

# Numerical Modelling

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*the anatomy of an ocean model*

- **Lesson 1 : [D109]**
  - Introduction
  - Equations of motions
  - *Activity 1 [run an ocean model]*
- **Lesson 2 : [B 012]**
  - Horizontal Discretization
  - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 3 : [D109]**
  - Presentation of the model CROCO
  - Dynamics of the ocean gyre
  - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 4 : [D109]**
  - Numerical schemes
  - *Activity 3 [Impacts of numerics]*
- **Lesson 5 : [D109]**
  - Vertical coordinates
  - *Activity 3 [Impact of topography]*
- **Lesson 6 : [D109]**
  - Boundary Forcings
  - *Activity 4 [Design a realistic simulation]*
- **Lesson 7 : [D109]**
  - Diagnostics and validation
  - *Activity 5 [Analyze a realistic simulation]*
- **Lesson 8 : [D109]**
  - *Work on your projet*

Presentations and material  
will be available at :

[jgula.fr/ModNum/](http://jgula.fr/ModNum/)

# Useful references

## Extensive courses:

- MIT:  
<https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/>
- Princeton: [https://stephengriffies.github.io/assets/pdfs/GFM\\_lectures.pdf](https://stephengriffies.github.io/assets/pdfs/GFM_lectures.pdf)

## Overview on ocean modelling and current challenges:

- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling.  
<http://jgula.fr/ModNum/Griffiesetal00.pdf>
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. [http://jgula.fr/ModNum/Griffies\\_Chapter.pdf](http://jgula.fr/ModNum/Griffies_Chapter.pdf)
- Fox-Kemper et al, 19, "Challenges and Prospects in Ocean Circulation Models"  
<http://jgula.fr/ModNum/FoxKemperetal19.pdf>

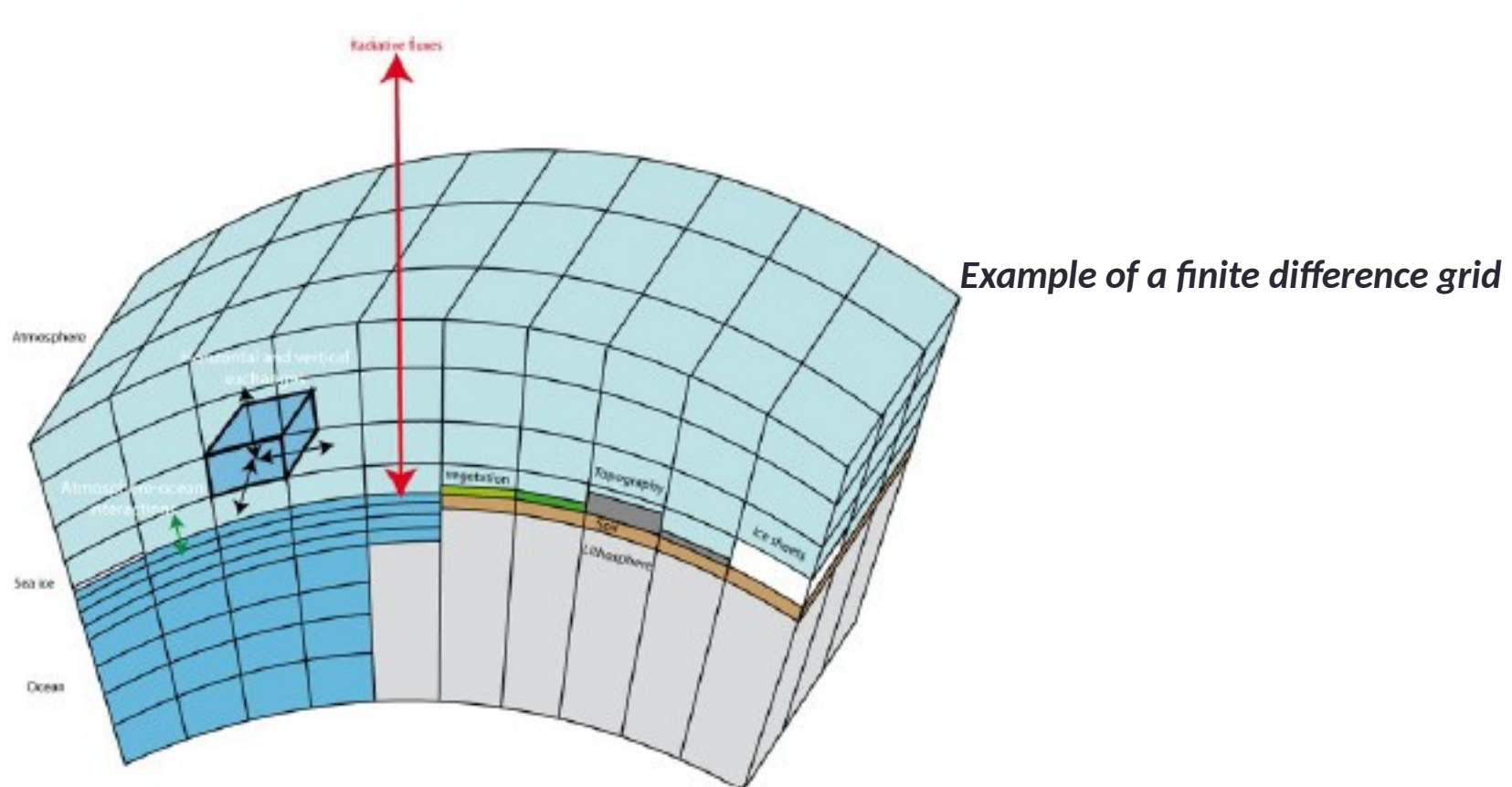
## ROMS/CROCO:

- <https://www.myroms.org/wiki/>
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell.  
<http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf>

## #3 Discretization

# Discretization

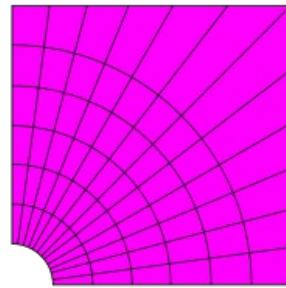
The ocean is divided into boxes : **discretization**



# Discretization

## Structured grids

Identified by regular connectivity  
= can be addressed by  $(i,j,k)$

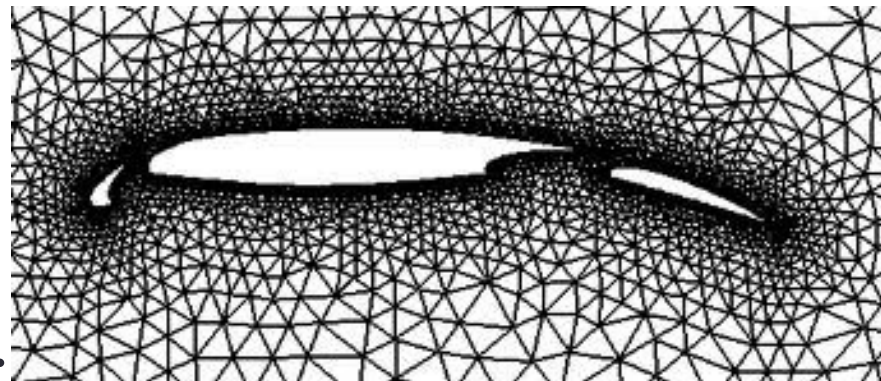


← ROMS

## Unstructured grids

The domain is tiled using more general geometrical shapes (triangles, ...) pieced together to optimally fit details of the geometry.

- ✓ Good for tidal modeling, engineering applications.
- ✓ Problems:  
geostrophic balance accuracy,  
conservation and positivity properties, ...



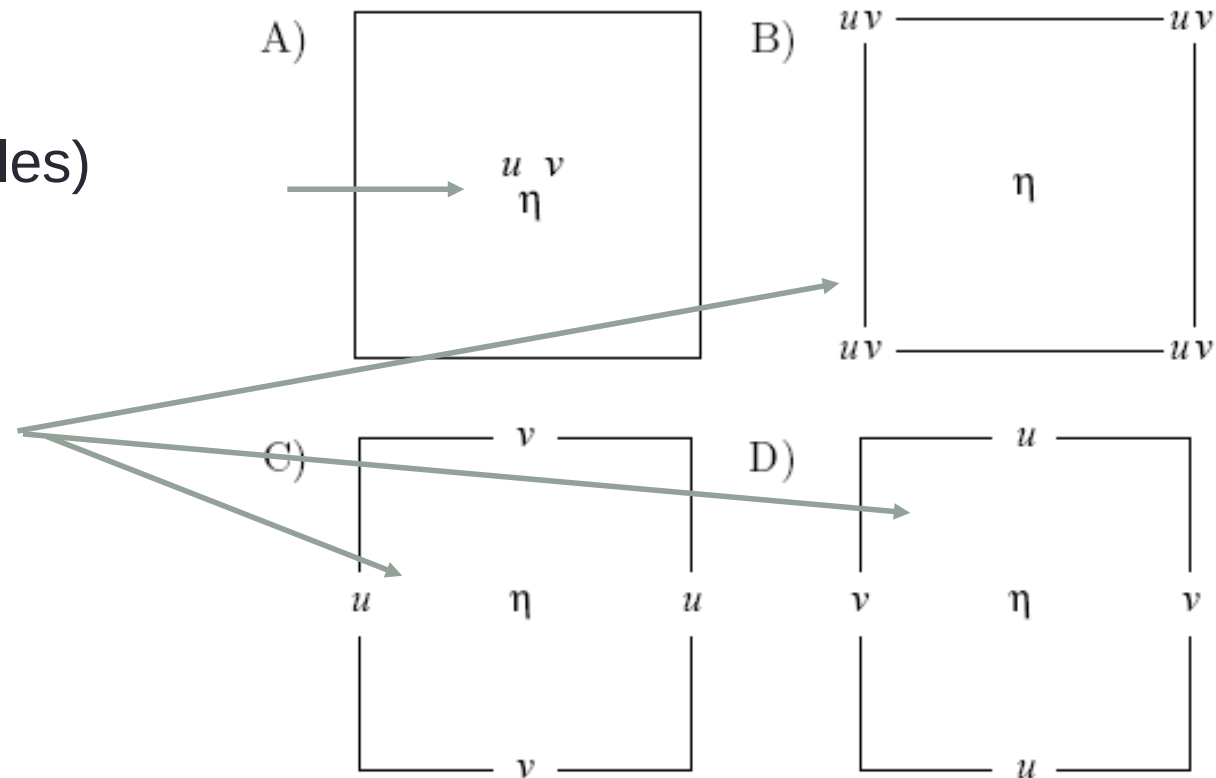
# Horizontal discretization

Different types of Horizontal Grids (Arakawa Grids):

Non-staggered  
(= collocated variables)

