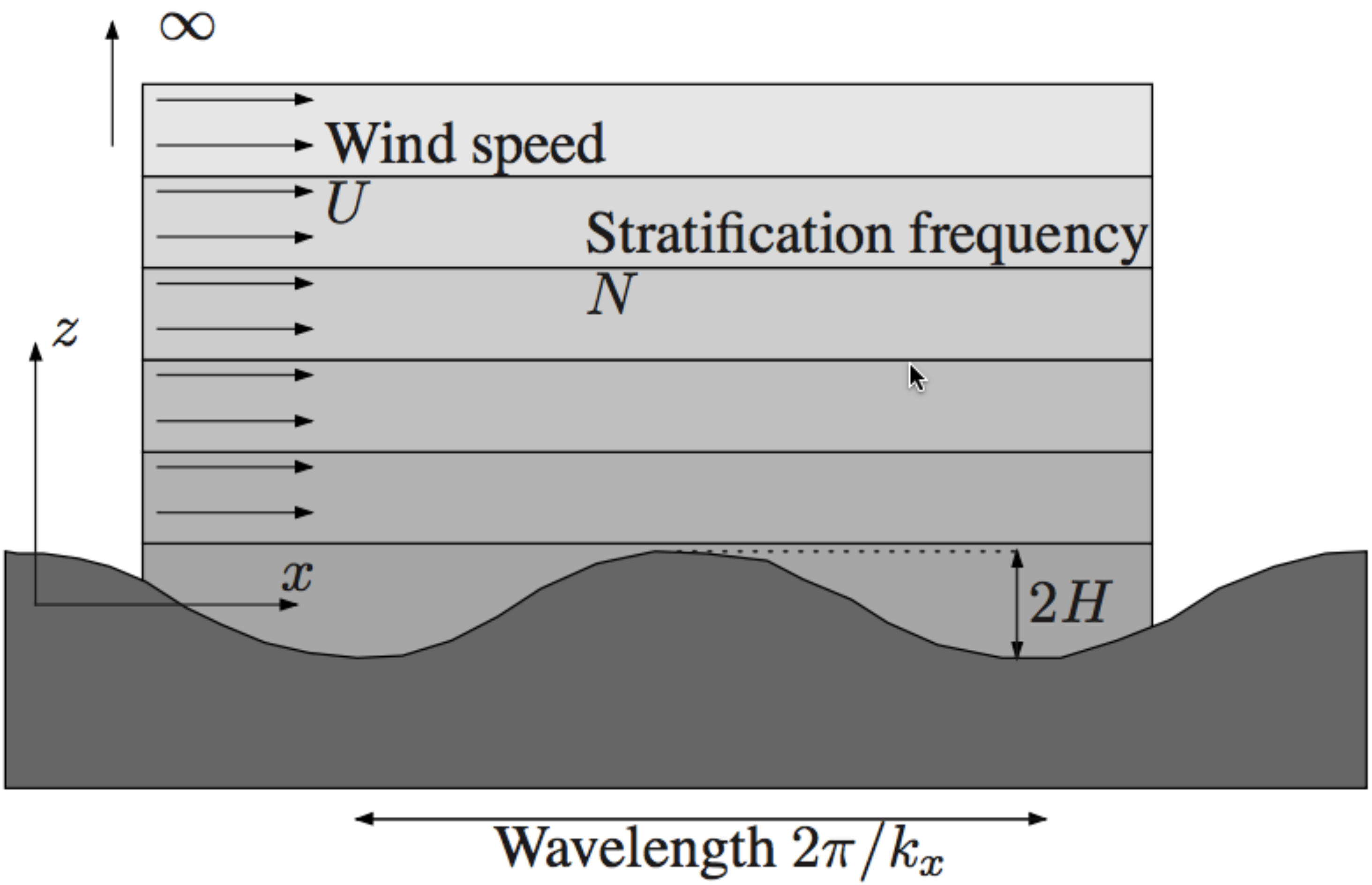
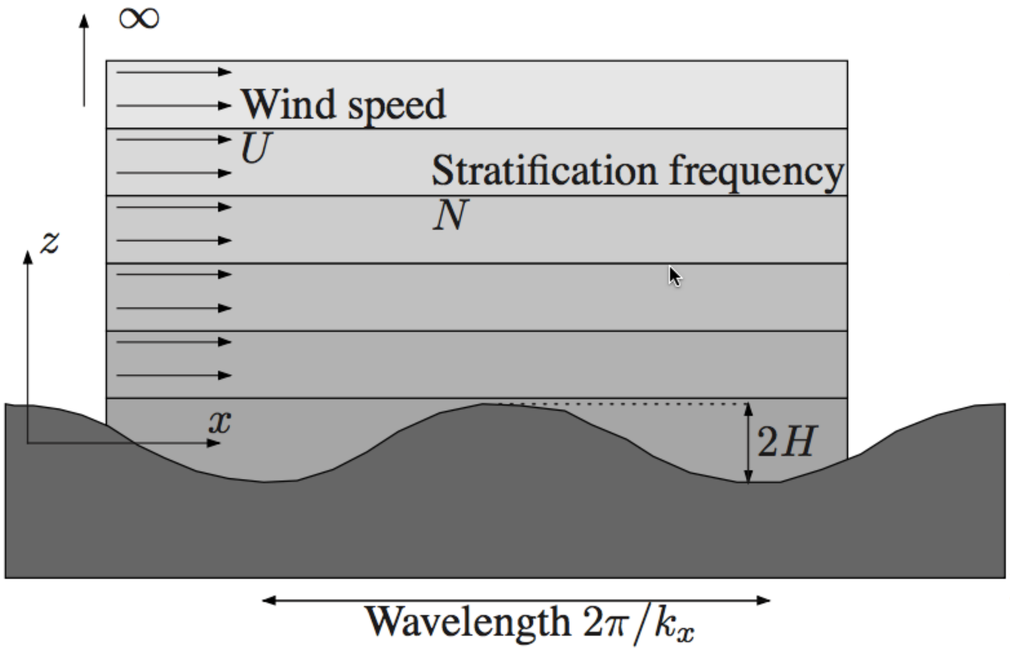
1.2 Generation of Lee waves

