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INTERNAL WAVES IN THE OCEAN

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INTRODUCTION

The roughness of the sea surface is a phenomenon of common experience. Distortions of a flat surface feel the restoring force of gravity and propagate as traveling surface waves, with periods of up to about 20 seconds, speeds up to tens of ms^{-1}, and amplitudes of up to several meters.

An interface between two superposed fluids of different densities can similarly support gravity waves. The restoring forces are weaker, periods and wavelengths are much longer than those of surface waves, and amplitudes can reach many meters. These waves are often observed over the continental shelf in summer, when solar heating produces a warm light surface layer, and in the case shown in Figure 1 appear to be launched as a packet each tidal cycle (Apel et al 1975). They are made visible at the sea surface through the effect of the internal wave currents on surface roughness (Gargett & Hughes 1972). These interfacial waves can be generated by moving ships; the drag associated with their generation at the freshwater/saltwater interface in Norwegian coastal waters was sometimes attributed to the attachment of "dead water" to the hull (Ekman 1904).

An abrupt, or fairly sharp, density interface within the fluid is not essential for the existence of waves that owe their restoring force to gravity; any hydrostatically stable density stratification can support "internal waves." These are a ubiquitous phenomenon in the ocean and have a surprisingly large amplitude (Figure 2). Vertical displacements of

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the order of ten meters are typical, with time scales from tens of minutes to many hours. The associated horizontal currents are typically 0.05 ms^{-1}, and horizontal displacements about a kilometer. The vertical structure is surprisingly complex (Figure 3). Much of the current structure reverses in half the inertial period.

For a historical discussion of early observations, theories, and laboratory models of internal waves, the reader is referred to Defant's (1961) book, and for a complete bibliography up to 1975 to the works by Roberts (1975) and Briscoe (1975a). The serious student of today could commence his studies with the treatment by Phillips (1977), Briscoe's (1975a) summary of work in the 1971–1974 quadrennium, and the 1975 collection of internal-wave papers in Volume 80 of *Journal of Geophysical Research* [described by Briscoe (1975a) as a kind of birthday party for the growing up of the study of internal waves]. Many of the post-birthday

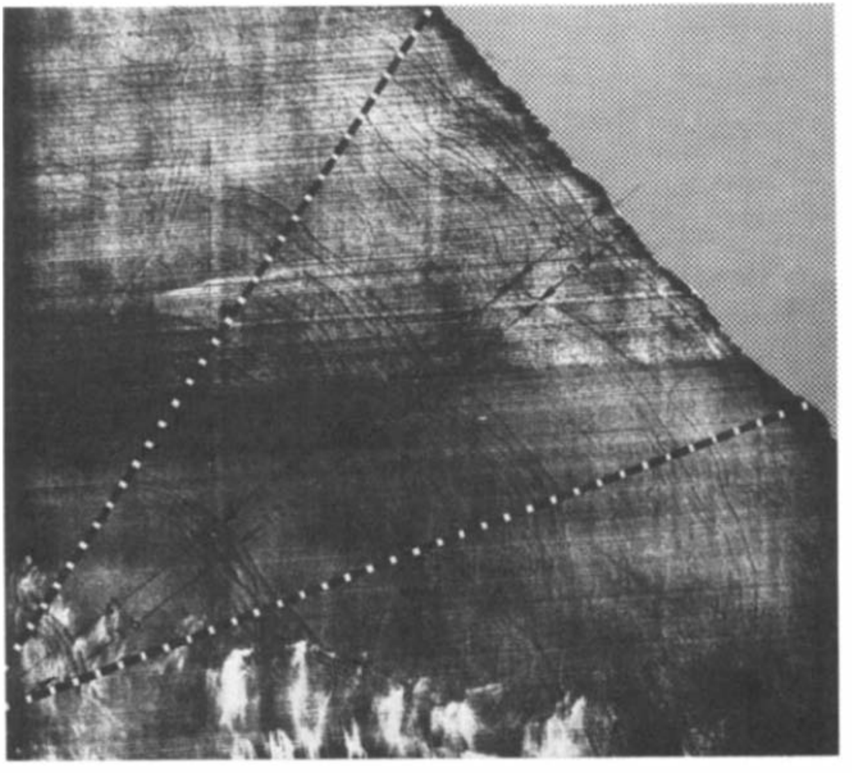


Figure 1 Satellite photograph (Apel et al 1975) of patterns of surface roughness corresponding to packets of internal waves apparently radiating from a source at the lower left corner. The coast of southwest Africa is at the top right corner.

papers are referred to in this review. A summary of the important Soviet literature can be found on pages 74 and 75 of Monin, Kamenkovich & Kort (1977).

Our review is not comprehensive, but we attempt to leave the non-specialist with some idea of how internal waves are measured, how the data are interpreted, what has been learned of the role of internal waves in the ocean, and what problems, both experimental and theoretical, remain. We concentrate on internal waves in the oceans proper, ignoring almost entirely the rather different properties and problems associated with internal waves over continental shelves, and in freshwater lakes.

The mere existence of water motions of the magnitude shown in Figures 2 and 3 provides grounds for their investigation, if only to remove the internal wave "noise" from the description of the mean, or slowly varying, state of the ocean. The modern view is that internal waves are more than just noise and play an important, if not vital, role in other ocean processes.

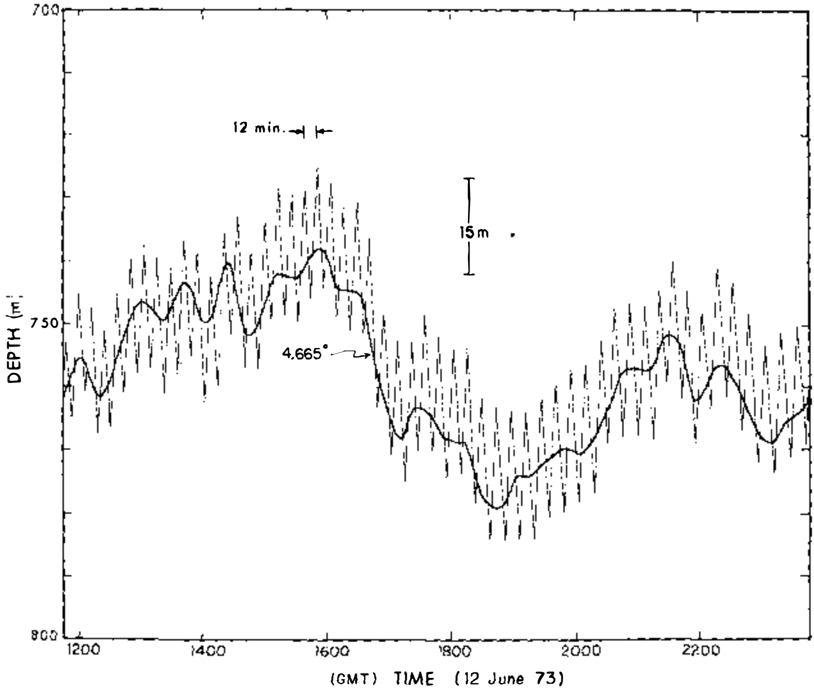


Figure 2 The heavy line shows the depth of a particular isotherm, measured by a drifting instrument package, yo-yoing vertically as shown by the zig-zag lines (Cairns 1975).

DYNAMICAL ROLE Cold, dense water that is formed in polar regions sinks and spreads over the bottom of the ocean basins; it must be slowly mixed with the waters above if a reasonably steady state is to be achieved (e.g. Munk 1966). Internal waves are the most likely cause of this mixing through sporadic instabilities and turbulence. They may play an even greater role in the transfer of momentum, with packets of internal waves acting rather like the molecules in a gas (Müller 1976). Quantifying the transfer of mass and momentum by internal waves, and their role in mixing processes in the ocean, is one of the central tasks of physical oceanography. A full understanding of internal wave dynamics may be essential for a comprehensive understanding of the mean ocean circulation and of the temperature and salinity structure.

BIOLOGICAL SIGNIFICANCE The intermittent uplift of phytoplankton into the sunlit layer of the upper ocean, where photosynthesis occurs, can lead to enhanced productivity (Kamykowski 1974). The wavy vertical displacement of a biologically stratified ocean is responsible for much of the biological variability found in horizontal tows (Denman 1976).

