Numerical Modelling

Jonathan GULA
gula@univ-brest.fr

the anatomy of an ocean model

#4
Numerical options in CROCO

Master's degree 2nd year Marine Physics

Jonathan GULA
gula@univ-brest.fr

Choice of numerics:

- cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!               REGIONAL (realistic) Configurations
!======================================================================
!
!----------------------
! BASIC OPTIONS
!----------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

- cppdefs_dev.h

```
======================================================================
    Select MOMENTUM LATERAL advection-diffusion scheme:
    (The default is third-order upstream biased)
======================================================================
*/
#ifdef UV_HADV_UP3      /* Check if options are defined in cppdefs.h */
#elif defined UV_HADV_C4
#elif defined UV_HADV_C2
#else
# define UV_HADV_UP3        /* 3rd-order upstream lateral advection */
# undef  UV_HADV_C4         /* 4th-order centered lateral advection */
# undef  UV_HADV_C2         /* 2nd-order centered lateral advection */
#endif
/*
   UV DIFFUSION: set default orientation
*/
#ifdef UV_MIX_S         /* Check if options are defined */
#elif defined UV_MIX_GEO
#else
# define UV_MIX_S       /* Default: diffusion along sigma surfaces */
#endif
/*
   Set keys related to Smagorinsky viscosity
*/
#ifdef UV_VIS_SMAGO
# define VIS_COEF_3D
#endif
/*
   Set UP3 scheme in barotropic equations for 2DH applications
*/
#if !defined SOLVE3D && !defined SOLITON
# define M2_HADV_UP3
#endif
/*
   If interior MOMENTUM LATERAL diffusion is defined, apply it
   over an anomaly with respect to a reference frame (climatology)
*/
#ifdef M3CLIMATOLOGY
# undef CLIMAT_UV_MIXH
#endif
```

cppdefs.h

```
#if defined REGIONAL
/*
!===========================================================
!               REGIONAL (realistic) Configurations
!===========================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Configuration name:

Used to define configuration specific part of the codes in param.h, ana_initial.F, analytical.F, etc.

```
#  elif defined PACIFIC
      parameter (LLm0=170,  MMm0=60,   N=30)   ! Pacific
#  elif defined  CORAL
      parameter (LLm0=81,   MMm0=77,   N=32)   ! CORAL sea
#  elif defined  BENGUELA_LR
      parameter (LLm0=41,   MMm0=42,   N=32)   ! BENGUELA_LR
#  elif defined  BENGUELA_HR
      parameter (LLm0=83,   MMm0=85,   N=32)   ! BENGUELA_HR
#  elif defined  BENGUELA_VHR
      parameter (LLm0=167,  MMm0=170,  N=32)   ! BENGUELA_VHR
#  elif defined  GULFSTREAM
      parameter (LLm0=134,  MMm0=112,  N=32)   ! GULFSTREAM
#  else
      parameter (LLm0=94,   MMm0=81,   N=40)
#  endif
```

cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!               REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
  # undef  OPENMP
  # undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

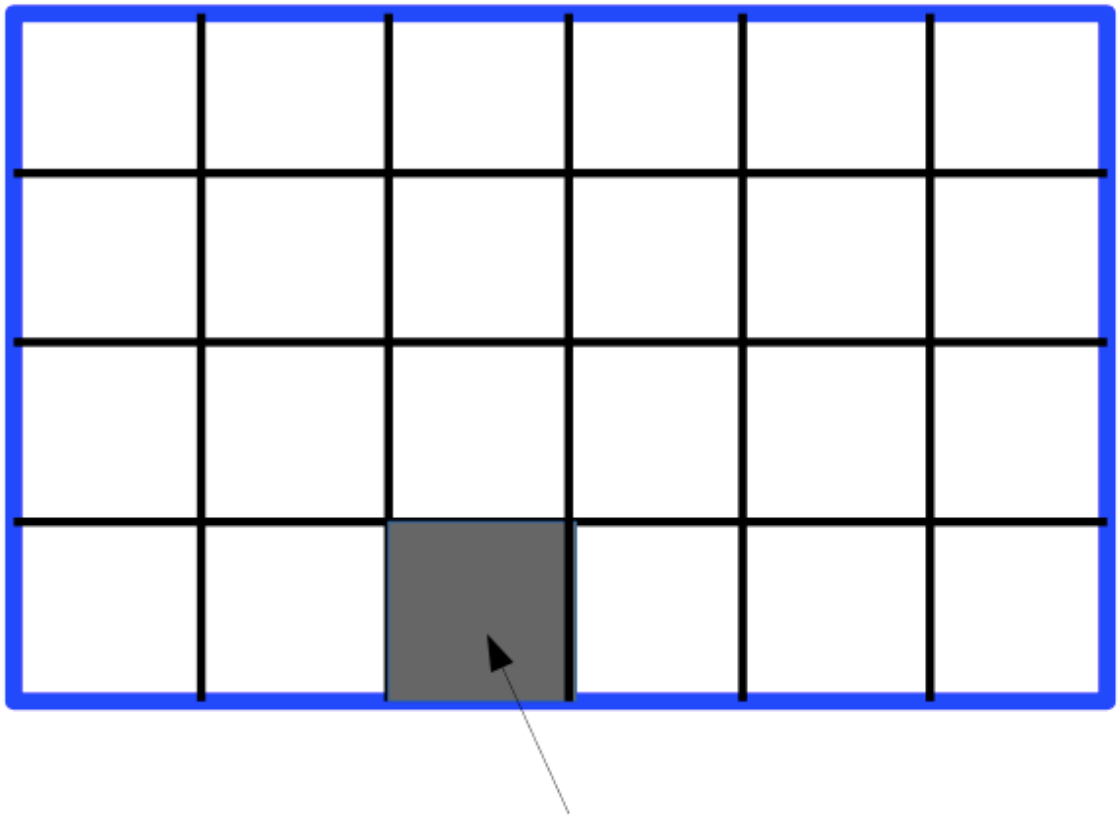
Parallelization options:

Activation of MPI and/or openMP parallelization:

Numbers of threads are defined in param.h

