FLUIDS 2

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II. INSTABILITIES

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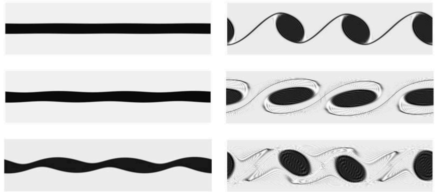
II. INSTABILITY

II.1. Concept of stability

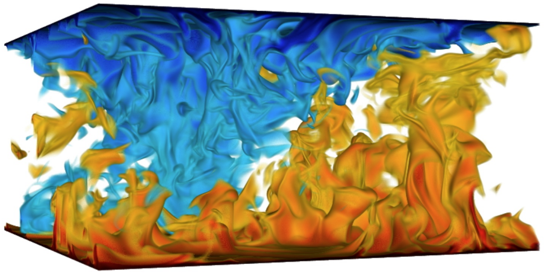
II.2. Kelvin-Helmholtz Instability



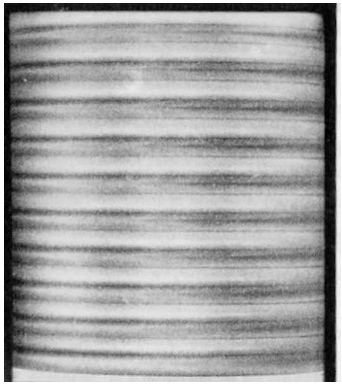
II.3. Parallel Shear instability

II.4. Convective instability

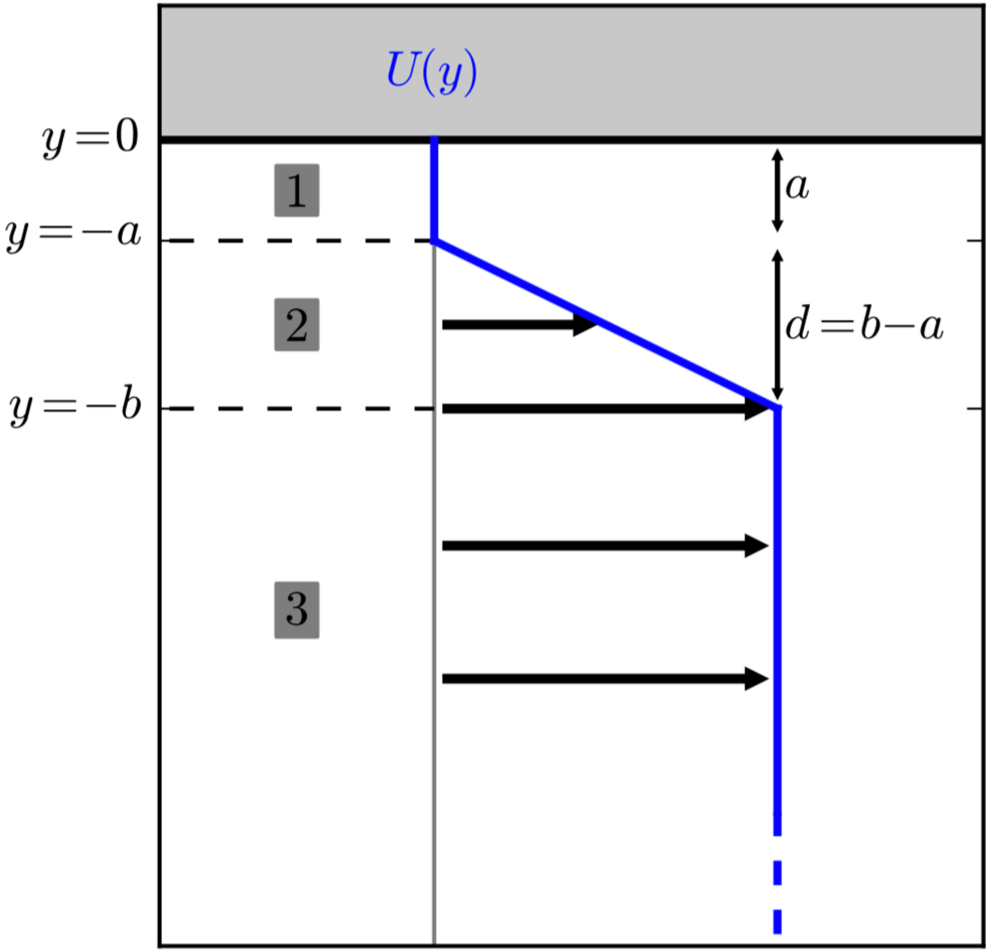
(Rayleigh–Bénard)



II.5. Taylor–Couette

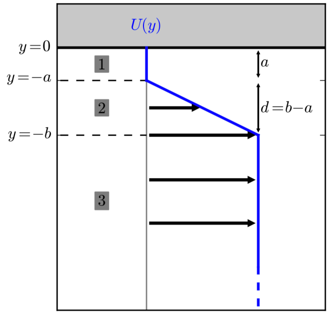


Rayleigh shear instability



$$\begin{cases} U_1 = 0, & -a < y < 0 \\ U_2 = -\frac{U_0}{d}(y+b) + U_0, & -b < y < -a \\ U_3 = U_0, & y < -b \end{cases}$$

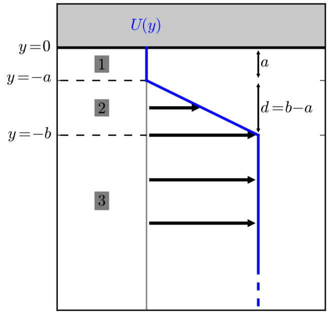
Rayleigh shear instability



- Solutions are:

$$
\begin{cases}
\psi_1 = A\,\mathrm{e}^{-k(y+a)} + B\,\mathrm{e}^{k(y+a)}, & -a < y < 0 \\
\psi_2 = C\,\mathrm{e}^{k(y+a)} + D\,\mathrm{e}^{k(y+b)}, & -b < y < -a \\
\psi_3 = E\,\mathrm{e}^{k(y+b)}, & y < -b
\end{cases}
$$

Rayleigh shear instability



- With matching conditions

1. Continuity of pressure across interfaces (namely, at $y = -a$ and $y = -b$) :

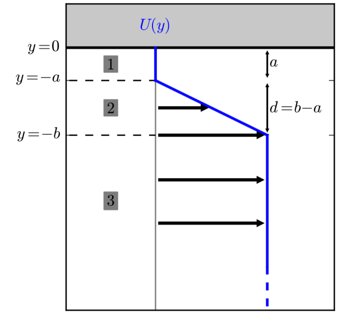
$$\Delta\left[(U-c)\frac{\partial\psi}{\partial y} - \psi\frac{\partial U}{\partial y}\right] = 0,$$