- Internal waves generated at horizontal boundary:



1.2 Generation of Lee waves

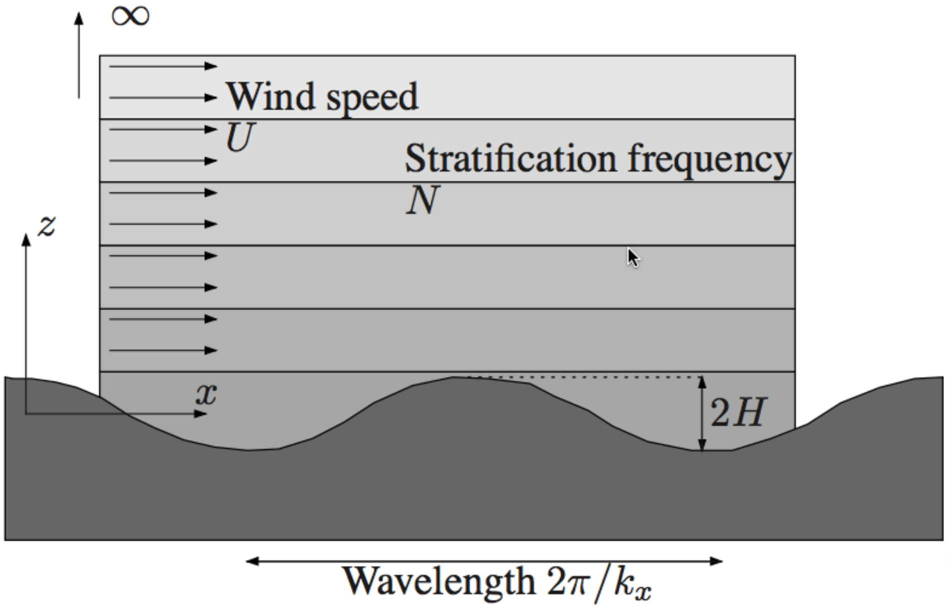


- Topography: $$z_b = H\sin[k_x x]$$
- *In the frame moving at constant speed U*: $$z_b = H\sin[k_x(x+Ut)]$$
- Frequency of the motion: $$\omega = -k_x U$$