```
!
! Domain subdivision parameters
! ====== =========== ==========
!
! NPP             Maximum allowed number of parallel threads;
! NSUB_X,NSUB_E   Number of SHARED memory subdomains in XI- and
!                                                   ETA-directions;
! NNODES         Total number of MPI processes (nodes);
! NP_XI,NP_ETA   Number of MPI subdomains in XI- and ETA-directions;
!
      integer NSUB_X, NSUB_E, NPP
#ifdef MPI
      integer NP_XI, NP_ETA, NNODES
      parameter (NP_XI=2, NP_ETA=1,  NNODES=NP_XI*NP_ETA)
      parameter (NPP=1)
      parameter (NSUB_X=1, NSUB_E=1)
#elif defined OPENMP
      parameter (NPP=8)
```

define OPENMP



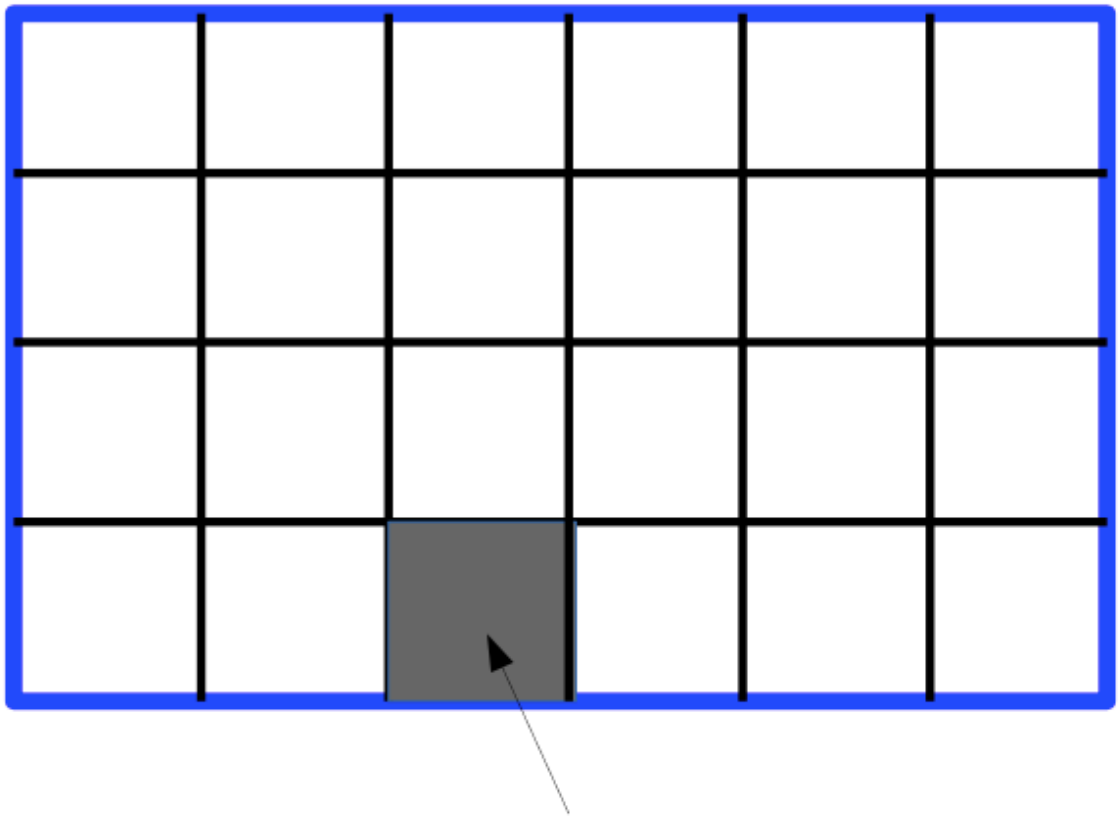
openMP = shared – memory

NPP = number of threads (cores)

NSUB_X , NSUB_E = number of tiles in both directions.

NSUB_X * NSUB_E has to be a multiple of NPP.

define OPENMP



Ex: NSUB_X = 6, NSUB_E = 4

openMP = shared – memory

NPP = number of threads (cores)

NSUB_X , NSUB_E = number of tiles in both directions.

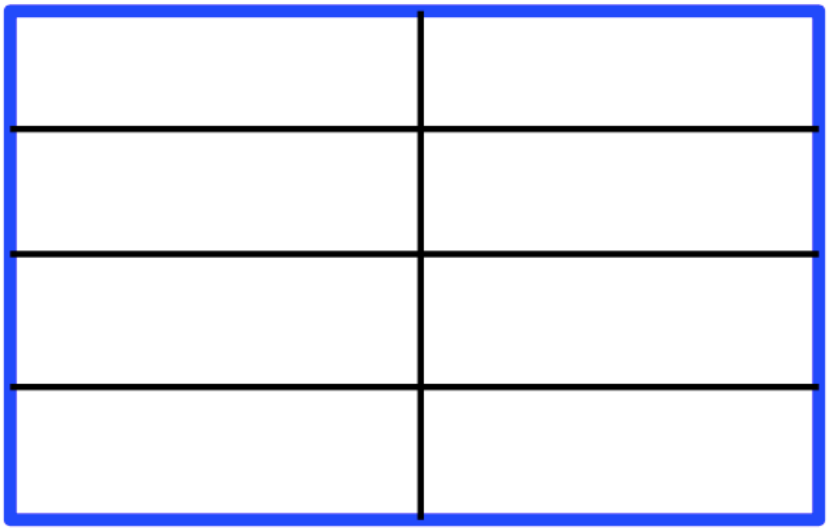
NSUB_X * NSUB_E has to be a multiple of NPP.

Default is (see param.h)

NSUB_X = 1, NSUB_E = NPP

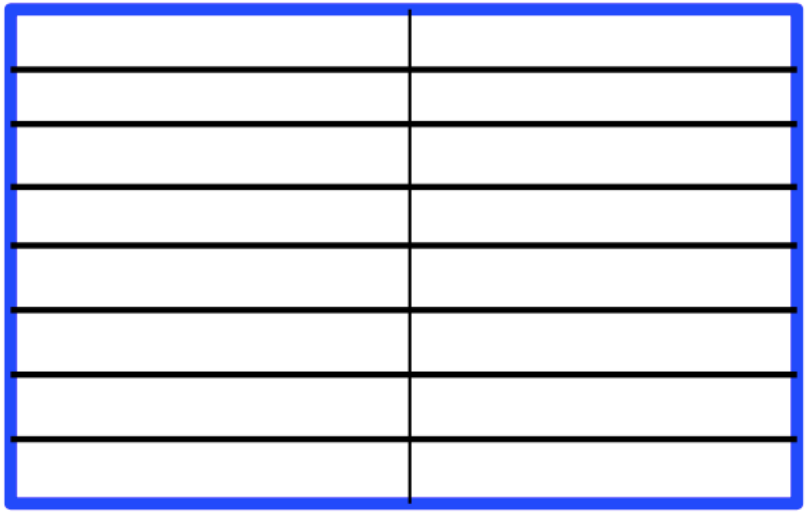
define OPENMP

Ex: 1 Node with 8 cores



NSUB_X = 2, NSUB_E = 4, NPP = 8

1 thread = 1 tile



NSUB_X = 2, NSUB_E = 8, NPP = 8

1 thread = 2 tiles

Multiple sub-domains can be assigned to each processor in order to optimize the use of processor cache memory.

define OPENMP

Each core can read/write global variables.

Has to ensure that different cores will not write the same indices of variables.

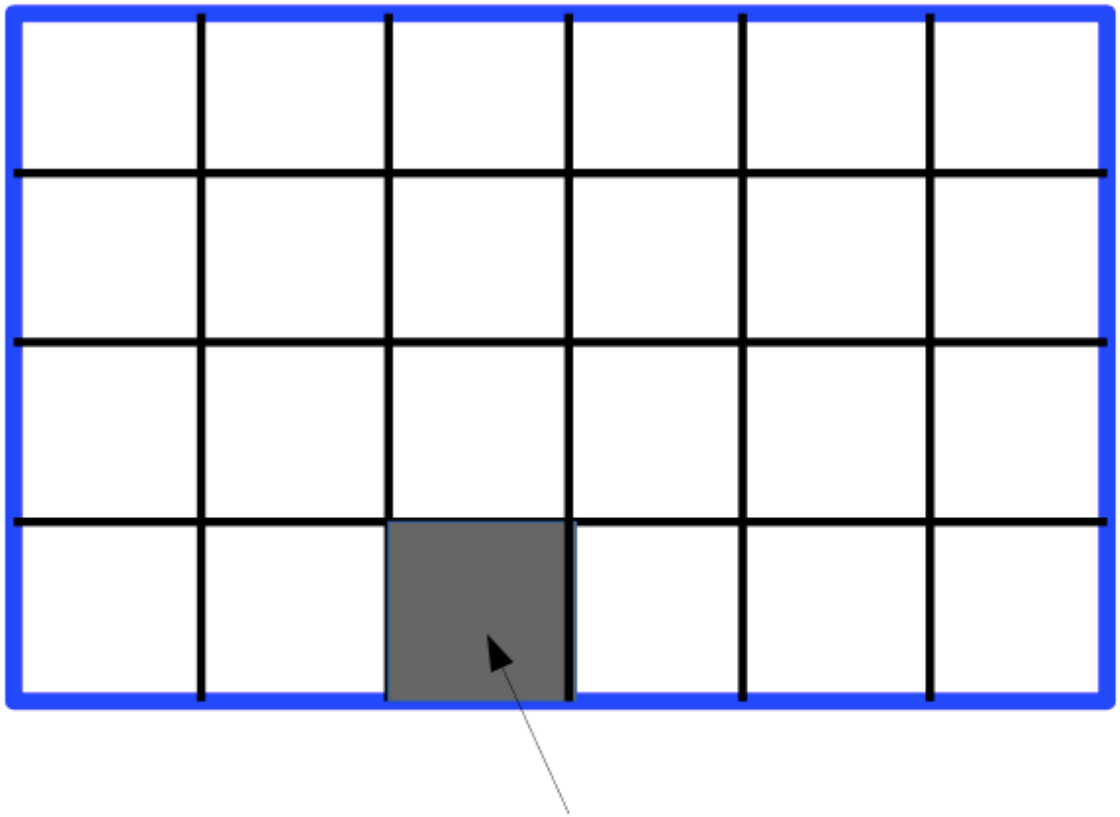
Also has to synchronize between different threads

Code structure example:

```
      Do tile=my_first,my_last

          Call compute_1(tile)
          Call compute_2(tile)
      Enddo
C$OMP BARRIER ! synchronisation

      Do tile=my_first,my_last

          Call compute_3(tile)
          Call compute_4(tile)
      Enddo
C$OMP BARRIER ! synchronisation
```

define MPI

MPI = distributed – memory



NP_XI*NP_ETA = number of threads (cores)

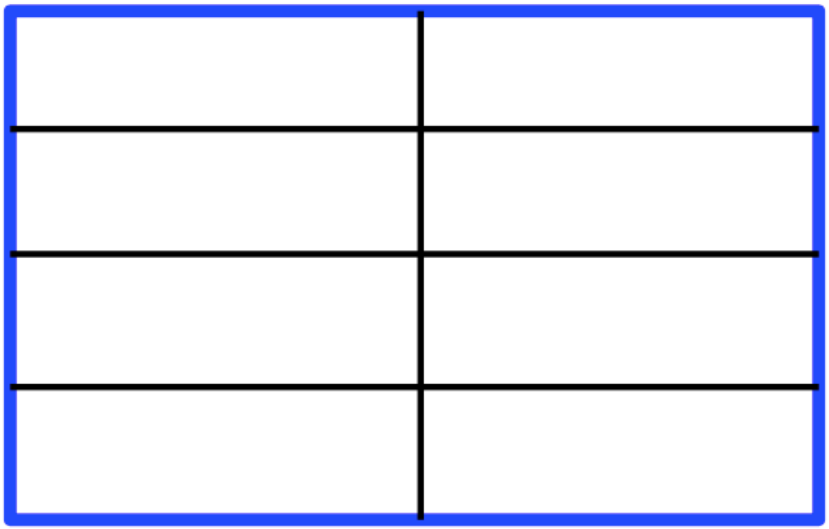
NPP = 1

NSUB_X * NSUB_E can be anything

Ex: NP_XI = 6, NP_ETA = 4

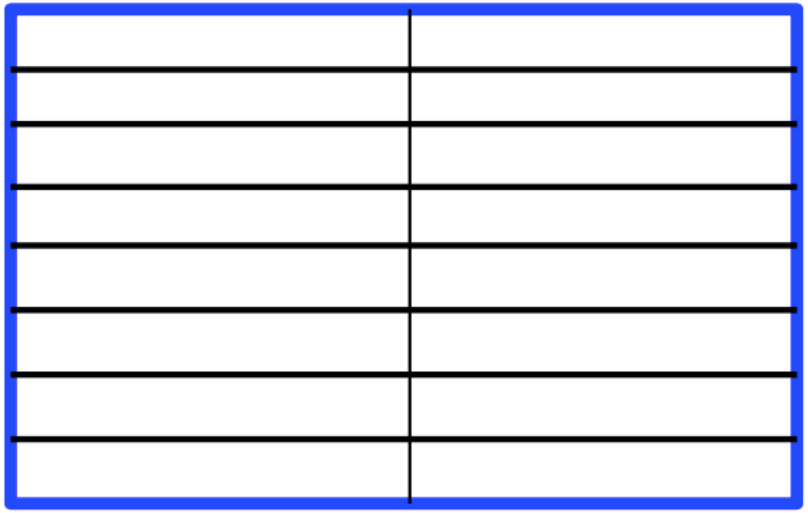
define MPI

Ex: 1 Node with 8 cores



NP_XI = 2, NP_ETA = 4, NPP = 1

1 thread = 1 tile



NP_XI = 2, NP_ETA = 4, NPP = 1

NSUB_X = 1, NSUB_E 2

1 thread = 2 tiles

define MPI

Each core has access to the variables only over the tile

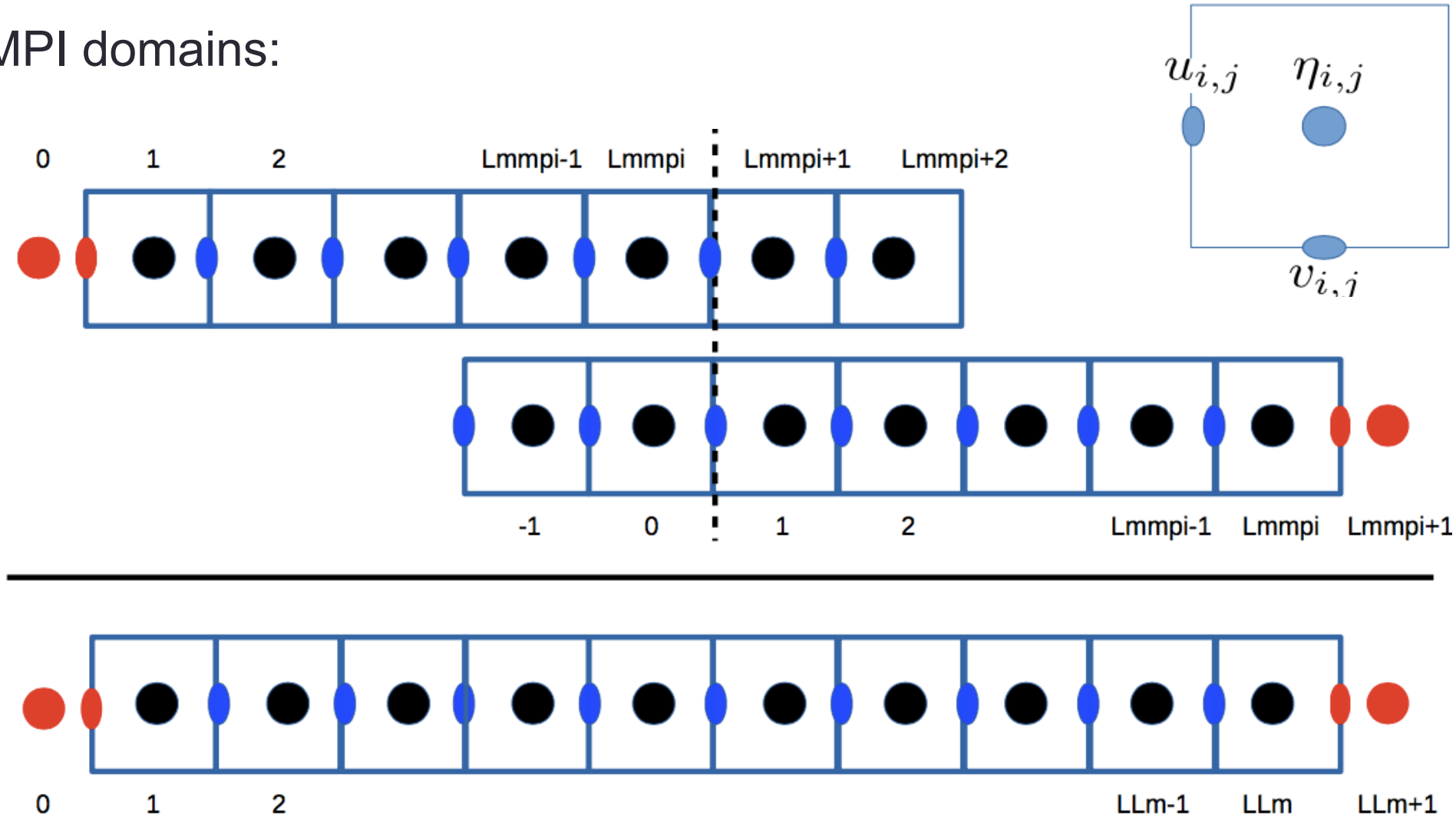
Cores have to communicate with each other to exchange information about boundaries

Code structure example:

```
# if defined EW_PERIODIC || defined NS_PERIODIC || defined MPI
      call exchange_u3d_tile (Istr,Iend,Jstr,Jend,
     &                                   u(START_2D_ARRAY,1,nnew))
      call exchange_v3d_tile (Istr,Iend,Jstr,Jend,
     &                                   v(START_2D_ARRAY,1,nnew))

      call exchange_u3d_tile (Istr,Iend,Jstr,Jend,
     &                                    Huon(START_2D_ARRAY,1))
      call exchange_v3d_tile (Istr,Iend,Jstr,Jend,
     &                                    Hvom(START_2D_ARRAY,1))

      call exchange_u2d_tile (Istr,Iend,Jstr,Jend,
     &                                    ubar(START_2D_ARRAY,knew))

      call exchange_v2d_tile (Istr,Iend,Jstr,Jend,
     &                                    vbar(START_2D_ARRAY,knew))
#   if defined TS_MIX_ISO || defined TS_MIX_GEO
      call exchange_u3d_tile (istr,iend,jstr,jend, dRdx  )
      call exchange_v3d_tile (istr,iend,jstr,jend, dRde  )
#  endif
```

define MPI

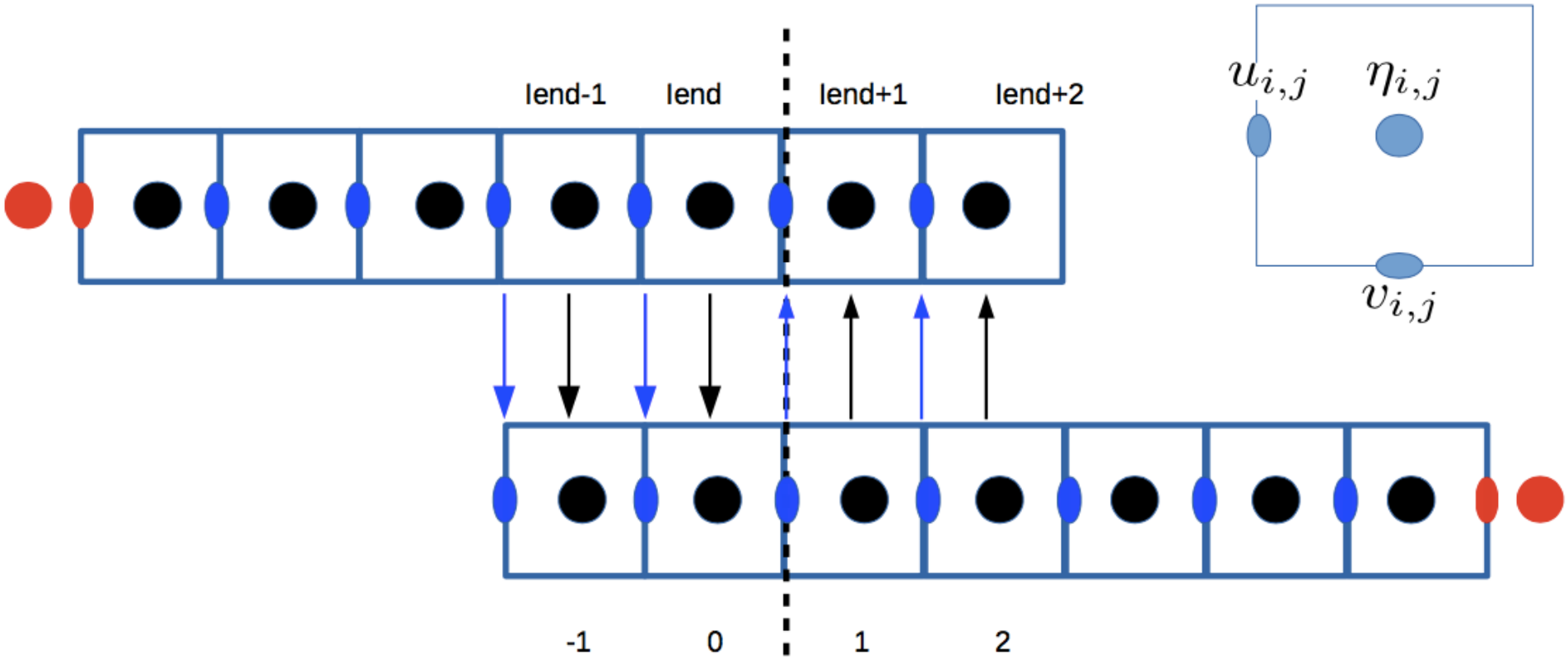
MPI domains:



Full domain:

define MPI

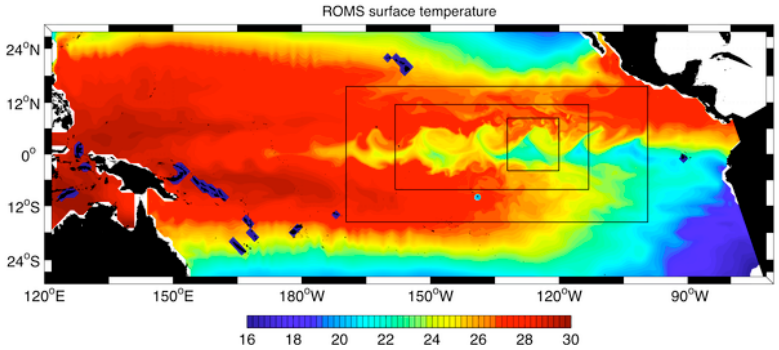
MPI exchanges:

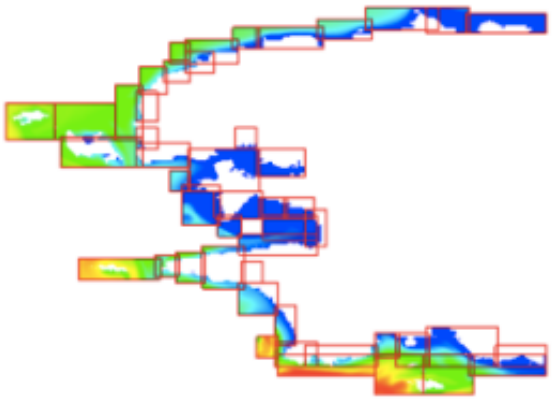


cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!               REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Online Nesting options:

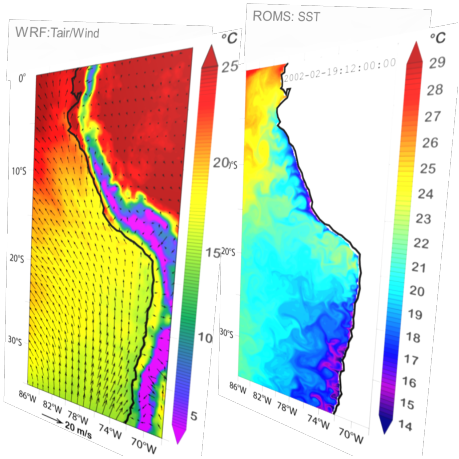




cppdefs.h

```
#if defined REGIONAL
/*
!===========================================================================
!               REGIONAL (realistic) Configurations
!===========================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Ocean - Atmosphere coupling



Ocean – Wave coupling

cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!                 REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

XIOS = XML – IO – SERVER

Management of IO by dedicated cores

cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!               REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Add barotropic tides (from a global tidal model) at the boundaries

cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!                 REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Use open boundary conditions

If undef then the domain is closed (see e.g. basin) or periodic.

cppdefs.h

```
#if defined REGIONAL
/*
!======================================================================
!               REGIONAL (realistic) Configurations
!======================================================================
!
!---------------------
! BASIC OPTIONS
!---------------------
!
*/
                        /* Configuration Name */
# define BENGUELA_LR
                        /* Parallelization */
# undef  OPENMP
# undef  MPI
                        /* Nesting */
# undef  AGRIF
# undef  AGRIF_2WAY
                        /* OA and OW Coupling via OASIS (MPI) */
# undef  OA_COUPLING
# undef  OW_COUPLING
                        /* I/O server */
# undef  XIOS
                        /* Open Boundary Conditions */
# undef  TIDES
# define OBC_EAST
# define OBC_WEST
# define OBC_NORTH
# define OBC_SOUTH
                        /* Applications */
# undef  BIOLOGY
# undef  FLOATS
# undef  STATIONS
# undef  PASSIVE_TRACER
# undef  SEDIMENT
# undef  BBL
/*!
```