2. Continuity of normal velocity across interfaces :

$$\Delta\left[\frac{\psi}{U-c}\right] = 0,$$

II.3. Parallel Shear instability

Rayleigh shear instability



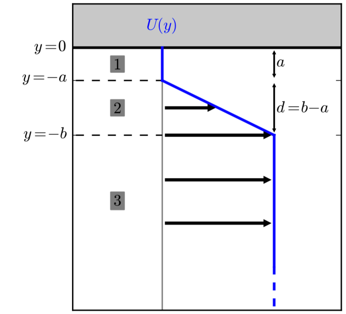
- With matching conditions

$$
\begin{aligned}
A(kc) + B(-kc) + C\left(kc - \tfrac{U_0}{d}\right) + D\left[\left(kc - \tfrac{U_0}{d}\right) \mathrm{e}^{-kd}\right] &= 0 \\
A + B - C - D\,\mathrm{e}^{-kd} &= 0 \\
C\left[\left((U_0 - c)k + \tfrac{U_0}{d}\right) \mathrm{e}^{-k(b-a)}\right] + D\left((U_0 - c)k + \tfrac{U_0}{d}\right) + E\left(-(U_0 - c)k\right) &= 0 \\
C\,\mathrm{e}^{-kd} + D - E &= 0
\end{aligned}
$$

$$A\,\mathrm{e}^{-ka} + B\,\mathrm{e}^{-ka} = 0$$

Rayleigh shear instability

- With matching conditions



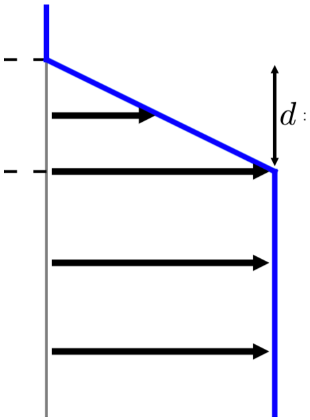
This set of 5 homogeneous equations (5-6) may be written in the form of a matrix equation :

$$\mathbf{M}\ (A\ B\ C\ D\ E)^T = 0 \tag{7}$$

with :

$$\mathbf{M} = \begin{pmatrix} \mathrm{e}^{ka} & \mathrm{e}^{-ka} & 0 & 0 & 0 \\ kc & -kc & kc - \frac{U_0}{d} & \left(kc - \frac{U_0}{d}\right)\mathrm{e}^{-kd} & 0 \\ 1 & 1 & -1 & -\mathrm{e}^{-kd} & 0 \\ 0 & 0 & \left((U_0 - c)k + \frac{U_0}{d}\right)\mathrm{e}^{-kd} & (U_0 - c)k + \frac{U_0}{d} & -(U_0 - c)k \\ 0 & 0 & \mathrm{e}^{-kd} & 1 & -1 \end{pmatrix} \tag{8}$$

Rayleigh shear instability



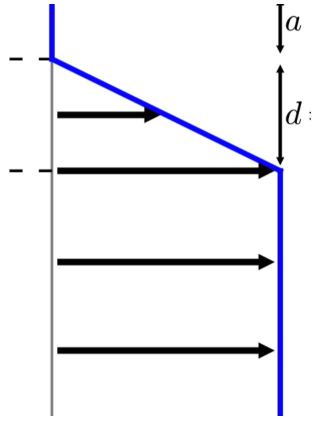
The determinant of the matrix must be zero for non-trivial solutions

(Assuming the wall is far away):

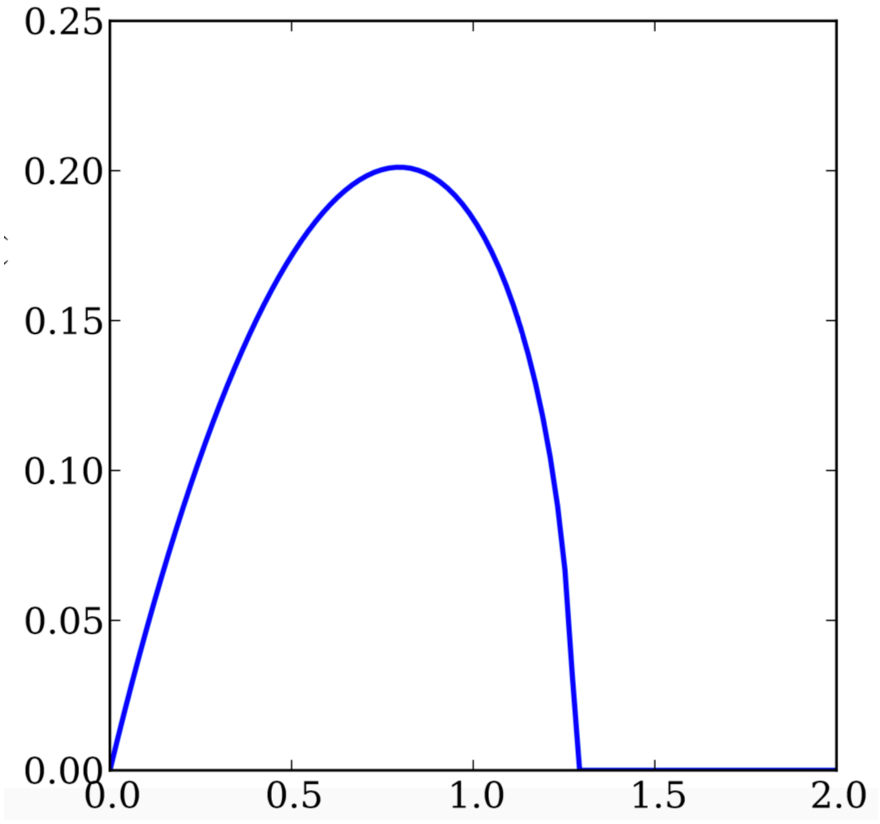
$$\det(\mathbf{M}') = \frac{4e^{2dk}}{d^2}\left[d^2k^2c^2 + \frac{U_0^2}{4}\left(e^{-2dk} - (1-dk)^2\right)\right]$$

$$\det(\mathbf{M}') = 0 \iff c^2 = \frac{U_0^2}{4d^2k^2}\left[(1-dk)^2 - e^{-2dk}\right]$$