UNDERWATER SOUND The acoustic signal received at one point in the ocean interior from a source at another can vary considerably in ampli-

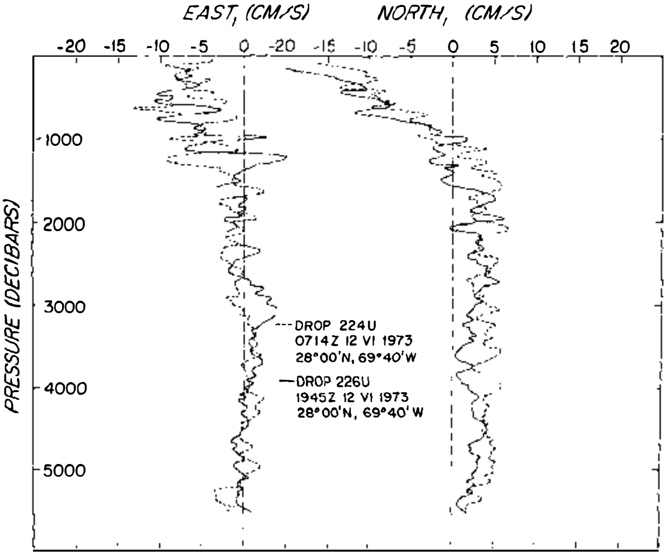


Figure 3 Vertical profiles of eastward and northward currents at an interval of 12.5 h at a fixed location (Sanford 1975). A decibar corresponds closely to 1 m of depth.

tude, travel time, and the direction from which it arrives. Only recently (Flatté et al 1979) has it been appreciated that internal wave-induced variations in sound speed are the dominant cause of these "acoustic scintillations."

MILITARY IMPORTANCE The existence of natural variations in acoustic characteristics of the ocean, associated with the presence of internal waves, clearly complicates the business of submarine detection. On the other hand, a moving submarine leaves behind it a wake of internal waves, and the detection of this wake remains an area of intensive, and well-funded, research.

OBSERVATIONAL TECHNIQUES

A wide variety of sensors have been used to collect data on internal waves. Together they have provided a measure of the relevant scales; not surprisingly, no single sensor has given a time history of the three-dimensional fluctuations.

MOORED SENSORS Much of what we know about internal waves in the ocean has come from current meters attached to a more or less vertical mooring line between an anchor on the seafloor and a buoyant float at or below the sea surface. The technique cannot be applied everywhere; for example, it is difficult to design a mooring to withstand the drag in regions of very high current, such as the Gulf Stream.

The standard current meter consists of a rotor for speed sensing and a vane for direction, with data recorded internally at set intervals (usually several minutes). When attached to a surface mooring, such a current meter can have its rotor "pumped up" by the high-frequency mooring motion (Gould & Sambuco 1975). Such difficulties can be avoided by instruments with an appropriate linear response, and in fact various acoustic and electromagnetic devices and other types of mechanical current meter are in an advanced state of development (McCullough 1978).

Temperature fluctuations are often measured at, or very close to, the current meters. If the local vertical temperature gradient $\partial T/\partial z$ can be measured sufficiently accurately, the instantaneous vertical velocity component can be estimated as $w = -(\partial T/\partial t)/(\partial T/\partial z)$ provided that horizontally advective and diffusive terms are unimportant in the temperature equation (Briscoe 1975b).

The step beyond a single sensor is the use of a number of sensors at different vertical spacings on the same mooring, or on several moorings separated by various distances horizontally. The interpretation of the statistical relationships between simultaneous measurements at different places is discussed later.

The three-dimensional IWEX (Internal Wave Experiment) array is the most ambitious to date. Twenty current meters, temperature sensors, and vertical temperature gradient sensors were deployed south of Bermuda in water of 5 km depth for 40 days in 1973 in a tripodal fashion, with three moorings, from anchors at the corners of an equilateral triangle of side 6 km, being brought together at a point 600 m below the sea surface (Briscoe 1975b).

TOWED SENSORS A thermistor chain, consisting of a cable with temperature sensors every few meters, can be suspended below a ship and towed slowly through the upper layers of the ocean, mapping out a two-dimensional section of temperature structure down to about 200 m below the surface. The displacement of a series of isotherms may be determined by interpolation (Charnock 1965, LaFond & LaFond 1971, Bell 1976). Some information in three dimensions is obtained by towing the chain in a two-dimensional pattern (Voorhis & Perkins 1966), although it becomes difficult to separate spatial from temporal variability.

Katz (1973, 1975) used a towed fish that had its depth continually adjusted to make it follow, as closely as possible, a particular isotherm or isopycnal (constant density surface), the vertical displacement of which was thus measured. Two sensors, at vertical separations of 11 and 21 m, provided a measure of the difference in vertical displacement at these average separations. The amount of data collected is less than with a thermistor chain, but the depth that can be achieved, 800 m, is considerably greater.

Another towed device is the Batfish, which measures temperature, salinity, and even chlorophyll fluorescence on a porpoising track through the water (Denman & Herman 1978) down to a maximum depth of 400 m below the surface.

DROPPED INSTRUMENTS Instruments lowered from a hove-to vessel, or dropped freely, are the traditional way to measure the vertical structure of the oceans. A single profile includes not only the mean structure, but also the distortion of the mean profile by internal waves. The XBT (expendable bathythermograph) can be dropped from a moving ship to measure the temperature profile down to 750 m. The CTD (conductivity, temperature, depth) records electrical conductivity and temperature (and hence salinity and density) as functions of depth as it is lowered from a stationary ship. The equivalent of an array measurement is to use two or more profiles separated horizontally (Stegen et al 1975) or in time (Hayes 1975).

Pinkel (1975) used temperature and density profiles from the stable platform FLIP to obtain a continuous picture of the fluctuations in the top 440 m of the Pacific. He obtained some limited information on horizontal variations by using three profilers about 50 m apart.

An important profiling instrument is the EMVP (electromagnetic velocity profiler) of Sanford, Drever & Dunlap (1978), which senses the electric potential in the sea due to the motion of the water through the Earth's magnetic field. Vertical profiles of both horizontal components of water velocity with a vertical resolution of about 10 m are obtained (Figure 3). An array type of measurement may be made with repeated, or simultaneous but horizontally spaced, drops. An expendable version for use from moving ships is now being developed.

YO-YOS The repeated temperature profiling of Pinkel (1975) mentioned above is a form of tethered yo-yo experiment. The self-contained capsule employed by Cairns (1975) and Cairns & Williams (1976) drifted with the mean current while yo-yoing sufficiently fast to give a time history of the vertical displacement of selected isotherms (Figure 2).

REMOTE SENSING The measurement from afar of oceanic fluctuations can provide extended spatial coverage. Ewing's (1950) time-lapse photography from a clifftop showed shoreward moving bands of variable roughness, which he correlated with the internal wave field. More recently (e.g. Apel et al 1975) photographs from aircraft and satellites have been used to describe internal waves in shallow seas (Figure 1).

High-frequency sound is effectively back-scattered from plankton in the upper layers of the ocean. The plankton are often concentrated along isopycnals, so that repeated soundings can reveal the vertical displacement of these surfaces (Proni & Apel 1975). Pinkel (1978) has used the Doppler shift of back-scattered sound in nearly horizontal beams to measure the current component along the beam as a function of range.

Fluctuations in intensity and phase of low-frequency acoustic transmission over a path of 1000 km provide a measure of certain statistical properties of the internal wave field (Dyson et al 1976).

It is usually the statistical properties of internal waves that are required, rather than a detailed description. Remote sensing can sometimes measure the statistical properties directly and with high precision; unfortunately, the moments measured may not be those required!

INTERPRETATION

Given a series of data points, in space or time, describing fluctuations in the ocean, the problem is now to relate them to the processes involved, and in particular to internal waves. To introduce this we need a brief discussion of the basic fluid-dynamical properties of internal waves; for a more detailed discussion the reader is referred to Phillips (1977).