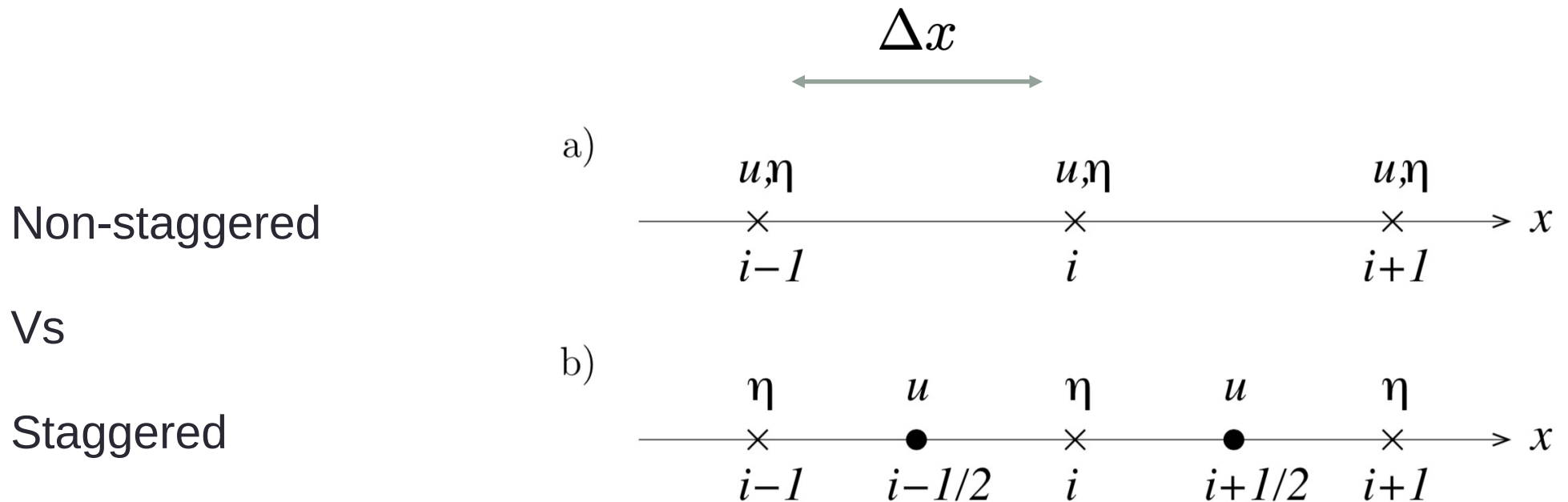
Or

Staggered



# Horizontal discretization

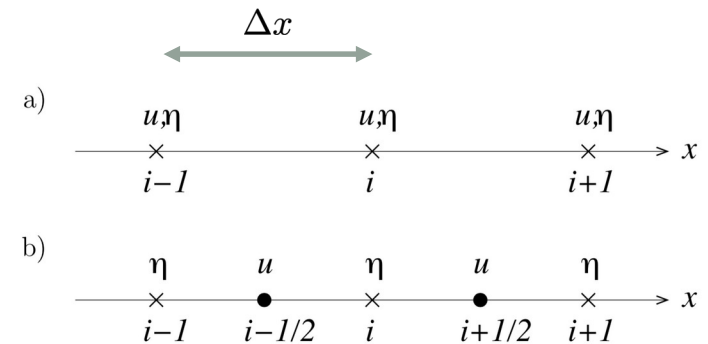
Staggered Vs unstaggered : the 1D problem





# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

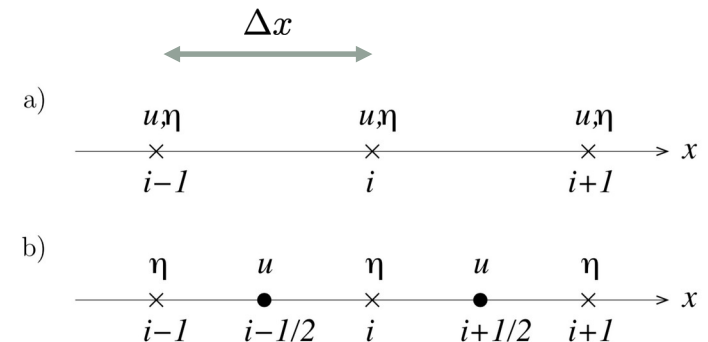


**1. The advection equation**  $\partial_t \theta + c \partial_x \theta = 0$

*Solutions of the continuous equations are non dispersive waves*  
 $\theta(x, t) = \theta_o e^{i(kx - \omega t)}$  with dispersion relation  $\omega = ck$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

Solutions of the **continuous equations** are non-dispersive waves

$\theta(x, t) = \theta_o e^{i(kx - \omega t)}$  with dispersion relation  $\omega = ck$

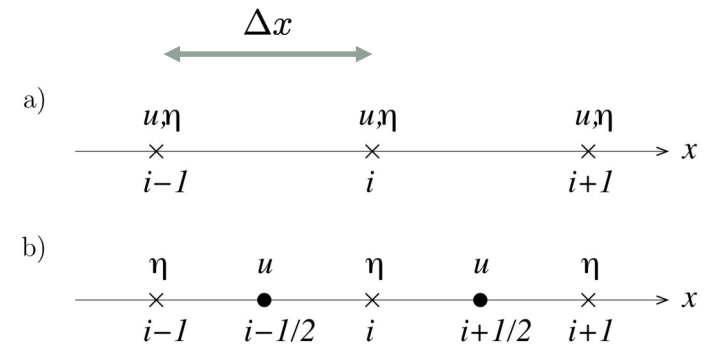
**Discretized equations** with the centered second order derivative are:

$$d_t \theta + \frac{c}{\Delta x} \delta_i \bar{\theta}^i = 0$$

$$d_t \theta_i + \frac{c}{2\Delta x} (\theta_{i+1} - \theta_{i-1}) = 0$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



**1. The advection equation**  $\partial_t \theta + c \partial_x \theta = 0$

Substituting in our solution:

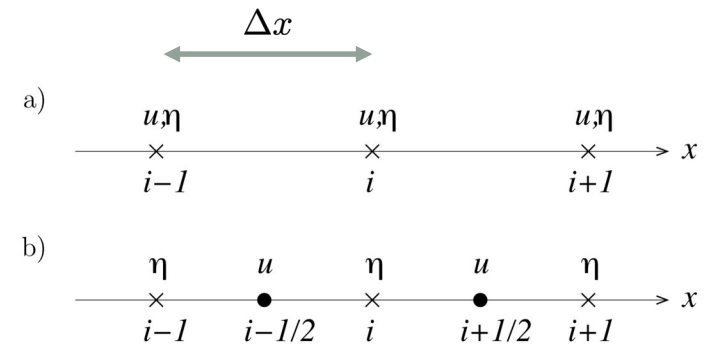
$$\theta_i(x, t) = \theta_0 e^{i(kx - \omega t)}$$

$$\theta_{i-1}(x, t) = \theta_0 e^{i(k(x - \Delta x) - \omega t)}$$

$$\theta_{i+1}(x, t) = \theta_0 e^{i(k(x + \Delta x) - \omega t)}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

Substituting in our solution gives:

$$\begin{aligned}
 -i\omega &= -\frac{c}{2\Delta x} \left( e^{ik\Delta x} - e^{-ik\Delta x} \right) \\
 &= -\frac{ci}{\Delta x} \sin k\Delta x
 \end{aligned}$$

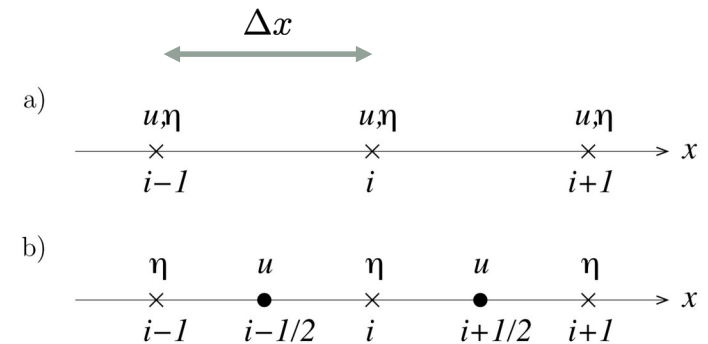
Now the solution is **dispersive!!!**

Even if it will converge to the non-dispersive solution in the limit of small  $\Delta x$

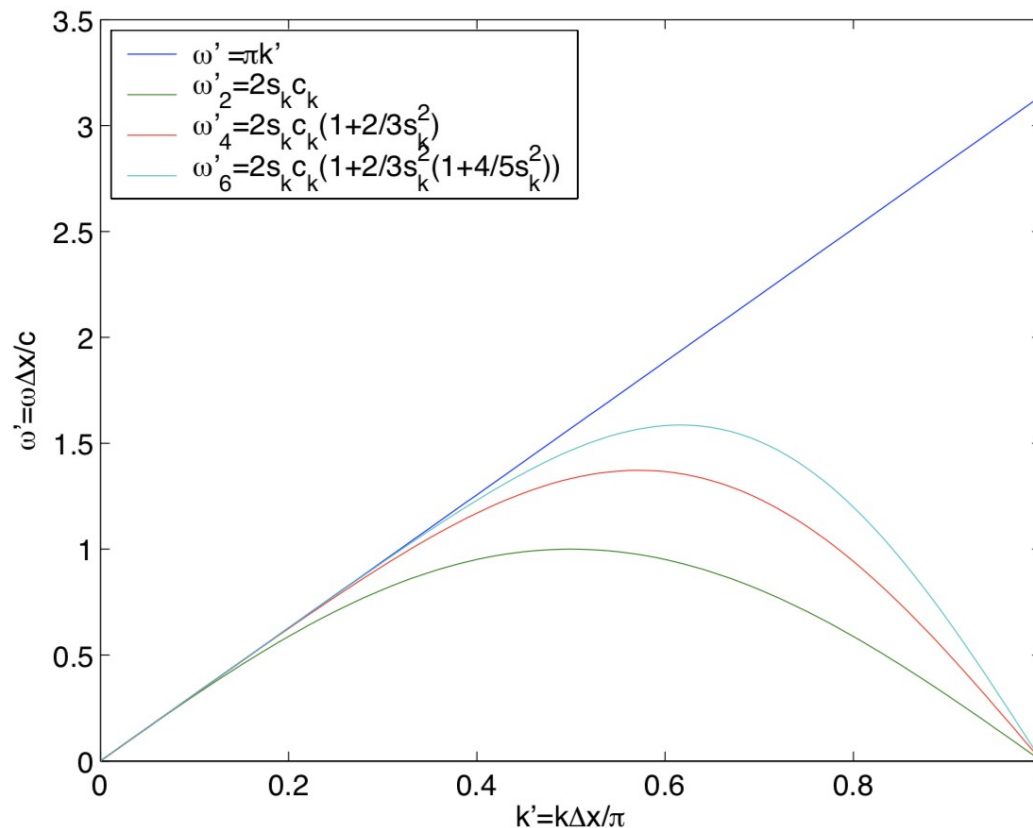
$$\omega = \frac{c}{\Delta x} \sin k\Delta x \xrightarrow{\Delta x \rightarrow 0} ck$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



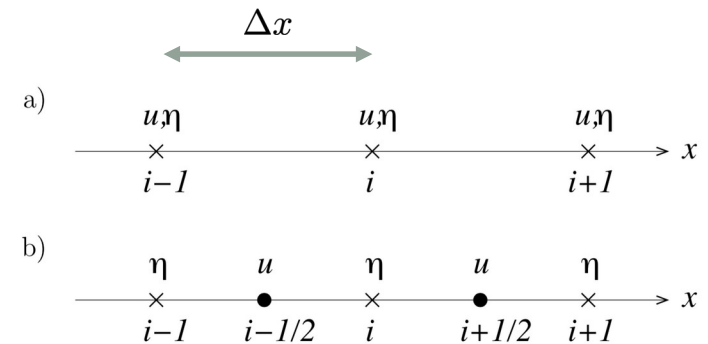
1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$



Dispersion relations for constant flow advection using second, fourth, and sixth order spatial differences.

