Numerical Modelling

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the anatomy of an ocean model

- **Lesson 1 :** [D109]
  - Introduction
  - Equations of motions
  - *Activity 1 [run an ocean model]*
- **Lesson 2 :** [D109]
  - Horizontal Discretization
  - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 3 :** [D109]
  - *Activity 2 [Dynamics of an ocean gyre]*
- **Lesson 4 :** [D109]
  - Numerical schemes
  - *Activity 3 [Impacts of numerics]*
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- **Lesson 5 :** [D109]
  - Vertical coordinates
  - Activity 4 [Impact of topography]
- **Lesson 6 :** [D109]
  - Boundary Forcings
  - Presentation of the model CROCO
  - *Activity 5 [Design a realistic simulation]*
- **Lesson 7 :** [D109]
  - Diagnostics and validation
  - *Activity 6 [Analyze a realistic simulation]*
- **Lesson 8 :** [D109]
  - *Project*

Presentations and material will be available at :

jgula.fr/ModNum/

Useful references

Extensive courses:

- MIT: https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/
- Princeton: https://stephengriffies.github.io/assets/pdfs/GFM_lectures.pdf

Overview on ocean modelling and current challenges:

- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling.http://jgula.fr/ModNum/Griffiesetal00.pdf
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. http://jgula.fr/ModNum/Griffies_Chapter.pdf
- Fox-Kemper et al, 19, “Challenges and Prospects in Ocean Circulation Models" http://jgula.fr/ModNum/FoxKemperetal19.pdf

ROMS/CROCO:

- https://www.myroms.org/wiki/
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell. http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf

#5 Vertical Discretization

Master's degree 2^{nd} year Marine Physics

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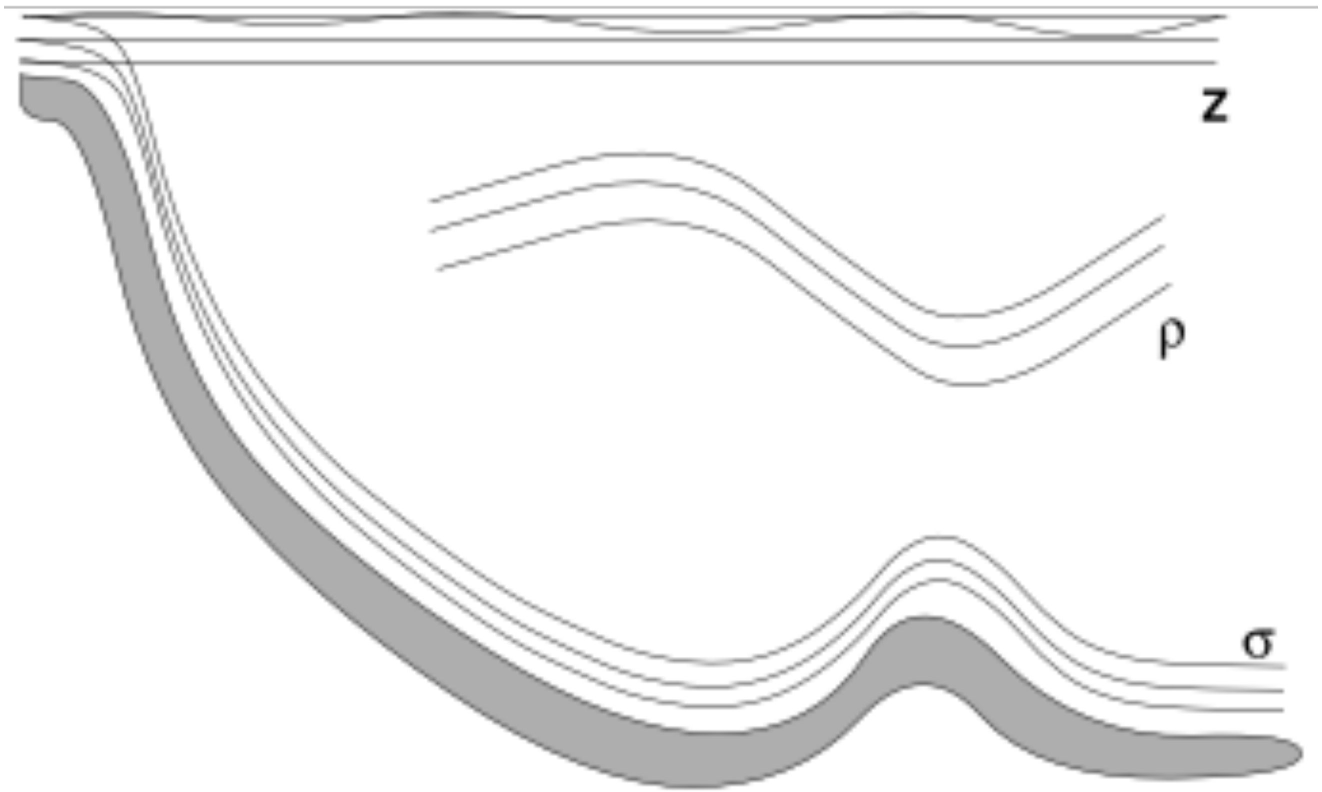
Vertical discretization

Depending on the depth, motions in the ocean will be mostly aligned along **geopotentials, isopycnal or topography.**



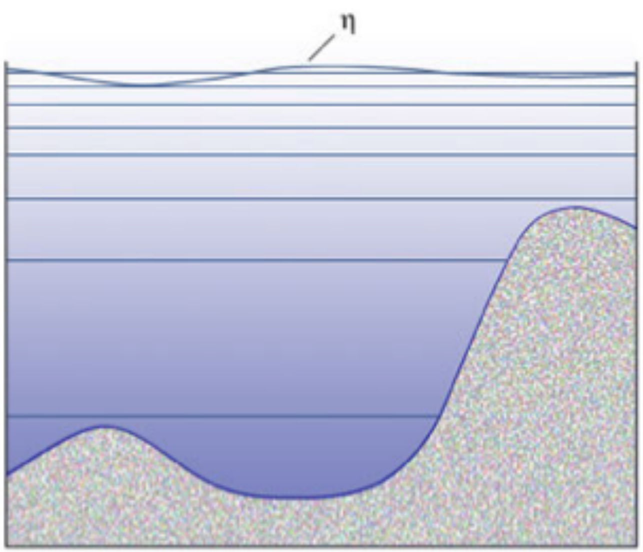
Surface mixed layer

Interior

Bottom

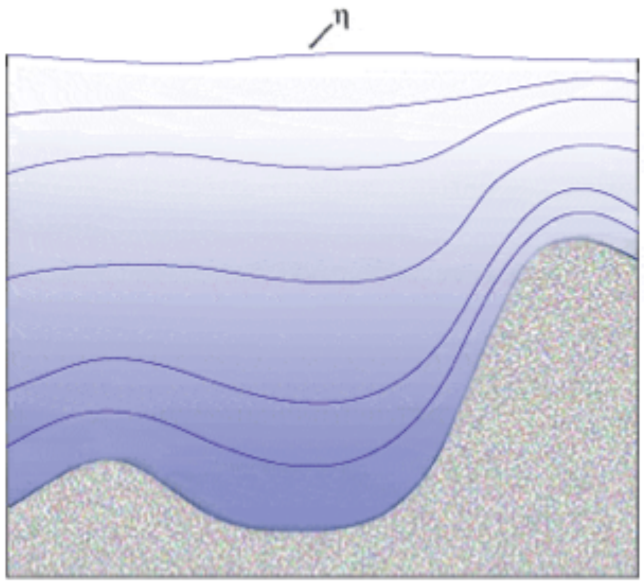
Vertical discretization

Several choices are possible to define the vertical coordinates system:



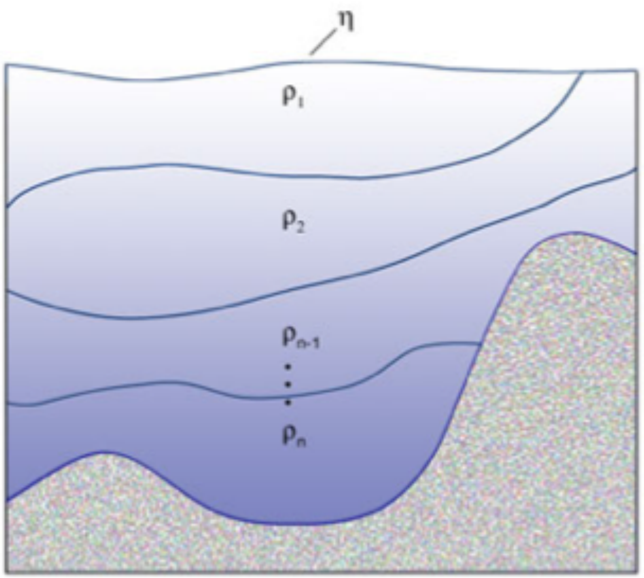
z-coordinates

Vertical coordinate is height (or depth)



sigma-coordinates

The vertical coordinate follows the bathymetry



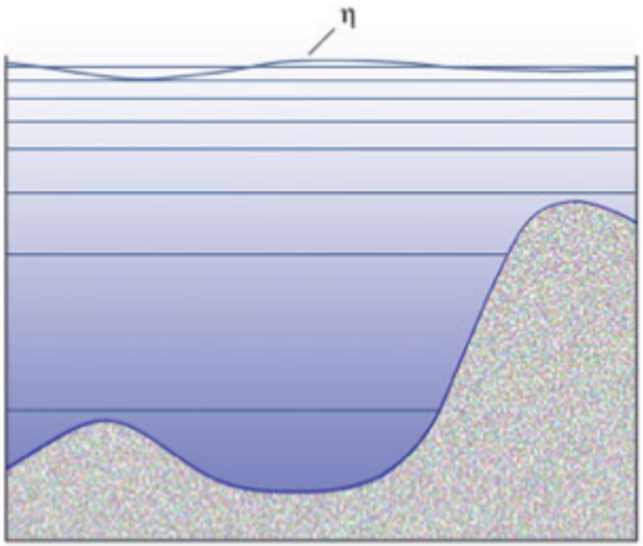
isopycnal-coordinates

The vertical coordinate is the potential density

The vertical coordinate is the major difference between models.

Vertical discretization

z-coordinates



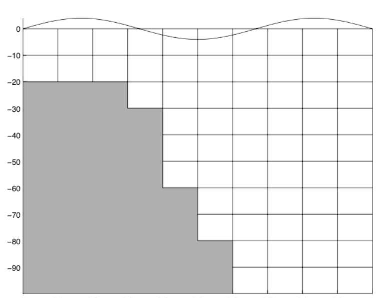
PROS

- Natural in the upper ocean and for mixed-layer processes
- Ideal to compute horizontal (pressure) gradients
- Easier to implement and use

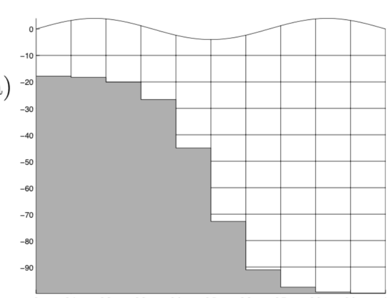
CONS

- The representation of tracer advection and diffusion along inclined density surfaces in the ocean interior is cumbersome.
- Representation of bottom topography is difficult.
- Need for bottom and lateral conditions
- Representation and parameterization of the BBL is unnatural.

a)

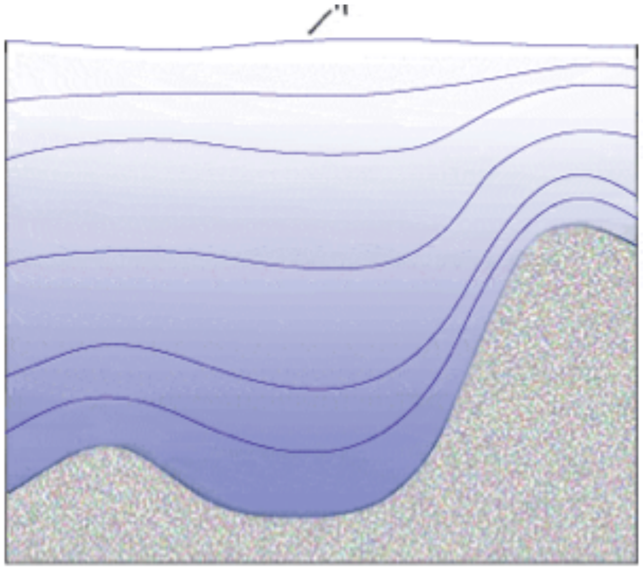


a)



Vertical discretization

sigma-coordinates



PROS

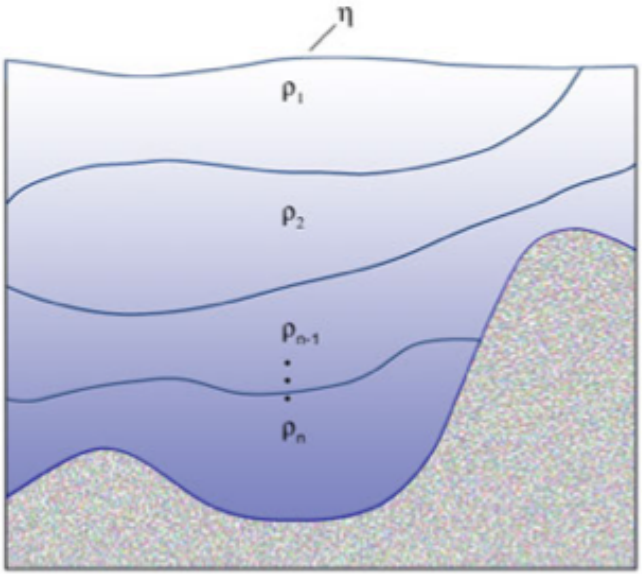
- Representation of bottom topography is natural (only bottom boundary condition)
- Representation and parameterization of the BBL is natural (more vertical res. in BBL)

CONS

- The representation of tracer advection and diffusion along inclined density surfaces in the ocean interior is cumbersome.
- Pressure gradient errors can be a problem

Vertical discretization

isopycnal-coordinates



PROS

- The representation of tracer advection and diffusion along inclined density surfaces in the ocean interior is natural.
- Water mass characteristics are preserved over long time scales

CONS

- Representing the effects of a realistic (non-linear) equation of state is cumbersome.
- Inappropriate for representing the surface mixed layer or BBL which are mostly unstratified.
- Non-hydrostatic effects/dynamics are not possible.
- Vertical and horizontal resolution are tightly connected in regions where isopycnals outcrop. This can lead to inadequate horizontal resolution in regions such as the ACC.

Vertical discretization

- Equations for a generalized coordinate system:

Consider a general vertical coordinate, r, which is assumed to be a monotonic function of height, z.