Dispersion and Particle Motion

The degree of density stratification of a fluid with average potential density $\rho(z)$ as a function of depth z is quantified by $N^2 = g\rho^{-1}\partial\rho/\partial z$. The Väisälä (or buoyant) frequency $N(z)$ is the frequency with which a vertically displaced fluid element would be expected to oscillate because of restoring buoyancy forces. If the displacement is not vertical, the restoring force is less and so the frequency of oscillation is reduced. These physical ideas are intrinsic to the dynamical equations that lead to the dispersion relation (Phillips 1977)

$$\omega^2 = (N^2k^2 + f^2m^2)/(k^2+m^2) = N^2\cos^2\theta + f^2\sin^2\theta, \tag{1}$$

connecting the frequency ω, horizontal wavenumber k, and vertical wavenumber m of a wave with variables proportional to $\exp\,[i(kx+mz-\omega t)]$. Here $f = 2\Omega\sin$ (latitude) is the Coriolis frequency associated with the Earth's angular velocity Ω. The permissible range of frequencies is $f \leqq \omega \leqq N$. At frequencies slightly greater than f the particle motion is almost horizontal and circular (Figure 4), with the velocity vector rotating anticyclonically (anticlockwise in the southern hemisphere when viewed from above). At higher frequencies the ellipse is increasingly inclined to the horizontal and more eccentric, tending to an up-and-down motion at N.

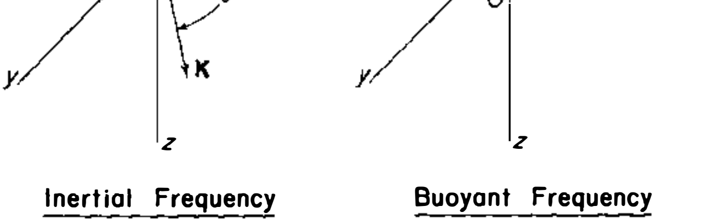


Figure 4 The wavenumber vector $\mathbf{K} = (k,m)$, group velocity $\mathbf{c}_g$, and the hodograph of the particle velocity $\mathbf{u}(t)$ near inertial frequency ($\omega = f+$) and buoyant frequency ($\omega = N-$), respectively. $\mathbf{K}$ is normal to both $\mathbf{c}_g$ and $\mathbf{u}$.

An important property of the dispersion relation (1) is that the frequency ω depends only on the angle to the horizontal θ of the vector wavenumber $\mathbf{K} = (k,m)$ and not on its magnitude.

The group velocity, the velocity with which a packet of these internal waves propagates, is given by

$$(\partial\omega/\partial k,\ \partial\omega/\partial m) = km\omega^{-1}(N^2 - f^2)(k^2 + m^2)^{-2}(m, -k), \tag{2}$$

which is orthogonal to the direction $\mathbf{K}$ of wave-phase propagation (Figure 4), i.e. a packet of waves would appear to slide sideways along the crests! This well-known property, nicely illustrated in the laboratory experiments of Mowbray & Rarity (1967), is indicated by considering the energy flux $\overline{p\mathbf{u}}$ of the wave, where p is the pressure and $\mathbf{u}$ the velocity vector. For an incompressible medium, such as the ocean at internal wave scales, $\nabla \cdot \mathbf{u} = 0$, so that for a wave with vector wavenumber $\mathbf{K}$ and $\mathbf{u} = \mathrm{Re}\,\mathbf{u}_0 \exp(i\mathbf{K}\cdot\mathbf{x})$ we have $\mathbf{K}\cdot\mathbf{u}_0 = 0$. This implies that $\mathbf{u}$ and hence $\overline{p\mathbf{u}}$ is perpendicular to $\mathbf{K}$. [This argument does not apply for all types of wave; $\overline{p\mathbf{u}}$ and the energy times group velocity can differ by a nondivergent vector (Longuet-Higgins 1964).]

MODES In the ocean, N is a function of depth. An internal wave propagating downward with a maintained frequency ω into a region in which the local N is less than ω experiences internal reflection. This, combined with reflection at the sea surface, or at some other level where N falls below ω, means that a discrete set of modes is possible (Figure 5), with the frequency for each given as a function of the horizontal wavenumber and mode number, i.e. the number of zero crossings in the vertical of some wavefunction.

Above the turning point, at which $\omega = N(z)$, the wavefunction is still very sinusoidal in appearance, at least for high modes, and its variation with depth is well described by a WKB-type approximation. This amounts to keeping ω, k fixed as a single wave propagates and changing its vertical wavenumber m to satisfy the local dispersion relation (1), with $N = N(z)$. The amplitude changes so that the energy flux [(amplitude)^2 times the vertical component of the group velocity] is conserved. For frequencies well below N the vertical wavenumber, vertical displacement, and horizontal currents scale accordingly with depth like N, $N^{-1/2}$, $N^{1/2}$ except near the turning point.

Power Spectra

A basic tool in the interpretation of a series of data points is the power spectrum. The data are split up into resolvable Fourier components, and the variance in each band is plotted as a function of frequency or wavenumber.

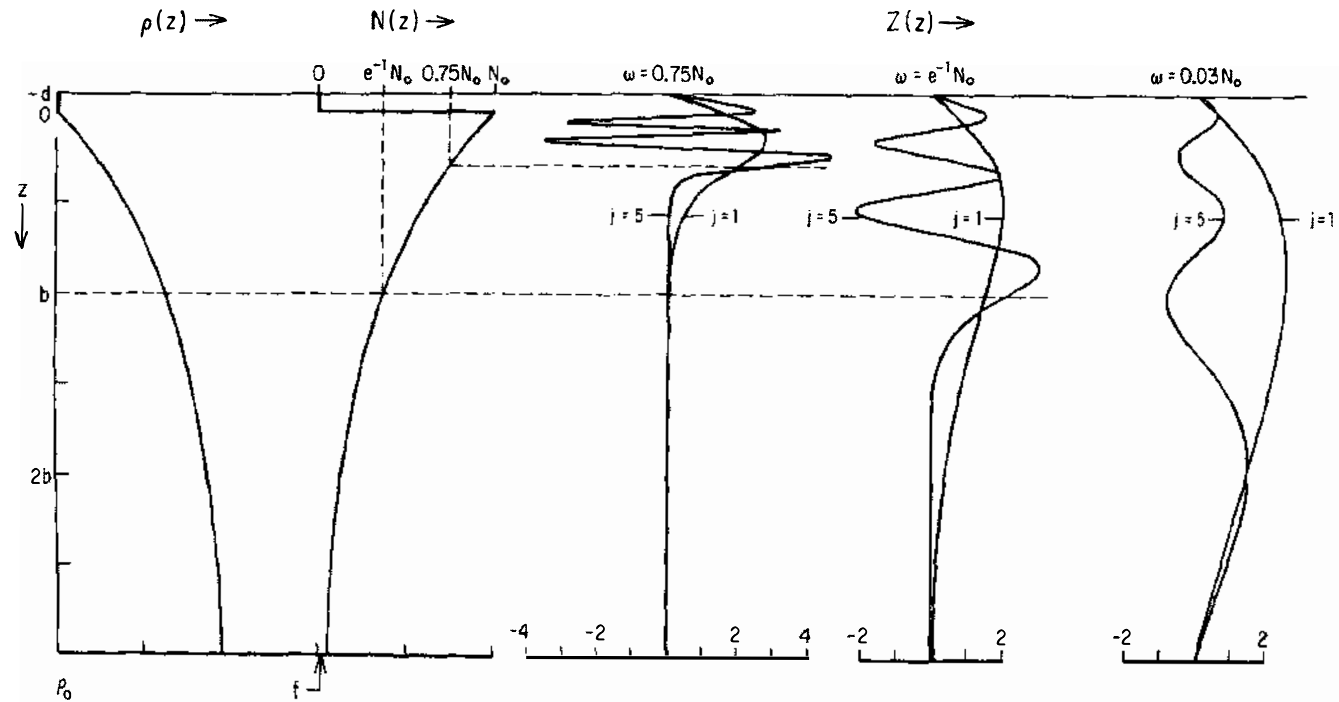


Figure 5 The wavefunctions for vertical displacement at three specified frequencies for modes 1 and 5 in a model ocean with density and Väisälä frequency as shown to the left (Garrett & Munk 1972a).

