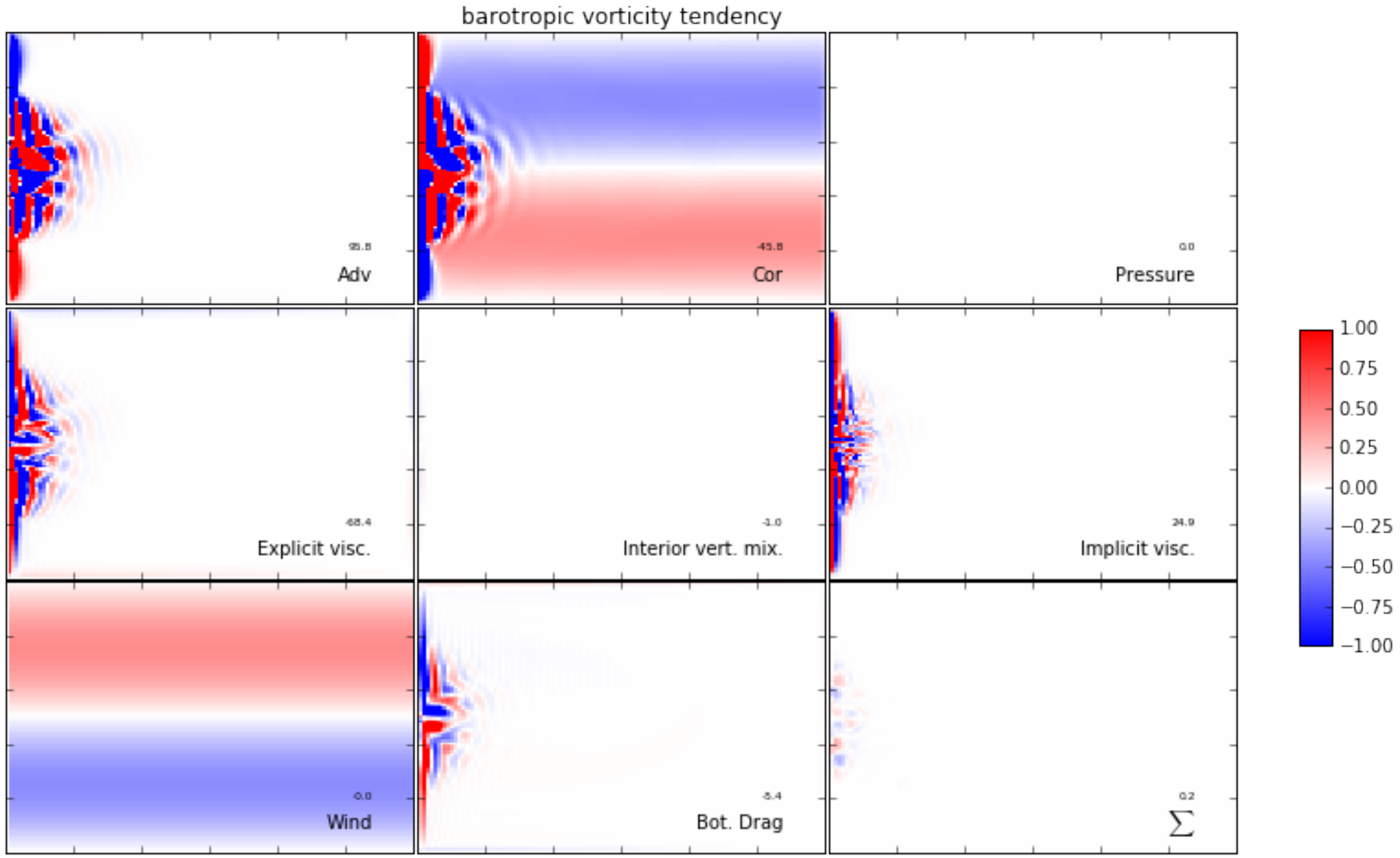
- more resolution and less viscosity:

- **param.h**

```
parameter (LLm0=120,    MMm0=100,    N=20)
```

- **Croco.in**

```
lateral_visc:   VISC2     [m^2/sec ]
                 100.   0.
tracer_diff2: TNU2          [m^2/sec]
                 100.  0.
```



Toward a more turbulent gyre

$$\underbrace{\frac{\partial\Omega}{\partial t}}_{\text{rate}} = \boxed{-\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\cancel{\frac{J(P_b,h)}{\rho_0}}}_{\text{bot. pres. torque}} + \boxed{\underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$
$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

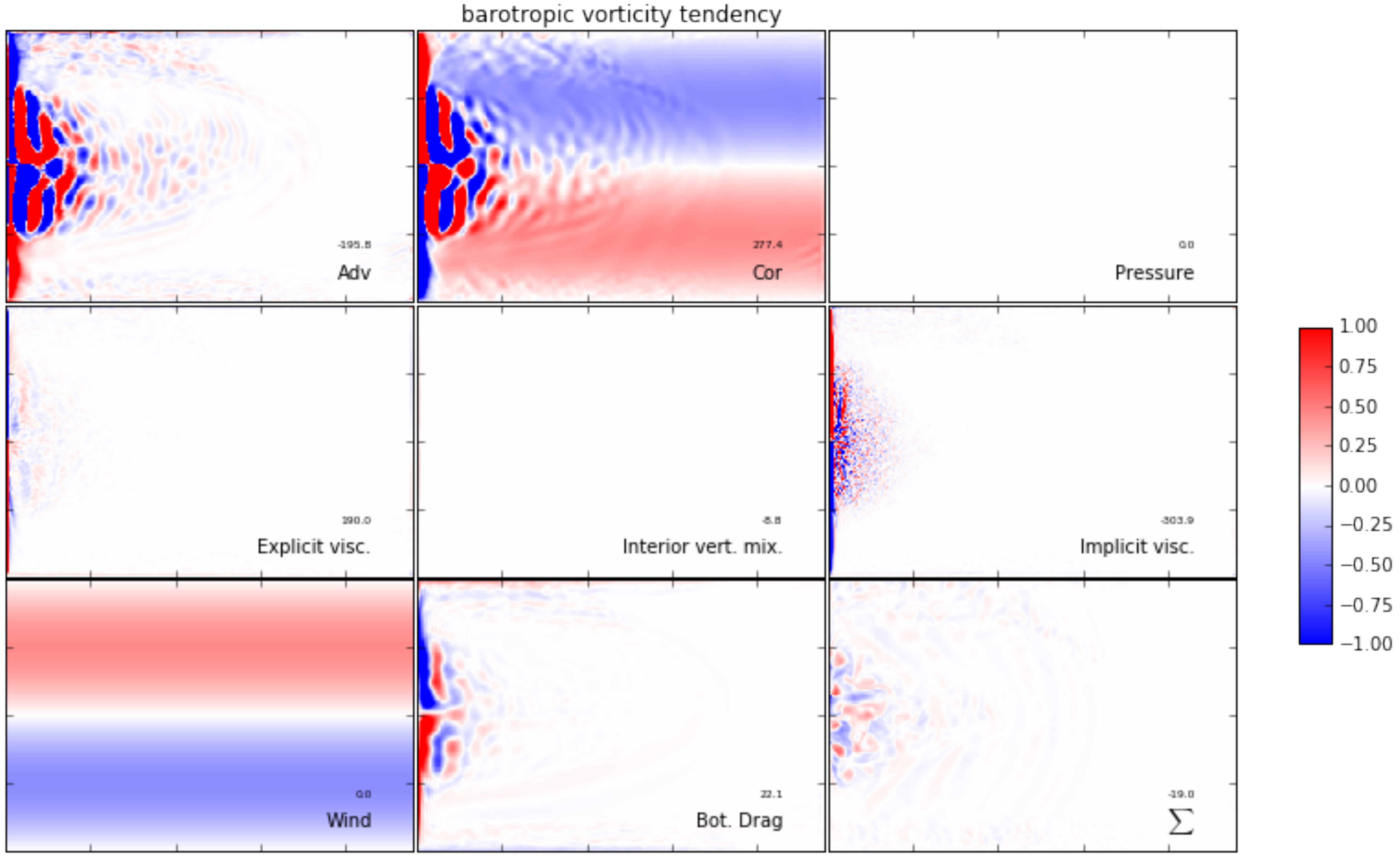
- more resolution and less viscosity:

- **param.h**

```
parameter (LLm0=240,   MMm0=200,   N=20)
```

- **Croco.in**

```
lateral_visc:   VISC2     [m^2/sec ]
                 10.   0.
tracer_diff2: TNU2          [m^2/sec]
                 10.  0.
```



Toward a more turbulent gyre

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{- \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \cancel{\underbrace{\frac{J(P,h)}{\rho_0}}_{\text{bot. pres. torque}}} + \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$

$$+ \boxed{\underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

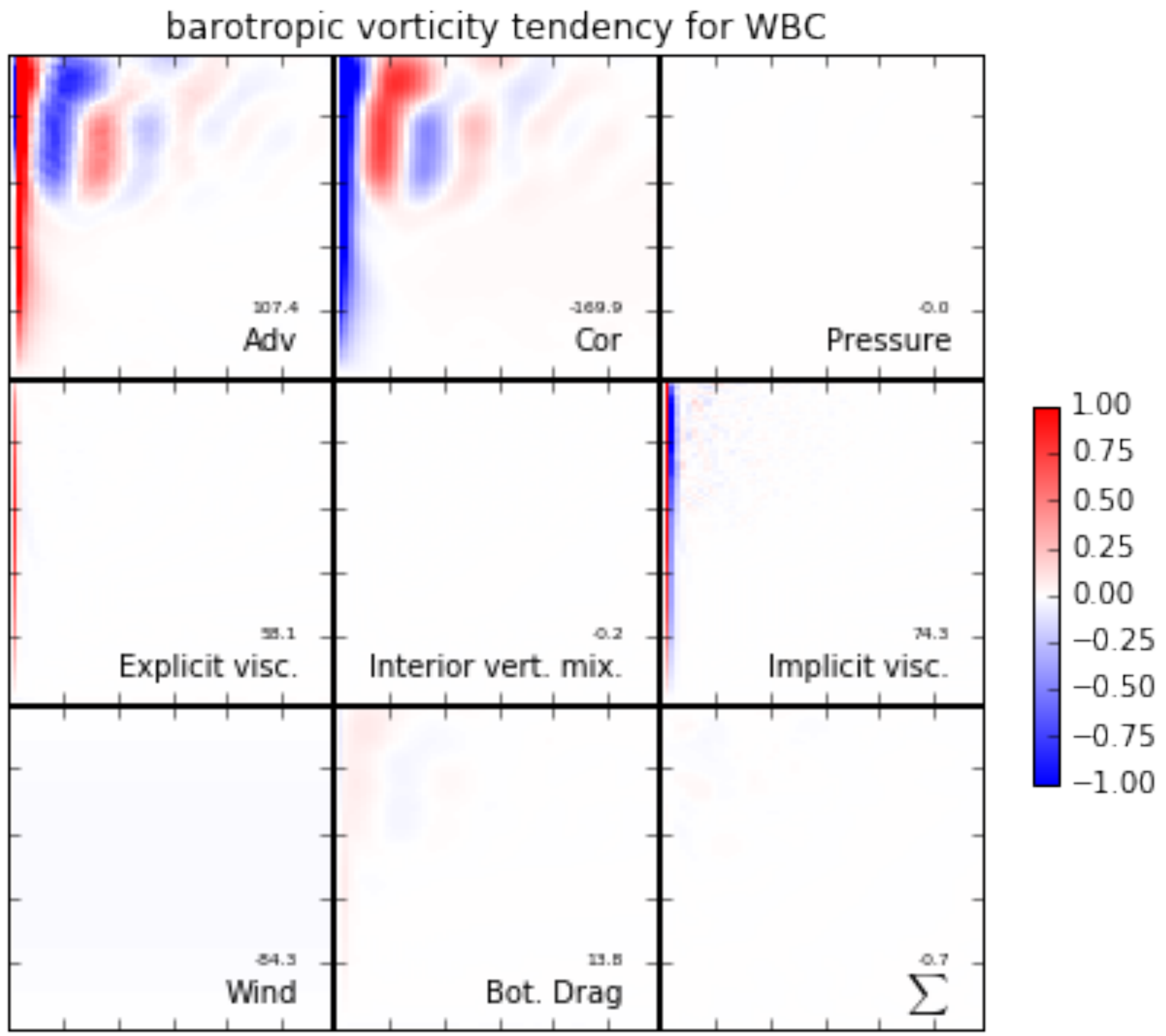
- more resolution and less viscosity:

- **param.h**

```
parameter (LLm0=240,   MMm0=200,   N=20)
```

- **Croco.in**

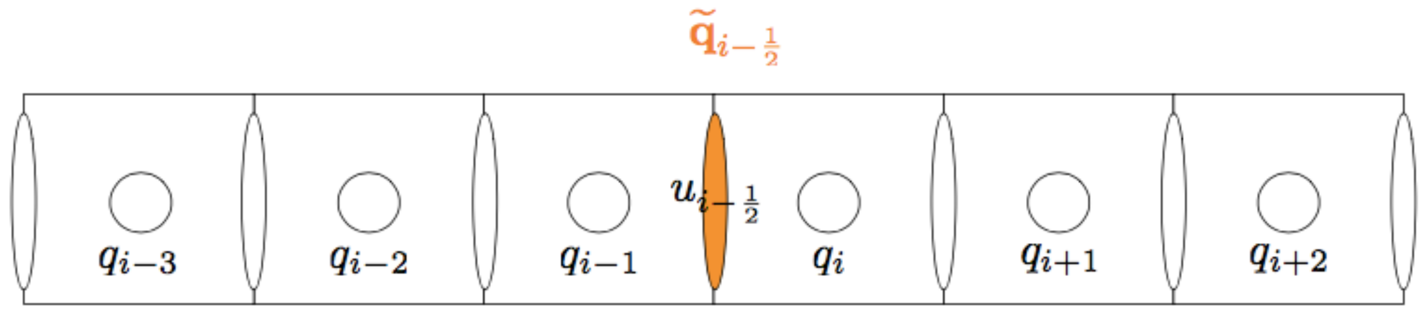
```
lateral_visc:   VISC2    [m^2/sec ]
                 10.  0.
tracer_diff2: TNU2          [m^2/sec]
                 10. 0.
```



DIfferent choices for advection/viscosity

- **The horizontal advective scheme:**

C2 = 2nd-order centered advection scheme
UP3 = 3rd-order upstream-biased advection scheme
C4 = 4th-order centered advection scheme
UP5 = 5th-order upstream-biased advection scheme
C6 = 6th-order centered advection scheme



$$\partial_x(uq)|_{x=x_i} = \frac{1}{\Delta x_i}\left\{u_{i+1/2}\widetilde{q}_{i+1/2} - u_{i-1/2}\widetilde{q}_{i-1/2}\right\}$$

$$\widetilde{q}^{C2}_{i-1/2} = \frac{q_i + q_{i-1}}{2}$$

$$\widetilde{q}^{C4}_{i-1/2} = (7/6)\widetilde{q}^{C2}_{i-1/2} - (1/12)(q_{i+1} + q_{i-2})$$

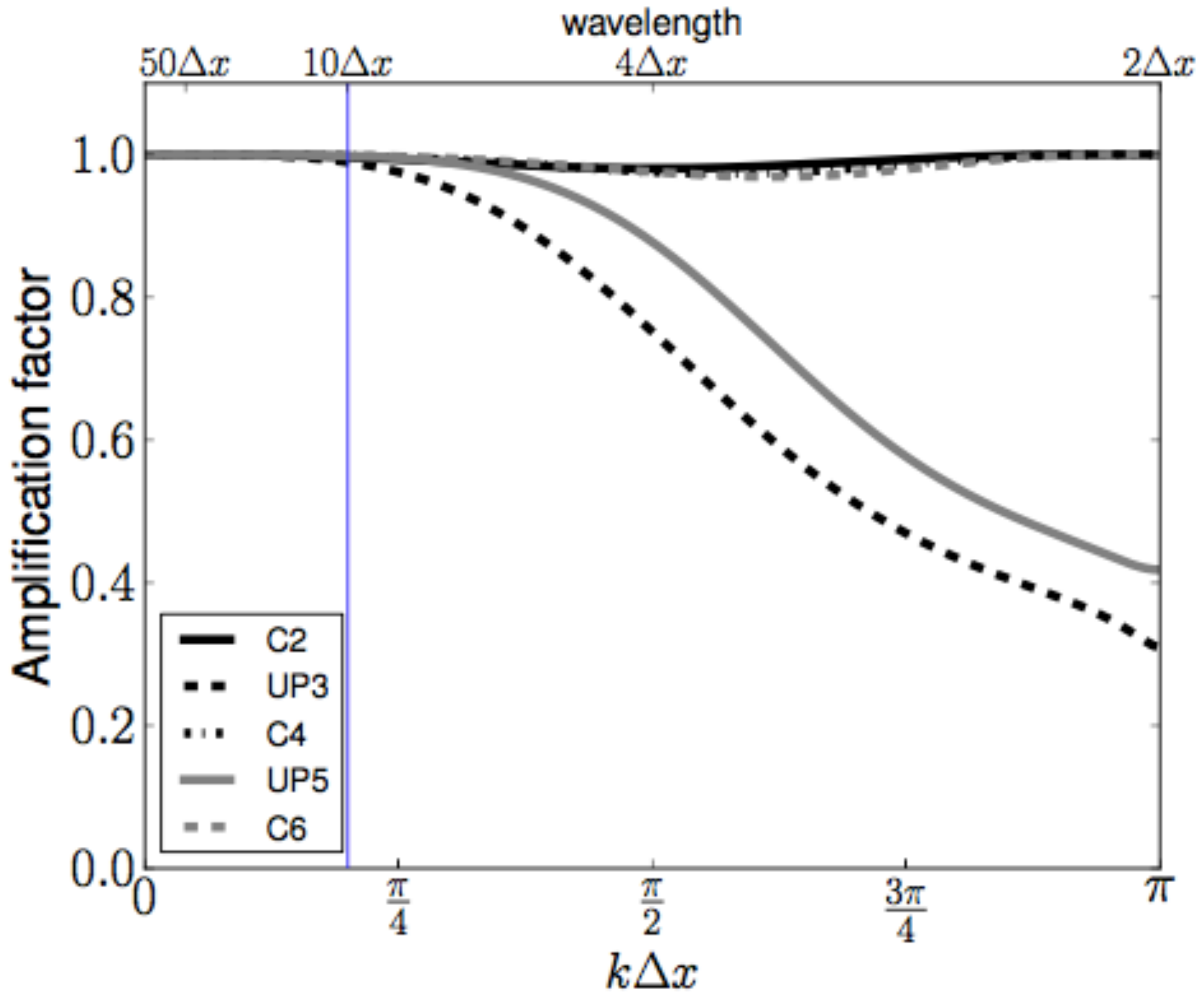
$$\widetilde{q}^{UP3}_{i-1/2} = \widetilde{q}^{C4}_{i-1/2} + \mathrm{sign}(1/12, u_{i-1/2})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{C6}_{i-1/2} = (8/5)\widetilde{q}^{C4}_{i-1/2} - (19/60)\widetilde{q}^{C2}_{i-1/2} + (1/60)(q_{i+2} + q_{i-3})$$

$$\widetilde{q}^{UP5}_{i-1/2} = \widetilde{q}^{C6}_{i-1/2} - \mathrm{sign}(1/60, u_{i-1/2})(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