Any variable can be written in the new coordinate system: $$A = A(x, y, z(x, y, r, t), t)$$

Vertical derivatives can be written $$\frac{\partial A}{\partial r} = \frac{\partial z}{\partial r}\frac{\partial A}{\partial z} \qquad \frac{\partial A}{\partial z} = \frac{\partial r}{\partial z}\frac{\partial A}{\partial r}$$

And other derivatives (for a horizontal coordinate s = x,y) can be written using the chain rule:

$$\left.\frac{\partial A}{\partial s}\right|_z = \left.\frac{\partial A}{\partial s}\right|_r - \frac{\partial A}{\partial z}\left.\frac{\partial z}{\partial s}\right|_r$$

So $$\left.\frac{\partial A}{\partial s}\right|_z = \left.\frac{\partial A}{\partial s}\right|_r - \frac{\partial r}{\partial z}\frac{\partial A}{\partial r}\left.\frac{\partial z}{\partial s}\right|_r$$

Vertical discretization

- Equations for a generalized coordinate system:

Using the chain rule, you can write the horizontal gradient: $$\nabla_z A = \nabla_r A - \frac{\partial A}{\partial r}\frac{\partial r}{\partial z}\nabla_r z$$

And the vertical velocity can be written:

$$\begin{aligned} w = D_t z &= \left.\frac{\partial z}{\partial t}\right|_r + \left.\frac{\partial z}{\partial x}\right|_r D_t x + \left.\frac{\partial z}{\partial y}\right|_r D_t y + \frac{\partial z}{\partial r} D_t r \\ &= \left.\frac{\partial z}{\partial t}\right|_r + \vec{v}\cdot\nabla_r z + \dot{r}\frac{\partial z}{\partial r} \end{aligned}$$

Vertical discretization

- Equations for a generalized coordinate system:

And the total derivative becomes:

$$
\begin{aligned}
D_t A &= \left.\frac{\partial A}{\partial t}\right|_z + \vec{v}\cdot\nabla_z A + w\frac{\partial A}{\partial z} \\
&= \left.\frac{\partial A}{\partial t}\right|_r + \vec{v}\cdot\nabla_r A + \left(w - \left.\frac{\partial z}{\partial t}\right|_r - \vec{v}\cdot\nabla_r z\right)\frac{\partial r}{\partial z}\frac{\partial A}{\partial r} \\
&= \left.\frac{\partial A}{\partial t}\right|_r + \vec{v}\cdot\nabla_r A + \dot{r}\frac{\partial A}{\partial r}
\end{aligned}
$$

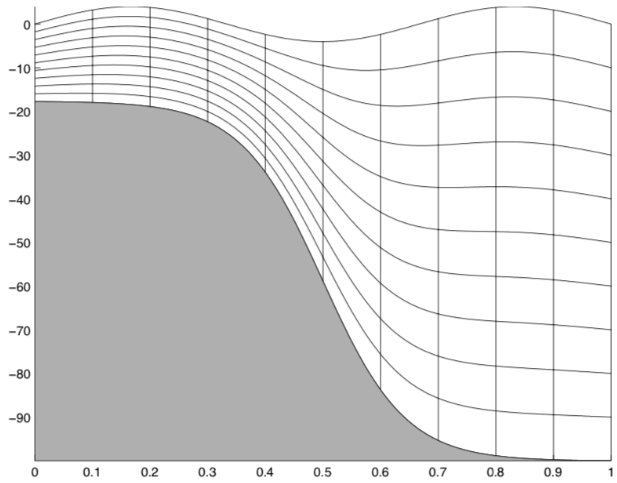
And the horizontal pressure gradient:

$$
\begin{aligned}
\nabla_z p &= \nabla_r p - \frac{\partial p}{\partial z}\nabla_r z \\
&= \nabla_r p + \rho\nabla_r g z
\end{aligned}
$$

Vertical grid : σ generalized coordinate

- Sigma-coordinate system:

Sigma-coordinates are terrain-following coordinates defined as:



$$\sigma = \frac{z}{H_z(x,y)}$$

$$\frac{\partial \sigma}{\partial z} = \frac{1}{H_z}$$

Vertical grid : σ generalized coordinate

- Horizontal and vertical derivatives can be written:

$$\left(\frac{\partial}{\partial x}\right)_z = \left(\frac{\partial}{\partial x}\right)_\sigma - \left(\frac{1}{H_z}\right)\left(\frac{\partial z}{\partial x}\right)_\sigma \frac{\partial}{\partial \sigma}$$

$$\left(\frac{\partial}{\partial y}\right)_z = \left(\frac{\partial}{\partial y}\right)_\sigma - \left(\frac{1}{H_z}\right)\left(\frac{\partial z}{\partial y}\right)_\sigma \frac{\partial}{\partial \sigma}$$

$$\frac{\partial}{\partial z} = \left(\frac{\partial s}{\partial z}\right)\frac{\partial}{\partial \sigma} = \frac{1}{H_z}\frac{\partial}{\partial \sigma}$$

Vertical grid : σ generalized coordinate

- Example: relative vorticity would be

$$\zeta = \vec{\nabla} \times \vec{u} \cdot \vec{k} = \left.\frac{\partial v}{\partial x}\right|_z - \left.\frac{\partial u}{\partial y}\right|_z$$

- Beware:

$$\left.\frac{\partial v}{\partial x}\right|_\sigma - \left.\frac{\partial u}{\partial y}\right|_\sigma \neq \left.\frac{\partial v}{\partial x}\right|_z - \left.\frac{\partial u}{\partial y}\right|_z$$

Vertical grid : σ generalized coordinate

- Example: (Hydrostatic) Potential vorticity (PV) is defined as

$$q = \left(f\vec{k} + \vec{\nabla} \times \vec{u} \right) . \vec{\nabla}\rho$$

$$q = \left[f + \left.\frac{\partial v}{\partial x}\right|_z - \left.\frac{\partial u}{\partial y}\right|_z \right] \frac{\partial \rho}{\partial z} - \frac{\partial v}{\partial z} \left.\frac{\partial \rho}{\partial x}\right|_z + \frac{\partial u}{\partial z} \left.\frac{\partial \rho}{\partial y}\right|_z$$

- Question: How do you compute PV in sigma-coordinates?
  - *Write the expression of q using only derivatives along sigma coordinates:*

$$\left.\frac{\partial}{\partial x}\right|_\sigma \qquad \left.\frac{\partial}{\partial y}\right|_\sigma \qquad \frac{\partial}{\partial \sigma}$$

Vertical grid : σ generalized coordinate

$$\left(\frac{\partial}{\partial x}\right)_z = \left(\frac{\partial}{\partial x}\right)_\sigma - \left(\frac{1}{H_z}\right)\left(\frac{\partial z}{\partial x}\right)_\sigma \frac{\partial}{\partial \sigma}$$

$$\left(\frac{\partial}{\partial y}\right)_z = \left(\frac{\partial}{\partial y}\right)_\sigma - \left(\frac{1}{H_z}\right)\left(\frac{\partial z}{\partial y}\right)_\sigma \frac{\partial}{\partial \sigma}$$

$$\frac{\partial}{\partial z} = \left(\frac{\partial s}{\partial z}\right)\frac{\partial}{\partial \sigma} = \frac{1}{H_z}\frac{\partial}{\partial \sigma}$$

-

$$q = \left[f + \left.\frac{\partial v}{\partial x}\right|_z - \left.\frac{\partial u}{\partial y}\right|_z\right]\frac{\partial \rho}{\partial z} - \frac{\partial v}{\partial z}\left.\frac{\partial \rho}{\partial x}\right|_z + \frac{\partial u}{\partial z}\left.\frac{\partial \rho}{\partial y}\right|_z$$

Vertical grid : σ generalized coordinate

- Example: (Hydrostatic) Potential vorticity (PV) is defined as

$$q = \left[f + \left.\frac{\partial v}{\partial x}\right|_\sigma - \left.\frac{\partial u}{\partial y}\right|_\sigma \right] \frac{\partial \rho}{\partial z} - \frac{\partial v}{\partial z} \left.\frac{\partial \rho}{\partial x}\right|_\sigma + \frac{\partial u}{\partial z} \left.\frac{\partial \rho}{\partial y}\right|_\sigma$$

- Even if: $$\left.\frac{\partial v}{\partial x}\right|_\sigma - \left.\frac{\partial u}{\partial y}\right|_\sigma \neq \left.\frac{\partial v}{\partial x}\right|_z - \left.\frac{\partial u}{\partial y}\right|_z$$

Vertical grid : isopycnal coordinates

- For isopycnal coordinates, the vertical coordinate is:

$$r = \rho$$

- Potential vorticity (PV) is easily defined as :

$$q = \frac{1}{h}\left[f + \left.\frac{\partial v}{\partial x}\right|_{\rho} - \left.\frac{\partial u}{\partial y}\right|_{\rho}\right]$$

with $$h = \frac{\partial z}{\partial \rho}$$

Vertical grid : σ generalized coordinate

- The vertical velocity in sigma-coordinates is:

$$\Omega(x, y, \sigma, t) = \frac{1}{H_z}\left[w - \left(\frac{z+h}{\zeta+h}\right)\frac{\partial \zeta}{\partial t} - u\frac{\partial z}{\partial x} - v\frac{\partial z}{\partial y}\right]$$

- And the “true” vertical velocity:

$$w = \frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} + v\frac{\partial z}{\partial y} + \Omega H_z.$$

Vertical grid : σ generalized coordinate

- Equations for ROMS/CROCO become:

$$\frac{\partial u}{\partial t} - fv + \vec{v}\cdot\nabla u = -\frac{\partial \phi}{\partial x} - \left(\frac{g\rho}{\rho_o}\right)\frac{\partial z}{\partial x} - g\frac{\partial \zeta}{\partial x} + \frac{1}{H_z}\frac{\partial}{\partial \sigma}\left[\frac{K_m}{H_z}\frac{\partial u}{\partial \sigma}\right] + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + fu + \vec{v}\cdot\nabla v = -\frac{\partial \phi}{\partial y} - \left(\frac{g\rho}{\rho_o}\right)\frac{\partial z}{\partial y} - g\frac{\partial \zeta}{\partial y} + \frac{1}{H_z}\frac{\partial}{\partial \sigma}\left[\frac{K_m}{H_z}\frac{\partial v}{\partial \sigma}\right] + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial C}{\partial t} + \vec{v}\cdot\nabla C = \frac{1}{H_z}\frac{\partial}{\partial \sigma}\left[\frac{K_C}{H_z}\frac{\partial C}{\partial \sigma}\right] + \mathcal{F}_T + \mathcal{D}_T$$

$$\rho = \rho(T, S, P)$$

$$\frac{\partial \phi}{\partial \sigma} = \left(\frac{-gH_z\rho}{\rho_o}\right)$$

$$\frac{\partial H_z}{\partial t} + \frac{\partial (H_z u)}{\partial x} + \frac{\partial (H_z v)}{\partial y} + \frac{\partial (H_z \Omega)}{\partial \sigma} = 0$$

Vertical grid + curvilinear grid

$$\frac{\partial}{\partial t}\left(\frac{H_z u}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{H_z u^2}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{H_z uv}{m}\right)+\frac{\partial}{\partial \sigma}\left(\frac{H_z u\Omega}{mn}\right)$$
$$-\left\{\left(\frac{f}{mn}\right)+v\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right)-u\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right\}H_z v=$$
$$-\left(\frac{H_z}{n}\right)\left(\frac{\partial \phi}{\partial \xi}+\frac{g\rho}{\rho_o}\frac{\partial z}{\partial \xi}+g\frac{\partial \zeta}{\partial \xi}\right)+\frac{1}{mn}\frac{\partial}{\partial \sigma}\left[\frac{K_m}{H_z}\frac{\partial u}{\partial \sigma}\right]+\frac{H_z}{mn}\left(\mathcal{F}_u+\mathcal{D}_u\right)$$

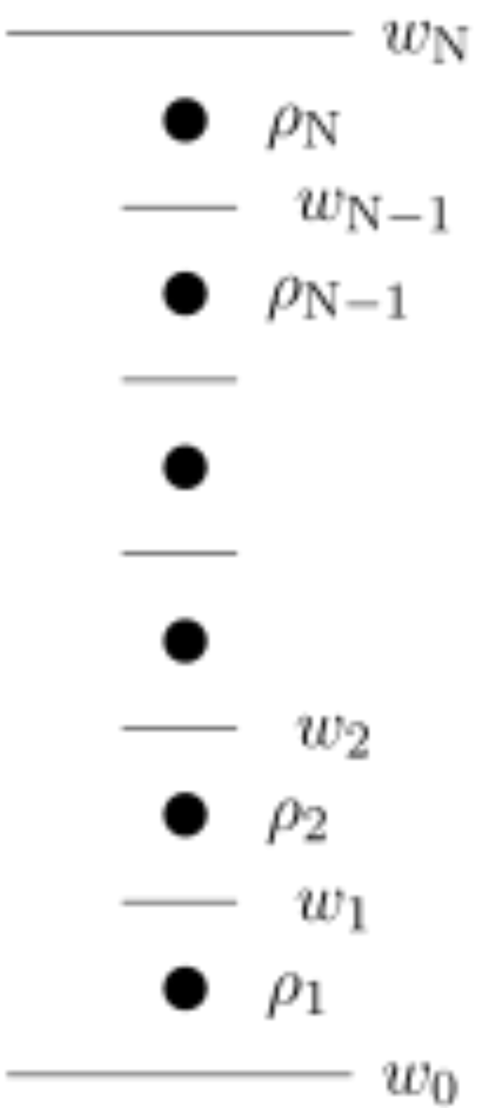
$$\frac{\partial}{\partial t}\left(\frac{H_z v}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{H_z uv}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{H_z v^2}{m}\right)+\frac{\partial}{\partial \sigma}\left(\frac{H_z v\Omega}{mn}\right)$$
$$+\left\{\left(\frac{f}{mn}\right)+v\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right)-u\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right\}H_z u=$$
$$-\left(\frac{H_z}{m}\right)\left(\frac{\partial \phi}{\partial \eta}+\frac{g\rho}{\rho_o}\frac{\partial z}{\partial \eta}+g\frac{\partial \zeta}{\partial \eta}\right)+\frac{1}{mn}\frac{\partial}{\partial \sigma}\left[\frac{K_m}{H_z}\partial v\partial \sigma\right]+\frac{H_z}{mn}\left(\mathcal{F}_v+\mathcal{D}_v\right)$$

$$\frac{\partial}{\partial t}\left(\frac{H_z C}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{H_z uC}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{H_z vC}{m}\right)+\frac{\partial}{\partial \sigma}\left(\frac{H_z \Omega C}{mn}\right)=$$
$$\frac{1}{mn}\frac{\partial}{\partial s}\left[\frac{K_C}{H_z}\frac{\partial C}{\partial \sigma}\right]+\frac{H_z}{mn}\left(\mathcal{F}_C+\mathcal{D}_C\right)$$

$$\rho=\rho(T,S,P)$$
$$\frac{\partial \phi}{\partial \sigma}=-\left(\frac{gH_z\rho}{\rho_o}\right)$$
$$\frac{\partial}{\partial t}\left(\frac{H_z}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{H_z u}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{H_z v}{m}\right)+\frac{\partial}{\partial \sigma}\left(\frac{H_z \Omega}{mn}\right)=0.$$

