M2 - Marine Physics **2018–2019**

Internal Waves Exam

Duration: 2 hours.

Exercise 1: Reflection of internal waves

We define a stably stratified and inviscid fluid in a resting state. We consider small perturbations of this basic state:

$$\vec{u} = (u', v', w'), \quad p = p_0 + p', \quad \rho = \rho_* + \rho_0(z) + \rho'(x, y, z, t),$$

where the perturbation (primed) quantities are functions of x, y, z, and t and are assumed to be small compared to the basic state variables. $p_0(z)$ and $\rho_* + \rho_0(z)$ denote the hydrostatic pressure and density. The hydrostatic density perturbation $\rho_0(z)$ is also assumed to be small compared to the background density ρ_*.

• Starting from the linearized momentum, density and continuity equations under the Boussineq approximation ($\rho' + \rho_0 << \rho_*$ the background density), show that the vertical perturbation velocity w' satisfies:

$$\frac{\partial^2}{\partial t^2}(\nabla^2 w') + f^2 \frac{\partial^2 w'}{\partial z^2} + N^2 \nabla_h^2 w' = 0, \tag{1}$$

where f is the Coriolis frequency, $N^2 = -g\rho_{0z}/\rho_*$, and ρ_* is a constant background density.

We restrict ourselves to the vertical plane (x, z), and assume solutions in the form of plane waves: $w(x, z, t) = w_0 e^{\mathrm{i}(\vec{k}\cdot\vec{x} - \omega t)}$, with $\vec{k} = (k_x, 0, k_z)$.

• Compute the dispersion relation $\omega(k_x, k_z)$.

• Compute the angle θ, between the horizontal and the wavenumber $\vec{k} = (k_x, 0, k_z)$, as a function of ω, f, and N.

• Discuss what happens when $\omega \to f$, and $\omega \to N$.

• Explain what happens when a wave propagating at an angle θ_i (direction of energy propagation) encounters a sloping topography of angle α in the 3 cases plotted figure 1: (a) $\alpha < \theta_i$, (b) $\alpha = \theta_i$, and (c) $\alpha > \theta_i$.

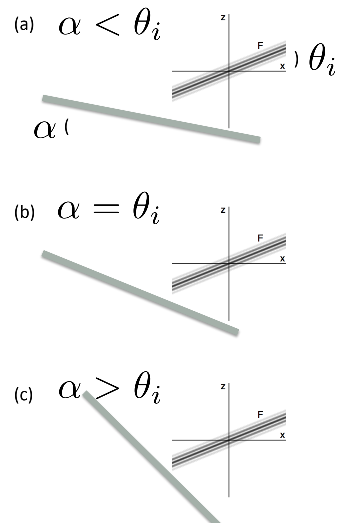

FIG. 1: (a) $\alpha < \theta_i$, (b) $\alpha = \theta_i$, and (c) $\alpha > \theta_i$. Grey contours show vertical velocity amplitude.

Exercise 2: Viscous damping of internal waves

We now consider a flow similar to the previous one but we restrict ourself to a two-dimensional flow ($v = 0, \frac{\partial .}{\partial y} = 0$) without rotation ($f = 0$). We also introduce, on the right hand sides of the momentum and density equations, the linear damping terms $-Ku'$, $-Kw'$, and $-K\rho'$.

- Write the new equation for the vertical velocity w'.

- Find the dispersion relation for a solution of the form: $w'(x, y, z, t) = w_0 e^{\mathrm{i}\vec{k}\cdot\vec{x}-\omega t}$, with $\vec{k} = (k_x, 0, k_z)$ and where ω has both real and imaginary parts.

- Explain what is the impact of the damping term.

Exercise 3:

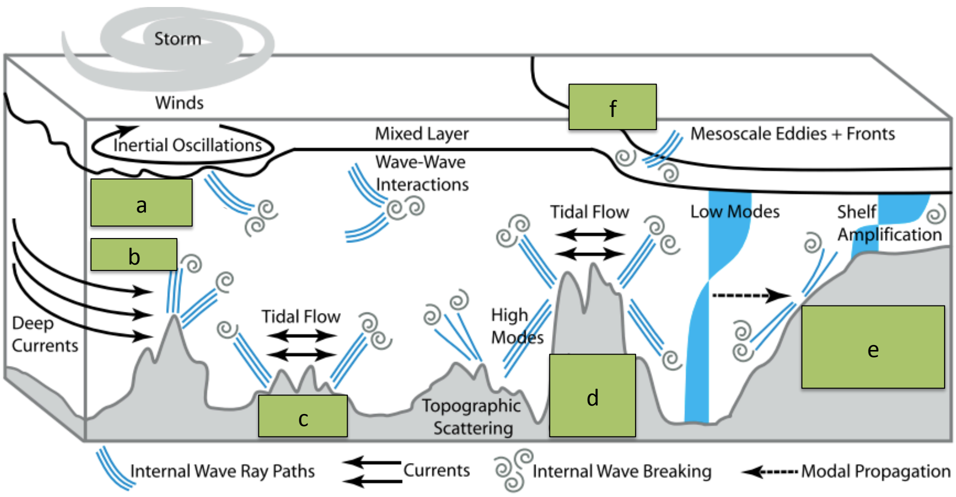

Figure 1: Pictural representation of some physical processes involved in the generation of internal waves (From MacKinnon et al., 2017).

- A number of processes related to interaction and dissipation of internal waves are shown figure 2. Identify and briefly describe the processes a,b,c,d,e, and f.