COASTAL DYNAMICS

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- **Outline:**
  1. Introduction
  2. General equations
  3. Surface Waves
     - 3.1. Surface gravity waves
     - 3.2. Inertia-gravity Waves
     - 3.3. Coastal waves
  4. Tides
  5. **Internal Waves**
  6. Geostrophy and thermal wind
  7. Bottom and surface boundary layers
  8. Coastal circulation and responses to meteorological forcing
  9. Frontal dynamics
  10. Estuary plumes and regimes

Presentations and material will be available at :

jgula.fr/Coastal/

4. Internal waves

- Generalities about internal waves
- Internal waves in the two-layer model
- Internal waves with continuous stratification

Bibliography

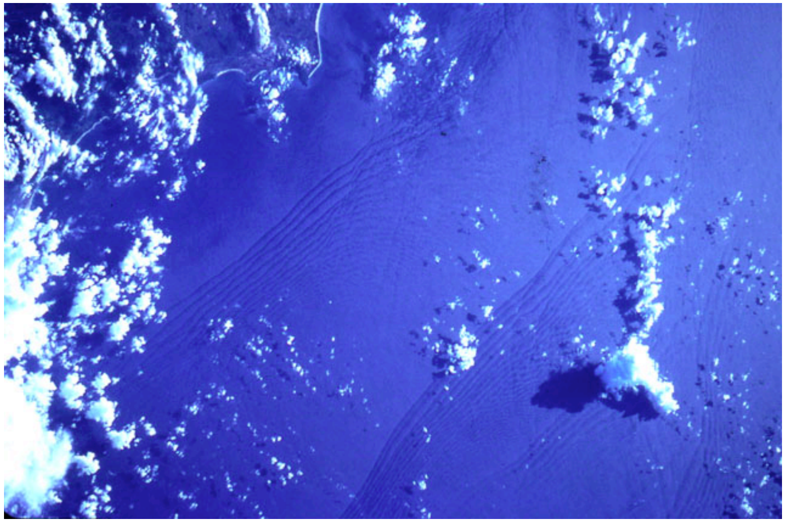
- Leblond-Mysak (1977) *: Waves in the ocean*
- Whitham (1974) *: Linear and nonlinear waves*
- Gill (1982) *: Atmosphère-Ocean Dynamics*
- Kundu-Cohen (1987). *Fluid Mechanics. Third edition*
- Cushman-Roisin*. Introduction to geophysical fluid Dynamics*
- Ardhuin (2024) *Waves in Geosciences*
- Gerkema- Zimmerman (2008). *An introduction to internal waves*
  - *http://stockage.univ-brest.fr/~gula/Ondes/gerkema.pdf*

Ocean waves

- **External waves** = surface gravity waves

- **Internal waves** = gravity waves that oscillate **in the interior of** a fluid
  - In a 2-layer stratification (density changes over a small vertical distance), they propagate horizontally
  - If the fluid is continuously stratified, they can also propagate vertically

Internal Waves in Rosario Strait North Puget Sound Washington



South China Sea Internal Waves as seen by NASA's Shuttle- June 1983

Internal waves

- Internal waves were discovered more than a century ago.
- One of the first observation by Helland-Hansen & Nansen:

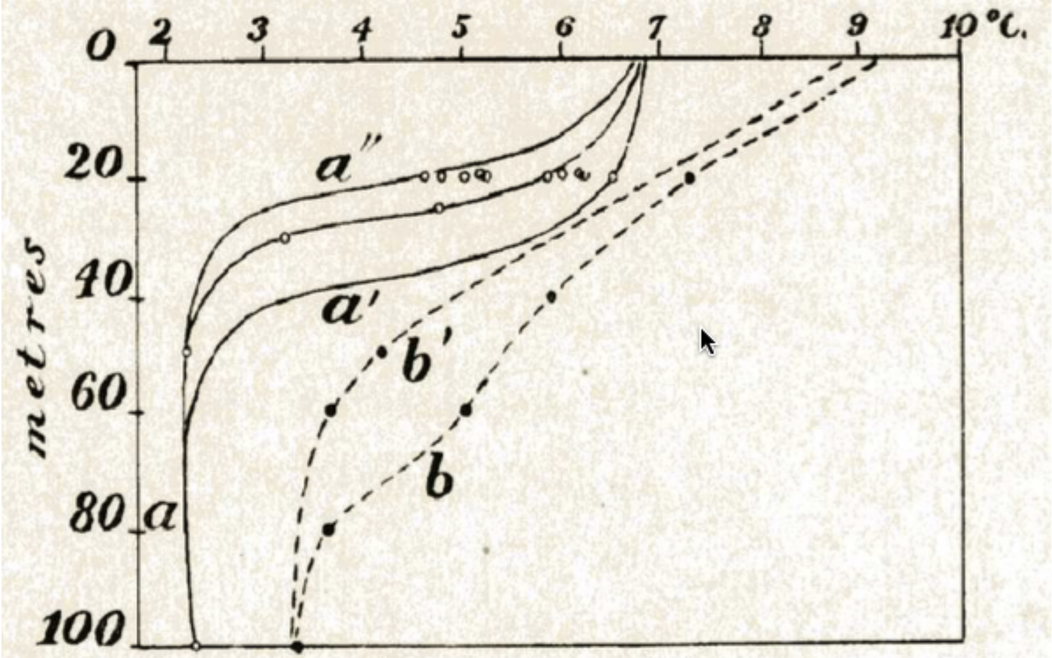


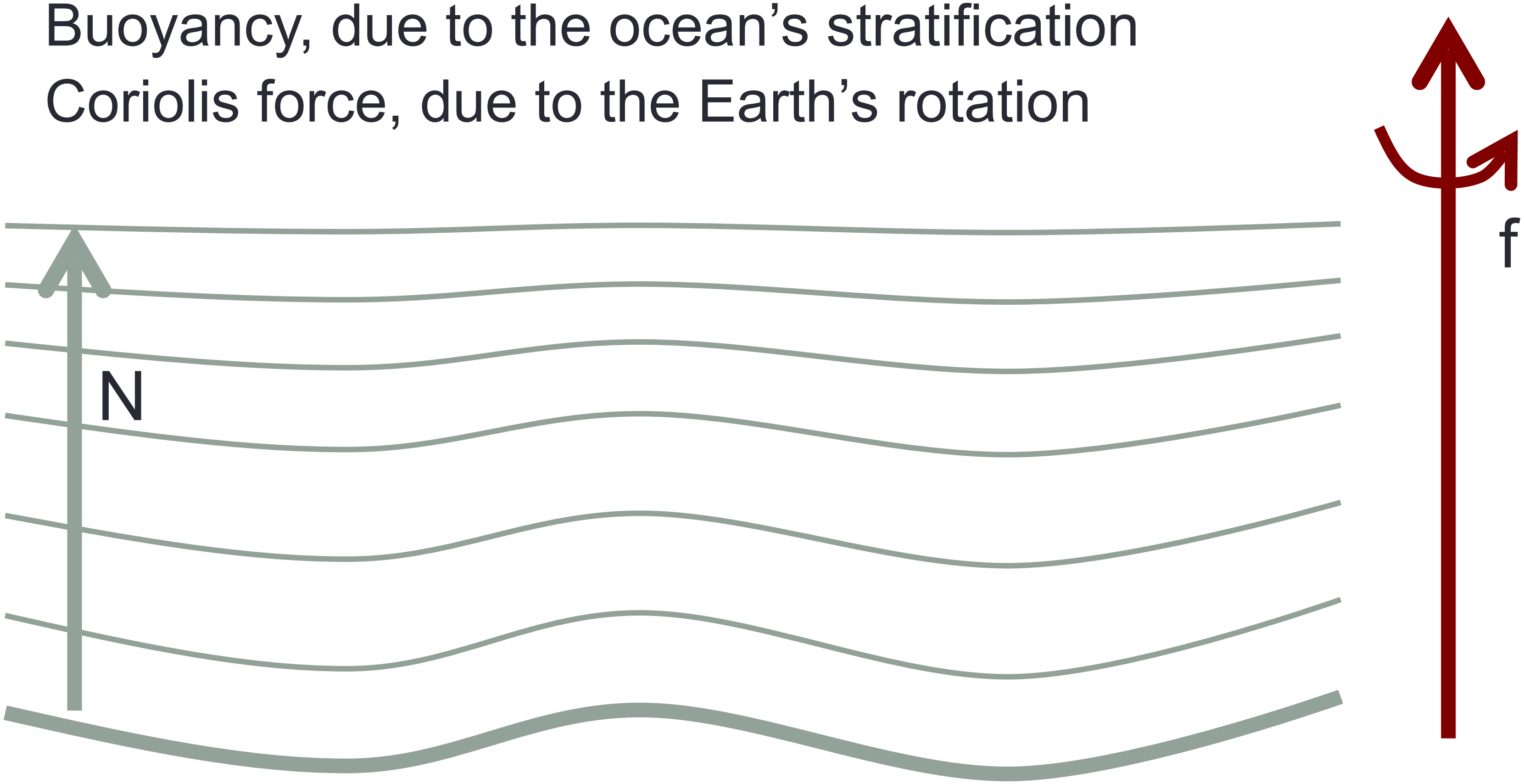
Fig. 14. Vertical Temperature-Curves at Stat. 15 (a) and Stat. 9 (b), 1900.

Temporal changes in temperature profiles, at two different locations with about 2 1/2 hours interval. a) northeast of Iceland, b) north of the Faeroes.

Internal waves

- Two restoring forces are at work for the internal waves:

1. Buoyancy, due to the ocean's stratification
2. Coriolis force, due to the Earth's rotation



Internal waves: some definitions

Internal waves go by many other names depending upon the fluid stratification, generation mechanism, amplitude, and influence of external forces.

- **Inertia-gravity wave** = internal waves that have a large enough wavelength / long enough period to be affected by the earth's rotation.
- **Near-Inertial waves** = internal waves with frequency close to f
- **Internal solitary waves (solitons)** = internal waves with large amplitude and small period (few minutes)
- **Internal tides** = Internal waves generated at the frequency of tides (forced by the interaction of the Barotropic tides with the bottom topography)
- **Lee waves (or mountain waves)** = internal waves generated by flow over topography

Internal waves: some definitions

Internal waves go by many other names depending upon the fluid stratification, generation mechanism, amplitude, and influence of external forces.

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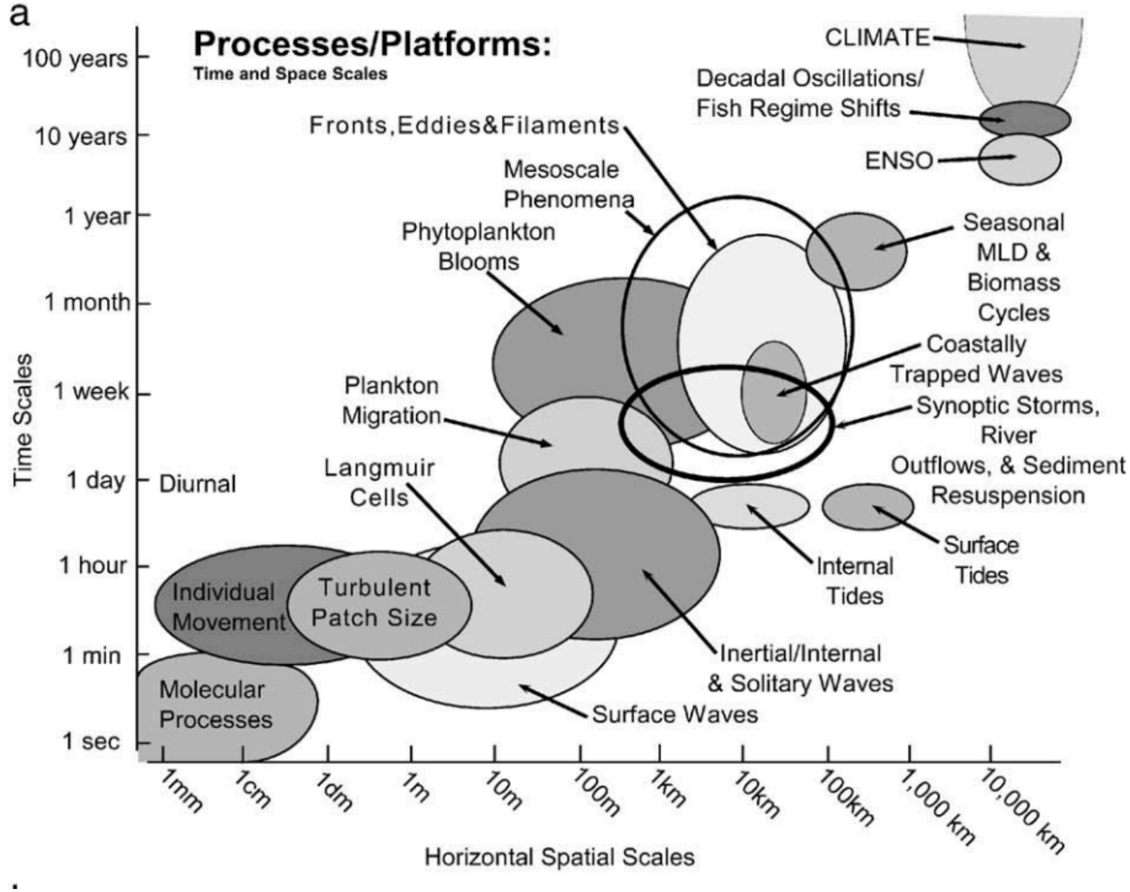
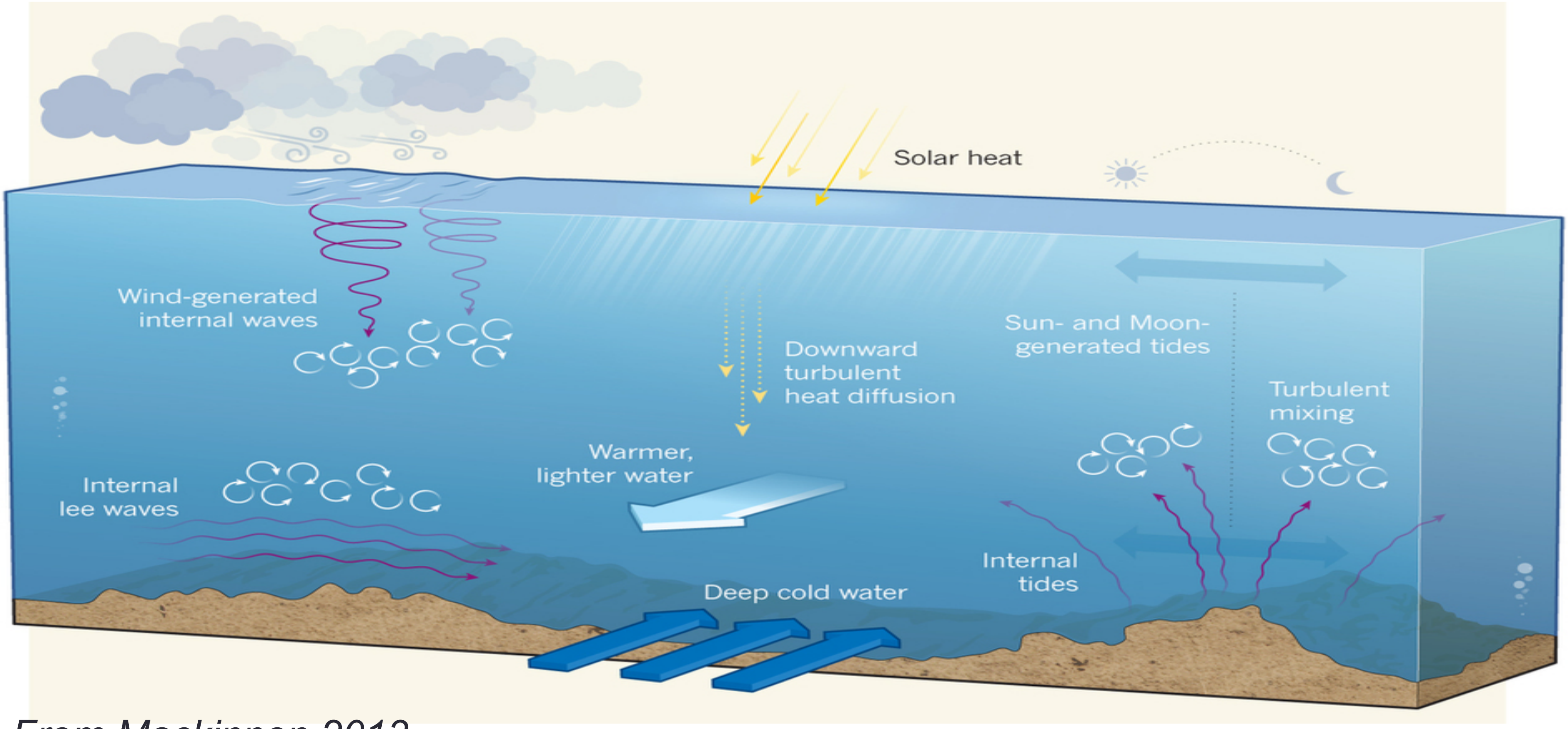


Internal waves generation

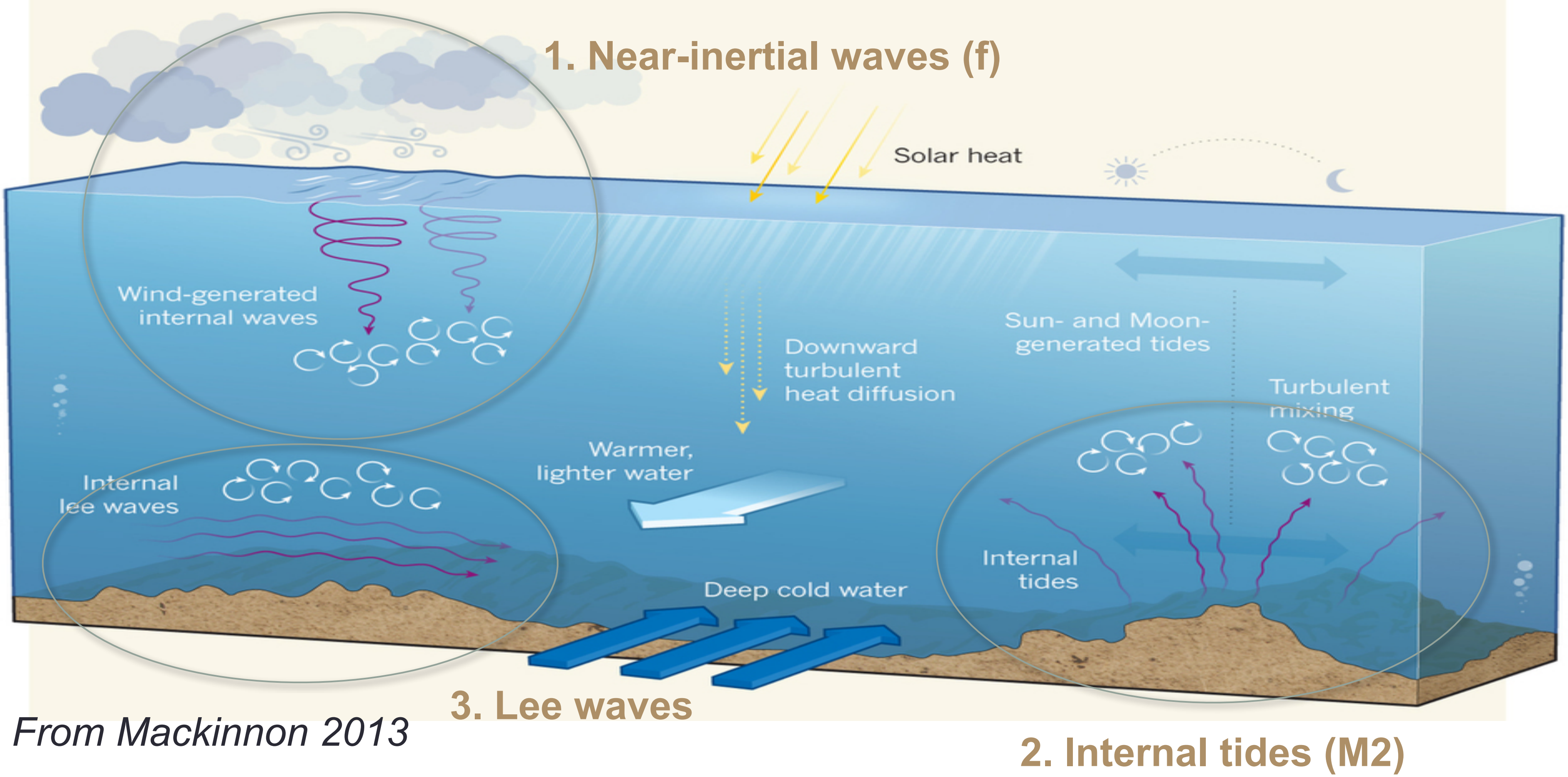
- Some mechanisms:



From Mackinnon 2013

Internal waves generation

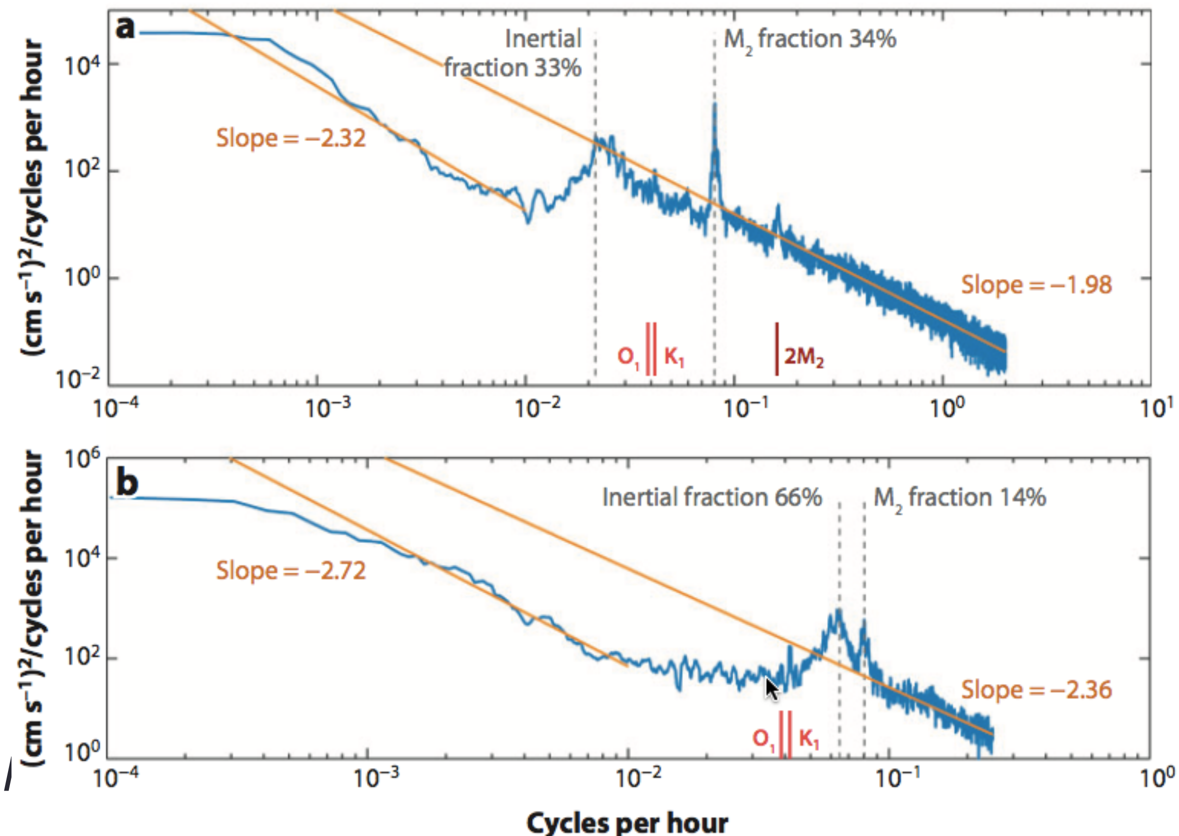
- Dominant mechanisms = wind and tides



From Mackinnon 2013

Internal waves generation

- Winds generate mostly near-inertial waves (frequency close to f)
- Barotropic tides generate internal tides at the frequency of tides

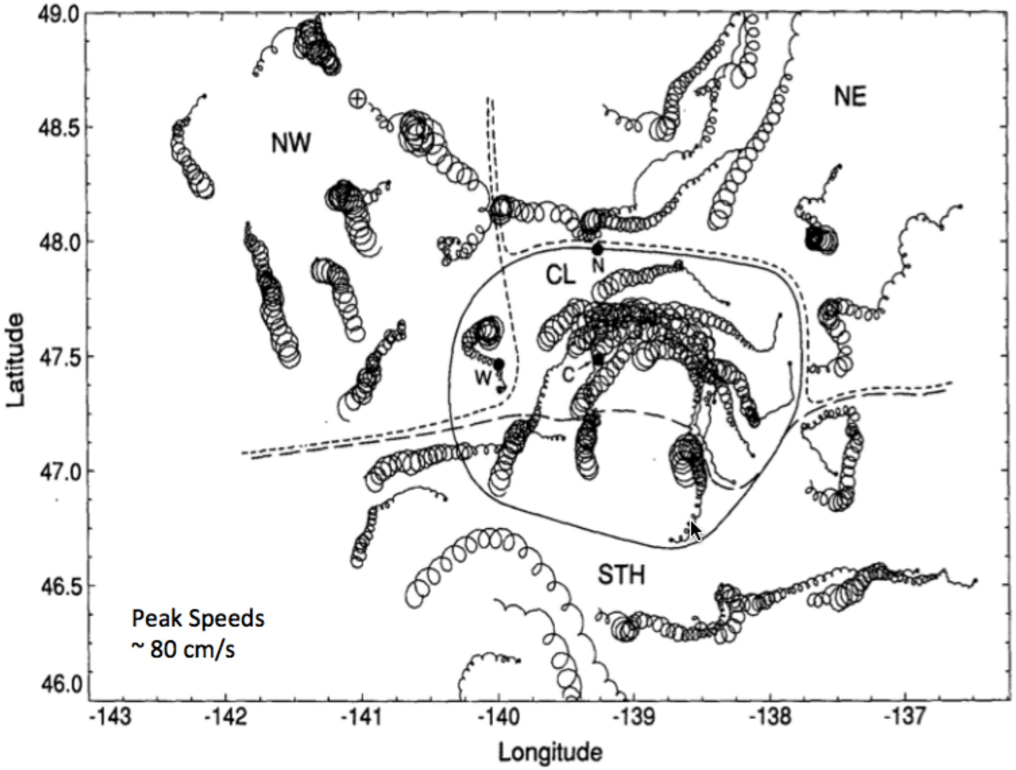


(*a*) Kinetic energy estimate for an instrument in the western North Atlantic near 15◦N at 500 m. (*b*) Power density spectral estimate from a record at 1000 m at 50.7◦S, 143◦W, south of Tasmania in the Southern Ocean

