COASTAL DYNAMICS

Jonathan GULA
gula@univ-brest.fr

- **Outline:**

1. Introduction
2. General equations
3. Waves
   - 3.1.Surface gravity waves
   - 3.2.Inertia-gravity Waves
   - 3.3.Coastal waves
   - 3.4.Internal Waves
4. Tides
5. Coastal circulation and responses to meteorological forcing
6. Bottom and surface boundary layers
7. Frontal dynamics
8. Estuary plumes and regimes

Presentations and material will be available at :

jgula.fr/Coastal/

3. WAVES

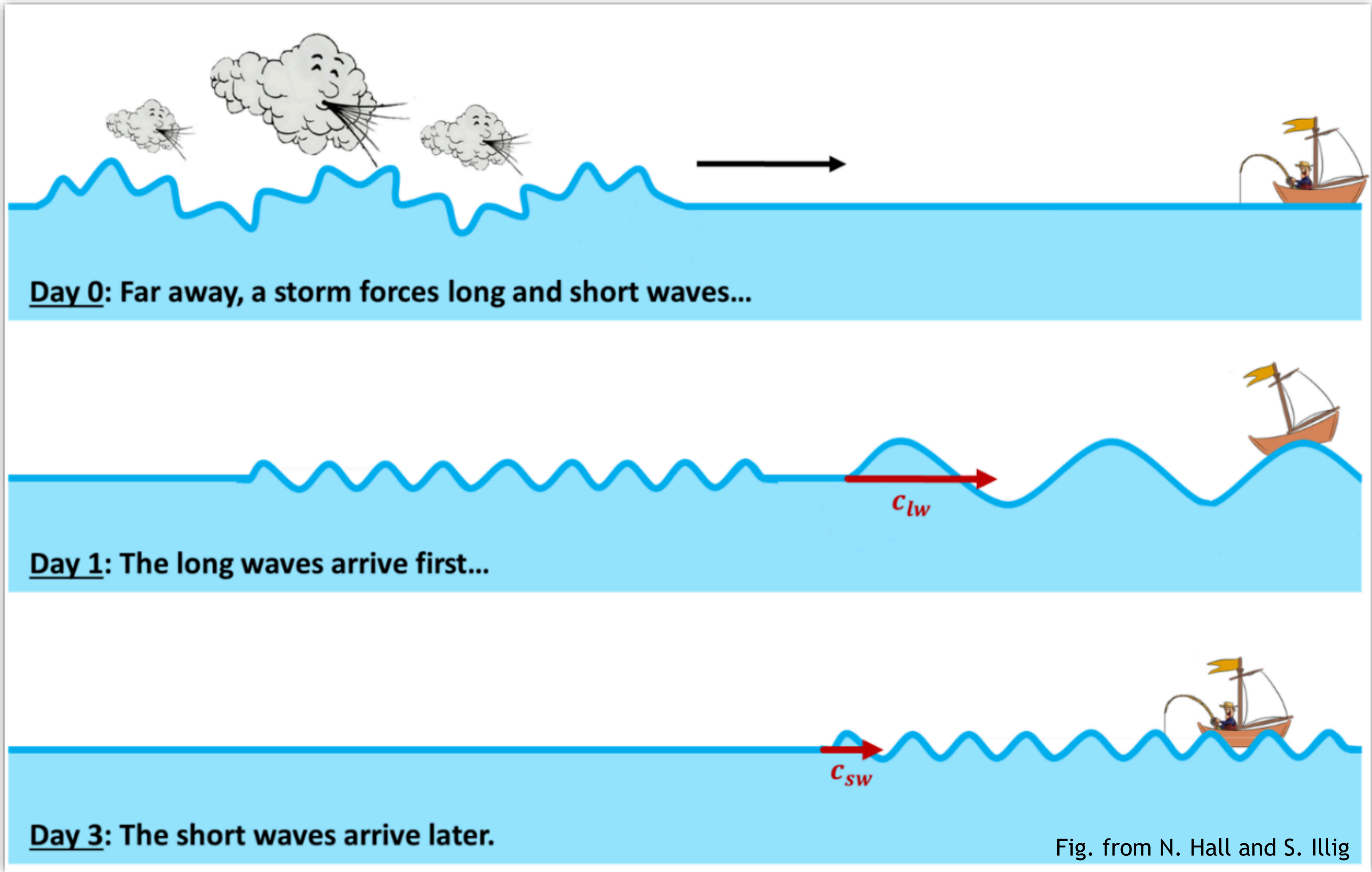
- 3.1. Introduction
  - 3.1.1. General properties of Wave
  - 3.1.2. Different types of ocean waves
- 3.2. Surface Gravity Waves
  - 3.2.1. Long waves
  - **3.2.2. Short waves**
- 3.3. Inertia-gravity Waves
- 3.4. Coastal waves
- 3.5. Internal Waves

3.2 Surface Gravity Waves

Wind waves versus Swells

Feature	Wind Waves	Swells
Origin	Generated by local winds	Formed from decayed wind waves traveling away from storms
Wavelength	Shorter (meters to ~100 m)	Longer (100–500 m)
Period	Short (1–10 s)	Long (10–30 s)
Appearance	Steep, irregular, choppy	Smooth, uniform, regular
Direction	Aligned with local wind	Can travel across ocean basins, independent of local wind
Energy	Confined near wind source	Can transport energy thousands of km
Persistence	Disappear quickly when wind stops	Can last for days to weeks

3.2 Surface Gravity Waves



3.2 Surface Gravity Waves

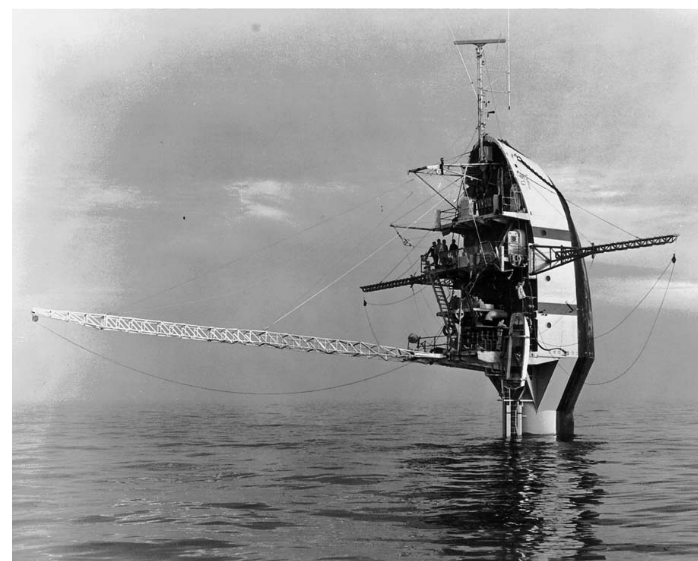
Munk's experiment in 1963: **"Waves Across the Pacific"**

His equations said that the swells hitting beaches in Mexico began some 9,000 miles away — somewhere in the southern reaches of the Indian Ocean, near Antarctica.

"Could it be?" he wrote in an autobiographical sketch. Could a storm halfway across the world produce a patch of moving water that traveled from near the South Pole, up past Australia, then past New Zealand, then across the vast expanse of the Pacific, arriving still intact — at a beach off Mexico?