Vertical discretization

ROMS: Staggered vertical grid



Vertical grid : σ generalized coordinate

ROMS: Generalized σ-Coordinate

Stretching & condensing of vertical resolution:

$$z(x,y,\sigma,t) = \zeta(x,y,t) + [\zeta(x,y,t) + h(x,y)]\,S(x,y,\sigma),$$

$$S(x,y,\sigma) = \frac{h_c\,\sigma + h(x,y)\,C(\sigma)}{h_c + h(x,y)}$$

$$S(x,y,\sigma) = \begin{cases} 0, & \text{if } \sigma = 0, \quad C(\sigma) = 0, \quad \text{at the free-surface;} \\ -1, & \text{if } \sigma = -1, \quad C(\sigma) = -1, \quad \text{at the ocean bottom.} \end{cases}$$

Surface refinement function:

$$C(\sigma) = \frac{1 - \cosh(\theta_S\,\sigma)}{\cosh(\theta_S) - 1}, \qquad \text{for } \theta_S > 0, \qquad C(\sigma) = -\sigma^2, \qquad \text{for } \theta_S \le 0$$

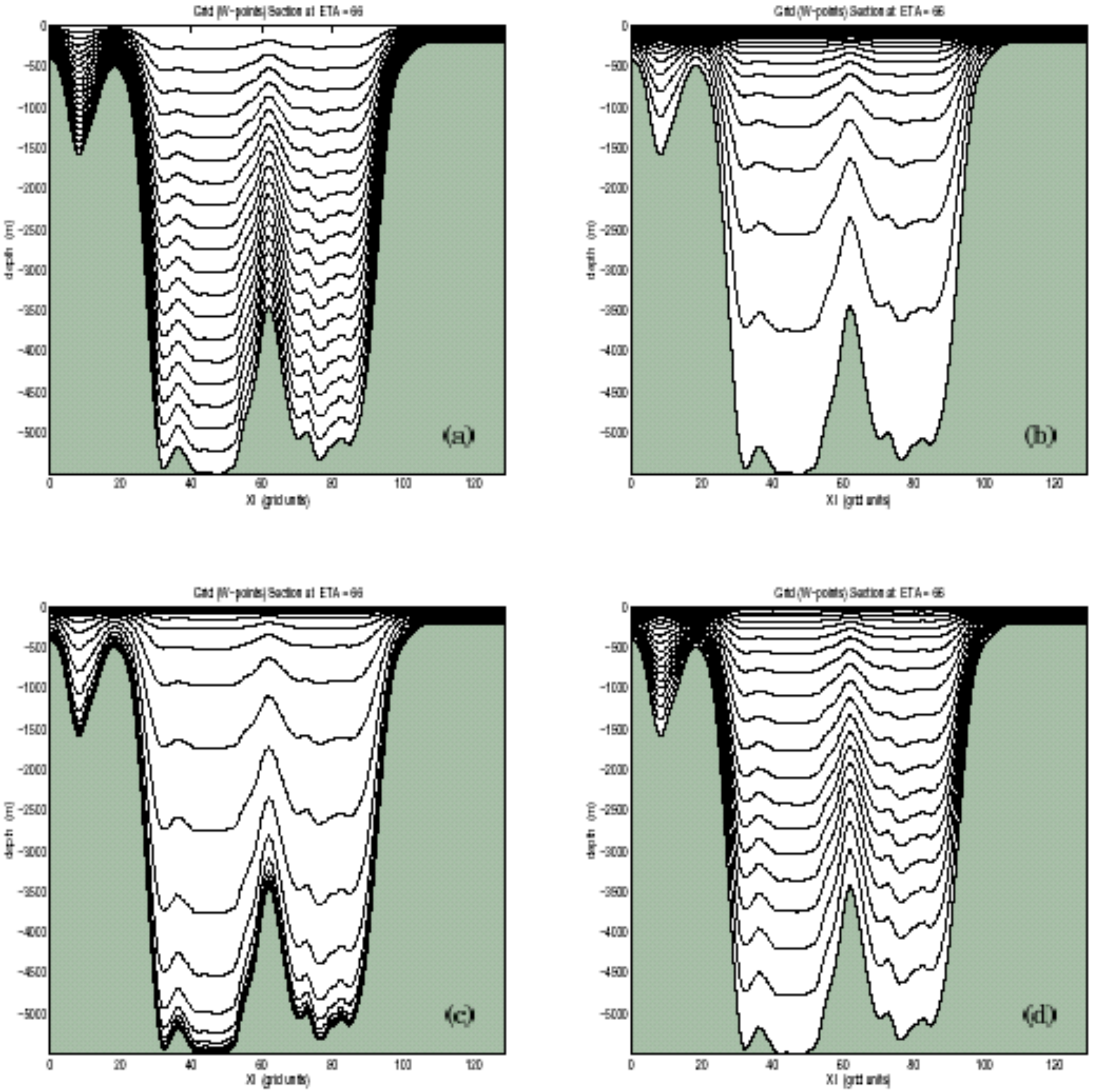
Bottom refinement function:

$$C(\sigma) = \frac{\exp(\theta_B\,C(\sigma)) - 1}{1 - \exp(-\theta_B)}, \qquad \text{for } \theta_B > 0$$

Vertical grid : σ generalized coordinate

ROMS: Generalized σ-Coordinate

Stretching & condensing of vertical resolution



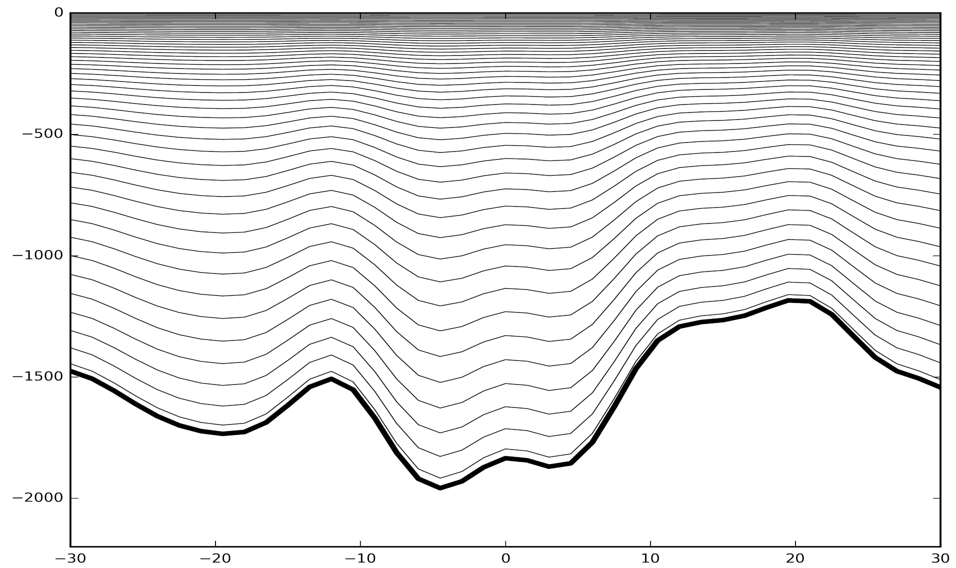
θ and b: suface and bottom parameters:

(a) θ=0, b=0
(b) θ=8, b=0
(c) θ=8, b=1
(d) θ=5, b=0.4

Vertical grid : σ generalized coordinate

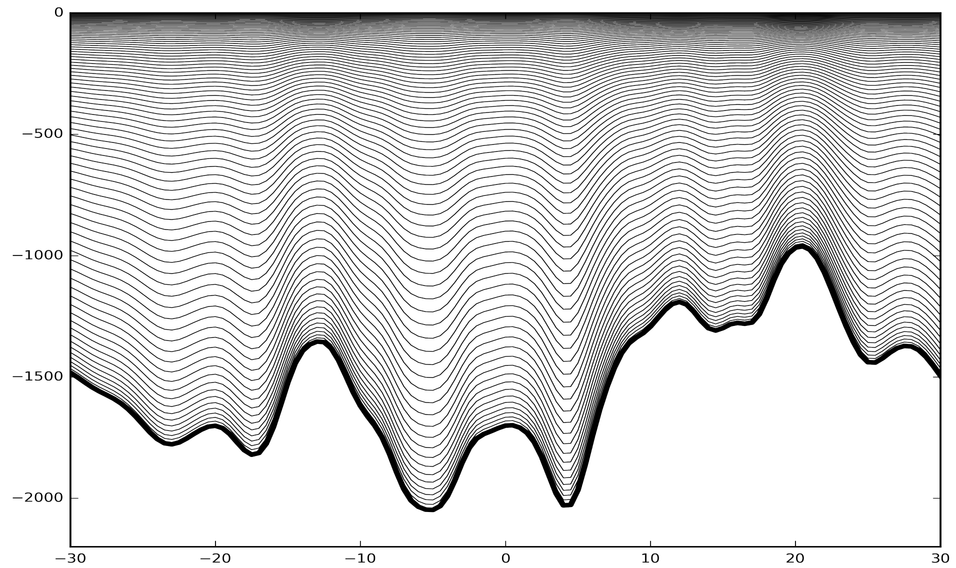
- 50 vertical levels

  $\theta=7$, $b=2$, $h_c=300$ m



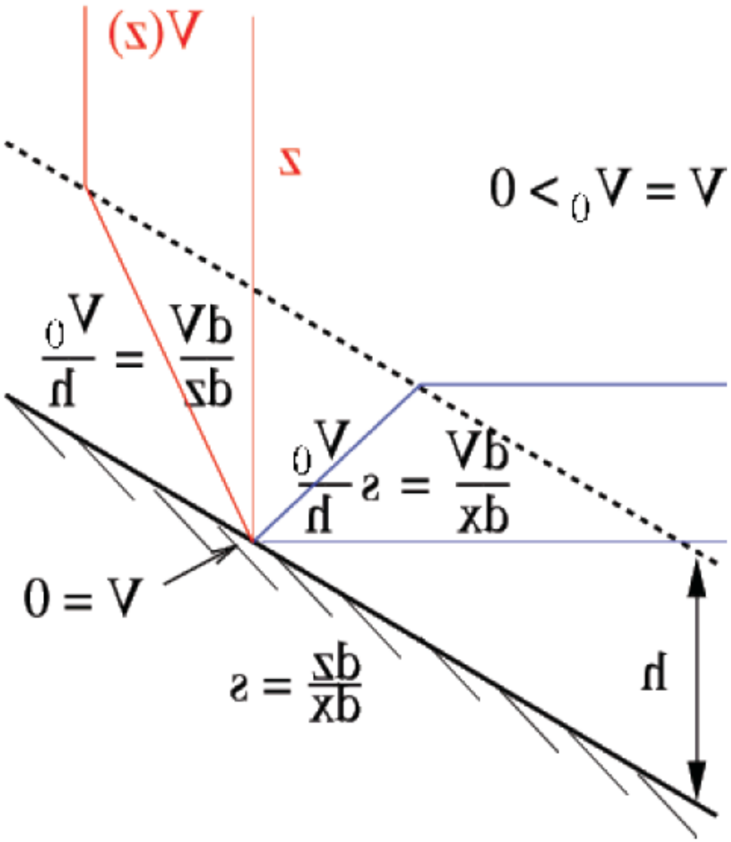
- 80 vertical levels

  $\theta=6$, $b=4$, $h_c=300$ m

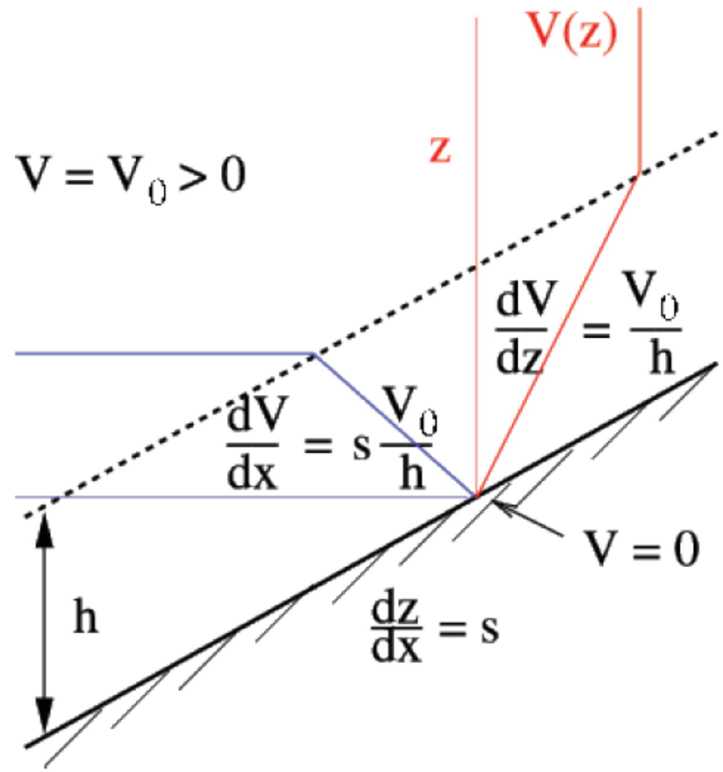


Ex: Topographic vorticity generation

Generation of vertical vorticity within the bottom boundary layer:



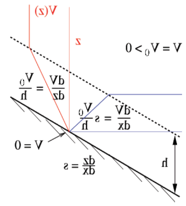
Current flowing with the coast on its left in the Northern hemisphere (opposite to Kelvin wave propagation)
= Positive vorticity generation



Current flowing with the coast on its right in the Northern hemisphere (same than Kelvin wave propagation)
= Negative vorticity generation

Ex: Topographic vorticity generation

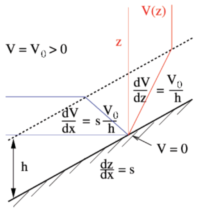
Positive vorticity generation :



- Horizontal shear instability
- Formation of submesoscale cyclones

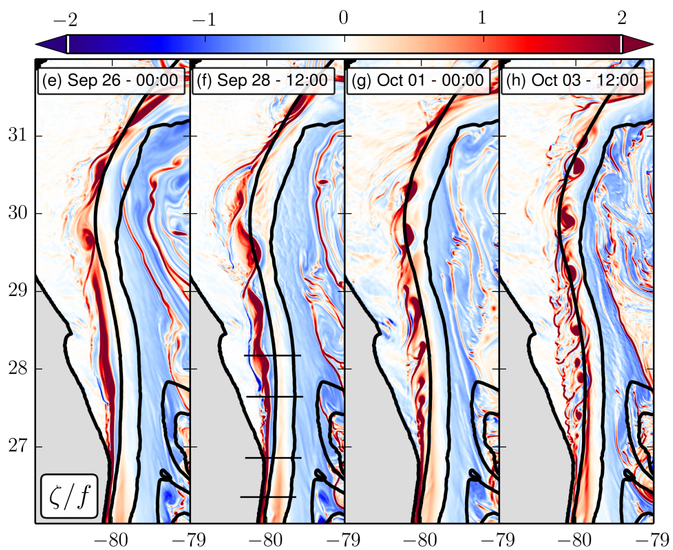
e.g.: Gulf Stream along the slope

Negative vorticity generation:

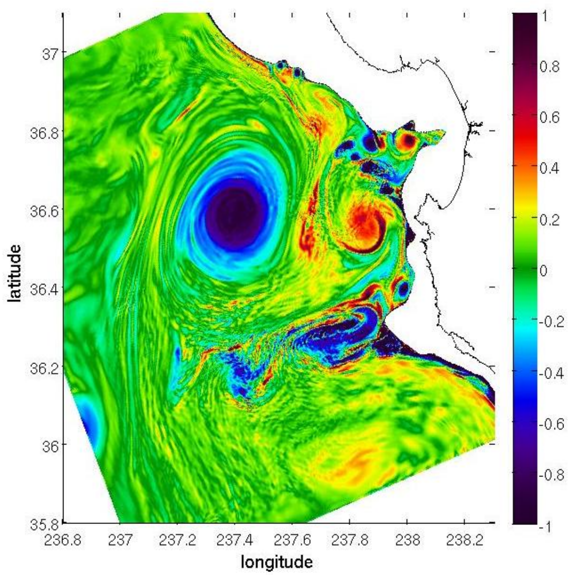


- Centrifugal instability
- Small-scale turbulence, Mixing and dissipation
- Formation of submesoscale anticyclones

e.g. California Undercurrent (formation of Cuddies)



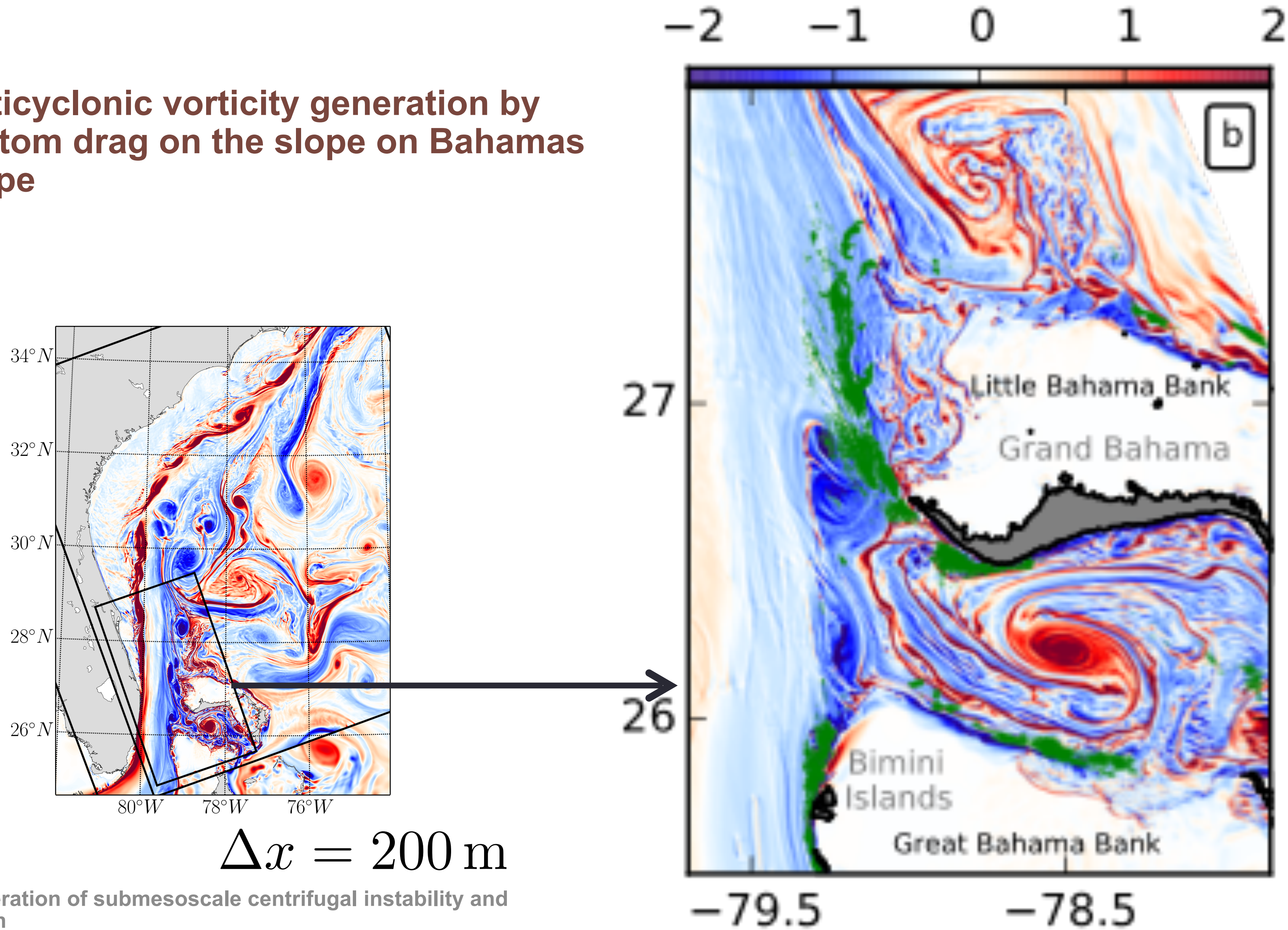
[Gula et al., GRL, 2015]



[Molemaker et al., JPO, 2015]

Ex: Topographic vorticity generation

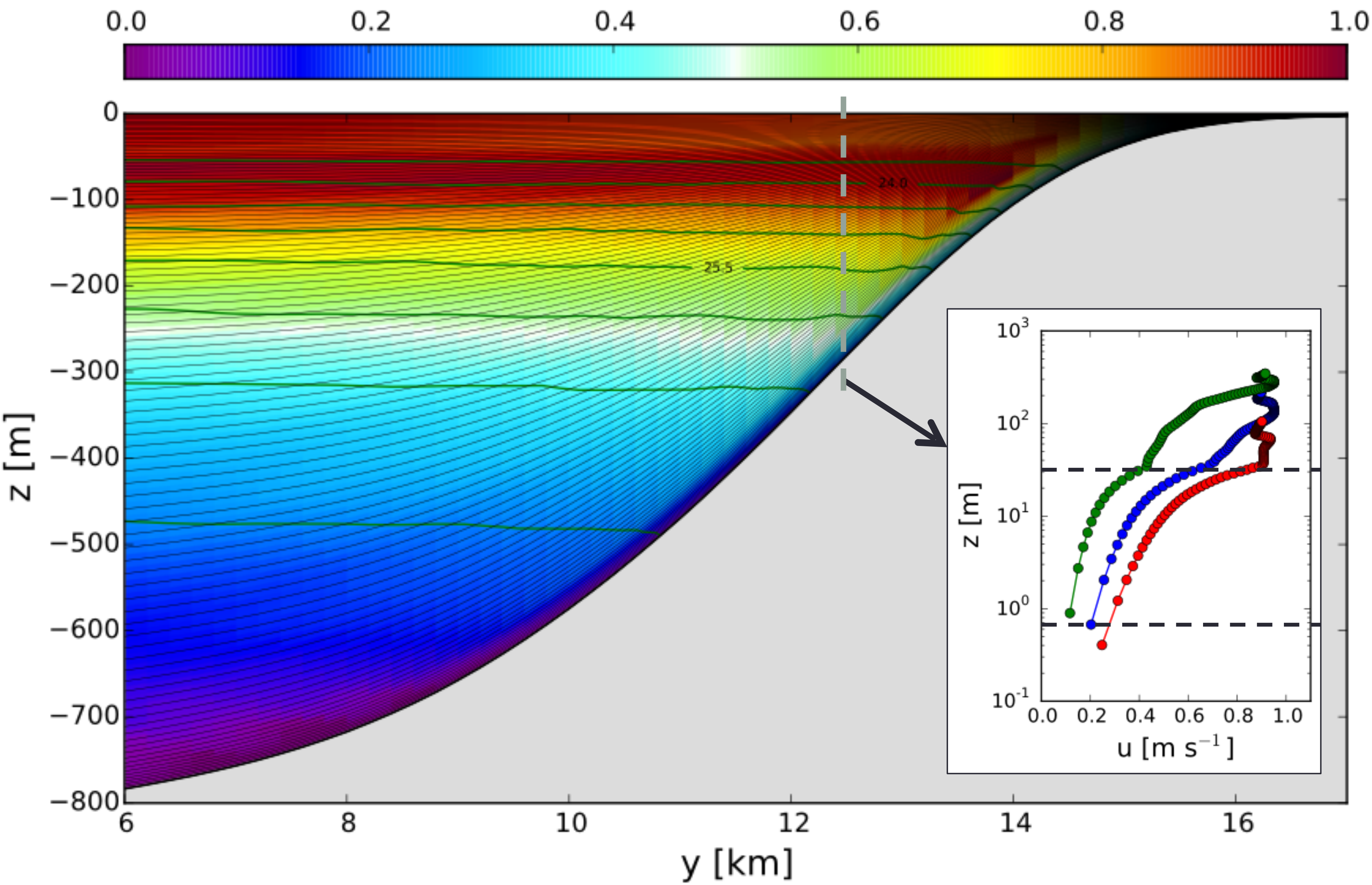
Anticyclonic vorticity generation by bottom drag on the slope on Bahamas slope



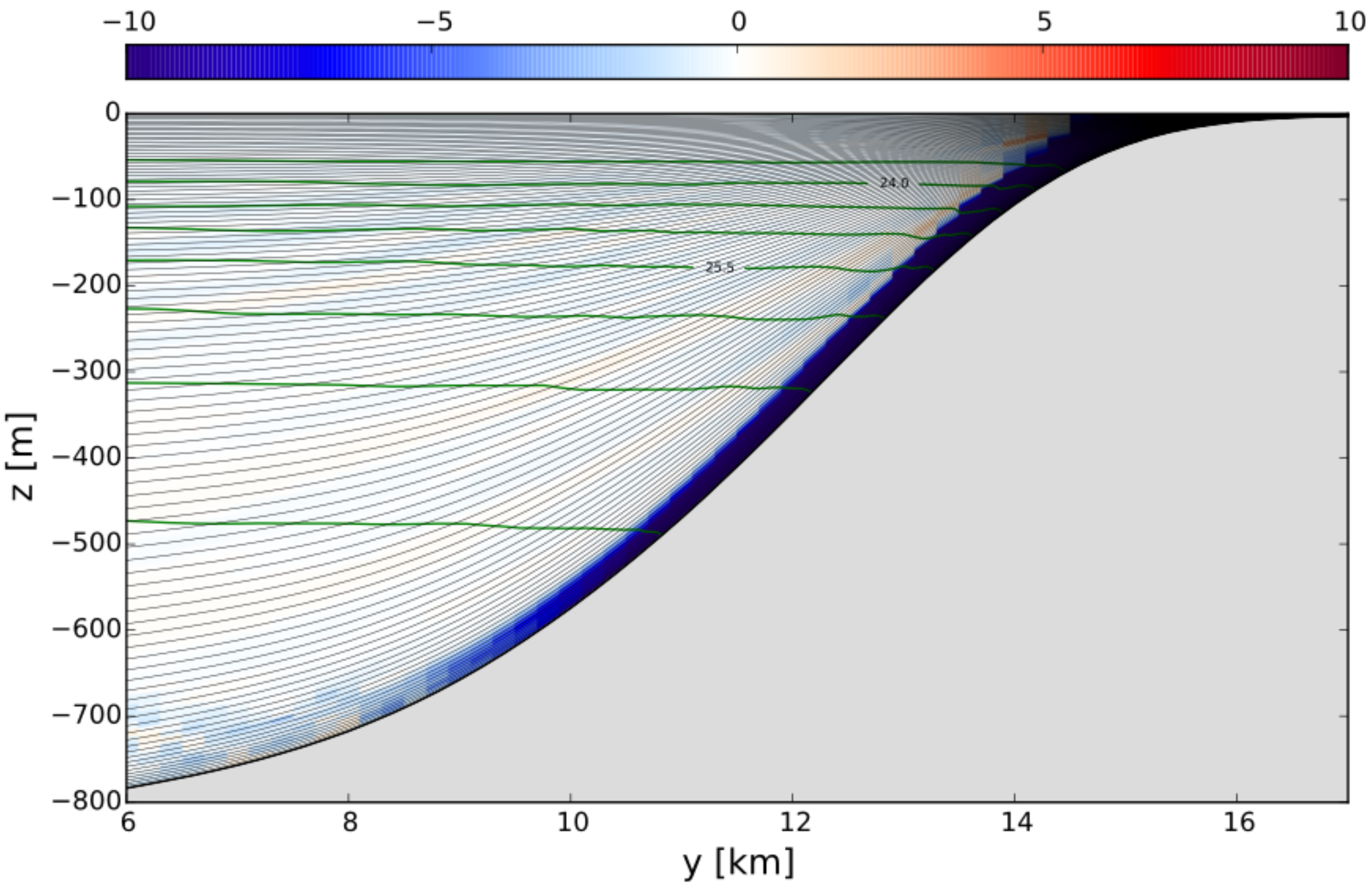
$$\Delta x = 200\,\mathrm{m}$$

Topographic generation of submesoscale centrifugal instability and energy dissipation