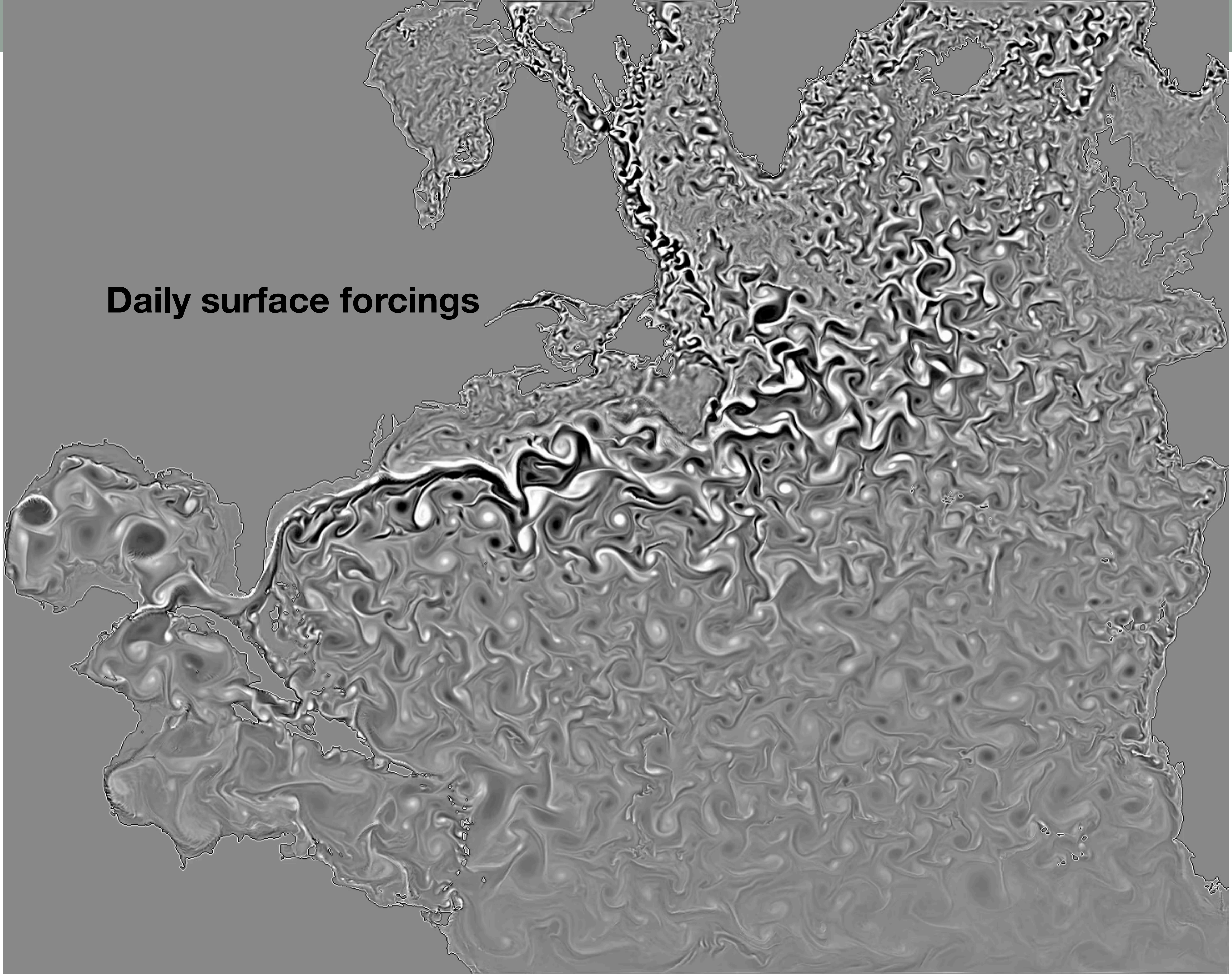
Internal waves generation

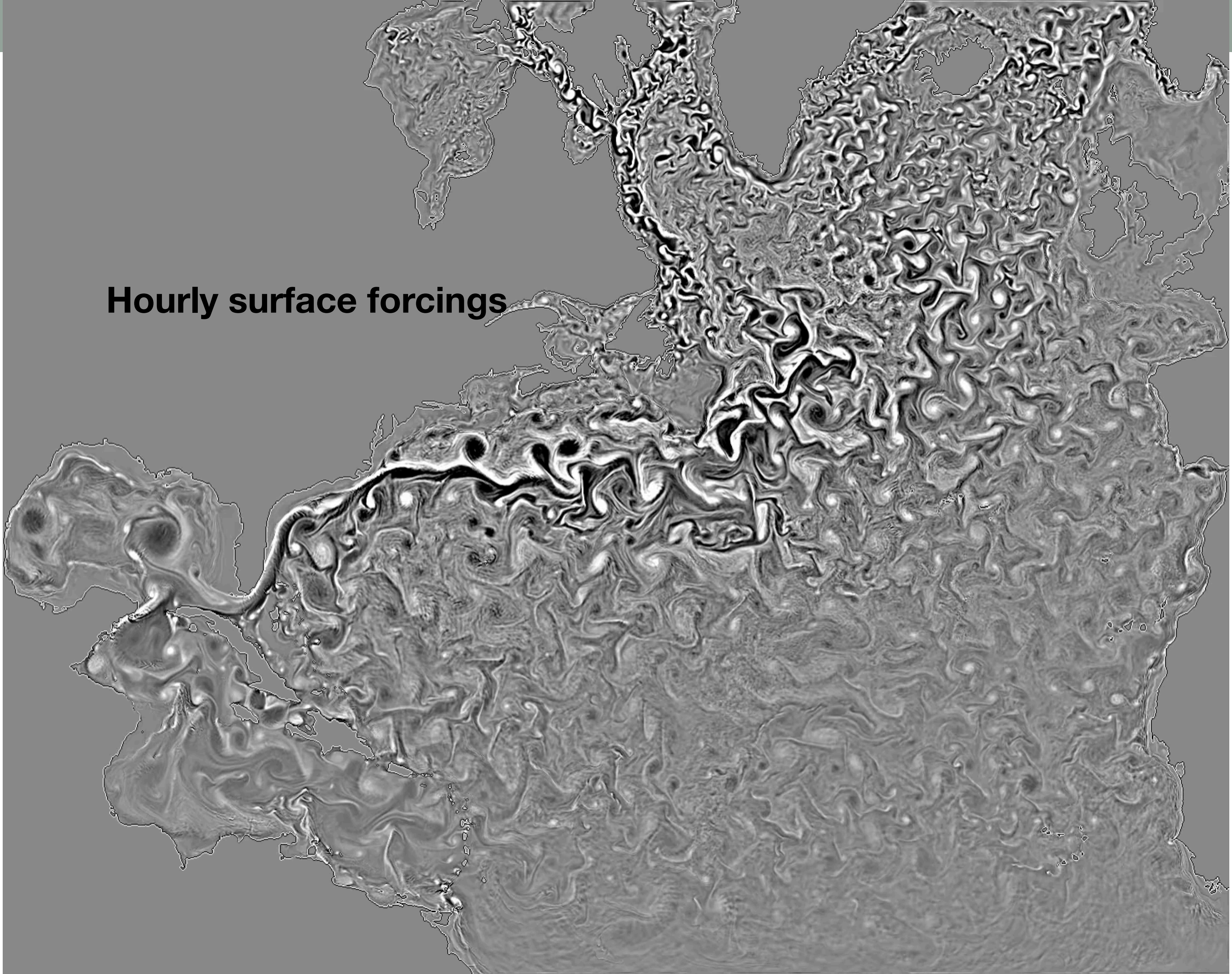
1. Near-inertial waves

- Winds generate mostly near-inertial waves (frequency close to f)
-



Twenty-five days of surface drifter trajectories after a storm in the eastern north Pacific. The drifters trajectories represent a combination of decaying inertial motions (circular oscillations) and weak geostrophic flow (the time-averaged drift). **[D'Asaro et al, 1995]**

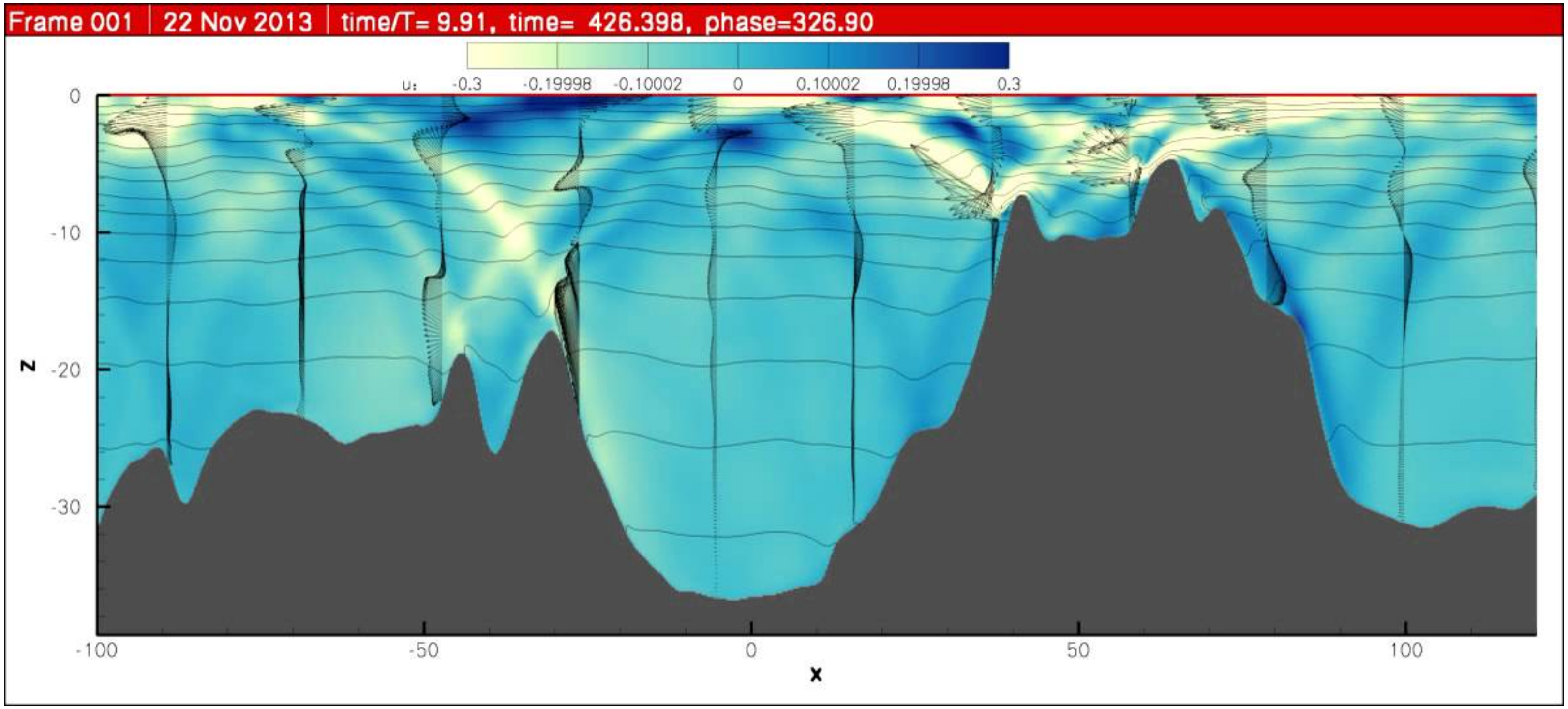

Daily surface forcings


Hourly surface forcings

Internal waves generation

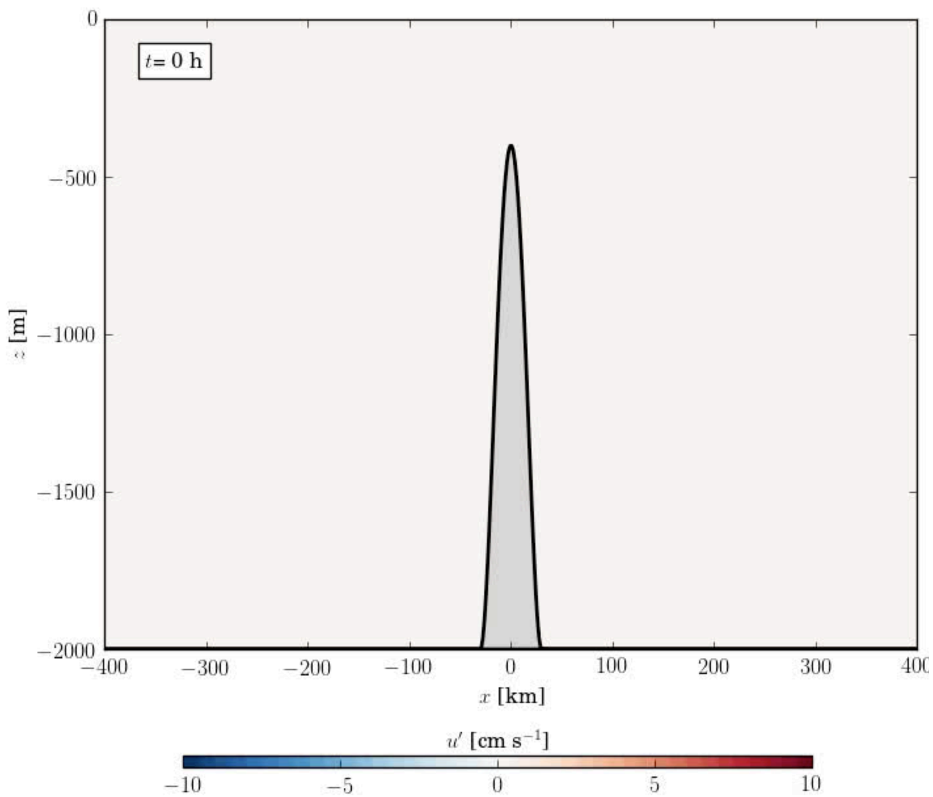
2. Internal tides

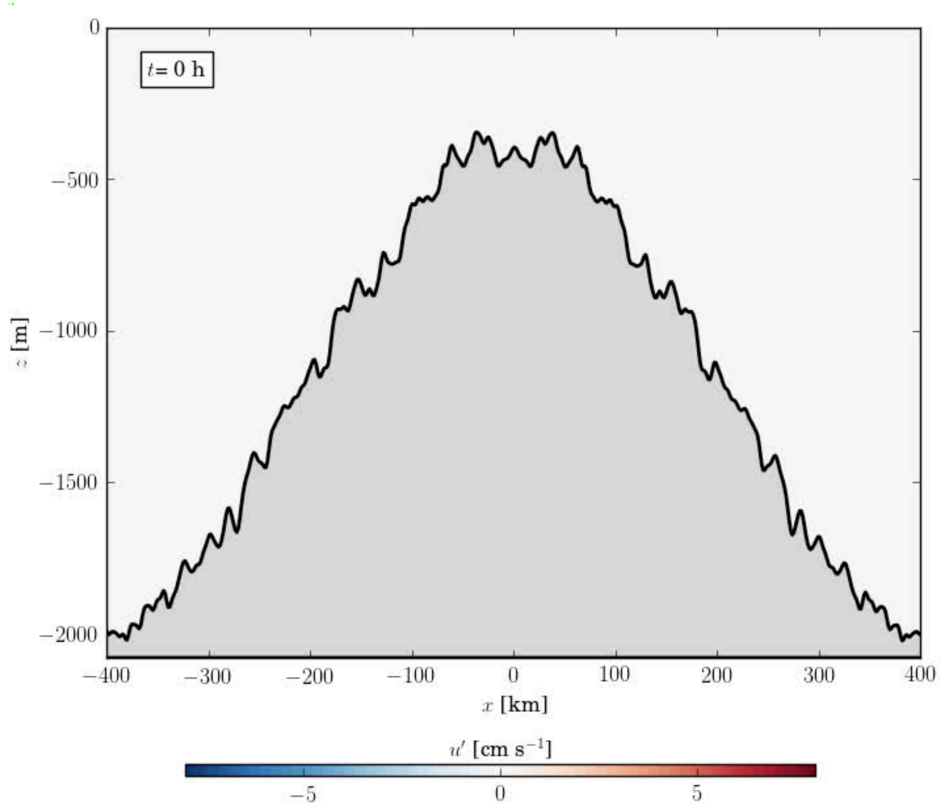
- Barotropic tides generate internal tides at the frequency of tides



Internal waves generation

2. Internal tides

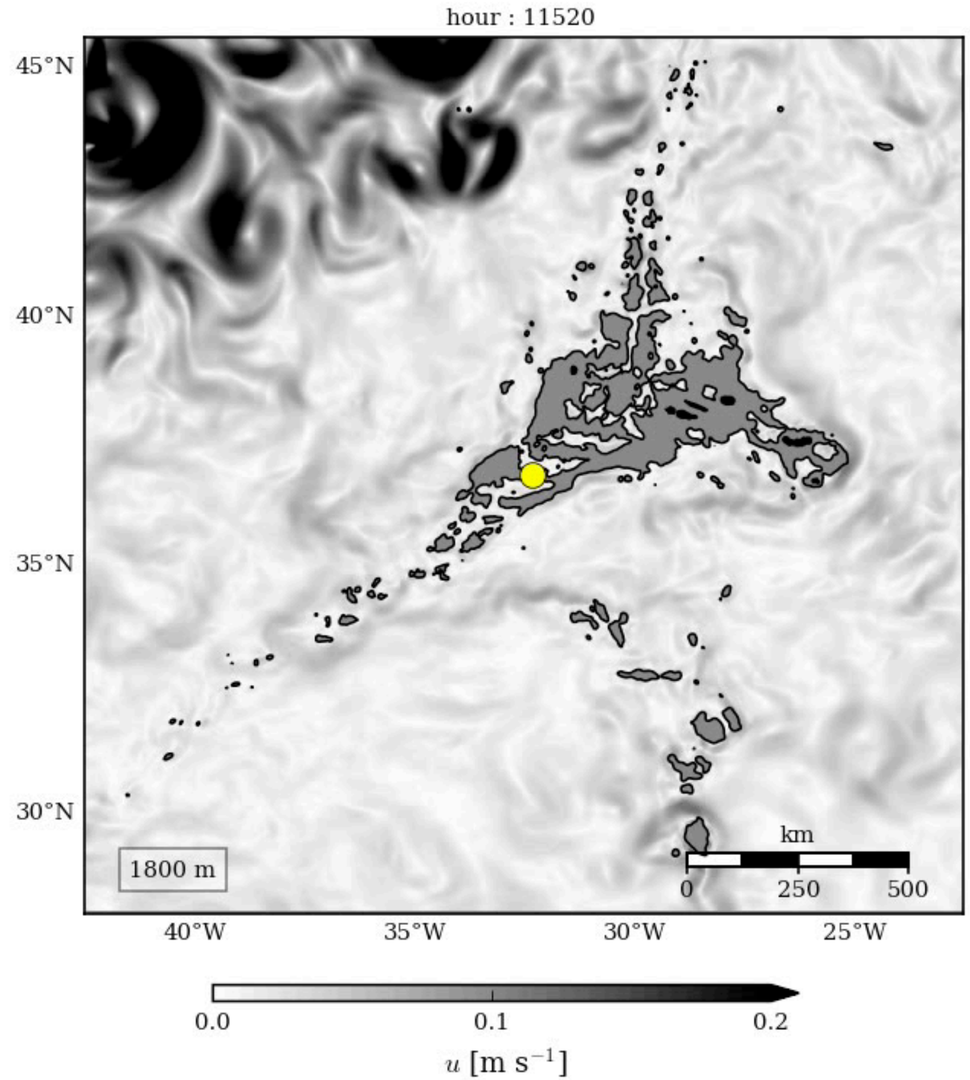




Example: Mid-Atlantic Ridge

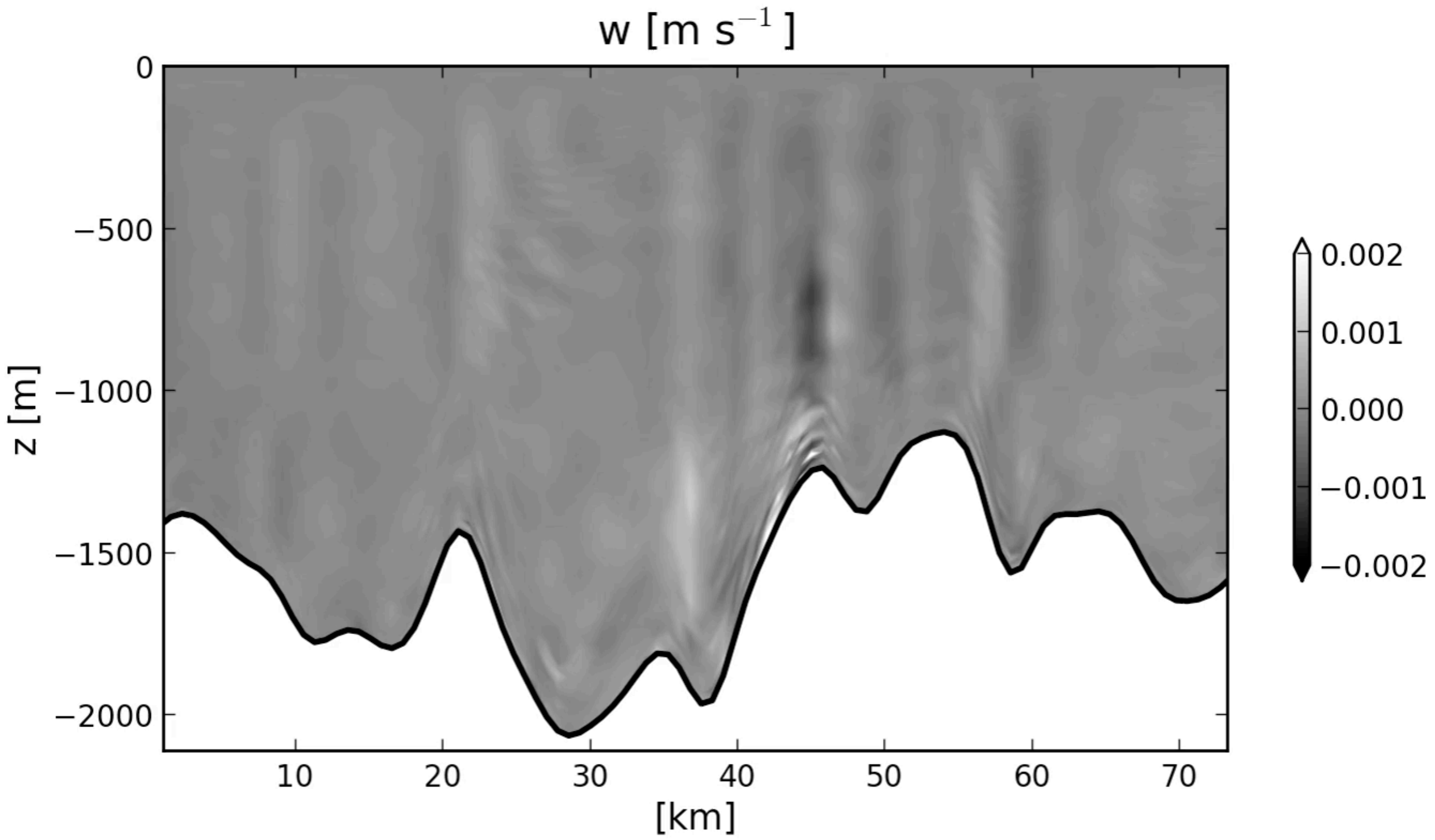
- Velocity amplitude at 1800m
- simulation ROMS (dx = 2km)

WITH TIDES:



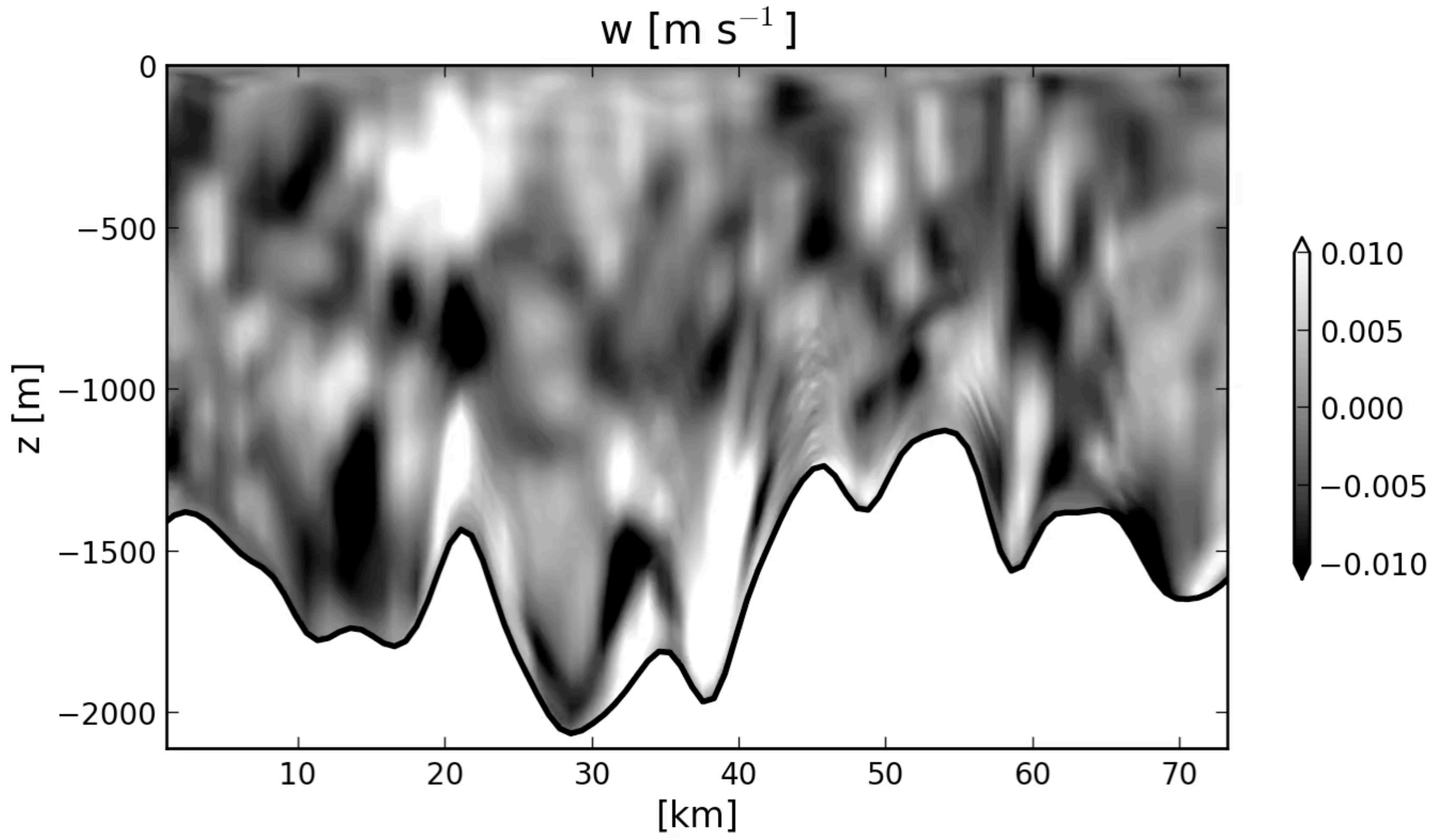
Example: Mid-Atlantic Ridge

NO TIDES:

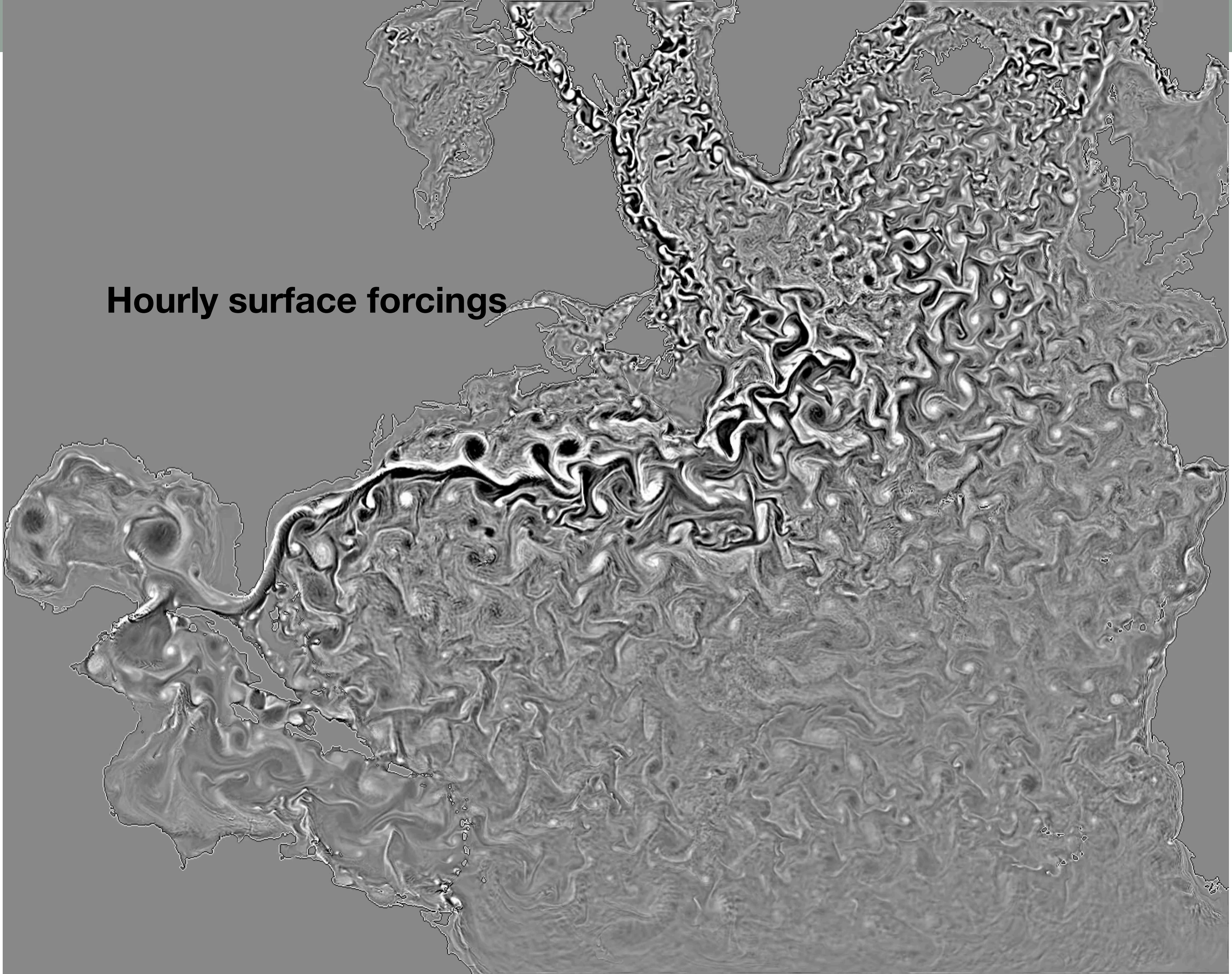


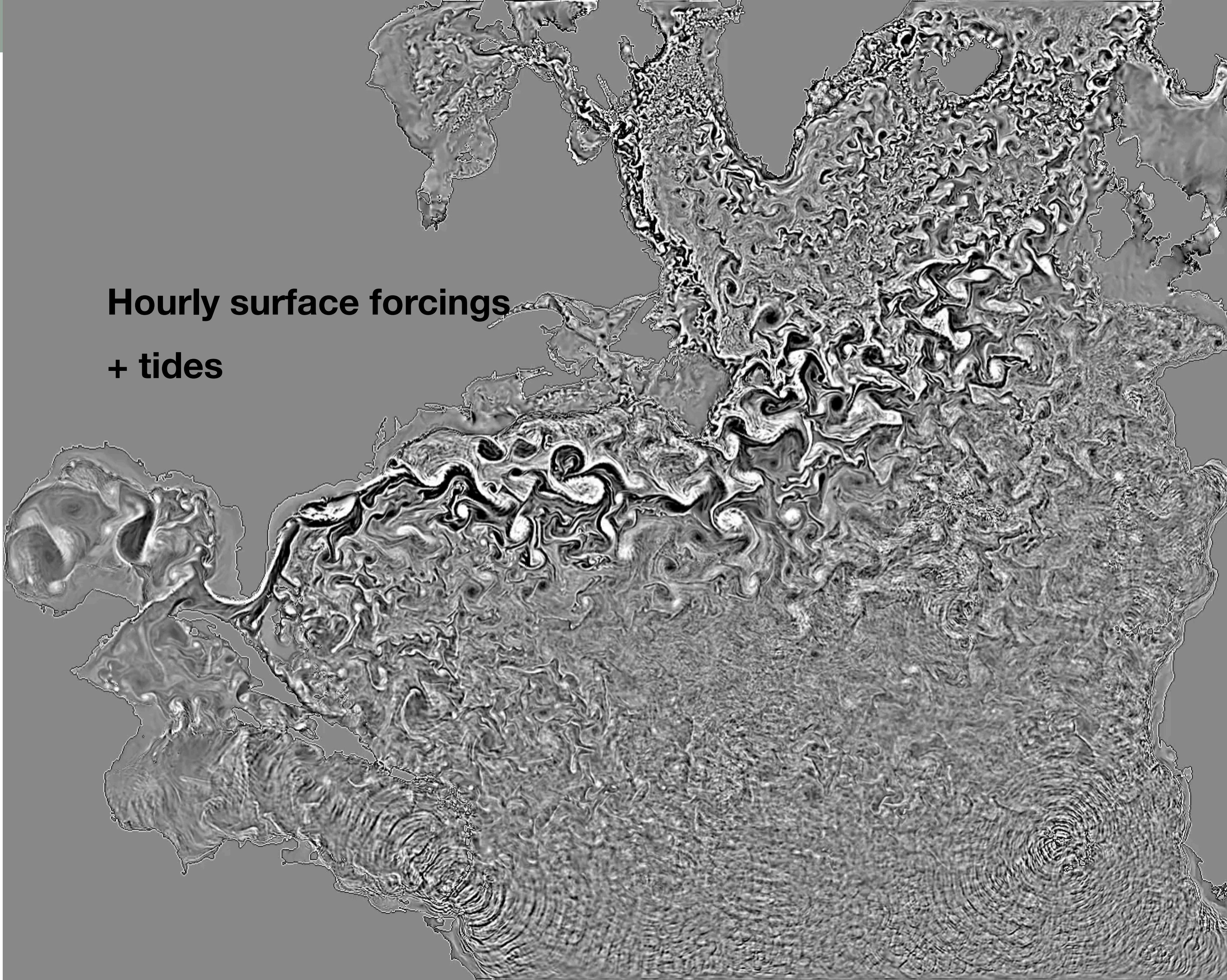
Example: Mid-Atlantic Ridge

WITH TIDES:



Hourly surface forcings

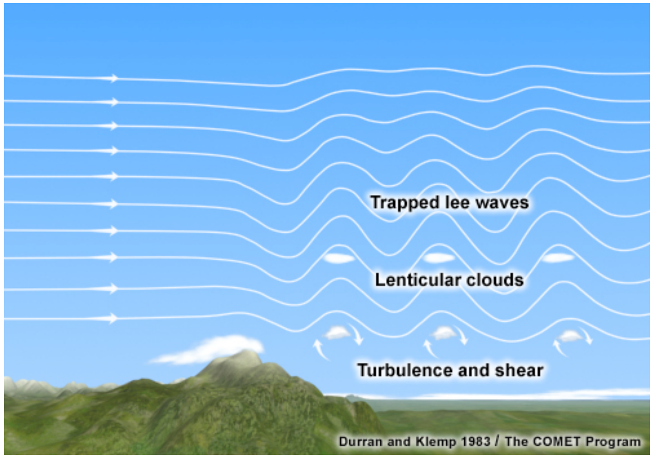



Hourly surface forcings
+ tides

Internal waves generation

3. Lee waves

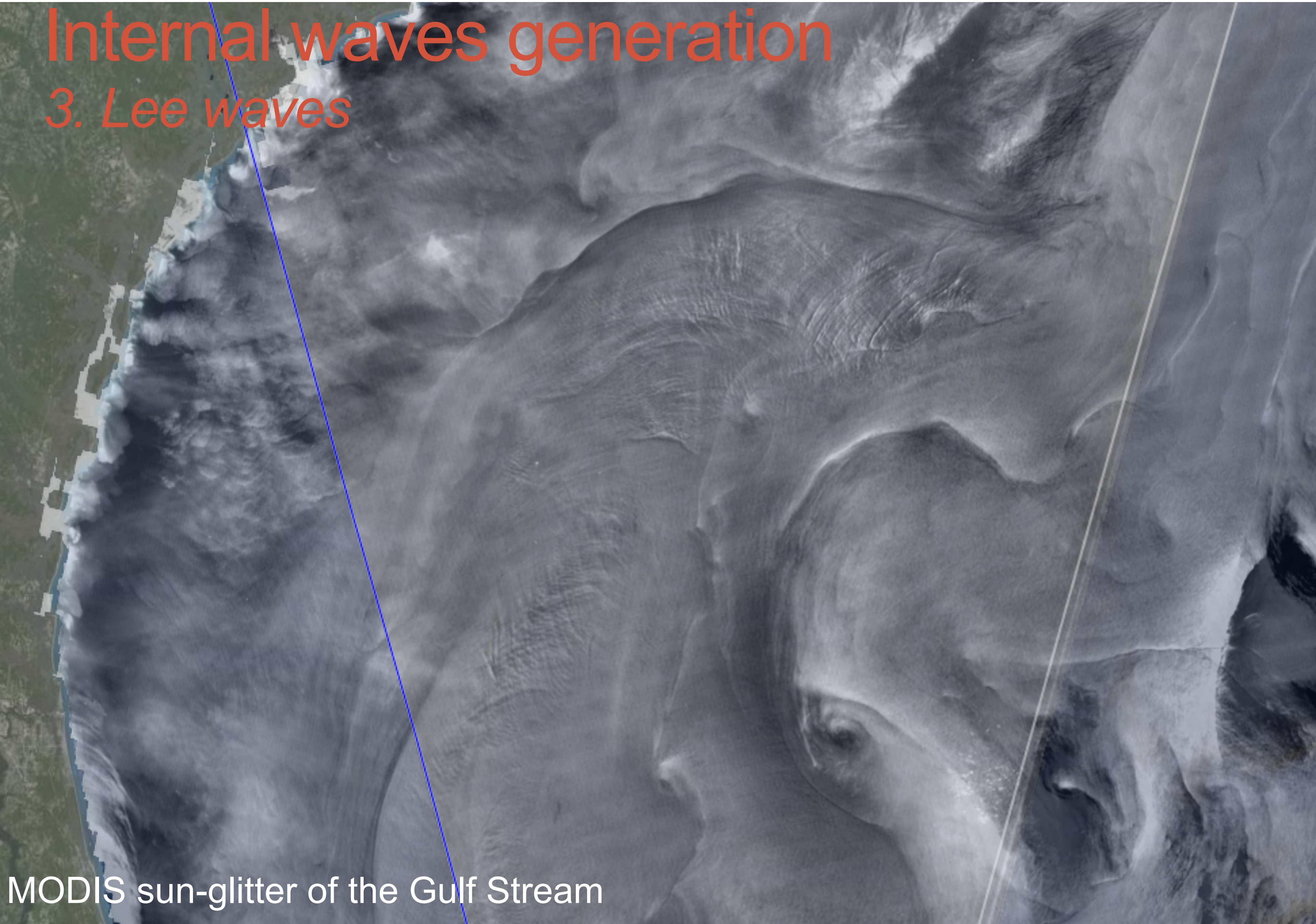
- **Lee waves** are similar to mountain waves in the atmosphere



Modis, 23/11/2009 - South Atlantic – Sandwich Islands

The lower atmosphere is drier – Downwind of the islands the waves are seen when the air goes up, condensate and form clouds

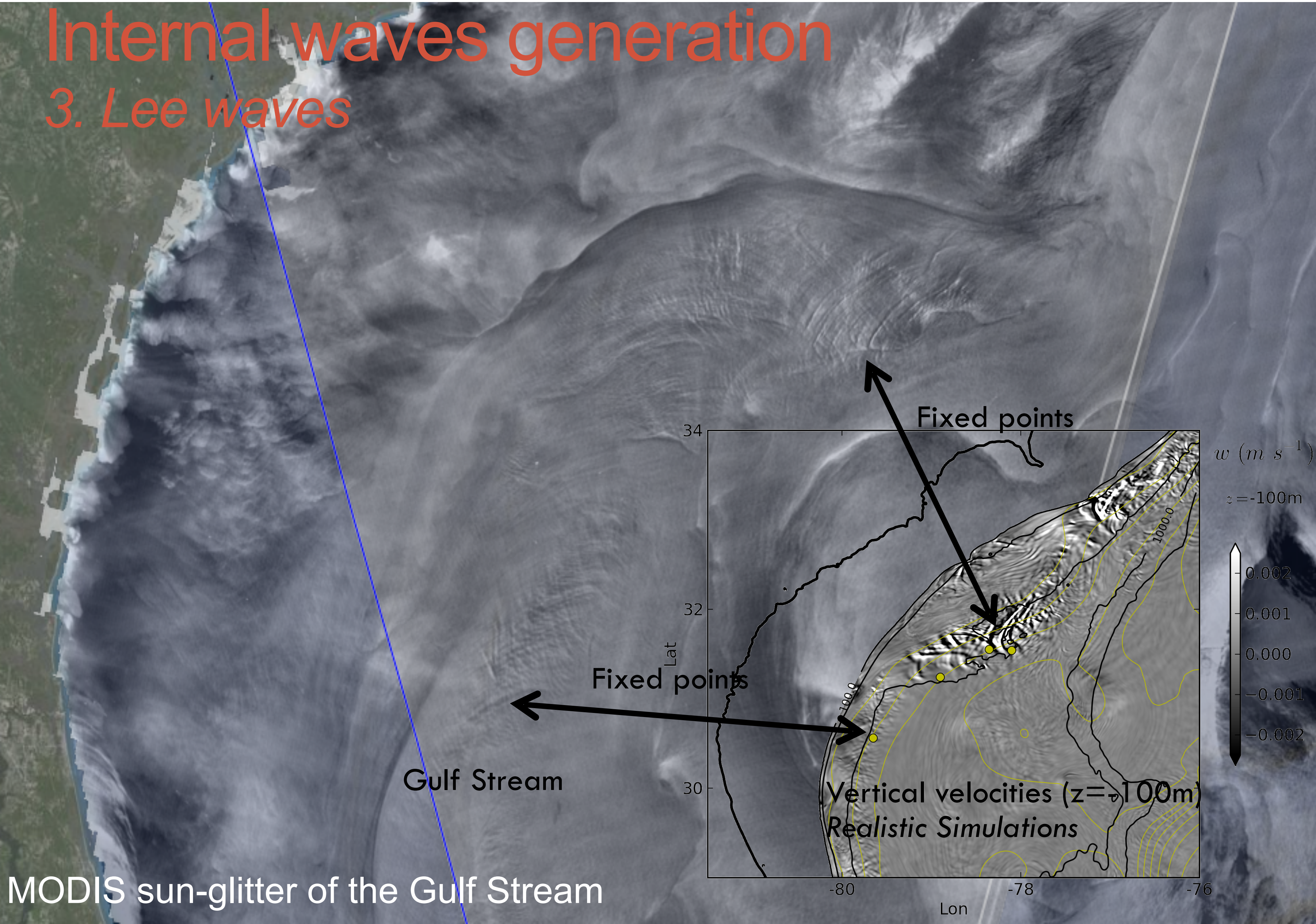
Internal waves generation
3. Lee waves



MODIS sun-glitter of the Gulf Stream

Internal waves generation

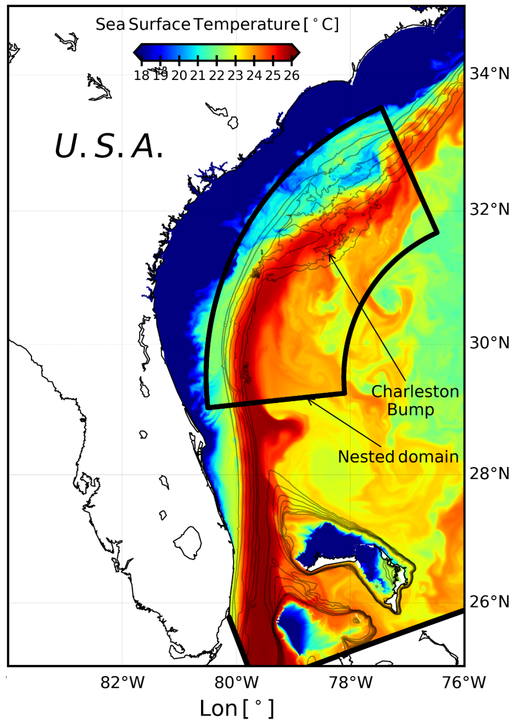
3. Lee waves

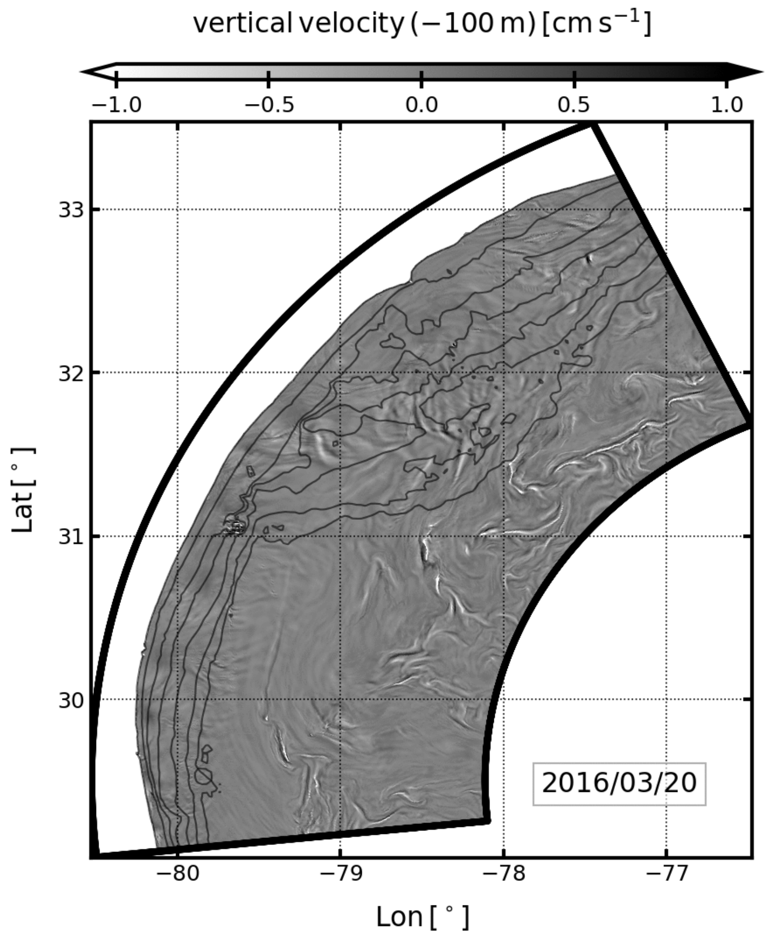


MODIS sun-glitter of the Gulf Stream

Internal waves generation

3. Lee waves



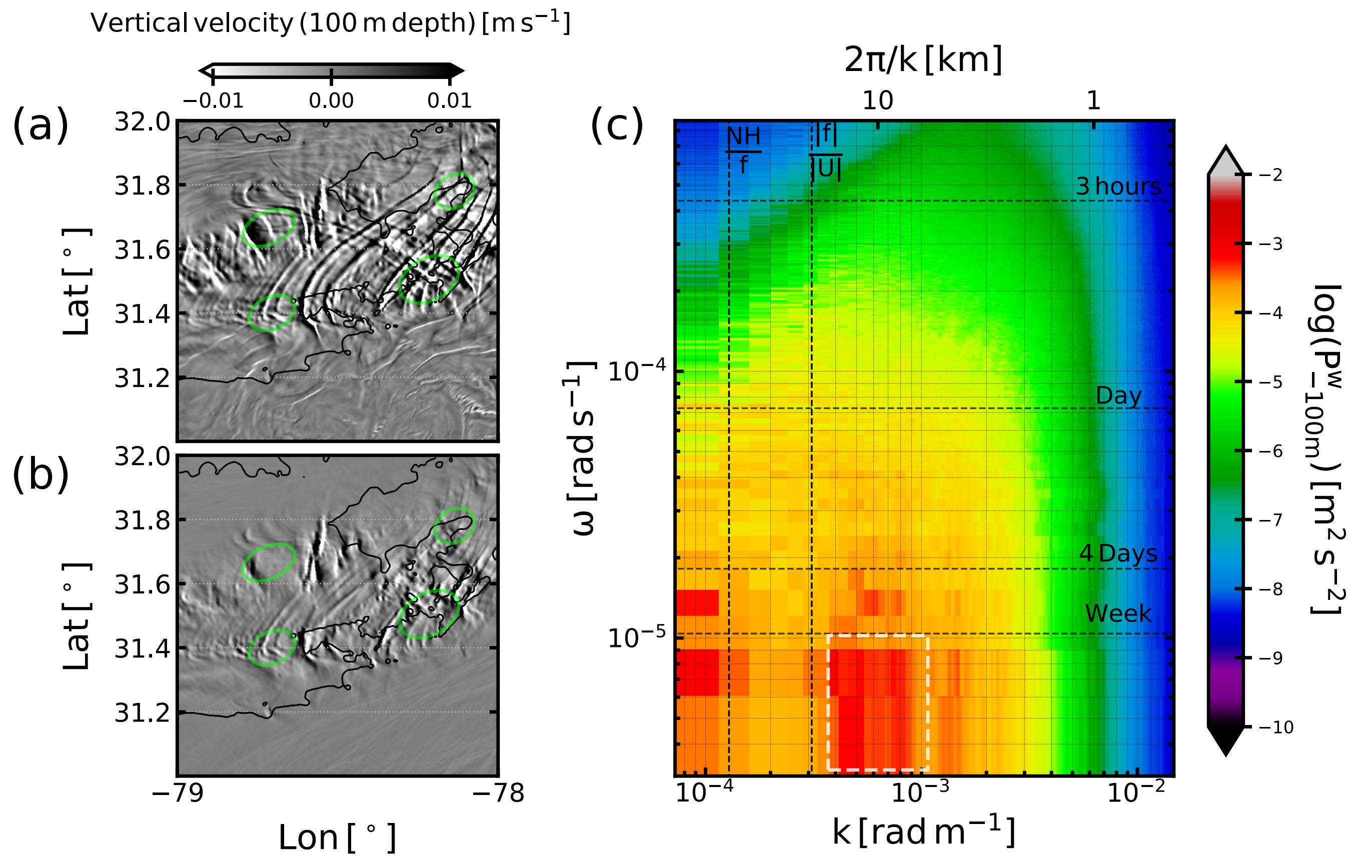


Simulation of the Gulf Stream

Internal waves generation

3. Lee waves

- We can isolate patterns related to Lee Waves by looking at time-low passed vertical velocities below the thermocline:

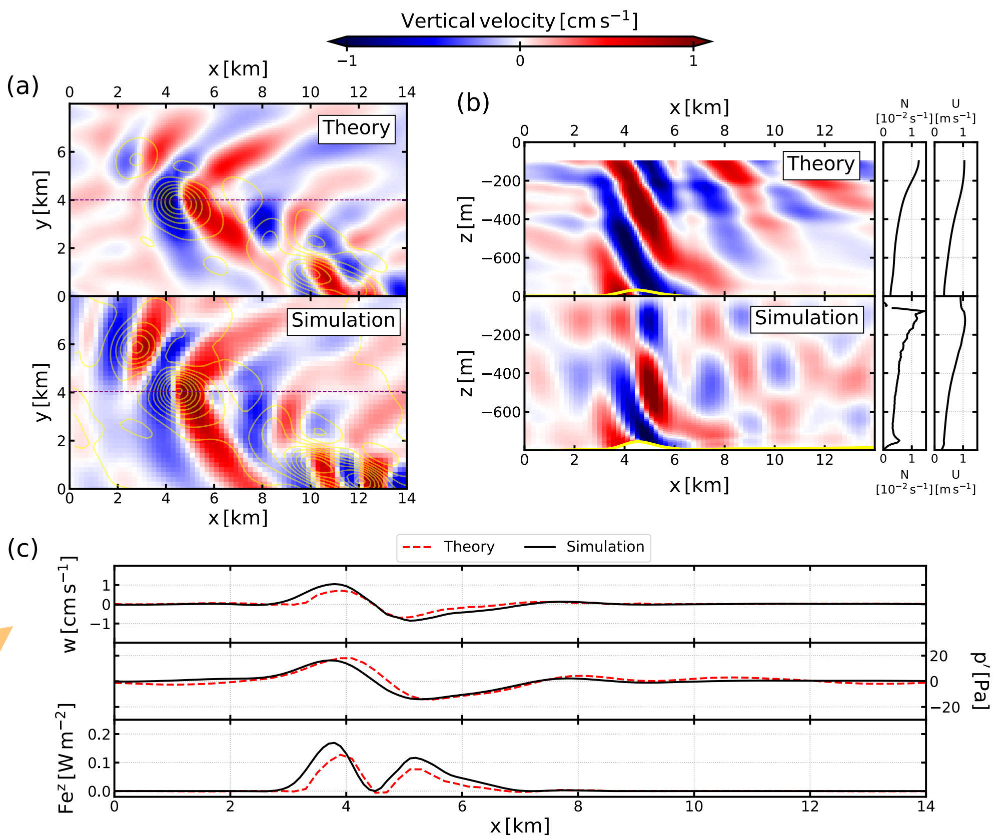


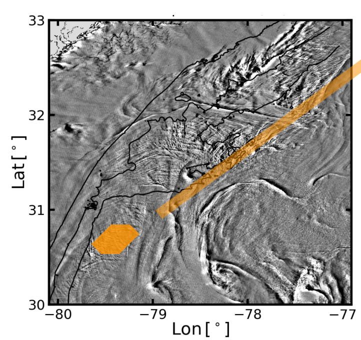
Internal waves generation

3. Lee waves

3D theoretical prediction of hydrostatic lee waves is obtained by numerically solving:

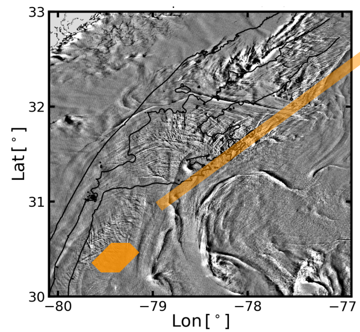
$$\partial_{zz}\,\tilde{\eta}(\mathrm{k,m,z}) + \mathrm{n}^2\,\tilde{\eta}(\mathrm{k,m,z}) = 0$$