- **The explicit viscosity:**

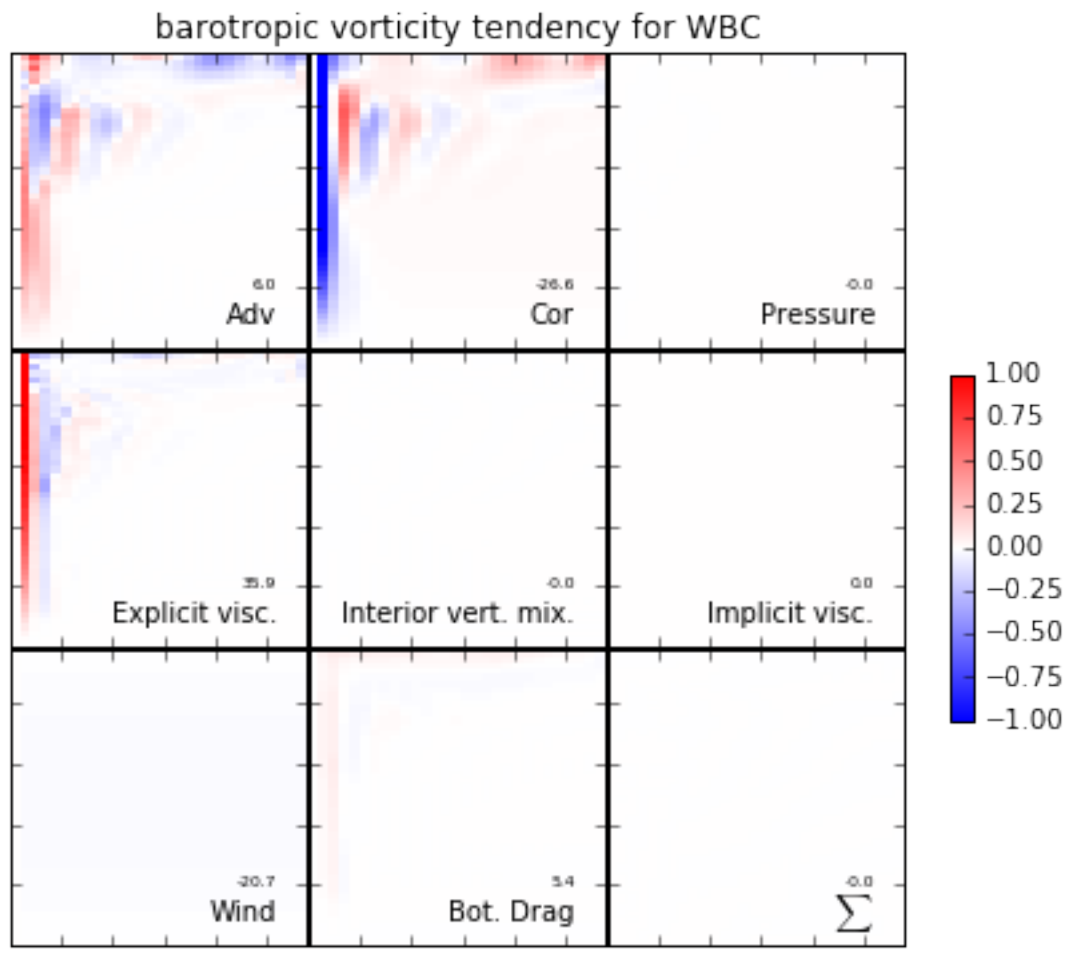
UV_VIS2 = Laplacian
UV_VIS4 = bilaplacian
UV_VIS_SMAGO = Smagorinsky parametrization of turbulent viscosity



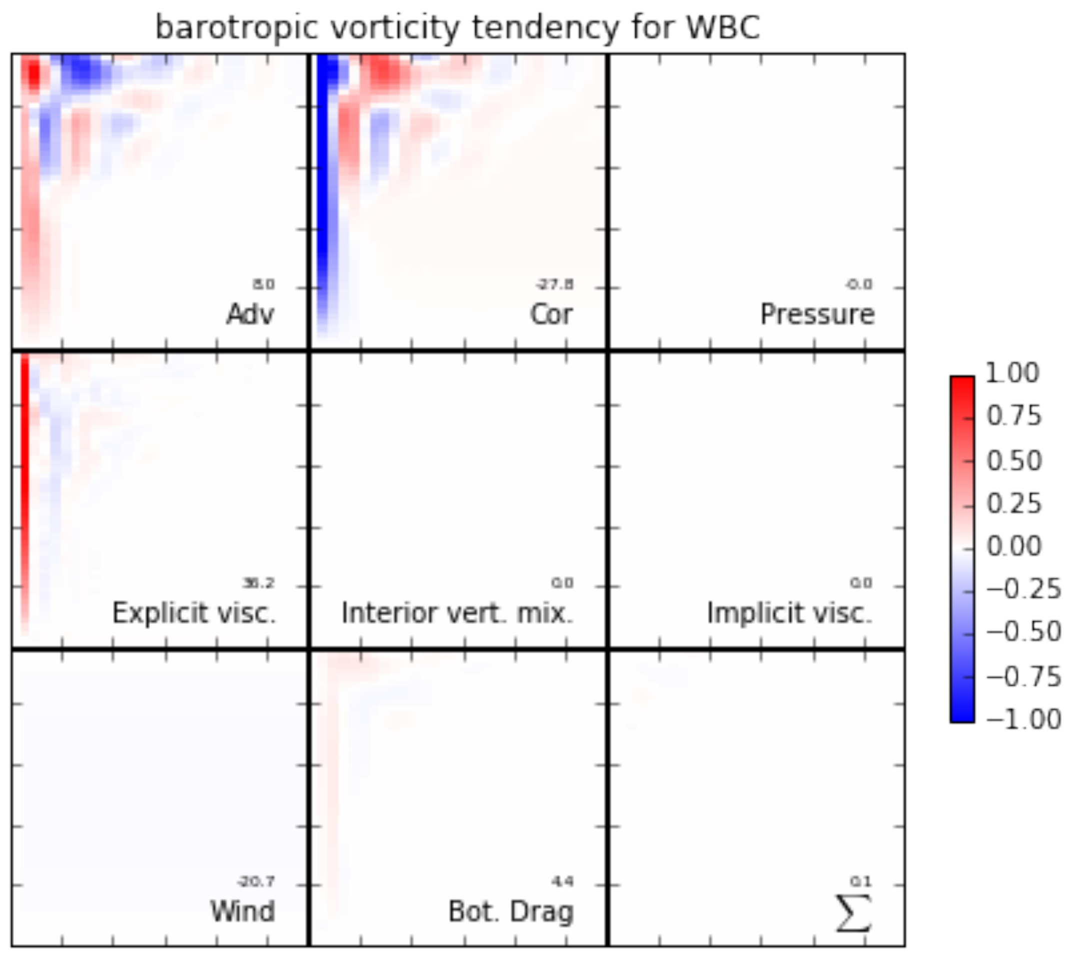
Implicit viscosity (UP3, UP5)
Or
Centered scheme (C2,C4,C6) + explicit viscosity

DIfferent choices for advection/viscosity

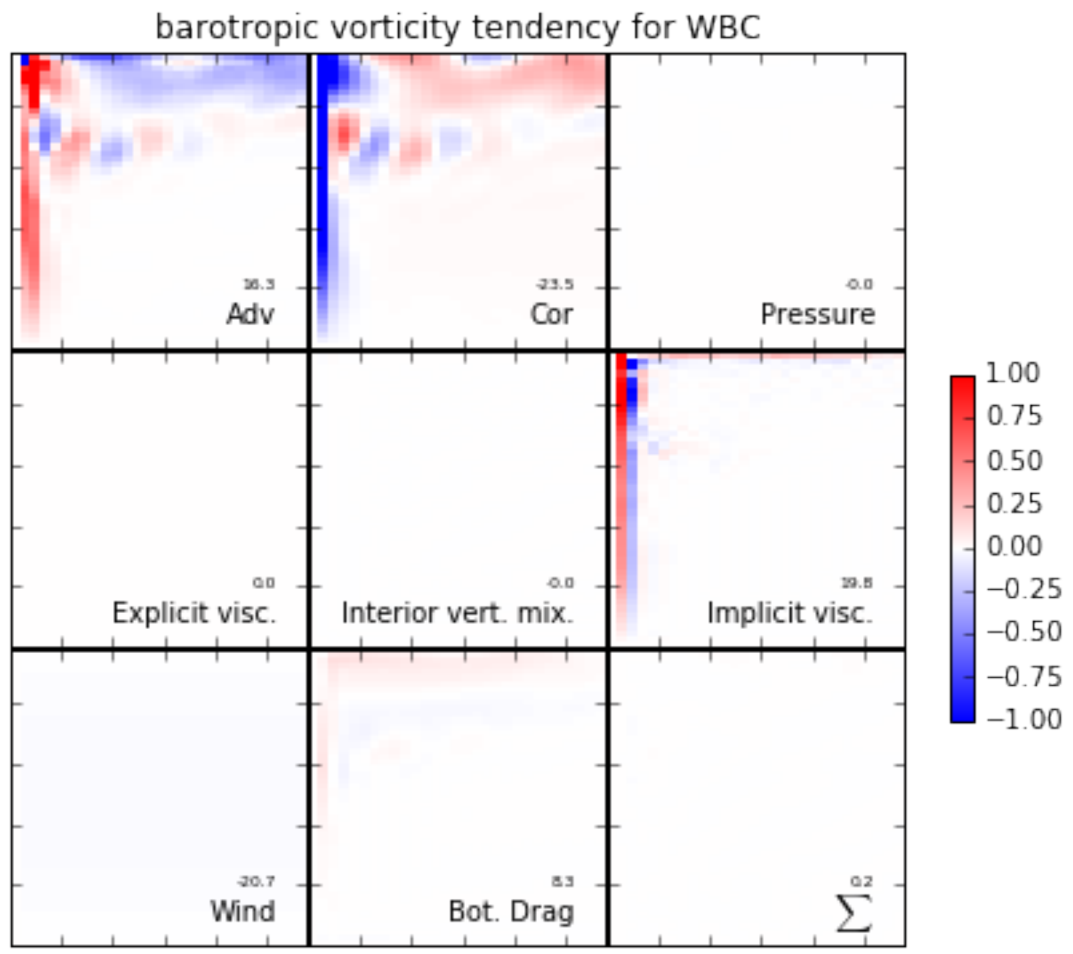
```
# define UV_HADV_C2
# define UV_VIS2
```



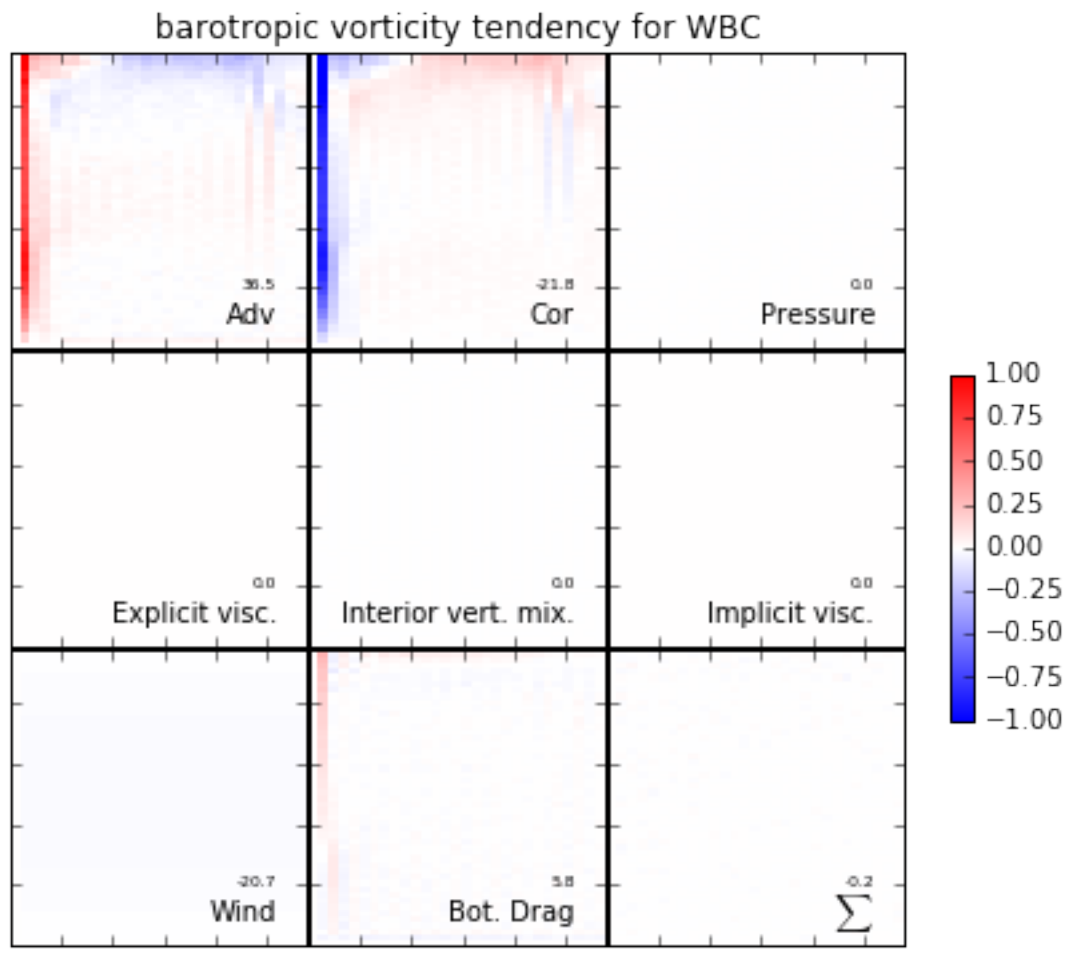

```
# define UV_HADV_C4
# define UV_VIS2
```



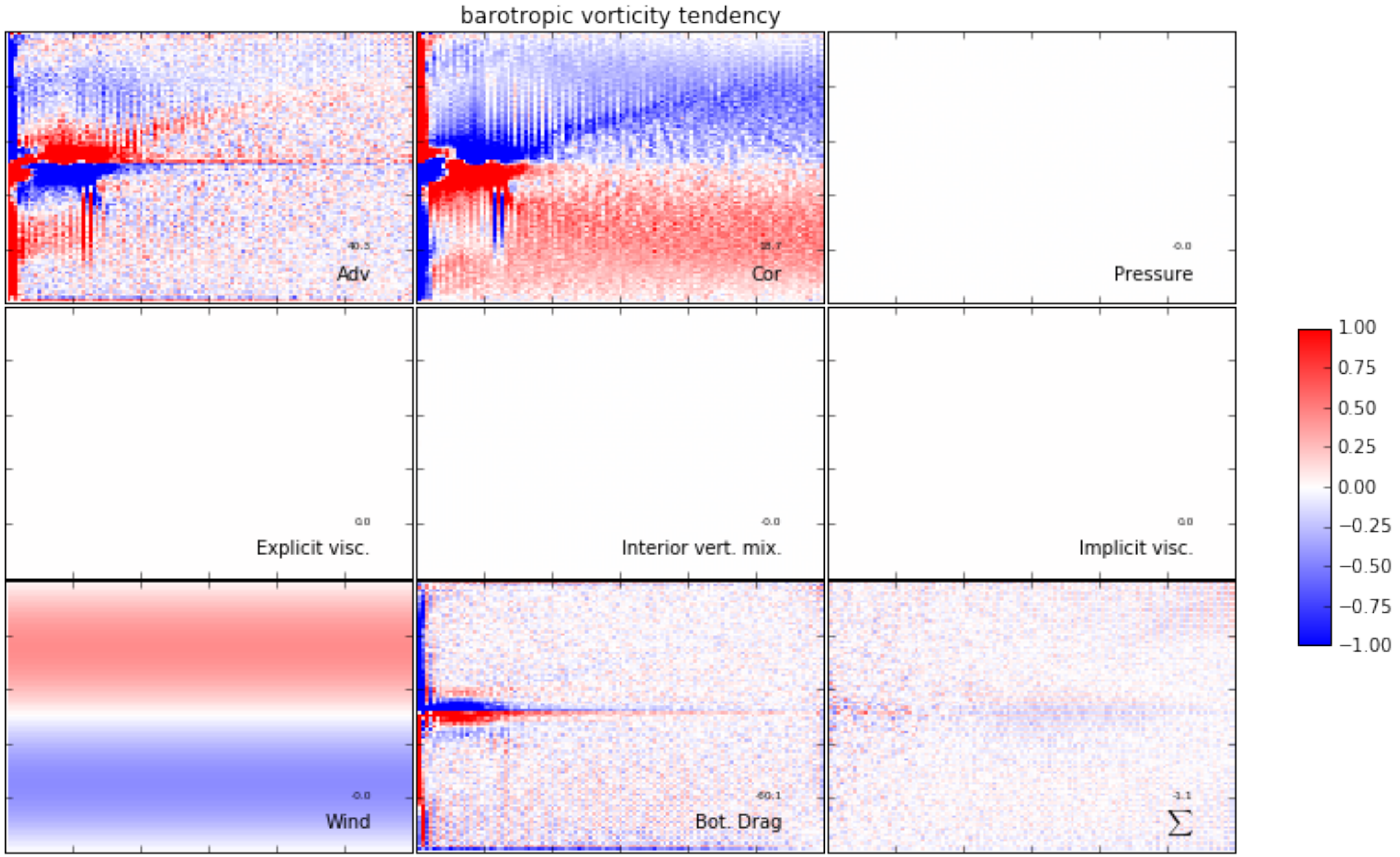

```
# define UV_HADV_UP3
# undef UV_VIS2
```




```
# define UV_HADV_C4
# undef UV_VIS2
```