FREQUENCY SPECTRA A time series of the sort displayed in Figure 2 can be analyzed to give a frequency spectrum. This usually shows significant drops in energy level below f and above N (Figure 6), suggesting that the fluctuations are due to internal waves. The evidence becomes almost overwhelming when combined with the success of WKB scaling for the comparison of spectra at different depths (Fofonoff 1969, Briscoe 1972) and the consistency of the data with certain relations between different variables (Fofonoff 1969, Müller & Siedler 1976). Absolutely conclusive evidence, requiring enough measurements from an array to verify the dispersion relation, is only occasionally and partially available (Pinkel 1975).

WAVENUMBER SPECTRA A series of data points in the horizontal or vertical is less obviously associated with internal waves as there is no bounded range of wavenumbers (equivalent to the f to N frequency band) to which internal waves are confined. An isopycnal might well be wrinkled in the horizontal due to lower-frequency, geostrophically balanced flows, and a wiggly vertical profile of temperature or horizontal current might likewise correspond to some quasisteady layered character of the oceans. The separation of internal waves and other phenomena requires information in the time domain as well as in space.

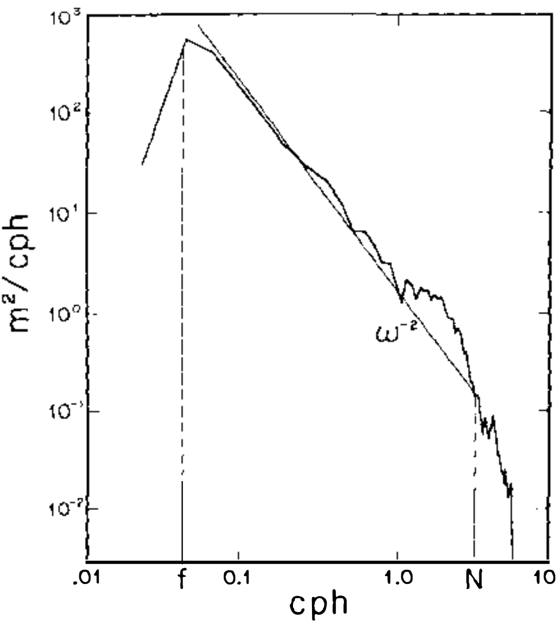


Figure 6 Power spectrum of vertical displacement of an isotherm (Cairns & Williams 1976).

CROSS SPECTRA The simplest array is a pair of sensors separated by a given horizontal or vertical distance. This provides important new information not available from single sensors. If the two sensors are sufficiently close together then the two time series will be nearly alike (except for a possible phase lag); if they are far enough apart, then the two records will bear no resemblance.

A statistical measure of the degree of relatedness is the normalized cross spectrum or coherence (Jenkins & Watts 1968). This is a function of separation as well as frequency. Suppose that within a narrow frequency band the range of vertical wavenumbers is $m \pm \frac{1}{2}\delta m$. Then for two sensors separated vertically by less than $(\delta m)^{-1}$ the normalized correlation (coherence) is high; for a separation exceeding $(\delta m)^{-1}$ the destructive interference between waves with different m leads to a low coherence. In this manner information is obtained concerning the bandwidth of the internal wave spectra. For example, the vertical coherence distance at mid-depth is typically a few hundred meters. The phase difference gives information about the mean direction of travel. Cross spectra may also be computed between two variables at the same location (such as the two horizontal components of current) as well as one variable at one location with the other variable at the other location.

A different but intriguing way of comparing data from different places (Frankignoul 1974) is the time-lagged correlation of the integrated energy within several different frequency bands.

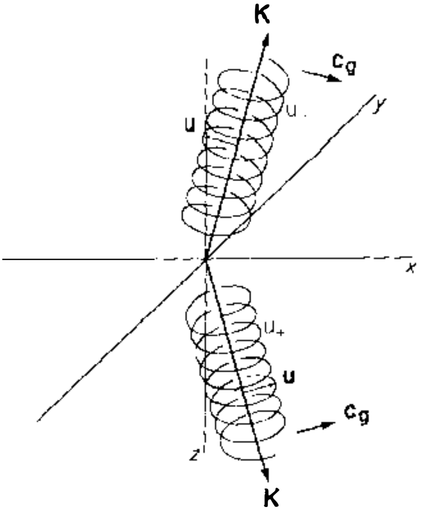


Figure 7 The oceanographer's double helix (Leaman & Sanford 1975). Corresponding to the lefthand diagram in Figure 4, the helices give the hodograph of the instantaneous particle velocity $\mathbf{u}(z)$ normal to $\mathbf{K}$.

THE DOUBLE HELIX A most revealing technique for the analysis of data from Sanford's (1975) electromagnetic velocity profiler has been employed by Leaman & Sanford (1975) and Leaman (1976). Given the eastward current u and northward current v as functions of depth z (with both currents and depth stretched according to WKB relationships), the Fourier amplitudes u_m and v_m define rotary components (Gonella 1972) by

$$u_m + iv_m = u_+ e^{imz} + u_- e^{-imz}, \tag{3}$$

thus representing the vector current by a double helix (Figure 7). Now current vectors associated with inertial waves rotate clockwise with time (in the northern hemisphere) so that the u_- helix, rotating clockwise with increasing depth, corresponds to a wave with upward phase progression and hence a downward component of group velocity. (For higher frequencies a description of the elliptical hodograph requires both clockwise and anticlockwise components even for a wave propagating solely in one vertical direction.)

Wavenumber-Frequency Spectra

MODEL SPECTRA Using data from power spectra and cross spectra from many different sources together with many simplifying assumptions, Garrett & Munk (1972a, 1975) patched together a simple algebraic representation of the distribution of internal wave energy in wavenumber-frequency space (Figure 8). The internal wave energy is assumed to be equally distributed in all horizontal directions, so that only a horizontal wavenumber magnitude, $\kappa = (k^2 + l^2)^{1/2}$, is used. The energy is smeared over all horizontal or vertical wavenumbers rather than confined to the discrete values appropriate for modes. This is convenient, and adequate for most purposes, but there are cases where a modal representation is required. For example, some integral properties of the spectra depend critically on the nonzero vertical wavenumber of the lowest mode. Model spectra with the energy distributed over discrete modes have been proposed by Watson, Siegmann & Jacobson (1977) and Milder (1978).

Let us focus on the presentation in (κ,ω) space. Integrating over κ yields the frequency spectrum MS(ω), which can be related to spectra from a single location (Figure 6). There is a peak at the inertial frequency and an ω^{-2} dependence out to a sharp cutoff at the Väisälä frequency. Information on the bandwidth $\kappa_*(\omega)$ in horizontal wavenumber comes from the moored horizontal coherence (MHC) between horizontally separated sensors. The high wavenumber shape of the spectrum, $\kappa^{-5/2}$ in this model, comes mainly from the wavenumber power spectra (TS and DS) of data from towed and dropped sensors.

From the dispersion relation the horizontal wavenumber bandwidth $\kappa_*(\omega)$ is related to the vertical wavenumber bandwidth, $m_*(\omega)$, in-