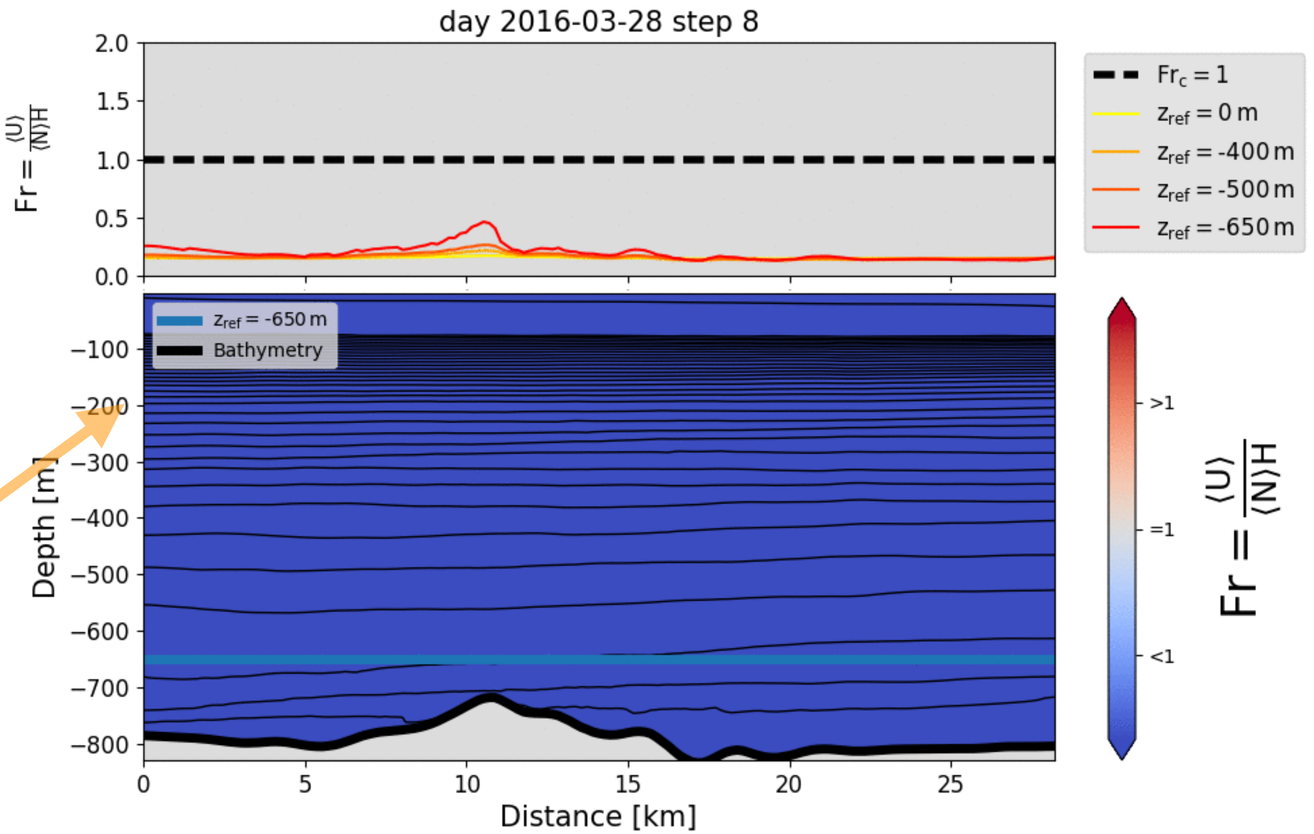




Internal waves generation

3. Lee waves (non-linear)





Internal waves generation

3. Lee waves

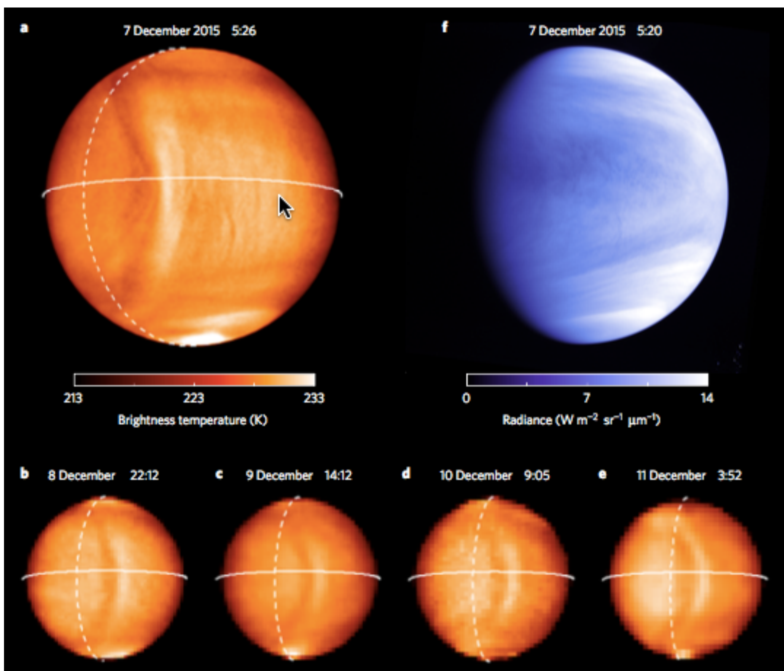
nature geoscience

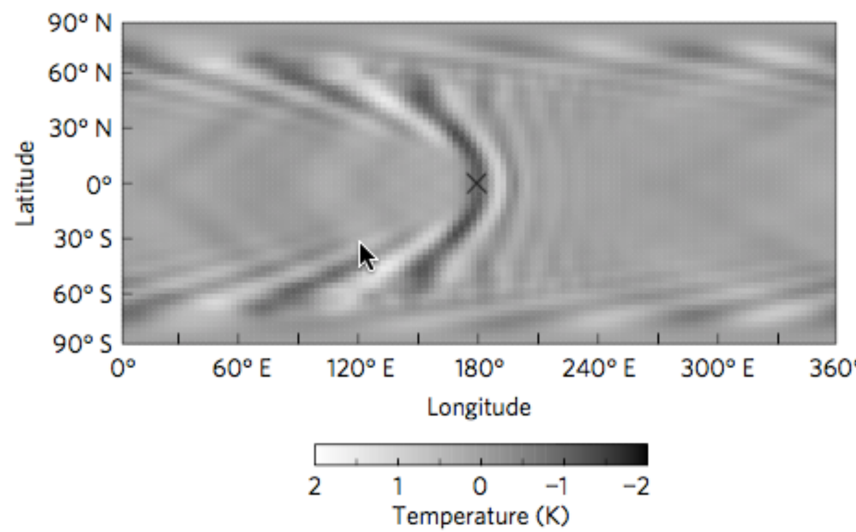
ARTICLES

PUBLISHED ONLINE: 16 JANUARY 2017 | DOI: 10.1038/NGEO2873

Large stationary gravity wave in the atmosphere of Venus

ara[1], Masahiko Futaguchi[2], George L. Hashimoto[3], Takeshi Horinouchi[4],
ra[5], Naomoto Iwagaimi[6], Toru Kouyama[7], Shin-ya Murakami[8], Masato Nakamura[8],
ara[9], Mitsuteru Sato[4], Takao M. Sato[8], Makoto Suzuki[8], Makoto Taguchi[1]*,
Munetaka Ueno[11], Shigeto Watanabe[12], Manabu Yamada[13] and Atsushi Yamazaki[8]





Internal waves generation

4. Solitary waves

Example: Strait of Messina

- Solitary waves train in the Strait of Messina

Picture from 11/08/2003 - Terra (NASA) ASTER radiometer - Sunglitter

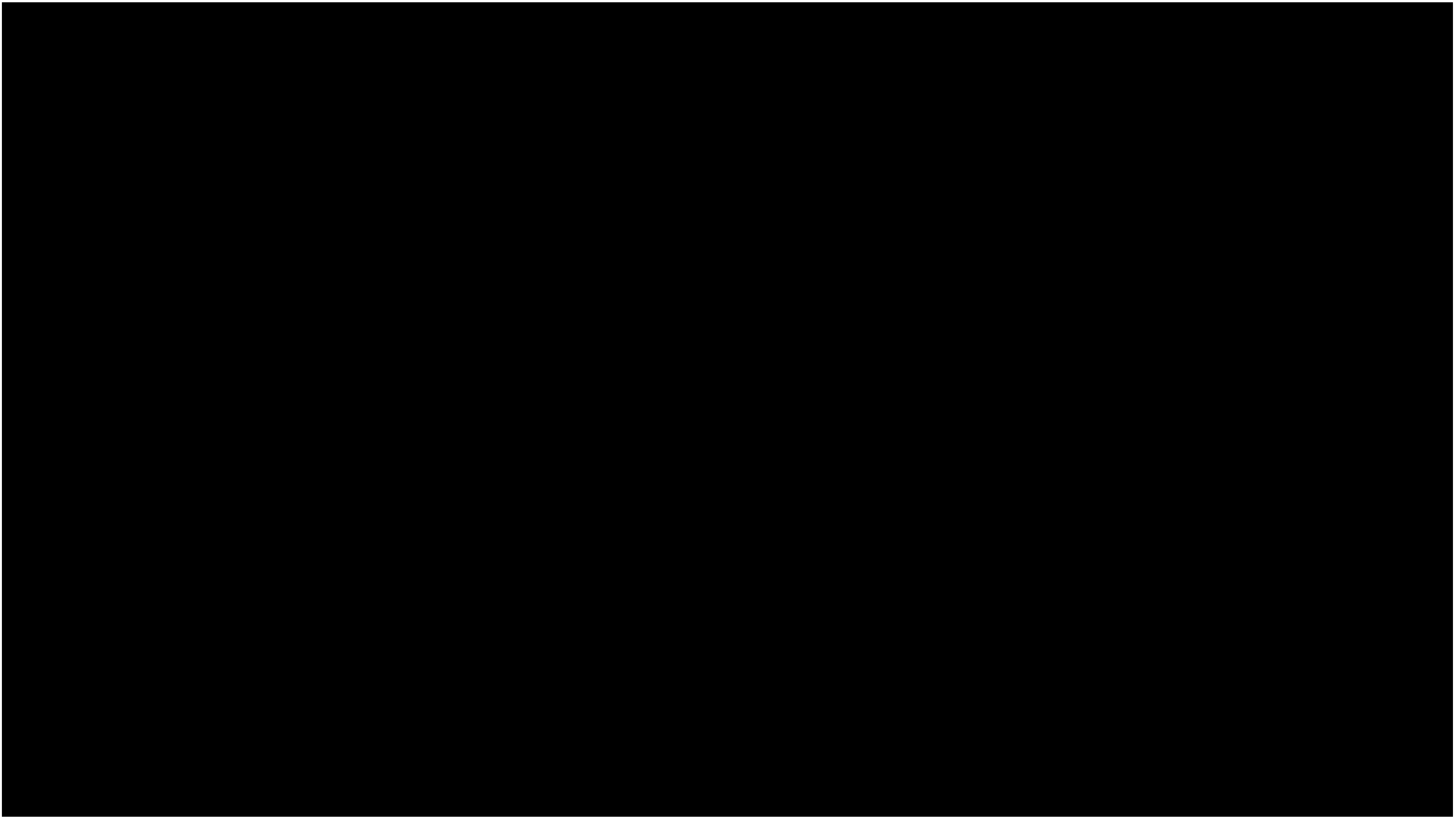
Internal waves generation
4. Solitary Waves

- Solitary waves train in the Strait of Gibraltar

Picture from 01/01/1993 – SAR image

Internal waves generation

4. Solitary Waves



Internal waves generation

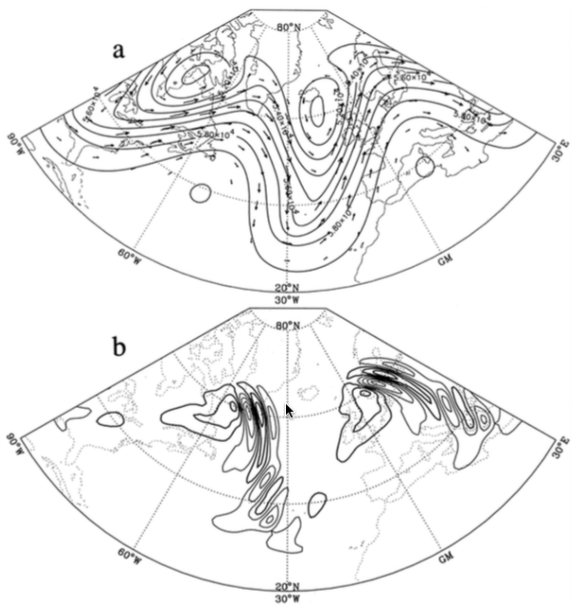
4. Solitary Waves

M.Mercier, L.Gostiaux, S.Saidi, J.Sommeria, H.Didelle, S.Viboud, K.Helfrich T.Dauxois,T.Peacock.

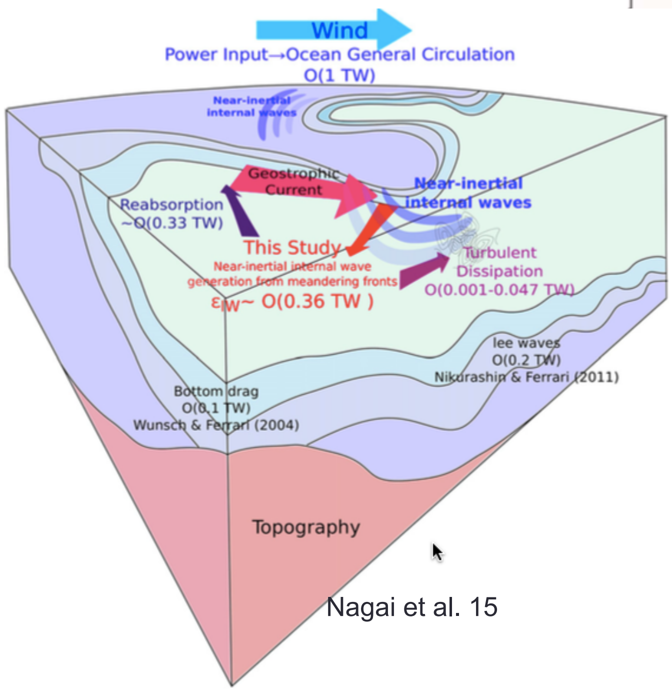
Internal waves generation

5. Spontaneously emitted wave

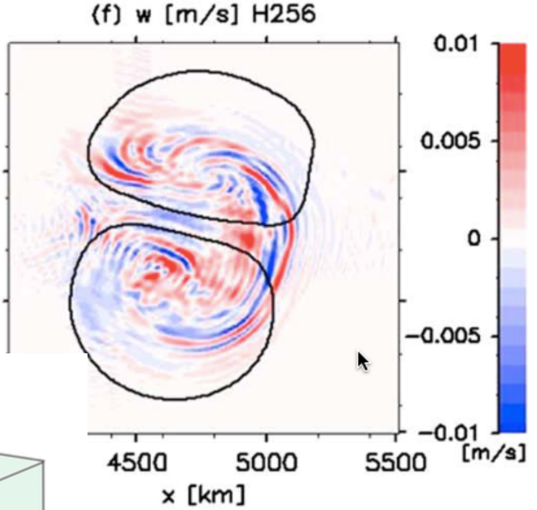
- **Spontaneous wave emission** from jets and fronts



Fritts & Alexander, 04



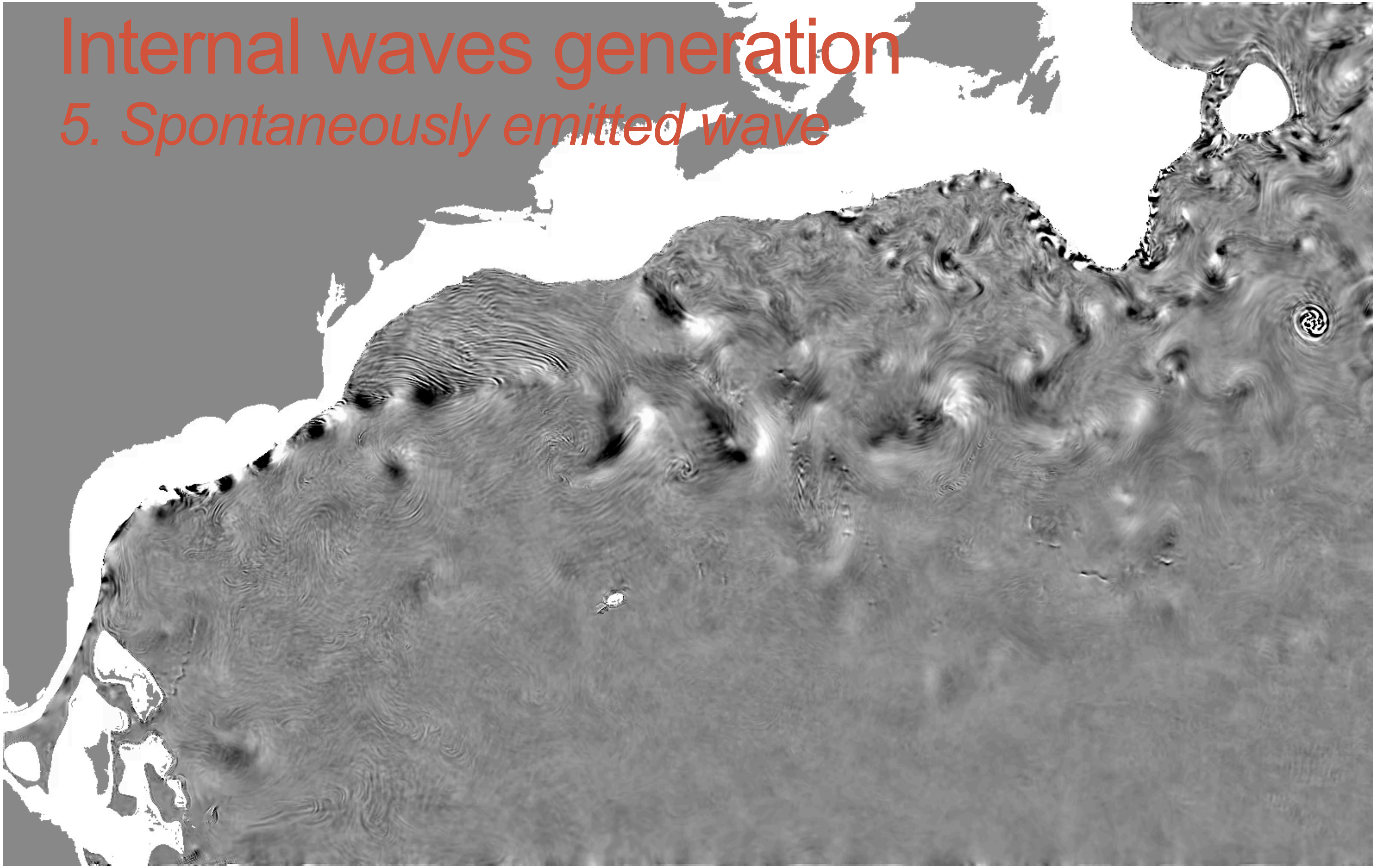
Nagai et al. 15



Sugimoto & Plougonven 16

Internal waves generation

5. Spontaneously emitted wave



Vertical velocities at z = -500 m in the Gulf Stream

Internal waves generation

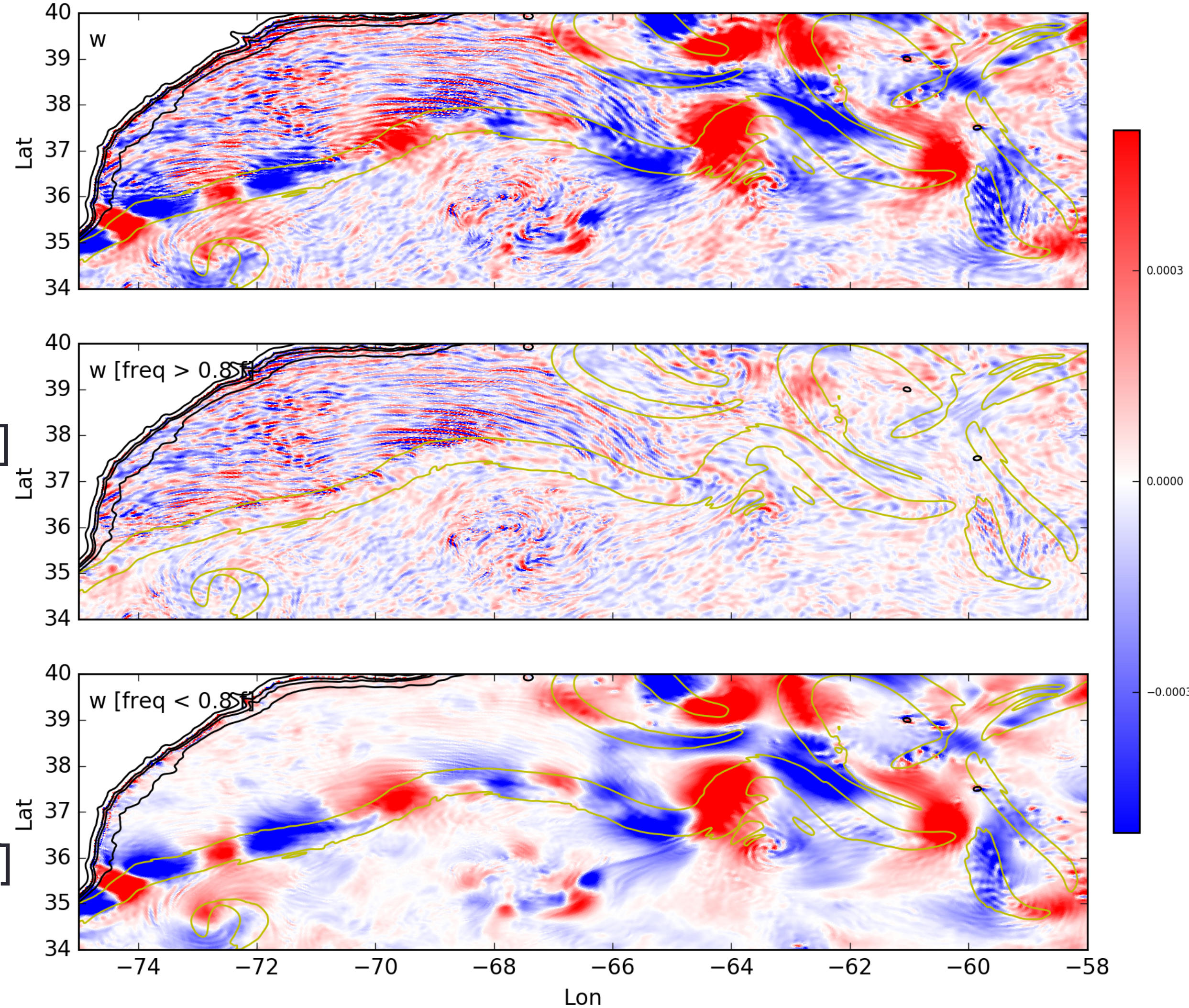
5. Spontaneously emitted wave

Total w

w [freq > 0.8 f]]

Vertical velocities at z = -500 m in the Gulf Stream

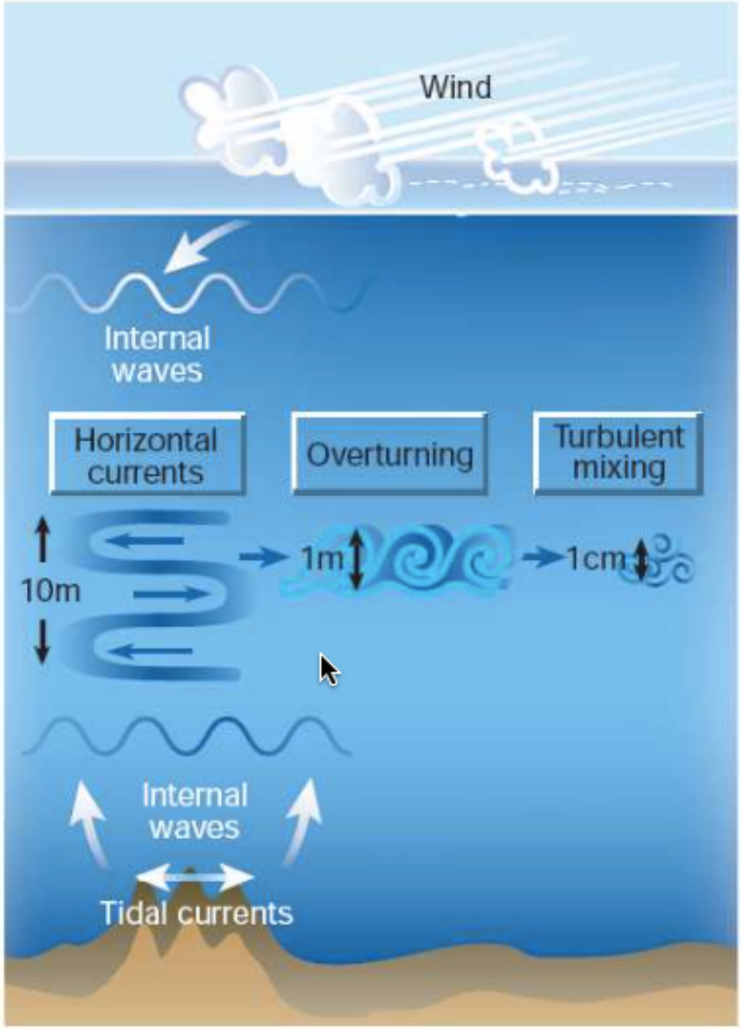
w [freq < 0.8 f]]



Dissipation and mixing

Internal waves can become unstable due to the presence of a background shear field, **leading to internal-wave breaking and mixing.**

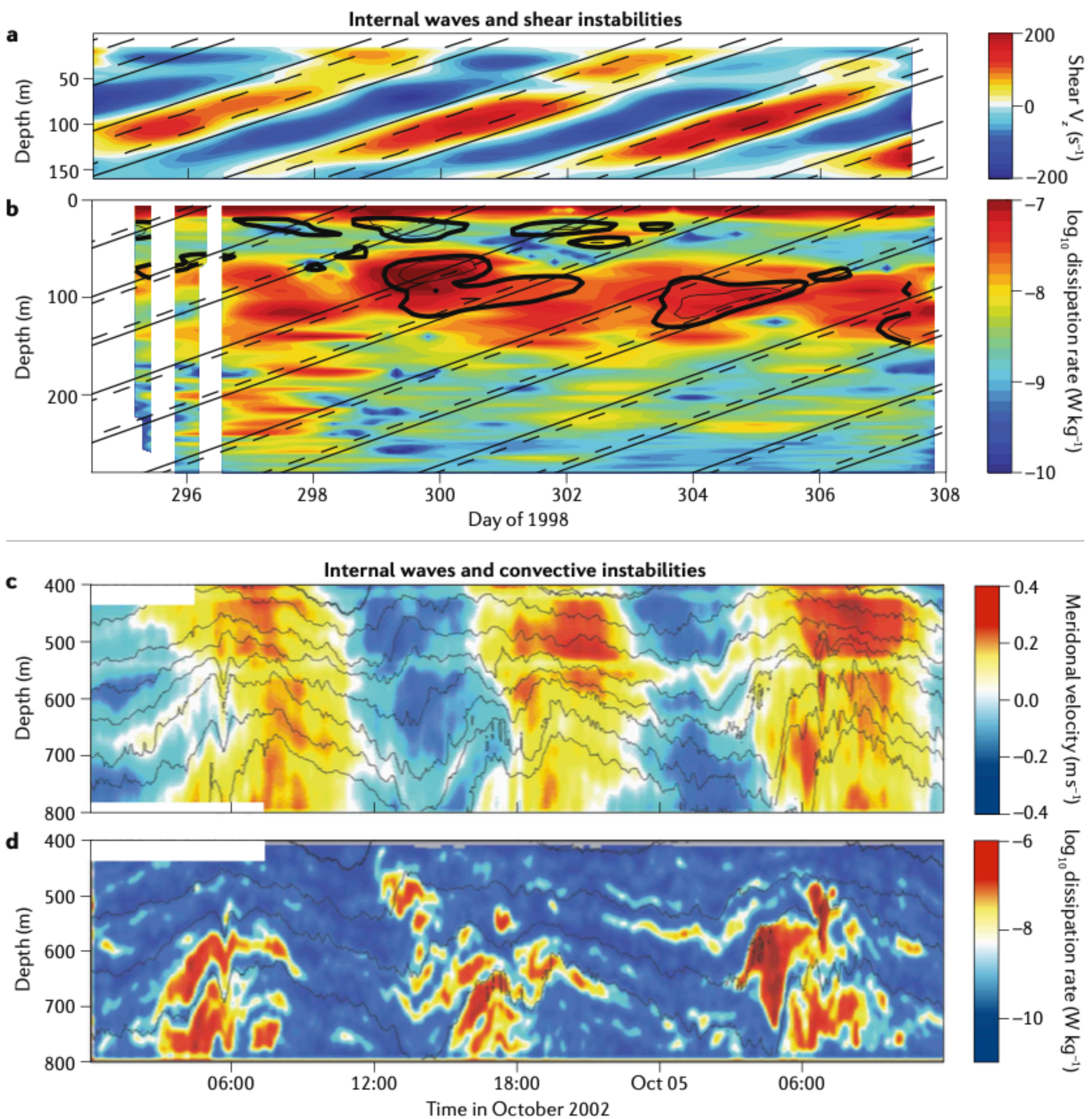
Pathway of internal-wave energy: from its origin, by the wind and by tidal flow over topography, to dissipation as small-scale mixing.