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

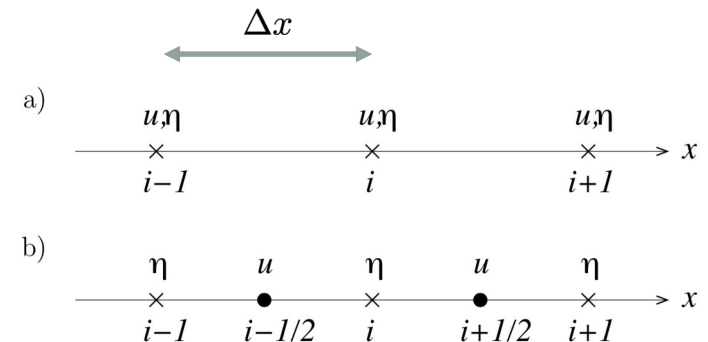
$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

*Solutions of the continuous equations are non-dispersive waves*

$$\eta = \eta_o e^{i(kx - \omega t)} \quad \text{with dispersion relation} \quad \omega = \pm \sqrt{gH} k$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

*Solutions of the continuous equations are non-dispersive waves*

$$\eta = \eta_o e^{i(kx - \omega t)} \quad \text{with dispersion relation} \quad \omega = \pm \sqrt{gH} k$$

Discretized equations with the centered second order derivative on the **unstaggered grid** are:

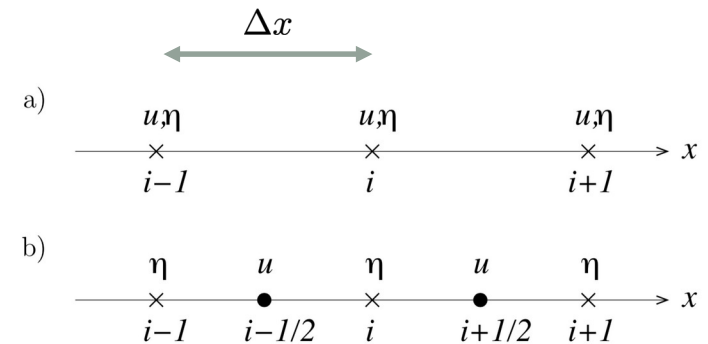
$$\partial_t u = -\frac{g}{\Delta x} \delta_i \bar{\eta}^i$$

$$\partial_t \eta = -\frac{H}{\Delta x} \delta_i \bar{u}^i$$

$$\longrightarrow \partial_{tt} \eta = \frac{gH}{\Delta x^2} \delta_{ii} \bar{\eta}^{ii} \quad \text{with} \quad \delta_{ii} \bar{\eta}^{ii} = \frac{1}{4} (\eta_{i-2} - 2\eta_i + \eta_{i+2})$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

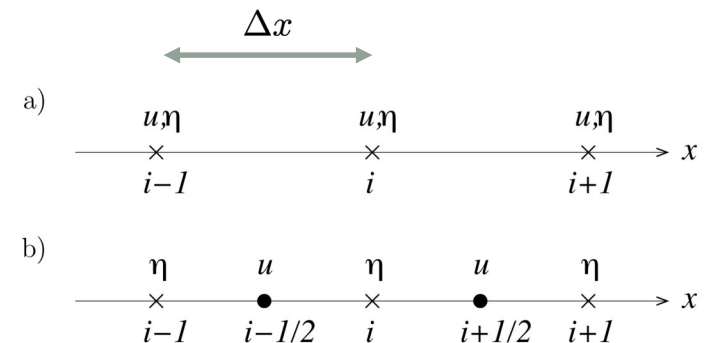
Substituting in our solution on the unstaggered grid gives :

$$\begin{aligned} -\omega^2 &= \frac{gH}{4\Delta x^2} \left( e^{-i2k\Delta x} - 2 + e^{i2k\Delta x} \right) \\ &= \frac{gH}{4\Delta x^2} (2 \cos 2k\Delta x - 2) \\ &= -\frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} \cos^2 \frac{k\Delta x}{2} \end{aligned}$$



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

Substituting in our solution on the unstaggered grid gives :

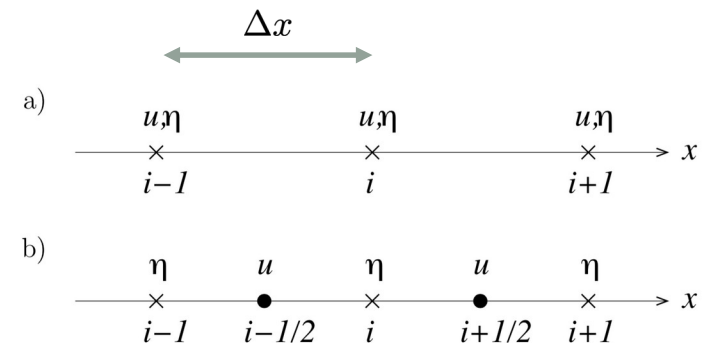
$$\begin{aligned} -\omega^2 &= \frac{gH}{4\Delta x^2} \left( e^{-i2k\Delta x} - 2 + e^{i2k\Delta x} \right) \\ &= \frac{gH}{4\Delta x^2} (2 \cos 2k\Delta x - 2) \\ &= -\frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} \cos^2 \frac{k\Delta x}{2} \end{aligned}$$

### • Question:

- What is the dispersion relation on the staggered grid?

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

Discretized equations with the centered second order derivative on the **staggered grid** are:

$$\partial_t u = -\frac{g}{\Delta x} \delta_i \eta$$

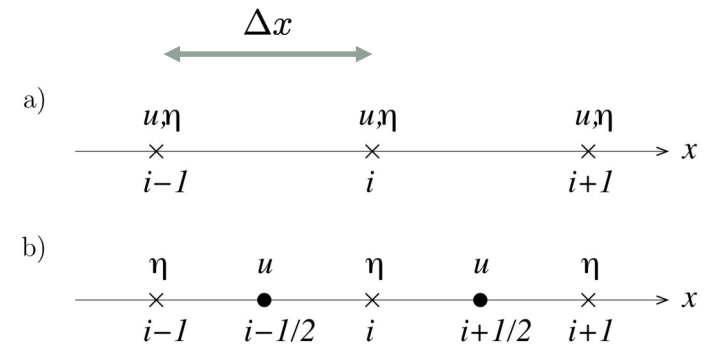
$$\partial_t \eta = -\frac{H}{\Delta x} \delta_i u$$

This can be written as a system:

$$\begin{pmatrix} \partial_t & \frac{g}{\Delta x} \delta_i \\ \frac{H}{\Delta x} \delta_i & \partial_t \end{pmatrix} \begin{pmatrix} u \\ \eta \end{pmatrix} = 0 \quad \begin{pmatrix} -i\omega & \frac{2ig}{\Delta x} \sin \frac{k\Delta x}{2} \\ \frac{2iH}{\Delta x} \sin \frac{k\Delta x}{2} & -i\omega \end{pmatrix} \begin{pmatrix} u \\ \eta \end{pmatrix} = 0$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$

Substituting in our solution on the staggered grid gives :

$$\omega^2 - \frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} = 0$$

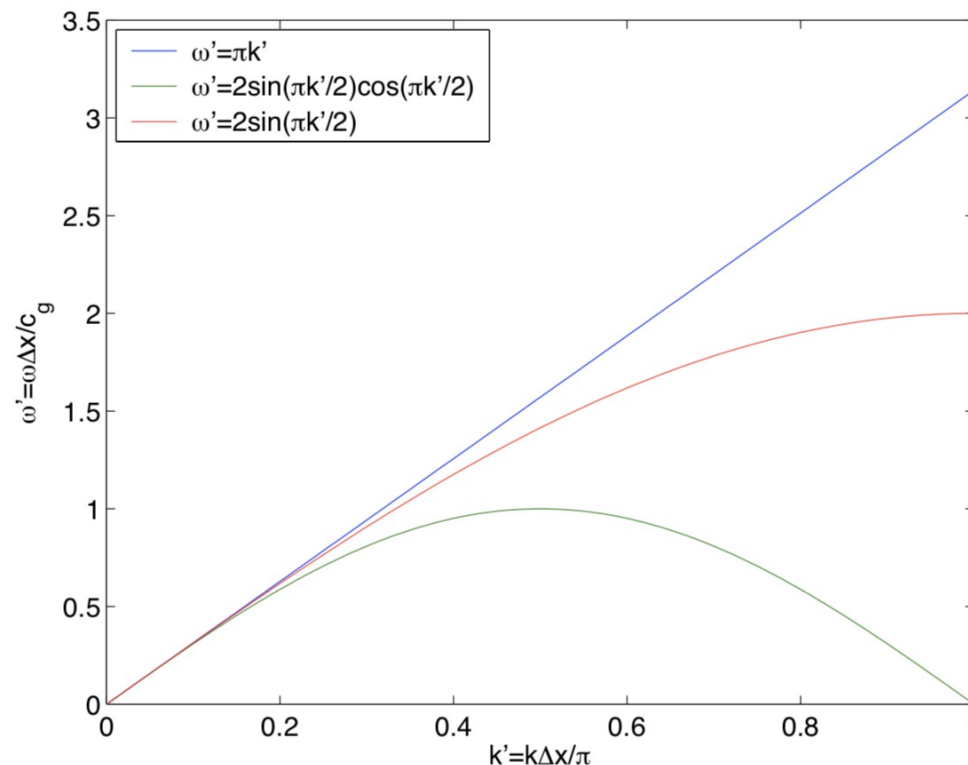
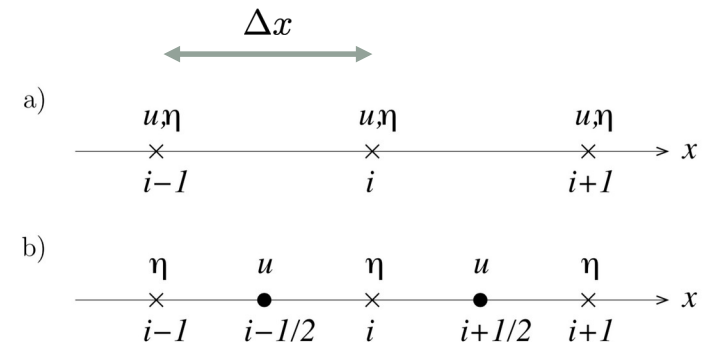
# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

$$\partial_t u = -g \partial_x \eta$$

$$\partial_t \eta = -H \partial_x u \longrightarrow \partial_{tt} \eta = gH \partial_{xx} \eta$$



When compared to the continuum we see that the numerical modes are still dispersive on the staggered grid, but:

there is no false extrema, unlike the non-staggered grid,

the group velocity  $v_g = \partial_k \omega$  is of the correct sign everywhere, even if reduced.

Dispersion of numerical gravity wave for the unstaggered grid (green) and the staggered grid (red). The continuum ( $= k$ ) is plotted for comparison (blue).

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

$$\partial_t u - f v + g \partial_x \eta = 0$$

$$\partial_t v + f u = 0$$

$$\partial_t \eta + H \partial_x u = 0$$

*Solutions of the continuous equations are waves following the dispersion relation:*

$$\left| \begin{pmatrix} -i\omega & -f & gik \\ f & -i\omega & 0 \\ Hik & 0 & -i\omega \end{pmatrix} \right| = 0 \Rightarrow \begin{cases} \omega = 0 \\ \omega^2 = f^2 + gHk^2 \end{cases}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

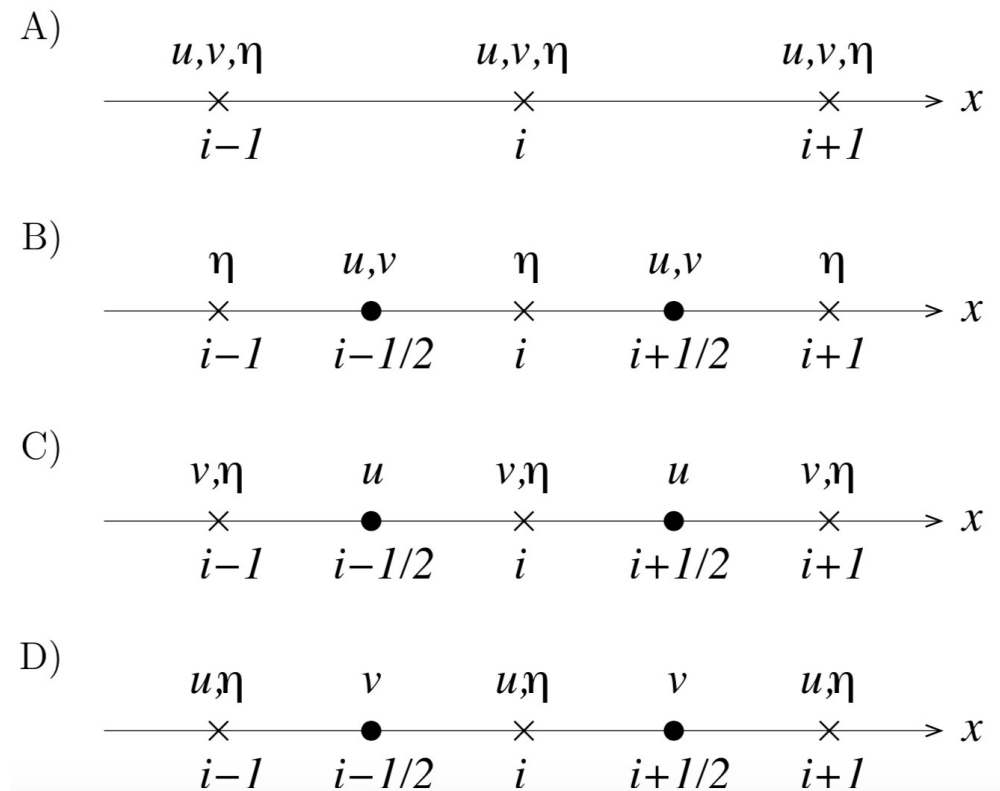
## 2. Inertia-Gravity waves

$$\partial_t u - fv + g\partial_x \eta = 0$$

$$\partial_t v + fu = 0$$

$$\partial_t \eta + H\partial_x u = 0$$

Now, 4 different grids are possible:



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

- A-grid model

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + f u &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i &= 0\end{aligned}$$

- B-grid model

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f u &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u &= 0\end{aligned}$$

- C-grid model

$$\begin{aligned}\partial_t u - f \bar{v}^i + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f \bar{u}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u &= 0\end{aligned}$$

- D-grid model

$$\begin{aligned}\partial_t u - f \bar{v}^i + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + f \bar{u}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i &= 0\end{aligned}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

The corresponding dispersion relations are :

$$\begin{aligned} \text{A:} \quad & \frac{\omega^2}{f^2} = 1 + \frac{4L_d^2}{\Delta x^2} s_k^2 c_k^2 \\ \text{B:} \quad & \frac{\omega^2}{f^2} = 1 + \frac{4L_d^2}{\Delta x^2} s_k^2 \\ \text{C:} \quad & \frac{\omega^2}{f^2} = c_k^2 + \frac{4L_d^2}{\Delta x^2} s_k^2 \\ \text{D:} \quad & \frac{\omega^2}{f^2} = c_k^2 + \frac{4L_d^2}{\Delta x^2} s_k^2 c_k^2 \end{aligned}$$

$$s_k = \sin \frac{k\Delta x}{2} \quad c_k = \cos \frac{k\Delta x}{2}$$

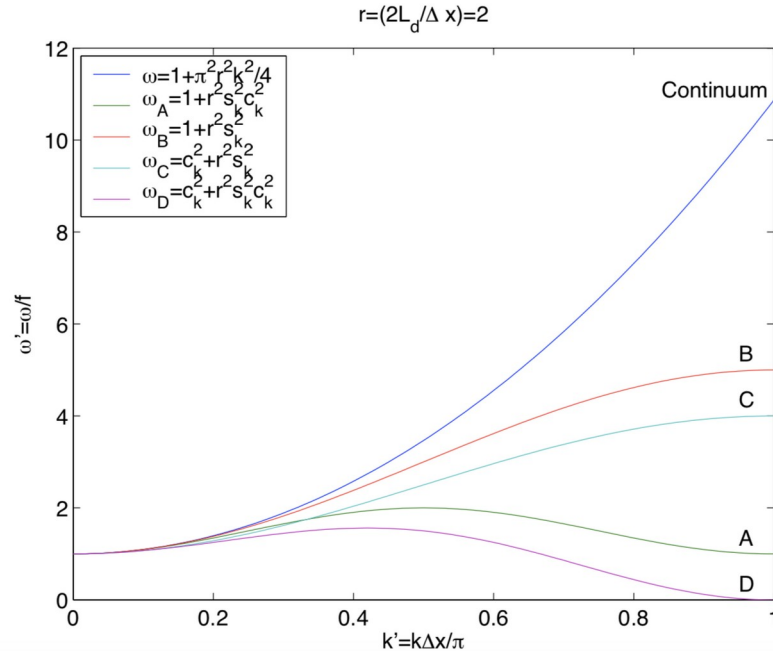
$$L_d = \sqrt{gH}/f$$



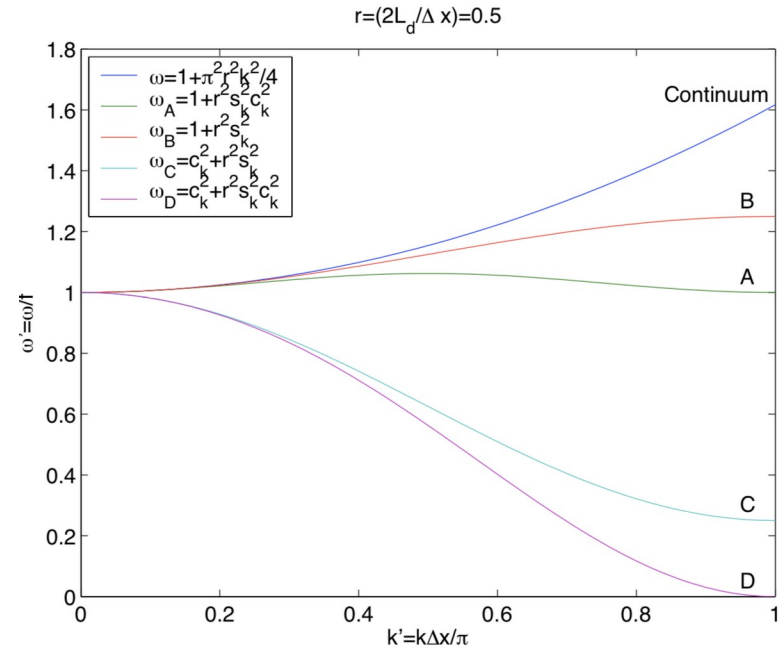
# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves



deformation radius is resolved

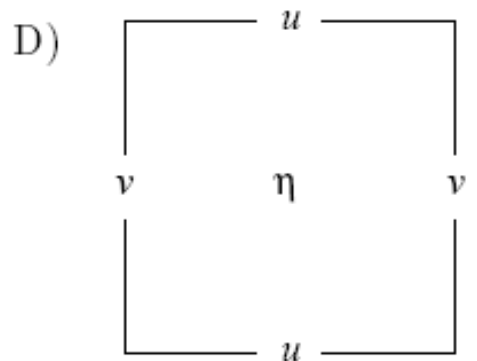
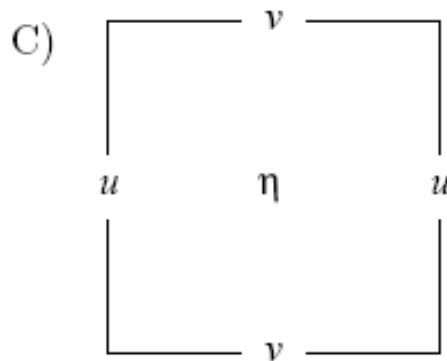
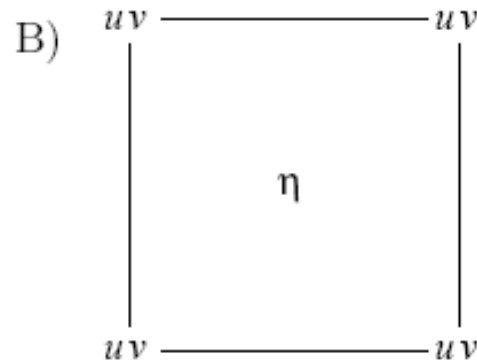
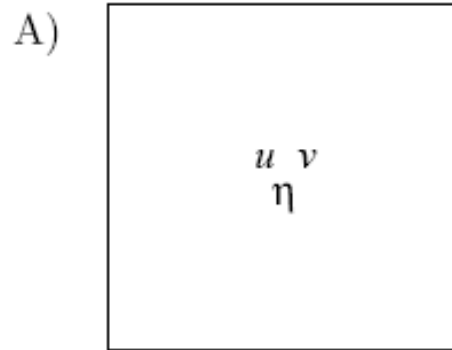


deformation radius is not resolved

Staggering variables in the form of the B grid is most likely to avoid computational modes when solving one-dimensional shallow water equations.

# Horizontal discretization

Horizontal Arakawa Grids:



Linear shallow water equation:

$$\partial_t u - f v + \frac{g}{\Delta x} \delta_i \eta = 0$$

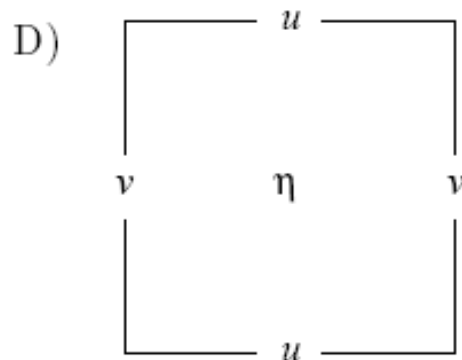
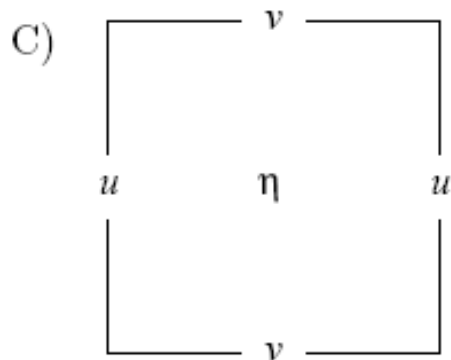
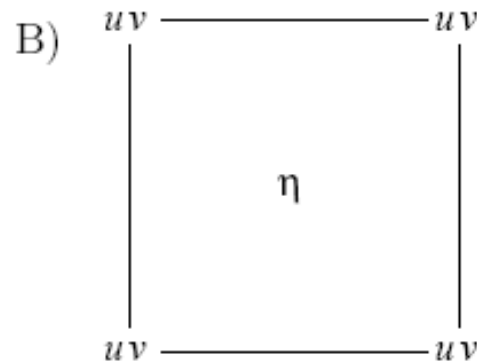
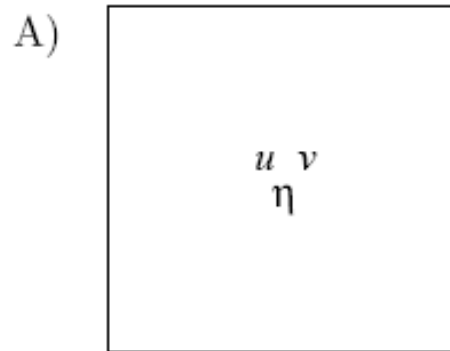
$$\partial_t v + f u + \frac{g}{\Delta y} \delta_j \eta = 0$$

$$\partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_j v = 0$$

# Horizontal discretization

Horizontal Arakawa Grids:

Linear shallow water equation:



$$\partial_t u - f v + \frac{g}{\Delta x} \delta_i \eta = 0$$

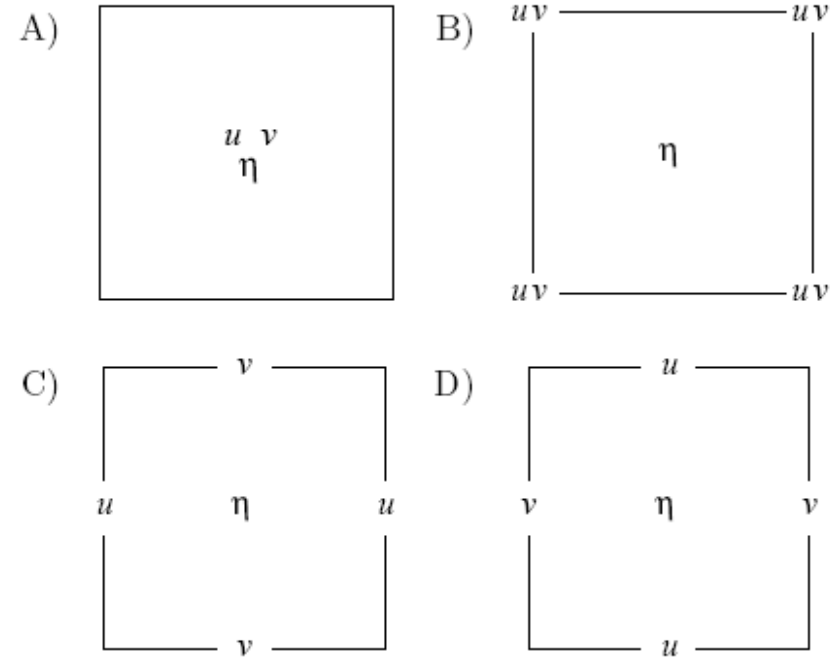
$$\partial_t v + f u + \frac{g}{\Delta y} \delta_j \eta = 0$$

$$\partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_j v = 0$$

## • Question:

- Which grid minimises the number of averaging between points when solving linear SW equations in 2d?

