Master OFFWIND

Coastal Dynamics

Problem: Ekman Layer and Upwelling

Consider a homogeneous, incompressible ocean with density $\rho = 1025$ kg/m^3. A steady wind blows over the surface in the x-direction, producing a surface stress of $\tau_0 = 0.1$ Pa. The water depth is large enough to assume a semi-infinite ocean. The Coriolis parameter at the location is $f = 10^{-4}$ s^{-1}, and the eddy viscosity in the ocean is $\nu = 10^{-2}$ m^2/s.

1. Assuming steady, linear flow, write the equations governing the horizontal velocities $u(z)$ and $v(z)$ in the Ekman layer.
2. Compute the Ekman layer depth δ_E.
3. Find the direction of the net transport in the Ekman layer relative to the wind direction.
4. Estimate the vertical velocity (Ekman pumping) at the base of the Ekman layer if the wind stress varies spatially as $\tau_x(y) = \tau_0 \cos\left(\frac{\pi y}{L}\right), \quad \tau_y = 0$ with $L = 100$ km.

Governing Equations For a steady, homogeneous Ekman layer over a semi-infinite ocean:

$$-fv = \nu \frac{d^2 u}{dz^2}, \qquad fu = \nu \frac{d^2 v}{dz^2},$$

where z is the vertical coordinate measured positive upward from the surface, $u(z)$ is the wind-aligned velocity, $v(z)$ is the cross-wind velocity.

Boundary conditions:

$$\text{at } z = 0 : \nu \frac{du}{dz} = \frac{\tau_0}{\rho}, \quad \nu \frac{dv}{dz} = 0$$
$$\text{as } z \to -\infty : u, v \to 0$$

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Ekman Layer Depth The characteristic Ekman depth is

$$\delta_E = \sqrt{\frac{2\nu}{f}}.$$

Substitute the given numbers:

$$\delta_E = \sqrt{\frac{2 \cdot 10^{-2}}{10^{-4}}} = \sqrt{200} \approx 14.1 \text{ m}.$$

$\boxed{\delta_E \approx 14 \text{ m}}$

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Direction of Net Transport The net transport in the Ekman layer is:

$$\mathbf{M} = \int_{-\infty}^{0} (u, v)\, dz = \frac{\tau_0}{\rho f} \hat{\mathbf{n}},$$

where $\hat{\mathbf{n}}$ is perpendicular to the wind direction:

- In the Northern Hemisphere, the transport is 90° to the right of the wind.

Magnitude:

$$M = \frac{\tau_0}{\rho f} = \frac{0.1}{1025 \cdot 10^{-4}} \approx 0.975 \text{ m}^2/\text{s}.$$

Direction: 90° to the right of the wind.

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Ekman Pumping (Vertical Velocity) The vertical velocity at the base of the Ekman layer is:

$$w_E = \frac{1}{\rho f} (\nabla \times \boldsymbol{\tau})_z = \frac{1}{\rho f} \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right).$$

Since $\tau_y = 0$ and $\tau_x = \tau_0 \cos(\pi y / L)$:

$$\frac{\partial \tau_x}{\partial y} = -\frac{\pi}{L} \tau_0 \sin\left(\frac{\pi y}{L}\right)$$

$$\Rightarrow (\nabla \times \boldsymbol{\tau})_z = -(-\frac{\pi \tau_0}{L} \sin(\pi y / L)) = \frac{\pi \tau_0}{L} \sin\left(\frac{\pi y}{L}\right)$$

$$\Rightarrow w_E(y) = \frac{1}{\rho f} \frac{\pi \tau_0}{L} \sin\left(\frac{\pi y}{L}\right)$$

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Numerical Estimate

$$w_E^{\max} = \frac{1}{1025 \cdot 10^{-4}} \cdot \frac{3.1416 \cdot 0.1}{10^5} = \frac{3.1416 \cdot 10^{-6}}{0.1025} \approx 3.06 \times 10^{-5} \text{ m/s}$$

Convert to m/day:

$$w_E^{\max} \approx 3.06 \times 10^{-5} \cdot 86400 \approx 2.64 \text{ m/day}.$$

Problem: Tea Leaves

It is observed that fragments of tea leaves at the bottom of a stirred tea cup conglomerate toward the center. Explain this phenomenon with Ekman-layer dynamics. Also explain why the tea leaves go to the center irrespectively of the direction of stirring (clockwise or counterclockwise).

Solution

Bottom boundary layer (Ekman layer): Viscous effects slow the flow near the bottom. The resulting balance between friction, pressure gradient, and rotation produces a radial flow toward the center at the bottom. Tea leaves, being heavier, are trapped in the bottom layer and are advected toward the center by the inward radial flow.

Independence of stirring direction: Changing the rotation direction reverses the surface vortex and pressure gradient. The bottom Ekman-layer flow, however, is always directed *toward the center*. Therefore, tea leaves accumulate at the center regardless of clockwise or counterclockwise stirring.