First year Master Marine Physics

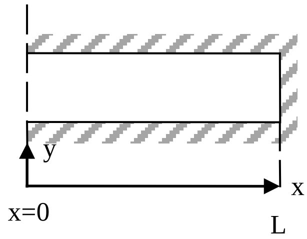
CC : take home exam due Tuesday April 2

Fluide II

1 –Acoustic waves

You want to find out the length L of an organ pipe with fundamental lowest frequency DO (C4)= 261.6 Hz

We do not consider the diameter of the pipe, only its length here. The main assumption is that the wave solution is linear and varies with x only (it is assumed independent of the transverse coordinate).



(i) Explain the chosen dynamics. Derive the wave equation in terms of the pressure p'.

(ii) Look for a free solution of the form $p'(x,t) = \mathrm{Real}[F(x) \exp i\omega t]$: derive a differential equation for F. Find out the boundary condition for F at x=L. At x=0 it is assumed that the pressure is close to constant atmospheric pressure p_0: find out the boundary condition for F.

(iii) Find out all possible solutions F(x) (eigenmodes) with their corresponding frequencies.

(iv) Give the lowest possible frequency. Compute the length of the pipe (use $c_0 = 340$ m/s). Compare L with the wavelength. Draw F(x).

(v) Think about the same problem but taking into account the circular shape of the pipe. Describe what you would do.

2. Phase portrait for competing species (Lotka-Volterra)

Here x represent the rabbit and y the sheep. When there is only one species, the logistic equation shows that the species goes to its equilibrium value. When rabbits and sheeps compete for the same grass, conflicts occur at a rate proportional to the size of the two populations and tend to reduce the population (more so for the rabbits). This is the origin of the nonlinear terms "xy" below. The rabbits have an intrinsic growth rate which is higher.

Consider the dynamical system:

$$dx/dt=x(3-x-2y)$$
$$dy/dt=y(2-x-y)$$

(i) Do a qualitative analysis of the phase portrait: identify the fixed points and their stabilities.

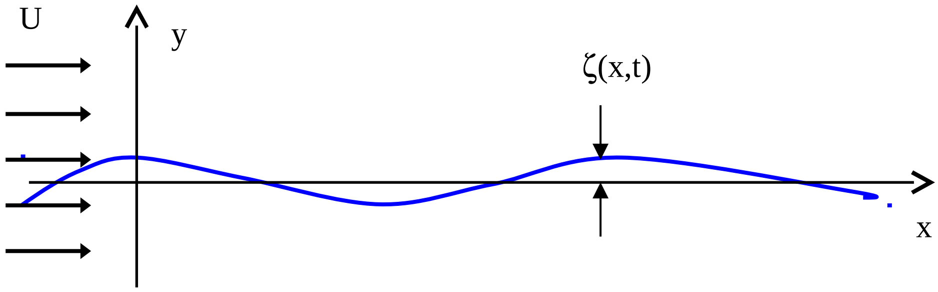
(ii) Use the Matsuno numerical scheme I gave you (with the Lorenz code) to integrate the system and draw the trajectories in x, y space. Discuss the basin of attraction of the fixed points. The ultimate fate of the two species is a very well known result in biology.

3. A flapping sail in the wind, an example of a coupled fluid-structure problem

We consider the instability of a membrane stretched by a tension T in a uniform flow (this could apply to a sail). The undisturbed infinite membrane lies in the x-z plane at y=0, the flow is parallel to the x-axis and the fluid has density ρ. The membrane has density per unit area ρ_m and uniform tension per unit length T. The displacement of the membrane obeys the equation:

$$\rho_m \frac{\partial^2 \varsigma}{\partial t^2} = p_2 - p_1 + T\left(\frac{\partial^2 \varsigma}{\partial x^2} + \frac{\partial^2 \varsigma}{\partial z^2}\right)$$

with p_1 and p_2 the pressures on each side of the membrane.



Assume a solution ζ=Real Part [ζ_0 exp(ik(x-ct))]. The pressure above p_1 and below p_2 are the same when the membrane is at rest but there is a pressure difference when the fluid moves.

(i) Determine the speed c of the waves of the membrane in the absence of wind (U=0 and pressure loading $p_1=p_2$).

(ii) When there is wind, recall that the fluid is irrotational. The undisturbed geopotentials are $\phi_1= \phi_2=$ Ux. The boundary condition on the moving surface is v=Dζ/Dt. You will need to use Bernouilli relation on each side to link pressure and potential. Linearize everything!

Assuming inviscid flow above and below the membrane, determine c in terms of T, ρ, ρ_m, U, k.

(iii) Discuss how each parameter T, ρ, ρ_m, U, k increase or decrease the instability of the flag.