Ex: Topographic vorticity generation

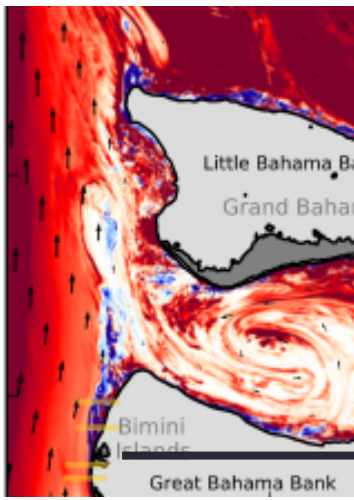


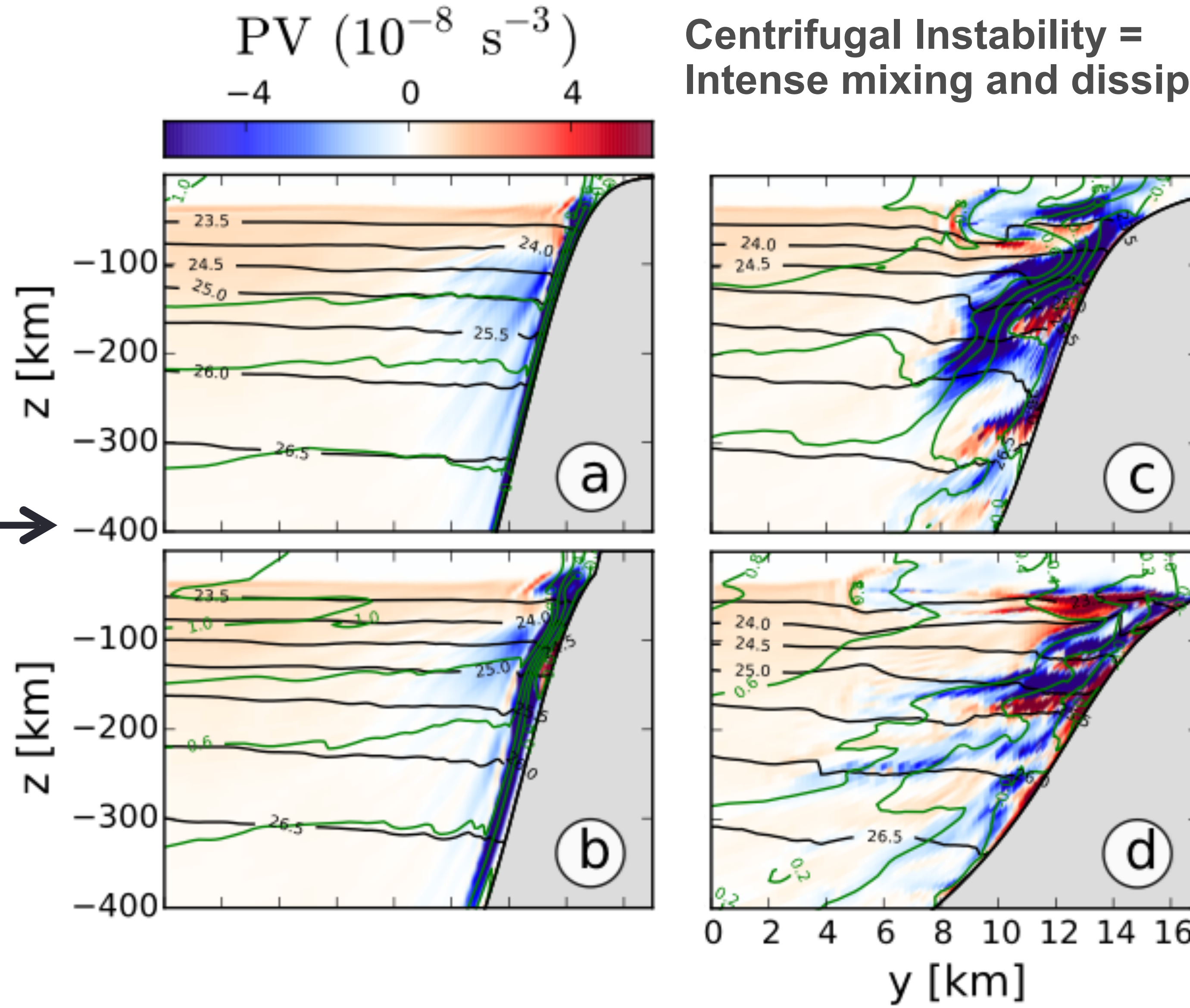
Ex: Topographic vorticity generation



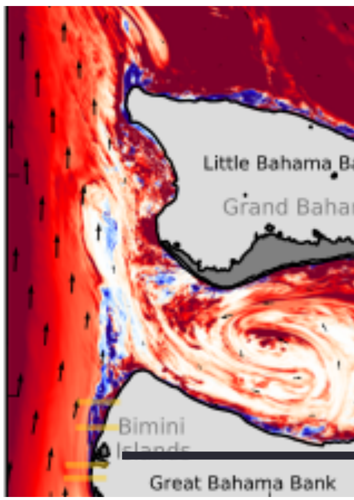
Ex: Topographic vorticity generation

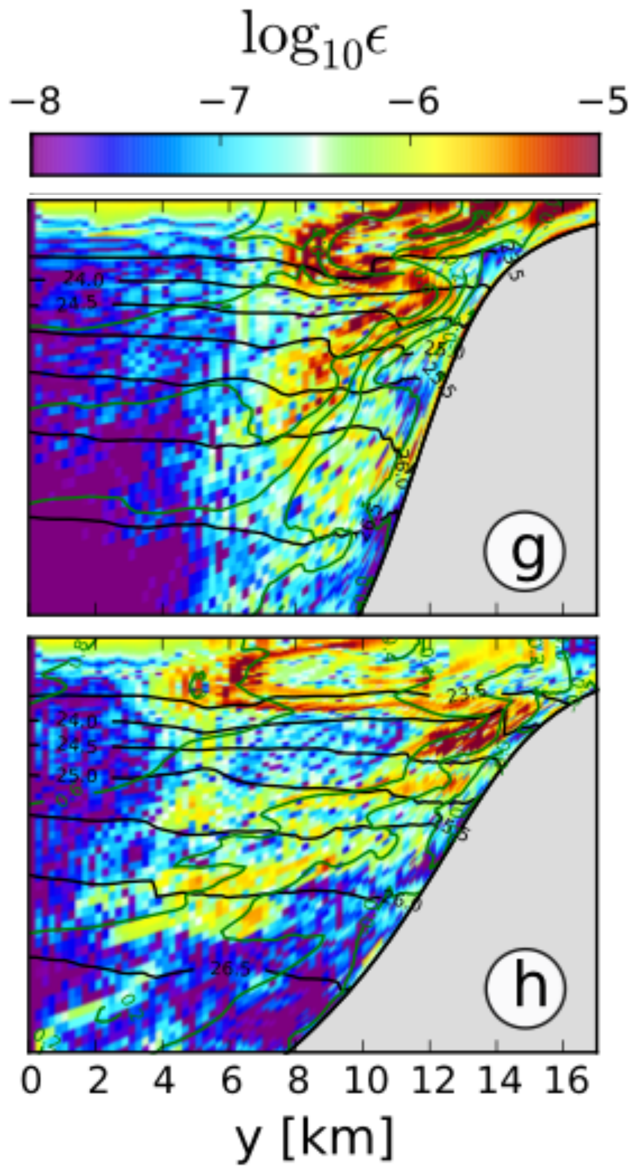
Centrifugal Instability =
Intense mixing and dissipation



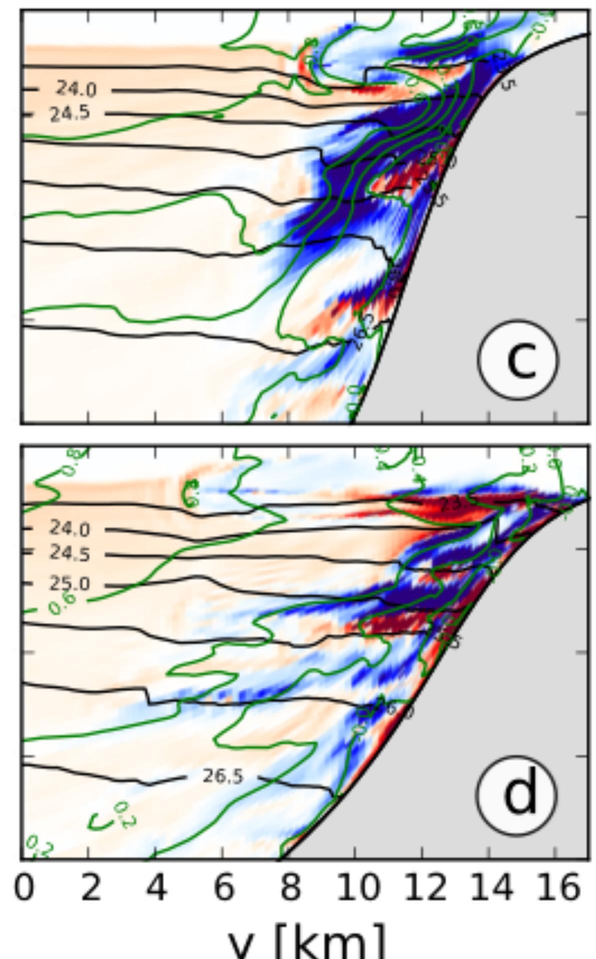


Ex: Topographic vorticity generation





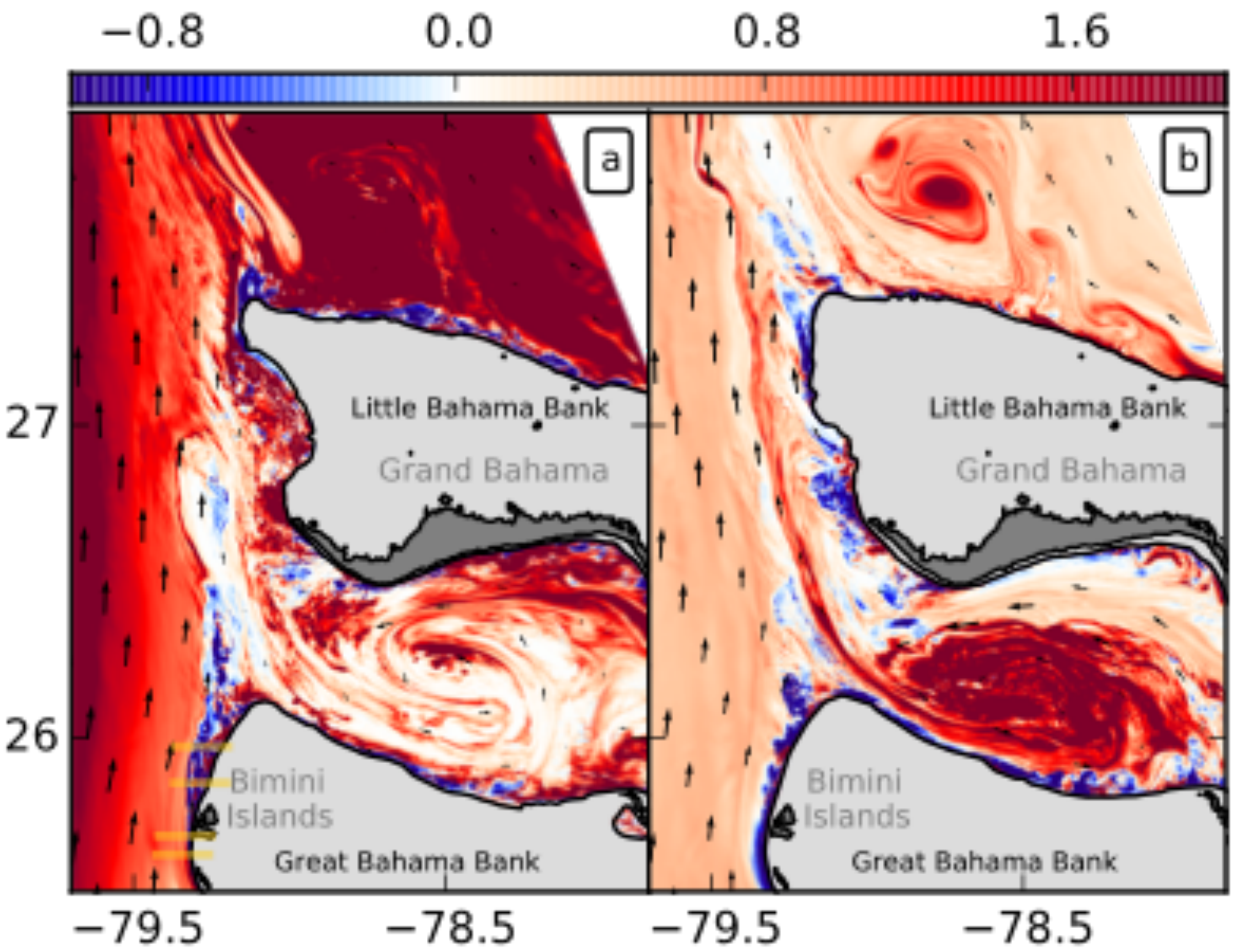
Centrifugal Instability =
Intense mixing and dissipation

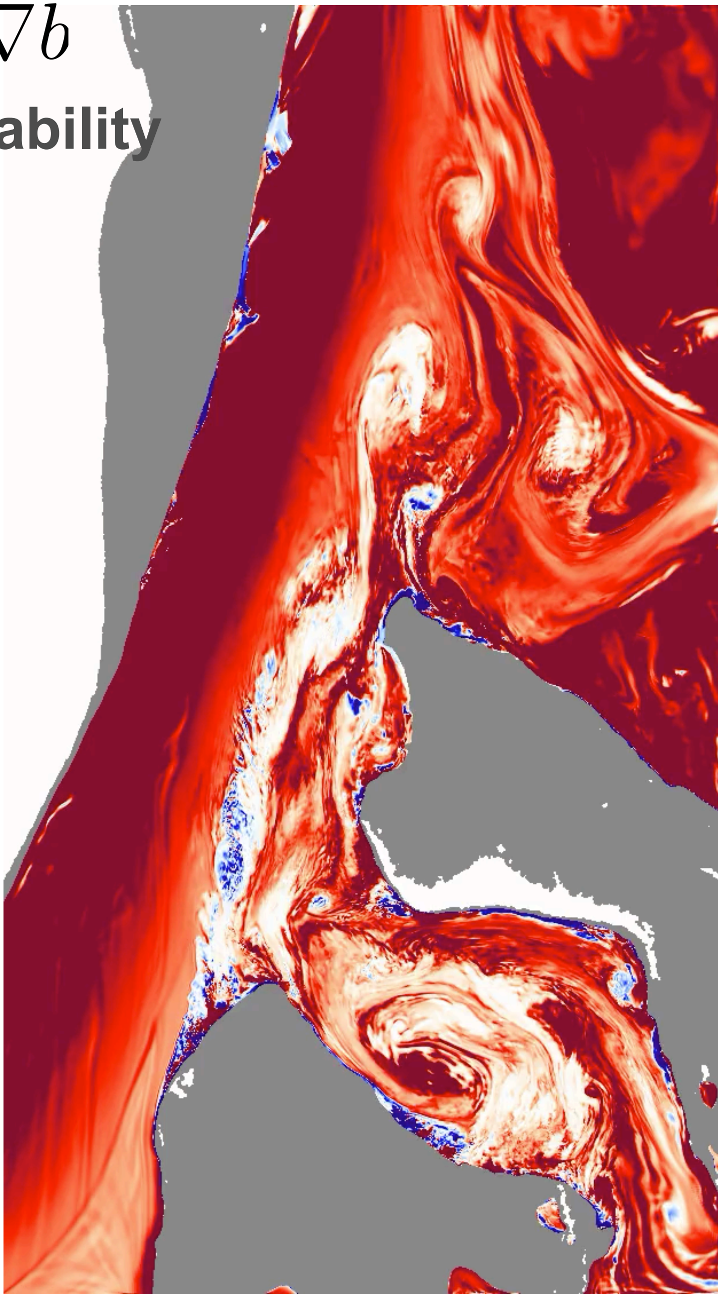


Ex: Topographic vorticity generation

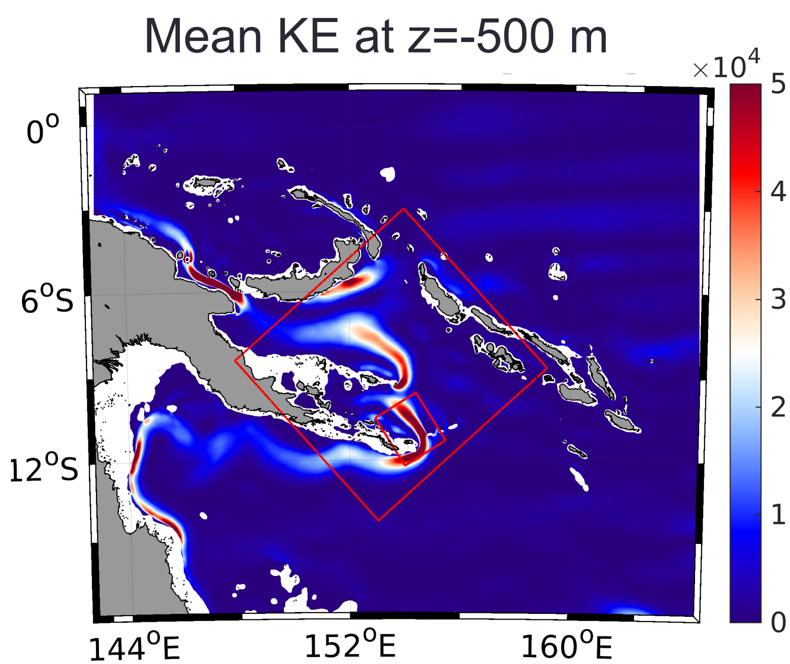
$$q = (f\vec{z} + \vec{\nabla} \times \vec{u}) \cdot \vec{\nabla} b$$