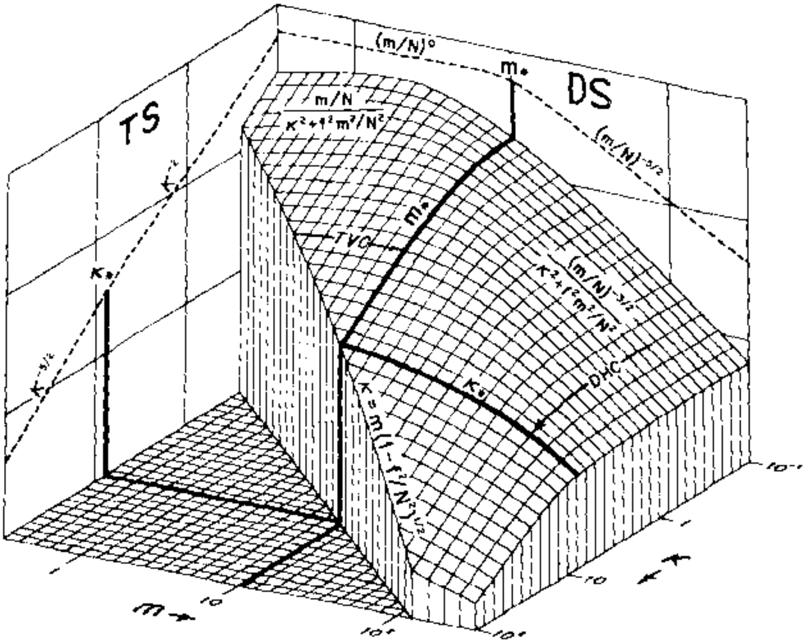
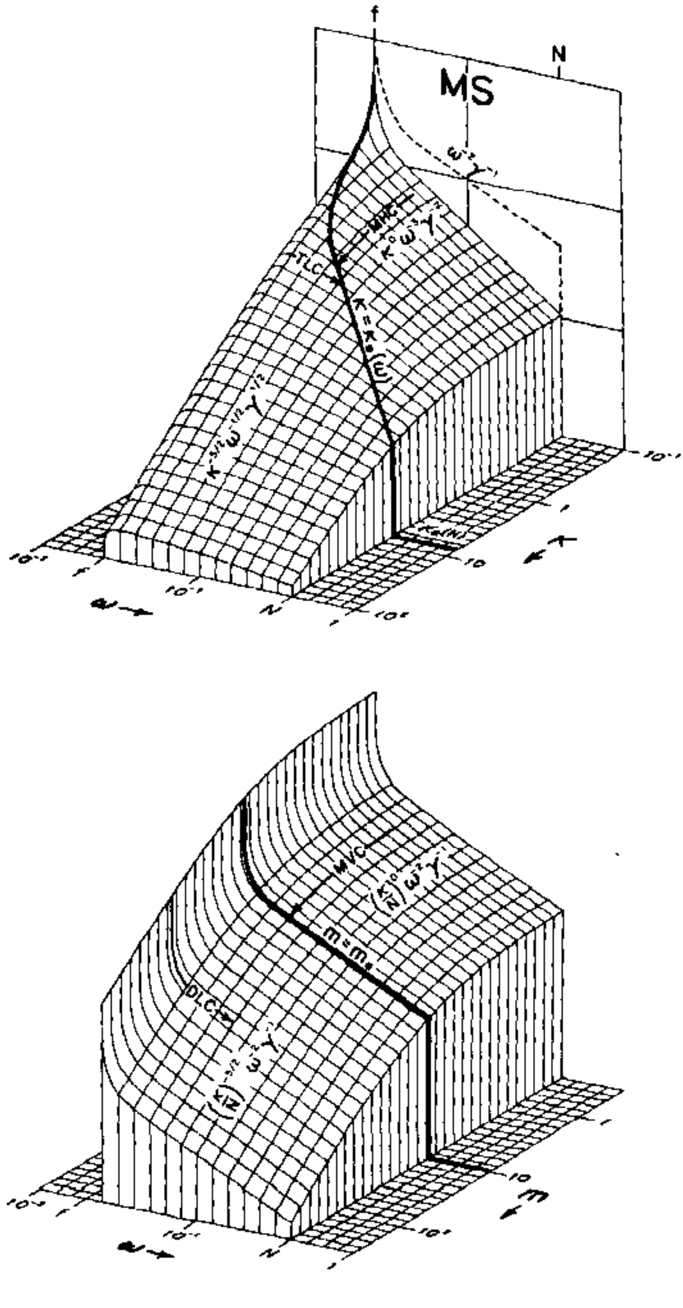


Figure 8 The distribution of energy in wavenumber-frequency space (Garrett & Munk 1975). The three presentations are equivalent, and related by the dispersion relation $\omega = \omega(\kappa,m)$. The two-dimensional spectrum can be integrated to give the one-dimensional moored spectrum MS(ω), towed spectrum TS(k), and dropped spectrum DS(m). Here, $\gamma = (1 - f^2\omega^{-2})^{1/2}$.

dependently derivable from moored vertical coherence (MVC) between vertically separated sensors. More refined MVC measurements led Cairns & Williams (1976) to propose slight modifications to the model, known as GM75, of Garrett & Munk (1975). Other measurements such as the towed vertical coherence (TVC) of Katz (1975), the dropped lagged coherence (DLC) of Hayes (1975), and the dropped horizontal coherence (DHC) of Stegen et al (1975) have provided useful checks on the model spectra.

Models of this kind involve a substantial subjective element in deciding what weight to attach to different measurements and in choosing algebraically convenient forms. Moreover, there was the underlying assumption that experiments at different places and times all described roughly the same universal wave field.

IWEX The most comprehensive measurements of internal waves have been made with the three-dimensional IWEX array. Nine current meters at horizontal and vertical separations of 7 to about 500 m, and 20 tem-


MS
N
f
TLC
MVC
DLC
ω →
k →
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perature sensors at spacings of 2 to 1500 m provided 38 autos 1406 cross spectra. Müller, Olbers & Willebrand (1978) performed an inverse analysis of this information by fitting to it a model somewhat similar to that of Garrett & Munk (1975), allowing for the presence of noise in the current meter data and for the finestructure contamination (see below) of both current and temperature data. At each frequency they fitted to the power and cross spectra a model with an arbitrary energy level, wavenumber bandwidth, low-wavenumber cutoff, slope of the spectrum at high wavenumbers, and a shape parameter. Moreover, unlike the GM models, which assumed horizontal and vertical isotropy, they allowed for the possibility of preferred directions of propagation.

The results were similar in most respects to the model shown in Figure 8. The wavenumber bandwidth was equivalent to about 20 modes being significantly excited at low frequencies, decreasing slightly to ten or so at high frequencies. The high-wavenumber slope (an important parameter in any discussion of shear instability) emerged as 2.4 ± 0.4, with no obvious frequency dependence. The large uncertainty results because the coherence is much more dependent on the energetic waves than on those with high wavenumbers. A slight vertical asymmetry in energy propagation was found at low (inertial and tidal) frequencies, with about 20% more energy propagating down than up. Some horizontal directionality was also found at low frequencies, away from the nearest continental shelf. The energy level corresponded to a greater concentration of energy near the inertial frequency than assumed by Garrett & Munk (1972a, 1975) in their simple $(1-f^2\omega^{-2})^{-1/2}$ representation of an inertial peak.

UPPER OCEAN Another rather complete description of the wavenumber-frequency spectrum has been obtained by Pinkel (1975) in his temperature profiling from FLIP. This work covered the top 440 m, whereas the IWEX measurements were at depths greater than 600 m. At low frequencies Pinkel's results were similar to those obtained at greater depth, but above 2 cph rather surprisingly he found larger vertical coherence, corresponding to the existence of only one or two modes.

Contamination

Quite apart from the obvious problems of instrumental response and noise, a number of oceanic factors can complicate the measurement of internal waves.

FINESTRUCTURE CONTAMINATION If an irregularly stratified medium is advected up and down past a temperature sensor, high-frequency components are produced that are not present in the vertical motion itself.

Phillips (1971), Reid (1971), Garrett & Munk (1971), McKean (1974), and others developed models for this contamination. Theories involve the statistics of the vertical displacement (a random function of time) of the temperature finestructure (a random function of depth). One can account for the existence of temperature fluctuations at frequencies above N and for some loss of vertical coherence. The theory can be applied to current as well as temperature (Briscoe 1977) and was incorporated into the interpretation of IWEX data (Müller, Olbers & Willebrand 1978). Of course, the irregular temperature or current profile advected past a sensor may owe much of its irregularity to the internal waves themselves, so that finestructure contamination can be regarded as a kind of kinematic nonlinearity of the wave field.

DOPPLER EFFECTS In the presence of a mean current past a moored sensor, the frequency of an internal wave is shifted by an amount proportional to the current and the wavenumber. Moreover, the interpretation of a temperature fluctuation at a point in terms of vertical temperature gradient and vertical displacement is invalidated (Ruddick & Joyce 1978). Garrett & Munk (1972a) computed the effect of mean currents of various speeds on the moored spectra, and found a small smearing of variance into the regions below f and above N. However, this model did not contain the more susceptible high wavenumbers. There is probably more to be learned from further consideration of the problem.