Various applications

Activate biogeochemical modeling

Activate floats

Store high frequency model outputs at stations

Add a passive tracer

Activate sediment modeling

Activate bottom boundary layer parametrization

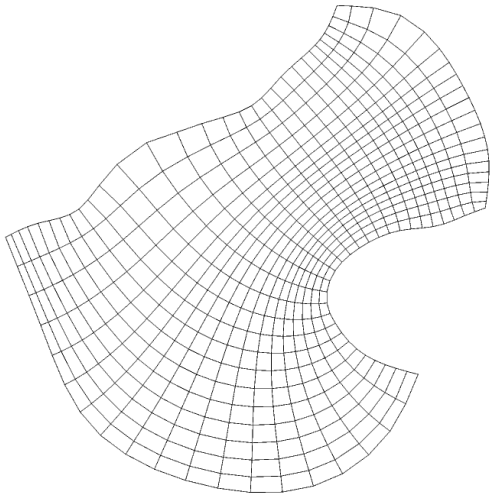
cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

Activate parallel I/O writing

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```



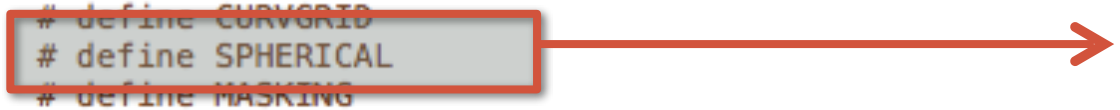
Activate curvilinear coordinate transformation

See rhs3d.F:

```
        do j=JstrV-1,Jend
          do i=IstrU-1,Iend
            cff=0.5*Hz(i,j,k)*(
#  ifdef UV_COR
     &               fomn(i,j)
#  endif
#  if (defined CURVGRID && defined UV_ADV)
     &             +0.5*( (v(i,j,k,nrhs)+v(i,j+1,k,nrhs))*dndx(i,j)
     &                   -(u(i,j,k,nrhs)+u(i+1,j,k,nrhs))*dmde(i,j))
#  endif
     &                                                          )
            UFx(i,j)=cff*(v(i,j,k,nrhs)+v(i,j+1,k,nrhs))
            VFe(i,j)=cff*(u(i,j,k,nrhs)+u(i+1,j,k,nrhs))
          enddo
        enddo
```

cppdefs.h

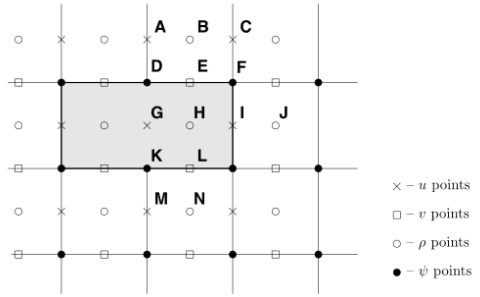
```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```



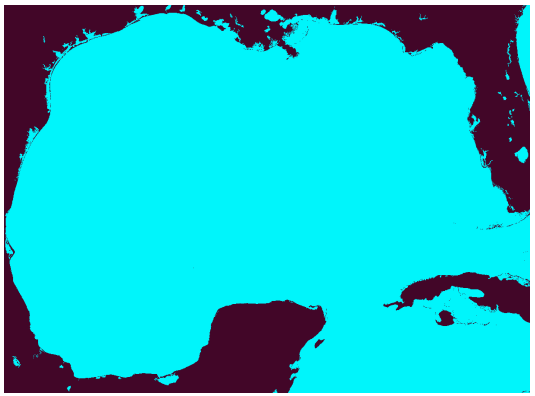
Activate longitude/latitude grid positioning

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```



Activate land masking



```
#  ifdef MASKING
            u(i,j,k,nnew)=u(i,j,k,nnew)*umask(i,j)
#  endif
# ifdef WET_DRY
            u(i,j,k,nnew)=u(i,j,k,nnew)*umask_wet(i,j)
# endif
          enddo
```

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

Activate wetting-Drying scheme

The Wetting-Drying scheme cancels the outgoing momentum flux (not the incoming) from a grid cell if its total depth is below a threshold value (5 cm).

```
#  ifdef MASKING
            u(i,j,k,nnew)=u(i,j,k,nnew)*umask(i,j)
#  endif
# ifdef WET_DRY
            u(i,j,k,nnew)=u(i,j,k,nnew)*umask_wet(i,j)
# endif
          enddo
```

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

Choose new (?) vertical S-coordinates

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

solve 3D primitive equations

See rhs3d.F:

```
#include "cppdefs.h"
#ifdef SOLVE3D

      subroutine rhs3d (tile)
!
      implicit none
      integer tile, trd, omp_get_thread_num
# include "param.h"
# include "private_scratch.h"
# include "compute_tile_bounds.h"
      trd=omp_get_thread_num()
      call rhs3d_tile (Istr,Iend,Jstr,Jend,
     &                                    A3d(1,1,trd), A3d(1,2,trd),
     &                        A2d(1,1,trd), A2d(1,2,trd), A2d(1,3,trd),
     &                        A2d(1,1,trd), A2d(1,2,trd), A2d(1,3,trd),
     &                        A2d(1,4,trd), A2d(1,5,trd), A2d(1,6,trd))
      return
      end
```

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

$$\frac{\partial u}{\partial t} + \vec{u}\cdot\vec{\nabla}_H u + w\frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u}\cdot\vec{\nabla}_H v + w\frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$

Activate Coriolis terms

See rhs3d.F:

```
        do j=JstrV-1,Jend
          do i=IstrU-1,Iend
            cff=0.5*Hz(i,j,k)*(
#  ifdef UV_COR
     &                fomn(i,j)
#  endif
#  if (defined CURVGRID && defined UV_ADV)
     &              +0.5*( (v(i,j,k,nrhs)+v(i,j+1,k,nrhs))*dndx(i,j)
     &                    -(u(i,j,k,nrhs)+u(i+1,j,k,nrhs))*dmde(i,j))
#  endif
     &                                                          )
            UFx(i,j)=cff*(v(i,j,k,nrhs)+v(i,j+1,k,nrhs))
            VFe(i,j)=cff*(u(i,j,k,nrhs)+u(i+1,j,k,nrhs))
          enddo
        enddo
```

cppdefs.h

```
!-----------------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

$$\frac{\partial u}{\partial t} + \vec{u}\cdot\vec{\nabla}_H u + w\frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u}\cdot\vec{\nabla}_H v + w\frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$

Activate advection terms

See rhs3d.F:

```
# ifdef UV_ADV
!
!=======================================================================
!
!  Add in horizontal advection of momentum.
!  Compute diagonal [UFx,VFe] and off-diagonal [UFe,VFx] components
!  of tensor of momentum flux due to horizontal advection; after that
!  add in horizontal advection terms.
!
!=======================================================================
!
#  if !(defined UV_HADV_UP3 || defined UV_HADV_C4 || defined UV_HADV_C2)
#   define UV_HADV_UP3
#  endif

#  if defined UV_HADV_UP3 || defined UV_HADV_C4
!
!-----------------------------------------------------------------
! Fourth or Third order advection scheme (default)
!-----------------------------------------------------------------
!
#   define uxx wrk1
#   define Huxx wrk2
!
#   ifdef EW_PERIODIC
#    define IU_EXT_RANGE IstrU-1,Iend+1
#   else
#    ifdef MPI
        if (WEST_INTER) then
          imin=IstrU-1
        else
          imin=max(IstrU-1,2)
        endif
        if (EAST_INTER) then
          imax=Iend+1
        else
          imax=min(Iend+1,Lmmpi)
        endif
#     define IU_EXT_RANGE imin,imax
#    else
#     define IU_EXT_RANGE max(IstrU-1,2),min(Iend+1,Lm)
#    endif
#   endif

        do j=Jstr,Jend
          do i=IU_EXT_RANGE
            uxx(i,j)=(u(i-1,j,k,nrhs)-2.*u(i,j,k,nrhs)
     &                +u(i+1,j,k,nrhs))   SWITCH umask(i,j)
            Huxx(i,j)=(Huon(i-1,j,k)-2.*Huon(i,j,k)
     &                 +Huon(i+1,j,k))   SWITCH umask(i,j)
          enddo
        enddo
```

cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

Define salinity as an active tracer

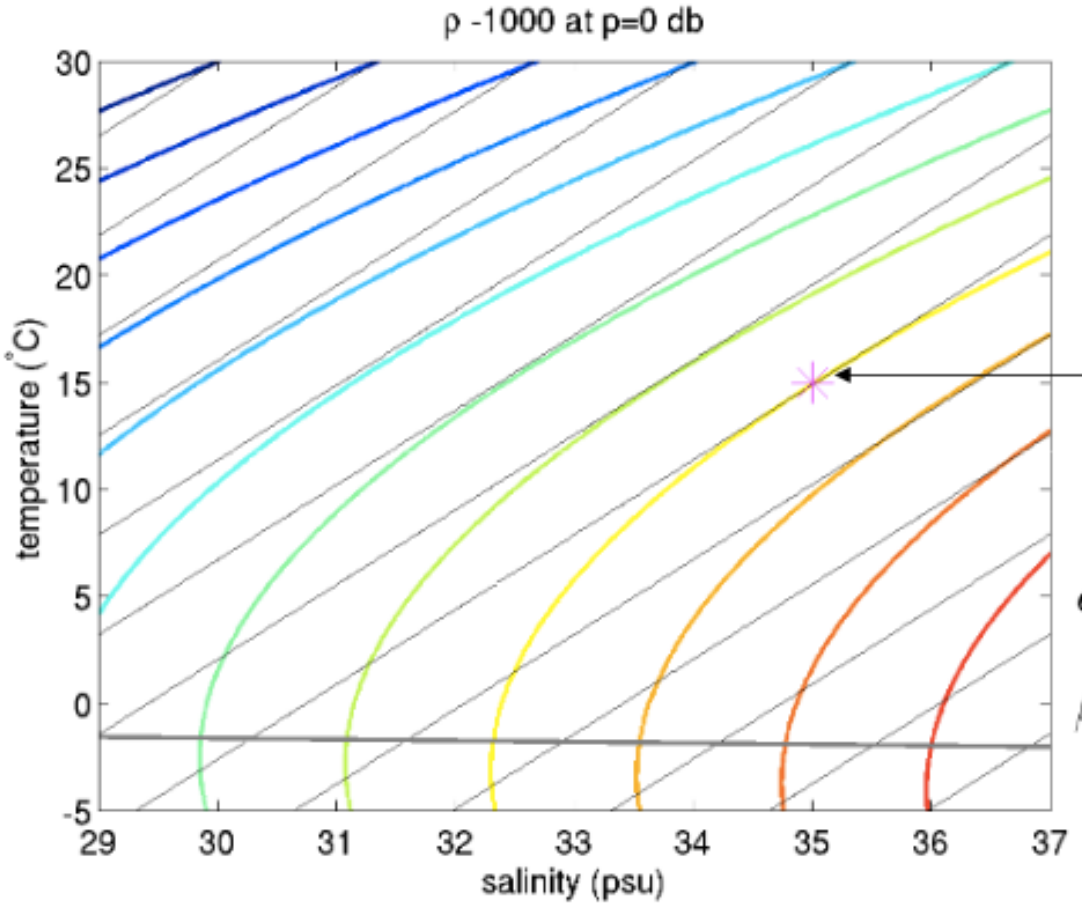
cppdefs.h

```
!-----------------------------------------------
! PRE-SELECTED OPTIONS
!
! ADVANCED OPTIONS ARE IN CPPDEFS_DEV.H
!-----------------------------------------------
*/
                        /* Parallelization */
# ifdef MPI
#  undef  PARALLEL_FILES
# endif
# undef  AUTOTILING
# undef  ETALON_CHECK
                        /* Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
# undef  WET_DRY
# undef  NEW_S_COORD
                        /* Model dynamics */
# define SOLVE3D
# define UV_COR
# define UV_ADV
                        /* Equation of State */
# define SALINITY
# define NONLIN_EOS
# define SPLIT_EOS
```

Choose non linear equation of state. *See rho_eos.F:*

```
      subroutine rho_eos(Lm,Mm,N, T,S, z_r,z_w,rho0, rho)

!
! Compute density anomaly from T,S via Equation Of State (EOS) for
!-------- ------- ------- ----  for seawater. Following Jackett and
! McDougall, 1995, physical EOS is assumed to have form
!
!                          rho0 + rho1(T,S)
!          rho(T,S,z) = ------------------------                (1)
!                       1 - 0.1*|z|/K(T,S,|z|)
!
! where rho1(T,S) is sea-water density perturbation[kg/m^3] at
! standard pressure of 1 Atm (sea surface); |z| is absolute depth,
! i.e. distance from free-surface to the point at which density is
! computed, and
!
!     K(T,S,|z|) = K00 + K01(T,S) + K1(T,S)*|z| + K2(T,S)*|z|^2.  (2)
!
! To reduce errors of pressure-gradient scheme associated with
! nonlinearity of compressibility effects, as well as to reduce
! roundoff errors, the dominant part of density profile,
!
!                         rho0
!                     ----------------                          (3)
!                     1 - 0.1|z|/K00
!
! is removed from from (1). [Since (3) is purely a function of z,
! it does not contribute to pressure gradient.]  This results in
!
!                    rho1 - rho0*[K01+K1*|z|+K2*|z|^2]/[K00-0.1|z|]
!    rho1 + 0.1|z| ------------------------------------------------
!                          K00 + K01 + (K1-0.1)*|z| + K2*|z|^2
!                                                              (4)
! which is suitable for pressure-gradient calculation.
!
! Optionally, if CPP-switch SPLIT_EOS is defined, term proportional
! to |z| is linearized using smallness 0.1|z|/[K00 + K01] << 1 and
! the resultant EOS has form
!
!              rho(T,S,z) = rho1(T,S) + qp1(T,S)*|z|            (5)
!
! where
!                             rho1(T,S) - rho0*K01(T,S)/K00
!            qp1(T,S)= 0.1 ---------------------------------   (6)
!                                  K00 + K01(T,S)
!
! is stored in a special array.
```

cppdefs.h



(S_o, T_o, σ_o)

=(35,15,26)

$\alpha_T = 2.1 \times 10^{-4}\ \mathrm{K}^{-1}$

$\beta_S = 7.5 \times 10^{-4}\ \mathrm{psu}^{-1}$

•The equation of state can be approximated using a linear function of temperature and salinity

$$\sigma = \sigma_o + \rho_{ref}\left[\beta_S(S - S_o) - \alpha_T(T - T_o)\right]$$

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Choose horizontal advective scheme for momentum