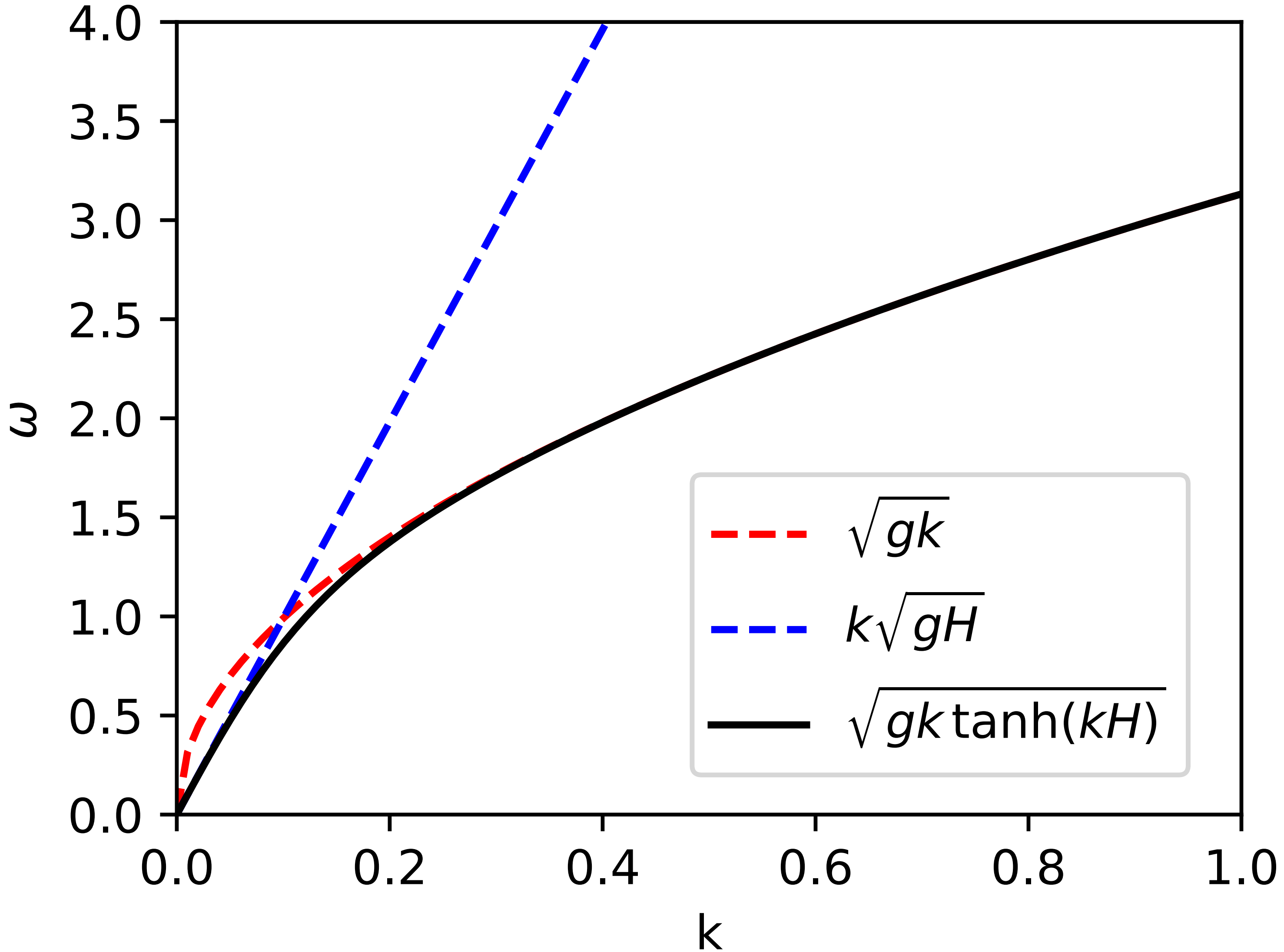
He decided to find out for himself.

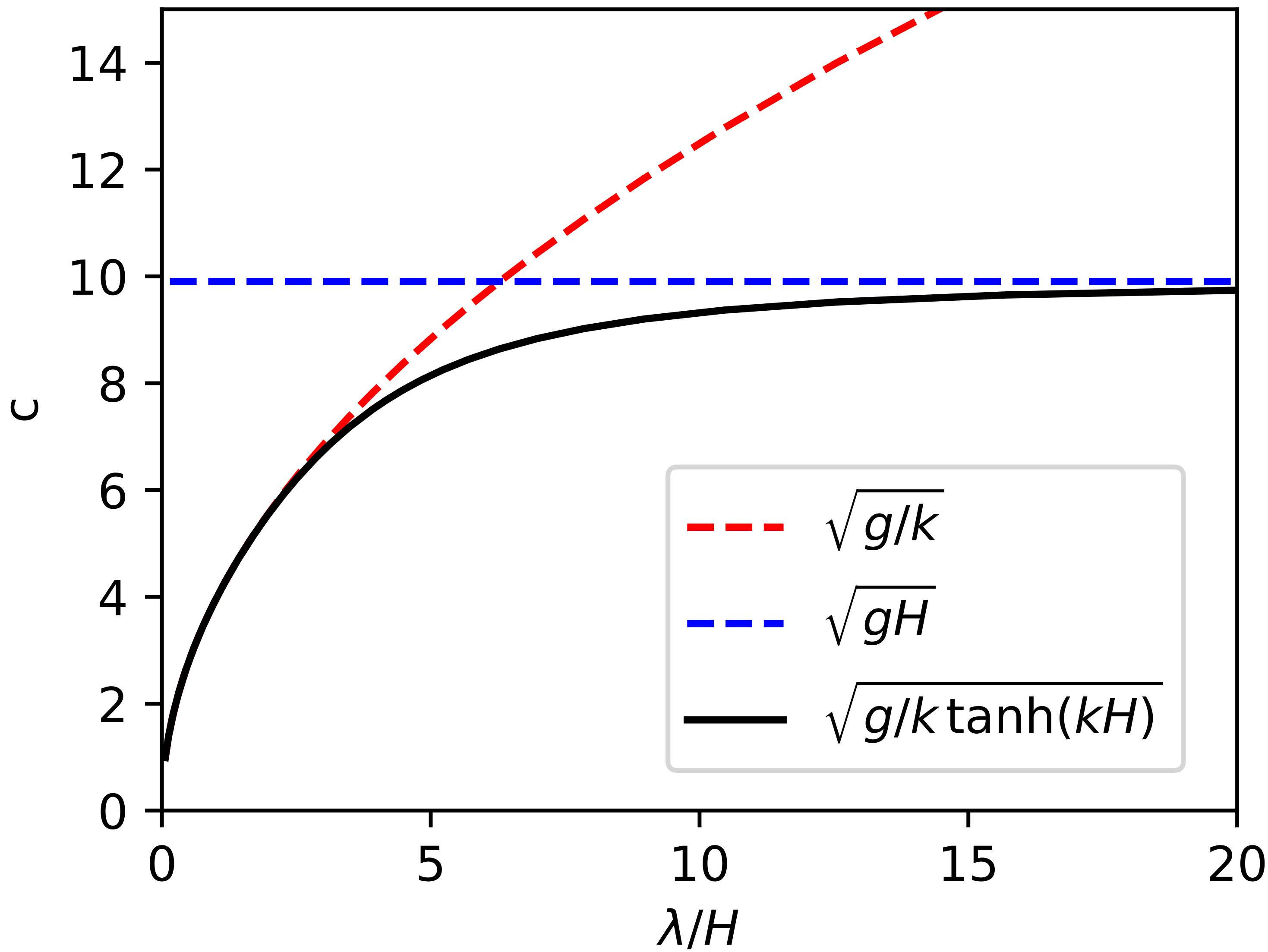
That is why, in 1957, Walter Munk designed a global, real-life wave-watching experiment.