OTHER PROCESSES We have already mentioned the problem of contributions from processes other than internal waves to vertical and horizontal profiles. It is particularly important to know whether fluctuations in vertical profiles on scales less than a few meters are typically due to internal wave straining of a smooth profile or due to an inherent, quasi-steady finestructure. Prompted by Lazier's (1973) evidence from repeated profiling in a freshwater lake, work in the ocean has led to general acceptance that much of the high-wavenumber variance in vertical profiles is due to internal waves, at least up to 0.1 cpm. Gregg (1977) found a break in spectral slope from -2 to -3 at 0.1 cpm, which suggests different physical mechanisms at higher wavenumbers. We favor the view that internal waves continue as the dominant process, in general, up to about 1 cpm, but the uncertainty remains and must be resolved if progress is to be made in understanding dissipative processes.

Turning Points

VERTICAL A prominent feature of frequency spectra (e.g. Figure 6) is the increased energy level, near the Väisälä frequency, above that expected from a continuation of the ω^{-2} spectrum at lower frequencies.

The feature occurs at a frequency that varies with depth, and is associated with the local $N(z)$. It is thus a non-WKB effect. It arises from the behavior of internal waves near their turning point, where $\omega = N(z)$ and internal reflection occurs. Locally the wave functions are better approximated by Airy functions than sinusoids (Desaubies 1975), as is apparent from the modal structure shown in Figure 4. The excess spectral level above a smooth background and the increased vertical coherence near N can in fact be used as a check on the wavenumber or modal bandwidth (Desaubies 1975, Cairns 1975).

This interpretation need not always apply (Pinkel, personal communication). In the top few hundred meters of the ocean a bump near N cannot be compared with a smooth spectrum at that frequency for higher N, as no higher N exists. It may just indicate that the low mode, high frequency waves in the upper thermocline (Pinkel 1975) are more energetic than implied by a smooth continuation of the spectrum from lower frequencies.

HORIZONTAL A very similar effect occurs in the north-south direction due to the variation of Coriolis parameter f with latitude. The energy near f represents waves near a latitude of reflection. The wave functions have Airy, rather than sinusoidal, behavior in the north-south direction. Munk & Phillips (1968) showed that the effect of this is to shift the measured peak to a slightly higher frequency than the local f, a well-documented effect. Their deduction that about ten vertical modes are significantly excited is in surprisingly good agreement with later, more direct estimates of the bandwidths. Further work is required to check the extent to which the observed inertial peak is just the turning point effect associated with waves that form part of a smooth ω^{-2} spectrum at lower latitudes.

Variability

A remarkable feature of recent work has been the success of models that originally, for want of any alternative, treated observations at different places and times as samples of some universal spectrum. For example, the frequency spectra of Cairns & Williams (1976) agreed very well with the curve fitted by Garrett & Munk (1972a, 1975) to earlier spectra, and the canonical IWEX spectrum, while less energetic by a factor of four at high frequencies, has much the same energy level as GM75 at low frequencies.

The measured spectra of Cairns & Williams (1976) are consistent with stationarity in time, and those of Zenk & Katz (1975) with stationarity from tow to tow except near the surface. Wunsch (1976) found little variation in the energy of intermediate frequencies at a wide variety of locations in the North Atlantic, apart from an increase of up to an order of magnitude very close to Muir Seamount. Ruddick & Joyce (1978) found a twofold increase in the level of moored spectra of currents associated with an increase in mean current.

Some variability of the wavenumber-frequency spectrum, both in energy level and spectral shape, does exist, but the variability is remarkable for how small it is. Certainly the "internal sea state" is more uniform than the surface sea state.

THEORY

A description of the internal wave field in the ocean is adequate for many applications, but it remains a major oceanographic problem to determine the dominant sources of internal waves, evaluate their interaction with each other and with motions of other scales and unravel the mechanisms and secondary effects of their dissipation. The wide variety of physical processes involved is illustrated in the masterly sketch of Thorpe (1975), reproduced here as Figure 9.

Energetics

Before introducing some of the proposed mechanisms of wave generation, interaction and decay, it seems useful to establish some reference values for the energy flux through the spectrum. The vertically integrated energy is typically 4×10^3 Jm^{-2} (Garrett & Munk 1975, Müller, Olbers & Willebrand 1978). [Recent equatorial measurements by Eriksen (personal communication) suggest that the energy density for $\omega \gg f$ is independent of latitude, so that the total energy scales roughly as f^{-1}.] If the dominant dissipative mechanism is by shear instability, and 25% of the lost energy is left behind as potential energy of the mean stratification (Thorpe 1973) due to the upward mixing of dense water, then the traditional (Munk 1966) vertical eddy diffusivity of 10^{-4} m^{2}s^{-1} requires that energy be lost from the internal wave spectrum at 7×10^{-3} Wm^{-2} (Garrett & Munk 1972b). This would dissipate the internal waves in seven days.

Another reference value for an energy flux is the vertical energy flux that would correspond to downward propagation (as opposed to vertical isotropy) of all the energy in a typically observed internal wave spectrum. For GM75 near the surface this is 0.05 Wm^{-2}, but this is strongly dependent on low wavenumbers.

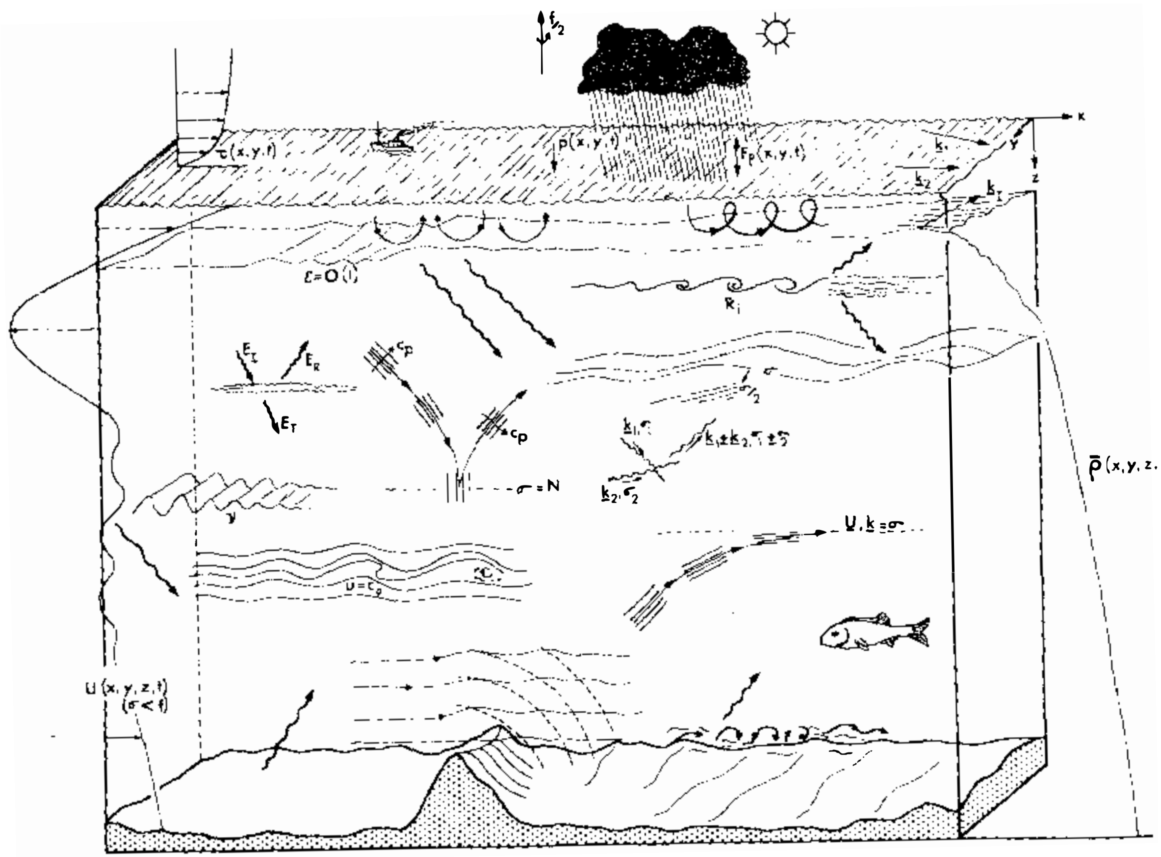

Figure 9 Thorpe's (1975) visualization of processes affecting internal waves. Identification of symbols and processes is left as an exercise for the reader.

