Numerical Modelling

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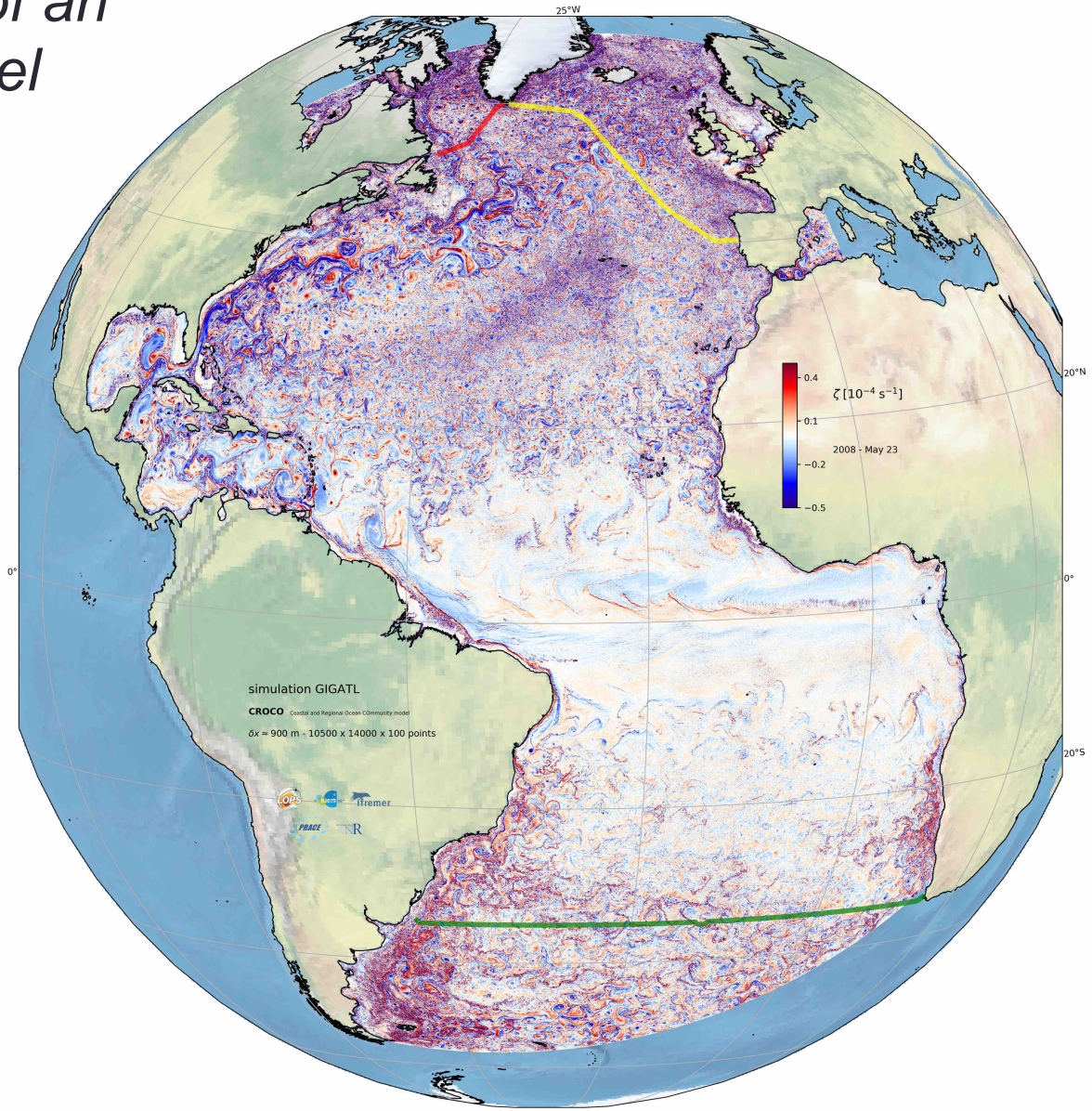
the anatomy of an ocean model



- **Lesson 1 :**
  - Introduction
  - Equations of motions
  - *Activity 1 [run an ocean model]*

- **Lesson 2 :** [B 012]
  - Horizontal Discretization
  - *Activity 2 [Dynamics of an ocean gyre]*

- **Lesson 3 :** [D109]
  - Numerical schemes
  - *Activity 3 [Impacts of numerics]*

- **Lesson 4 :** [D109]
  - Vertical coordinates
  - Model parameterizations
  - *Activity 4 [Impact of topography]*