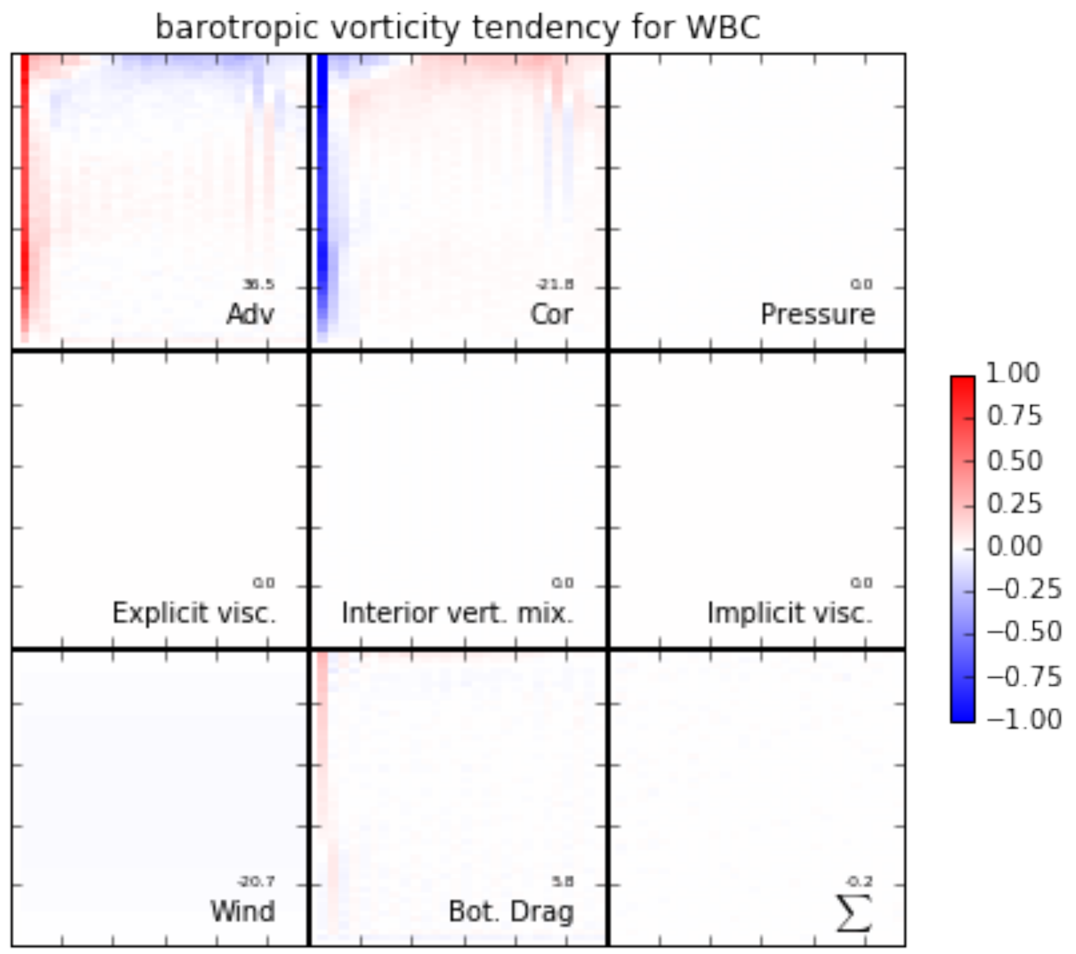


DIfferent choices for advection/viscosity

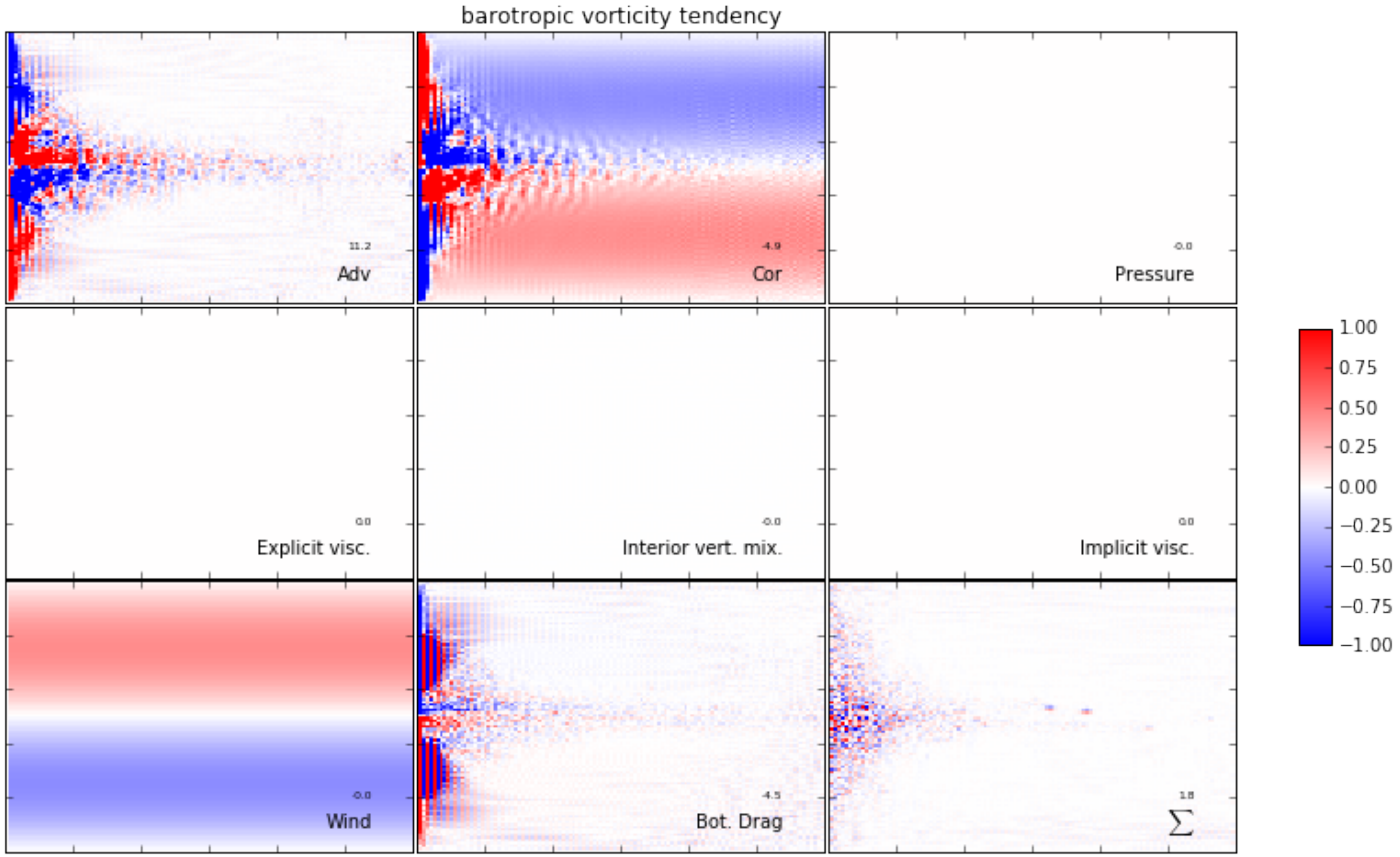


What happens
with no viscosity?

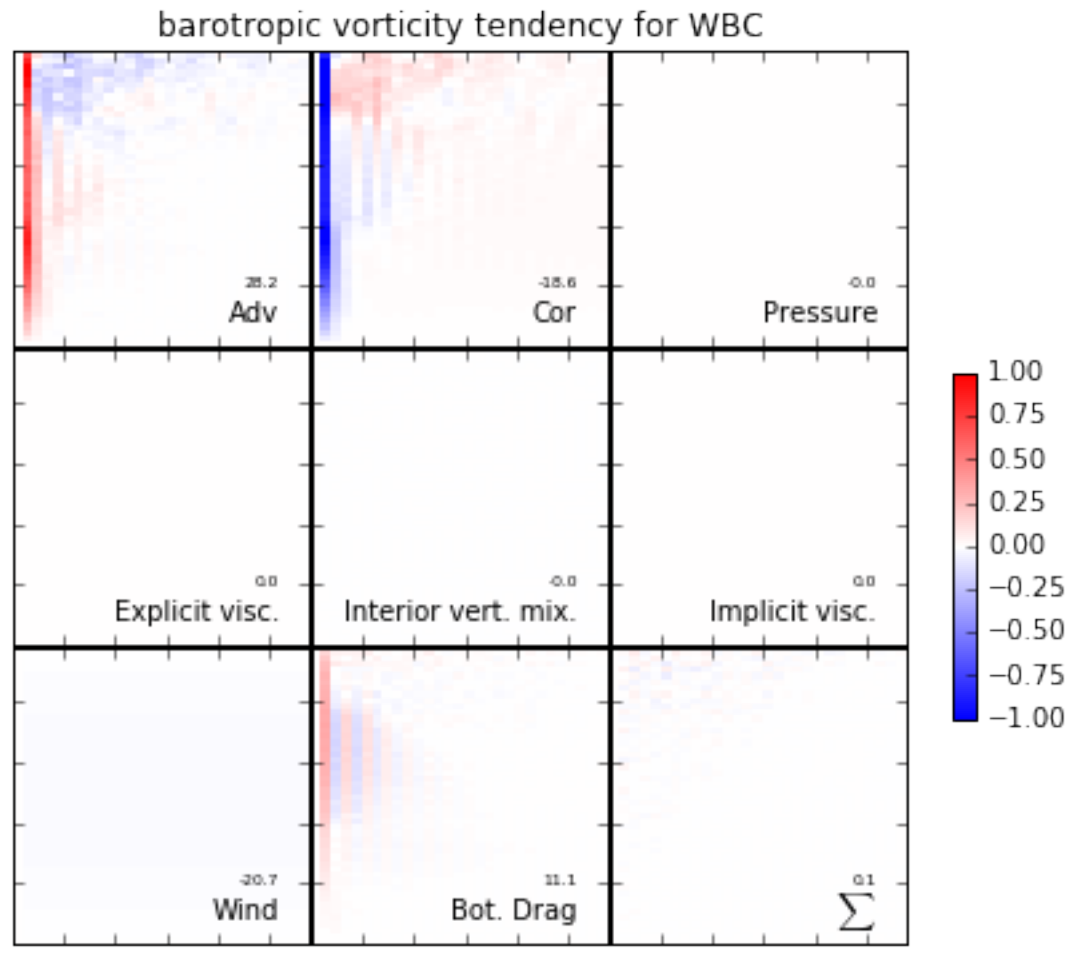
```
# define UV_HADV_C4
# undef UV_VIS2
```



DIfferent choices for advection/viscosity



No horizontal viscosity but more vertical mixing



Toward a more turbulent gyre

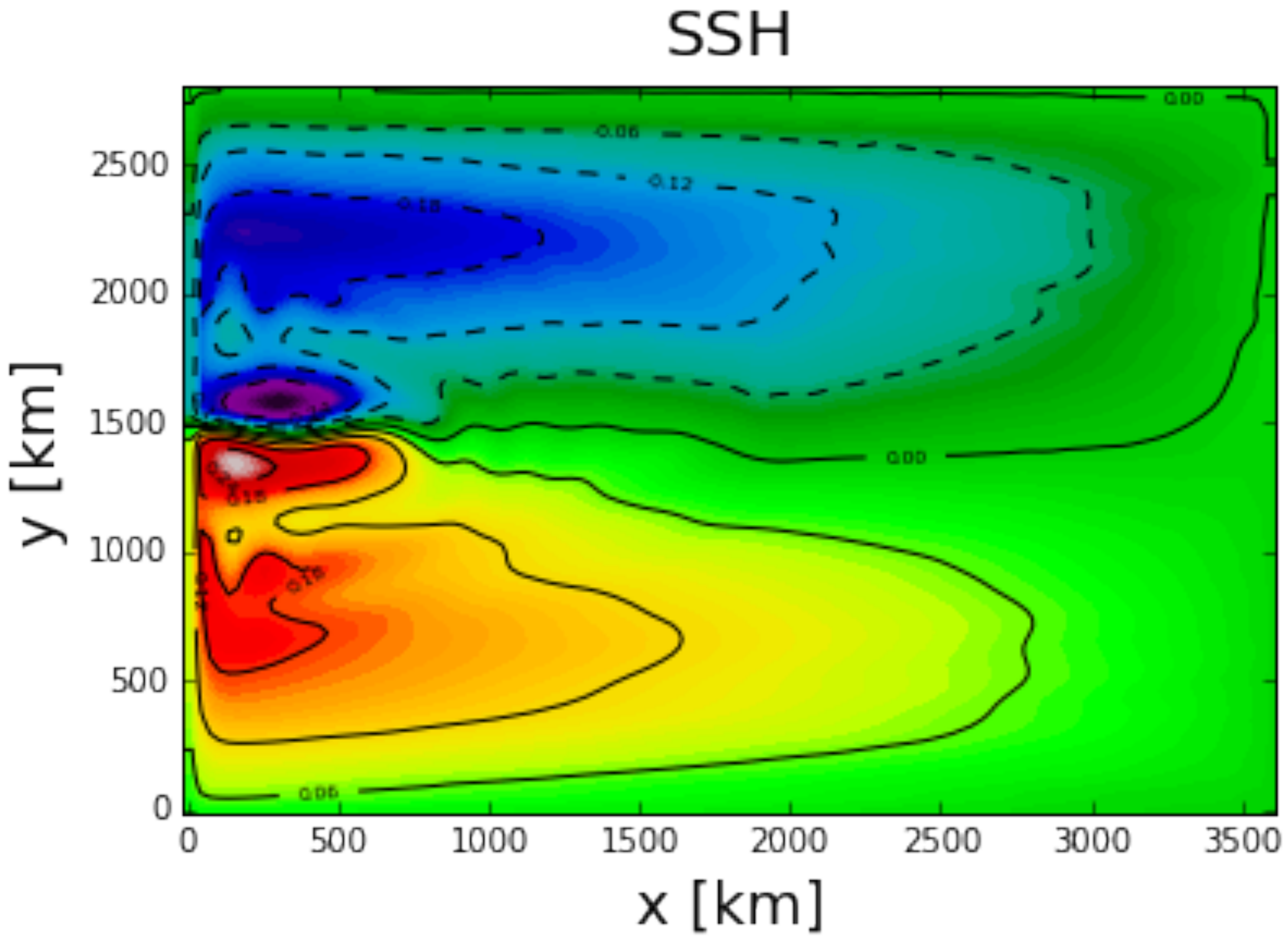
- **Standard choices:**

- **cppdefs.h** (implicit viscosity)

```
# define UV_HADV_UP3
# undef UV_VIS2
```

- **croco.in** (low drag, free-slip)

```
bottom_drag:     RDRG(m/s),      RDRG2, Zob [m],  Cdb_min, Cdb_max
                 3.e-4             0.       0.          0.       0.
gamma2:
                 1.
```



ROMS simulation after 200 years

Toward a more turbulent gyre

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{\underbrace{-\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\cancel{\frac{\mathbf{J}(P_b,h)}{\rho_0}}}_{\text{bot. pres. torque}} \boxed{\underbrace{+\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} \underbrace{-\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

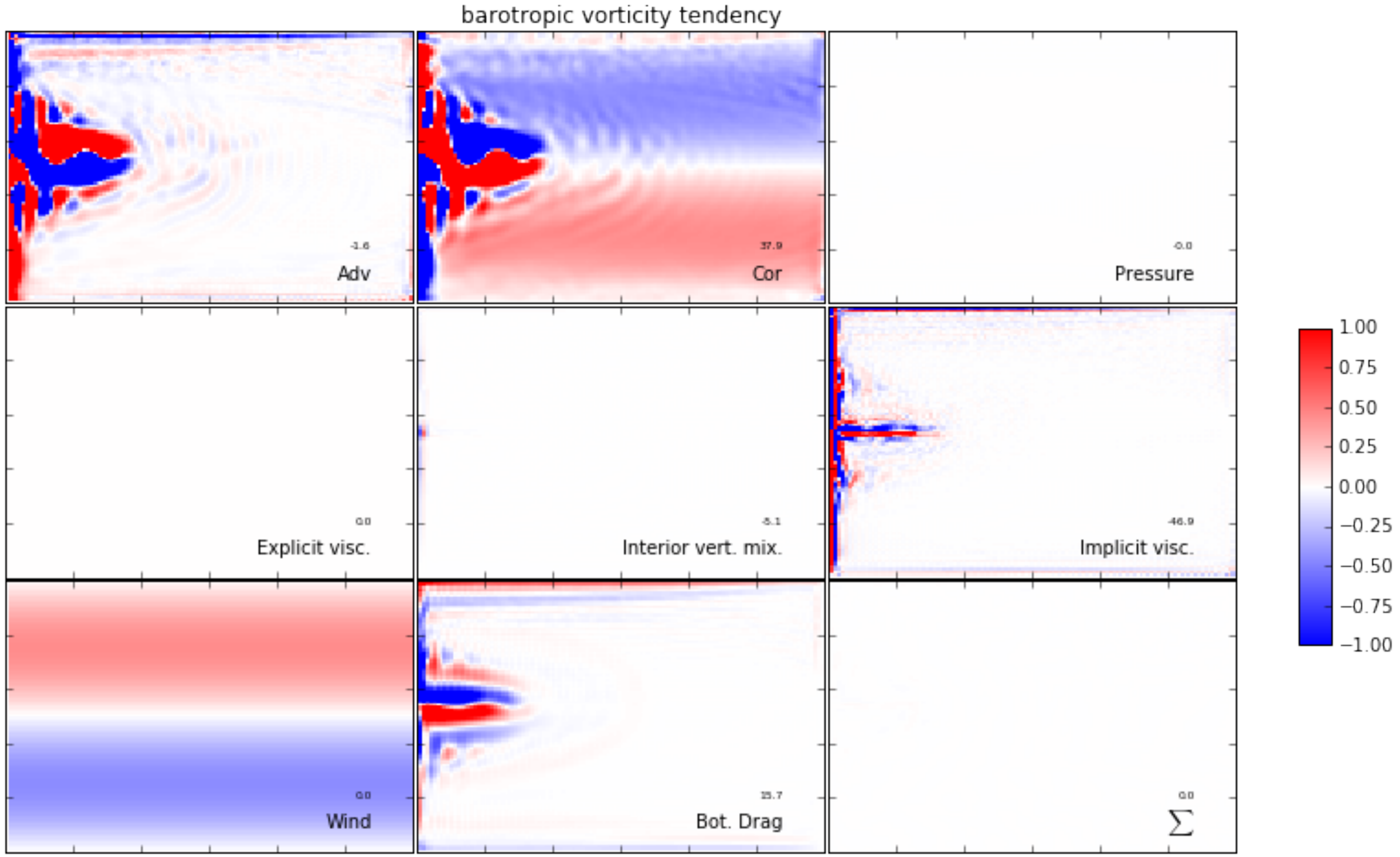
- **Standard choices:**

- **cppdefs.h** (implicit viscosity)

```
# define UV_HADV_UP3
# undef UV_VIS2
```

