Master OFFWIND

Coastal Dynamics #1

Exercise 1: Bottom pressure

(a) Find the deep-water speed and wavelength of a wave of period 12 s.

(b) Find the speed and wavelength of a wave of period 12 s in water of depth 3 m using the general full dispersion relation (you will need to solve it numerically).

(c) Compare with the shallow-water approximation.

Exercise 2: Tides in the Bay of Fundy

The Bay of Fundy in New Brunswick (Canada) has a very large tidal range (17 m), a depth $H = 75$ m and a length $L \approx 320$ km. Gravity is $g = 9.8$ m s^{-2}.

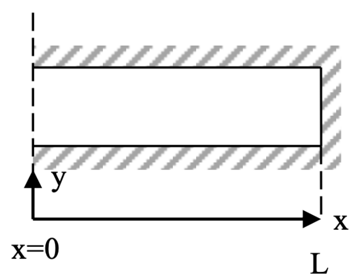

Figure 1: An illustration of the Bay of Fundy.

The main assumptions here are that the wave solution is linear and independent of y. Recall that the linearized long gravity wave equations are:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x}, \tag{1}$$

$$\frac{\partial \eta}{\partial t} + H\frac{\partial u}{\partial x} = 0, \tag{2}$$

where η is the elevation of the free surface.

The flow in the bay is forced at the entrance of the bay by a (small) tide of O(1 m) coming from the open ocean, so:

$$\eta(x = 0, t) = \eta_0 \cos(\omega \mathrm{t})$$

where the forcing period above is $T = 12h25$' for the principal lunar constituent called M2.

(i) Derive the wave equation in terms of the elevation η.

(ii) Look for a forced solution of the form $\eta(x,t) = F(x)\cos(\omega t)$ which has the same shape (in time) as the forcing: derive a differential equation for F and then find $F(x)$ given boundary conditions at $x = 0$ and $x = L$.

(iii) Show that the solution can become infinitely large for certain frequencies to be found, illustrating a resonance phenomenon. Discuss.

(iv) Calculate the period of the gravest resonant mode (longest wavelength, longest period) for the Bay of Fundy when the amplitude in (iii) becomes infinite. Compare with the period of the M2 tide and discuss. Estimate the amplification factor (ratio of maximum amplitude in the Bay to amplitude at the mouth) for the M2 tide. Estimate the wavelength and draw a sketch of the surface elevation along the Bay (as a function of x).

Exercise 3: Short Waves

You know that a storm has occurred on January 1, 2018 in the North Atlantic. You observe waves on a beach in Brest which have a 12" period. The next day you observe waves that have an 8" period. Estimate the distance of the storm (assuming of course that the waves that you observe have been forced by the same storm).

Exercise 4: Long wave

The free surface of a long gravity wave is given by : $\eta(x,t) = a\,cos(kx - \omega t)$ in a fluid of depth H.

1 Compute the horizontal velocity u. Is there a phase lag with the pressure field ? Compare the amplitude of u with the phase velocity c. Derive a criterion for the validity of the linear approximation in term of the amplitude a.

2 Compute the tendency (time rate of change) $\partial\eta/\partial t$ and $\partial u/\partial t$ and try to explain the propagation of the wave through a sketch (drawing).

3 Compute the vertical velocity and discuss its variation as a function of z.

Exercise 5: Seiche

Calculate the period of free oscillations (the basin modes are called seiches in this context) of a narrow lake of length L and depth h. ' Narrow ' implies that the modes are taken to vary along x only, (with x the coordinate axis along the length of the lake). Compare your solutions with the longest period of a few lakes which have been observed :

- Geneva lake : L=70 km, h=160m and T=73.5 mn
- Loch Earn (Scotland) : L=10 km, h=60m, T=14.5 mn
- Lake Baikal : L=665 km, h=680m, T=4.64 h