# Horizontal discretization



- A grid:

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + f u + \frac{g}{\Delta y} \delta_j \bar{\eta}^j &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i + \frac{H}{\Delta y} \delta_j \bar{v}^j &= 0\end{aligned}$$

- B grid:

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \bar{\eta}^j &= 0 \\ \partial_t v + f u + \frac{g}{\Delta y} \delta_j \bar{\eta}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^j + \frac{H}{\Delta y} \delta_j \bar{v}^i &= 0\end{aligned}$$

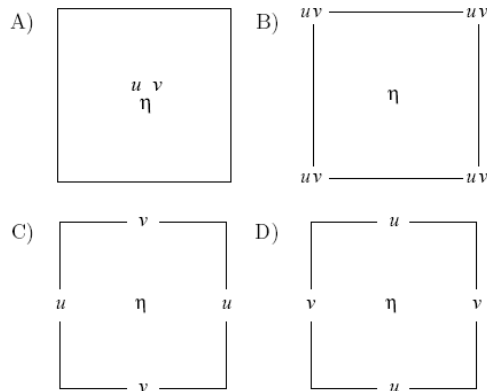
- C grid:

$$\begin{aligned}\partial_t u - f \bar{v}^{ij} + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f \bar{u}^{ij} + \frac{g}{\Delta y} \delta_j \eta &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_j v &= 0\end{aligned}$$

- D grid:

$$\begin{aligned}\partial_t u - f \bar{v}^{ij} + \frac{g}{\Delta x} \delta_i \bar{\eta}^{ij} &= 0 \\ \partial_t v + f \bar{u}^{ij} + \frac{g}{\Delta y} \delta_j \bar{\eta}^{ij} &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^{ij} + \frac{H}{\Delta y} \delta_j \bar{v}^{ij} &= 0\end{aligned}$$

# Horizontal discretization



Response of each operator:

$$\begin{aligned}
 R(\delta_i \phi) &= 2i \sin \frac{k\Delta x}{2} = 2is_k \\
 R(\delta_j \phi) &= 2i \sin \frac{l\Delta y}{2} = 2is_l \\
 R(\bar{\phi}^i) &= \cos \frac{k\Delta x}{2} = c_k \\
 R(\bar{\phi}^j) &= \cos \frac{l\Delta y}{2} = c_l
 \end{aligned}$$

Dispersion relations:

- A grid:

$$\begin{aligned}
 \omega^2 &= f^2 + \frac{4gH}{\Delta x^2} s_k^2 c_l^2 + \frac{4gH}{\Delta y^2} s_l^2 c_k^2 \\
 \text{or} \quad \left(\frac{\omega}{f}\right)^2 &= 1 + r_x^2 s_k^2 c_l^2 + r_y^2 s_l^2 c_k^2
 \end{aligned}$$

- B grid:

$$\begin{aligned}
 \omega^2 &= f^2 + \frac{4gH}{\Delta x^2} s_k^2 c_l^2 + \frac{4gH}{\Delta y^2} s_l^2 c_k^2 \\
 \text{or} \quad \left(\frac{\omega}{f}\right)^2 &= 1 + r_x^2 s_k^2 c_l^2 + r_y^2 s_l^2 c_k^2
 \end{aligned}$$

- C grid:

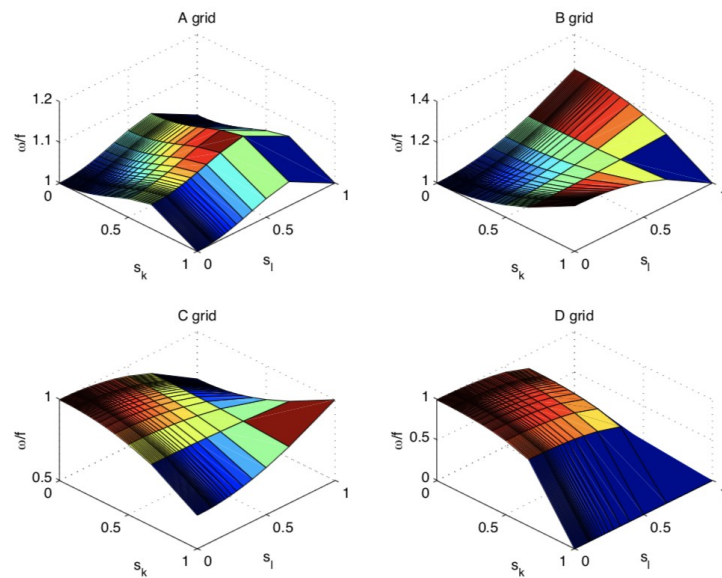
$$\begin{aligned}
 \omega^2 &= f^2 c_k^2 c_l^2 + \frac{4gH}{\Delta x^2} s_k^2 + \frac{4gH}{\Delta y^2} s_l^2 \\
 \text{or} \quad \left(\frac{\omega}{f}\right)^2 &= c_k^2 c_l^2 + r_x^2 s_k^2 + r_y^2 s_l^2
 \end{aligned}$$

- D grid:

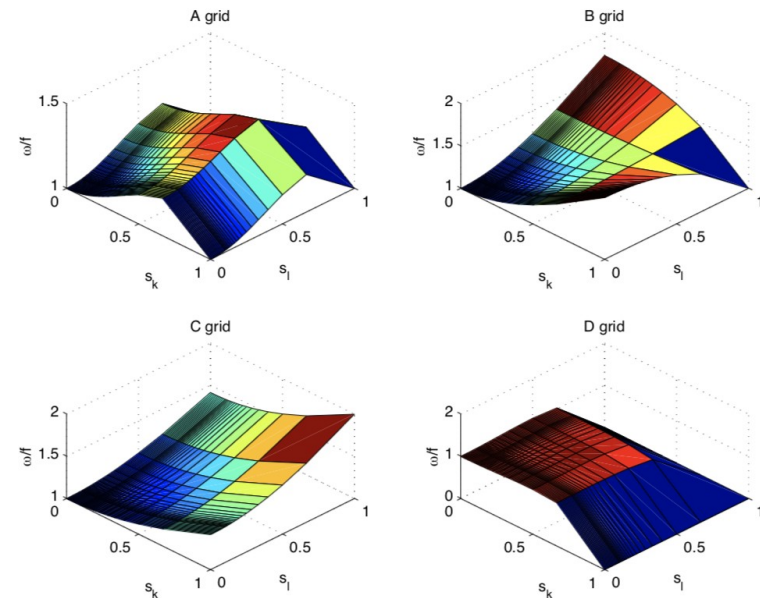
$$\begin{aligned}
 \omega^2 &= f^2 c_k^2 c_l^2 + \frac{4gH}{\Delta x^2} s_k^2 c_k^2 c_l^2 + \frac{4gH}{\Delta y^2} s_l^2 c_k^2 c_l^2 \\
 \text{or} \quad \left(\frac{\omega}{f}\right)^2 &= (1 + r_x^2 s_k^2 + r_y^2 s_l^2) c_k^2 c_l^2
 \end{aligned}$$

# Horizontal discretization

Coarse resolution:



High resolution:



D is always bad.

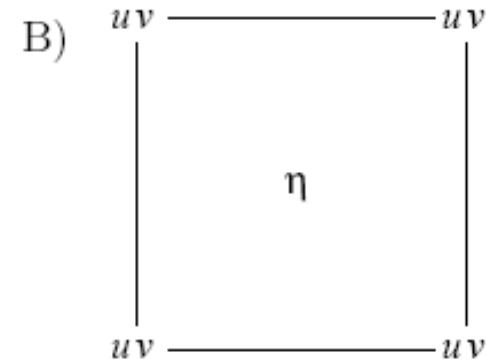
B underestimates frequency for short two-dimensional waves

C is the only grid with monotonically increasing frequency (i.e. right sign of group velocity) at high res.

# Horizontal discretization

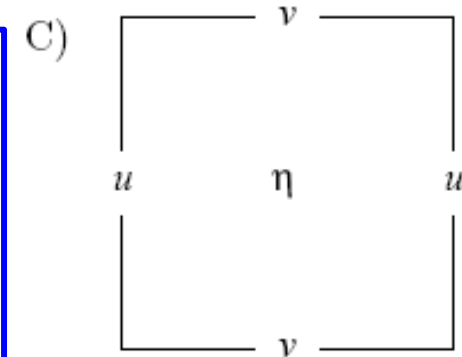
■ B grid is preferred at coarse resolution,  
when Coriolis is important:

- Superior for poorly resolved inertia-gravity waves.
- Good for Rossby waves: collocation of velocity points.
- Bad for gravity waves: computational checkerboard mode



■ C grid is preferred at fine resolution,  
when Coriolis is less important

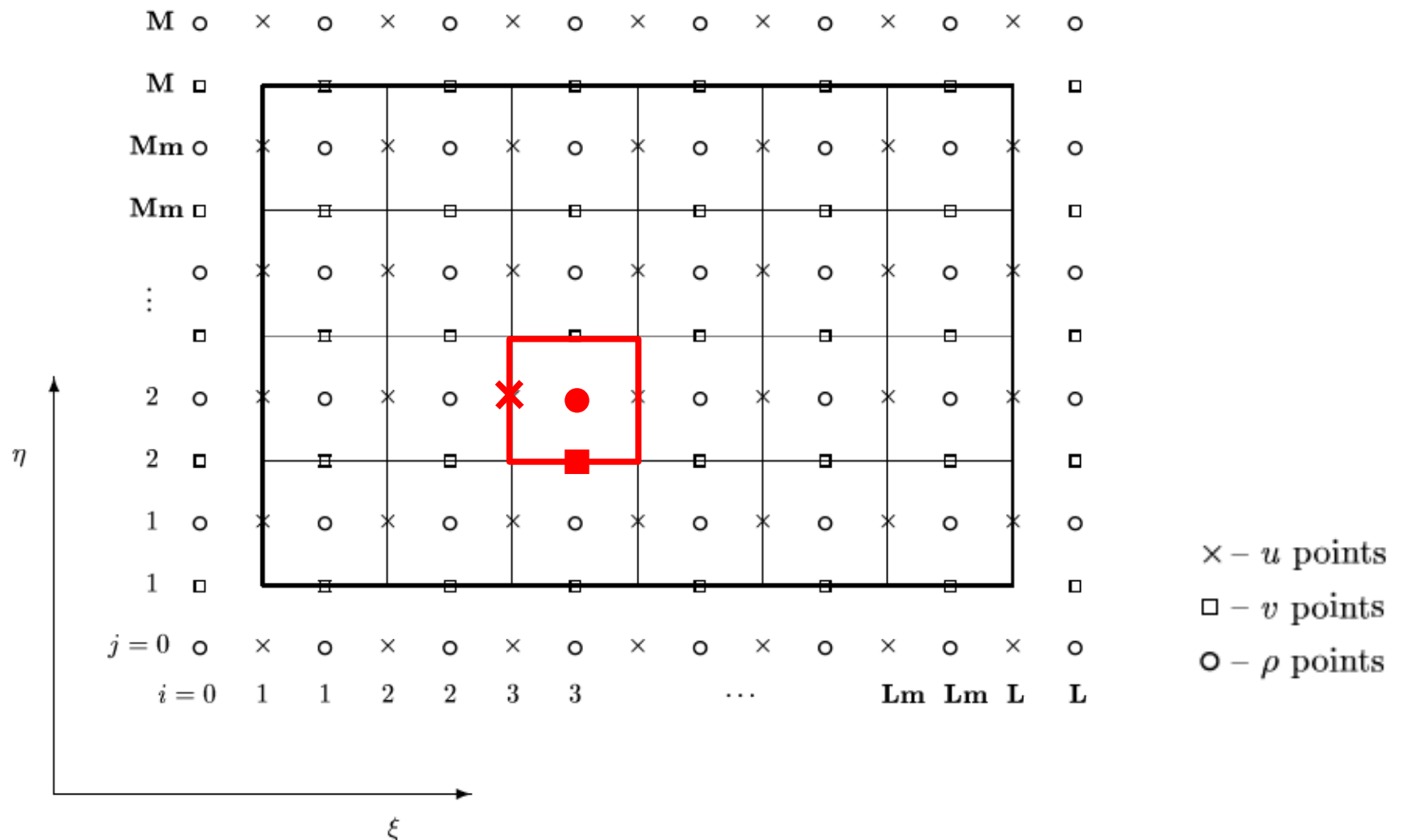
- Superior for gravity waves.
- Good for well resolved inertia-gravity waves.
- Bad for poorly resolved waves: Rossby waves (computational checkerboard mode) and inertia-gravity waves due to averaging the Coriolis force.