Rayleigh shear instability



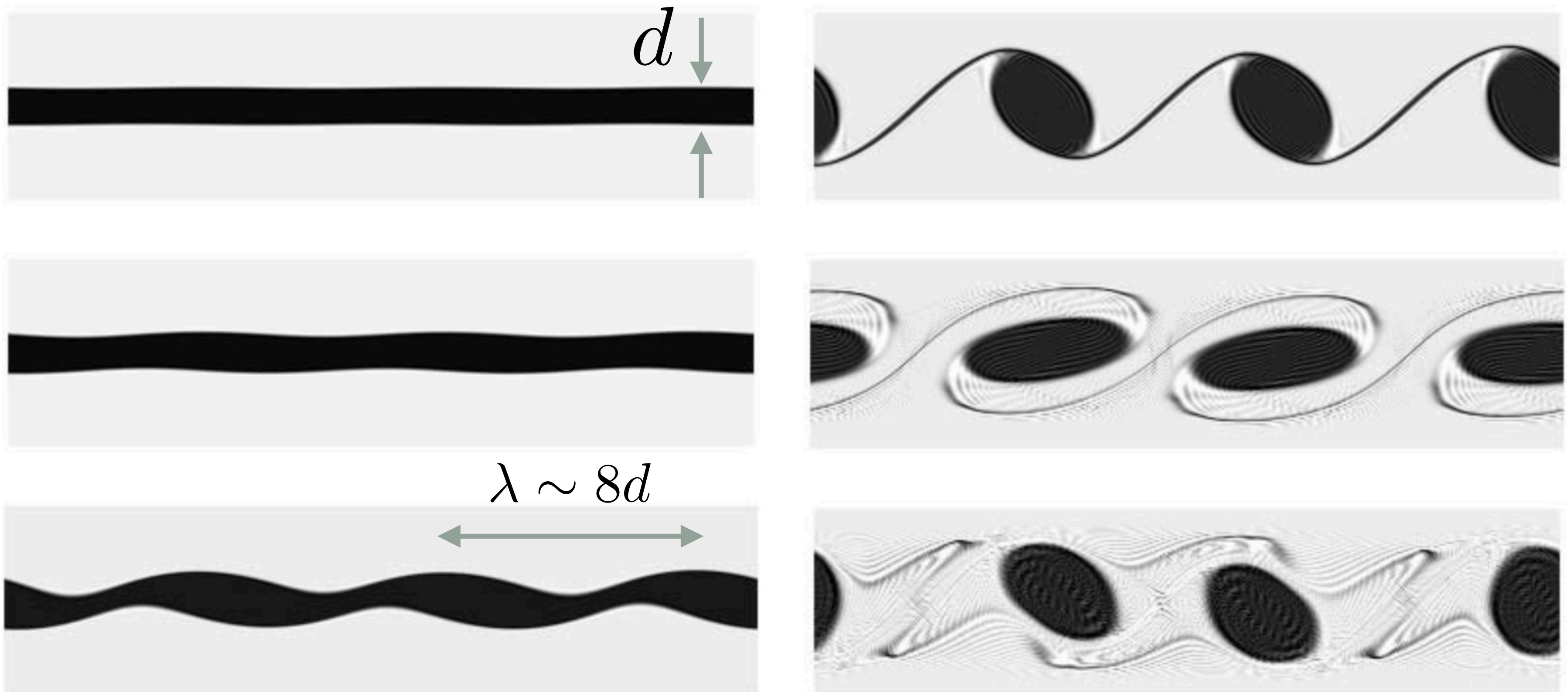
Growth rate as a function of wavenumber:



$$d\,k \sim 0.8$$

$$\lambda \sim 8d$$

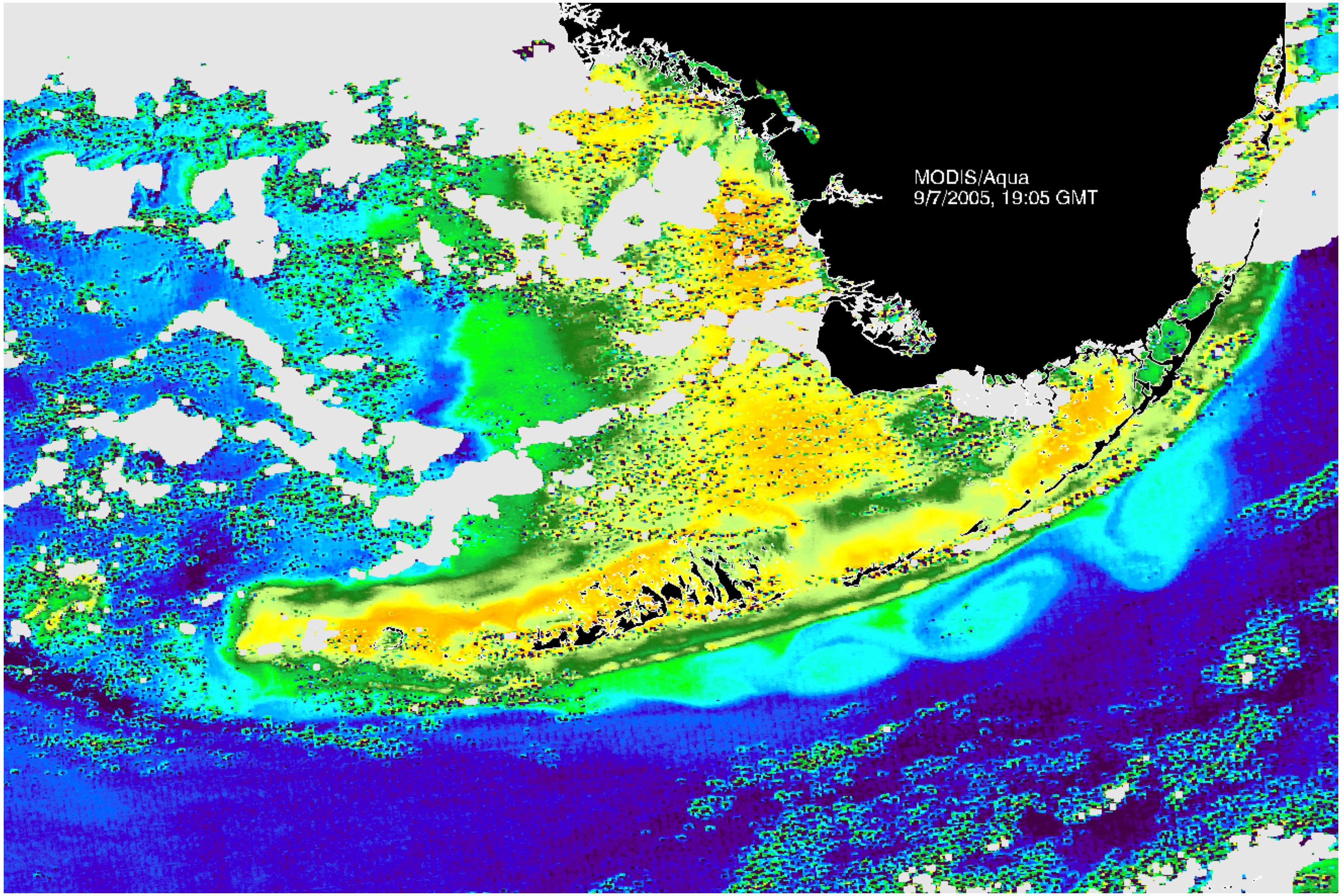
Rayleigh shear instability



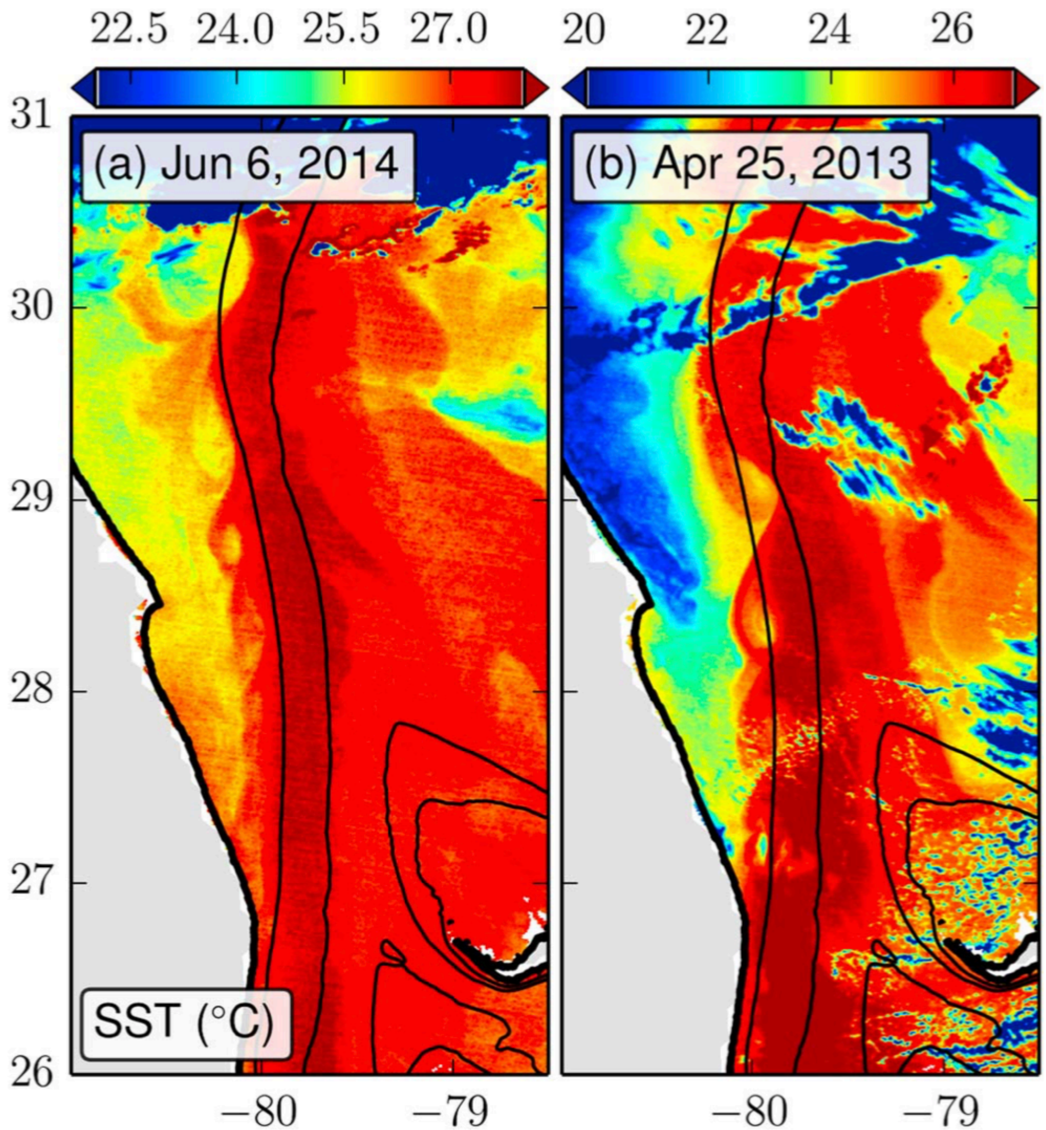
II.3. Parallel Shear instability

Realistic flows

Satellite Chlorophyl observations along the Florida Keys



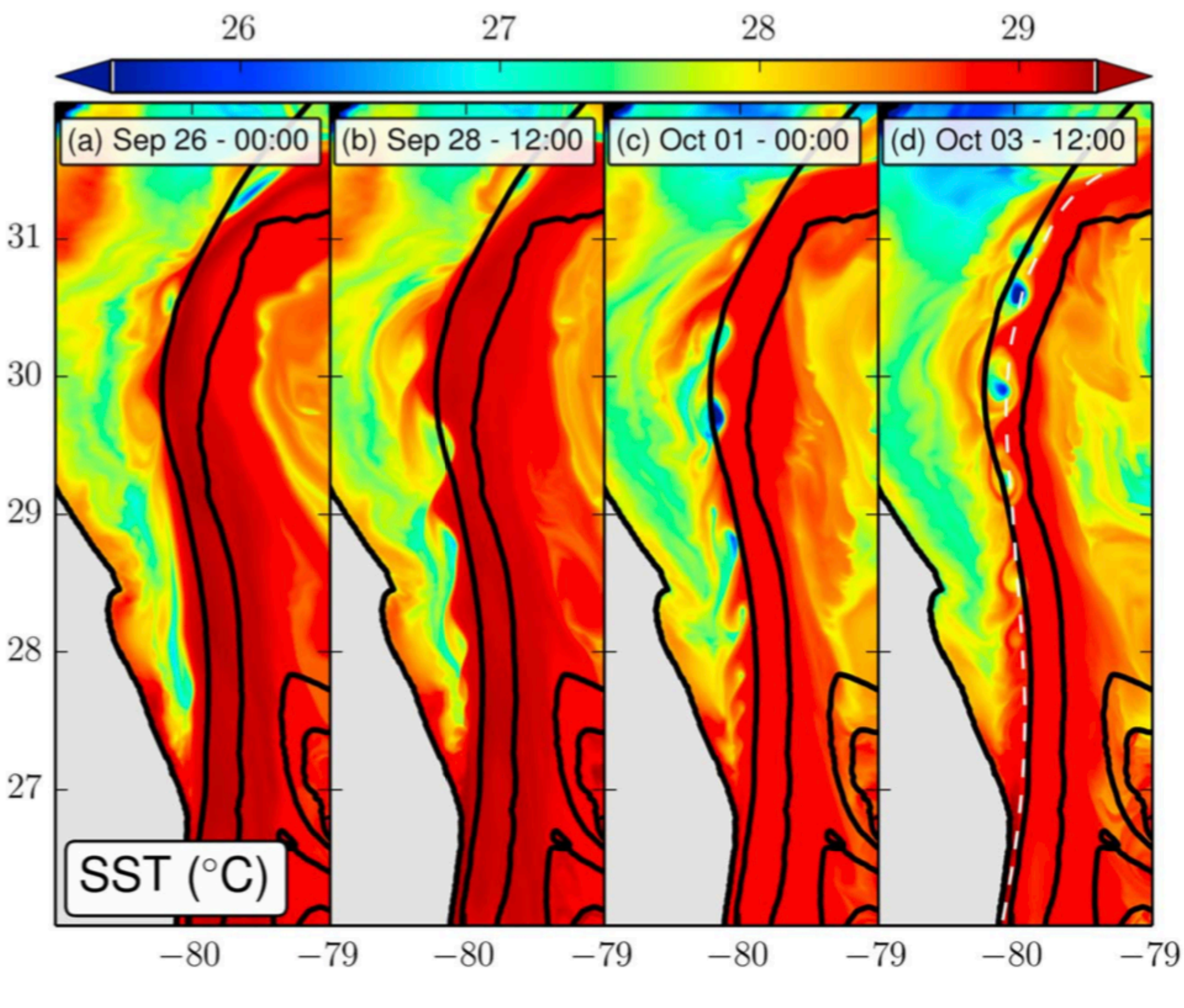
Realistic flows