1.2 Generation of Lee waves

- Equation for linear internal waves:

$$(w_{xx} + w_{yy} + w_{zz})_{tt} + f^2 w_{zz} + N^2(w_{xx} + w_{yy}) = 0$$



- Topography *in the frame moving at constant speed U*:

$$z_b = H \sin[k_x(x + Ut)]$$

- Frequency of the motion:

$$\omega = -k_x U$$

- Activity: Solve the equation for linear internal waves without rotation (f=0) in the x-z plane (also assuming H<<U/N).

1. Write the bottom boundary condition for the vertical velocity $w(z_b) \approx w(0)$
2. Solve for a solution in the form $w = w_0 \exp(i(k_x x + k_z z - \omega t))$

1.2 Generation of Lee waves

- Case 1 = **Radiating waves**

$$\frac{N}{U} > k_x \qquad k_z = \pm\sqrt{\frac{N^2}{U^2} - k_x^2}$$

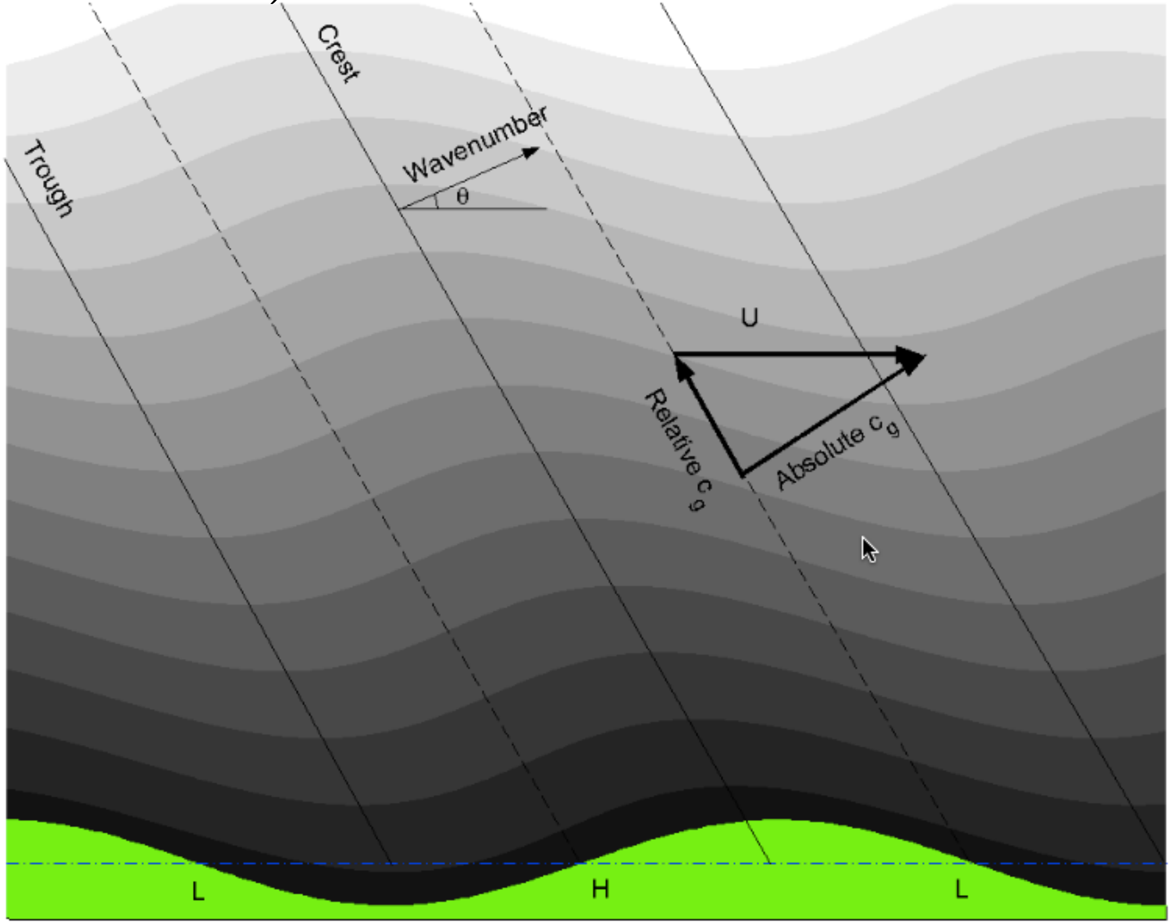
$$w = k_x U H \cos(k_x x + k_z z - \omega t)$$