- **croco.in** (low drag, free-slip)

```
bottom_drag:     RDRG(m/s),      RDRG2, Zob [m],  Cdb_min, Cdb_max
                 3.e-4              0.      0.         0.      0.
gamma2:
                 1.
```



Toward a more turbulent gyre

$$\frac{1}{2}\frac{\partial u_i^2}{\partial t} + u_j \frac{\partial \frac{1}{2} u_i^2}{\partial x_j} + w \frac{\partial \frac{1}{2} u_i^2}{\partial z} = -\frac{u_i}{\rho_0}\frac{\partial P}{\partial x_i} + \mathcal{V}_i u_i + \mathcal{D}_i u_i + \mathcal{S}_i u_i$$

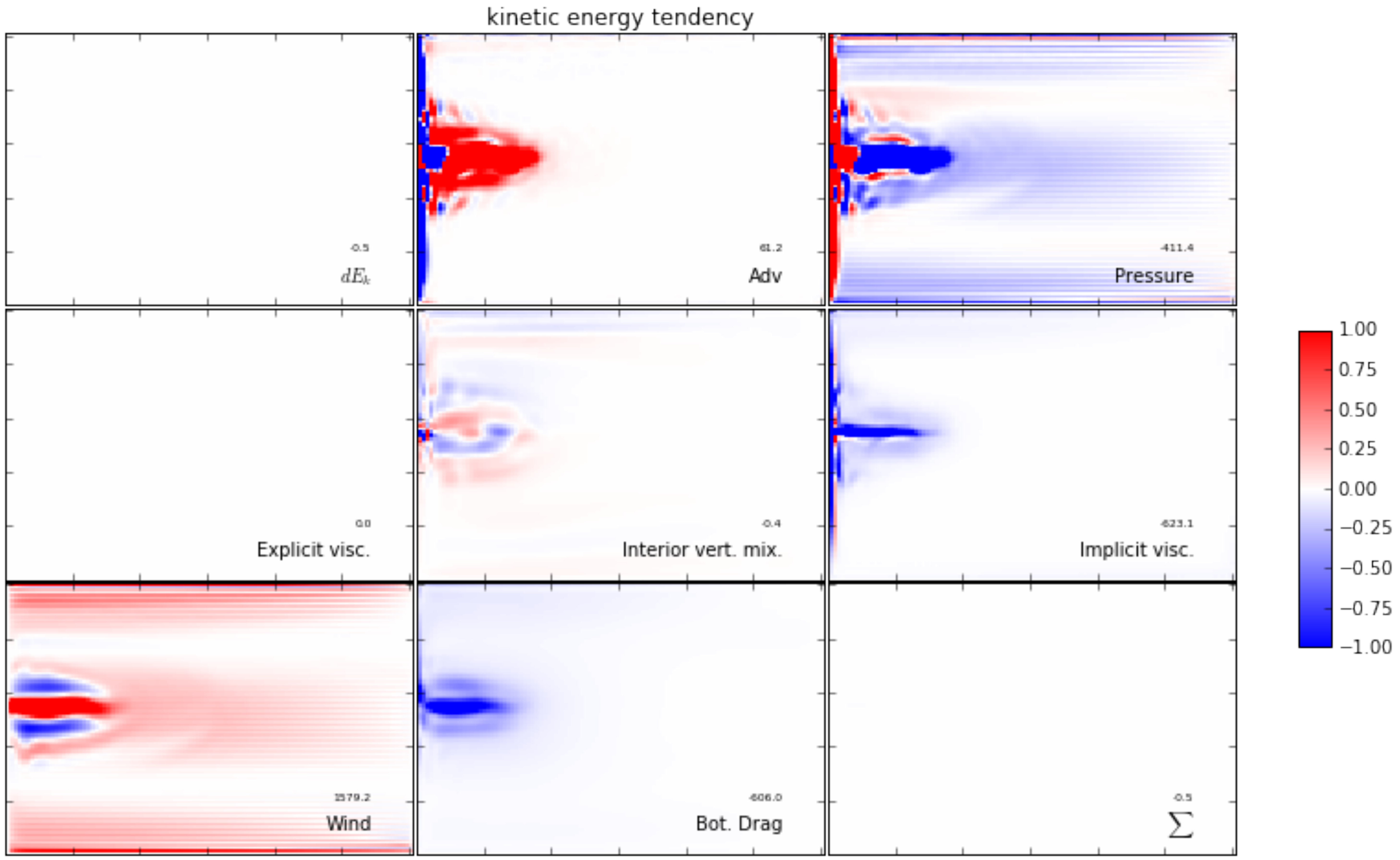
- **Standard choices:**

- **cppdefs.h** (implicit viscosity)

```
# define UV_HADV_UP3
# undef UV_VIS2
```

- **croco.in** (low drag, free-slip)

```
bottom_drag:     RDRG(m/s),      RDRG2, Zob [m],   Cdb_min, Cdb_max
                 3.e-4             0.       0.          0.       0.
gamma2:
                 1.
```



Leaving flatland!

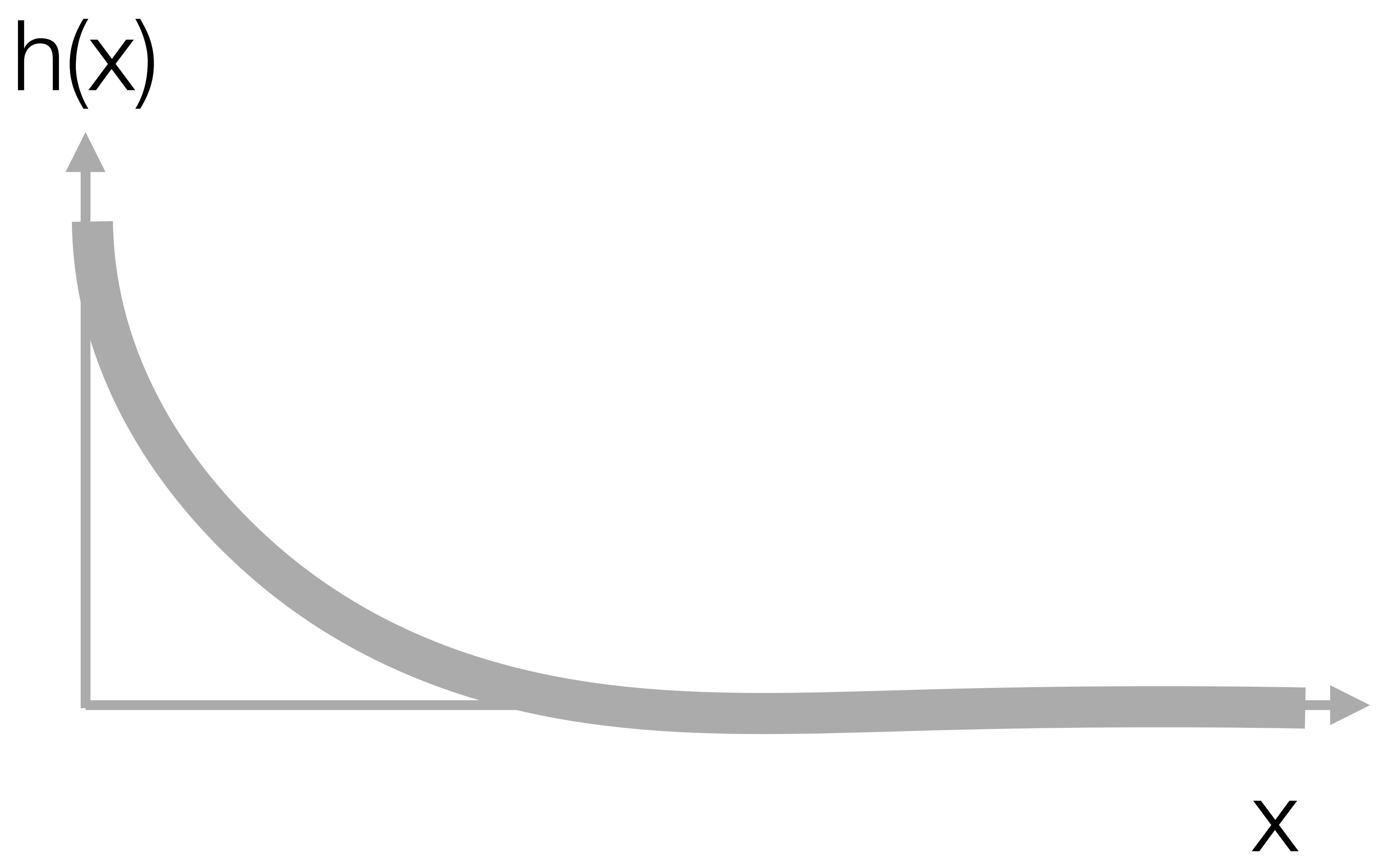
- The ocean bottom is not flat.
- What happens if we had some topography?

Fig. 6. Heinrich Caesar Berann, 'Northern Atlantic Ocean', 1968. Based on the Heezen–Tharp maps, which by the 1960s came to include all ocean basins, Berann's paintings brought the Heezen–Tharp physiographic diagram to worldwide audiences. Versions of Berann's maps were featured in *National Geographic*; his world oceans depiction became one of the most recognizable maps of the twentieth century. Lateral ridge-bisecting faults (known as transform faults) are depicted here although not in the 1957 map, their existence and extent not yet then discerned. Courtesy http://www.berann.com, reproduced with permission.

Leaving flatland!

- **ana_grid.F** (add topo)

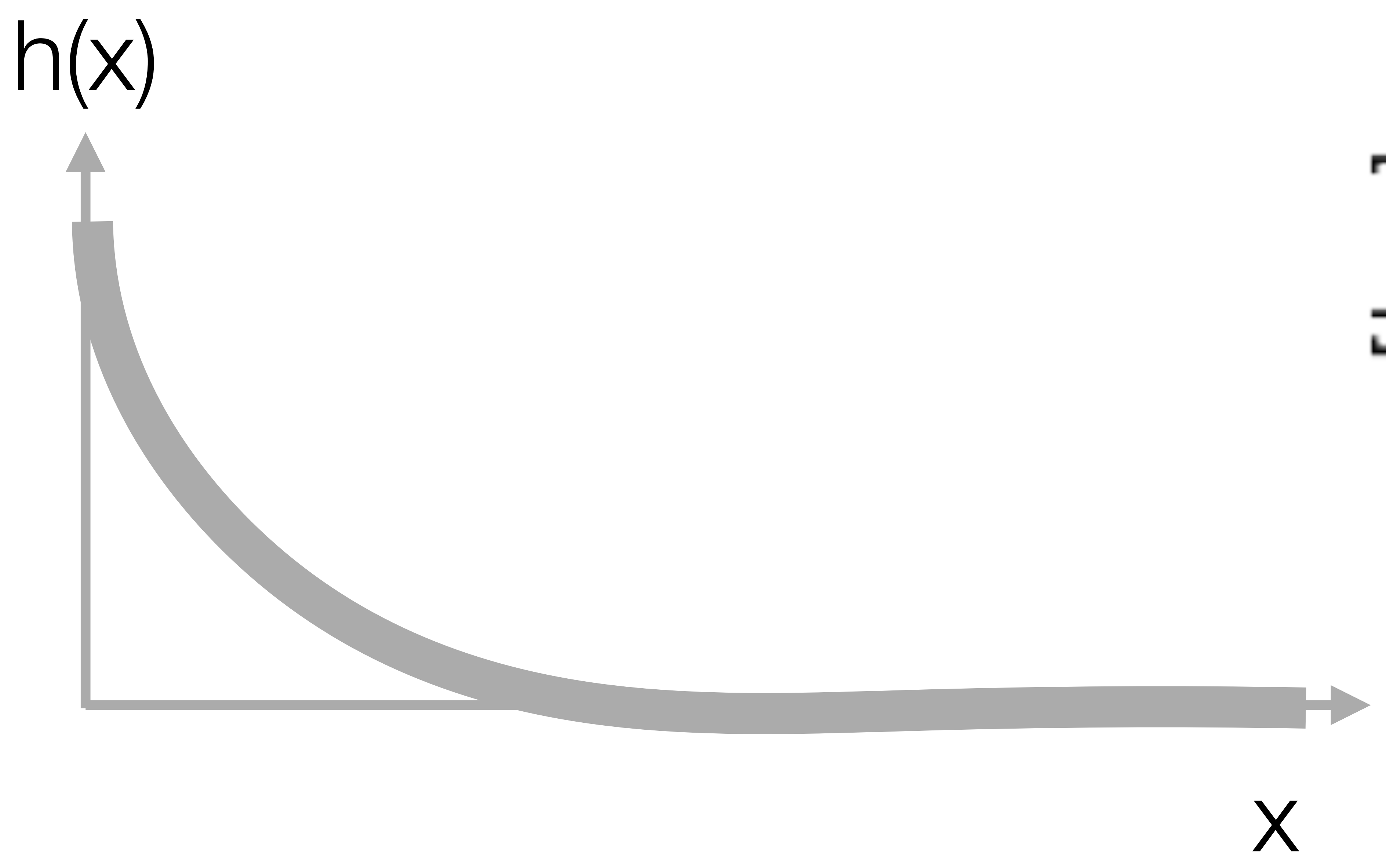
```
h(i,j)=depth * ( 1. - exp(-10.*(xr(i,j)/xl)))
```

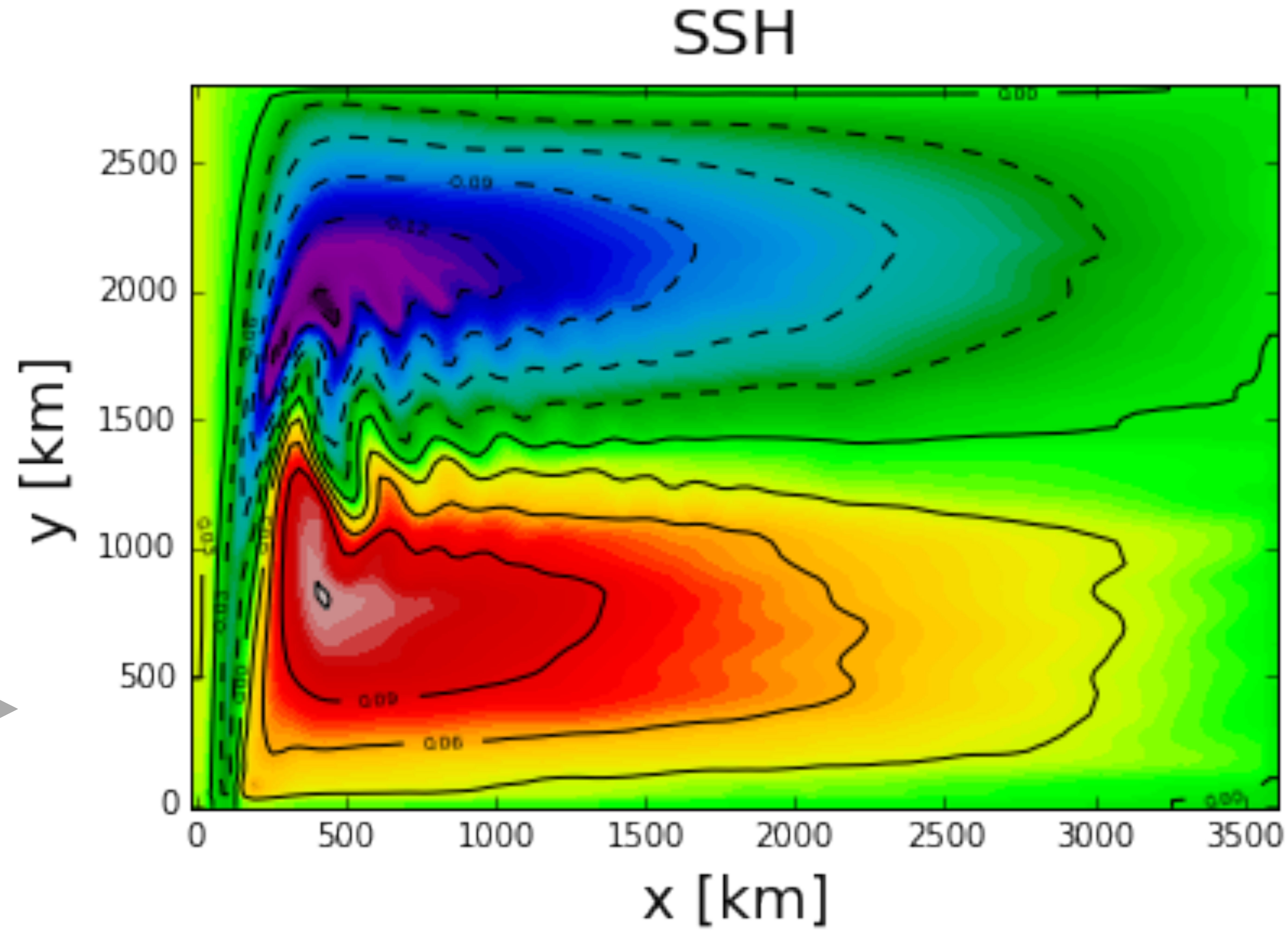


Leaving flatland!