Negative PV (f q<0) = centrifugal (inertial) instability



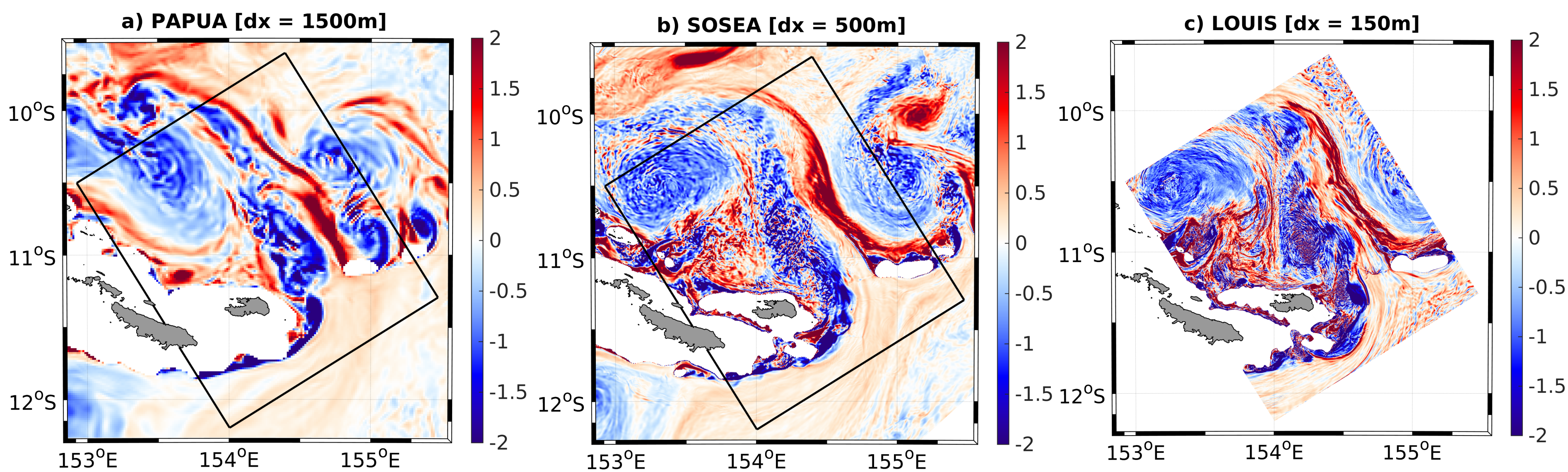


Ex: Topographic vorticity generation

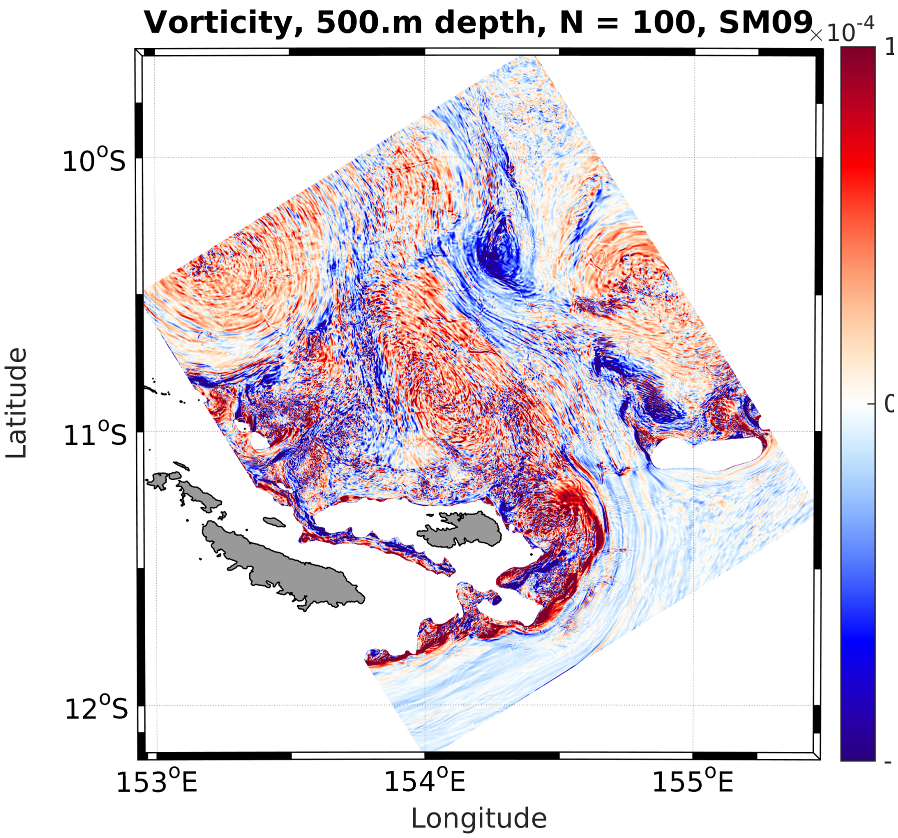


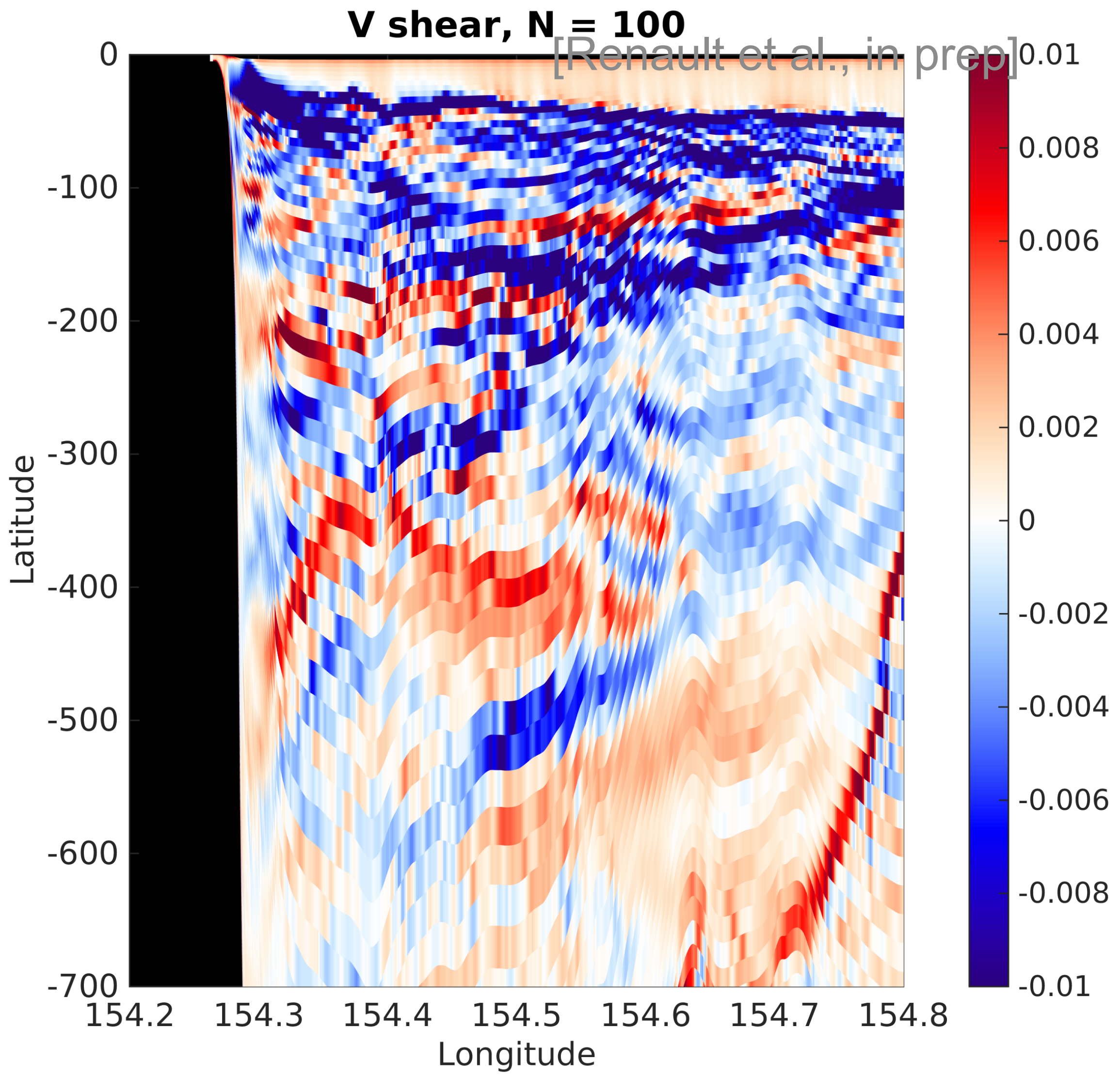
Gulf of Papua Undercurrent

Relative vorticity at z=-500 m



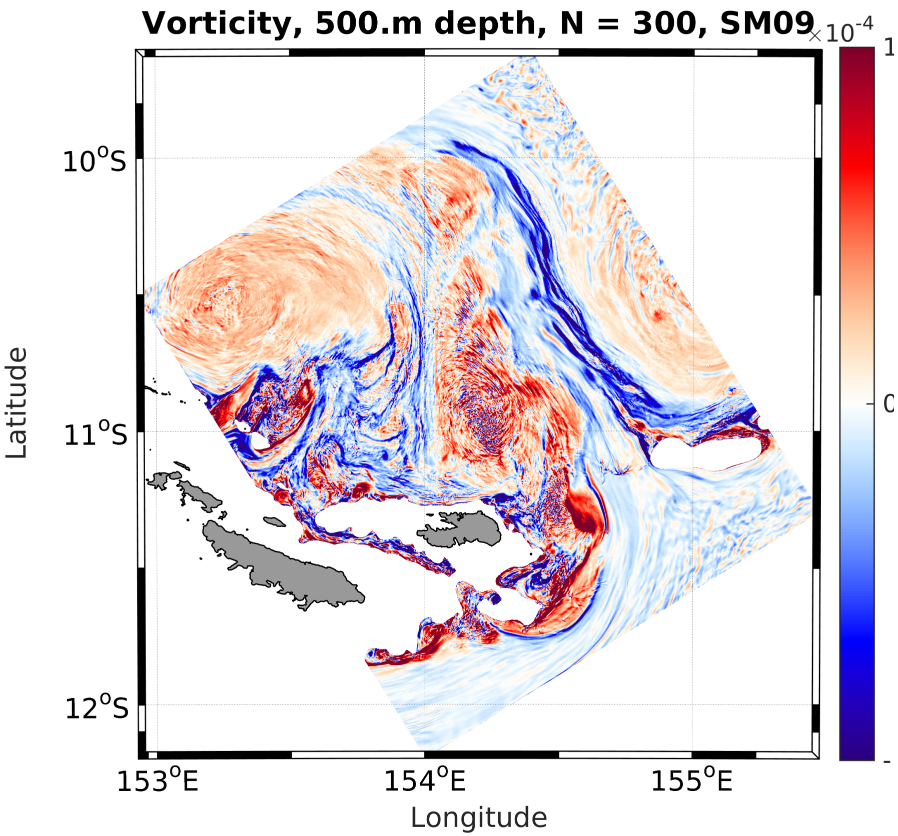
Ex: Need in Vertical resolution?

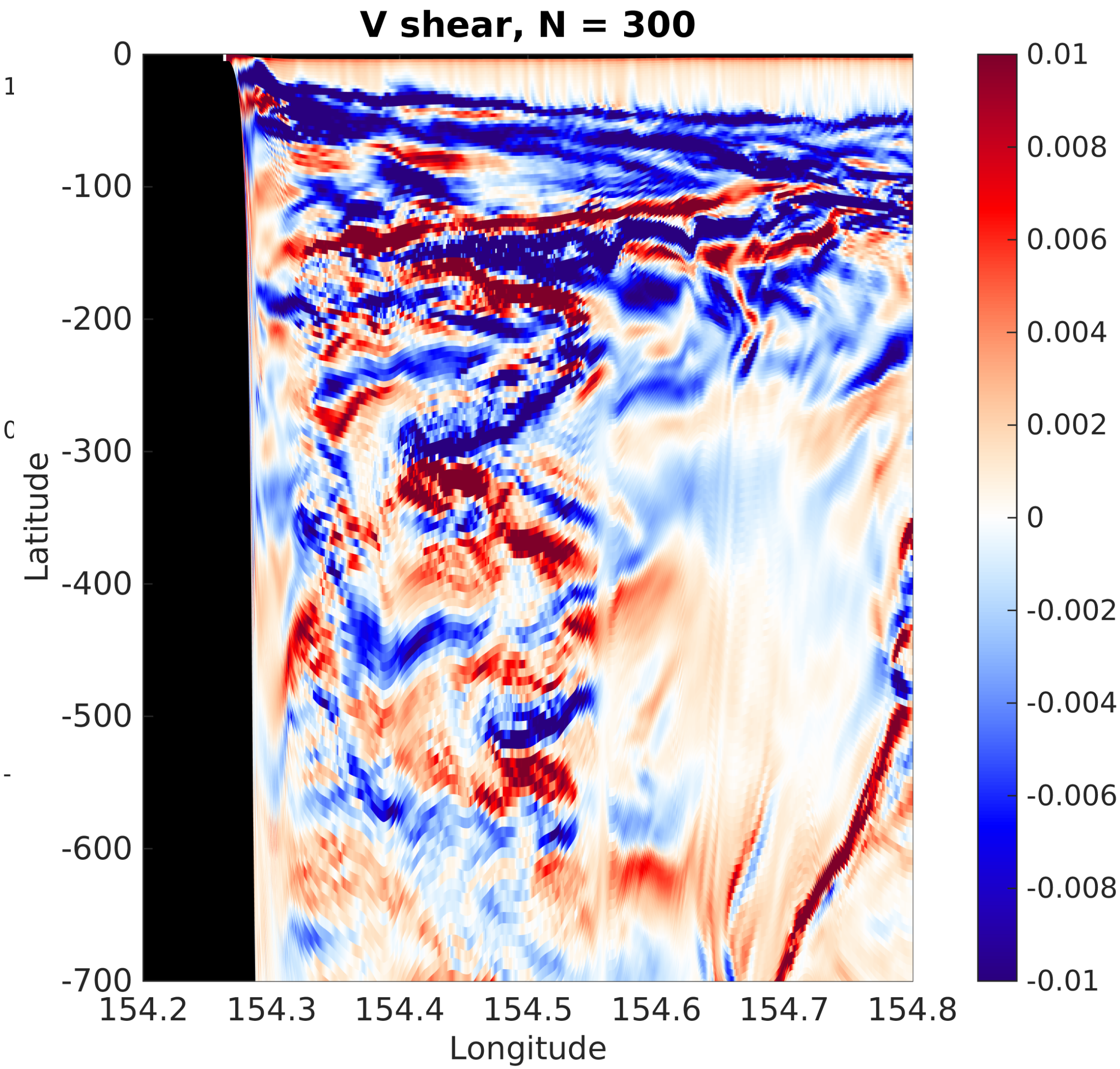




Ex: What happens when underresolving vertical scales with HR simulations (Solomon Sea)