Angle between wave fronts and vertical:

$$\cos\theta = \frac{k_x U}{N}$$

Group velocity in the vertical:

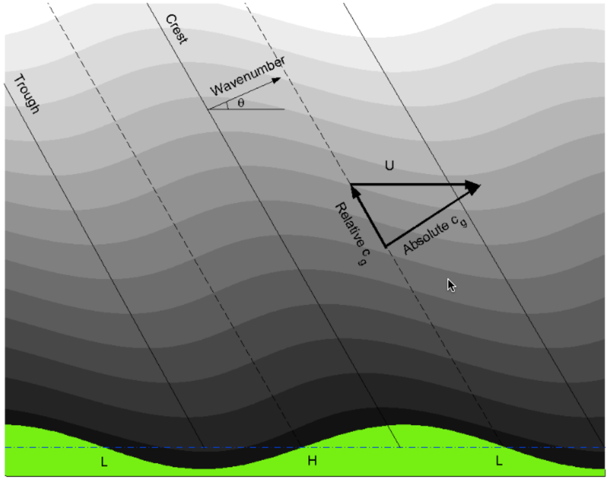
$$c_{gz} = \frac{\partial \omega}{\partial k_z} = \frac{k_x k_z}{k^2} U$$



1.2 Generation of Lee waves

- Case 1 = **Radiating waves**

$$\frac{N}{U} > k_x$$



- **Drag force**:

$$\rho_0 \overline{uw}|_{z=0} = -\frac{1}{2}\rho_0 k_k k_z U^2 H^2$$

The terrain exerts a drag force on the flowing air mass.

- **Energy flux:**

$$\mathbf{F}' = \overline{p'\mathbf{u}} = E\mathbf{c_g}$$

In the vertical

$$F'_z = \frac{1}{2}k_x\rho_0 H^2 U^2 \sqrt{N^2 - U^2 k_x^2}$$

Continual upward propagation of energy. Supply of energy from the ground to upper levels where it will be absorbed.