- **ana_grid.F** (add topo)

```
h(i,j)=depth * ( 1. - exp(-10.*(xr(i,j)/xl)))
```



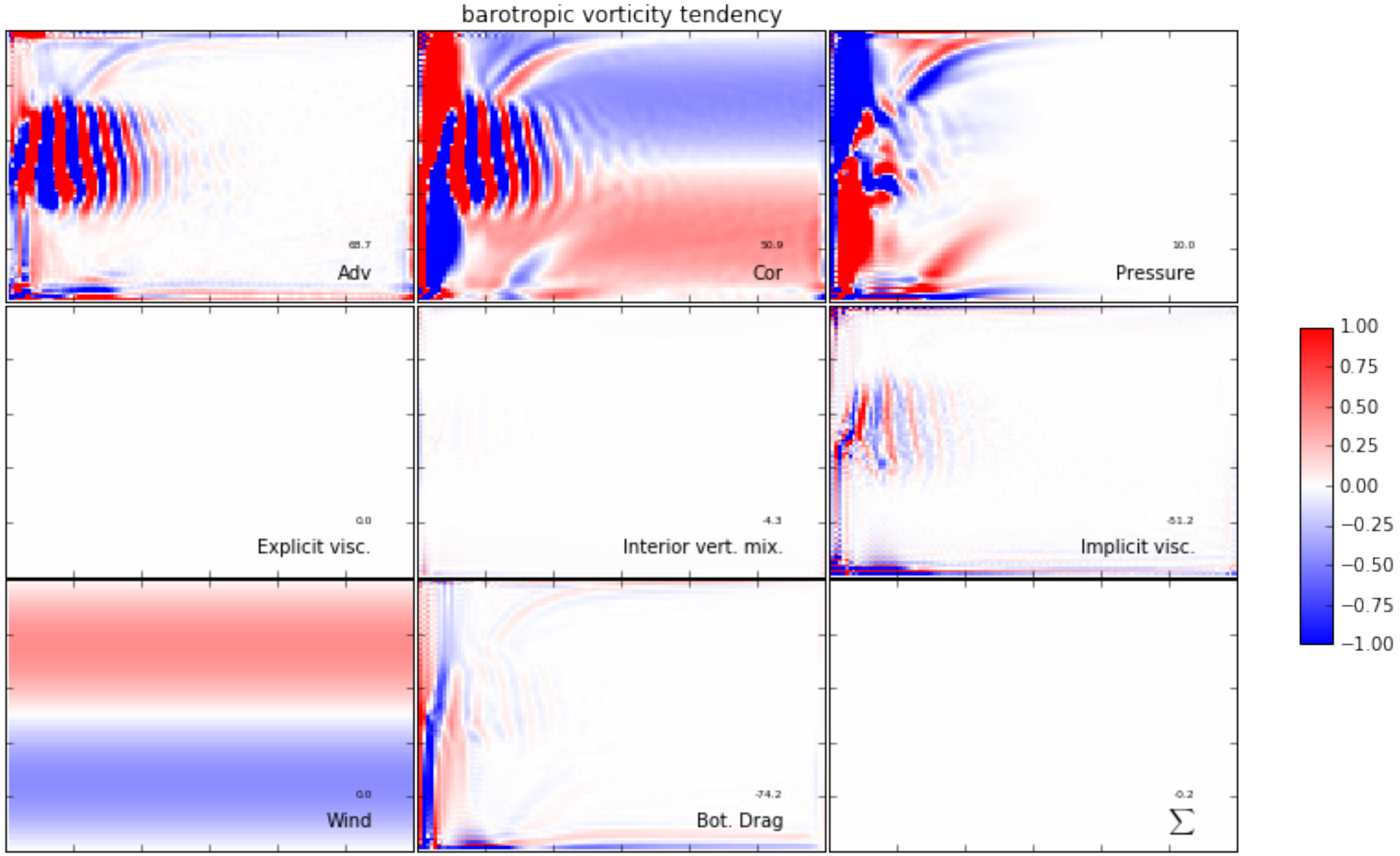


ROMS simulation after 500 years

Leaving flatland!

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{\underbrace{-\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}} \boxed{\underbrace{+\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} \underbrace{-\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

```
ana_grid.F
h(i,j)=depth * ( 1. - exp(-10.*(xr(i,j)/xl)))
```



Toward a more turbulent gyre

- The western boundary current balance is now **inviscid.** No need for viscous effects to close the gyre.

$$\underbrace{\frac{\partial\Omega}{\partial t}}_{\text{rate}} = \boxed{\underbrace{-\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \boxed{\underbrace{\frac{\mathbf{J}(P_b,h)}{\rho_0}}_{\text{bot. pres. torque}}} + \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{\mathcal{A}_\Sigma}_{\text{NL advection}}$$

ELSEVIER
Ocean Modelling 2 (2000) 73–83
Ocean Modelling
www.elsevier.com/locate/omodol

A theoretical reason to expect inviscid western boundary currents in realistic oceans

Chris W. Hughes
CCMS-Proudman Oceanographic Laboratory, Bidston Observatory, Bidston Hill, Prenton, Merseyside CH43 7RA, UK

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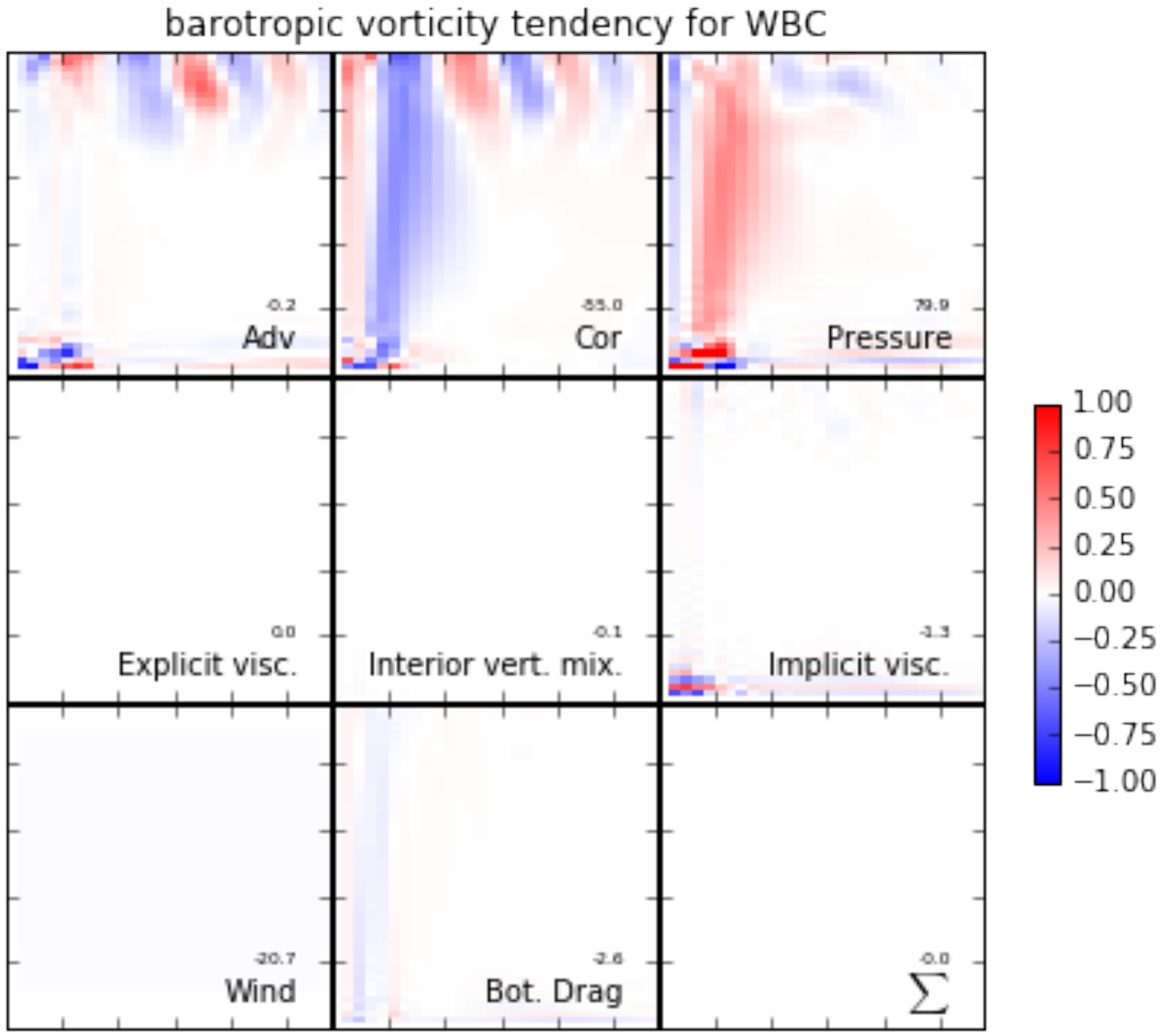
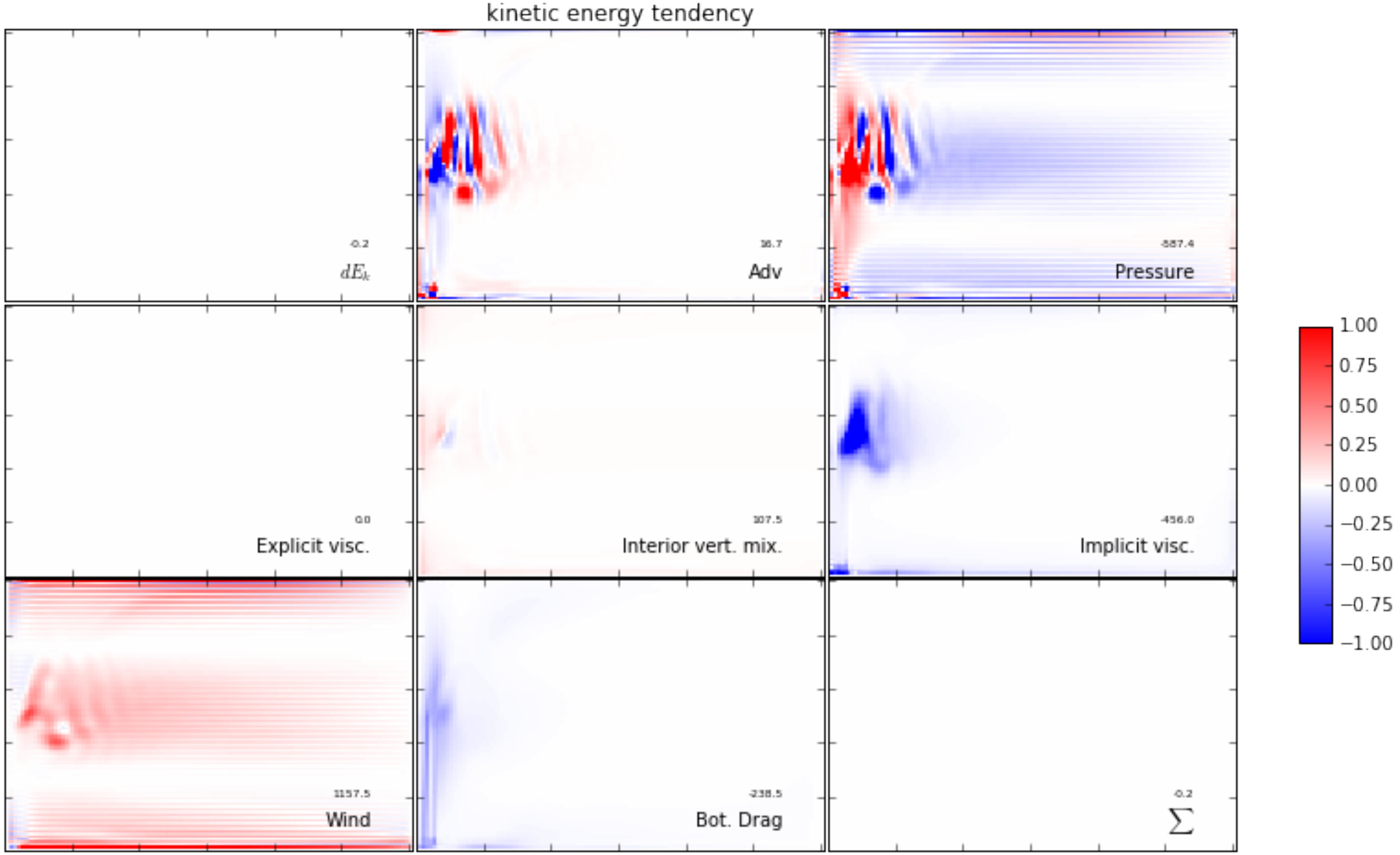


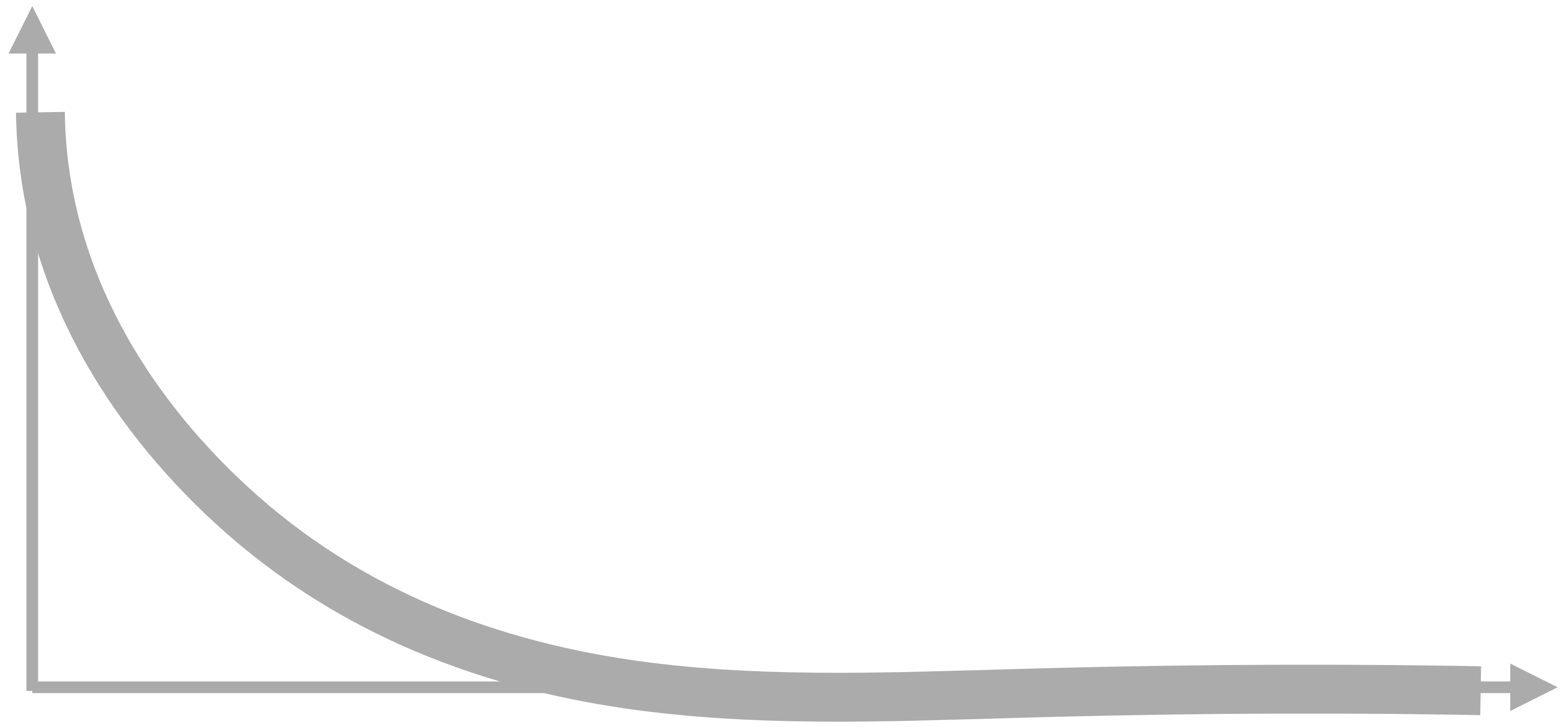
Leaving flatland!

$$\frac{1}{2}\frac{\partial u_i^2}{\partial t} + u_j \frac{\partial \frac{1}{2} u_i^2}{\partial x_j} + w \frac{\partial \frac{1}{2} u_i^2}{\partial z} = -\frac{u_i}{\rho_0}\frac{\partial P}{\partial x_i} + \mathcal{V}_i u_i + \mathcal{D}_i u_i + \mathcal{S}_i u_i$$