← ROMS

# Horizontal discretization

## ROMS: Arakawa C-grid





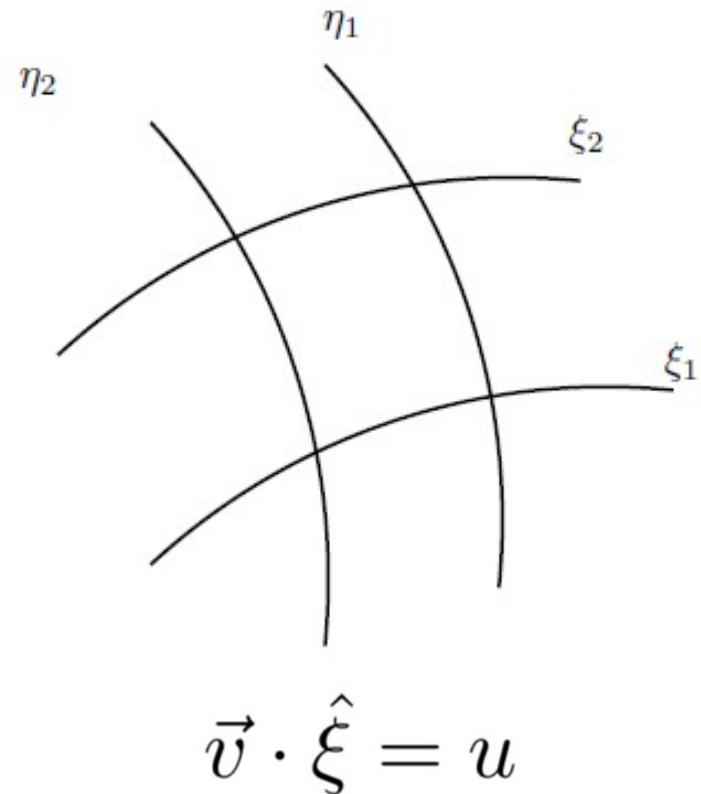
# Horizontal curvilinear grid

- **ROMS:** is formulated in general horizontal curvilinear coordinates:

$$(ds)_\xi = \left( \frac{1}{m} \right) d\xi$$

$$(ds)_\eta = \left( \frac{1}{n} \right) d\eta$$

$m, n$ : scale factors relating the differential distances to the physical arc lengths



$$\vec{v} \cdot \hat{\eta} = v$$

# Horizontal curvilinear grid

- **ROMS:** is formulated in general horizontal curvilinear coordinates:

$$(ds)_\xi = \left( \frac{1}{m} \right) d\xi$$

$$(ds)_\eta = \left( \frac{1}{n} \right) d\eta$$

*With classical formulas for div, grad, curl and lap in curvilinear coordinates:*

$$\nabla \phi = \hat{\xi} m \frac{\partial \phi}{\partial \xi} + \hat{\eta} n \frac{\partial \phi}{\partial \eta}$$

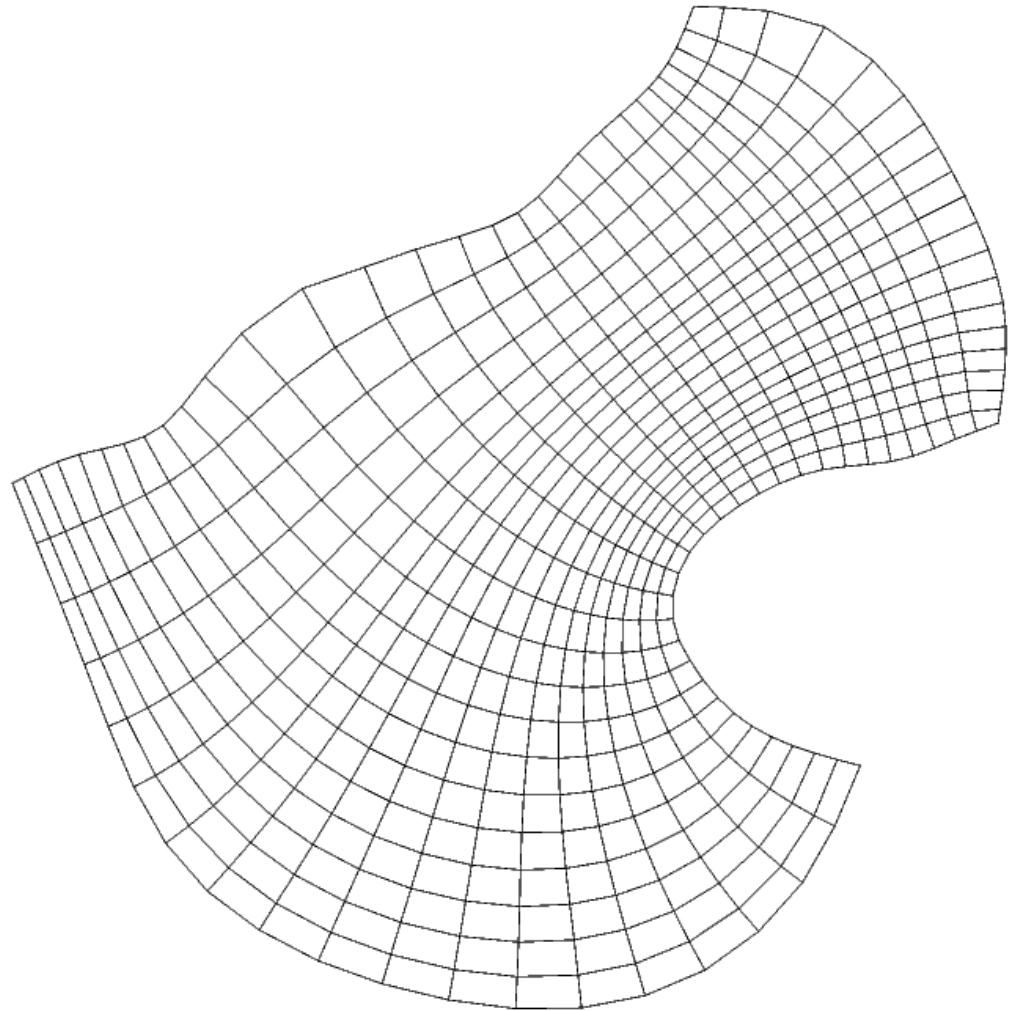
$$\nabla \cdot \vec{a} = mn \left[ \frac{\partial}{\partial \xi} \left( \frac{a}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{b}{m} \right) \right]$$

$$\nabla \times \vec{a} = mn \begin{vmatrix} \frac{\hat{\xi}_1}{m} & \frac{\hat{\xi}_2}{n} & \hat{k} \\ \frac{\partial}{\partial \xi} & \frac{\partial}{\partial \eta} & \frac{\partial}{\partial z} \\ \frac{a}{m} & \frac{b}{n} & c \end{vmatrix}$$

$$\nabla^2 \phi = \nabla \cdot \nabla \phi = mn \left[ \frac{\partial}{\partial \xi} \left( \frac{m}{n} \frac{\partial \phi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{n}{m} \frac{\partial \phi}{\partial \eta} \right) \right]$$

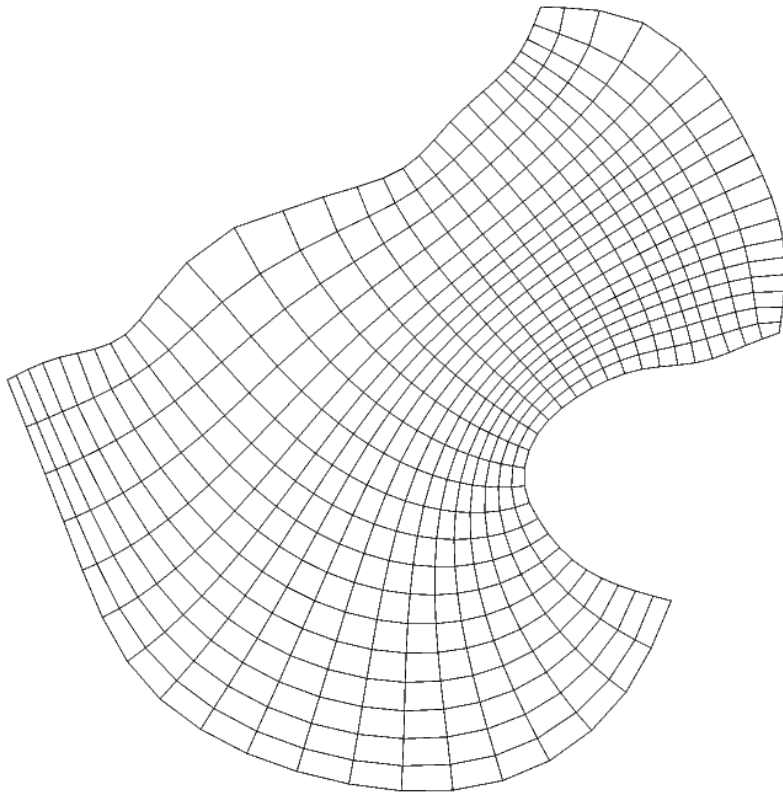
# Horizontal curvilinear grid

- This is a possible grid:



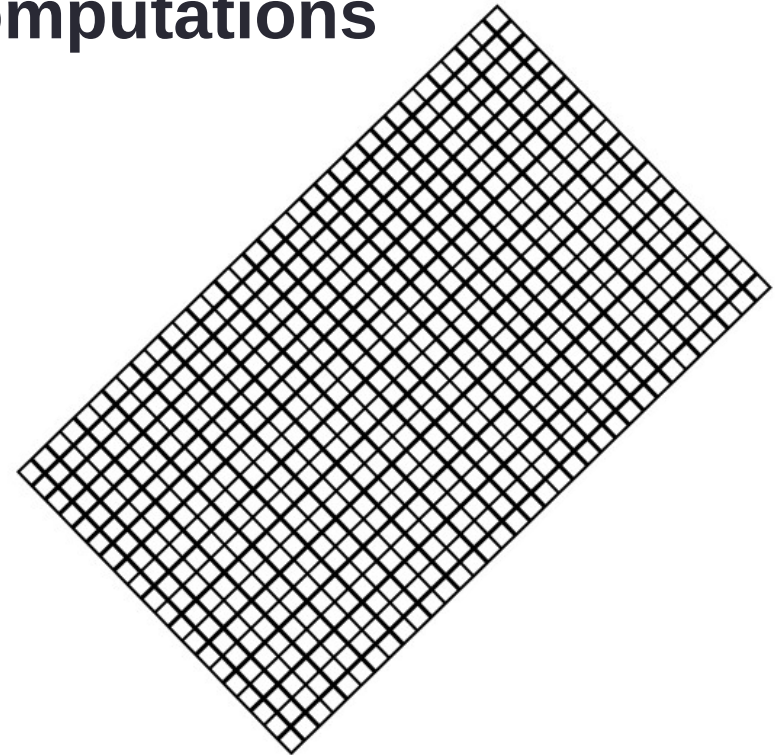
# Horizontal curvilinear grid

- This is a possible grid:



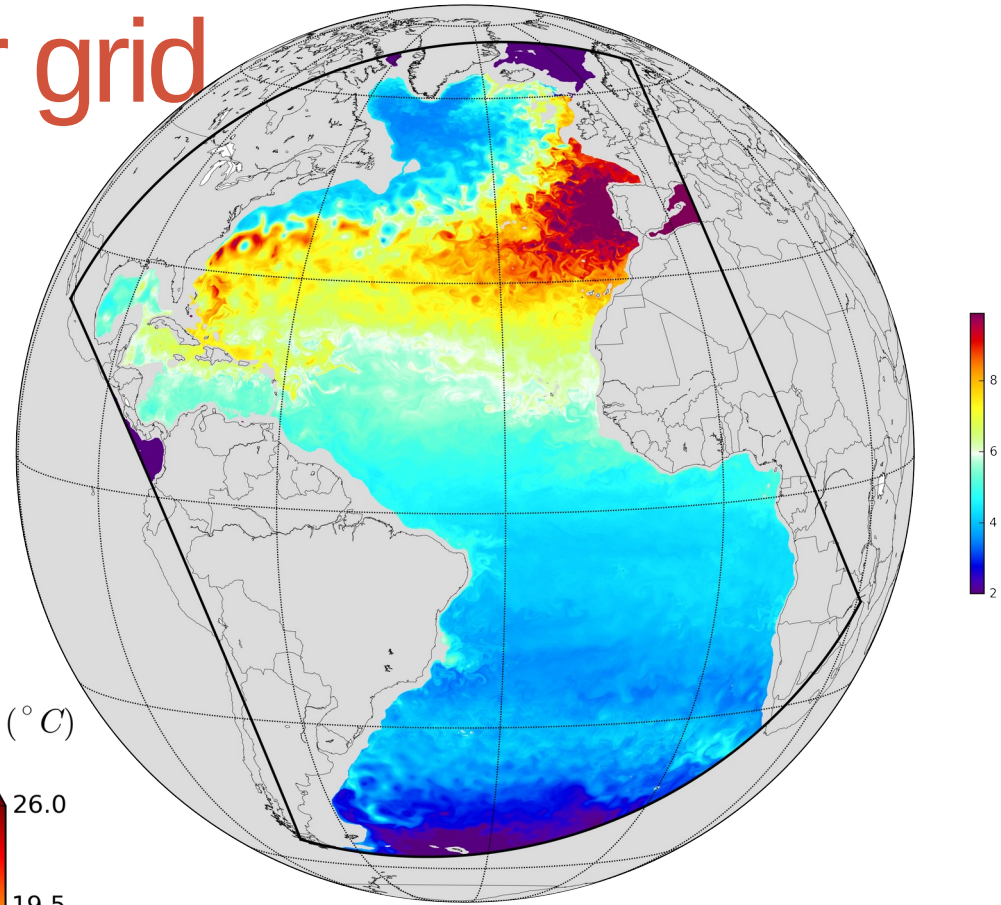
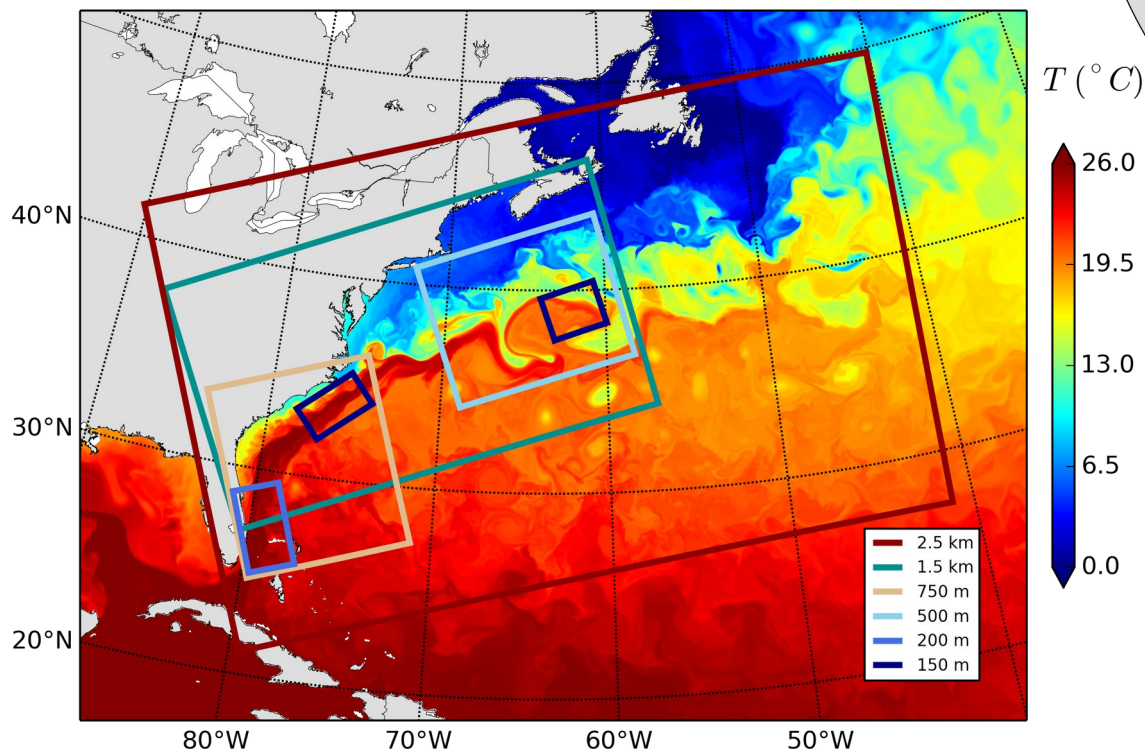
In practice variations in  $dx$  and  $dy$  should be minimized to minimize errors and optimize computation time.

**So avoid extreme distortions and be as close as rectangular grids as possible (+ use land masks) to optimise computations**



# Horizontal curvilinear grid

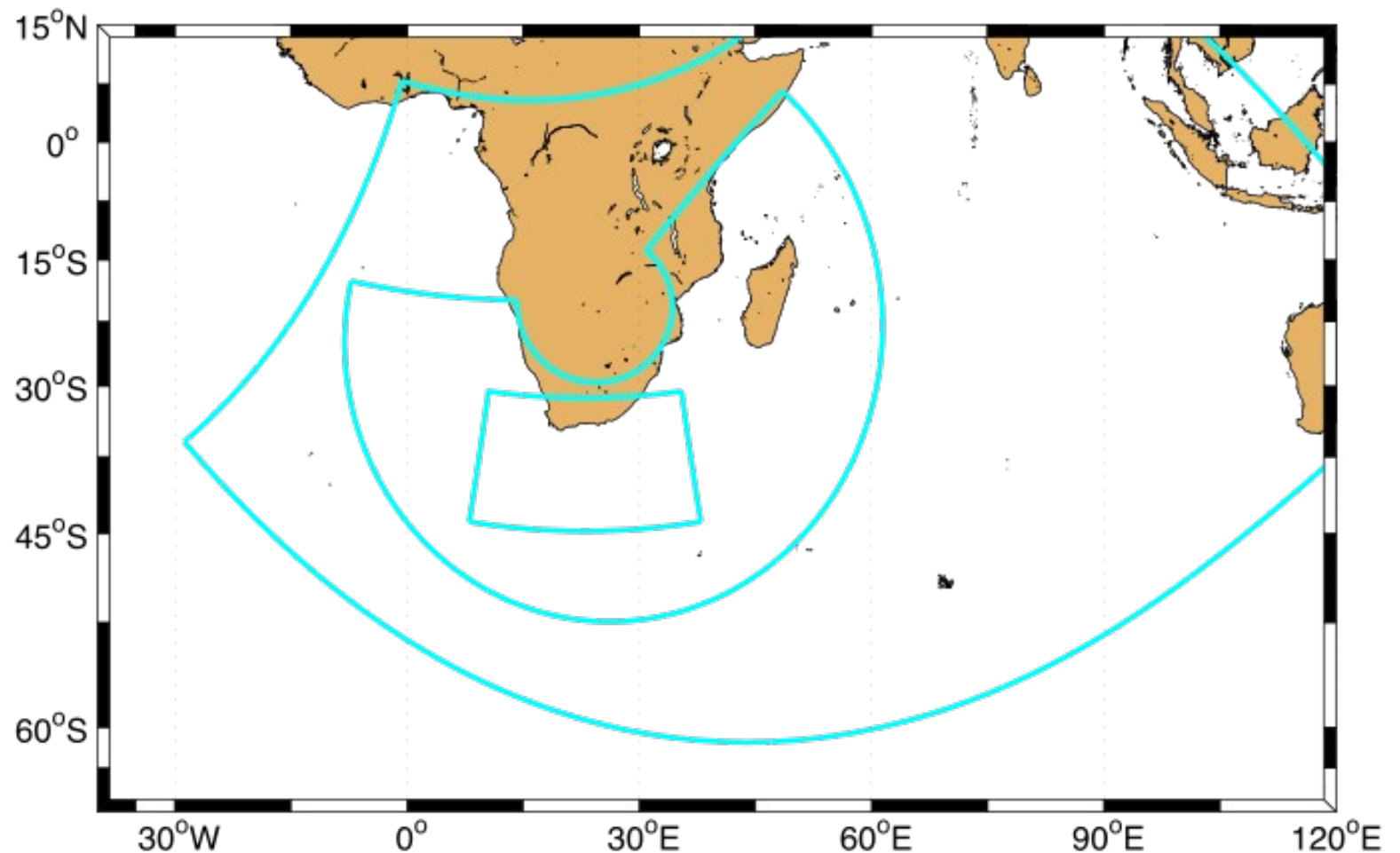
- Example of realistic domains:





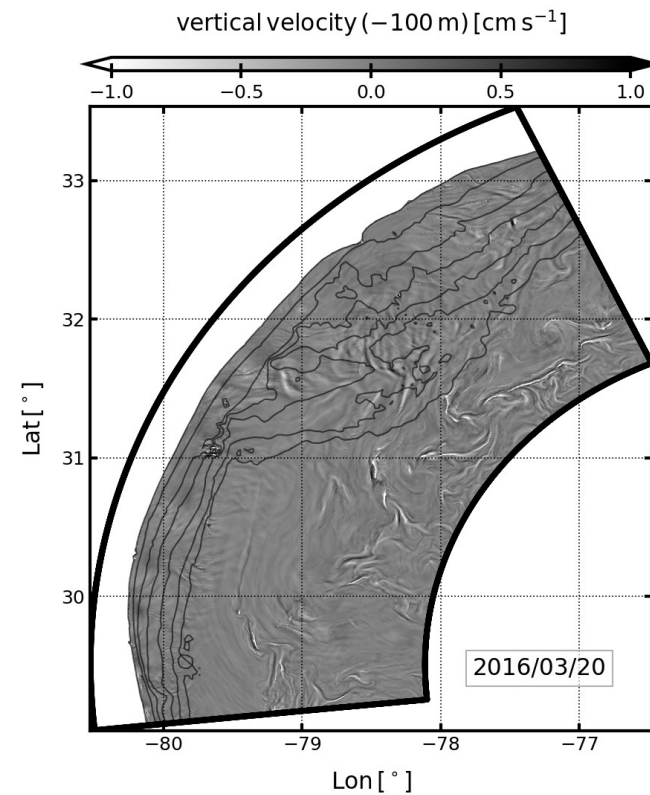
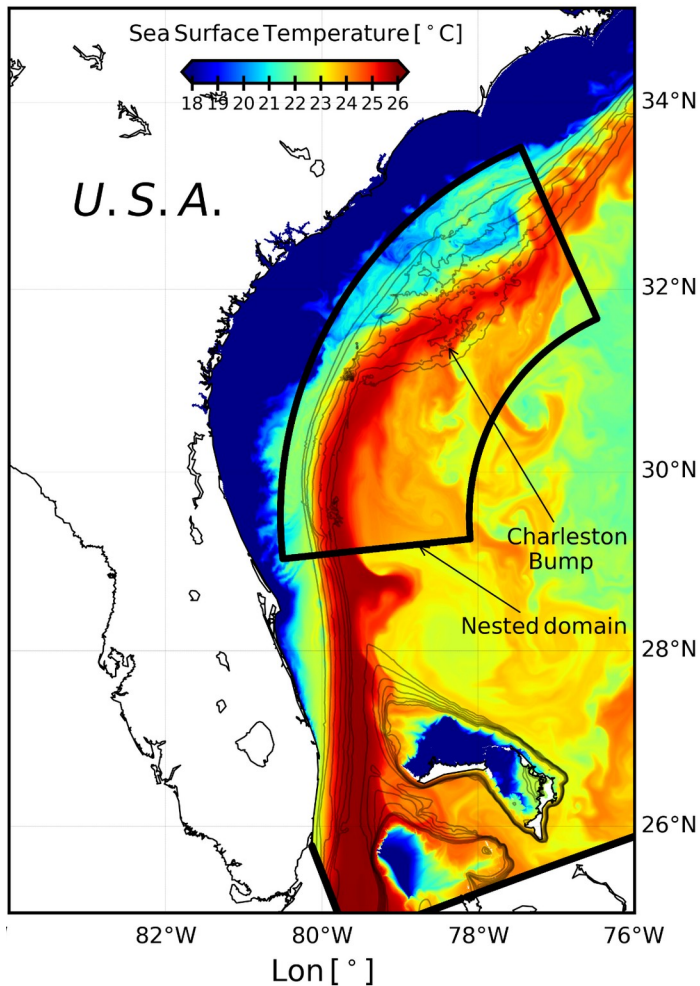
# Horizontal curvilinear grid