- **Lesson 5 :** [D109]
  - Boundary Forcings
  - Presentation of the model CROCO
  - *Activity 4 [Design a realistic simulation]*

- **Lesson 6 :** [D109]
  - Diagnostics and validation
  - *Activity 5 [Analyze a realistic simulation]*

- **Lesson 7 :** [D109]
  - *Work on your projet*

Presentations and material will be available at **:**

jgula.fr/ModNum/

Useful references

Extensive courses:

- MIT: https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/
- Princeton: https://stephengriffies.github.io/assets/pdfs/GFM_lectures.pdf

Overview on ocean modelling and current challenges:

- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling.http://jgula.fr/ModNum/Griffiesetal00.pdf
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. http://jgula.fr/ModNum/Griffies_Chapter.pdf
- Fox-Kemper et al, 19, “Challenges and Prospects in Ocean Circulation Models" http://jgula.fr/ModNum/FoxKemperetal19.pdf

ROMS/CROCO:

- https://www.myroms.org/wiki/
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell. http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf

#1
Which Equations?

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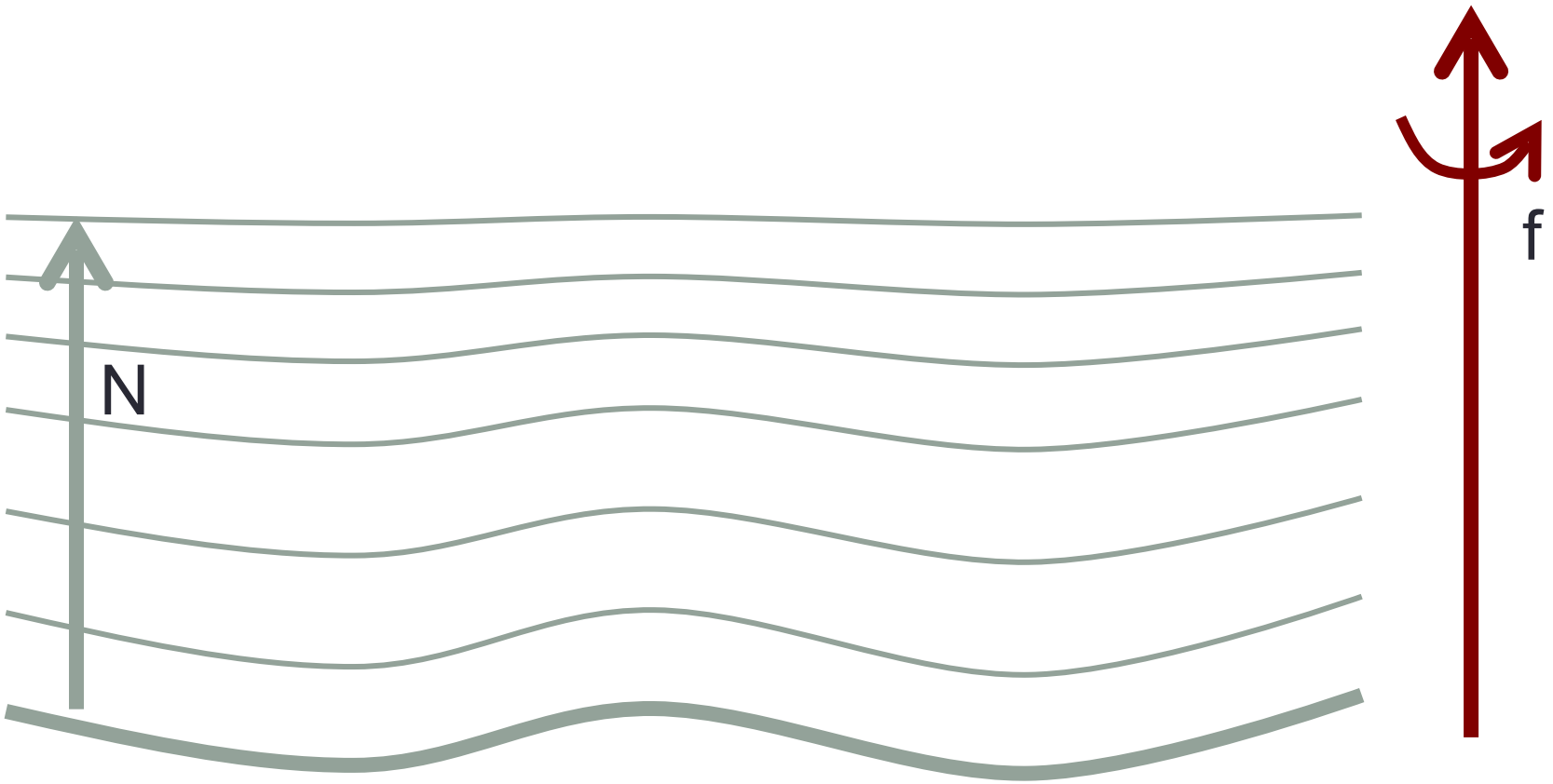
Which Equations?

