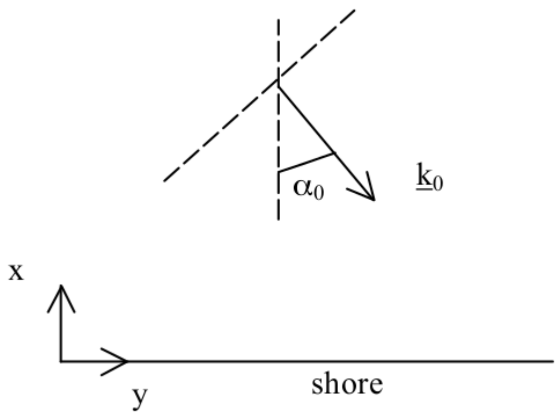
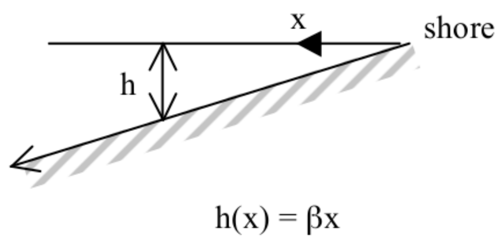
Marine Physics Master (M1)
Fluid 2
Ray Theory

The goal is to compute how waves are modified when approaching a beach. To simplify things, we will assume linear long waves.





The incident wavenumber vector is defined as :

$$\underline{k}_0 = \begin{pmatrix} k_0 \\ l_0 \end{pmatrix} = |\underline{k}_0| \begin{pmatrix} -\cos\alpha_0 \\ \sin\alpha_0 \end{pmatrix} \quad \text{with} \quad 0 \leqslant \alpha_0 < \pi/2$$

(1) Explain why the frequency is constant (independent of time). Deduce that l = constant :

$$|\underline{k}|\sin\alpha = |\underline{k}_0|\sin\alpha_0$$

Show next that the phase velocity obeys the Descartes's law :

$$\frac{c}{\sin\alpha} = \frac{c_0}{\sin\alpha_0}$$

Describe (drawing) how crests deform when the wave train approaches the beach.

(2) We now want to calculate precisiely the wave solution. Show that the equations for the rays is :

$$\frac{dx}{dy} = \frac{k}{l_0}$$

To find the rays (parallel to the group velocity) this equation must be integrated. To simplify things assume proximity of the shore (h small) so that

$$\frac{\omega^2}{ghl_0^2} \gg 1 \quad \text{Illustrate your solution.}$$

(3) We now want to find the shape of the crests. Show that they obey $\frac{dx}{dy} = -\frac{l_0}{k}$ Integrate this equation under the same approximation as previously and show that the crests are parabolas.