- Example of realistic domains (with gentle bendings):



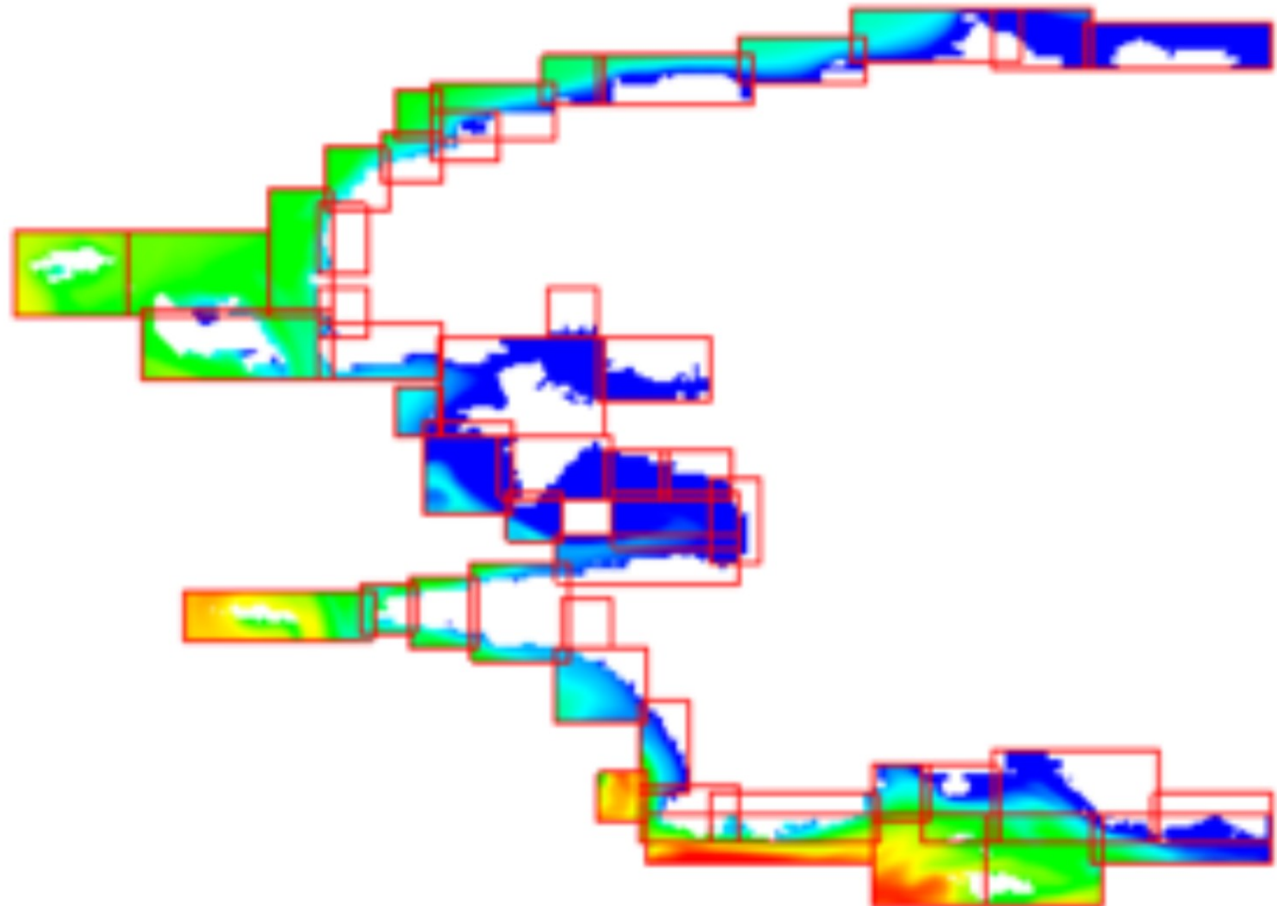
# Horizontal curvilinear grid

- Example of realistic domains (with gentle bendings):



# Horizontal curvilinear grid

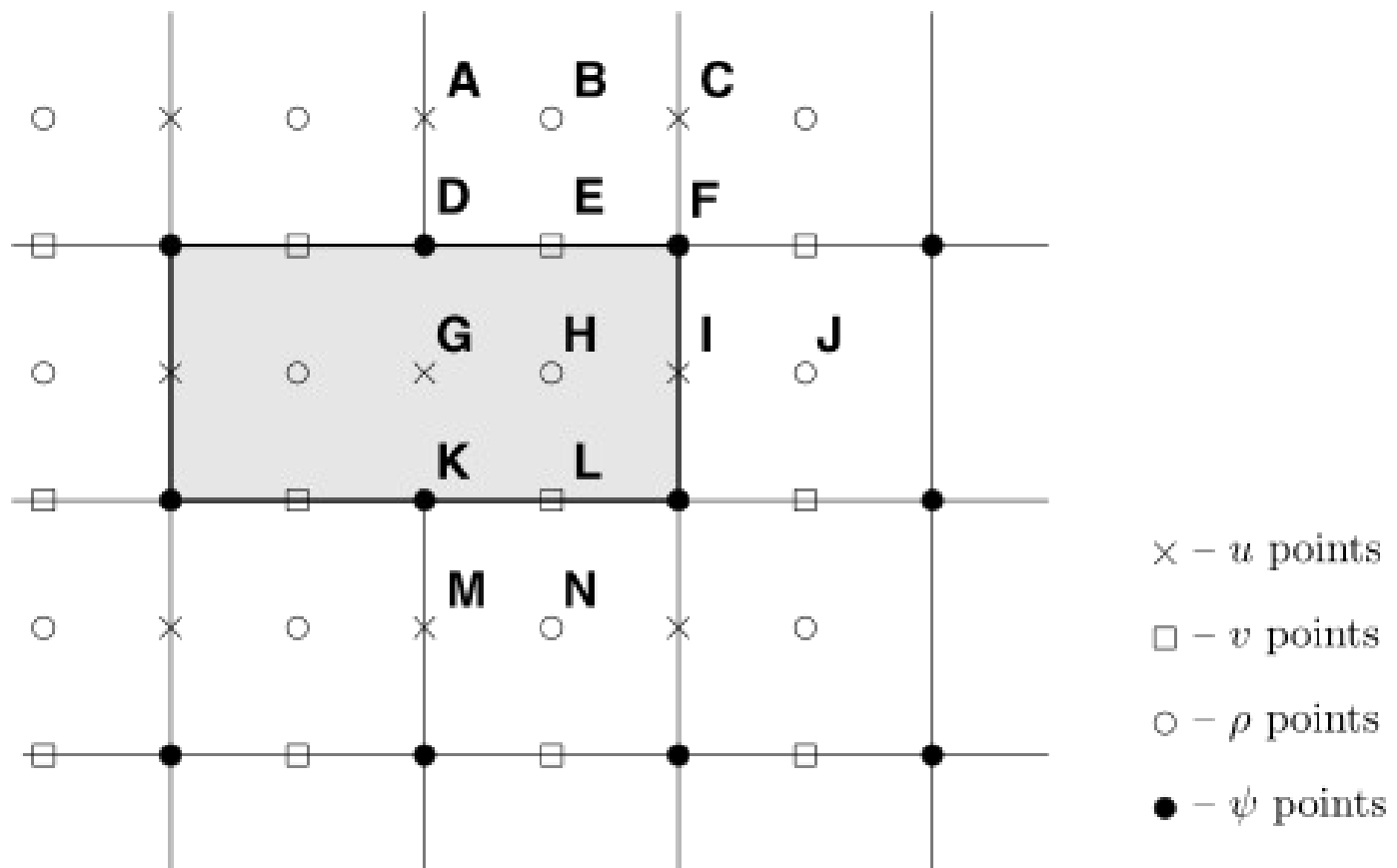
- Another method = massive multigrain





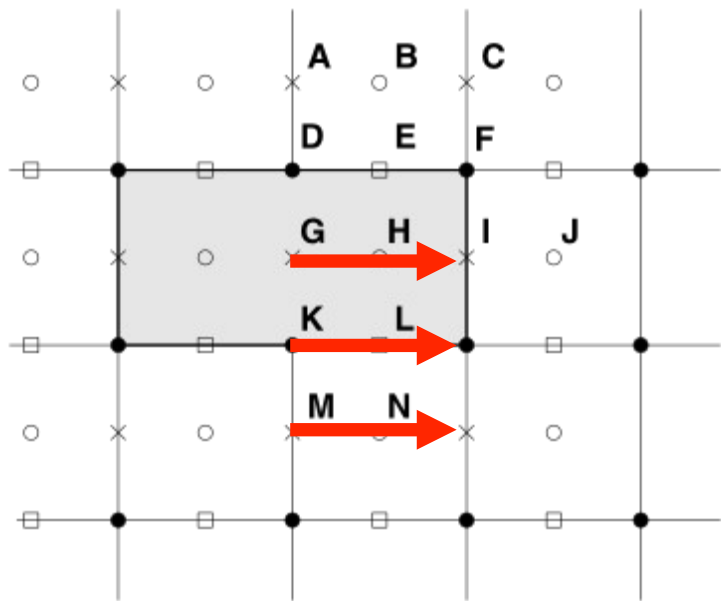
# Land/sea Mask

Variables within the masked region are set to zero by multiplying by the mask for either the  $u$ ,  $v$  or  $\rho$  points :

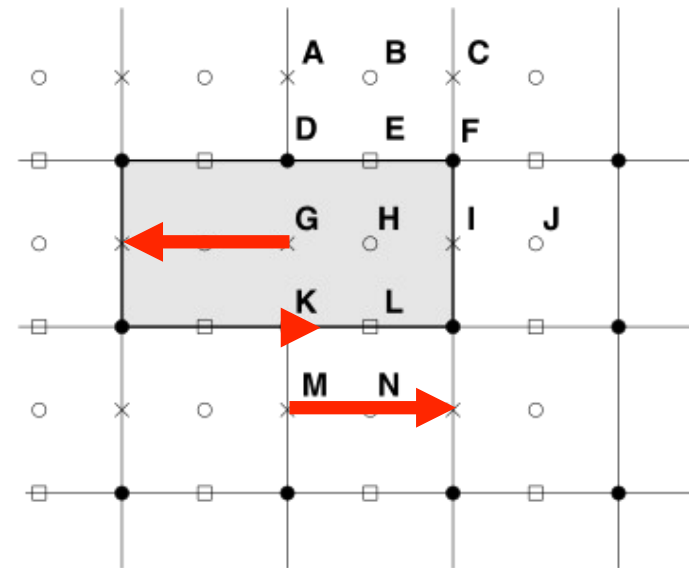


# Land/sea Mask

## Free-slip versus No-Slip



$\times$  -  $u$  points  
 $\square$  -  $v$  points  
 $\circ$  -  $\rho$  points  
 $\bullet$  -  $\psi$  points



$\times$  -  $u$  points  
 $\square$  -  $v$  points  
 $\circ$  -  $\rho$  points  
 $\bullet$  -  $\psi$  points

# Land/sea Mask

Variables within the masked region are set to zero by multiplying by the mask for either the u, v or rho points :



# Land/sea Mask

See the code routines:

```
#ifndef MASKING
# define SWITCH *
#else
# define SWITCH !
#endif

!#####

    do k=1,N
        do i=IstrU,Iend
            u(i,j,k,nnew)=(DC(i,k)-DC(i,0)) SWITCH umask(i,j)
```

## Activity 2 – Run an idealized ocean basin II

SSH