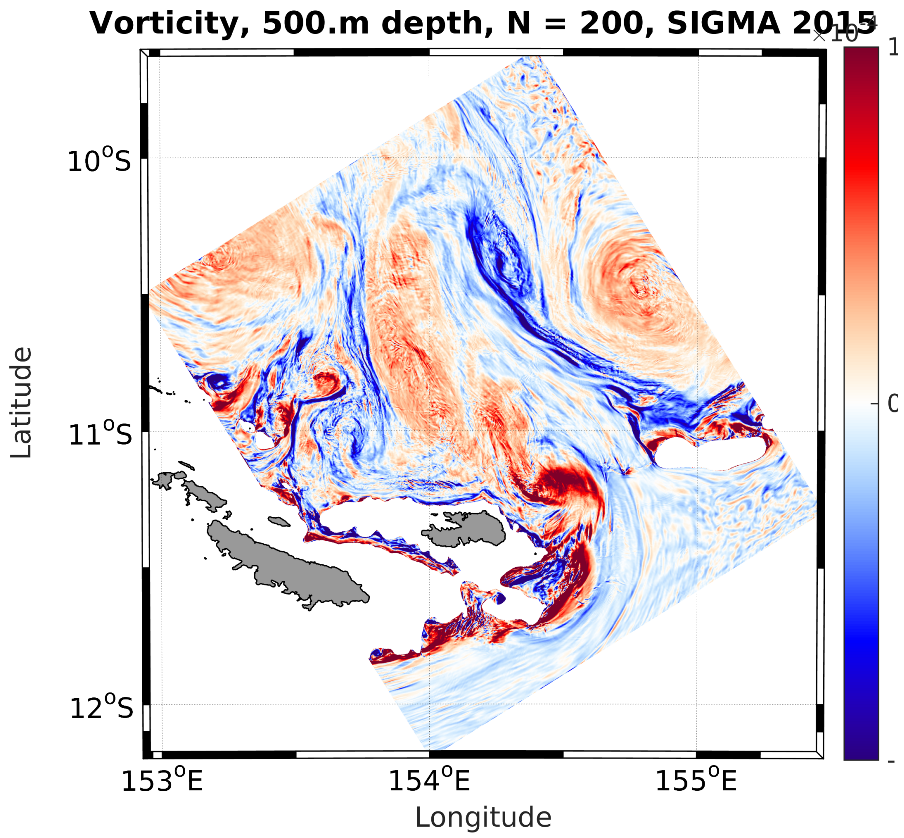
Ex: Need in Vertical resolution?

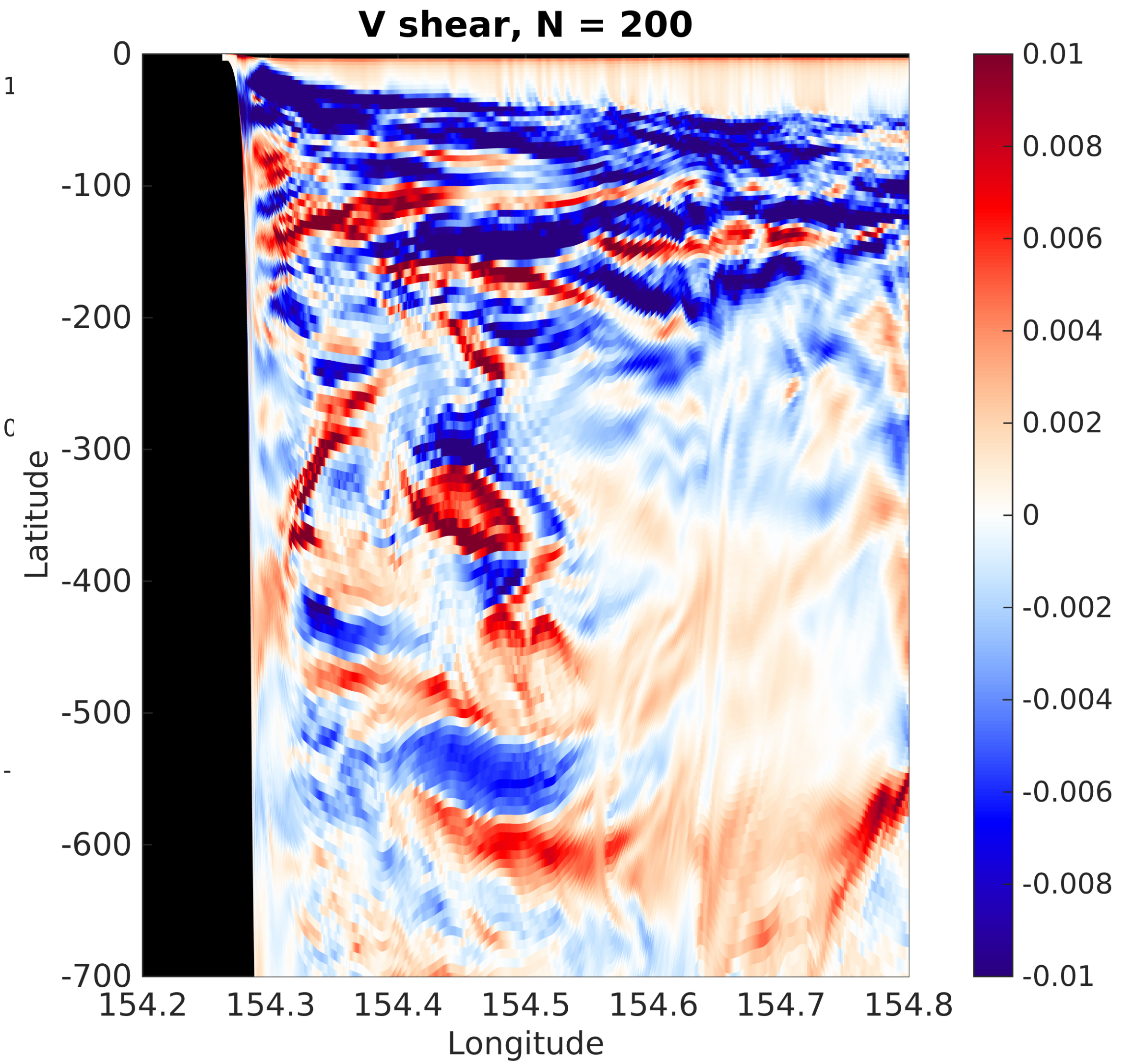




From N=100 levels to N=300 levels…

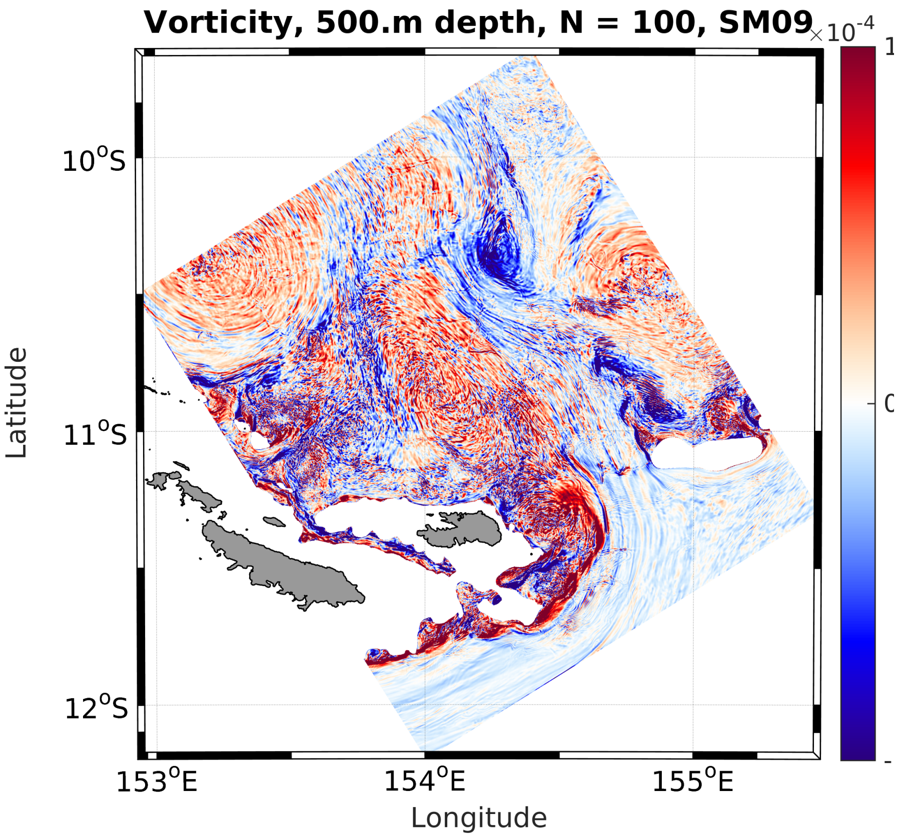
Ex: Need in Vertical resolution?

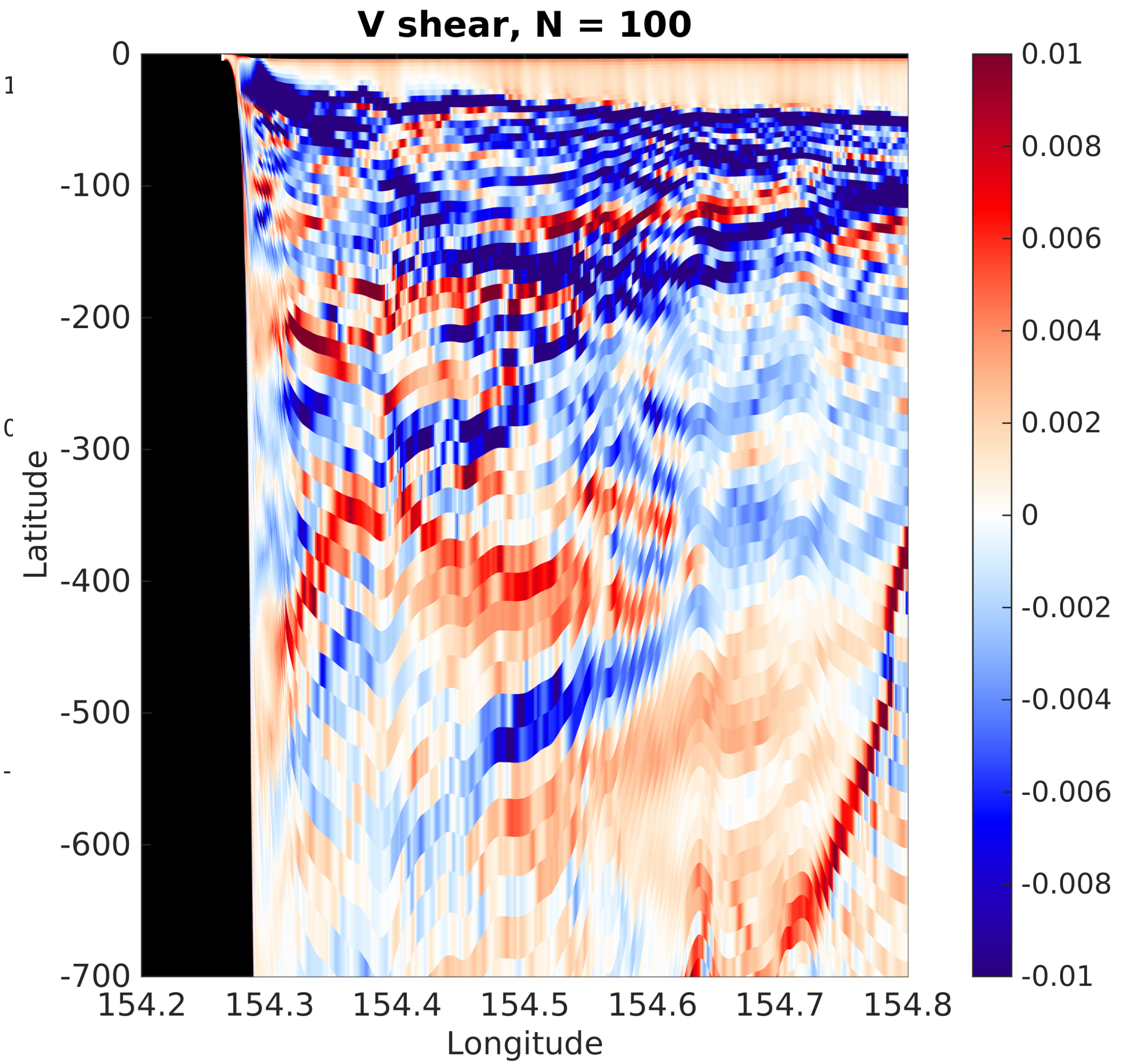




N=200 levels with a more balanced distribution

Ex: Need in Vertical resolution?