See rhs3d.F:

cppdefs.h

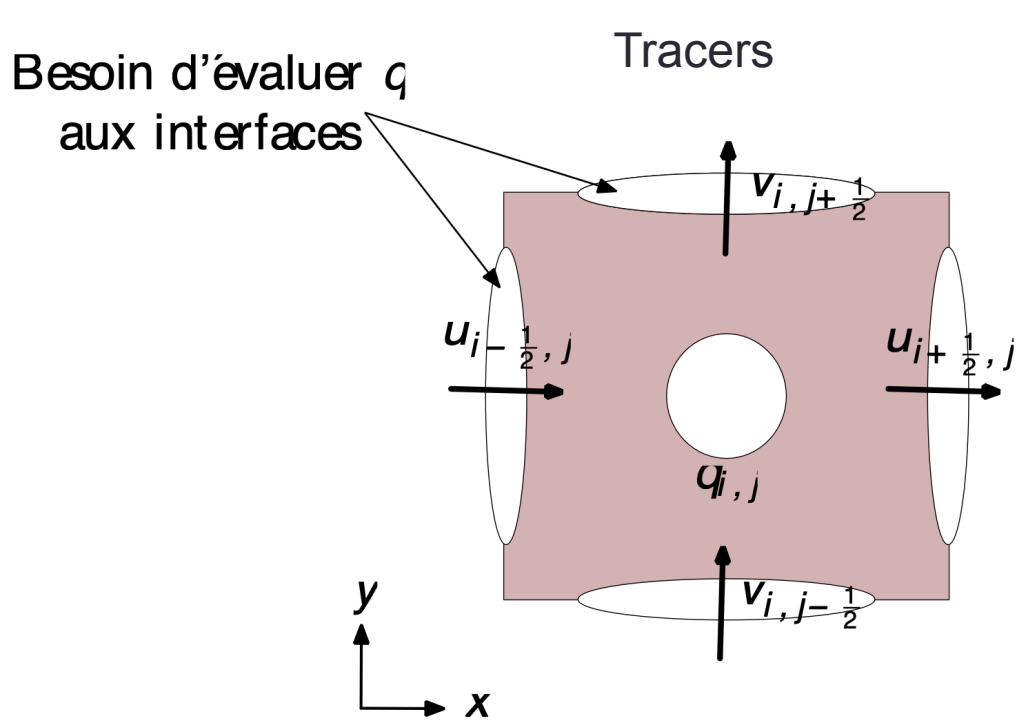
```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

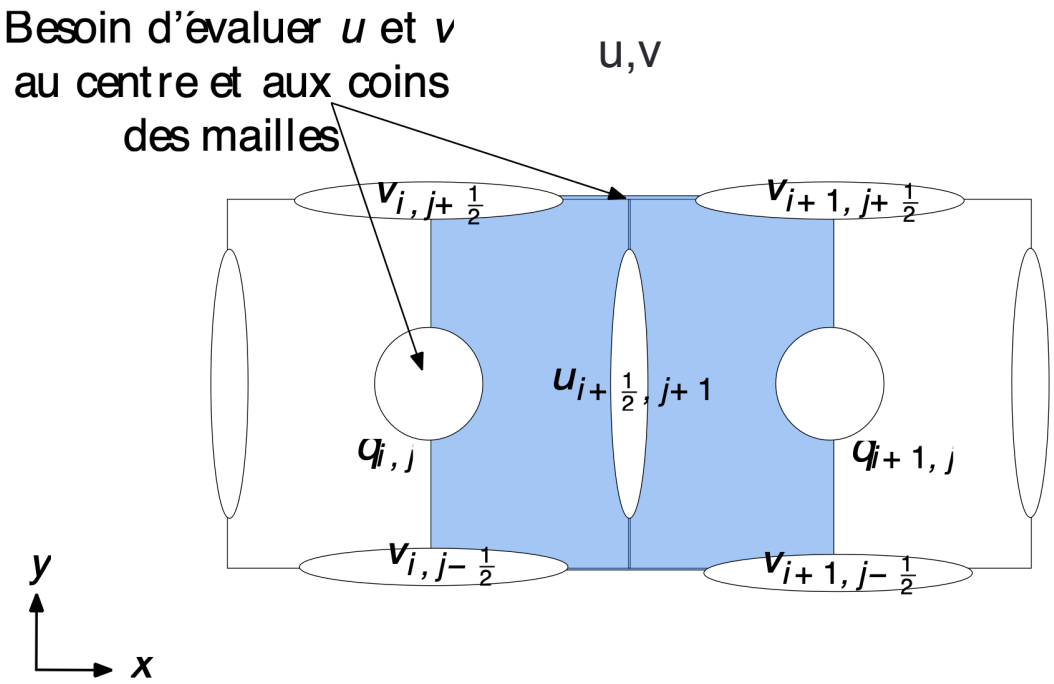
Choose horizontal advective scheme for tracers

See step3d_t.F and compute_horiz_tracer_fluxes.h

Horizontal Advective Schemes

- Advective scheme = interpolation problem



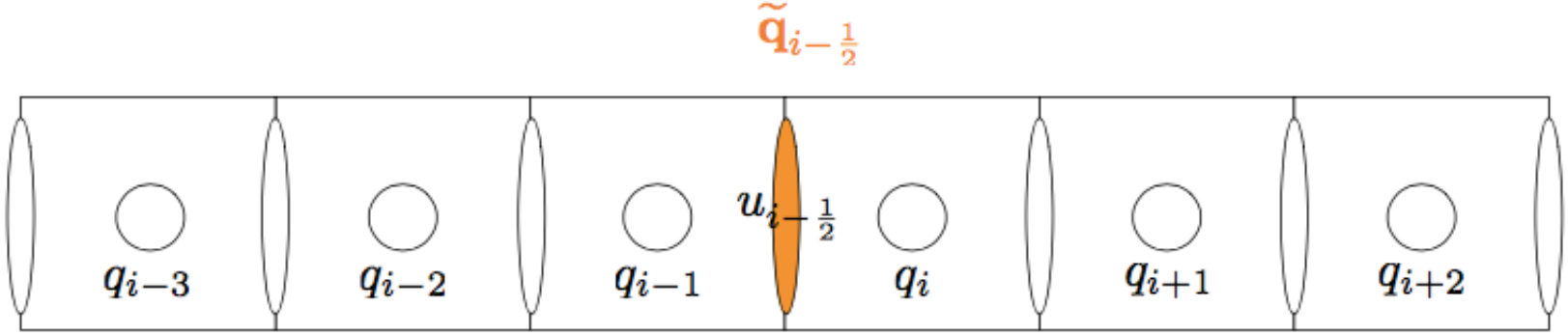


Horizontal Advective Schemes

- C2 = 2nd-order centered advection scheme
- UP3 = 3rd-order upstream-biased advection scheme
- C4 = 4th-order centered advection scheme
- UP5 = 5th-order upstream-biased advection scheme
- C6 = 6th-order centered advection scheme
- RSUP3 = Split and rotated 3rd-order upstream-biased advection scheme
- RSUP5 = Split and rotated 3rd-order upstream-biased advection scheme with reduced dispersion/diffusion
- HADV_WENO5 = 5th-order WENOZ quasi-monotone advection scheme for all tracers

Horizontal Advective Schemes

[HADV_C2, HADV_UP3, HADV_C4, HADV_UP5, HADV_C6]



$$\partial_x(uq)|_{x=x_i} = \frac{1}{\Delta x_i}\left\{u_{i+1/2}\widetilde{q}_{i+1/2} - u_{i-1/2}\widetilde{q}_{i-1/2}\right\}$$

$$\widetilde{q}^{\,C2}_{i-1/2} = \frac{q_i + q_{i-1}}{2}$$

$$\widetilde{q}^{\,C4}_{i-1/2} = (7/6)\widetilde{q}^{\,C2}_{i-1/2} - (1/12)(q_{i+1} + q_{i-2})$$

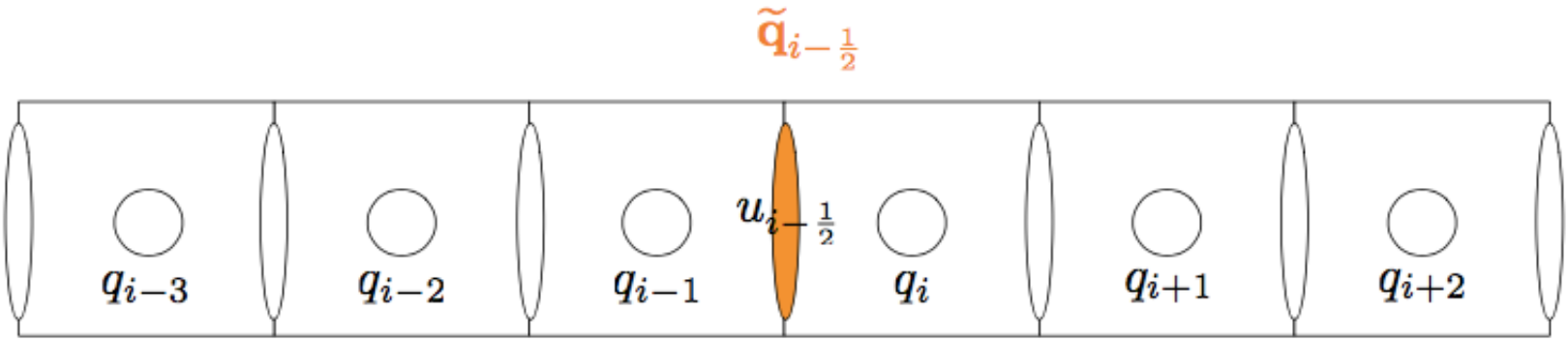
$$\widetilde{q}^{\,UP3}_{i-1/2} = \widetilde{q}^{\,C4}_{i-1/2} + \mathrm{sign}(1/12, u_{i-1/2})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{\,C6}_{i-1/2} = (8/5)\widetilde{q}^{\,C4}_{i-1/2} - (19/60)\widetilde{q}^{\,C2}_{i-1/2} + (1/60)(q_{i+2} + q_{i-3})$$

$$\widetilde{q}^{\,UP5}_{i-1/2} = \widetilde{q}^{\,C6}_{i-1/2} - \mathrm{sign}(1/60, u_{i-1/2})(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

Horizontal Advective Schemes

[HADV_RSUP3,HADV_RSUP5]



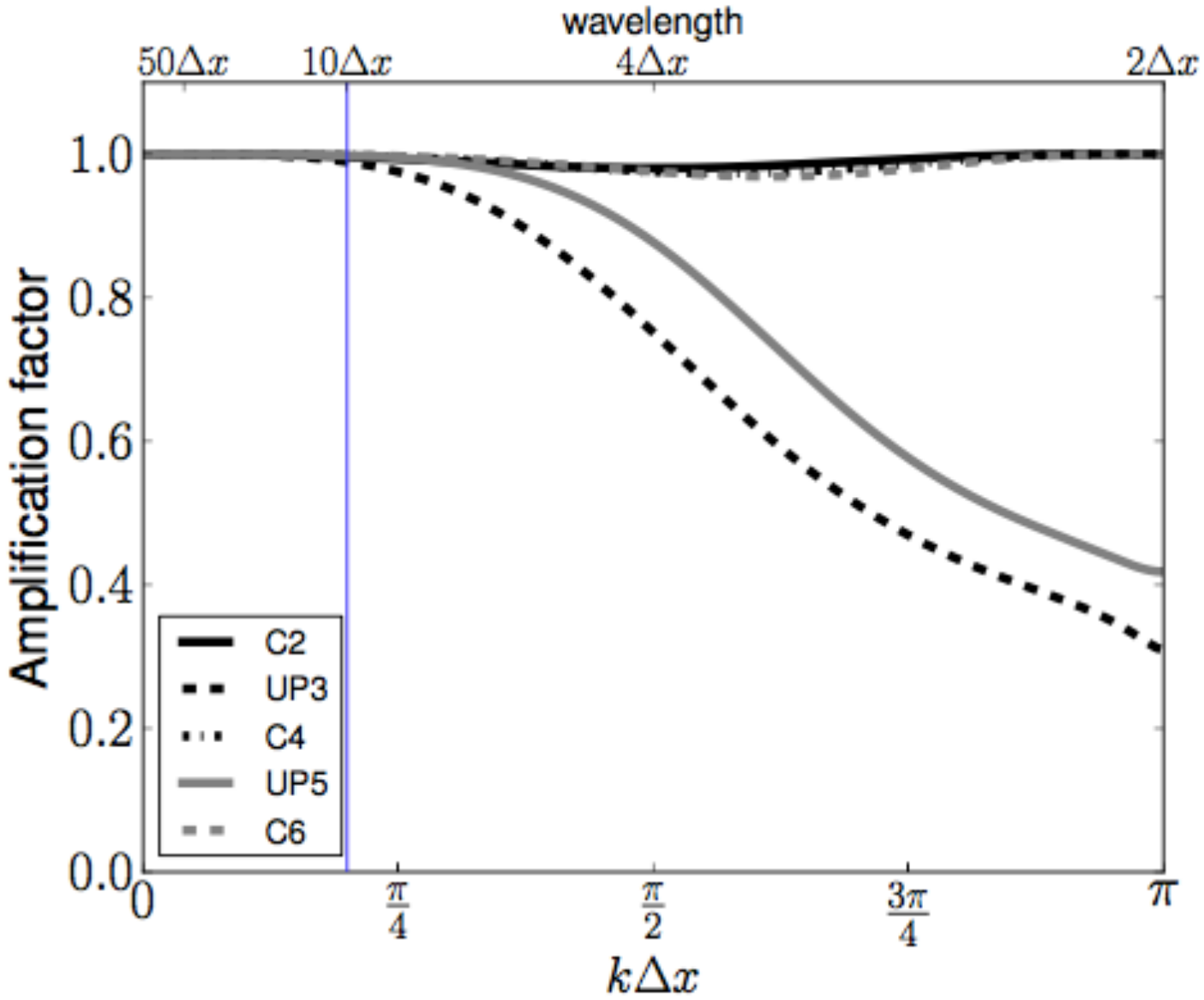
$$\partial_x(uq)|_{x=x_i} = \frac{1}{\Delta x_i}\left\{u_{i+1/2}\widetilde{q}_{i+1/2} - u_{i-1/2}\widetilde{q}_{i-1/2}\right\}$$

$$\widetilde{q}^{\,\mathrm{UP3}}_{i-1/2} = \widetilde{q}^{\,\mathrm{C4}}_{i-1/2} + \mathrm{sign}(1/12, u_{i-1/2})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{\,\mathrm{UP5}}_{i-1/2} = \widetilde{q}^{\,\mathrm{C6}}_{i-1/2} - \mathrm{sign}(1/60, u_{i-1/2})(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

+ SPLIT – UP3 and SPLIT – UP5 schemes ()

Horizontal Advective Schemes



Vertical Advective Schemes

- cppdefs_dev.h