Dissipation and mixing

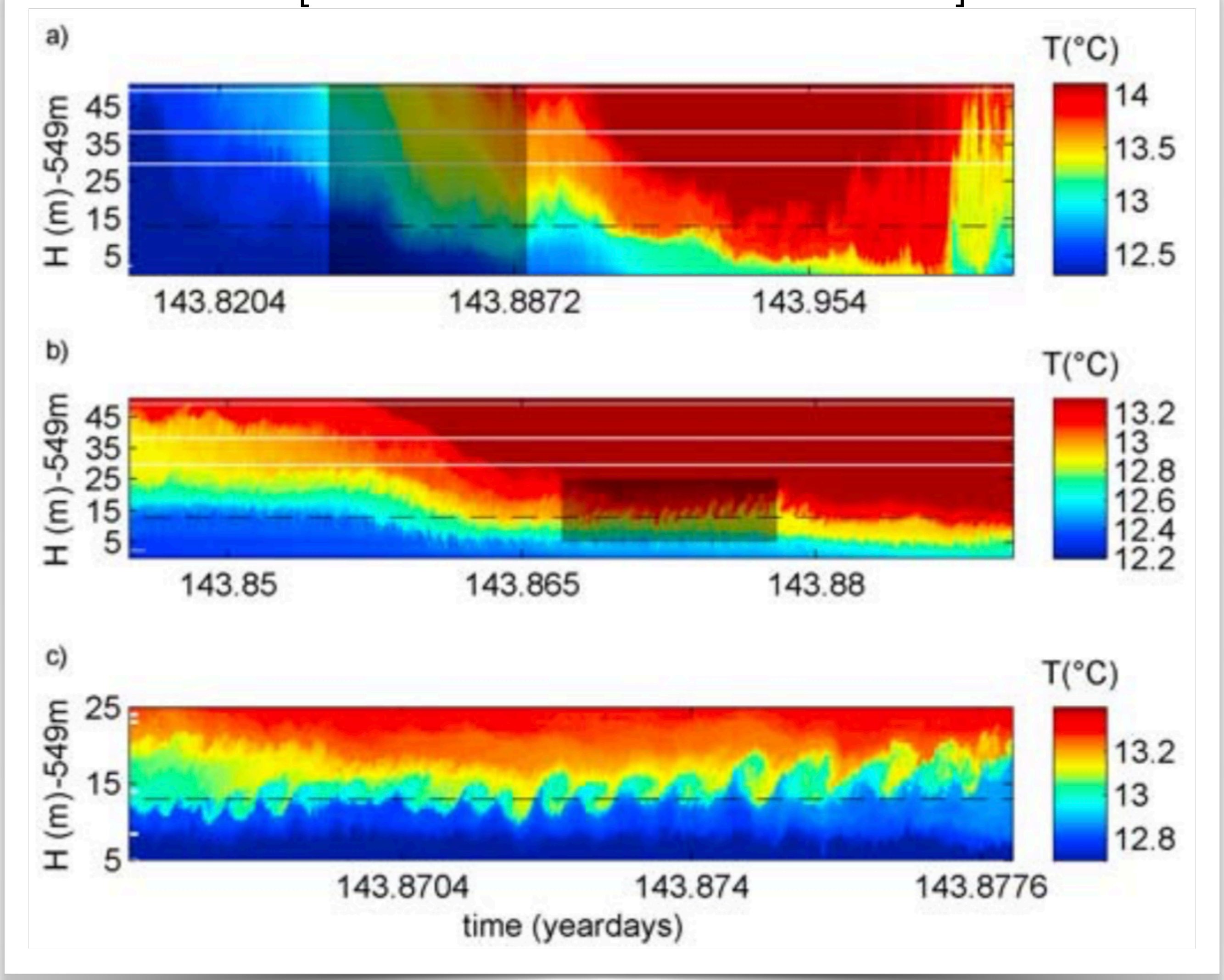
Turbulent mixing in two different internal wave environments.

[See Whalen et al, 20]

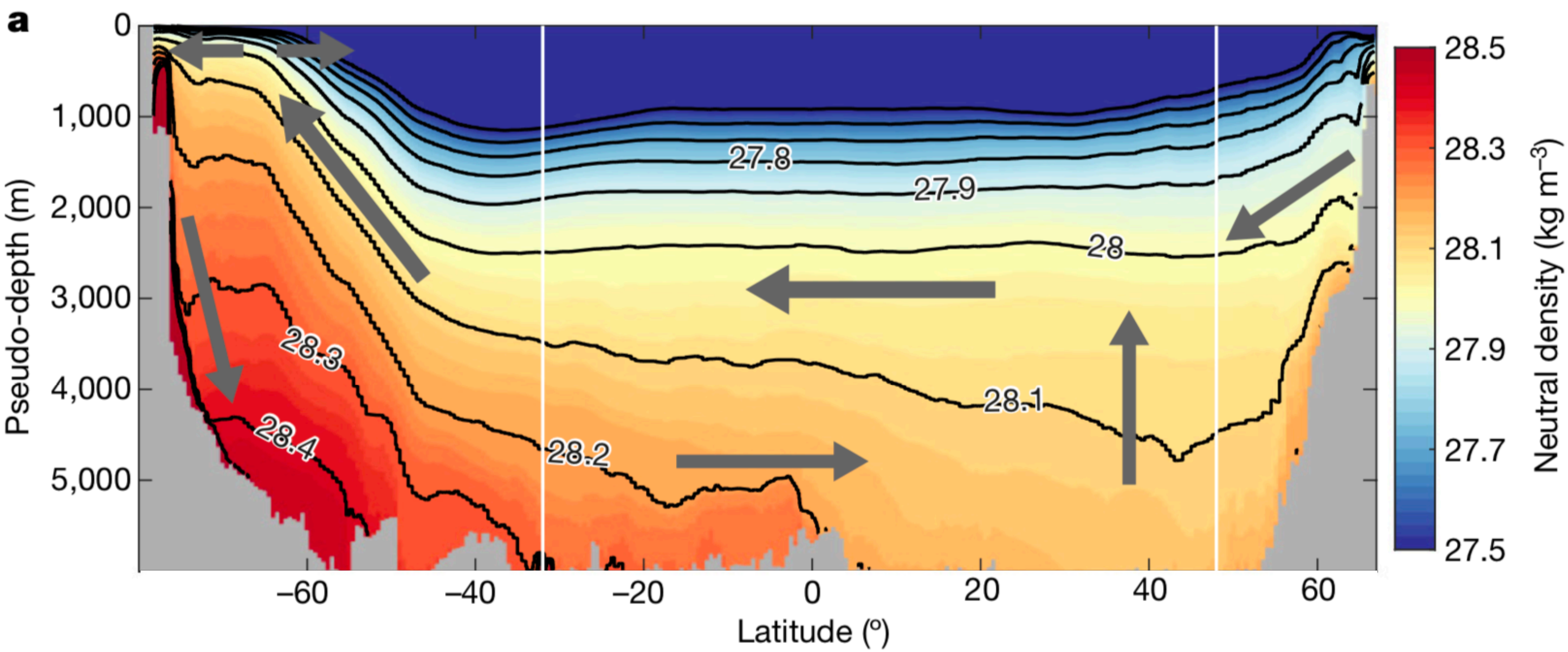


Dissipation and mixing

[Van Haren and Gostiaux 2010]



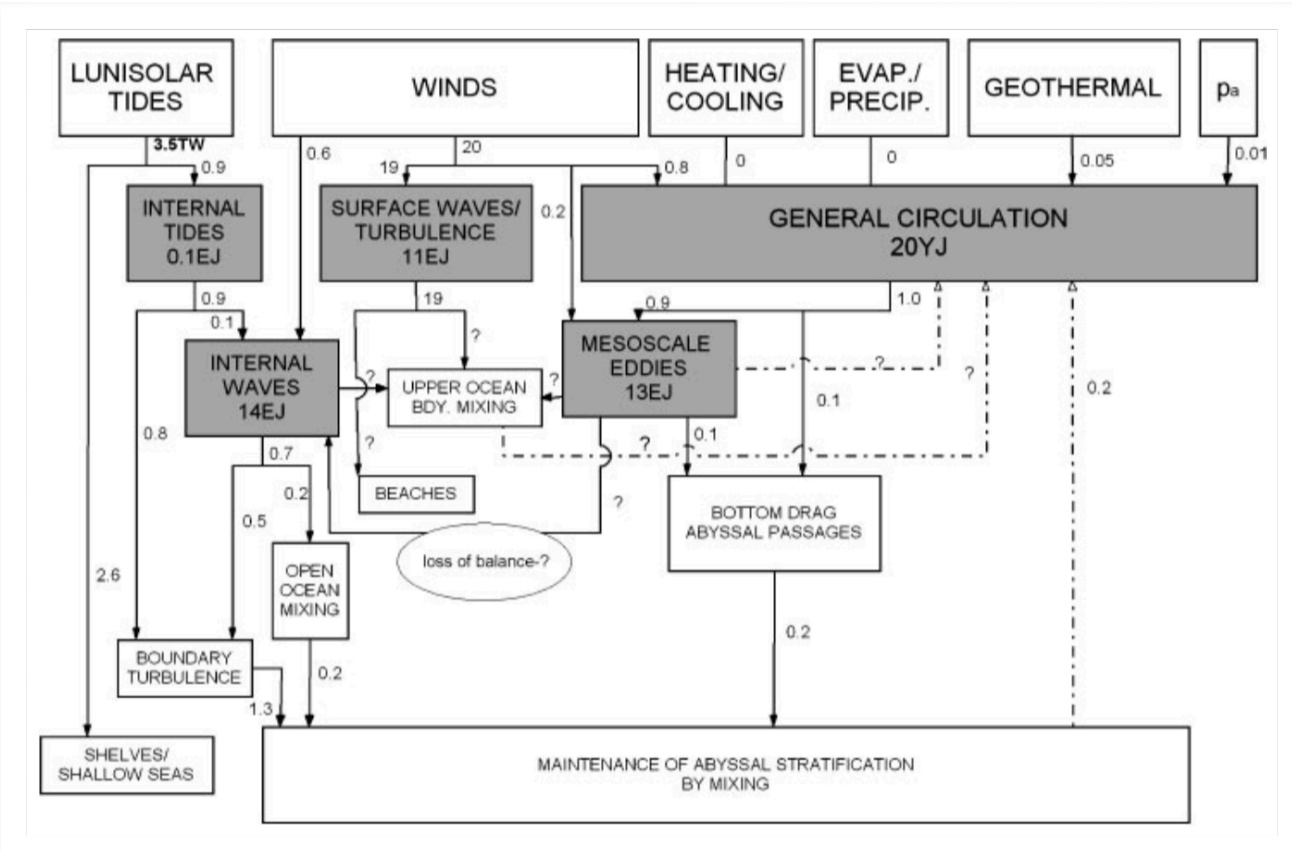
Dissipation and mixing



Turbulent mixing in the ocean regulates:

- the upwelling rate of deep waters and impacts the deep branch of the Meridional Overturning Circulation (MOC)
- exchanges between water masses, i.e., heat, salt, carbon, nutrients…

Dissipation and mixing



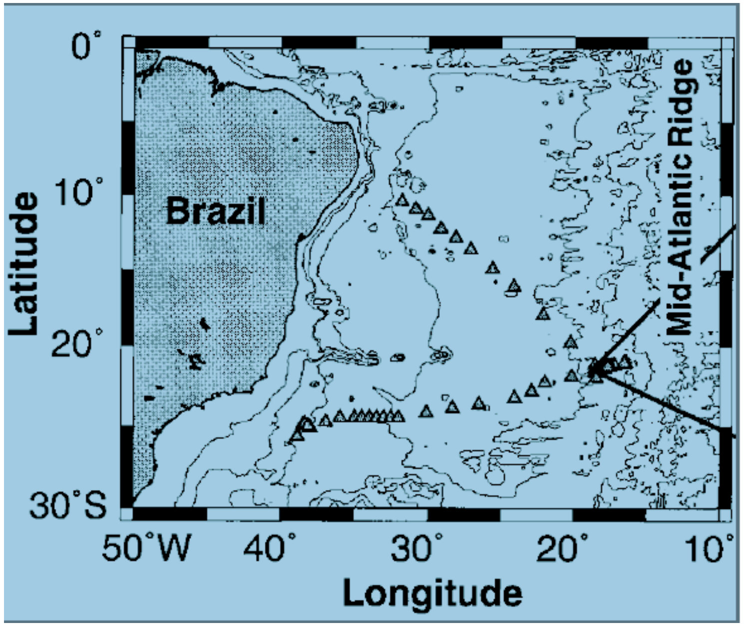
<— forcing

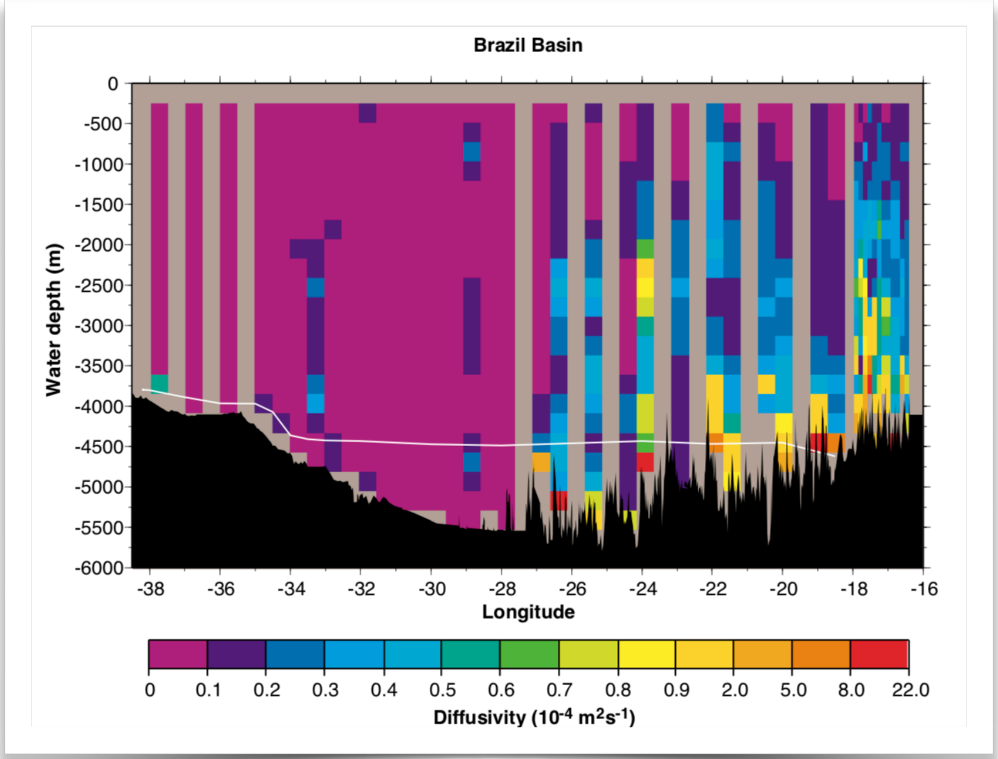
<— oceanic motions directly forced

<— oceanic motions resulting from instabilities + directly forced e.g., internal waves

[Wunsch and Ferrari 2004]

Dissipation and mixing

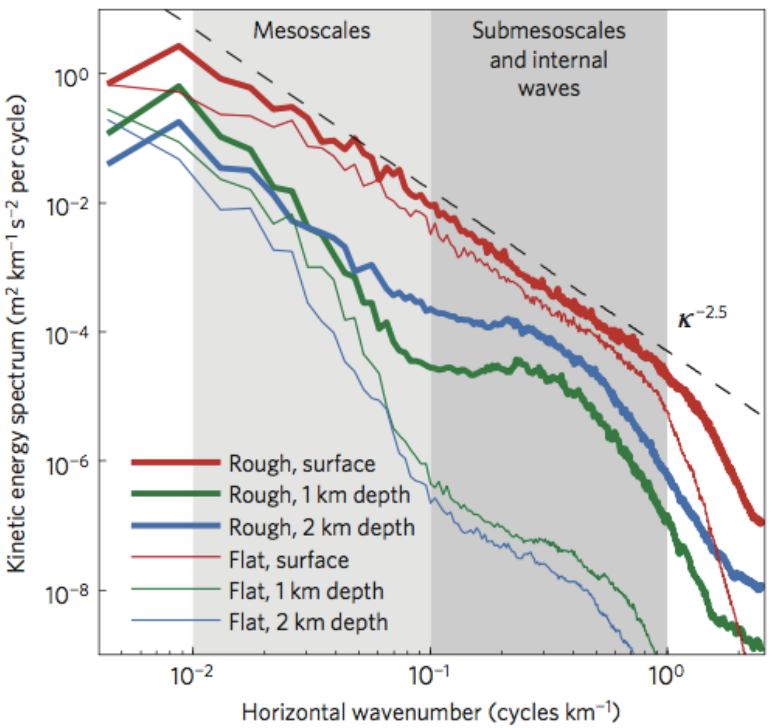


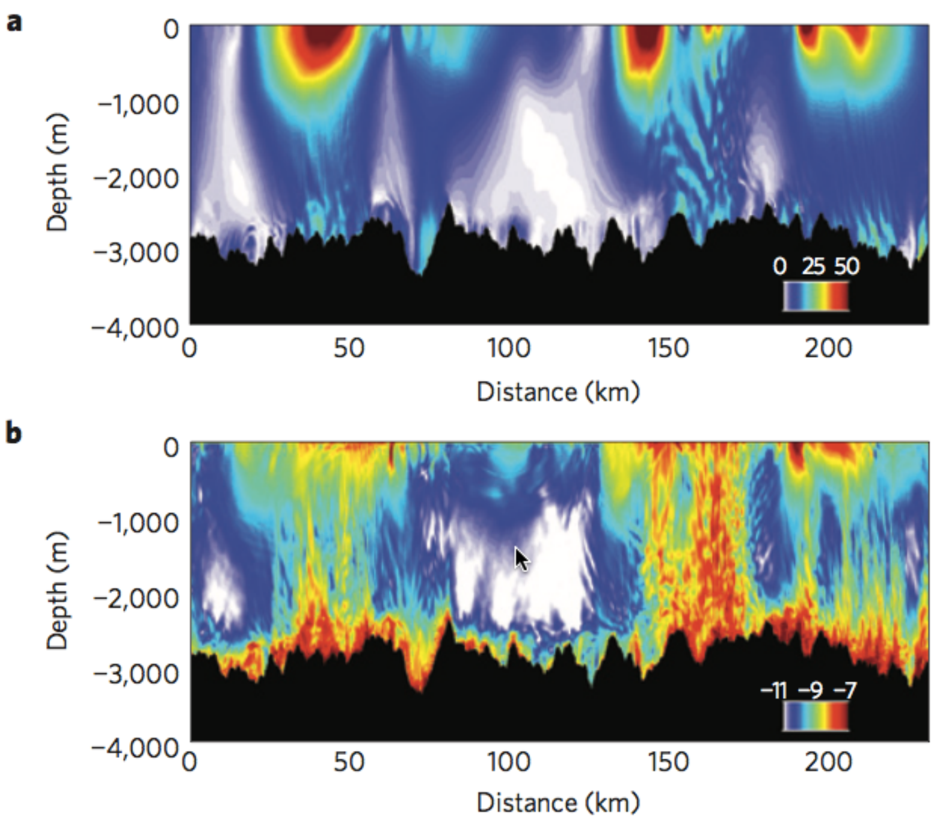


Vertical section of diapycnal diffusivity (m2/s) in the Brazil Basin (from Polzin et al., 1997). Mixing, believed to be sustained by upward radiating waves, is observed to be enhanced in the bottom O(1km) over the rough topography of the MAR.

Polzin et al., 1997

Dissipation and mixing



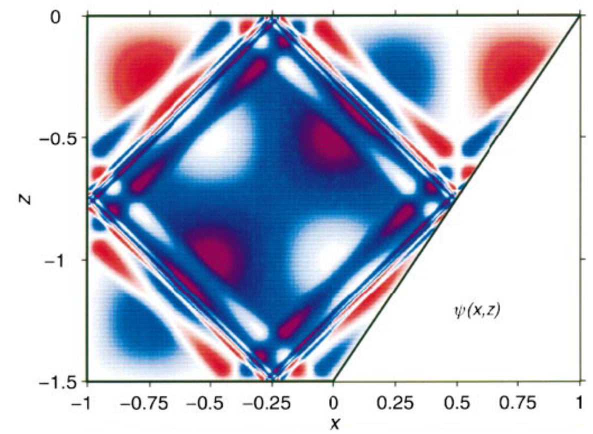


Kinetic energy dissipation is enhanced close to the bottom and in the interior due to breaking internal waves.

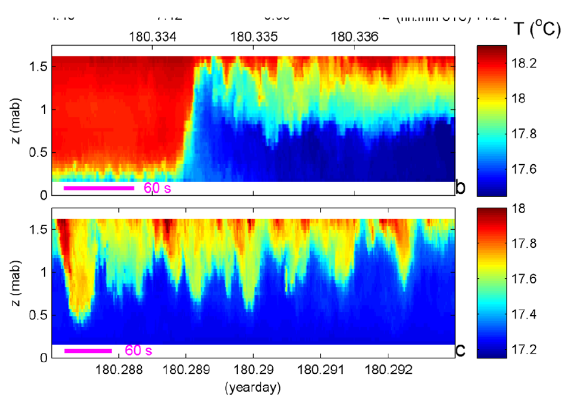
Nikurashin et al., 2013

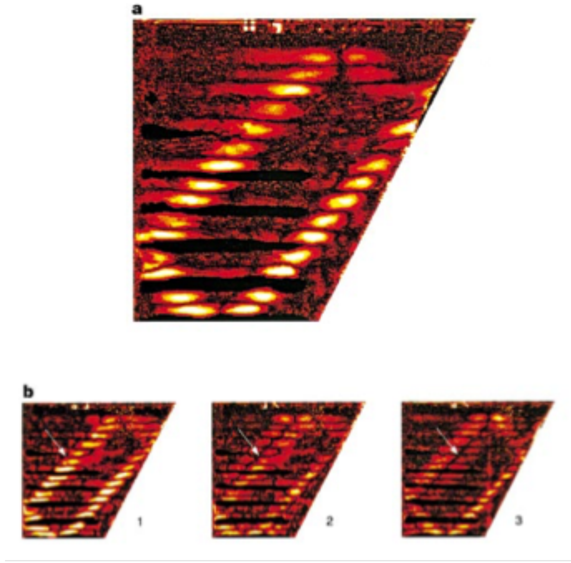
Observation of internal waves

- Satellite observations
- In-situ measurements
- Laboratory experiments



Maas et al, Nature, 1997





Satellite observations

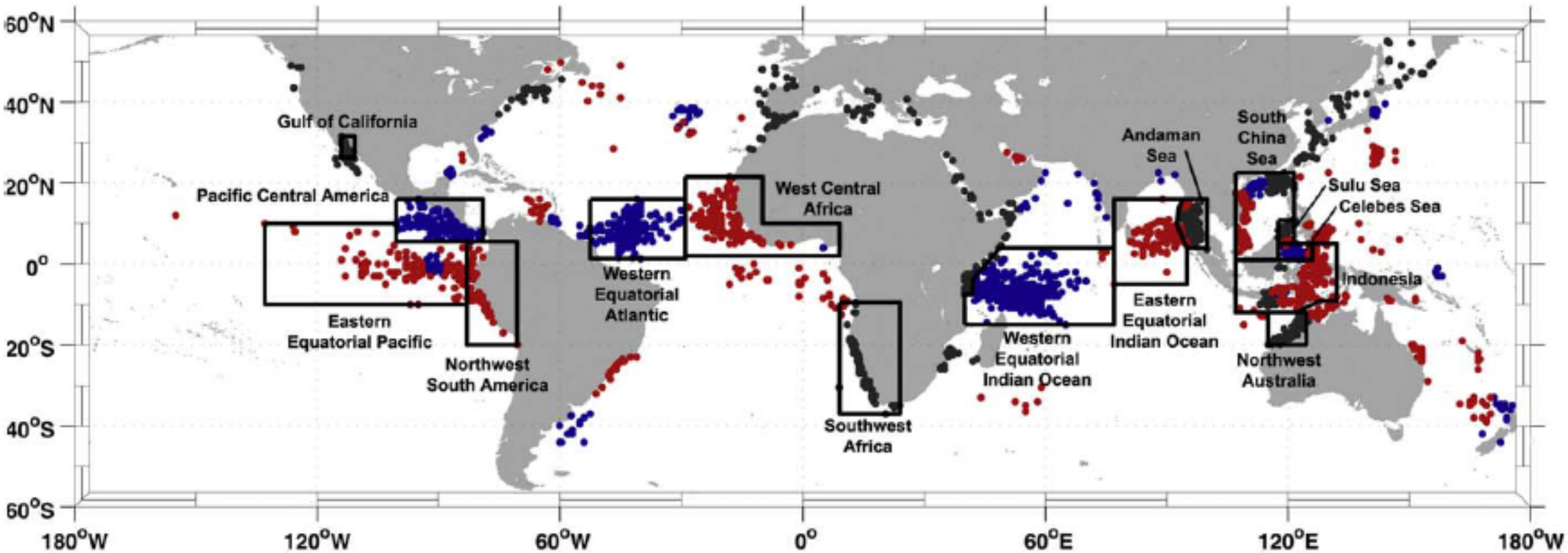
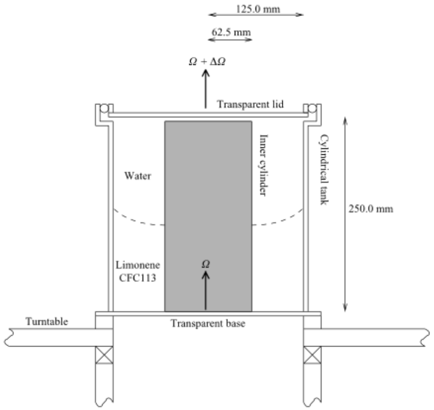
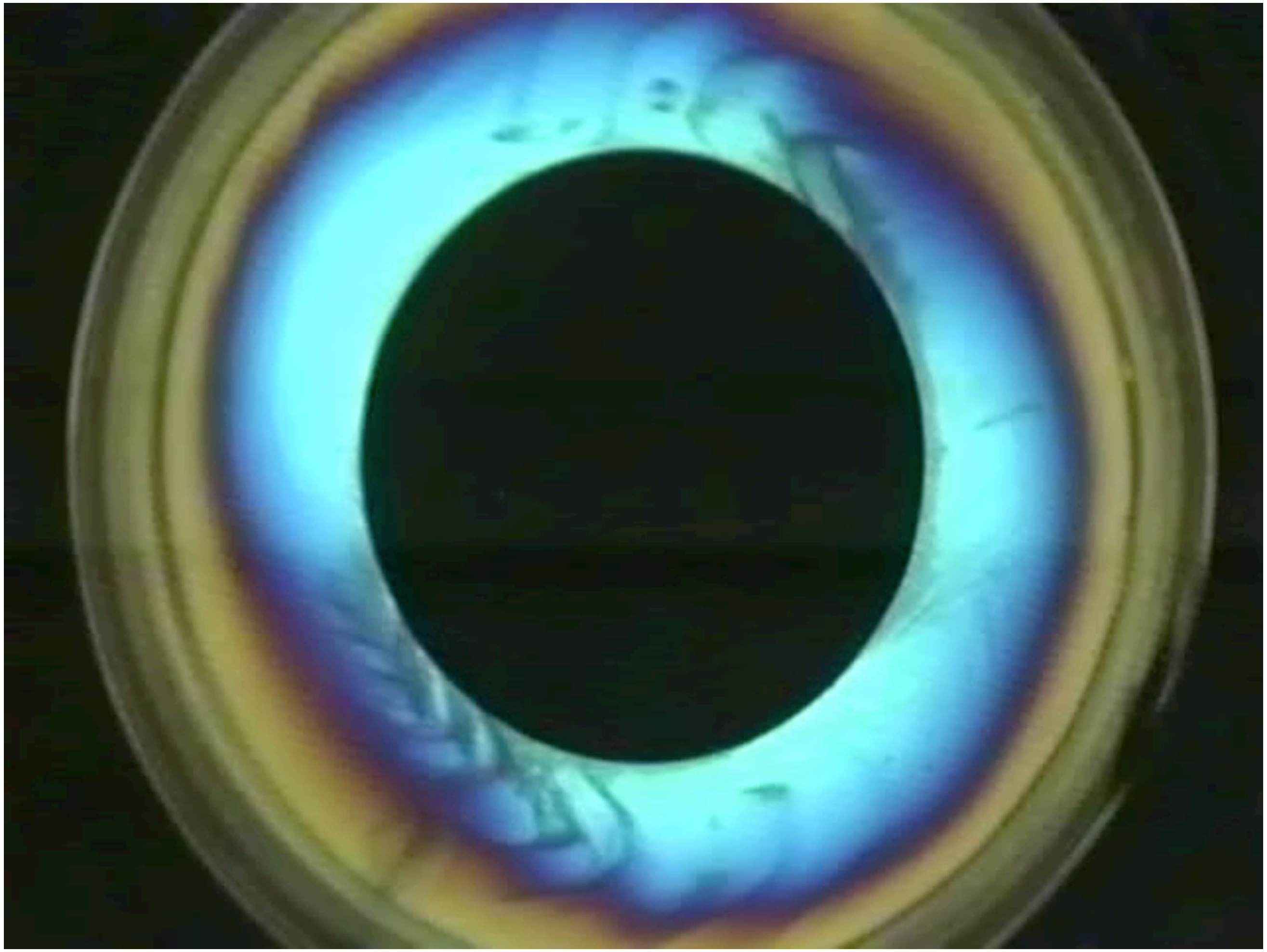


Figure 1. Location of internal waves observed in MODIS imagery from August 2002 through May 2004 along with the geographic boundary for the 15 regions listed in Table 1. The survey identified a total number of 3581 wave occurrences which combine to create 2774 distinct region, area, and date occurrences. Well-known occurrence sites are shown in gray, new areas of activity are shown in red, and areas of geographically expanded activity are shown in blue.

