Master Physique Marine (M1)
Fluides II

Linear non dispersive waves : an example, the long gravity waves

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Homework n°1

The free surface of a long gravity wave is given by : $\eta(x,t)=a \cos(kx-\omega t)$ in a fluid of depth h.

1/ Compute the horizontal velocity u. Is there a phase lag with the pressure field ? Compare the amplitude of u with the phase velocity c. Derive a criterion for the validity of the linear approximation in term of the amplitude a.
2/ Compute the tendency (time rate of change) $d\eta/dt$ and du/dt and try to explain the propagation of the wave through a sketch (drawing).
3/ Compute the vertical velocity and discuss its variation as a function of z.

Homework n°2

Calculate the period of free oscillations (the basin modes are called *seiches* in this context) of a narrow lake of length L and depth h. « *Narrow* » implies that the modes are taken to vary along x only, (with Ox the coordinate axis along the length of the lake). Compare your solutions with the longest period of a few lakes which have been observed :
Geneva lake : L=70 km, h=160m and T=73.5 mn
Loch Earn (Scotland) : L=10 km, h=60m, T=14.5 mn
Lake Baikal : L=665 km, h=680m, T=4.64 h

Homework n°3

1/ Show that the energy equation in the one dimensional case reduces in the linear approximation to :

$$1/2\ \partial_t (Hu^2 + g\eta^2) = -H\partial_x (ug\eta)$$

where H is the constant fluid depth. Interpret.

2/ Take a *real* wave solution of the form $\eta(x,t)=\eta_o \exp i(kx-\omega t)$ +c.c (complex conjugate). Compute the kinetic and potential energy averaged over a period (or wave length) and show equipartition.

3/ The energy flux for a wave is simply the work of pressure forces per unit time and per unit surface across a surface normal to the propagation. Identify this flux term in the above equation and show that for the wave solution given in 2/ the flux F= c E with $c =(gh)^{1/2}$ and E the total energy. Why is it constant here?

Homework n°4

We consider oscillations of a harbor known as the Pumping Mode (or Helmholtz resonator in acoustics). The harbor (of arbitrary shape) has a surface S and connects to the sea by a narrow channel of width a and length L.

(i) If $\eta(t)$ is the water level in the harbor (assumed uniform in space) and U(t) the current in the channel (also assumed uniform in space), indicate the origin of the two equations :

$$S\ d_t\eta = -a\ h\ U$$
$$d_t U = g\eta / L$$

(we neglect motions of the sea *outside*).

(ii) Show that the solution is oscillatory. Find the period of oscillations in the harbor and discuss its variation with respect to the main model parameters. Estimate the period of « La rade de Brest » connected to the open sea by the goulet de Brest. You will need to find rough estimates of S, L and h.