Ingredients : rotation + stratification



Which Equations?

- Momentum equations (3d) $\frac{D\vec{u}}{Dt} = \ldots$
- Conservation of mass $\frac{D\rho}{Dt} = \mathcal{S}_\rho$

- Conservation of heat $\frac{DT}{Dt} = \mathcal{S}_T$
- Conservation of salinity $\frac{DS}{Dt} = \mathcal{S}_S$
- Equation of state : $\rho = \rho(T, S, p)$

[7 equations for the 7 variables: u,v,w,p,T,S,ρ]

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[7 equations for the 7 variables: u,v,w,p,T,S,ρ]

Equations for momentum/mass?

- Navier-Stokes Equations (NS)
- Non-hydrostatic Primitive Equations (NH)
- Hydrostatic Primitive Equations (PE)
- Shallow-water (SW)
- Quasi-geostrophic (QG)
- 2D Euler equations
- Etc.

Type of models

Navier Stokes

- DNS = Direct Numerical Simulation
- LES = Large Eddy Simulation

PE

- PE = Primitive Equations models

SW

- SW = Shallow-Water models

SQG

- SQG = Surface Quasi-Geostrophic models

QG

- QG = Quasi-Geostrophic models
- Etc.

CFD

Process studies

Ocean Circulation Models

Idealized models

Equations for momentum/mass?

Navier-Stokes Equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + g\vec{k} = -\frac{\vec{\nabla} P}{\rho} + \vec{\mathcal{F}}$$

Momentum equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$

Mass conservation
(no source/sink)

Equations for momentum/mass?

Navier-Stokes Equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + g\vec{k} = -\frac{\vec{\nabla} P}{\rho} + \vec{\mathcal{F}}$$

Time variation	Advection (inertia)	Rotation	Gravity	Pressure gradient	Forcings + Dissipation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$

Mass conservation (no source/sink)

Equations for momentum/mass?

Navier-Stokes Equations:

Linearized momentum equations

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla} P$$

+ continuity equation

$$\frac{\partial P}{\partial t} = -\rho_0 c_s^2 \vec{\nabla} P \cdot \vec{u}$$

+ adiabatic motion :

= Acoustic modes (sound waves)

$$\partial_{tt} P = c_s^2 \nabla^2 P$$

With $c_s \approx 1500\,\mathrm{m\,s^{-1}}$ in water, a model requires a very small time-step to solve these equations.

Equations for momentum/mass?

Boussinesq Approximation:

Density perturbations small compared to mean background value:

$$\rho = \rho_0 + \rho' \qquad\qquad \rho' << \rho_0$$

Linearize all terms involving a product with density,

except the gravity term which is already linear:

$$\begin{aligned} \rho\vec{u} &\rightarrow \rho_0\vec{u} \\ \rho g &\rightarrow \rho g \end{aligned}$$

Equations for momentum/mass?

Boussinesq Approximation :

[+ incompressibility or adiabatic]

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$

Continuity equation

$$\vec{\nabla} \cdot \vec{u} = 0$$

Equations for momentum/mass?

Non hydrostatic boussinesq (NH):

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + \frac{\rho}{\rho_0} g\vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \frac{\vec{\mathcal{F}}}{\rho_0} + \frac{\vec{\mathcal{D}}}{\rho_0}$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

Easier to solve than Navier-Stokes, but still requires to invert a 3d elliptic equation for P (computationally expansive)

Equations for momentum/mass?