https://www.youtube.com/watch?v=MX5cKoOm6Pk&pbjreload=10

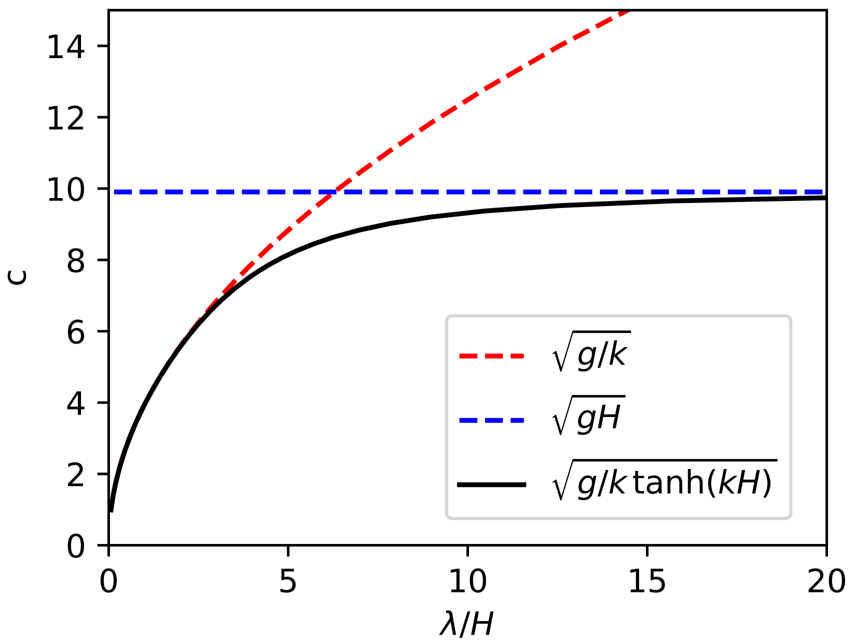
3.2 Surface Gravity Waves

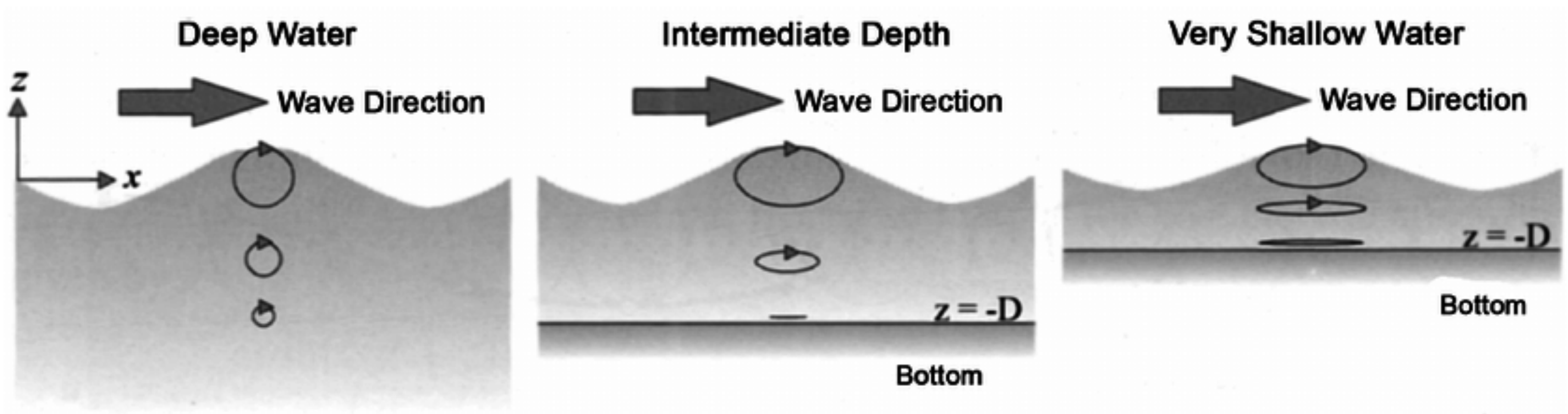




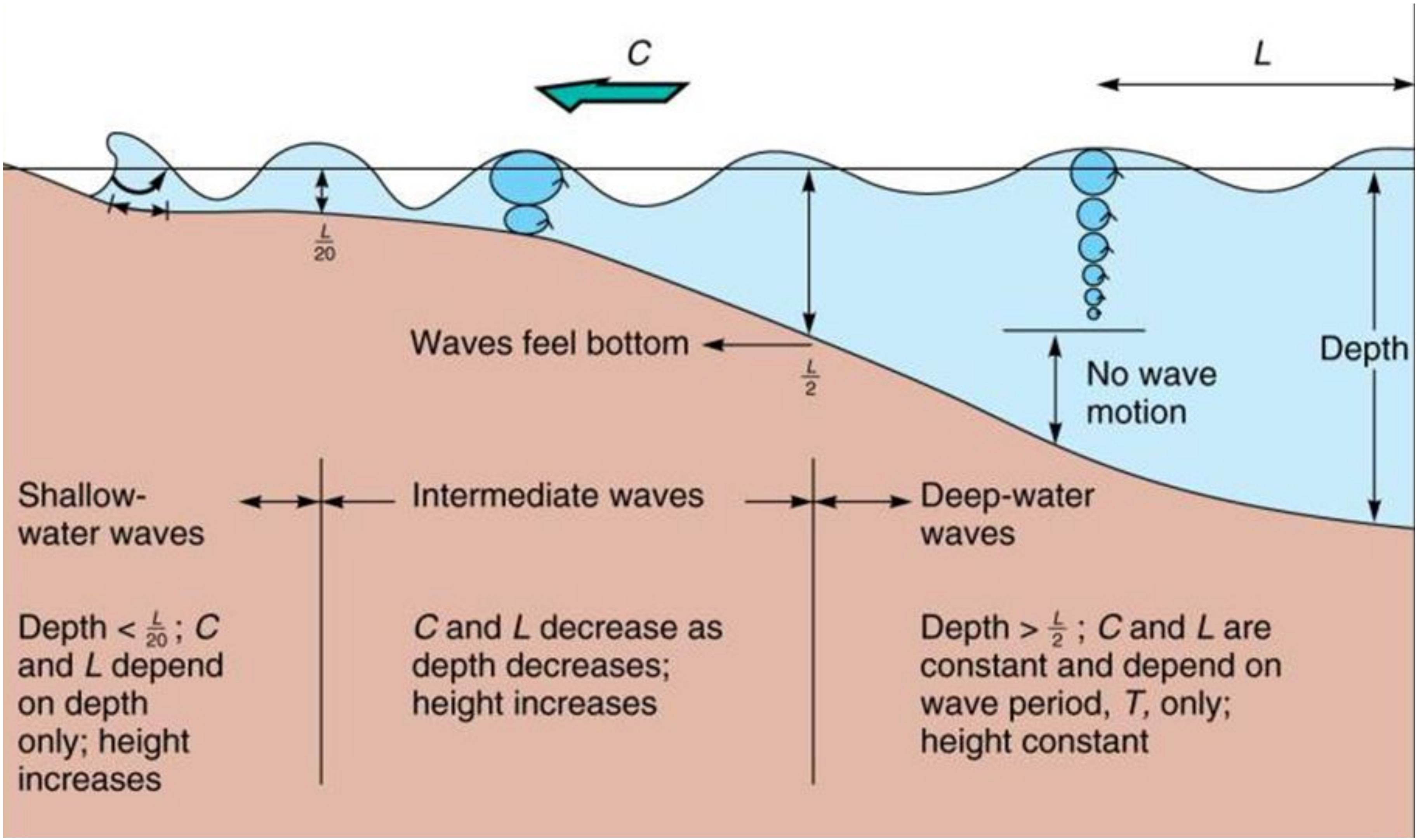
c
14
12
10
8
6
4
2
0
0
5
10
15
20
λ/H
$\sqrt{g/k}$
$\sqrt{gH}$
$\sqrt{g/k\tanh(kH)}$