Satellite SST observations in the Florida Strait

II.3. Parallel Shear instability

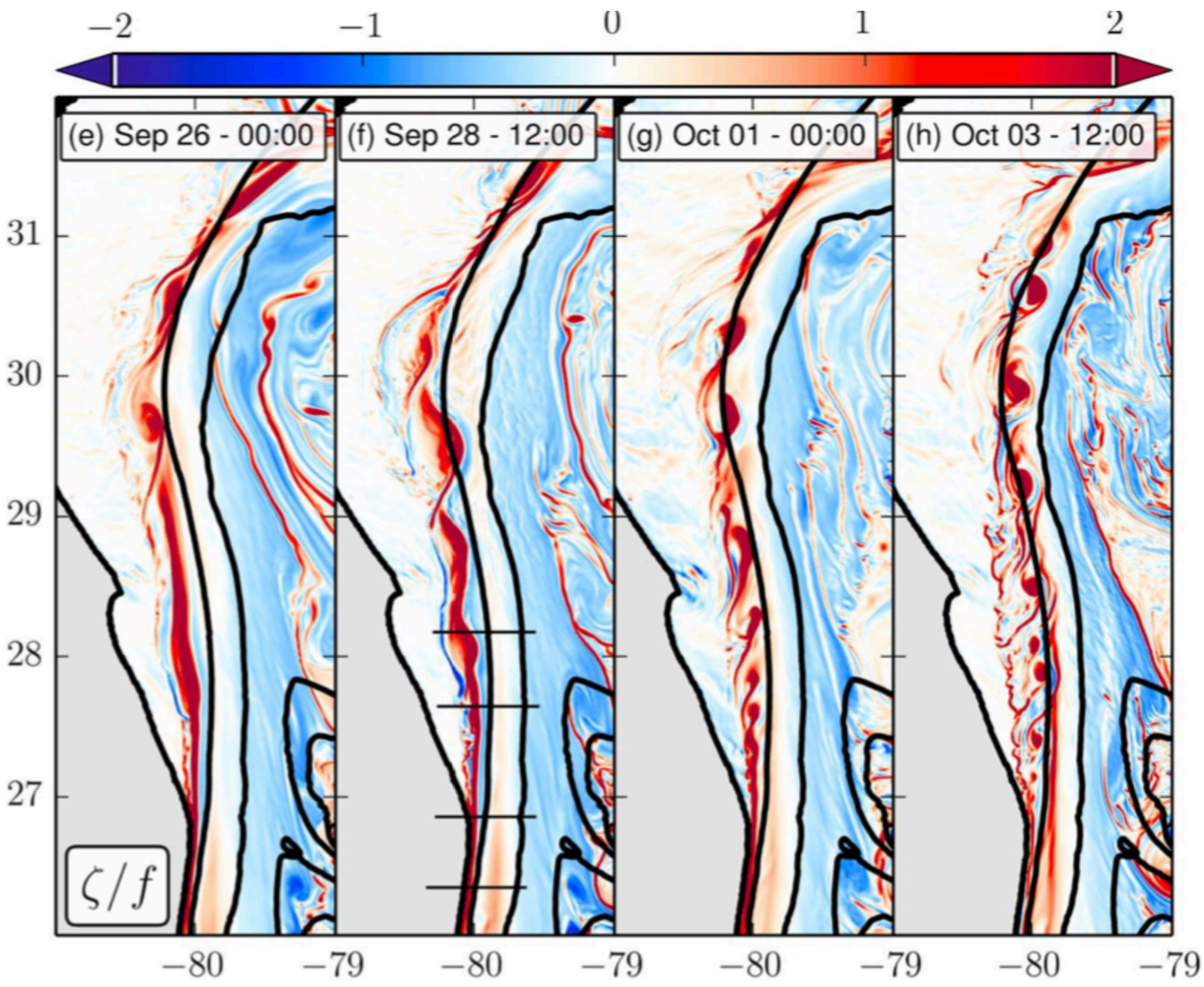
Realistic flows



Model simulations

II.3. Parallel Shear instability

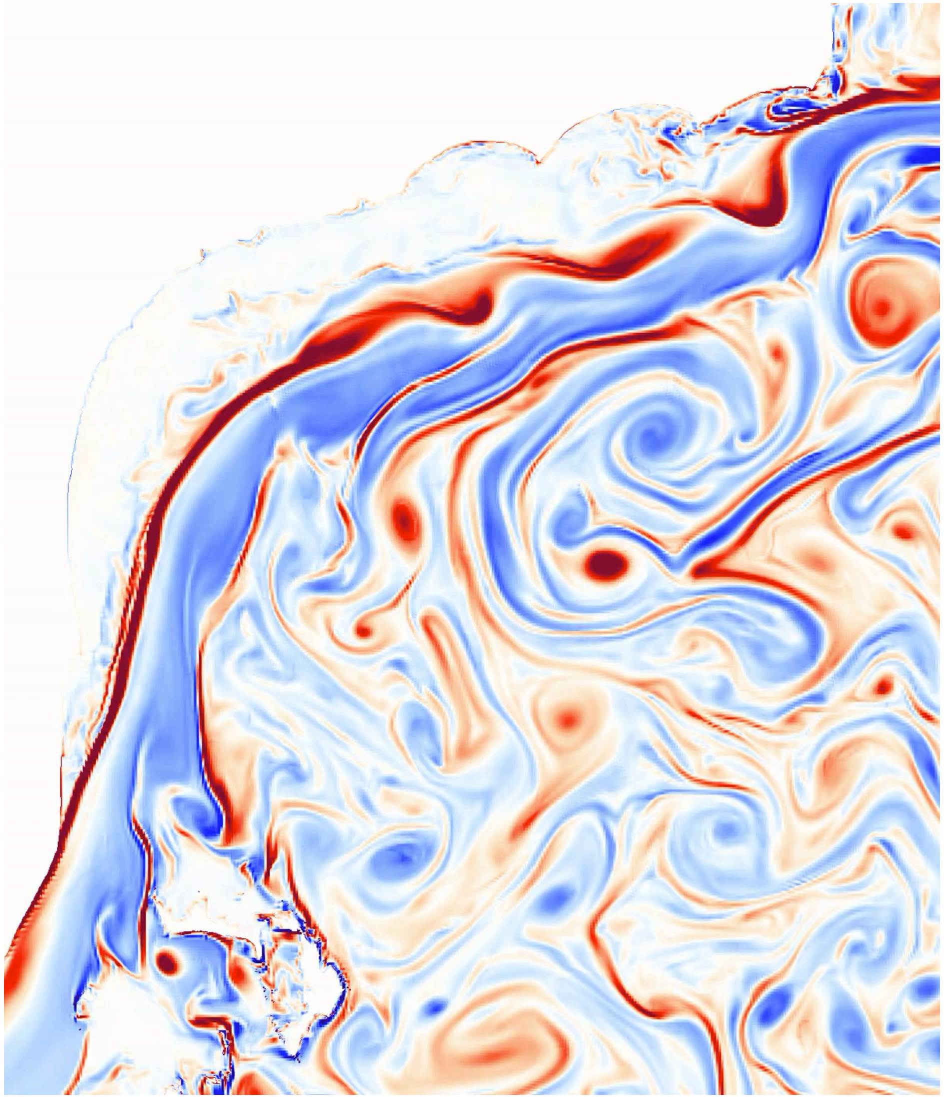
Realistic flows



Model simulations

II.3. Parallel Shear instability

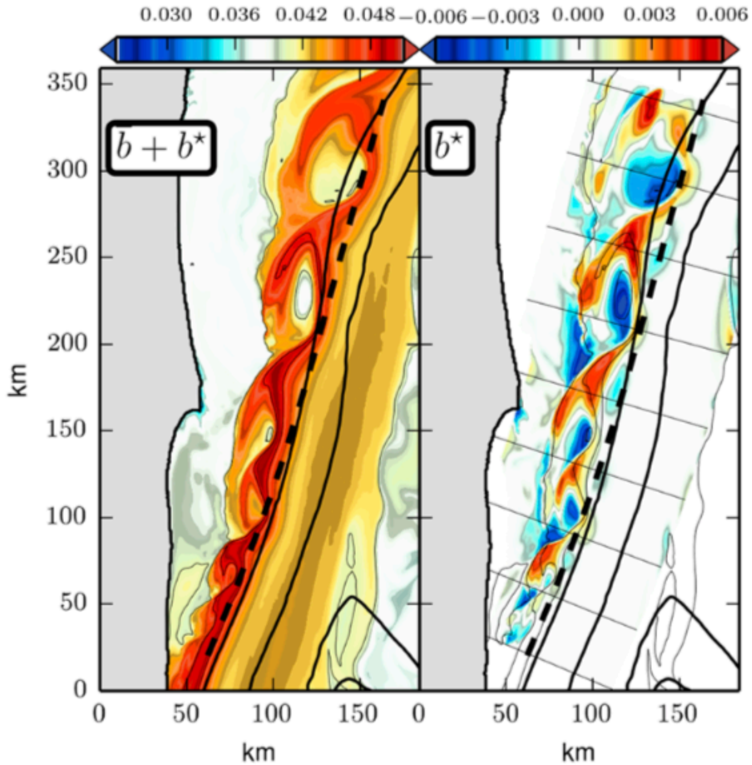
Realistic flows