Hydrostatic balance:

The vertical component of the Boussinesq momentum equations is

$$\frac{\partial w}{\partial t} + \vec{u} \cdot \vec{\nabla} w + 2\Omega \cos\phi v + \frac{\rho}{\rho_0} g = -\frac{\partial_z P}{\rho_0} + \frac{\mathcal{F}_w}{\rho_0} + \frac{\mathcal{D}_w}{\rho_0}$$

For long horizontal motions (L >> H) the dominant balance is

H ~3000 m
L ~3000 km

$$\frac{\partial P}{\partial z} = -\rho g$$

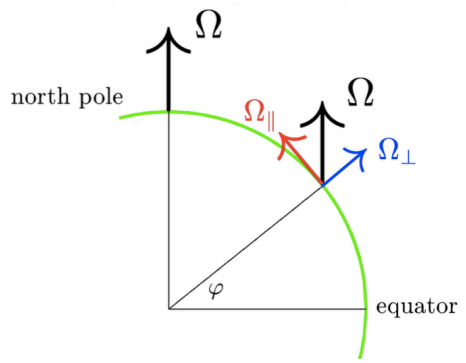
Such that pressure is just a vertical integral:

$$P = \int_z^\eta g \rho dz$$

Equations for momentum/mass?

Traditional approximation

= neglect horizontal Coriolis term



$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + \frac{\rho}{\rho_0} g\vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \vec{\mathcal{F}} + \vec{\mathcal{D}}$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f\vec{k} \times \vec{u} + \frac{\rho}{\rho_0} g\vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \vec{\mathcal{F}} + \vec{\mathcal{D}}$$

Equations for momentum/mass?

Hydrostatic Primitive Equations (PE)

- 2d momentum with Boussinesq approximation:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \vec{\nabla}_H v + w \frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$

- Hydrostatic:

$$\frac{\partial P}{\partial z} = -\rho g$$

- Continuity equation for an incompressible fluid:

$$\vec{\nabla} \cdot \vec{u} = 0$$

Equations for PE Ocean Models

Hydrostatic Primitive Equations (PE)

- 2d momentum with Boussinesq approximation:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \vec{\nabla}_H v + w \frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$

- Hydrostatic: $\frac{\partial P}{\partial z} = -\rho g$
- Continuity equation for an incompressible fluid: $\vec{\nabla} \cdot \vec{u} = 0$
- Conservation of heat and salinity $\frac{DT}{Dt} = \mathcal{S}_T \qquad \frac{DS}{Dt} = \mathcal{S}_S$
- Equation of state : $\rho = \rho(T, S, z)$

Equations for PE Ocean Models

Hydrostatic Primitive Equations (PE)

- 4 prognostics equations for **u, v, T, S**
- 3 diagnostics equations for **w, ρ, P**

Equations for PE Ocean Models

Hydrostatic Primitive Equations (PE)

- 2d momentum with Boussinesq approximation:

?