3.2 Surface Gravity Waves

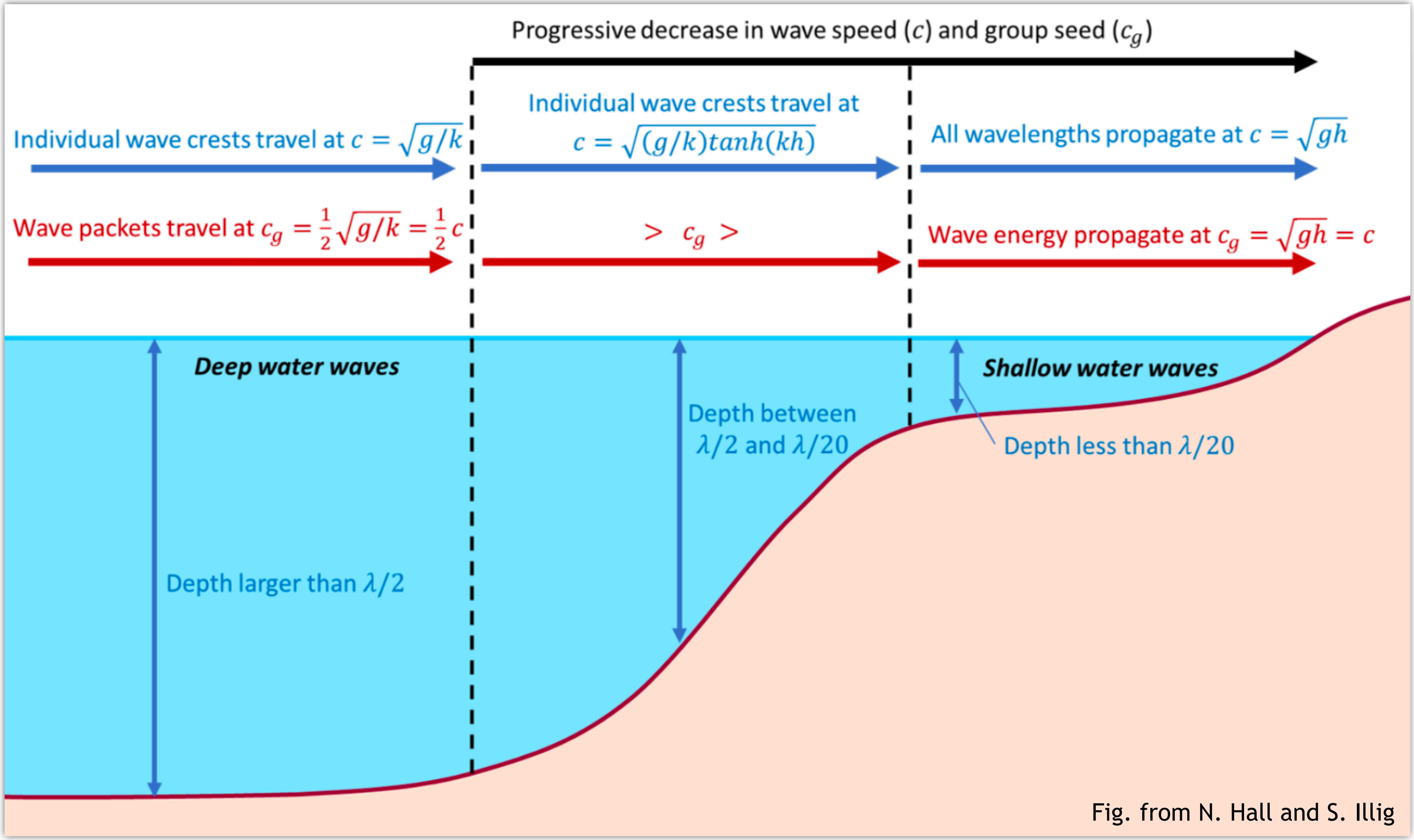




3.2 Surface Gravity Waves



3.2 Surface Gravity Waves



Waves change properties approaching the coast

3.2 Surface Gravity Waves

Water gets shallower so waves must slow down.

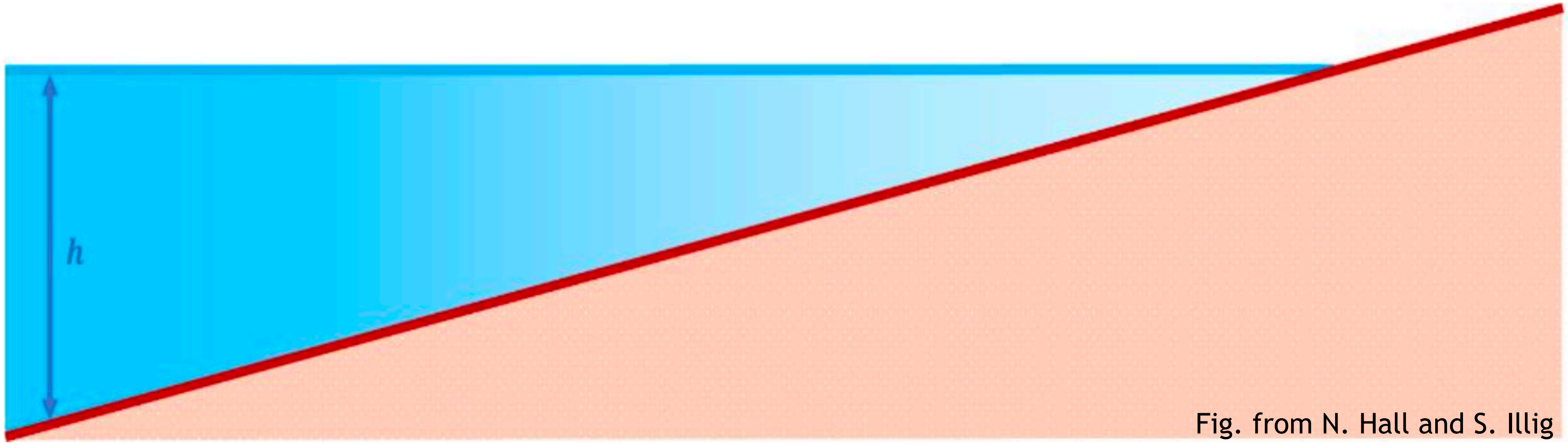


Fig. from N. Hall and S. Illig

So what happens?

3.2 Surface Gravity Waves

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{h_1}{h_2}}$$

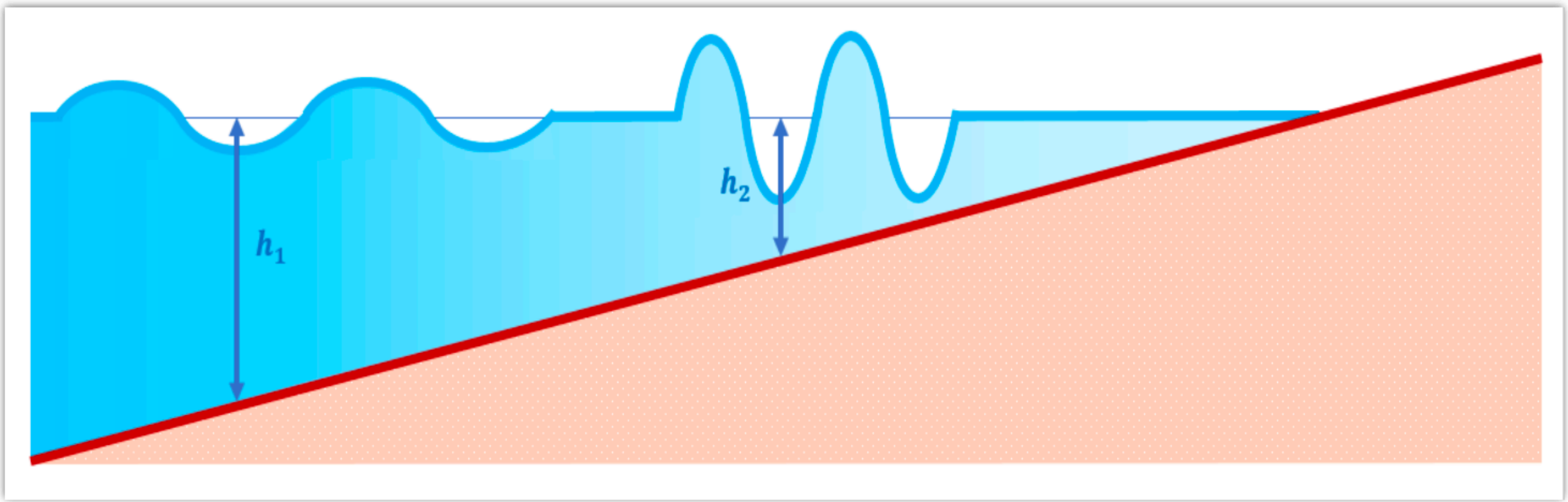
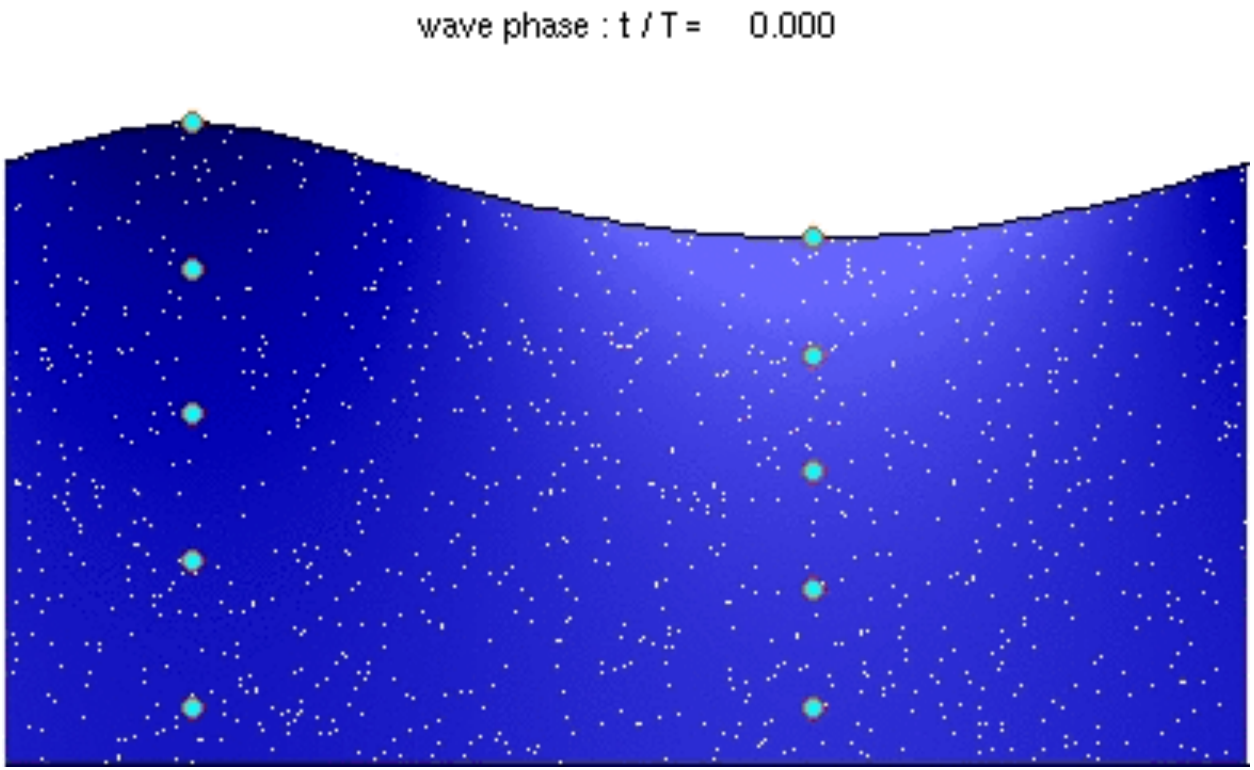