Generation Mechanisms

There is no shortage of proposed mechanisms for getting energy into internal waves from the top, bottom (and sides), and interior of the ocean. Many of these were discussed by Thorpe (1975) and are illustrated in Figure 9.

SURFACE GENERATION Internal waves can, in principle, be generated by the atmosphere through traveling pressure fields, variable buoyancy flux, or variable wind stress. The latter is the most plausible (Thorpe 1975), although the calculation of a transfer rate is hampered by the lack of appropriate meteorological information.

Surface waves are another possible source. A pair of surface waves with nearly equal wavenumber and frequency produces a forcing term at the difference (and sum) frequency and wavenumber through non-linearities in the equations of motion and surface boundary conditions. This difference frequency and wavenumber can satisfy the dispersion relation of an internal wave nearly at right angles, causing it to grow resonantly. Watson, West & Cohen (1976) find a possible energy flux of several times 10^{-3} Wm^{-2} into internal waves by this mechanism, although Olbers & Herterich (1978) find very much smaller transfer rates. The reason for this discrepancy is not yet resolved, and in any event, the flux depends critically on the density stratification in the upper 100 m of the ocean.

Bell (1978) recently discussed the role of corrugations at the base of the surface mixed layer. As these are advected by the currents associated with mixed-layer inertial oscillations, they will generate internal waves of high frequency and wavenumber that propagate downward.

Distinguishing between different surface generation mechanisms may be difficult if each is associated with the strength of the wind. In any case, the important thing will be to associate a downward energy flux with intermittent storms. Leaman (1976) estimated a downward energy flux of 2–4 $\times$ 10^{-4} Wm^{-2} on one occasion through use of the rotary decomposition of current profiles discussed earlier, but some uncertainty resulted from lack of resolution of the frequency and hence group velocity.

BOTTOM GENERATION A quasisteady current advecting a stratified ocean over bottom topography generates standing internal waves, known as lee waves. Bell (1975) claimed that these could be particularly important in the bottom kilometer of the ocean.

The possibility of the surface tides as a source of internal wave energy really deserves more detailed discussion (Garrett 1977). The loss of tidal energy from astronomical considerations is about 5×10^{12} W (Lambeck 1977). Much of this loss may occur in shallow seas, but if uniformly distributed over the oceans, it corresponds to a dissipation rate of about 2×10^{-2} Wm^{-2}. Bell (1975) analyzed the generation of internal tides (internal waves of tidal frequency) by barotropic tidal currents over bottom topography and estimated, very roughly, an energy flux of 10^{-3} Wm^{-2}. Olbers (1974) calculated a time of 4–40 days for the transfer of internal tide energy to internal waves of other frequencies. The problem requires further investigation.

It is quite possible that the bottom, with its rough topography and turbulent boundary layer, may act as an energy sink for internal waves of sufficiently low frequency to reach it through the decreasing N, as suggested by Leaman's (1976) observations.

INTERIOR GENERATION Müller (1976) introduced the important idea that internal wave packets in a shear flow in the ocean behave much like molecules in a gas, so that the internal waves extract energy from the mean flow while acting as a large effective vertical viscosity. Interactions between waves of different frequencies and wavenumbers play the role of molecular collisions in smoothing out any spectral anisotropy that results from the effect of the shear flow. Assuming that the spectrum tends to relax to some canonical form, and with an estimate of the relaxation time from the study of nonlinear interactions by Olbers (1976), Müller (1976) predicted the very large eddy viscosity 0.4 m^2s^{-1}, and hence a large energy input into internal waves.

McComas & Bretherton (1977) argue that Müller has greatly overestimated the relaxation time, and hence the eddy viscosity. They also point out that observed internal wave spectra seem to be near a form in which vertical isotropy is preserved even in the presence of shear. However, the strongest evidence against Müller's (1976) theory has come from direct measurements of the Reynolds stress $\overline{uw}$ in the internal wave frequency band by Ruddick & Joyce (1978). According to Müller this should be correlated with the mean shear dU/dz; the constant of proportionality is the eddy viscosity A_v. Ruddick & Joyce (1978) find that A_v cannot be more than 0.02 m^2s^{-1} and estimate an energy input to the internal waves, for a typical value of mean shear, of only about 2×10^{-4} Wm^{-2} or less.

Internal Processes

CRITICAL LAYERS The horizontal phase speed of the waves in the typical internal wave spectrum is about 2 ms^{-1} for the lowest mode, decreasing in inverse proportion to the wavenumber. Hence the waves with a high vertical wavenumber are so slow that, in a typical shear flow, they can be expected to encounter critical levels and dissipate (Booker & Bretherton 1967), giving up their momentum to the mean flow. This would act like a negative eddy viscosity, with values up to $-10^{-2}\ \mathrm{m^2 s^{-1}}$ according to Ruddick & Joyce (1978), although they found no supporting evidence in current meter data.

WAVE-WAVE INTERACTIONS A triad of internal waves with frequencies ω_1, ω_2, ω_3 and wavenumbers k_1, k_2, k_3 can transfer energy (or rather action = energy/frequency) among themselves through the nonlinear terms in the equations of motion, if $\omega_1 \pm \omega_2 \pm \omega_3 = 0$ and $k_1 \pm k_2 \pm k_3 = 0$. Olbers (1976) evaluated this interaction for a typical internal wave spectrum and found an energy transfer to high wavenumbers at a rate $3 \times 10^{-3}\ \mathrm{Wm^{-2}}$. McComas & Bretherton (1977) have repeated the calculations with similar results, but they found that the energy transfer rates are very dependent on the spectral shape, and hence it is not possible to identify a definite, quantifiable cascade of energy to the dissipative high wavenumbers. Basically, while the transfer rate to high wavenumbers would drain the whole spectrum in a time of the order ten days, the time scale for the more energetic parts of the spectrum is several times greater which suggests that the rather uncertain high-wavenumber parts of the spectrum may in fact adjust themselves to reduce the transfer rate to dissipative scales.

Most importantly for future work, McComas & Bretherton (1977) identify the three dominant types of resonant wave-wave interaction that move wave action around in the two-dimensional space defined by vertical wavenumber m and horizontal wavenumber κ (they assume

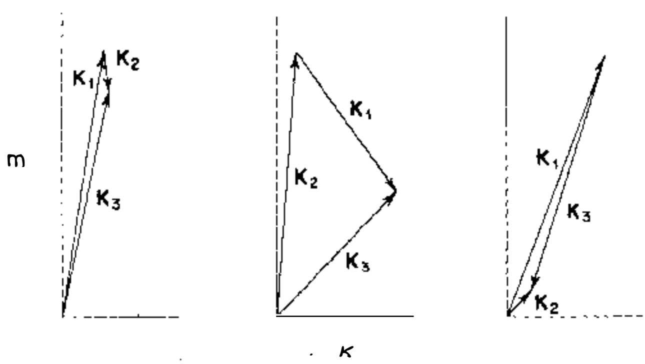

Figure 10 Wavenumber vectors for resonantly interacting triads, corresponding to induced diffusion (*left*), elastic scattering (*center*), and parametric subharmonic instability (*right*) (McComas & Bretherton 1977).

horizontal isotropy). The wavenumber representation of these three classes of triad (which must, of course, satisfy the resonance conditions) is illustrated in Figure 10.

In induced diffusion, two waves of nearly equal wavenumber and frequency interact with a much-larger-scale low-frequency flow in such a way that wave action is diffused in vertical-wavenumber space. McComas & Bretherton (1977) find that models of $E(m,\omega)$, the energy density in vertical-wavenumber and frequency space, proportional to m^{-t} are unchanged by induced diffusion if $t = 2$ or 3. In currently accepted models for $E(m,\omega)$, such as that of Figure 8, t ranges from 2 to 3 for high wavenumbers, so that the effect of induced diffusion is small. However, McComas (1977) found that a small bump superposed on an equilibrium slope diffuses away in a few periods or less, except near the inertial frequency, and of course a -2 or -3 slope cannot continue to infinite wavenumbers without producing infinite shear, so induced diffusion must be important at some high-wavenumber cutoff. The recognition of the importance of induced diffusion adds weight to the interpretation that most of the high-wavenumber wiggles on vertical profiles are due to internal waves.