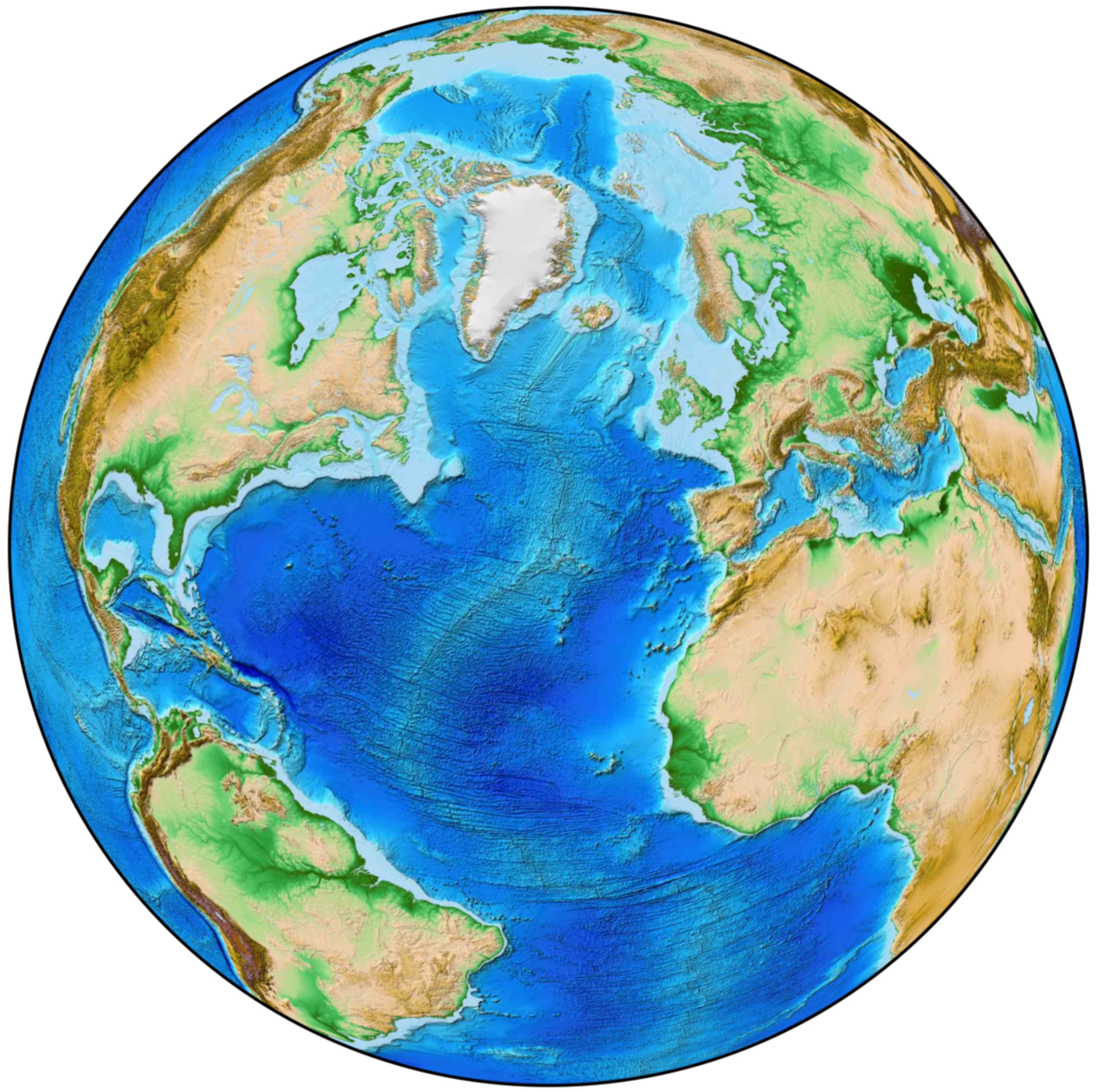


Realistic gyre

- Forcings and data
  - Bottom topography + Land mask
  - Atmospheric surface boundary forcing
  - Initial oceanic conditions
  - Lateral oceanic boundary conditions
- h(x)



- **Realistic topography (SRTM30)**



Realistic gyre

Satellite geodesy data + soundings

Available Datasets:

- SRTM30 (http://topex.ucsd.edu/WWW_html/srtm30_plus.html)

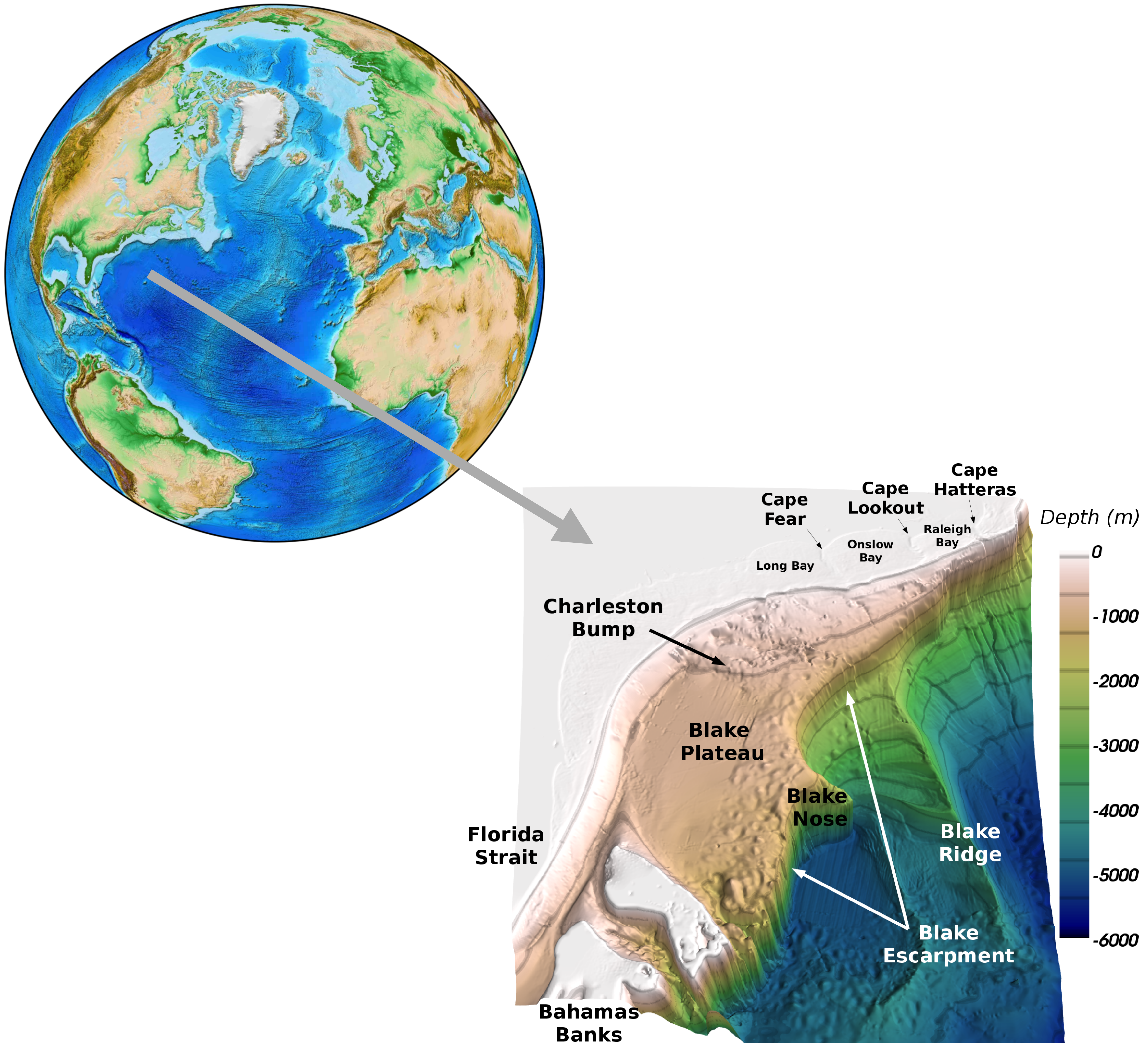
  Smith, W. H. F., and D. T. Sandwell, Global seafloor topography from satellite altimetry and ship depth soundings, Science, v. 277, p. 1957-1962, 26 Sept., 1997.

- Etopo1 (https://www.ngdc.noaa.gov/mgg/global/global.html)

  Amante, C. and B.W. Eakins, 2009. ETOPO1 1 Arc-Minute Global Relief Model: Procedures, Data Sources and Analysis. NOAA Technical Memorandum NESDIS NGDC-24. National Geophysical Data Center, NOAA. doi:10.7289/V5C8276M.

- Gebco30 (http://www.gebco.net/data_and_products/gridded_bathymetry_data/)

  British Oceanographic Data Centre (BODC), 2008, The GEBCO_08 Grid, version 20091120, General Bathymetric Chart of the Oceans (GEBCO)



Realistic gyre

- Forcings and data
  - Bottom topography + Land mask
  - Atmospheric surface boundary forcing
  - Initial oceanic conditions
  - Lateral oceanic boundary conditions

- **Realistic topography (SRTM30)**
- **Climatological fluxes/wind stress**

Realistic gyre

- Surface boundary conditions:

- $$\frac{\partial \eta}{\partial t} = w$$

$$K_{Mv}\frac{\partial u}{\partial z} = \frac{\tau_x}{\rho_0}$$

$$K_{Mv}\frac{\partial v}{\partial z} = \frac{\tau_y}{\rho_0}$$

Wind stress

$$K_{Tv}\frac{\partial T}{\partial z} = \frac{Q}{\rho_0 C_p}$$

Heat flux

$$K_{Sv}\frac{\partial S}{\partial z} = \frac{S(E-P)}{\rho_0}$$

Salt flux : evap - rain

Heat fluxes & Freshwater fluxes:

- Directly read the forcing files
- Or use of a bulk formulae :
  - Heat flux : compute total heat flux from latent, sensible, solar and longwave fluxes and model SST
  - Freshwater flux : compute from evap, prate and model SSS

Wind stress:

- Directly read the forcing files
- Or compute the windstress from the Cd drag coefficient, model SST and wind (use of bulk formulae)

+ Rivers runoff

Available Datasets:

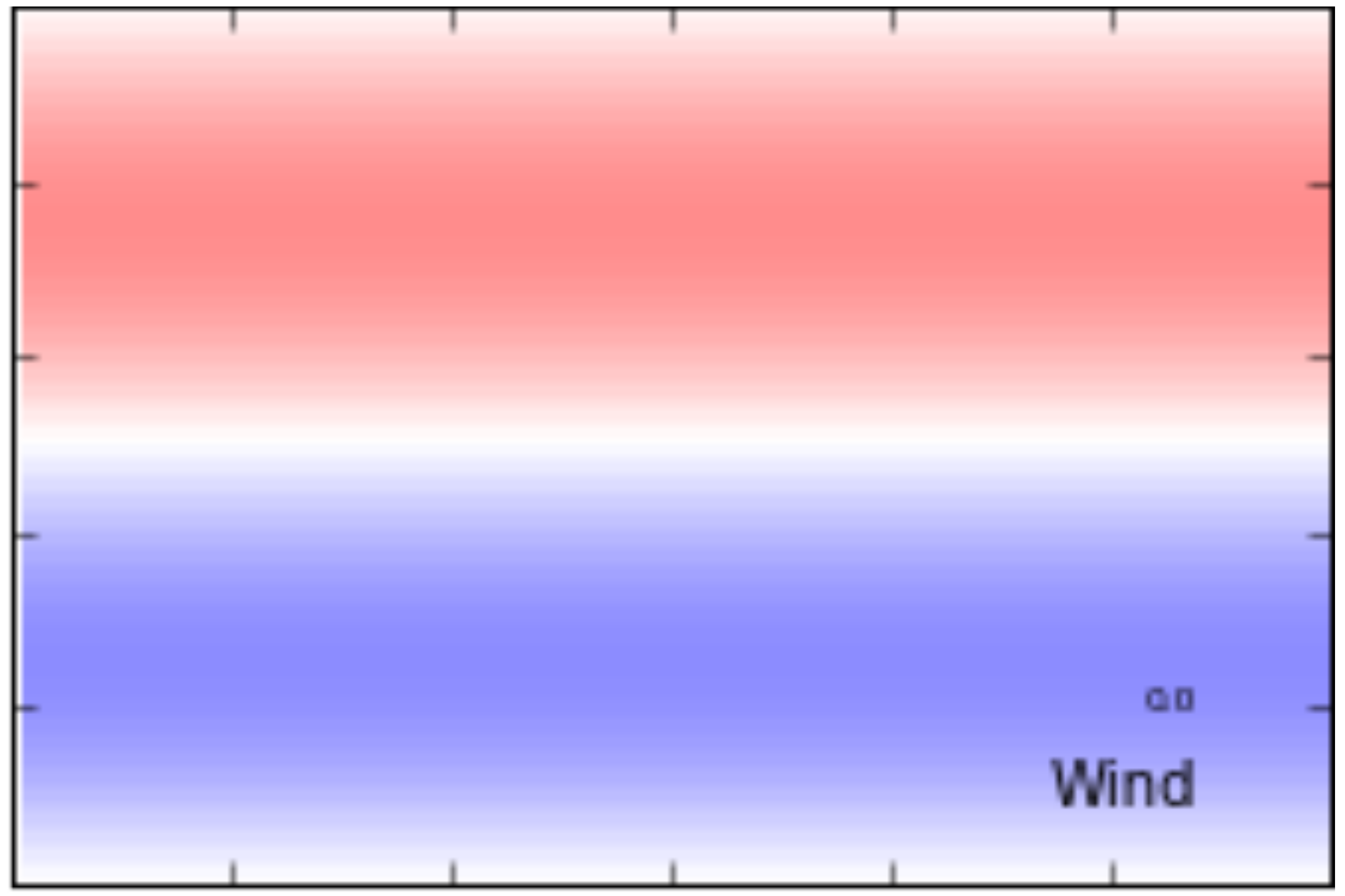
- Heat and freshwater fluxes : COADS, …
- Wind Stress: QuickScat, SCOW, …

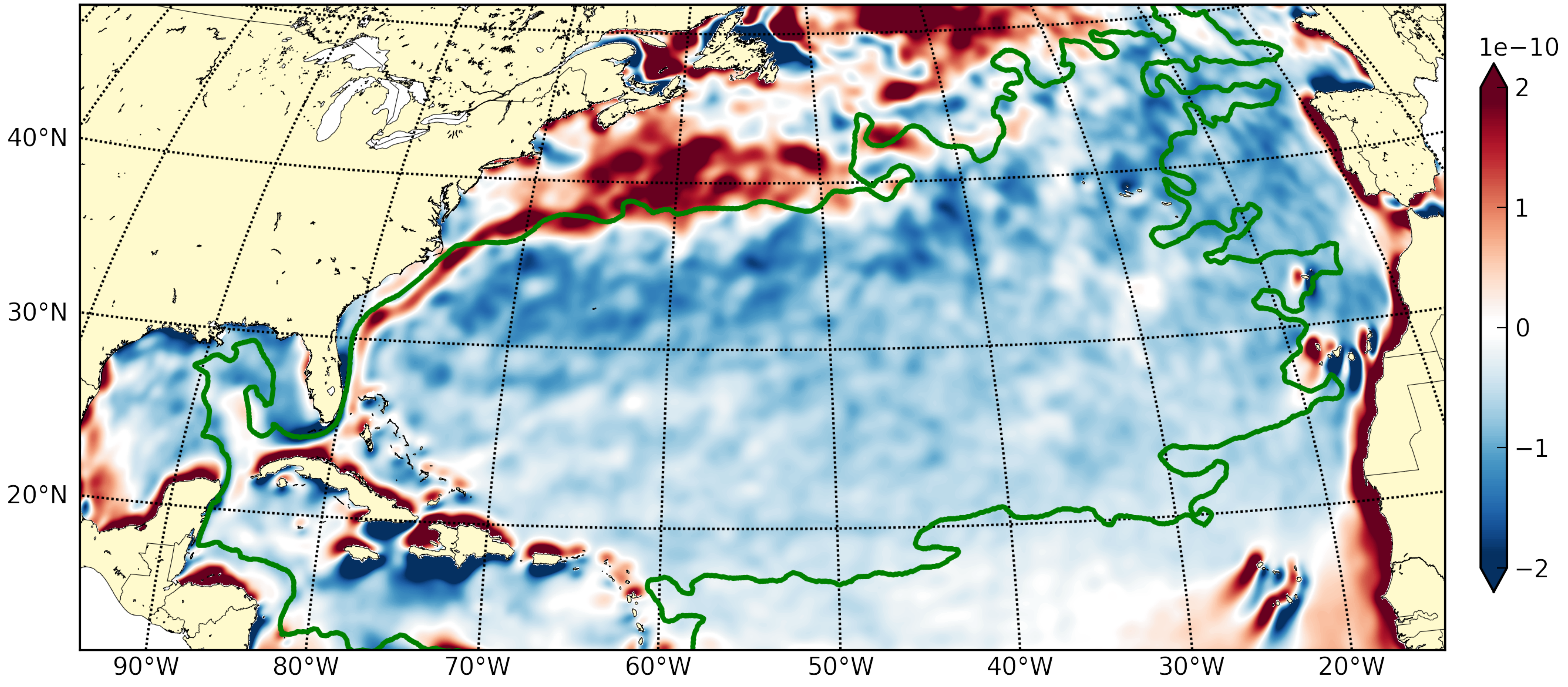
- Reanalysis (Model +obs.) : CFSR, ERA interim, etc.