Fig. from N. Hall and S. Illig

3.2 Surface Gravity Waves



A **nonlinear wave** also carries energy and can lead to a net transport of material (for example, Stokes drift at the ocean surface)

Animation from https://en.wikipedia.org/wiki/Wind_wave

Stokes drift

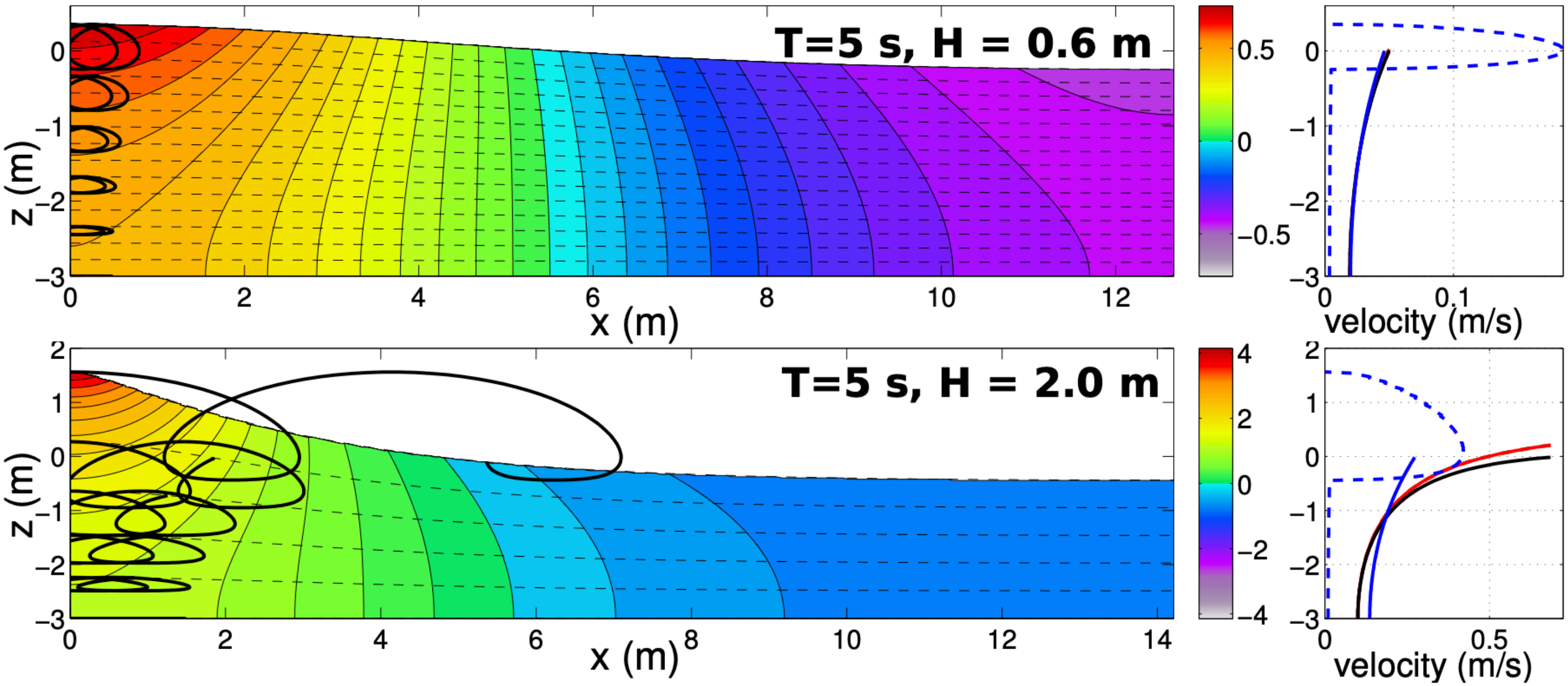
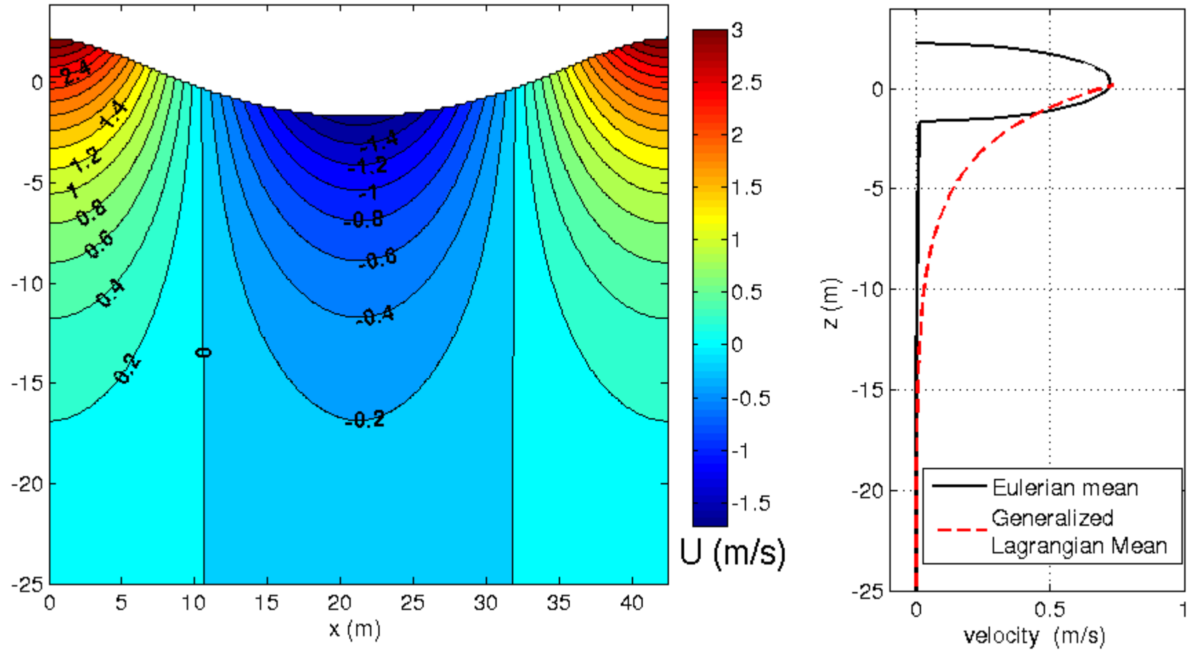