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

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$$\rho = \rho(T, S, z)$$

Equations for PE Ocean Models

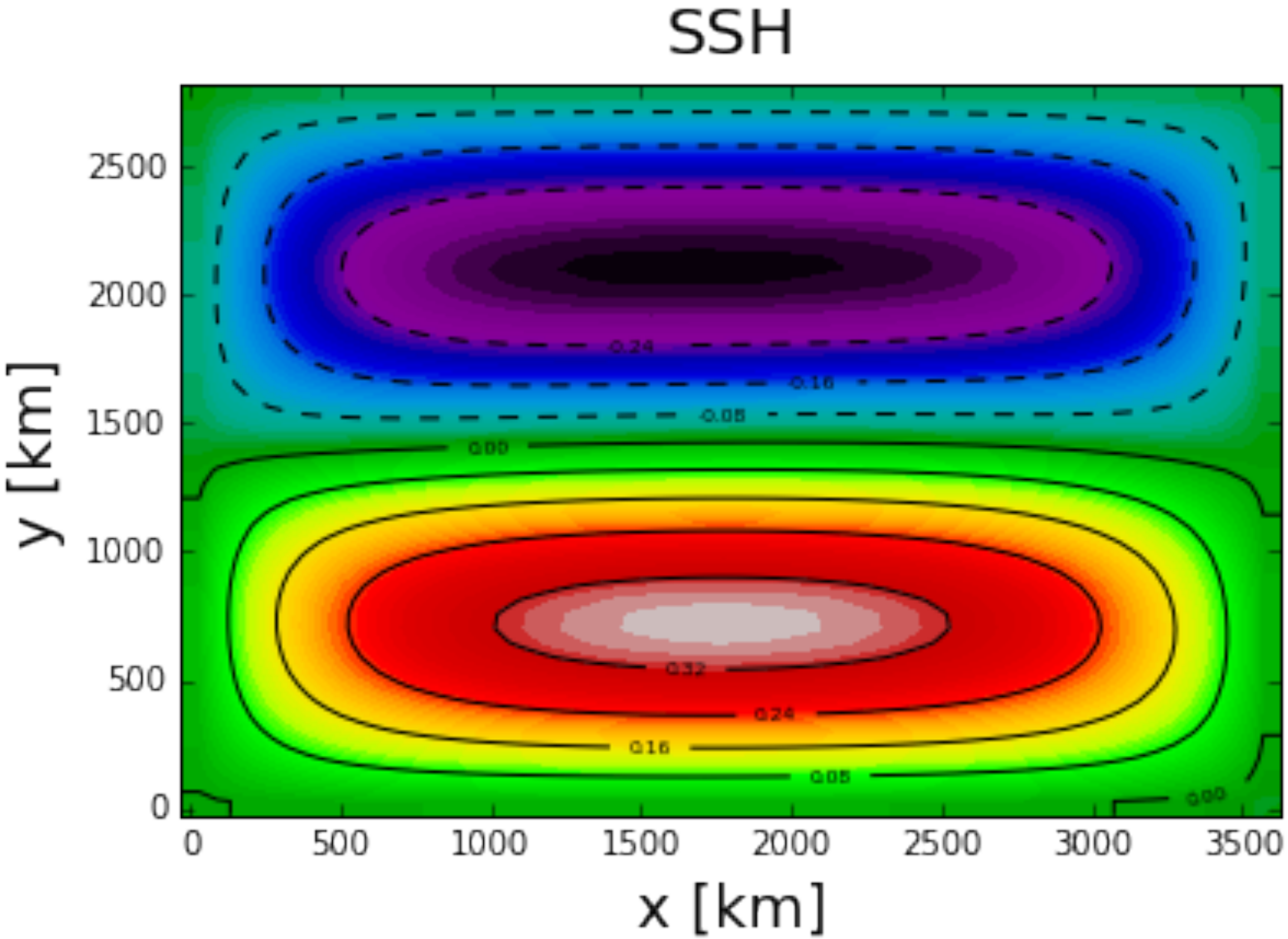
Hydrostatic Primitive Equations (PE)

- 4 prognostics equations for **u, v, T, S**
- 3 diagnostics equations for **w, ρ, P**

+ Forcings (wind, heat flux)

+ sub-grid scale parameterizations (bottom drag, mixing, etc.)

Activity 1 – Run an idealized ocean basin



Activity 1 – Run an idealized ocean basin

- **Jobcomp** (compilation)
- **cppdefs.h** (Numerical/physical options)
- **param.h** (gris size/ parallelisation)
- **croco.in** (choice of variables, parameter values, etc.)

1) Preparing and compiling the model

For that use the the jobcomp bash file
./jobcomp

1. Set library path
2. Automatic selection of option accordingly the platform used
3. Use of makefile
   - C-preprocessing step : .F → .f using the CPP keys defintions (in cppdefs.h file, customization of the code)
   - Compilation step : .f → .o (object) using Fortran compiler
   - Linking step : link all the .o file and the librairy (Netcdf, MPI, AGRIF)
     --
   - --> produce the executable roms

1) Preparing and compiling the model

Edit the param.h and cppdefs.h file to set-up the model

param.h defines the size of the arrays in ROMS:

```
. . .
#elif defined REGIONAL
#  if defined  BENGUELA
     parameter (LLm0=23, MMm0=31, N=32)
#  else
     parameter (LLm0=??,  MMm0=??,  N=??)
#  endif
. . .
```

Given by running make_grid

<---- Southern Benguela test Model

Defined in romstools_param.m

cppdefs.h: → Basic options

→ More advanced options

- Define CPP keys used by the C-preprocessor when compiling the model.
- Reduce the code to its minimal size: fast compilation.
- Avoid FORTRAN logical statements: efficient coding.