Laboratory experiments





P.D. Williams, P.L. Read, and T. W. N. Haine, *Spontaneous generation and impact* of *inertia-gravity waves in a stratified*, *two-layer shear flow*,. Geophys. Res. Lett., 30(24), 2255

In-situ observations

Mooring for real-time observation of internal waves

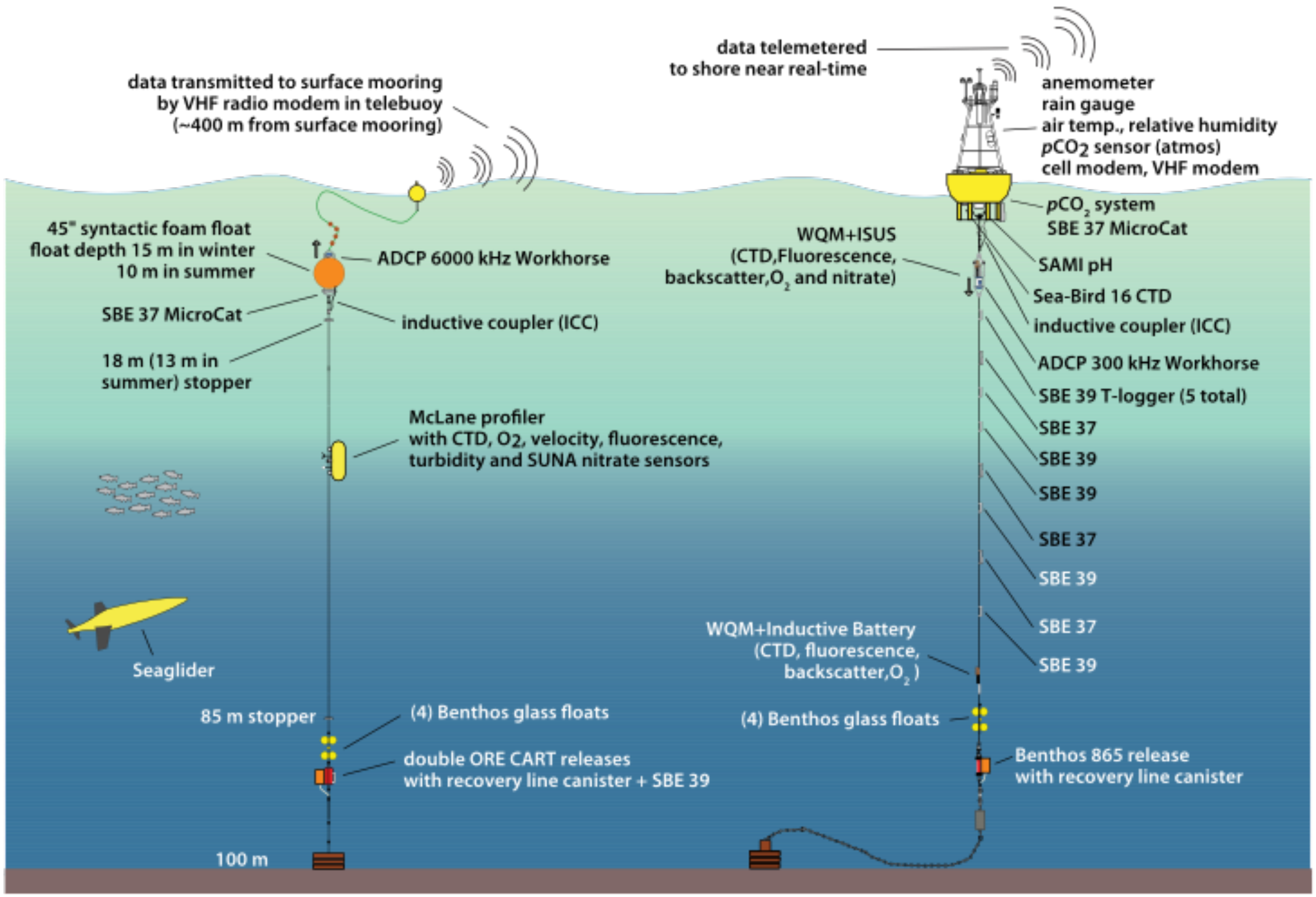


Figure 2. Schematic of the NEMO system showing the surface mooring (right), the subsurface profiling mooring (left), and the glider.

In-situ observations

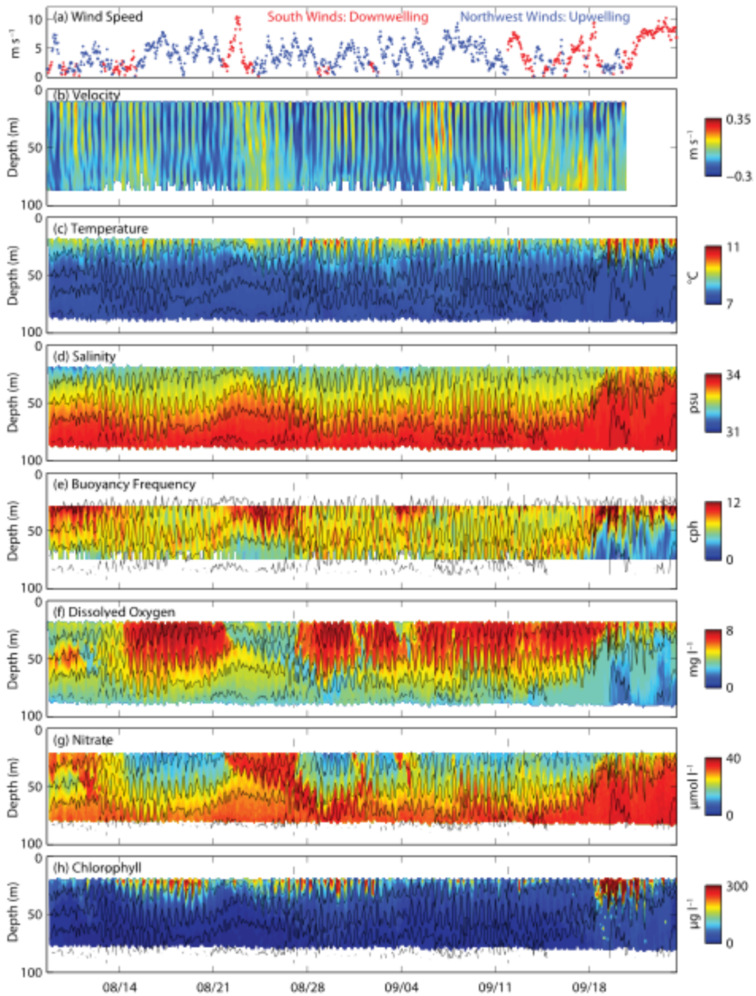


Figure 4. Time series of data from the subsurface mooring, corresponding to the last 46 days of the period plotted in the previous figure. Panels are wind speed colored by (a) direction as in Figure 3, (b) velocity toward 315° true, (c) temperature, (d) salinity, (e) buoyancy frequency, (f) dissolved oxygen, (g) nitrate, and (h) chlorophyll. Isopycnals whose mean spacing is 10 m are over-plotted in each panel in black.

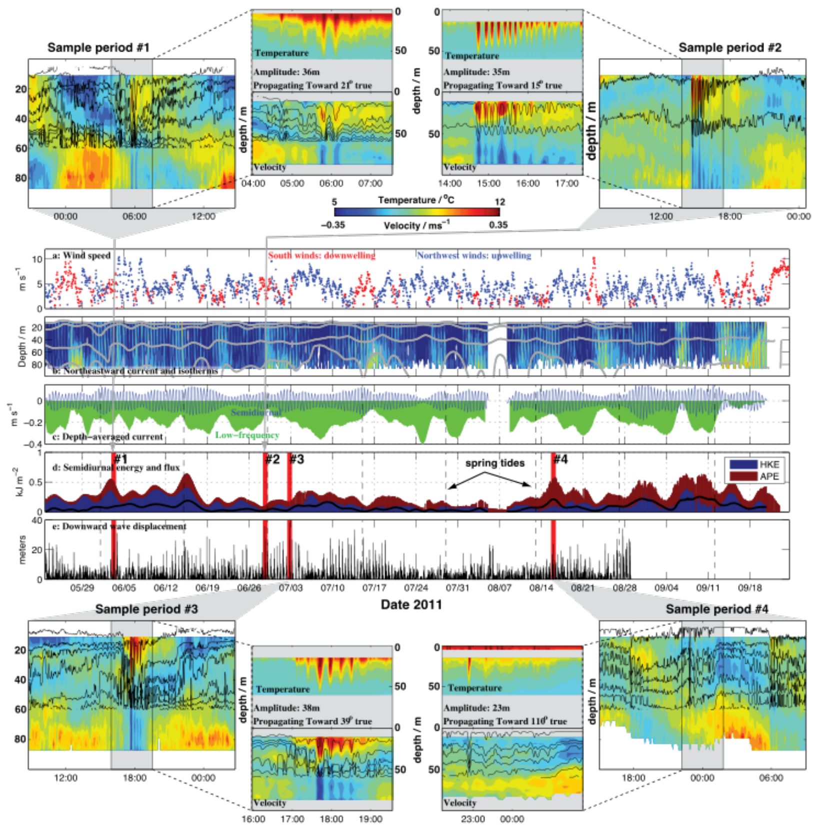
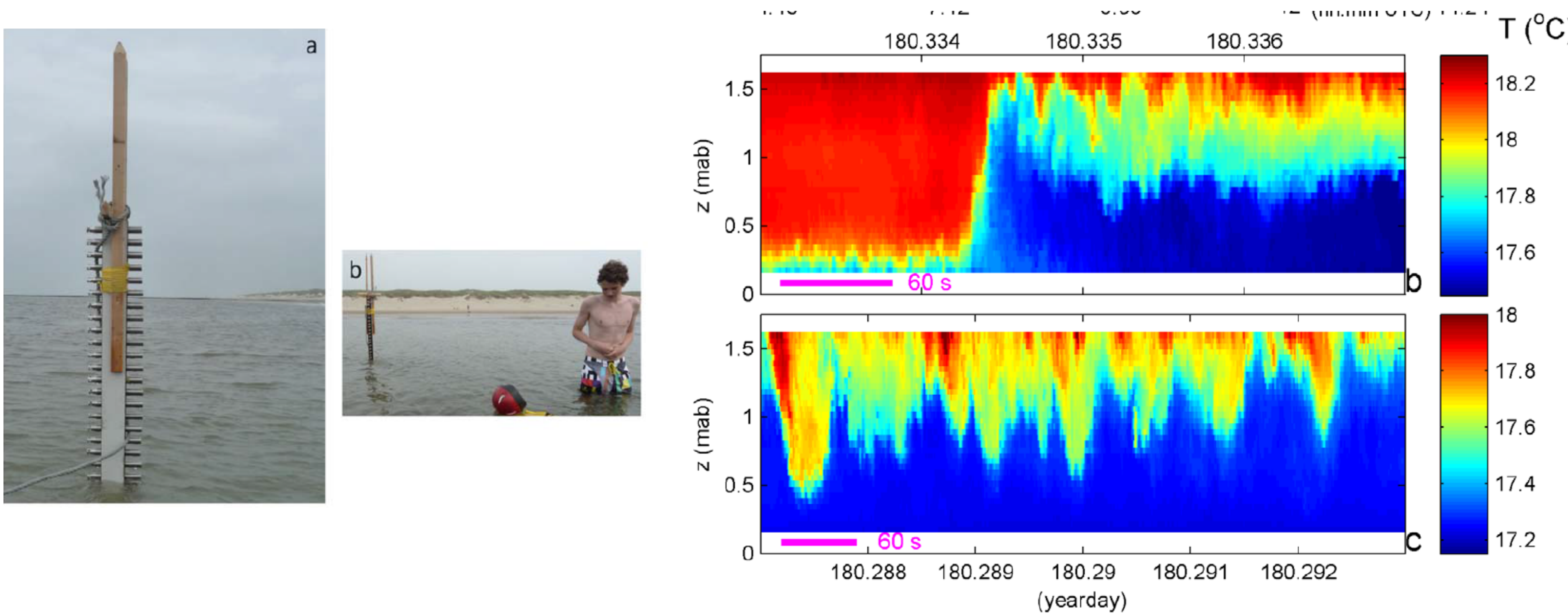


Figure 3. A 2011 time series from the surface mooring, with outer insets showing zoom-ins of 18-hour periods around the four times marked in (d) to demonstrate the variability in the internal tide. Inner insets are zoomed in further on the period indicated in each outer inset to illustrate the temperature (top) and baroclinic northward velocity (bottom) of the nonlinear internal waves. (a) Wind speed, colored by direction (south = red, northwest = blue). (b) Velocity toward 315° true (northwest), with blue colors indicating flow to the southeast. The 8°, 9°, and 10°C isotherms are contoured in gray. (c) Depth-averaged velocity towards 315° true (northwest) of the low-frequency flow (green) and the semidiurnal tidal band (blue). (d) Mode-1 semidiurnal tide: Horizontal kinetic energy (HKE) and available potential energy (APE) are plotted as stacked histograms, with their sum indicating total energy. Energy flux magnitude in kW m^{-1}, which is always toward the north-northeast, is plotted in black. The times of the insets are indicated in red in (d,e). Black dashed lines in (c–e) show full and new moons, which should correspond with spring tides or maximal semidiurnal tidal forcing. (e) Amplitude of the nonlinear internal waves computed by tracking the depth of the isotherm normally at 15 m depth.

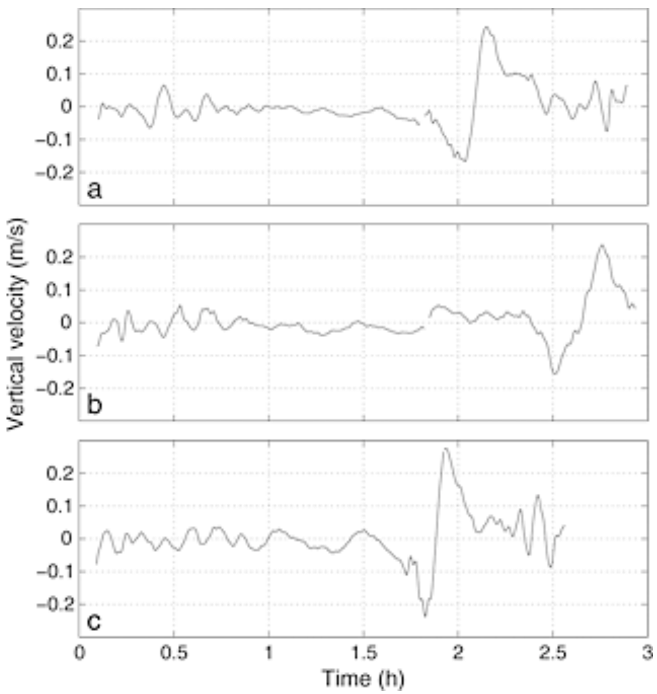
Alford, M.H., J.B. Mickett, S. Zhang, P. MacCready, Z. Zhao, and J. Newton. **2012.** Internal waves on the Washington continental shelf. *Oceanography*. 25(2):66–79.

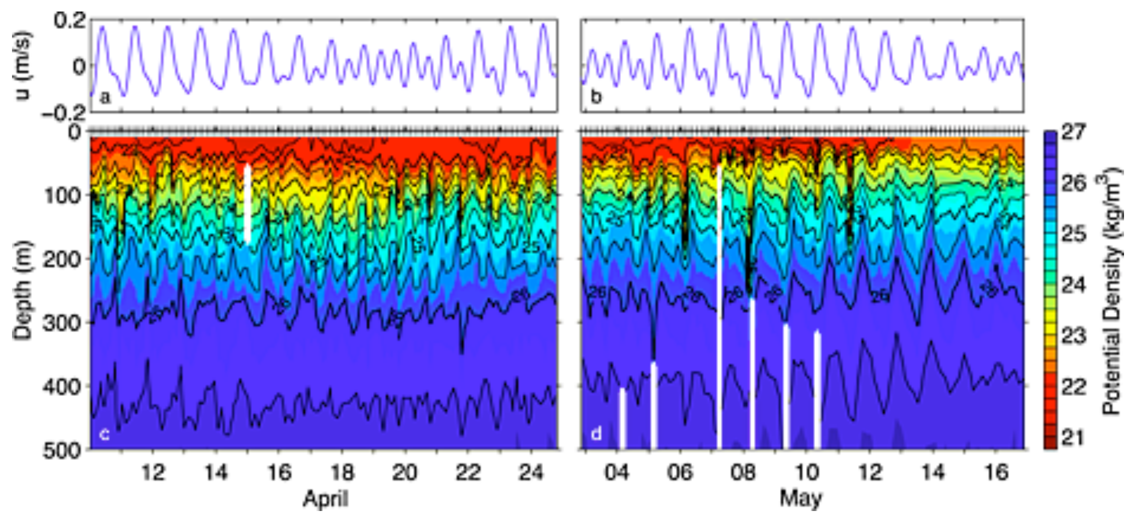
In-situ observations



van Haren H, Gostiaux L, Laan M, van Haren M, van Haren E, Gerringa LJA (**2012**) Internal Wave Turbulence Near a Texel Beach. *PLoS ONE* 7(3): e32535. doi:10.1371/journal.pone.0032535

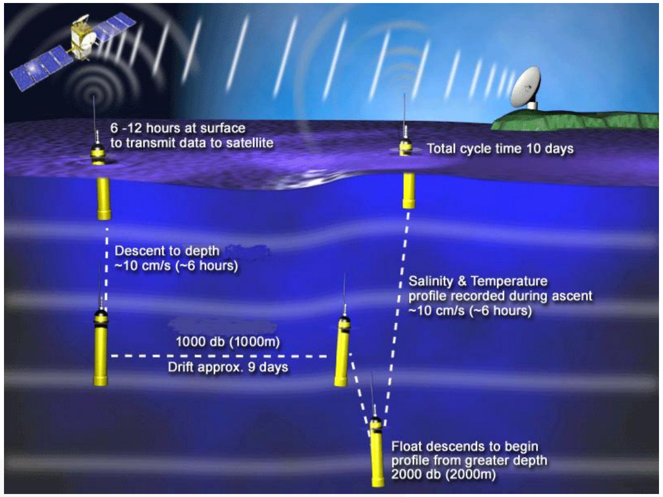
gliders





Rudnick, DL, Johnston TMS, Sherman JT. **2013**. High-frequency internal waves near the Luzon Strait observed by underwater gliders. *J. Geophys. Res. Oceans*. 118

ARGO floats



Observations of Internal Gravity Waves by Argo Floats

By: Hennon, Tyler D.; Riser, Stephen C.; Alford, Matthew H.
JOURNAL OF PHYSICAL OCEANOGRAPHY Volume: 44 Issue: 9 Pages: 2370-2386 Published: SEP 2014

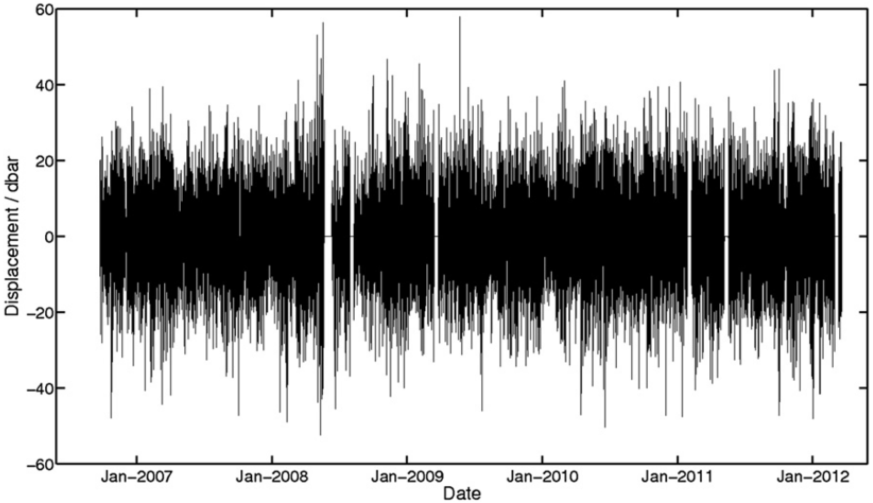


FIG. 4. The synthesized time series of vertical internal wave displacement for float 5135, located in the Indian Ocean. The small gaps of data (zeroes) correspond to park phases where the vertical temperature profile was not linear enough to meet quality control (see section 2c).

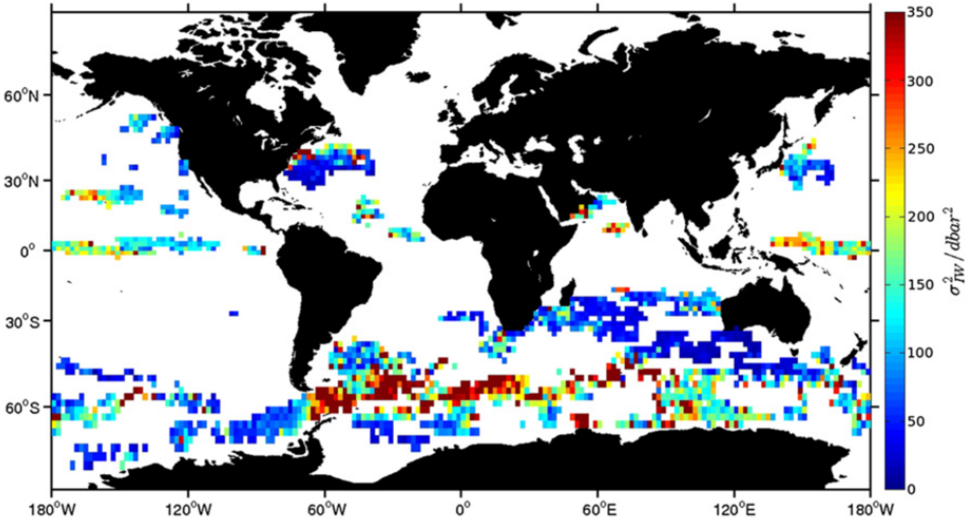


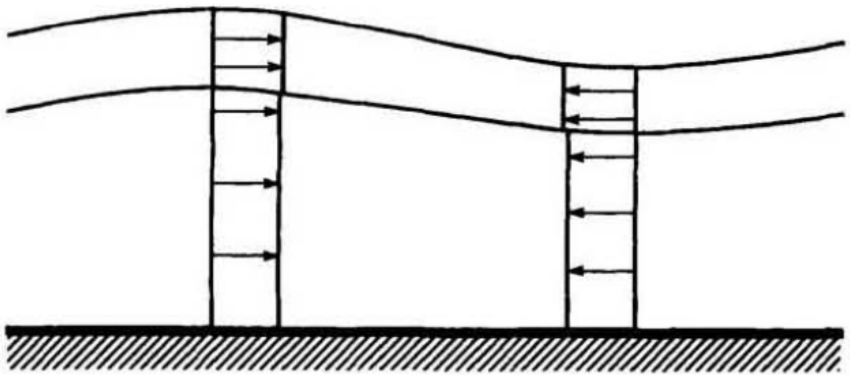
FIG. 11. Estimates of internal gravity wave vertical displacement variance from the 194 profiling floats used in this study. Values are averaged into 2° by 2° bins.

4. Internal waves

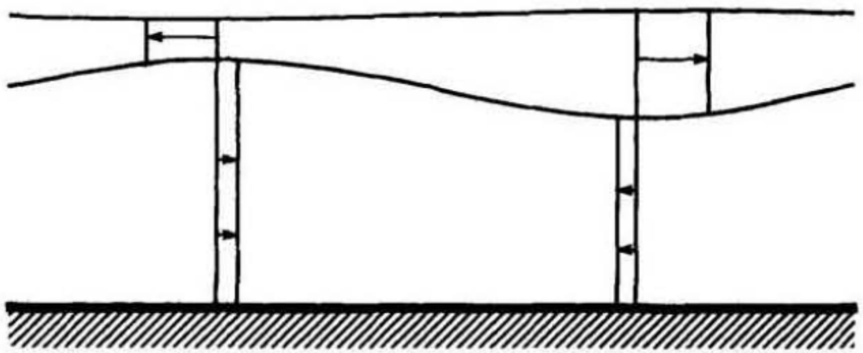
- Generalities about internal waves
- **Internal waves in the two-layer model**
- Internal waves with continuous stratification

- **See notes:**
  - *2-layer SW model*
  - *Solution for baroclinic / barotropic waves*

2-layer approximation



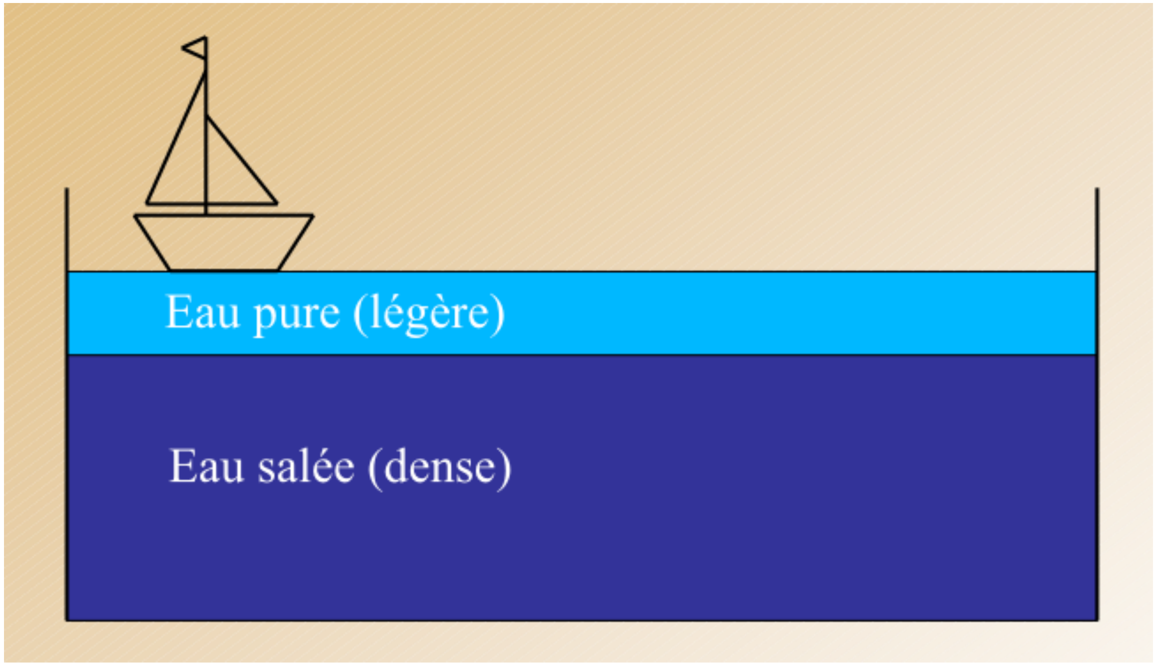
barotropic mode



baroclinic mode

- 2-layers fluid: 1 barotropic mode + 1 baroclinic mode
- N-layers fluid: 1 barotropic mode + N-1 baroclinic mode
- Continuously stratified fluid: 1 barotropic mode + infinity of baroclinic modes

Dead water



"When caught in dead water Fram appeared to be held back, as if by some mysterious force, and she did not always answer the helm. In calm weather, with a light cargo, Fram was capable of 6 to 7 knots. When in dead water she was unable to make 1.5 knots. We made loops in our course, turned sometimes right around, tried all sorts of antics to get clear of it, but to very little purpose."