Figure 9.1: Left: Horizontal velocity field at $t = 0$ and particle trajectories integrated over 2 Eulerian periods. Solid lines are isotachs and dashed lines are streamlines in the frame of reference moving with the wave phase speed. Right: vertical profile of Eulerian mean water velocity (dashed, velocity is set to zero in the air for computing the average), Generalized Lagrangian mean (red), Lagrangian mean (black) and Lagrangian mean from the linear approximation (blue). Both top and bottom panels are computed with streamfunction theory to 80th order (Dalrymple, 1974). Non-linear terms are only significant in the bottom case.

Stokes drift

For a long wave, with surface elevation: $\eta = a\cos(\mathrm{kx} - \omega \mathrm{t})$

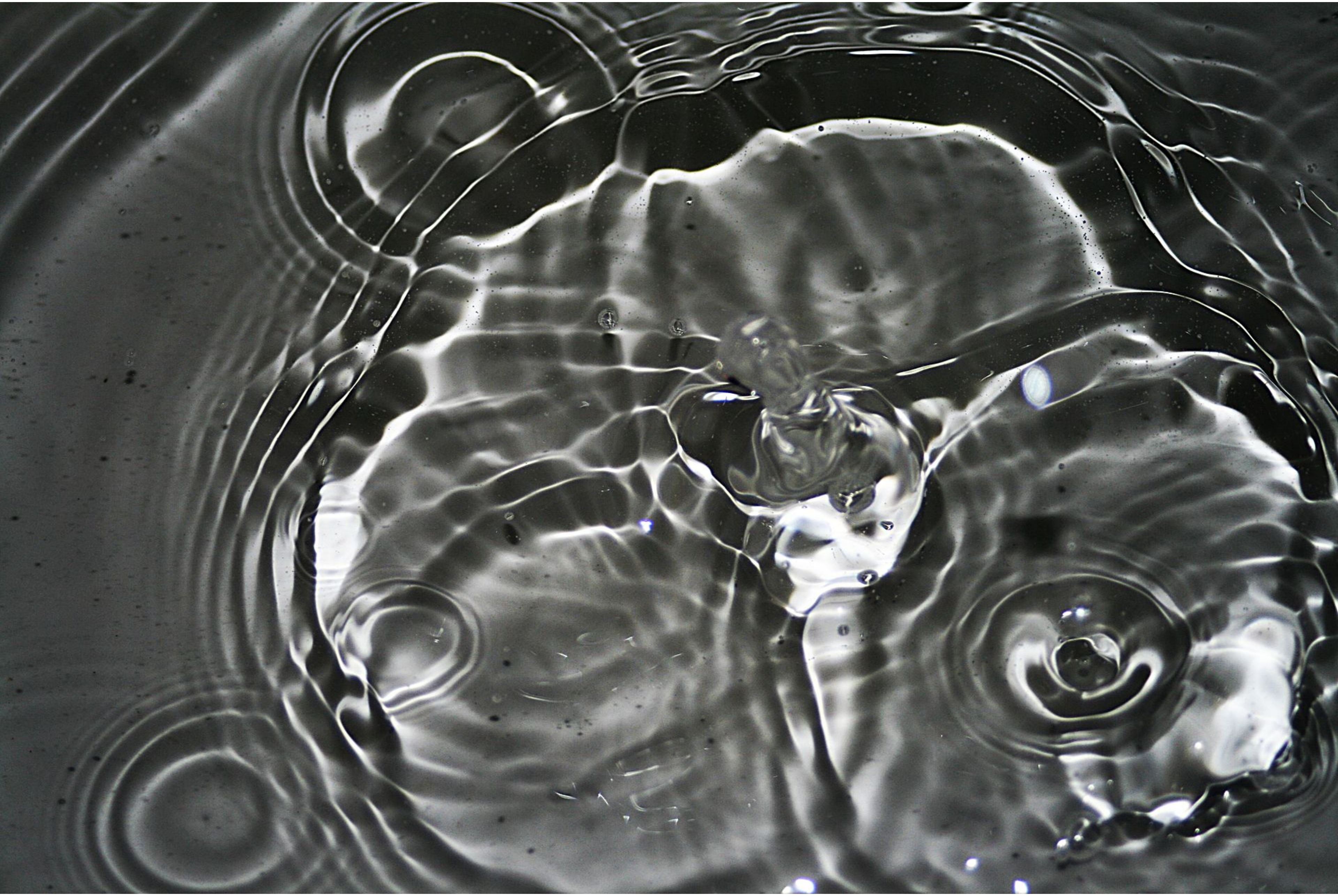
The velocity potential is $\phi = \dfrac{\omega}{k} a e^{kz} \sin(\mathrm{kx} - \omega \mathrm{t})$



The Stokes drift velocity (Lagrangian minus Eulerian mean) is: $\overline{u_s} = \omega k a^2 e^{2kz}$

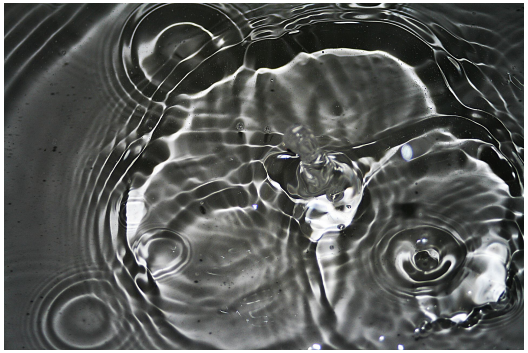
see https://en.wikipedia.org/wiki/Stokes_drift