Realistic gyre

- Forcings and data
  - Bottom topography + Land mask
  - Atmospheric surface boundary forcing
  - Initial oceanic conditions
  - Lateral oceanic boundary conditions

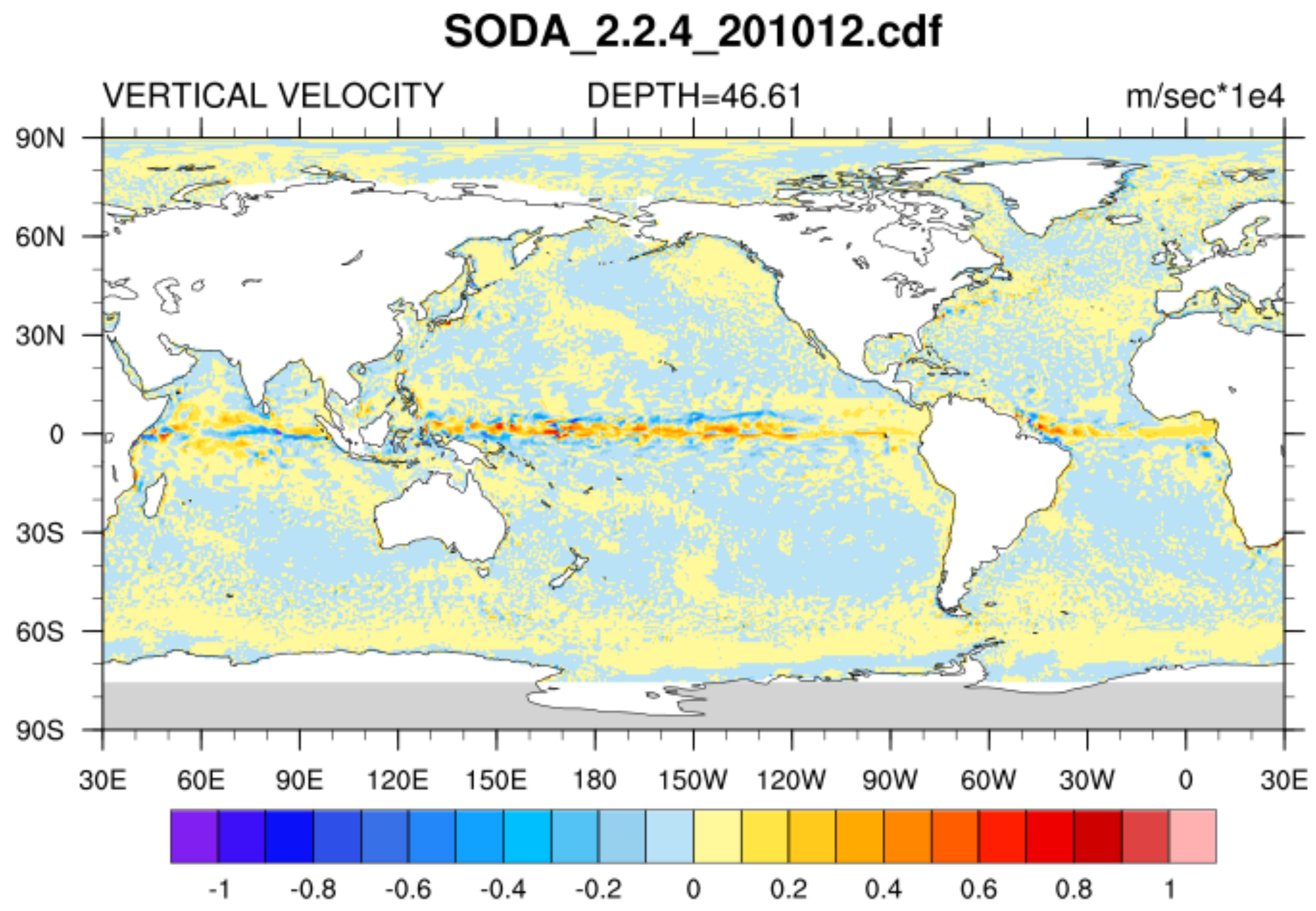
- **Realistic topography (SRTM30)**
- **Climatological fluxes/wind stress**





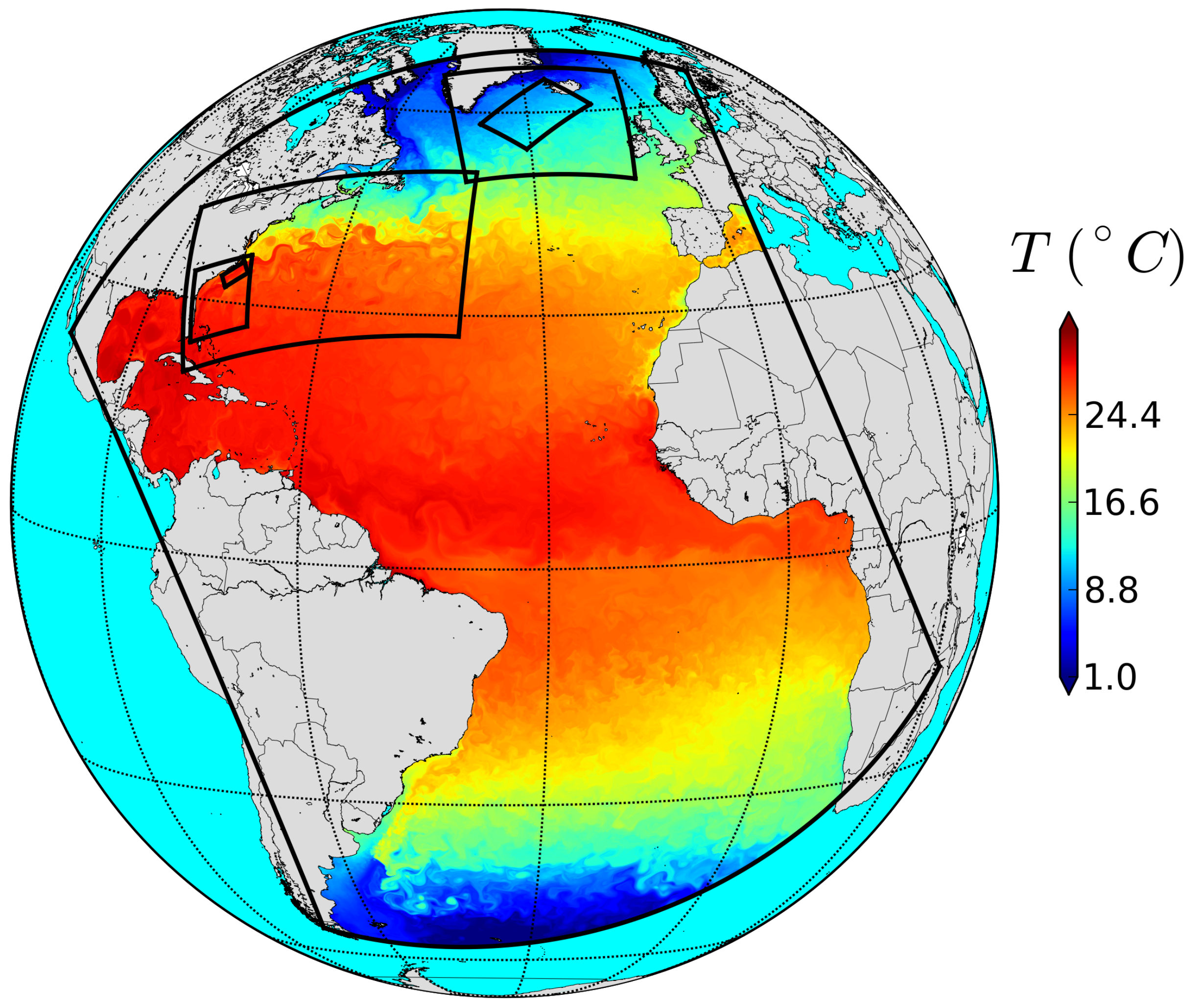
Realistic gyre

- Forcings and data
  - Bottom topography + Land mask
  - Atmospheric surface boundary forcing
  - Initial oceanic conditions
  - Lateral oceanic boundary conditions

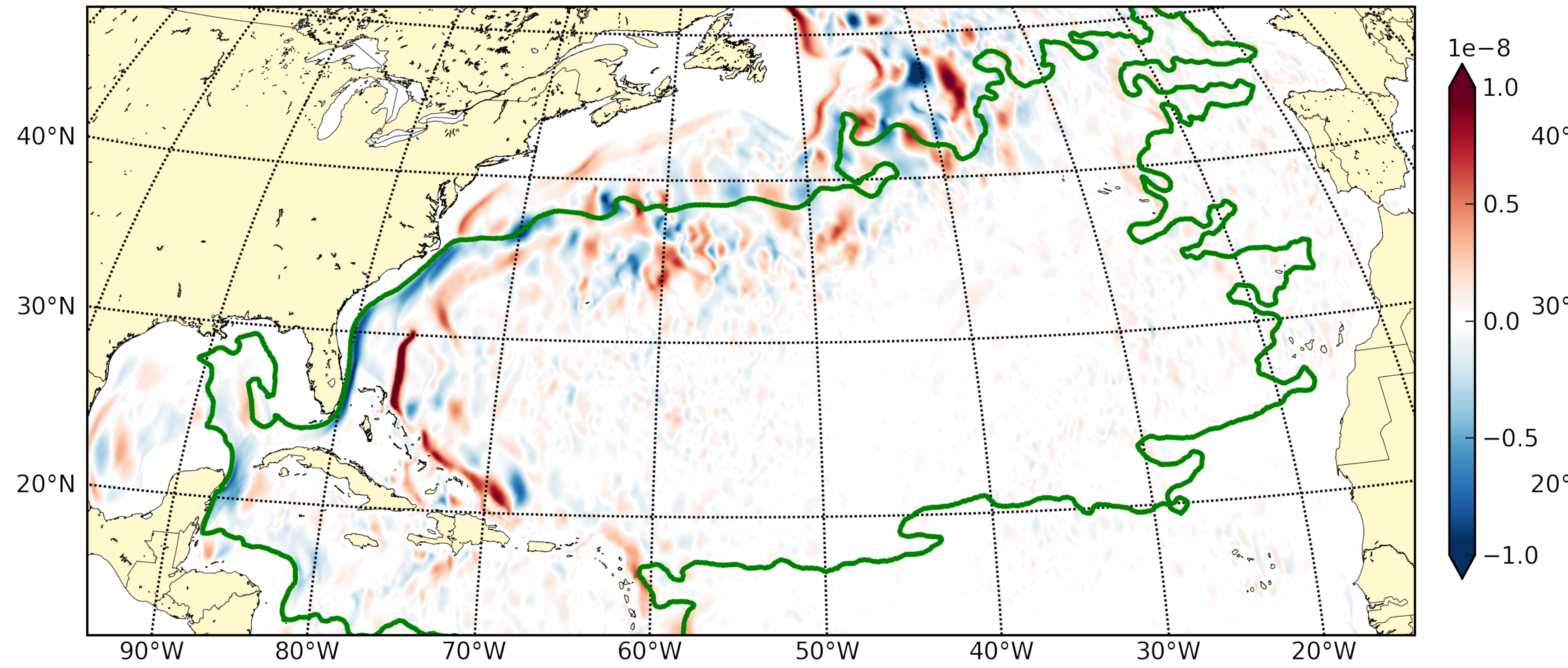


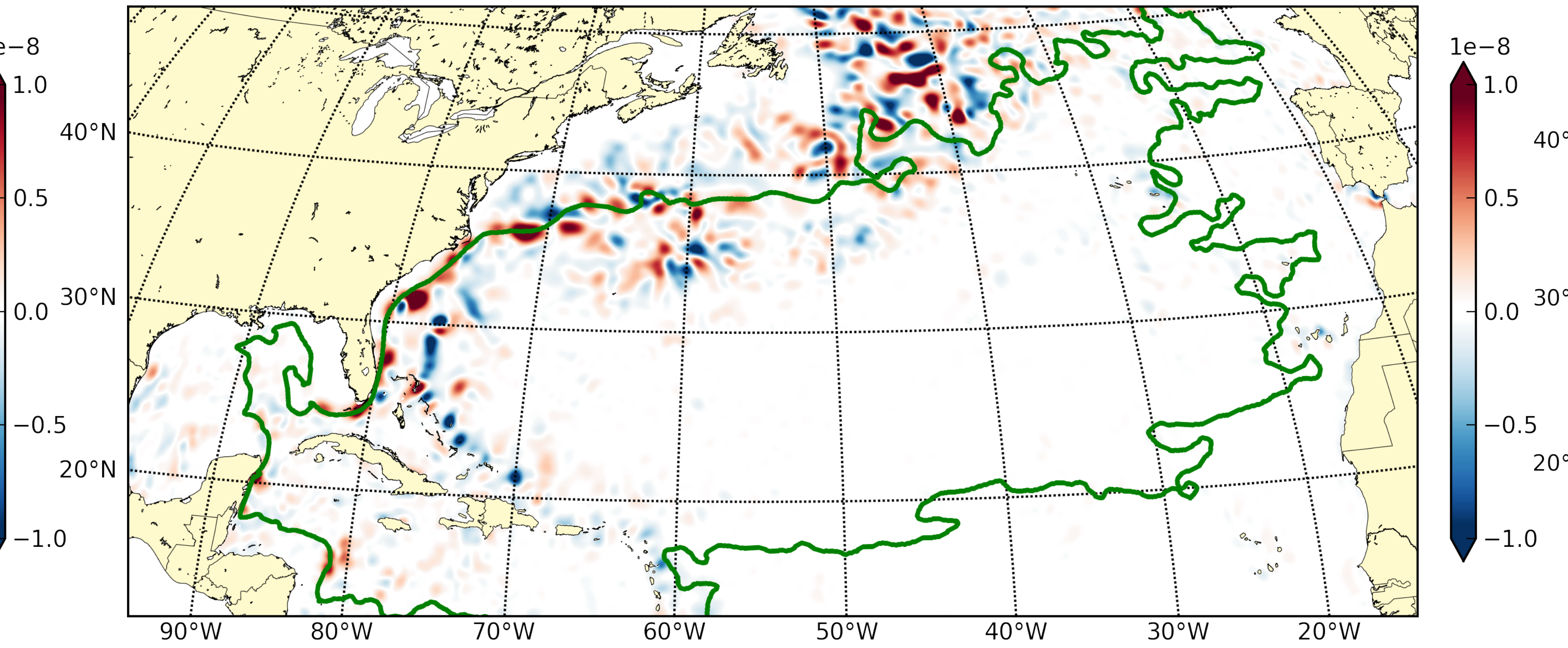
- **Realistic topography (SRTM30)**
- **Climatological fluxes/wind stress**

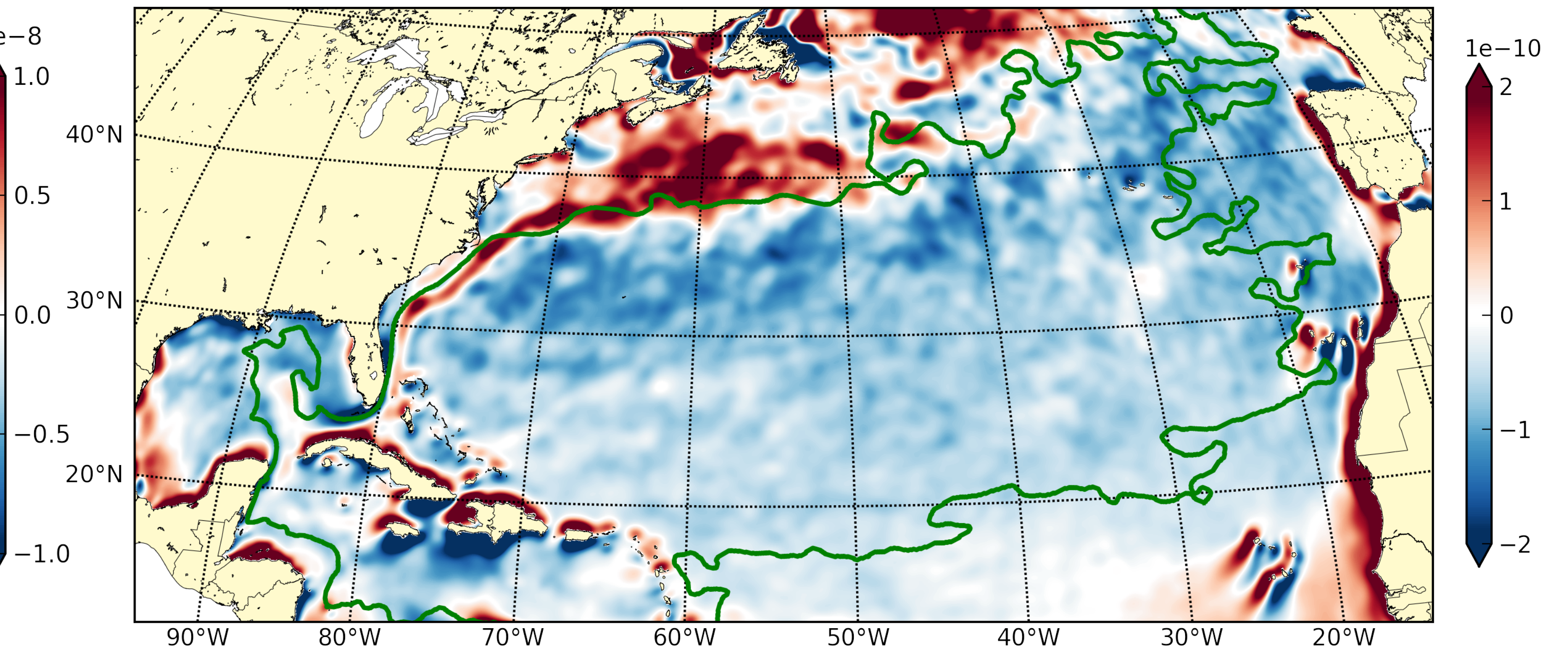
- **Reanalysis (e.g. SODA, ECCO, GLORIS, etc.)**
- **Or ROMS/CROCO parent**