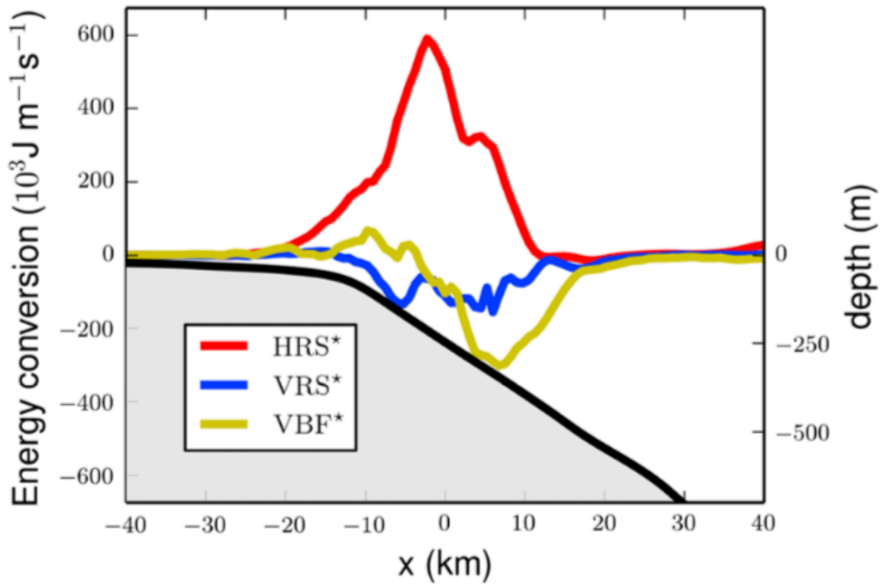


Model simulations

II.3. Parallel Shear instability

Realistic flows



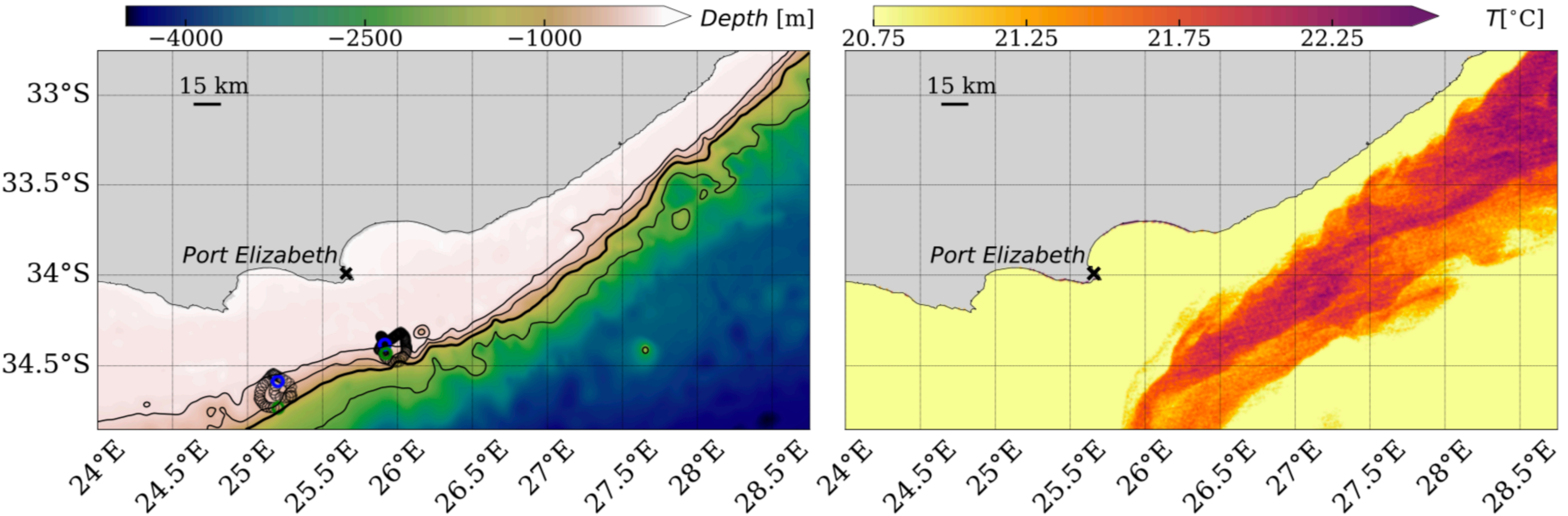


$$\mathrm{HRS} = -\langle \mathbf{u}' v' \rangle \frac{\partial \langle \mathbf{u} \rangle}{\partial y} - \langle \mathbf{u}' u' \rangle \frac{\partial \langle \mathbf{u} \rangle}{\partial x}$$