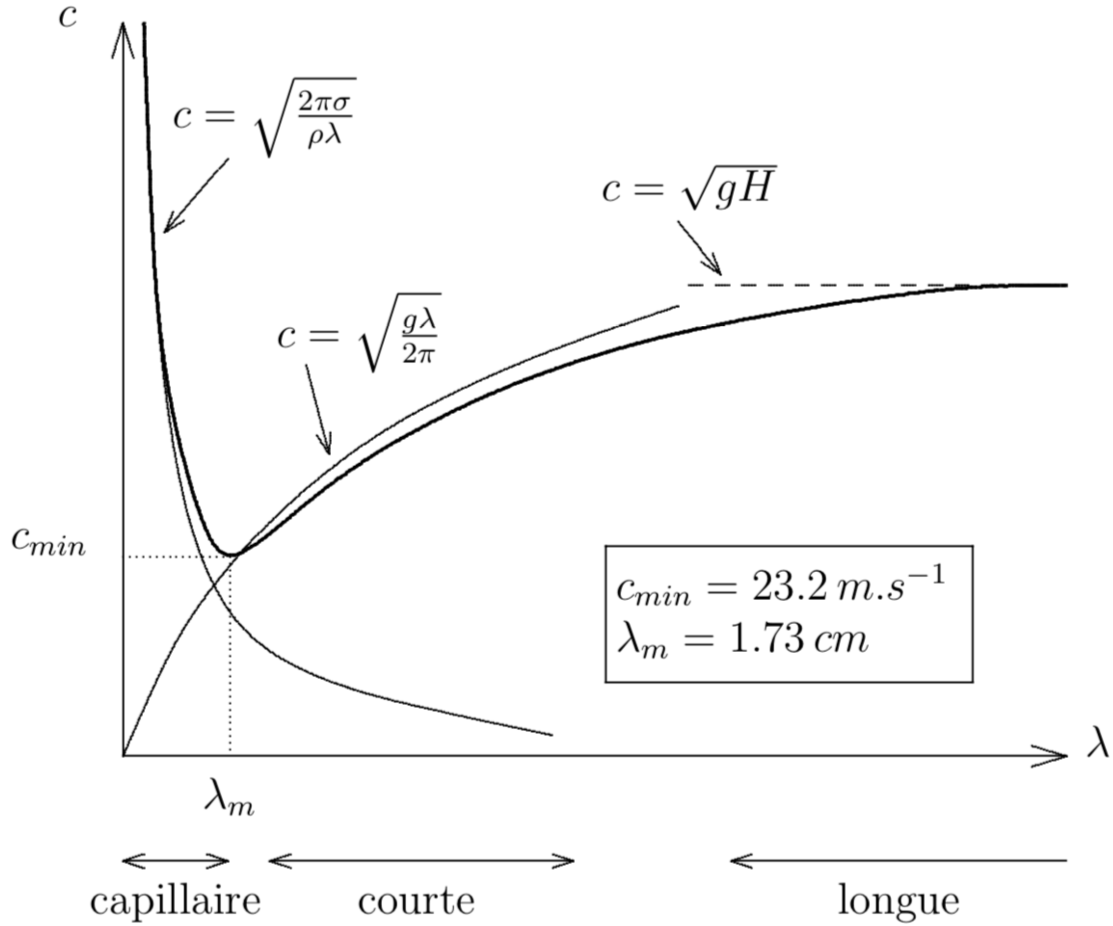
(bonus) Capillary waves



(bonus) Capillary waves

With surface tension effects:





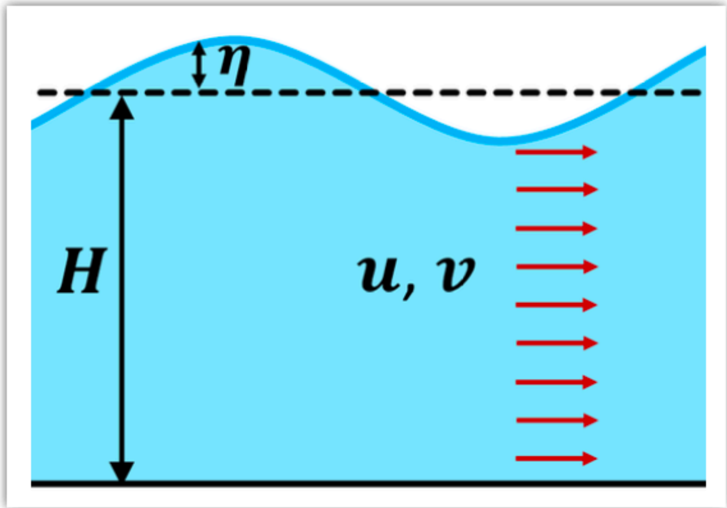
3. WAVES

- 3.1.Introduction
  - 3.1.1.General properties of Wave
  - 3.1.2. Different types of ocean waves
- 3.2.Surface Gravity Waves
  - 3.2.1.Long waves
  - 3.2.2.Short waves
- **3.3.Inertia-gravity Waves**
- 3.4.Coastal waves
- 3.5.Internal Waves

3.3 Inertia-gravity Waves

Let's go back to the linear shallow water equations, but keep the rotation:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$



What is the dispersion relation now?

3.3 Inertia-gravity Waves

Linear shallow water equations on an f-plane

$$u_t - fv + g\eta_x = 0 \qquad (1)$$

$$v_t + fu + g\eta_y = 0 \qquad (2)$$

$$\eta_t + H(u_x + v_y) = 0 \qquad (3)$$

vorticity equation: $\frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(1) \rightarrow \quad \frac{\partial}{\partial t}(v_x - u_y) + f(u_x + v_y) = 0 \qquad (V)$

divergence equation: $\frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(2) \rightarrow \quad \frac{\partial}{\partial t}(u_x + v_y) - f(v_x - u_y) + g\nabla^2\eta = 0 \qquad (D)$

substitute (V) into (3) $\quad \eta_t - \frac{H}{f}\frac{\partial}{\partial t}(v_x - u_y) = 0$

3.3 Inertia-gravity Waves

subsitute (D) into $\dfrac{\partial}{\partial t}$ (3)

$$\eta_{tt} + fH(v_x - u_y) - gH\nabla^2\eta = 0 \qquad \frac{\partial}{\partial t} \rightarrow$$

$$\eta_{ttt} + fH\frac{\partial}{\partial t}(v_x - u_y) - gH\nabla^2\eta_t = 0$$

substitute from $\quad \eta_t - \dfrac{H}{f}\dfrac{\partial}{\partial t}(v_x - u_y) = 0$ gives $\quad \eta_{ttt} + f^2\eta_t - gH\nabla^2\eta_t = 0$

With appropriate initial condition at *t = 0*, the departure from geostrophic disequilibrium follows:

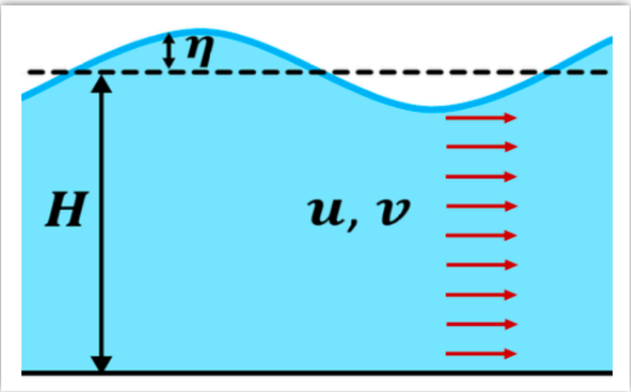
$\eta_{tt} - gH\nabla^2\eta + f^2\eta = 0$ substitute solution $\eta = \tilde{\eta}e^{i(\mathbf{k}.\mathbf{x} - \omega t)}$

leads to dispersion relation

$$\boxed{\omega = \pm\sqrt{f^2 + gHk^2}}$$

3.3 Inertia-gravity Waves

⇨ *flat bottom, f-plane, linear perturbations u, v, η*



$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Wave solution: $(u, v, \eta) = (\tilde{u}, \tilde{v}, \tilde{\eta})e^{i(lx+my-\omega t)}$

With: $\frac{\partial}{\partial x} \rightarrow il\times \quad \frac{\partial}{\partial y} \rightarrow im\times \quad \frac{\partial}{\partial t} \rightarrow -i\omega\times$

⇨ *substitute wave solution: differential equations become linear algebraic equations*

$$-i\omega\tilde{u} - f\tilde{v} = -igl\tilde{\eta}$$
$$-i\omega\tilde{v} + f\tilde{u} = -igm\tilde{\eta}$$
$$-i\omega\tilde{\eta} + H(il\tilde{u} + im\tilde{v}) = 0$$

The unknowns are the wave amplitudes $\tilde{u}, \tilde{v}, \tilde{\eta}$

The parameters are the wave properties l, m, ω
and the geophysical constants f, g, H

3.3 Inertia-gravity Waves

Alternative derivation:

We need to solve algebraic system

$$\begin{pmatrix} -i\omega & -f & igl \\ f & -i\omega & igm \\ ilH & imH & -i\omega \end{pmatrix} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\eta} \end{pmatrix} = 0$$

⇨ *trivial solution* $\tilde{u} = \tilde{v} = \tilde{\eta} = 0$ *(no flow)*

⇨ *The condition for havina non-trivial solutions is hat the determinant of the matrix is zero.*

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

This leads to $\omega\left[\omega^2 - f^2 - gH(l^2 + m^2)\right] = 0$

this is a more complicated dispersion relation !

