Master OFFWIND

Coastal Dynamics

Problem 1: Baroclinic Coastal Kelvin Wave (two-layer ocean)

Consider a straight vertical coastline at $x = 0$ with the ocean occupying $x > 0$. The fluid is on an f-plane (constant Coriolis parameter f). The ocean has two homogeneous layers: an upper layer of thickness H_1 and density ρ_1, over a deeper layer of thickness H_2 and density $\rho_2 > \rho_1$. Use linearized two-layer shallow-water theory (no background flow), with alongshore coordinate y (positive north) and offshore coordinate x (positive seaward). Neglect friction.

Let $\eta_1(x, y, t)$ and $\eta_2(x, y, t)$ be the layer-thickness perturbations (or equivalently interface displacement for internal motions). You may use the reduced gravity

$$g' = g\frac{\rho_2 - \rho_1}{\rho_2}. \tag{1}$$

(a) Starting from the linearized two-layer shallow-water momentum and continuity equations, derive the governing equations for small-amplitude internal (baroclinic) motions in the long-wave limit. Show that there exists a trapped coastal Kelvin wave solution in which alongshore velocity and interface displacement propagate alongshore with no cross-shore velocity, and all fields decay exponentially offshore.

(b) Find the dispersion relation for the baroclinic Kelvin wave and show that the alongshore phase speed equals

$$c = \sqrt{g' \frac{H_1 H_2}{H_1 + H_2}}.$$

Show that the offshore decay scale is $L_d = c/f$. Write the solution as

$$\eta(x, y, t) = \hat{\eta}\, e^{-x/L_d}\, e^{i(ky - \omega t)}.$$

(c) Explain why baroclinic Kelvin waves are trapped more strongly than barotropic waves. Discuss the role of g' and stratification.

(d) Using $H_1 = 50$ m, $H_2 = 150$ m, $\rho_2 - \rho_1 = 2$ kg/m^3, $\rho_2 = 1025$ kg/m^3, and latitude 45° ($f = 2\Omega \sin\phi$ with $\Omega = 7.2921 \times 10^{-5}$ s^{-1}), compute g', c, and L_d.

Problem 2: Tides on an Ocean-Covered Planet

Imagine a spherical planet with radius $r = 4727$ km, mass $M = 2.6792 \times 10^{24}$ kg, entirely covered by ocean. The planet rotates once every 20 hours. A satellite of mass $m = 4.63 \times 10^{21}$ kg is in a circular orbit in the same direction as rotation. The orbital distance is $d = 500{,}000$ km, and its period is 200 hours.

The universal gravitational constant is $G = 6.672 \times 10^{-11}$ N m^{2}kg^{-2}.

1. Show that g at the surface is 8 N/kg.
2. Find the period (h) and wavelength (km) of the tide at the equator.
3. With ocean depth $H = 3200$ m, compute the maximum speed of surface gravity waves.
4. Find the speed at the equator needed to maintain equilibrium tide.
5. What is the lowest latitude for equilibrium tide?
6. If the orbit were elliptical, which tidal properties would change? Why?
7. The planet orbits a star of mass $M_\star = 5 \times 10^{30}$ kg at distance 3.0×10^8 km. Is the stellar tide stronger than the satellite tide?