II.3. Parallel Shear instability

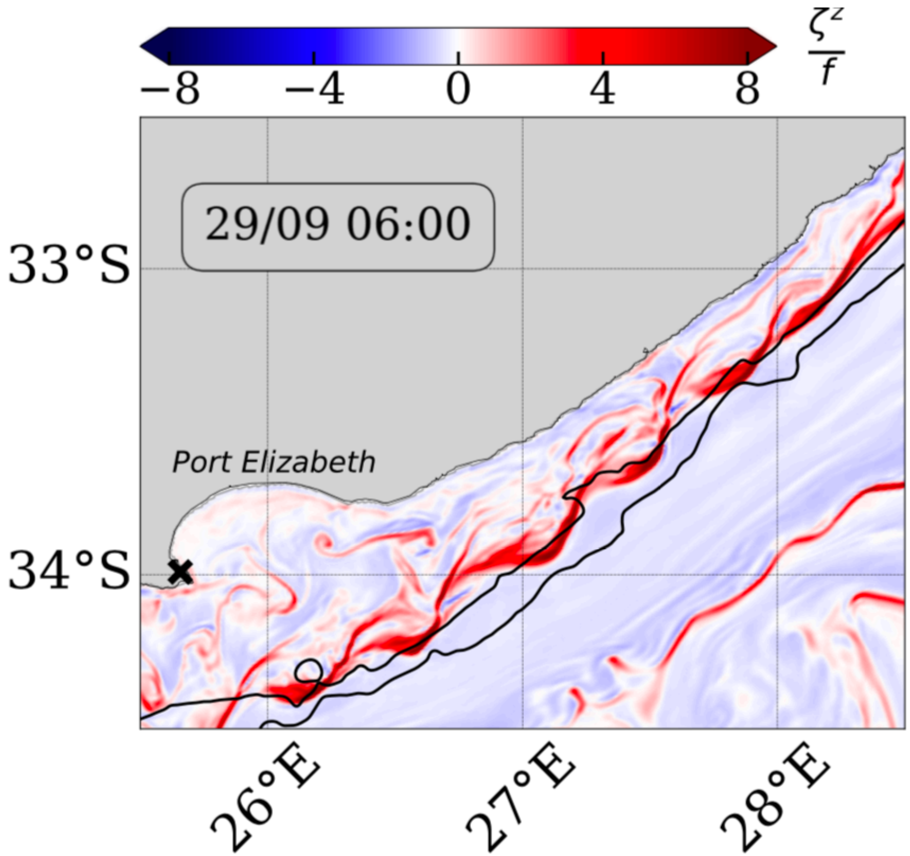
Realistic flows

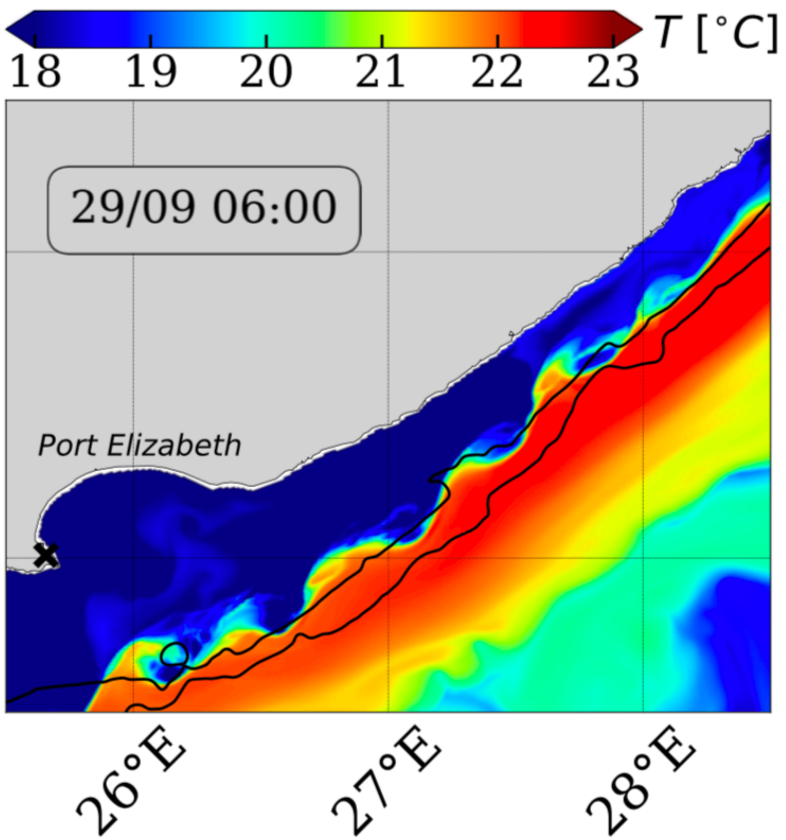
Satellite SST observations
in the Agulhas Current