three solutions: $\omega = 0$ *steady geostrophic flow*

$$\omega = \pm\sqrt{f^2 + gHk^2}$$ *inertia-gravity waves*

3.3 Inertia-gravity Waves

short wave limit: $k^2 \gg f^2/gH$

i.e. the wavelength $\lambda^2 \ll 2\pi L_d^2$
(L_d is the radius of deformation)

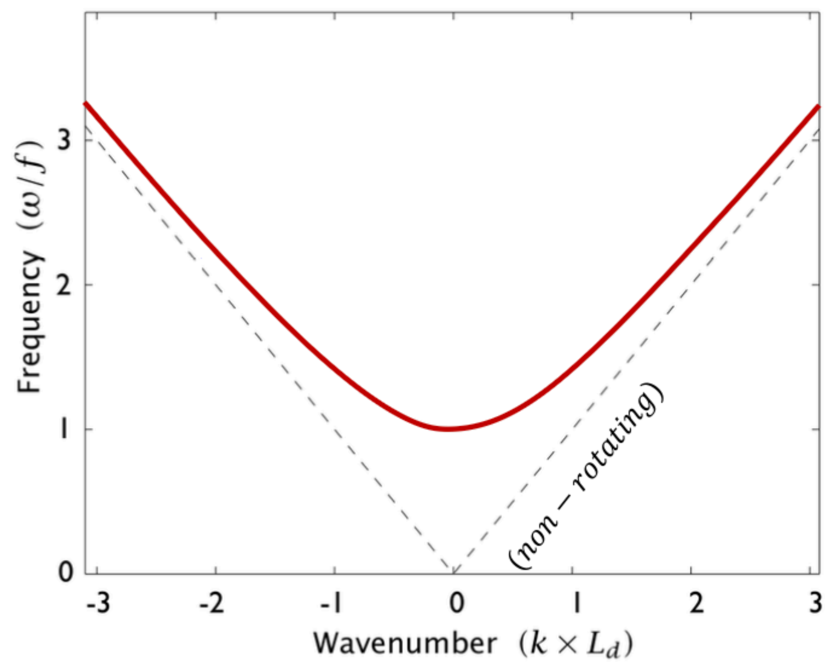
waves behave as in non-rotating case (provided shallow water condition is not violated)

long wave limit: $k^2 \ll f^2/gH$

dispersion relation reduced to

$$\omega = f$$

free inertial oscillations



3. WAVES

- 3.1. Introduction
  - 3.1.1. General properties of Wave
  - 3.1.2. Different types of ocean waves
- 3.2. Surface Gravity Waves
  - 3.2.1. Long waves
  - 3.2.2. Short waves
- 3.3. Inertia-gravity Waves
- **3.4. Coastal waves**
- 3.5. Internal Waves

Kelvin waves

What happens if you have a gravity wave along a solid boundary?

William Thompson (Lord Kelvin) was the first to wonder what happens if one imposes $u = 0$ (normal to the boundary) everywhere.

Starting from the linear SW equations, check what happens if $u = 0$:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Kelvin waves

$$\cancel{\frac{\partial u}{\partial t}} \boxed{- fv = -g\frac{\partial \eta}{\partial x}}$$

Geostrophic balance

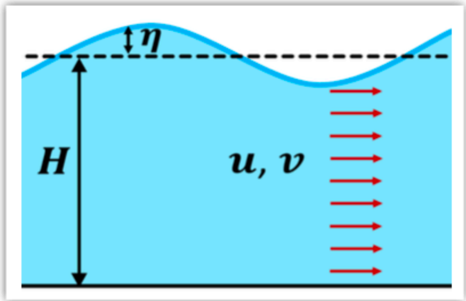
$$\boxed{\begin{aligned} \frac{\partial v}{\partial t} + \cancel{fu} &= -g\frac{\partial \eta}{\partial y} \\ \frac{\partial \eta}{\partial t} + H\left(\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y}\right) &= 0 \end{aligned}}$$

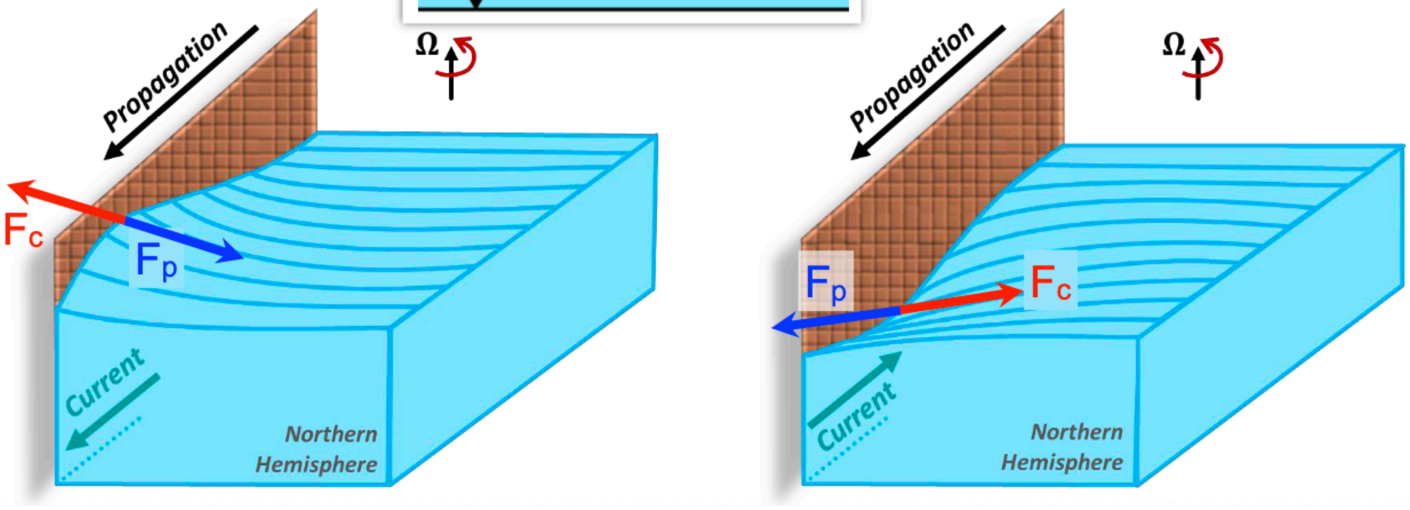
non-dispersive waves

In the x direction we have geostrophic balance, with pressure and Coriolis forces alternating in direction as crests and troughs propagate meridionally.

In the y direction we have non-dispersive gravity waves with a fixed phase speed independent of horizontal scale

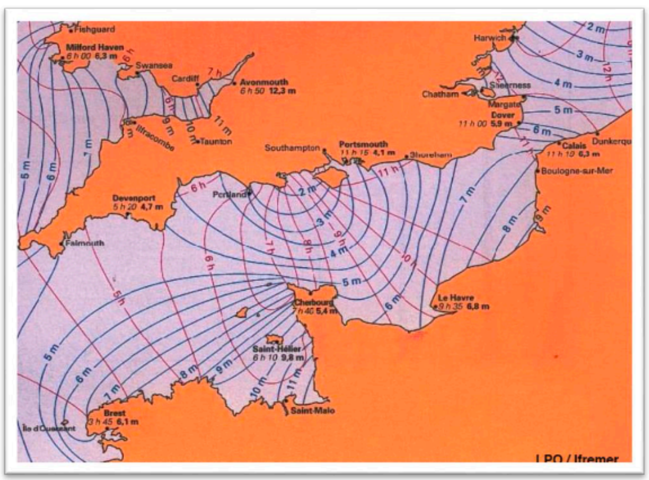
$$|c| = \sqrt{gH}$$





Kelvin waves

Since the only admissible solution is V_1, we conclude that for a system bounded on the west (x positive) the wave propagates in the negative y direction, i.e. to the south (in the northern hemisphere). If x is negative this reverses. Kelvin waves thus propagate south on the western boundary and north on the eastern boundary. They circuit ocean basins in an anticlockwise (cyclonic) direction. In the southern hemisphere the direction is clockwise (but still cyclonic).



Tides are higher on the French side because the signal propagates in from the west