1) Preparing and compiling the model

View cppdef.h file

```
!---------------------------------------------------------
!                 BASIC OPTIONS
!---------------------------------------------------------
*/
/*                      Configuration Name */
# define BENGUELA
/*                      Parallelization */
# undef  OPENMP
# undef  MPI
/*                      Embedding */
# undef AGRIF
/*                      Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# undef OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
*/                       Embedding conditions */
# ifdef AGRIF
#  undef AGRIF_OBC_EAST
#  define AGRIF_OBC_WEST
#  define AGRIF_OBC_NORTH
#  define AGRIF_OBC_SOUTH
# endif
/*                     Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENTS
# undef  BBL
```

```
!---------------------------------------------------------
!         MORE ADVANCED OPTIONS
!---------------------------------------------------------
*/
/*                      Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
# ifdef TIDES
#  define SSH_TIDES
#  define UV_TIDES
#  define TIDERAMP
# endif
/*                      Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
/*                              Input/Output & Diagnostics */
# define AVERAGES
# define AVERAGES_K
# define  DIAGNOSTICS_TS
# define  DIAGNOSTICS_UV
/*                              Equation of State  */ ...
/*                              Surface Forcing */ ...
/*                              Lateral Forcing  */ ...
/*                              Input/Output & Diagnostics */ ...
*                               Bottom Forcing  */ ...
/*                              Point Sources - Rivers */ ...
/*                              Lateral Mixing  */ ...
/*                              Vertical Mixing */ ...
/*                              Open Boundary Conditions  */ ...
/*                              Embedding conditions  */ ...
```

2) Running the model

The namelist roms.in

roms.in provides the run time parameters for ROMS:

```
title:
      Southern Benguela
time_stepping: NTIMES   dt[sec]  NDTFAST  NINFO
          480     5400     60     1
S-coord: THETA_S,   THETA_B,    Hc (m)
        6.0d0     0.0d0     10.0d0
grid:  filename
                  ROMS_FILES/roms_grd.nc
forcing: filename
                  ROMS_FILES/roms_frc.nc
bulk_forcing: filename
                  ROMS_FILES/roms_blk.nc
climatology: filename
                  ROMS_FILES/roms_clm.nc
boundary: filename
                  ROMS_FILES/roms_bry.nc
initial: NRREC  filename
        1
                  ROMS_FILES/roms_ini.nc
restart:        NRST, NRPFRST / filename
              480   -1
                  ROMS_FILES/roms_rst.nc
```

Warning ! These should be identical to the ones in romstools_param.m

```
history: LDEFHIS, NWRT, NRPFHIS / filename
          T    480     0
                  ROMS_FILES/roms_his.nc
averages: NTSAVG, NAVG, NRPFAVG / filename
          1     48     0
                  ROMS_FILES/roms_avg.nc

primary_history_fields: zeta UBAR VBAR  U  V   wrtT(1:NT)
                    T   F   F   F  F    10*F
auxiliary_history_fields:   rho Omega  W  Akv  Akt  Aks  HBL  Bostr
                      F  F    F  F    F    F    F    F
primary_averages: zeta UBAR VBAR  U  V   wrtT(1:NT)
                T   T   T   T  T  10*T
auxiliary_averages: rho Omega  W  Akv  Akt  Aks  HBL  Bostr
                 F  T    T  F    T    F    T    T
rho0:
     1025.d0
lateral_visc:   VISC2,    VISC4    [m^2/sec for all]
              0.       0.
tracer_diff2: TNU2(1:NT)          [m^2/sec for all]
            10*0.d0
bottom_drag:     RDRG [m/s],  RDRG2,  Zob [m],  Cdb_min, Cdb_max
              0.0d-04      0.d-3    1.d-2     1.d-4    1.d-1
gamma2:
             1.d0
sponge:          X_SPONGE [m],    V_SPONGE [m^2/sec]
               100.e3          800.

nudg_cof:    TauT_in, TauT_out, TauM_in, TauM_out  [days for all]
             1.       360.      10.      360.
```

Activity 1 – Run an idealized ocean basin

- **param.h**

```
      parameter (LLm0=60,    MMm0=50,    N=10)
```

- **cppdefs.h**

```
# define UV_COR
# define UV_VIS2
# define TS_DIF2

# define ANA_GRID
# define ANA_INITIAL
```

- **ana_grid.F**

```
f0=1.E-4
beta=0.
```

- **croco.in**

```
bottom_drag:     RDRG(m/s),      RDRG2, Zob [m],  Cdb_min, Cdb_max
                 3.e-4             0.      0.         0.      0.
gamma2:
                 1.
lin_EOS_cff:  R0 [kg/m3], T0 [Celsius], S0 [PSU], TCOEF [1/Celsius], SCOEF [1/PSU]
              30.         0.            0.        0.28               0.
lateral_visc:   VISC2    [m^2/sec ]
                 1000.  0.
tracer_diff2: TNU2         [m^2/sec]
                 1000. 0.
```

Homework

- For next time:
  - Read https://www.jgula.fr/ModNum/Stommel48.pdf
  - Read https://www.jgula.fr/ModNum/Munk50.pdf
  -