1.2 Generation of Lee waves

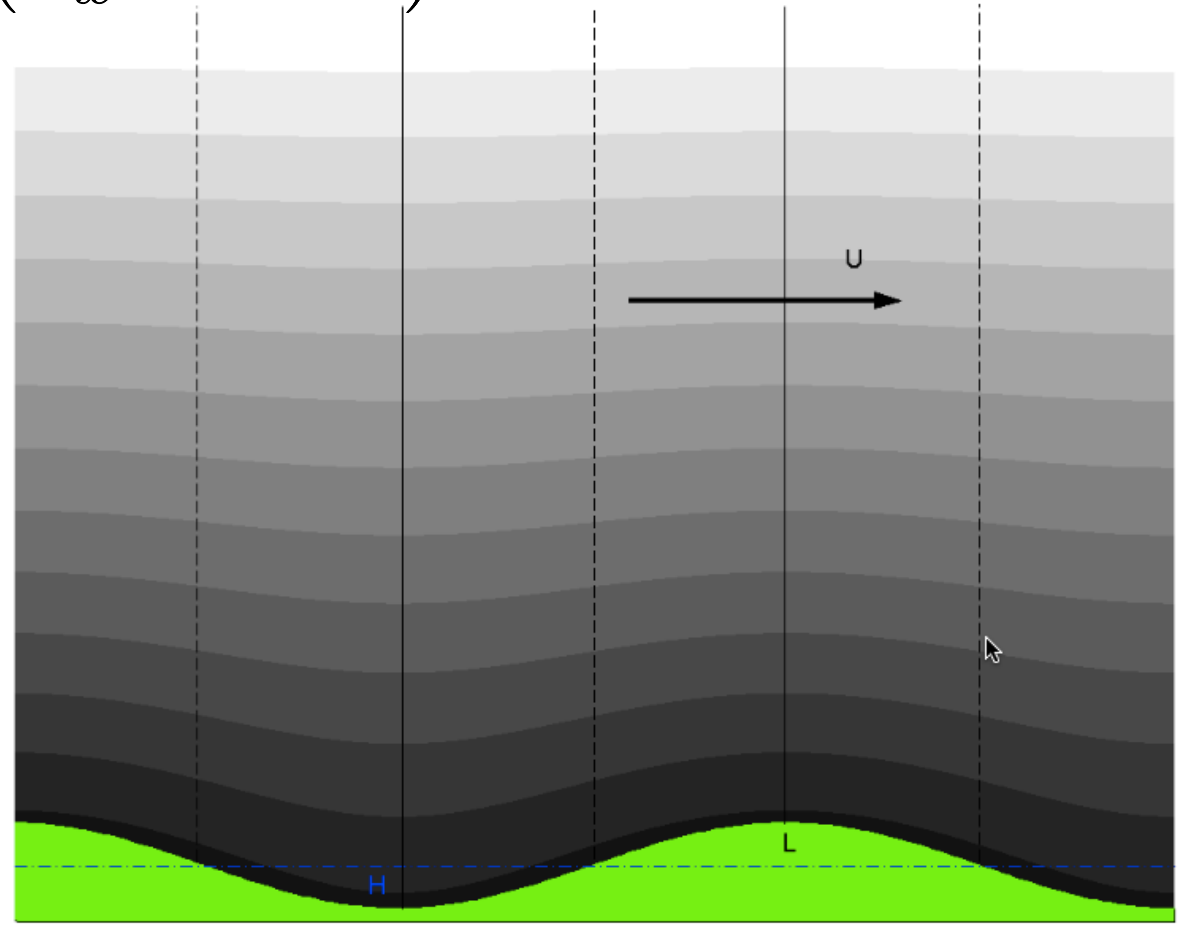
- Case 2 = **Trapped waves**

$$\frac{N}{U} < k_x \qquad \gamma = \sqrt{k_x^2 - \frac{N^2}{U^2}}$$

$$w = k_x U H e^{-\gamma z} \cos(k_x x - \omega t)$$

The solution now contains exponential functions in z.

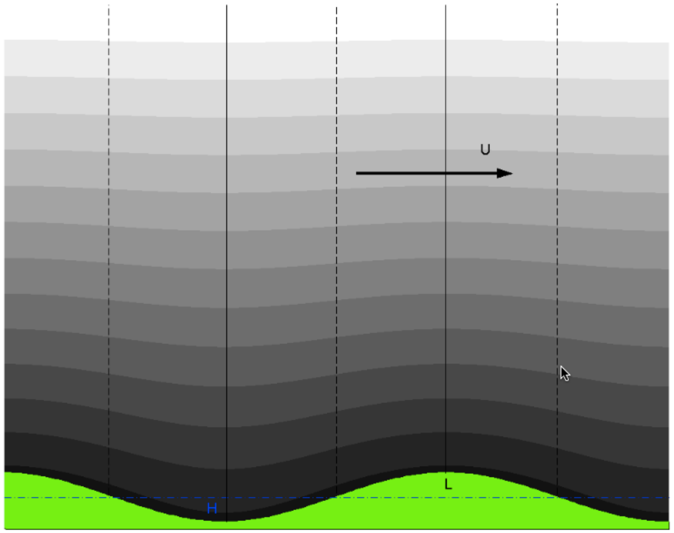
Density surfaces undulate at the same wavelength as the terrain, but the amplitude decays with height.



1.2 Generation of Lee waves

- Case 2 = **Trapped waves**

$$\frac{N}{U} > k_x$$



- **Drag force**:

$$\rho_0 \overline{uw}\big|_{z=0} = 0$$

u and w in quadrature = **No Drag**

- **Energy flux:**

$$\mathbf{F}' = \overline{p'\mathbf{u}} = E\mathbf{c_g}$$

p and w in quadrature = **no vertical energy flux**