In elastic scattering two waves of the triad have almost equal and opposite vertical wavenumbers, and almost equal horizontal wavenumbers and frequencies. The third wave is of low frequency and horizontal wavenumber and has a vertical wavenumber nearly twice that of the other two waves. It acts essentially as a layered structure which, by Bragg scattering (Mysak & Howe 1976) tends to equalize the amplitudes of upward and downward propagating waves. McComas (1977) finds that noninertial waves tend to vertical symmetry in a few periods or less, and that inertial waves actually become more asymmetric, though on a very long time scale.

Parametric subharmonic instability describes the excitation of a simple pendulum if its point of support is oscillated vertically with twice the natural frequency of the pendulum. Similarly, as found by McEwan & Robinson (1975) both theoretically and in a laboratory experiment, small scale internal waves may be excited by larger scale waves of twice the frequency. McComas & Bretherton (1977) find this mechanism particularly effective at feeding energy into near-inertial waves with a small vertical scale.

A recognition of these three main classes of triad interactions has considerably clarified what McComas & Bretherton (1977) describe as "an otherwise mystifying computational exercise." For example, a possible reason for the high-wavenumber slope of observed spectra is beginning to emerge, and attention has been focussed on the high-wavenumber inertial waves. Unfortunately an unacceptably large amount of data would be required for direct verification, by bispectral analysis, that wave-wave interactions do occur (McComas 1978).

Calculations to date of nonlinear interactions among oceanic internal waves have assumed horizontal isotropy, mainly to keep the algebra and computation within reasonable bounds. A future study of the time scale and mechanisms for changes in a directional spectrum is particularly important for any consideration of the horizontal spreading of a local patch of high energy.

SOLITONS The calculations of wave-wave interactions discussed above rely on the assumption that nonlinearities are small and can be dealt with by a perturbation procedure. However, strongly nonlinear internal waves of permanent form, "solitons," are known theoretically to be possible for a shallow ocean, or on a thin thermocline in the deep ocean (e.g. Case & Rosenbluth 1977). Their existence is well documented in the laboratory (Davis & Acrivos 1967) and in groups on the continental shelf (Lee & Beardsley 1974). Proni, Ostapoff & Sellers (1978) have detected similar groups in the upper thermocline of the deep ocean by acoustic back-scatter, and Pinkel (1978) has measured the speed of single solitons with a Doppler sonar system. Further theoretical and experimental work on their existence, formation mechanism, and role in the ocean, can be expected.

Dissipation

The most likely way for internal waves to dissipate is by the occasional Kelvin-Helmholtz shear instability due to the random superposition of different internal waves bringing the local Richardson number $N^2/(\partial u/\partial z)^2$ down below the critical value of 0.25. Now if the frequency-wavenumber spectrum $E(m,\omega) \propto m^{-t}$ for large m, and $t \leqq 3$, an upper bound to the vertical wavenumber is required if the Richardson number based on the mean-square shear is to be finite. Garrett & Munk (1975) fixed this cutoff at 2 cpm to give a Richardson number of 0.4 in the main thermocline. The vertical distance over which mixing should occur is related to the high-wavenumber cutoff (Garrett & Munk 1972b) and is then 0.8 m, consistent with the small vertical scale of mixing events found by Gregg, Cox & Hacker (1973) but implying a vertical diffusivity less than 10^{-4} m^2s^{-1} even if breaking occurs very frequently (Garrett 1977).

Eriksen (1978) finds evidence for vertical mixing at times of high shear, and Richardson numbers as low as 0.25 (based on the shear over 7 m), but Hogg, Katz & Sanford (1978) find no correlation between internal wave shear and the intensity of small-scale temperature fluctuations. Both of these studies were near an island, though, and future open ocean investigations of the connection between shear and mixing are desirable. Further theoretical work is also required, aimed in particular at a spectral representation of the loss of energy by shear instability and mixing.

Overall Balances

One would like to have a clearcut picture of a balance between wave generation, wave propagation, redistribution of energy in wavenumber-frequency space by wave-wave interactions, and dissipation. This requires a full solution of the radiation balance equation (Müller & Olbers 1975), which is not yet available but will surely be attempted in the next few years.

UNIVERSALITY The evidence for some degree of uniformity in time and space, or "universality," of the spectral shape and level presents a major puzzle. The most obvious interpretation is in terms of "saturation," whereby a slight increase in energy leads to a sharp increase in dissipation. This would be a plausible consequence of the existence of a critical Richardson number, provided that the internal wave spectrum had a fixed shape and high-wavenumber cutoff. In that case a slight increase in energy level, and hence in mean-square shear, would greatly increase the probability of shear instability and dissipation. However, the same critical shear could equally well be produced by a low energy density with a large high-wavenumber cutoff, as by a high energy density with a smaller cutoff. Unless the cutoff wavenumber is imposed by some independent process, there seems to be no reason for expecting saturation of the energy level.

The results of McComas & Bretherton (1977) make it seem more likely that, if an equilibrium spectral shape exists, the high-wavenumber cutoff adjusts itself so that the dissipation at high wavenumbers balances energy flux through the spectrum from lower wavenumbers, much as for homogeneous turbulence. The dependence of the strength of wave-wave interactions on the energy (Cox & Johnson 1978, submitted for publication) implies a spectral level proportional to $\varepsilon^{1/2}$, for dissipation ε, rather like the $\varepsilon^{2/3}$ for turbulence. This is hardly saturation, and it is difficult to believe that variations in energy input, and hence ε, are small enough for $\varepsilon^{1/2}$ to approach universality.

Another interpretation, essentially as proposed by Cox & Johnson (1978, submitted for publication), is that there exists some equilibrium spectral shape, not yet fully determined, for which the time constant is much longer than the time between generation events. It is supposed that the change in this spectrum during a generation event is partly removed, in a few days or less, by rapid wave-wave interactions and greatly enhanced dissipation, as suggested by the results of McComas (1977), and partly "tops up" the equilibrium spectrum. In order to be compatible with the very low turbulence levels found by Gregg, Cox & Hacker (1973) and Gargett (1976), which were equivalent to a vertical diffusivity of no more than 10^{-6} m^2s^{-1}, the dissipation time for this equilibrium spectrum must be several hundred days, possibly reduced by loss of energy in the benthic boundary layer.

This could account for rather constant energy levels as a function of time, but the problem of spatial uniformity remains. If the equilibrium spectral level is higher in one part of the ocean than another, nonequilibrium conditions quickly develop due to the propagation of the low modes. Cox & Johnson (1978, submitted for publication) argue that the departure from equilibrium is rapidly removed by wave-wave interaction, and that the spread of a patch of high energy is by propagation out to a certain distance and by diffusion beyond it. With an interaction time of three days they estimate this distance to be 70 km and the horizontal energy diffusivity to be of order 10^4 m^2s^{-1}. In this case, even with a dissipation time of several hundred days for the equilibrium spectrum, the diffusion distance is about 1000 km and one might still expect to see major differences in equilibrium spectral level between stormy and calm parts of the ocean.

However, the time scale for the energetic, low-frequency, low modes is more like 100 days, according to the calculations of McComas (1977) (although anisotropy needs to be taken into account), so that energy can spread by propagation out 2000 km and by a diffusivity of about 3×10^5 m^2s^{-1} beyond that. On this basis the oceans would be filled to almost the same equilibrium spectral level everywhere.

FUTURE WORK

Further study of the assumptions and consequences of wave-wave interaction will probably constitute the main theoretical task for the next few years. This will involve considerable numerical work, but hard thinking and careful analysis will also be required. In particular, for high vertical wavenumbers the strength of the interactions invalidates the assumption of weak interaction, and some improvement must be sought. Also, the connections between waves, fine structure, and turbulence need further consideration and spectral models of dissipation are required.

Other theoretical problems connected with generation, interaction with boundaries, and latitudinal turning points also suggest themselves, and we cannot help feeling that some new understanding of internal-wave dynamics in the ocean should be possible from the clues already available.

Experimentally the identification of generation mechanisms and studies of the interrelation of internal waves, shear instability, and turbulence, particularly at times or places of strong generation, require further attention.

While we hope that the later sections of this review will already be out of date by the time of publication, we are sure that the study of internal waves will provide important, challenging, and exciting puzzles for years to come.

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