Realistic gyre

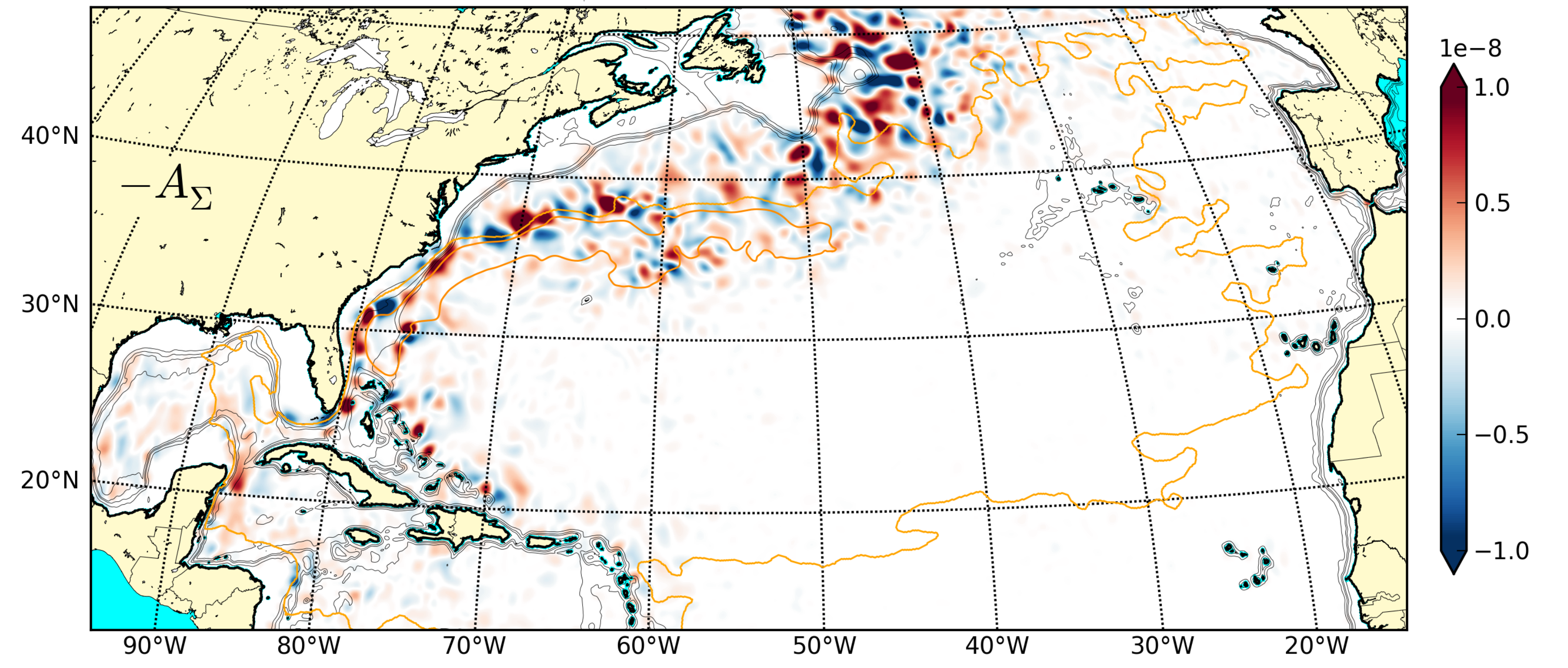


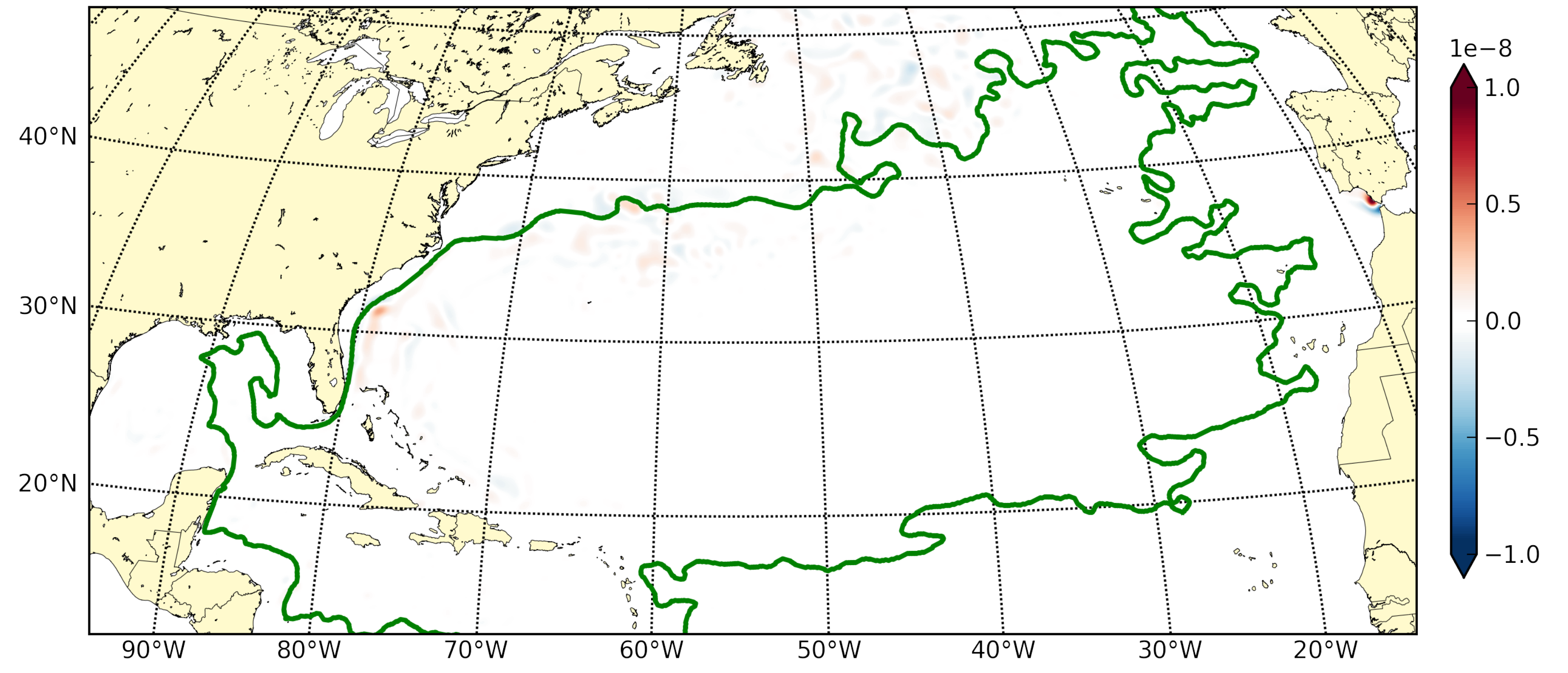




$$\underbrace{\frac{\partial\Omega}{\partial t}}_{\text{rate}} = -\underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}} + \underbrace{\frac{\mathbf{J}(P_b,h)}{\rho_0}}_{\text{bot. pres. torque}} + \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla}\times\frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$

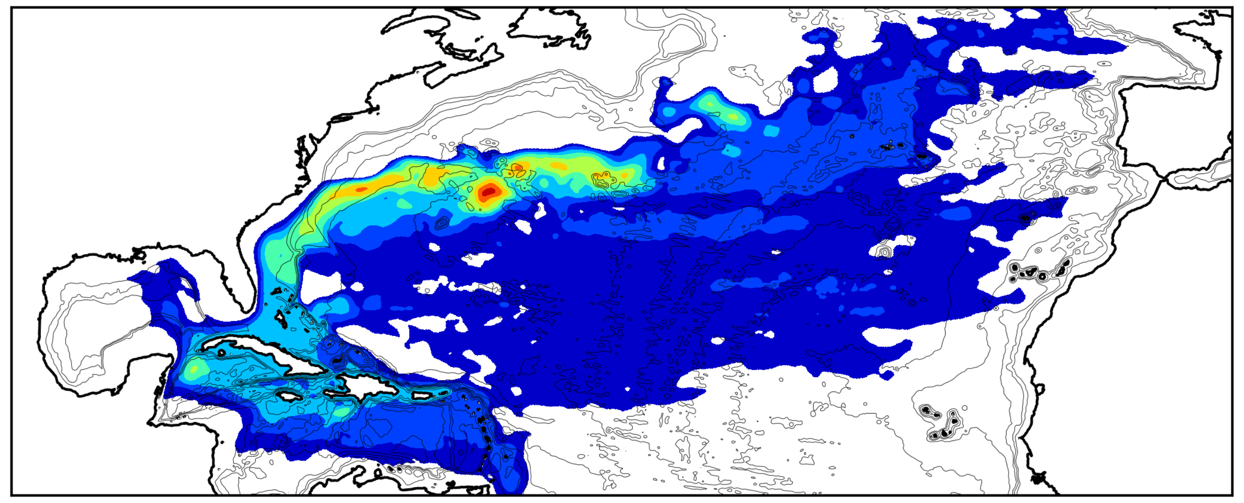
$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$

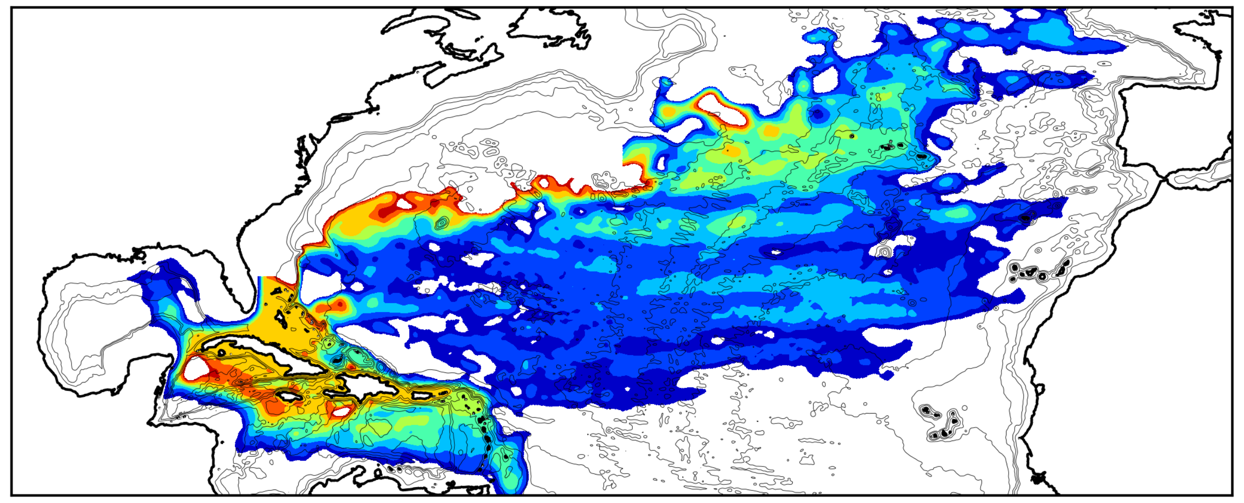


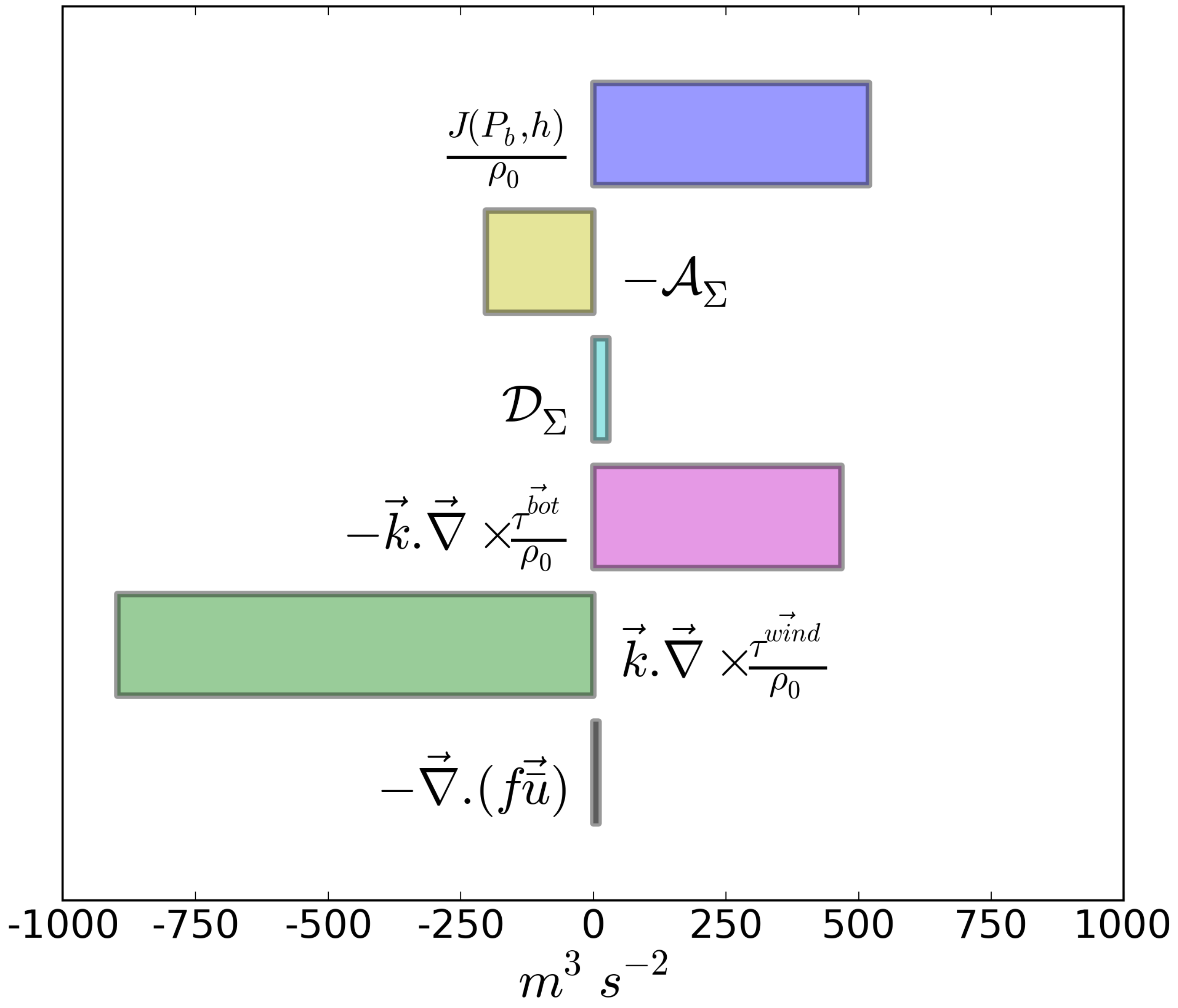


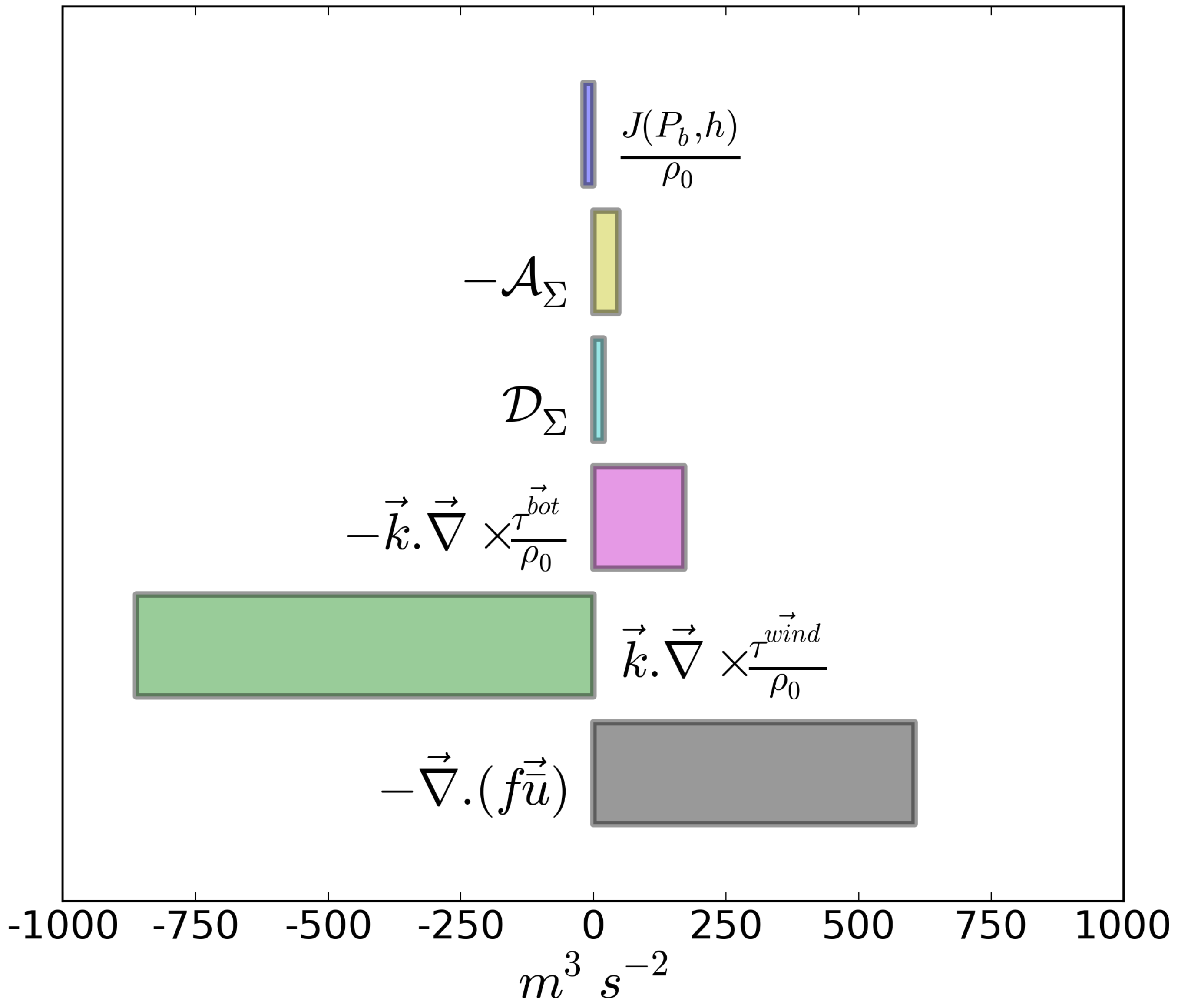
Leaving flatland!

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{- \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}} \boxed{+ \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}} + \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{A_\Sigma}_{\text{NL advection}}$$









Toward a more turbulent gyre

- The western boundary current balance is now **inviscid.** No need for viscous effects to close the gyre.

$$\underbrace{\frac{\partial \Omega}{\partial t}}_{\text{rate}} = \boxed{- \underbrace{\vec{\nabla}.(f\vec{u})}_{\text{planet. vort. adv.}}} + \boxed{\underbrace{\frac{\mathbf{J}(P_b, h)}{\rho_0}}_{\text{bot. pres. torque}}} + \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{wind}}{\rho_0}}_{\text{wind curl}} - \underbrace{\vec{k}.\vec{\nabla} \times \frac{\vec{\tau}^{bot}}{\rho_0}}_{\text{bot. drag curl}}$$
$$+ \underbrace{\mathcal{D}_\Sigma}_{\text{horiz.diffusion.}} - \underbrace{\mathcal{A}_\Sigma}_{\text{NL advection}}$$