Realistic flows

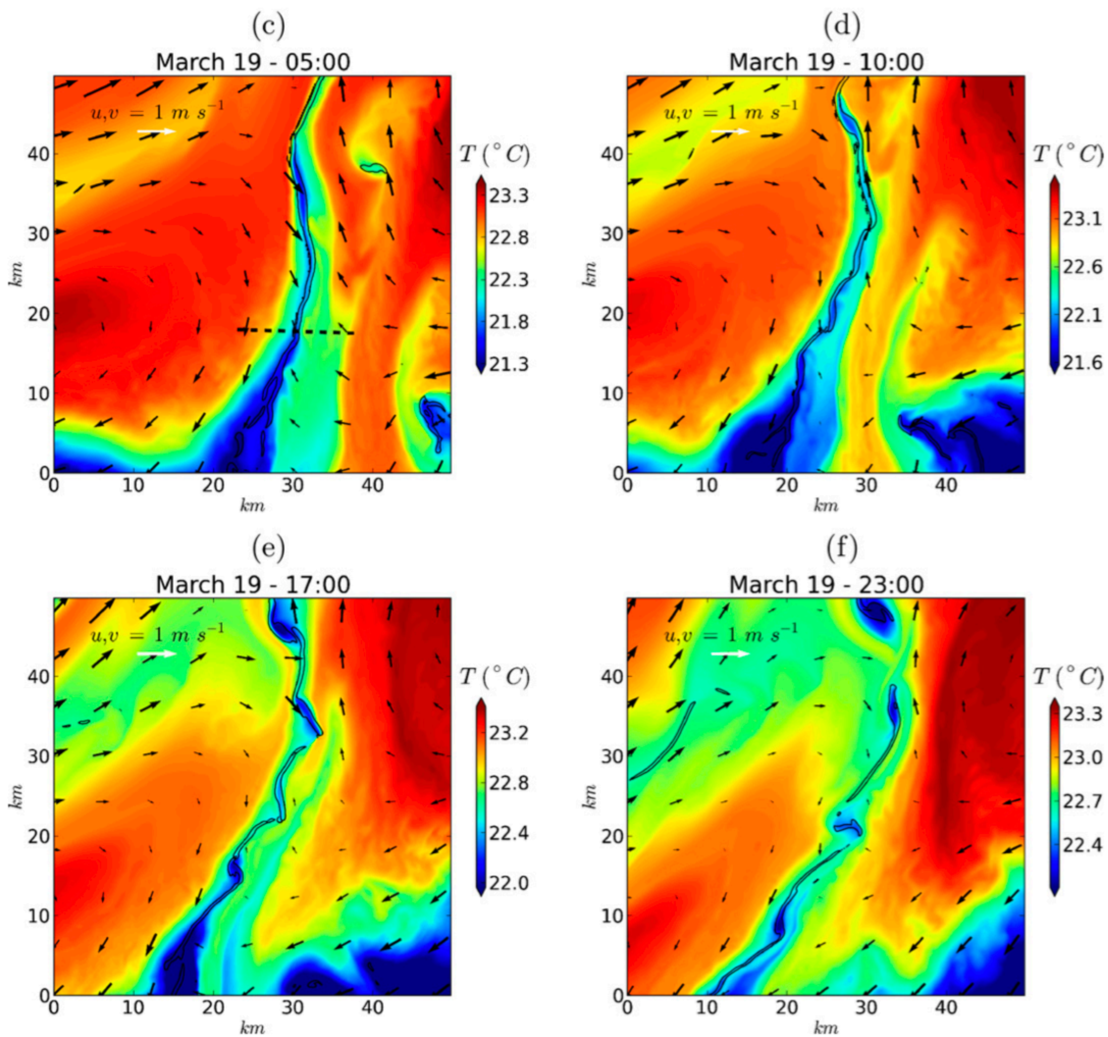
Simulation of the
Agulhas Current

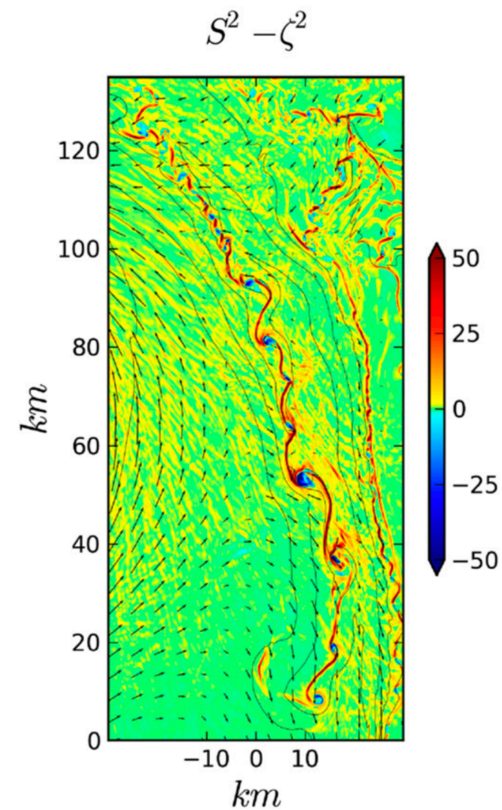




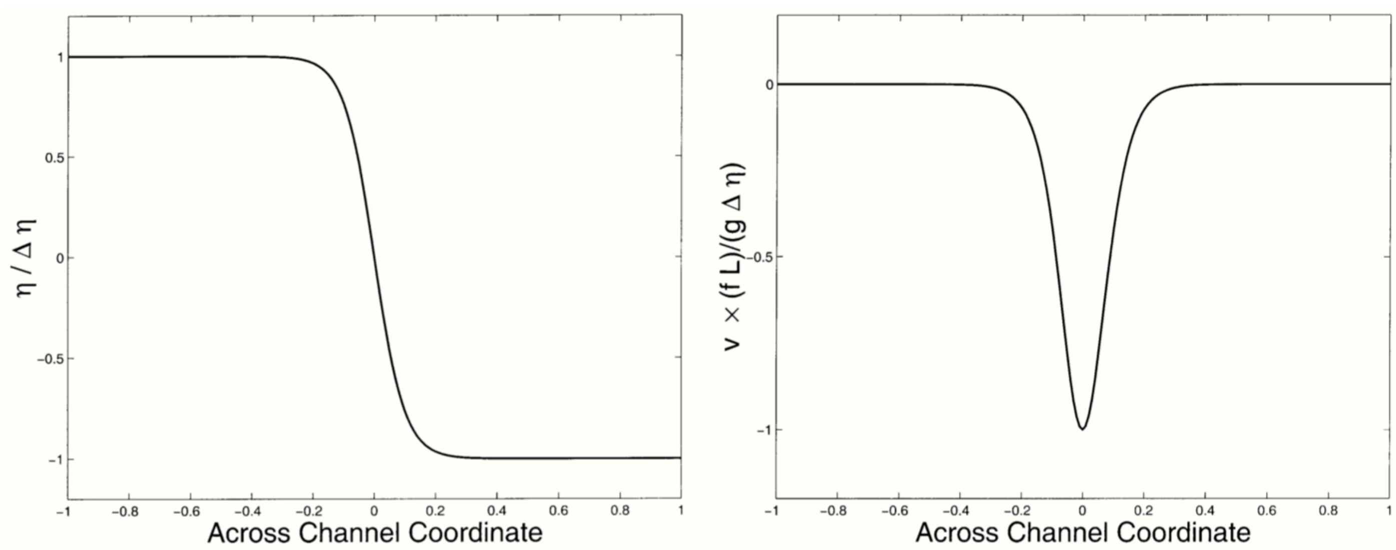
Realistic flows

Instability of a surface vorticity filament





The Bickley Jet



$$\overline{\eta} = -\Delta\eta \tanh\left(\frac{x}{L}\right)$$

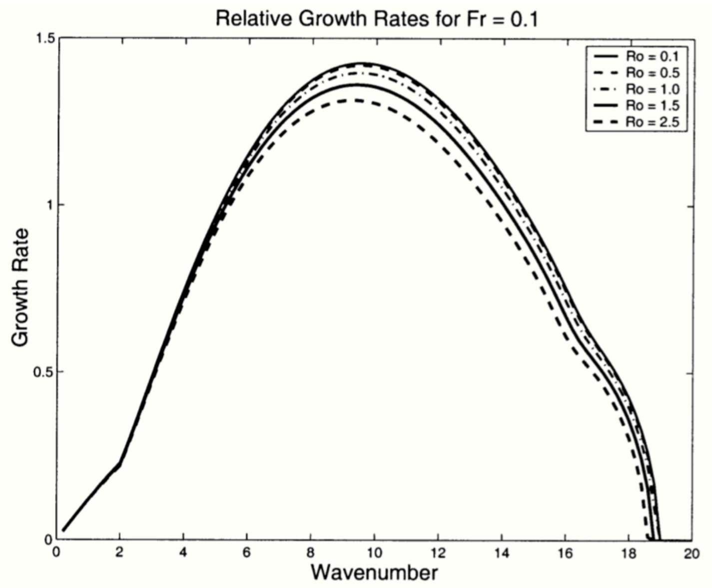
$$\overline{u} = 0$$

$$\overline{v} = -\frac{g'\Delta\eta}{fL} \operatorname{sech}^2\left(\frac{x}{L}\right).$$

See Poulin et Flierl, JFM, 2003

The Bickley Jet

$$\begin{bmatrix} \bar{v} & \left(\dfrac{dH}{dx} + H\dfrac{d}{dx}\right) & H \\ -\dfrac{g'}{k^2}\dfrac{d}{dx} & \bar{v} & \dfrac{f}{k^2} \\ g' & \left(\dfrac{d\bar{v}}{dx} + f\right) & \bar{v} \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{u} \\ \hat{v} \end{bmatrix} = c \begin{bmatrix} \hat{\eta} \\ \hat{u} \\ \hat{v} \end{bmatrix}$$



The Bickley Jet