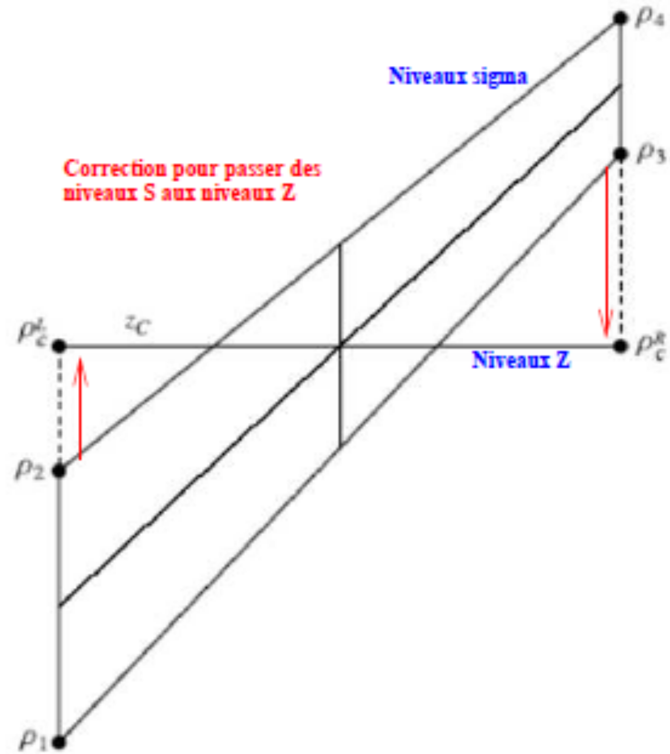




Ex: What happens when underresolving vertical scales with HR simulations (Solomon Sea)

Pressure Gradient force

- Truncation errors are made from calculating the baroclinic pressure gradients across sharp topographic changes such as the continental slope



- Difference between 2 large terms

$$\boxed{-\frac{1}{\rho_0}\left.\frac{\partial P}{\partial x}\right|_z = -\frac{1}{\rho_0}\left.\frac{\partial P}{\partial x}\right|_s + \frac{1}{\rho_0}\cdot\frac{\partial P}{\partial z}\cdot\left.\frac{\partial z}{\partial x}\right|_s,}$$

- Errors can appear in the unforced flat stratification experiment

$$\epsilon \equiv \frac{\left|\left.\frac{\partial P}{\partial x}\right|_s - \frac{\partial P}{\partial z}\cdot\left.\frac{\partial z}{\partial x}\right|_s\right|}{\left|\left.\frac{\partial P}{\partial x}\right|_s\right| + \left|\frac{\partial P}{\partial z}\cdot\left.\frac{\partial z}{\partial x}\right|_s\right|} \ll 1,$$

Reducing PGF Truncation Errors

- Smoothing the topography using a nonlinear filter and a criterium: $\boxed{r = \Delta h / h < 0.2}$
- Using a "density formulation"

$$-\frac{1}{\rho_0}\frac{\partial P}{\partial x}\bigg|_z = -\frac{1}{\rho_0}\frac{\partial P}{\partial x}\bigg|_{z=\zeta} - \frac{g}{\rho_0}\int_z^\zeta \frac{\partial \rho}{\partial x}\bigg|_z \mathrm{d}z'$$

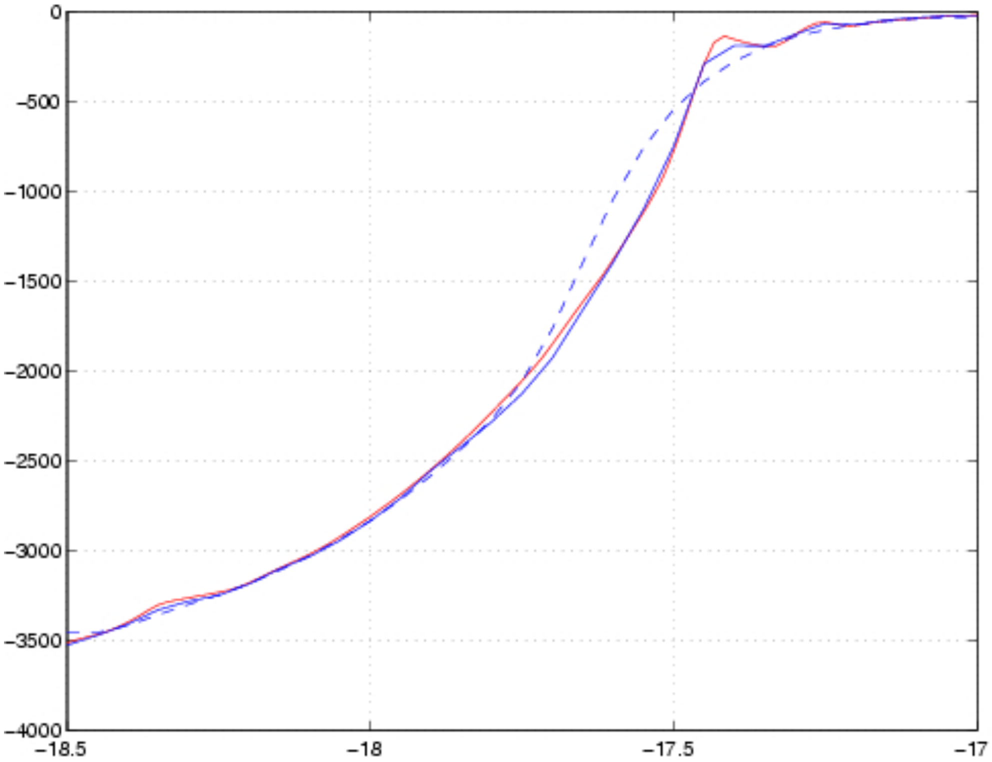
$$= -\frac{g\rho(\zeta)}{\rho_0}\frac{\partial \zeta}{\partial x} - \frac{g}{\rho_0}\int_z^\zeta \left[\frac{\partial \rho}{\partial x}\bigg|_s - \frac{\partial \rho}{\partial z'}\frac{\partial z'}{\partial x}\bigg|_s\right]\mathrm{d}z',$$

- Using high order schemes to reduce the truncation error (4th order, McCalpin, 1994)
- Gary, 1973: substracting a reference horizontal averaged value from density ($\rho' = \rho - \rho_a$) before computing pressure gradient
- Rewritting Equation of State: reduce passive compressibility effects on pressure gradient

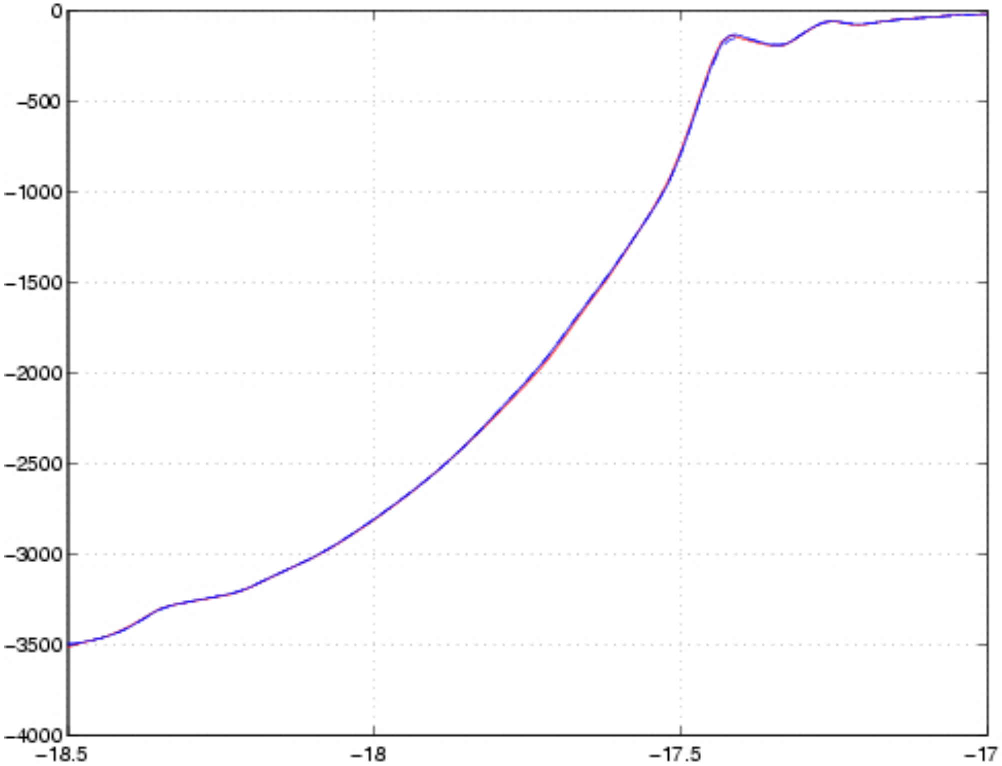
Smoothing methods

- $r = \Delta h \,/\, h$ is the slope of the logarithm of h
- One method (ROMS) consists of smoothing $ln(h)$ until $r < r_{max}$

Res: 5 km ; $r_{max} = 0.25$



Res: 1 km ; $r_{max} = 0.25$



Turbulence closure

In ROMS:

$$\mathcal{F}_u = \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial u}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h u)$$

$$\mathcal{F}_v = \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial v}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h v)$$

$$\mathcal{S}_T = \frac{\partial}{\partial z}\left(K_{Tv}\frac{\partial T}{\partial z}\right) + \nabla_h(K_{Th}.\nabla_h T)$$

$$\mathcal{S}_S = \frac{\partial}{\partial z}\left(K_{Sv}\frac{\partial S}{\partial z}\right) + \nabla_h(K_{Sh}.\nabla_h S)$$

Vertical mixing

Horizontal diffusion

Turbulence closure

Vertical Mixing: $$K_{Mv}, K_{Tv}, K_{Sv}$$

- Local Turbulence closure: GLS, k-kl (MY2.5), k-ε, κ-ω, etc. *[e.g. Warner et al, 2005, Ocean Modelling]*
- Non local K-profile parameterization (KPP) *[Large et al, 1994, Rev. of Geophysics]*

Horizontal diffusion:

$$K_{Mh}, K_{Th}, K_{Sh}$$

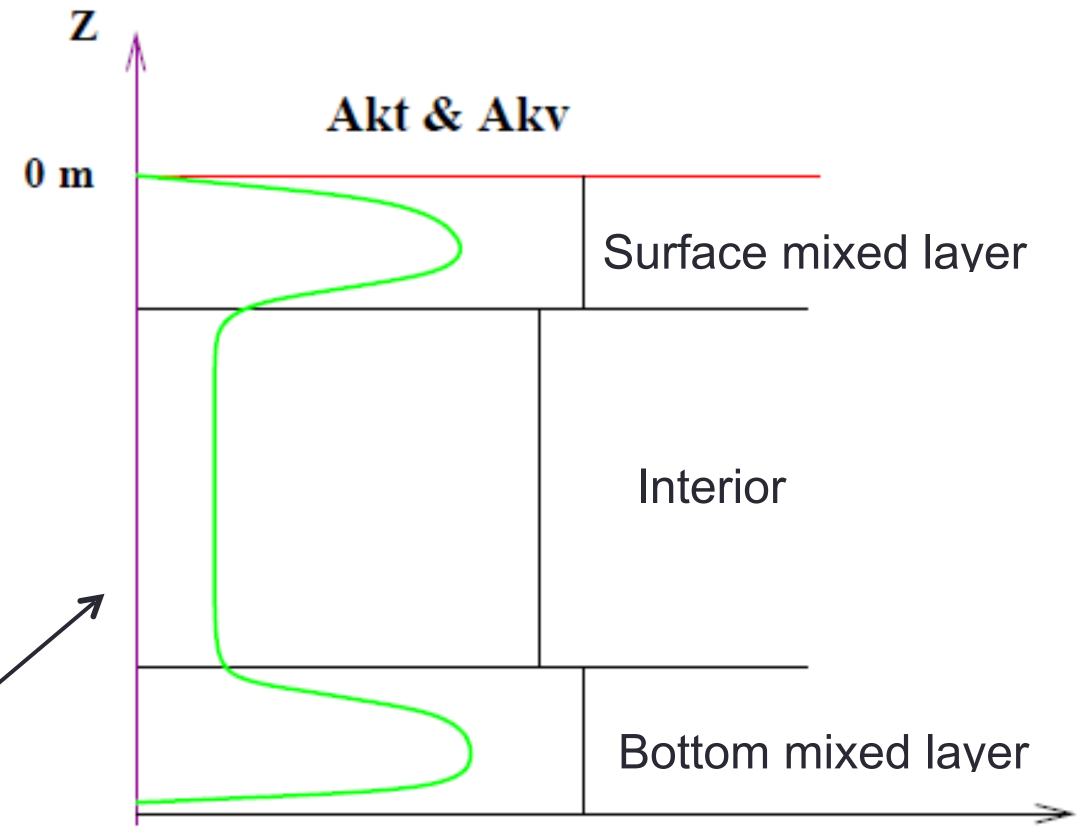
- Explicit diffusion
- Implicit (comes from the advective scheme)

Non local K-profile parameterization

- ❑ Mixed layer schemes are often based on one-dimensional « column physics »

- ❑ Boundary layer parameterizations are based either on:
  - Turbulent closure (Mellor-Yamada, TKE)
  - **K profile (KPP)**

Mixing coefficient profils



Principle scheme of KPP turbulent

Vertical grid : σ generalized coordinate

- Boundary conditions become:

top ($\sigma = 0$)

$$\left(\frac{K_m}{H_z}\right)\frac{\partial u}{\partial \sigma} = \tau_s^x(x,y,t)$$
$$\left(\frac{K_m}{H_z}\right)\frac{\partial v}{\partial \sigma} = \tau_s^y(x,y,t)$$
$$\left(\frac{K_C}{H_z}\right)\frac{\partial C}{\partial \sigma} = \frac{Q_C}{\rho_o c_P}$$
$$\Omega = 0$$

and bottom ($\sigma = -1$)

$$\left(\frac{K_m}{H_z}\right)\frac{\partial u}{\partial \sigma} = \tau_b^x(x,y,t)$$
$$\left(\frac{K_m}{H_z}\right)\frac{\partial v}{\partial \sigma} = \tau_b^y(x,y,t)$$
$$\left(\frac{K_C}{H_z}\right)\frac{\partial C}{\partial s} = 0$$
$$\Omega = 0.$$

Bottom friction parametrization

1. Linear friction, with r ***friction velocities*** [m/s] $\rightarrow$ $$(\tau_b^x, \tau_b^y) = -r \, (u_b, v_b)$$

2. Quadratic friction, controled by a constant drag coefficient **Cd** $\rightarrow$ $$(\tau_b^x, \tau_b^y) = C_d \sqrt{u_b^2 + v_b^2} \, (u_b, v_b)$$

3. Quadratic friction coefficient, using variable **Cd** (Von Karman log. layer) $\rightarrow$

$$\begin{cases} (\tau_b^x, \tau_b^y) = C_d \sqrt{u_b^2 + v_b^2} \, (u_b, v_b) \\ C_d = \left(\frac{\kappa}{\log[\Delta z_b / z_r]}\right)^2 \text{ si } C_d^{min} < C_d < C_d^{max} \\ C_d = C_d^{min} \text{ ou } C_d^{max} \\ \kappa = 0.41 \\ z_r \text{ = Roughness Length} \\ \Delta z_b \text{ = thickness of the first bottom level} \end{cases}$$

For Activity : Computing z and vertical interpolations

- See example in :

  https://github.com/Mesharou/mesharou.github.io/blob/master/ModNum/example_croco.ipynb

```python
# some tools to interpolate croco outputs vertically

import tools as to
```

```python
##################################################################################
#Load variables and parameters
##################################################################################

nc = Dataset(ncfile, 'r')

temp3d=np.array(nc.variables['temp'][-1,:,:,:])


#Load some parameters

zeta=nc.variables['zeta'][-1,:,:]
topo=nc.variables['h'][:]
pm=nc.variables['pm'][:]
pn=nc.variables['pn'][:]

hc = nc.hc
Cs_r = nc.Cs_r
Cs_w = nc.Cs_w

#close netcdf file
nc.close()

##################################################################################
#Compute vertical coordinates
##################################################################################

(z_r,z_w) =  to.zlevs(topo,zeta, hc, Cs_r, Cs_w)

##################################################################################
#Interpolate a variable on a given depth
##################################################################################

t400 = to.vinterp(temp3d,z_r,-500,topo=topo,cubic=1)
```