Fridtjof Nansen (Norwegian Arctic explorer in 1893)

Publications

- **Dead-water:**

  M. J. Mercier, R. Vasseur and T. Dauxois, Resurrecting Dead-Water Phenomenon, Nonlinear Processes in Geophysics 18, pp.193-208 (2011).

- **Internal waves VIP:**
  - Garrett, C. J., and W. Munk, 1975: Space–time scales of internal waves: A progress report. *J. Geophys. Res.*, **80,** 291–297
- **Internal waves reviews**
  - Garrett, C. J., and W. Munk, 1979: Internal waves in the ocean. *Annual Review of Fluid Mechanics*, **11: 339-369**
  - Matthew H. Alford, Jennifer A. MacKinnon, Harper L. Simmons, and Jonathan D. Nash, 2016, Near-Inertial Internal Gravity Waves in the Ocean, *Annual Review of Marine Science*, Vol. 8: 95 -123

4. Internal waves

- Generalities about internal waves
- Internal waves in the two-layer model
- **Internal waves with continuous stratification**

We use the Boussinesq approximation: variations in density ρ' are small compared to the background density ρ_0. The linearised equations are:

$$\frac{\partial \vec{u}}{\partial t} + f\vec{k} \times \vec{u} + \frac{\rho'}{\rho_0} g\vec{k} = -\frac{\overrightarrow{\nabla} p}{\rho_0},$$

$$\overrightarrow{\nabla} \cdot \vec{u} = 0,$$

$$-\frac{g}{\rho_0}\frac{\partial \rho'}{\partial t} + N^2 w = 0.$$

We introduce buoyancy $b = -g\frac{\rho'}{\rho_0}$.

4. Internal Waves

We have a set of 5 equations for 5 variables:

$$u_t - fv = -\frac{p_x}{\rho_0} \quad (1)$$

$$v_t + fu = -\frac{p_y}{\rho_0} \quad (2)$$

$$w_t - b = -\frac{p_z}{\rho_0} \quad (3)$$

$$u_x + v_y + w_z = 0 \quad (4)$$

$$b_t + N^2 w = 0 \quad (5)$$

1. Combine the momentum equations to get rid of p :

$$\partial_z(1) - \partial_x(3): u_{zt} - fv_z - w_{xt} + b_x = 0 \quad (6)$$

$$\partial_z(2) - \partial_y(3): v_{zt} + fu_z - w_{yt} + b_y = 0 \quad (7)$$

$$\partial_y(1) - \partial_x(2):$$

$$u_{yt} - fv_y - v_{xt} - fu_x = 0 \iff (u_y - v_x)_t - f(u_x + v_y) = 0$$

2. Use the continuity equation into the third equation :

$$(u_y - v_x)_t + fw_z = 0 \quad (8)$$

4. Internal Waves

3. Express vorticity and divergence in terms of w :

$\partial_{yt}(7) + \partial_{xt}(6)$: (…)

use (8) use (4) use (5)

$$f(u_y - v_x)_{zt} + (u_x + v_y)_{ztt} - w_{xxtt} - w_{yytt} + b_{xxt} + b_{yyt} = 0$$

After some rearrangement, we get : $$(\nabla^2 w)_{tt} + f^2 w_{zz} + N^2 \nabla_h^2 w = 0$$
(9)

4. Internal Waves

Two methods can be used to tackle Equation (9) :

—> **method of characteristics**

—> **method of vertical modes**

Method of characteristics

Assumptions:

- stratification is constant, $N = cst$
- the domain is infinite, that is, there is no boundary effect
- waves are sinusoidal in time and propagate in the $x - z$ plane (no variation in the y direction): $w(x, y, z, t) = \hat{w}(x, z)\exp(i\omega t)$

The equation for w becomes: $\hat{w}_{xx} - \boxed{\frac{\omega^2 - f^2}{N^2 - \omega^2}}\hat{w}_{zz} = 0$ (10)

The method of characteristics consists in changing the variables x, z into ξ_+, ξ_- :

$\xi_\pm = \mu_\pm x - z$ with $\mu_\pm = \pm\left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2}$.

Method of characteristics

Now we use $\hat{w}(x,z) = \bar{w}(\xi_+, \xi_-)$ with the properties:

$$\frac{\partial \hat{w}}{\partial x} = \frac{\partial \bar{w}}{\partial \xi_+}\frac{\partial \xi_+}{\partial x} + \frac{\partial \bar{w}}{\partial \xi_-}\frac{\partial \xi_-}{\partial x}$$, with $\frac{\partial \xi_+}{\partial x} = \mu_+$ and $\frac{\partial \xi_-}{\partial x} = \mu_-$

.

After some reorganising,

$$\frac{\partial^2 \hat{w}}{\partial x^2} = \mu_+^2 \left(\frac{\partial^2 \bar{w}}{\partial \xi_+^2} + \frac{\partial^2 \bar{w}}{\partial \xi_-^2} - 2\frac{\partial^2 \bar{w}}{\partial \xi_+ \partial \xi_-} \right)$$ and

$$\frac{\partial^2 \hat{w}}{\partial z^2} = \frac{\partial^2 \bar{w}}{\partial \xi_+^2} + \frac{\partial^2 \bar{w}}{\partial \xi_-^2} + 2\frac{\partial^2 \bar{w}}{\partial \xi_+ \partial \xi_-}.$$

Method of characteristics

The equation for $\hat{w}$ reduces to:

$$\hat{w}_{xx} - \mu_+^2 \hat{w}_{zz} = \ldots = -4\mu_+^2 \frac{\partial^2 \bar{w}}{\partial \xi_+ \partial \xi_-} = 0.$$ That is the new PDE to solve.

Any function of the form $\bar{w}(\xi_+, \xi_-) = F(\xi_+) + G(\xi_-)$ is a solution of the PDE,

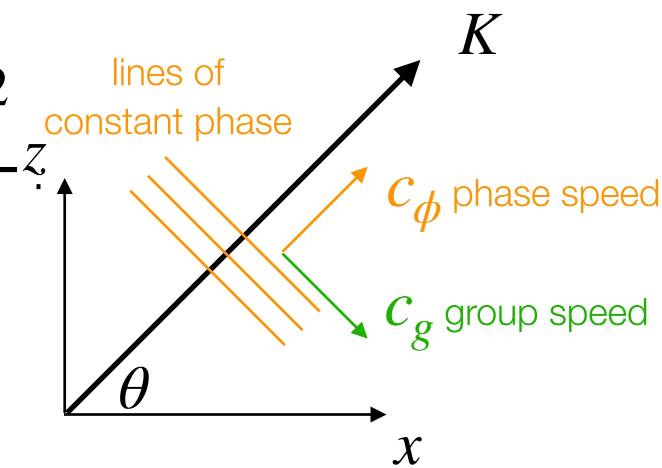
with F and G arbitrary functions.

The general solution is thus $\hat{w}(x, z) = F(\mu_+ x - z) + G(\mu_- x - z)$.

Method of characteristics

To find the dispersion relation, we set $w(x, y, z, t) = \hat{w}(x, y, z)\exp(i\omega t)$ with $\hat{w} = w_0 \exp(i(kx + mz))$.

From Equation (10), $$\omega^2 = \frac{N^2k^2 + f^2m^2}{k^2 + m^2}$$



Using polar coordinates, the wave vector is

$$\overrightarrow{K} = (k, m) = K(\cos\theta, \sin\theta),$$

we have $$\omega^2 = N^2\cos^2\theta + f^2\sin^2\theta$$

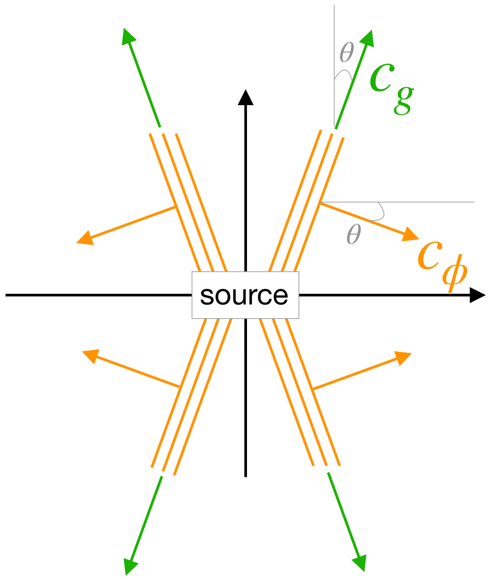
$\longrightarrow$ The wave frequency depends only on the stratification, the rotation and the direction of the wave. Conversely, the direction of the wave can be inferred from the stratification, rotation and the frequency of the wave.

Method of characteristics

The two extreme cases are:

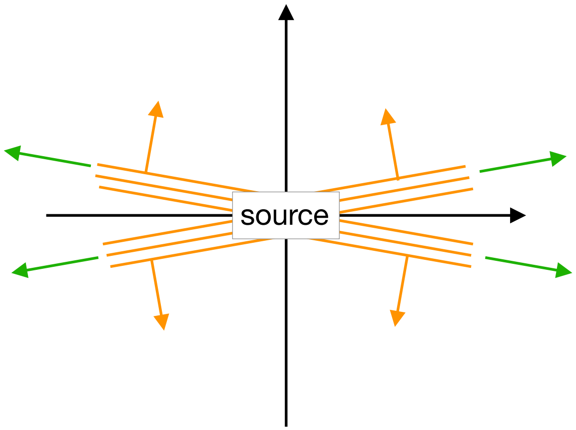
- $\omega \to N \Leftrightarrow \theta \to 0$

  rapidly oscillating waves propagate almost vertically

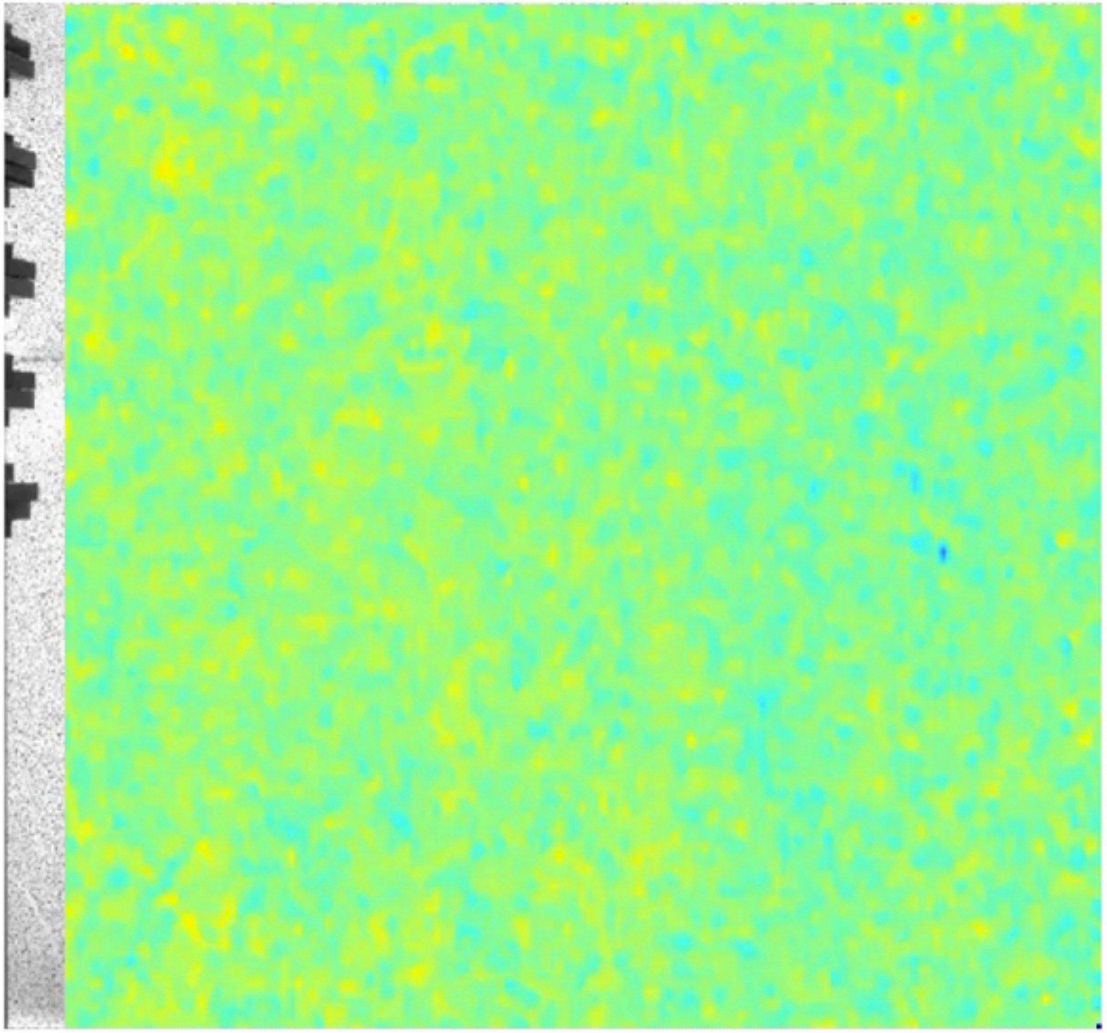


- $\omega \to f \Leftrightarrow \theta \to \dfrac{\pi}{2}$

  "near-inertial" waves propagate almost horizontally

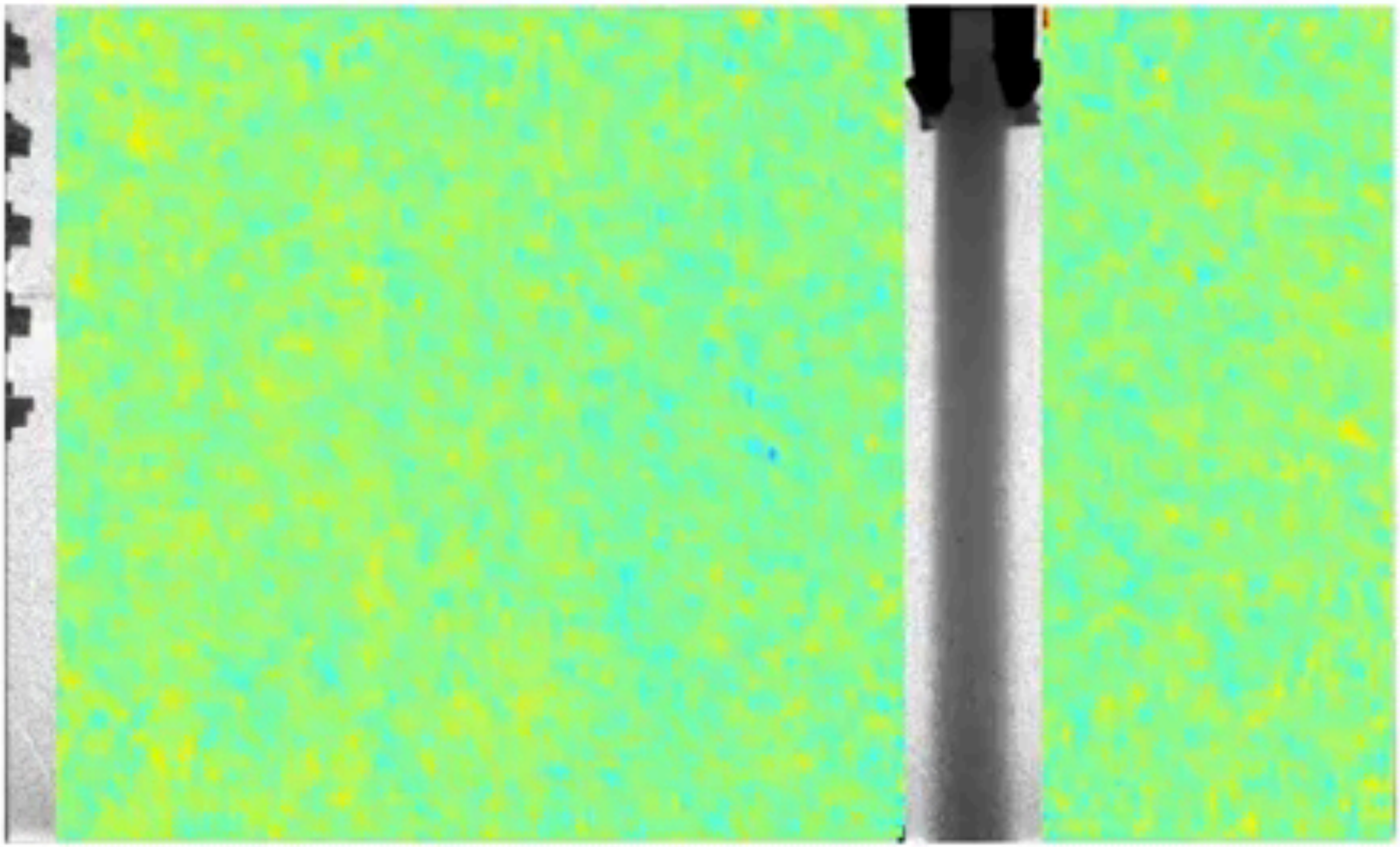


4. Internal Waves



[(c) E. Horne]

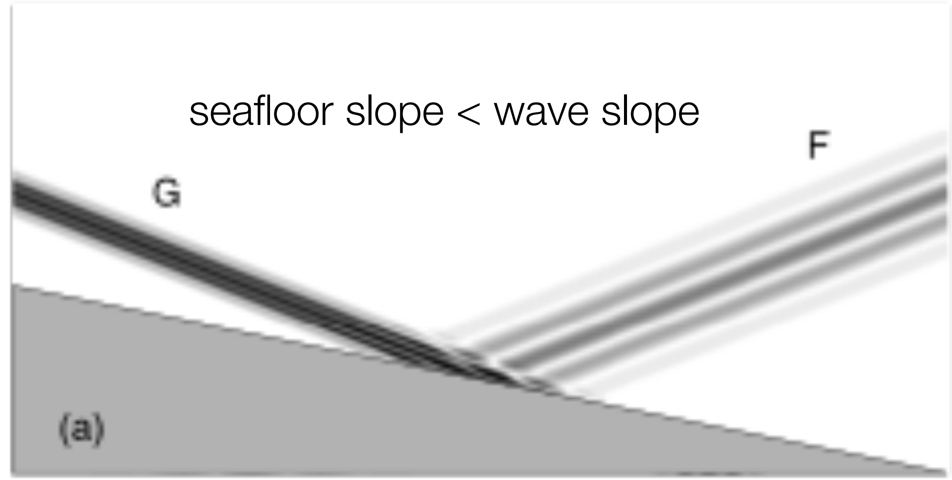
4. Internal Waves

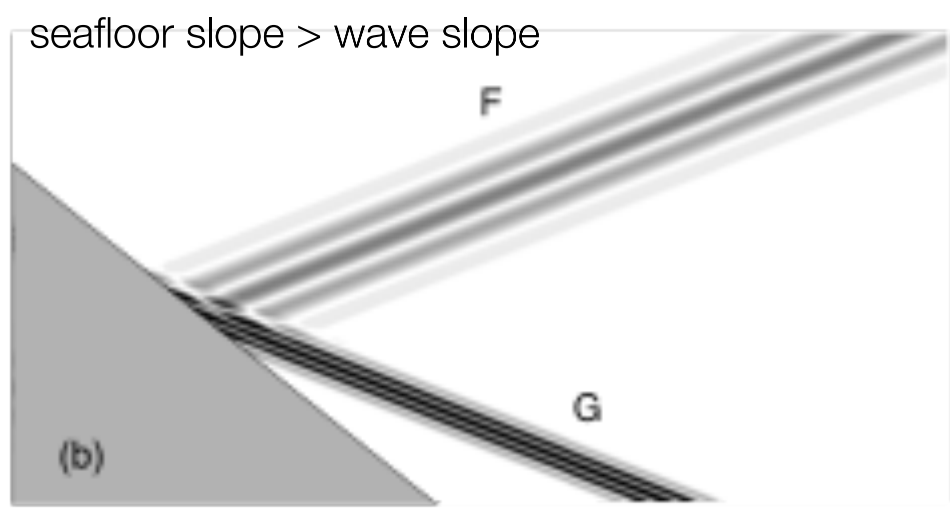


[(c) E. Horne]

Method of characteristics

When a wave impinges on a seafloor slope, its frequency ω is conserved, hence its propagating slope is conserved.

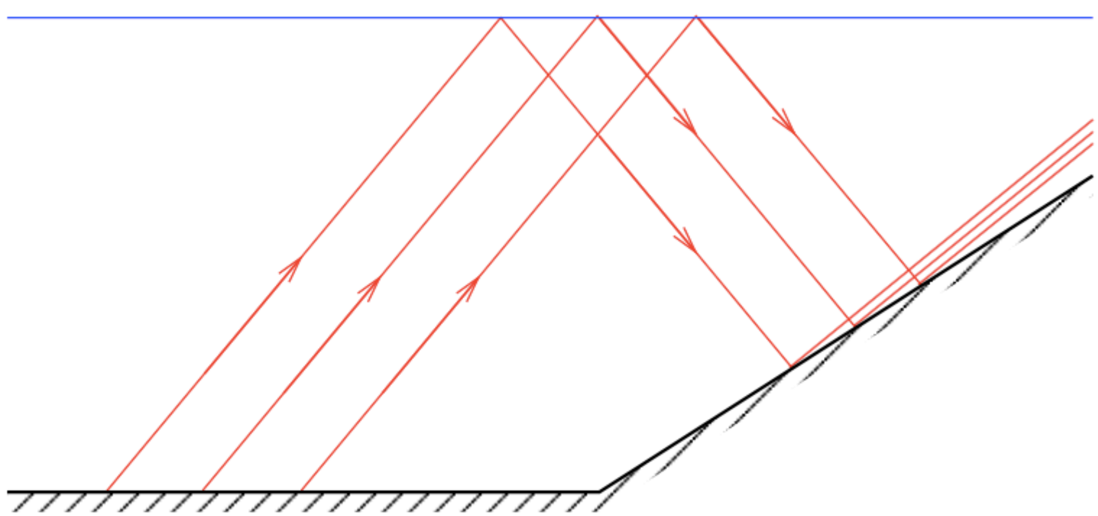




Method of characteristics

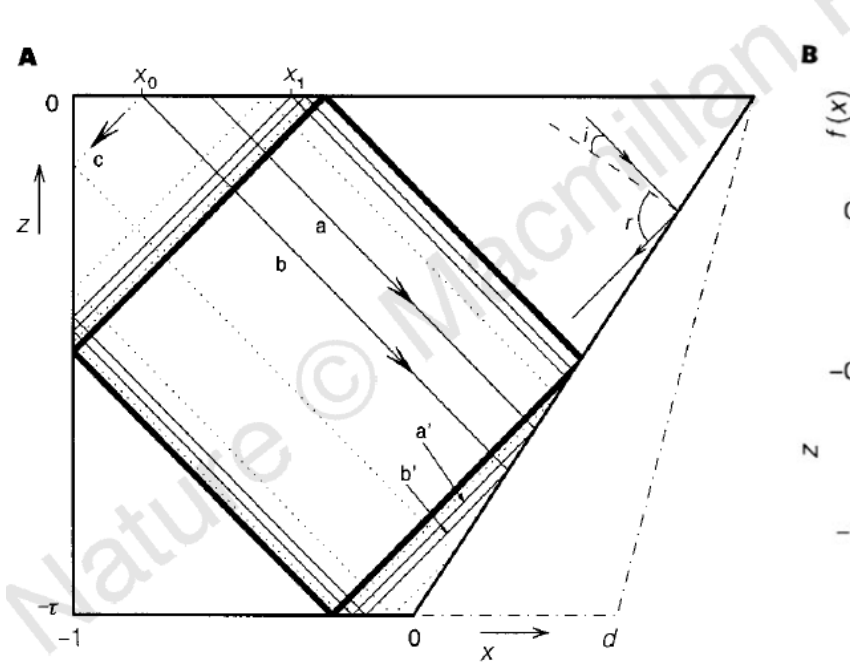
When a wave impinges on a seafloor slope, its frequency ω is conserved, hence its propagating slope is conserved.

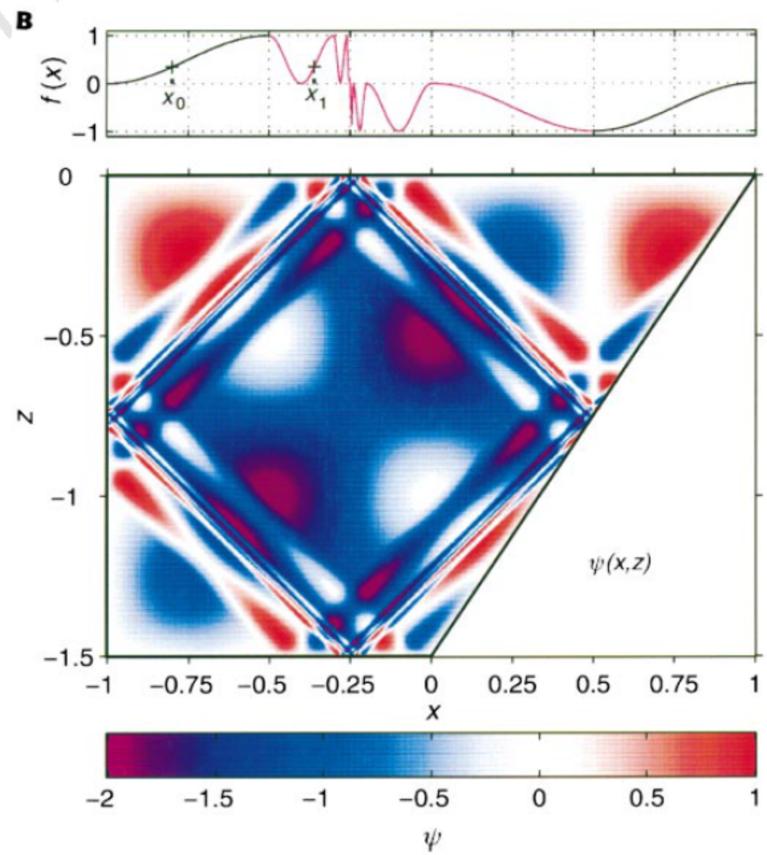
Energy can focus in narrow bands.



Method of characteristics

Specific geometries can work as wave attractors.

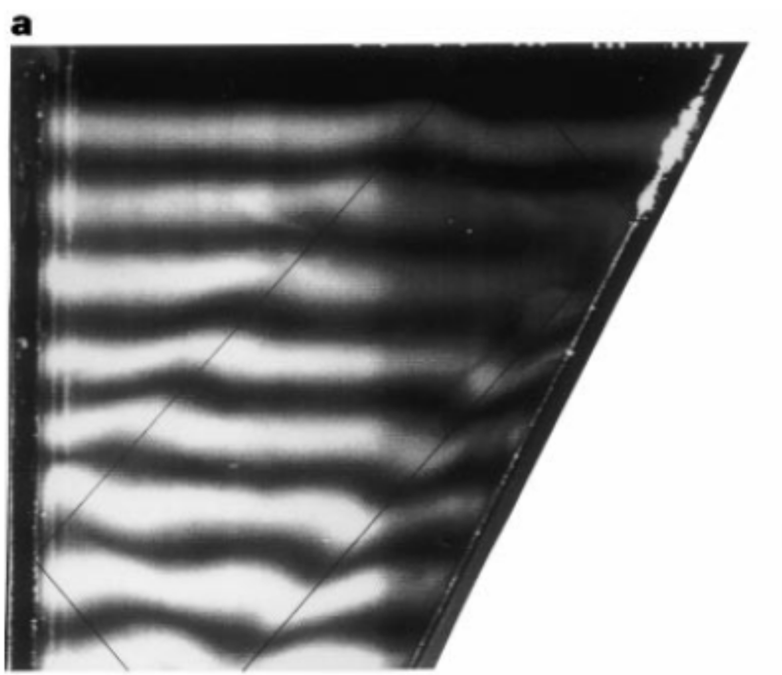


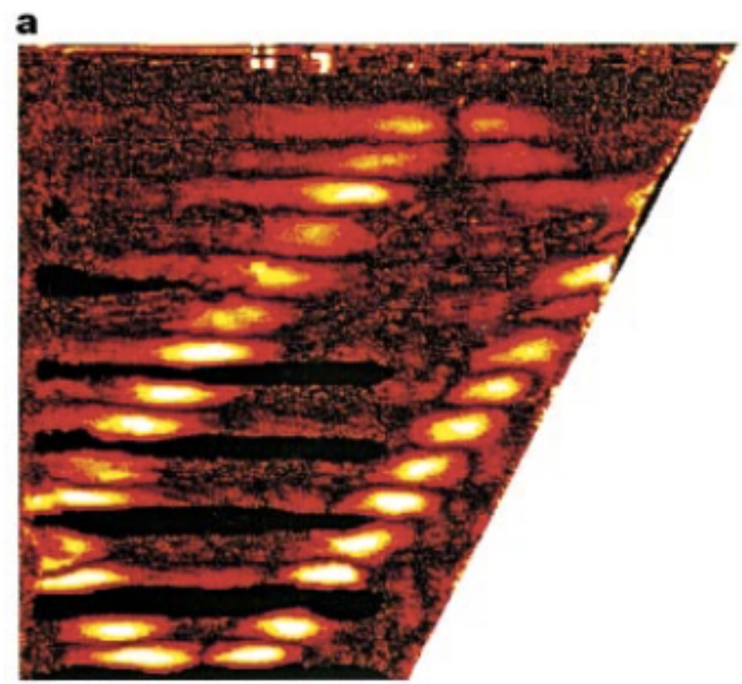


Maas et al. 1997

Method of characteristics

Specific geometries can work as wave attractors.





Maas et al. 1997

Method of characteristics

Main limit of the method of characteristics: no top or bottom boundary condition is specified. The ocean is supposed to be “infinite” ! What happens with a finite-depth ocean ? $\longrightarrow$ **method of modes**

Method of modes

Assumptions:

- $N = N(z)$
- flat sea surface height (*rigid lid*) and flat seafloor

We start back from equation (9): $(\nabla^2 w)_{tt} + f^2 w_{zz} + N^2 \nabla_h^2 w = 0$,

and assume a solution in the form: $w = W(z)\exp(i(\omega t - kx))$, that is, the vertical structure is decoupled from the horizontal structure (+ no variation in y).

The equation for w becomes $W_{zz} + k^2 \dfrac{N^2 - \omega^2}{\omega^2 - f^2} W = 0$, which can be written as:

$$W_{zz} + m^2 W = 0$$ with

Method of modes

If $N(z) = cst$, the solution can be written:

$W(z) = A\cos(mz) + B\sin(mz)$.

The top and bottom boundary conditions are: $w(0) = w(-H) = 0$.

They give: $m = \pm n\dfrac{\pi}{H}$, which translates into the dispersion relation:

$$k_n = \pm\frac{n\pi}{H}\left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2} \qquad \text{or}$$

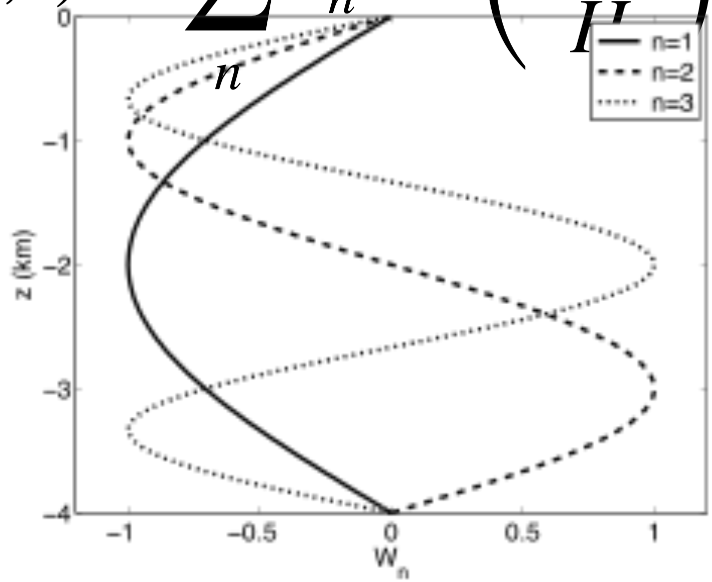
$$\omega^2 = \frac{N^2 k_n^2 + \frac{n^2\pi^2}{H^2} f^2}{k_n^2 + \frac{n^2\pi^2}{H^2}}$$

Method of modes

The general solution for w is the superposition of modes:

$$w(x,z,t) = \sum_n W_n(z) a_n \cos(k_n x - \omega t).$$

If $N = cst$, $w(x,z,t) = \sum_n a_n \sin\left(\frac{n\pi z}{H}\right) \cos(k_n x - \omega t).$



Gerkema & Zimmerman (2008)

Method of modes

From the initial set of equations, we get the vertical structure of u, v, p, b:

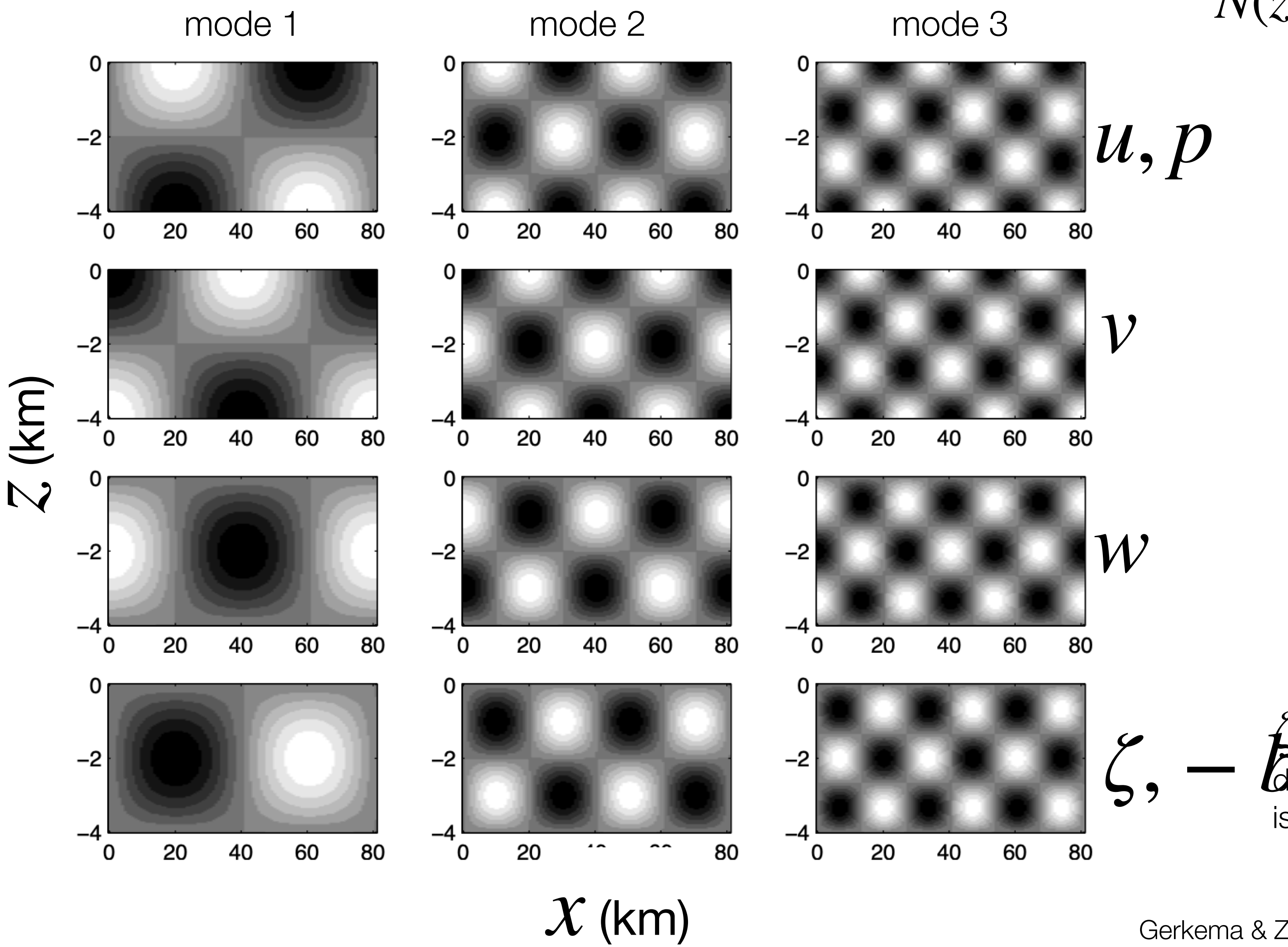
$$U(z) = \frac{i}{k}\frac{\partial W}{\partial z}\ ;\ V(z) = \frac{f}{\omega k}\frac{\partial W}{\partial z}\ ;\ P(z) = i\rho_0 \frac{\omega^2 - f^2}{\omega k^2}\frac{\partial W}{\partial z}$$

$$;\ B(z) = -\frac{iN^2}{\omega}W(z)$$

Gerkema & Zimmerman (2008)

Method of modes

$N(z) = cst$



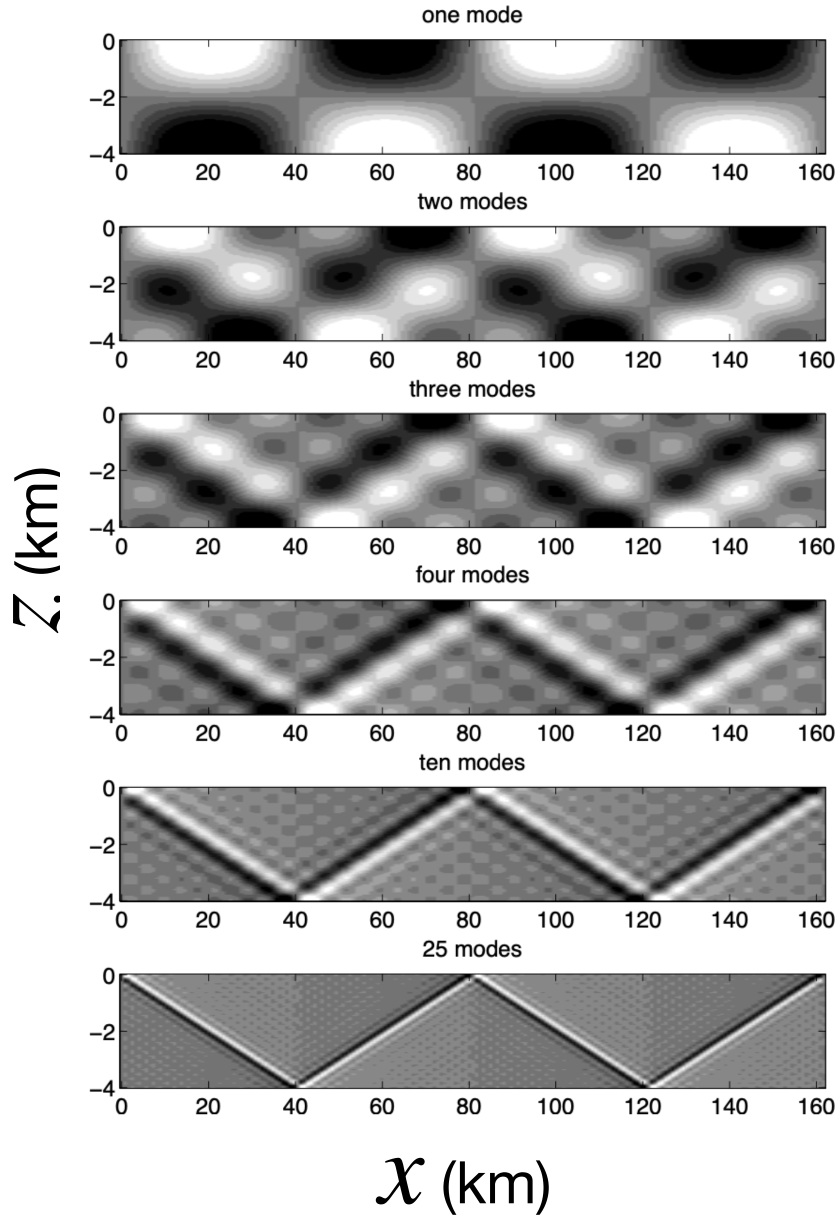
ζ is the vertical displacement of isopycnals.

Gerkema & Zimmerman (2008)

Method of modes

The superposition of modes shows up as beams

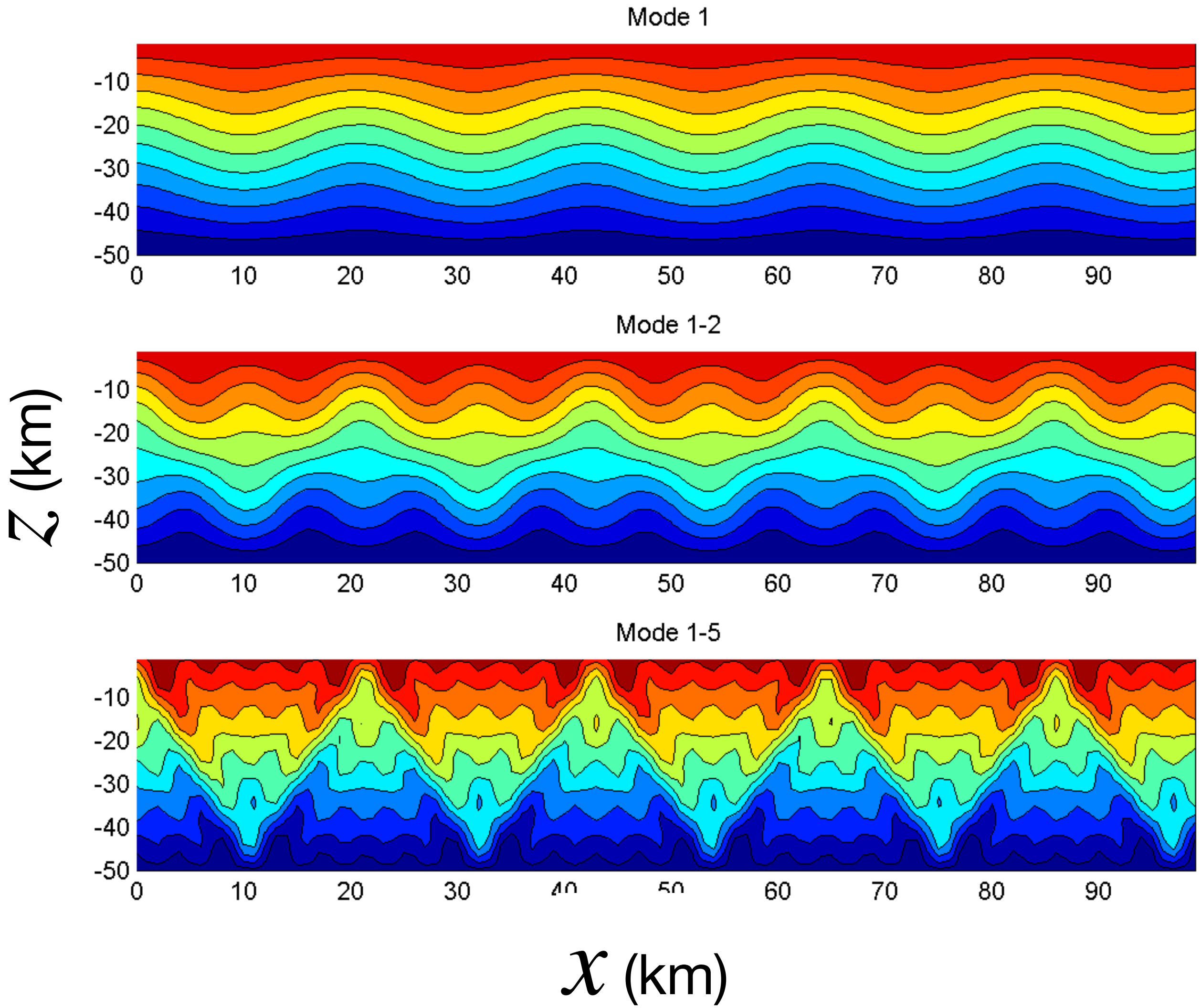
NB: $N(z) = cst$ in the figure



Gerkema & Zimmerman (2008)

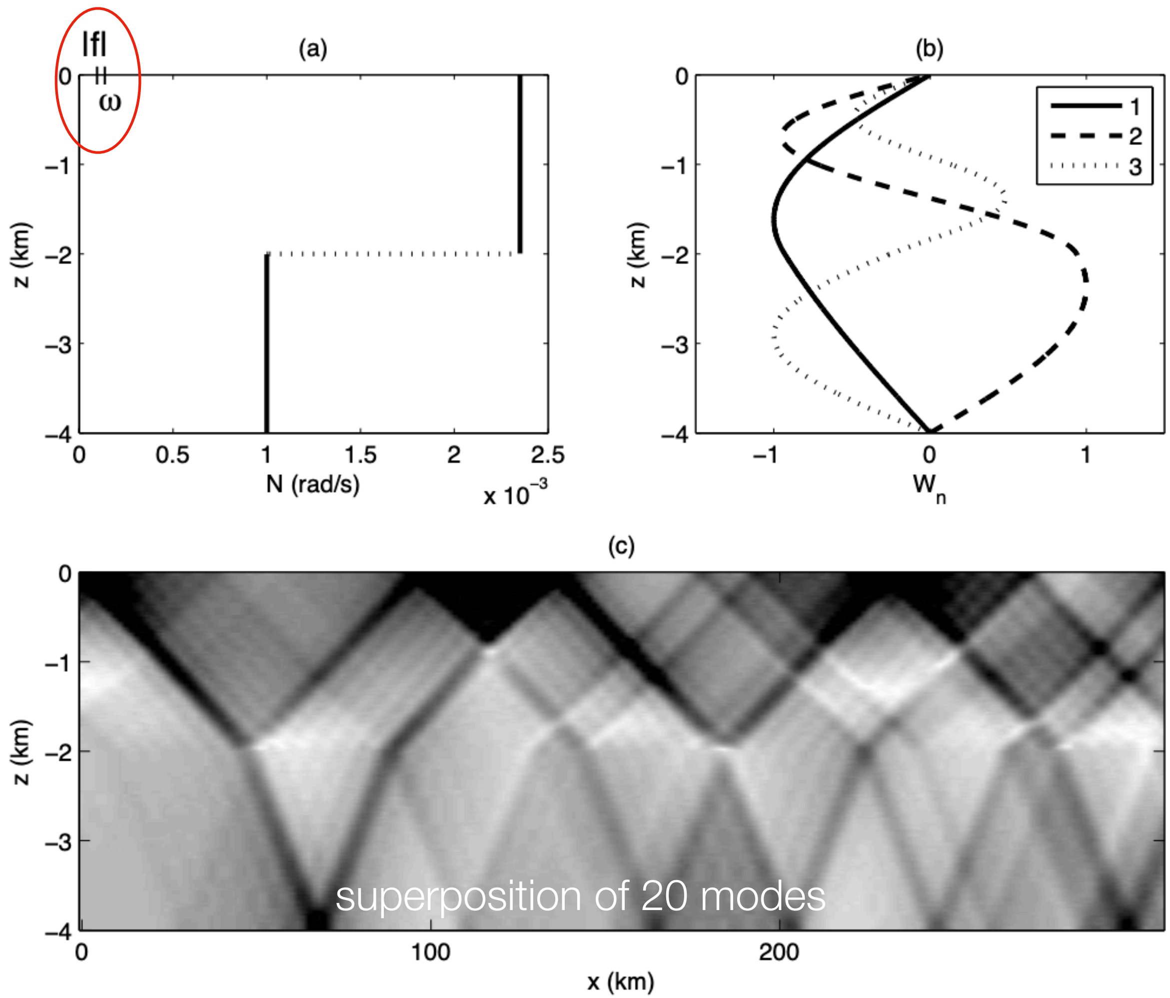
Method of modes

$$N(z) = cst$$



Method of modes

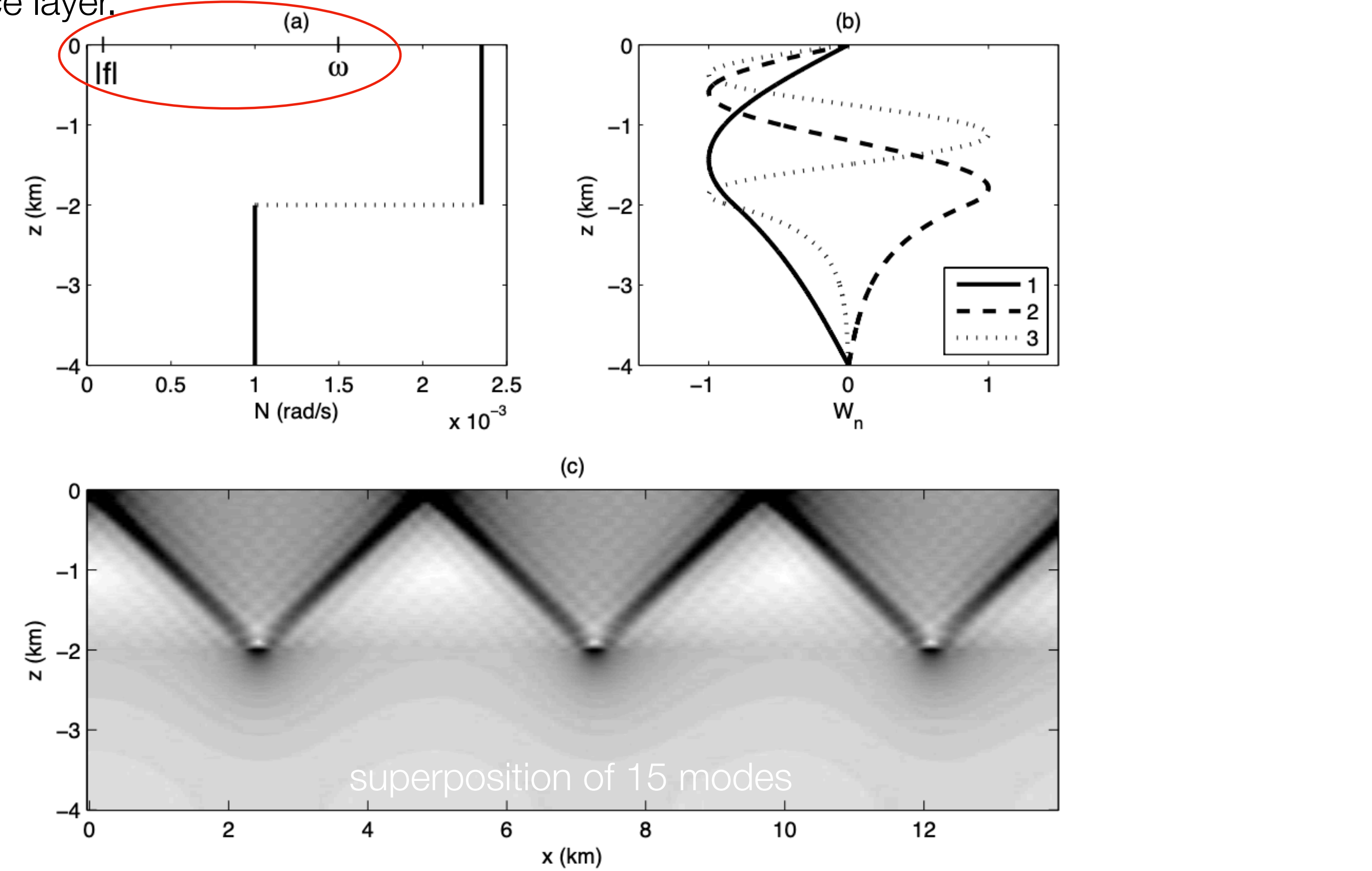
case of a piecewise-constant $N(z)$



Gerkema & Zimmerman (2008)

Method of modes

case of a piecewise-constant $N(z)$, with $\omega > N$ in the bottom layer: wave trapping in the surface layer.

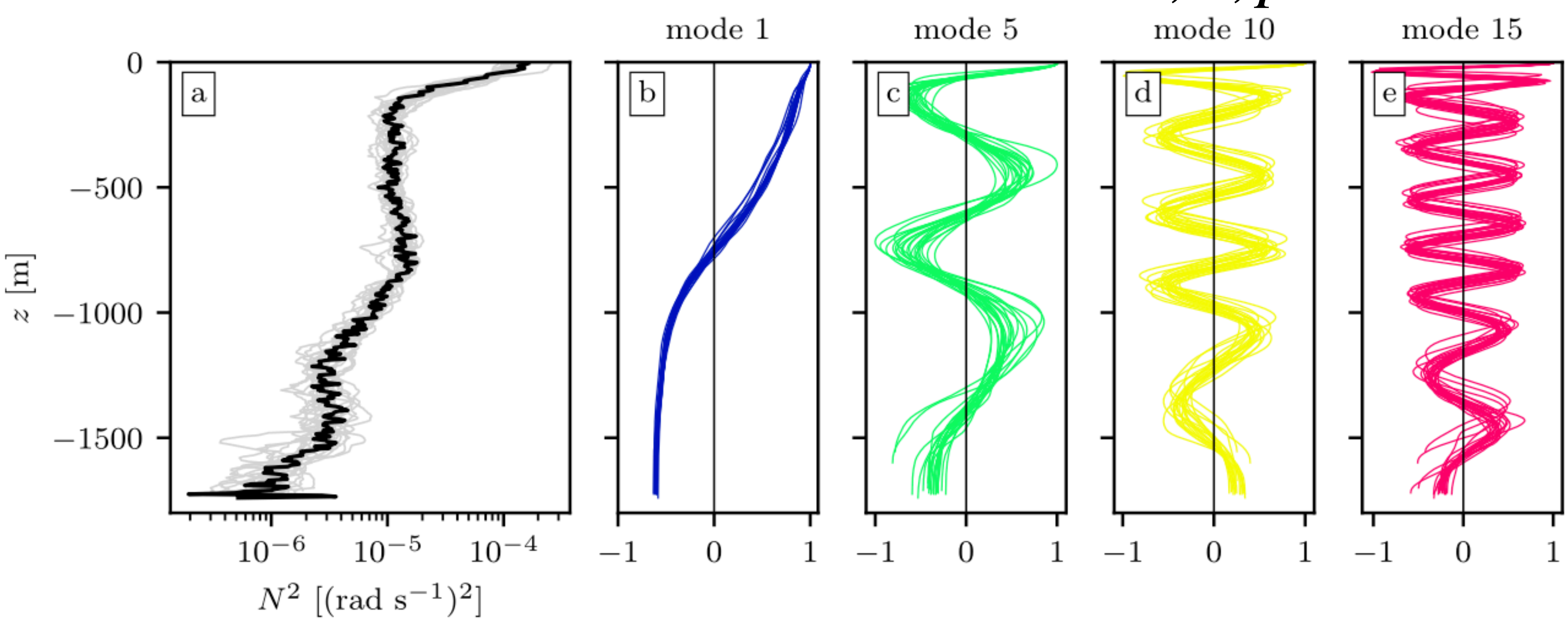


Gerkema & Zimmerman (2008)

Method of modes

Case of a real stratification $N(z)$ over the Mid-Atlantic Ridge

modes for u, v, p



Vic & Ferron (JGR 2023)

References used in this document:

- Gerkema & Zimmerman, 2008 textbook, *An introduction to internal waves*
- Maas et al., Nature 1997, *Observation of an internal wave attractor in a confined, stably stratified fluid*
- Vic & Ferron, JGR 2023, *Observed Structure of an Internal Tide Beam Over the Mid-Atlantic Ridge*