```
/*
=======================================================================
    Select MOMENTUM VERTICAL advection scheme:
=======================================================================
*/
#ifdef UV_VADV_SPLINES  /* Check if options are defined in cppdefs.h */
#elif defined UV_VADV_C2
#else
# define UV_VADV_SPLINES    /* Splines vertical advection             */
# undef  UV_VADV_C2         /* 2nd-order centered vertical advection  */
#endif

#ifdef VADV_ADAPT_IMP       /* Semi-implicit vertical advection       */
# undef  VADV_ADAPT_PRED    /* apply to both pred/corr steps (choice) */
# define UV_VADV_SPLINES    /* Impose splines advection (no choice)   */
# undef  UV_VADV_C2
#endif
```

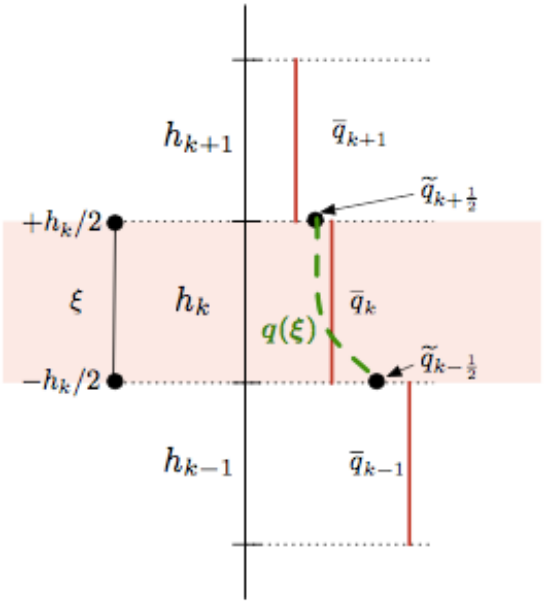
```
/*
=======================================================================
    Select model dynamics for TRACER vertical advection
    (The default is 4th-order centered)
=======================================================================
*/
#ifdef TS_VADV_SPLINES  /* Check if options are defined in cppdefs.h */
#elif defined TS_VADV_AKIMA
#elif defined TS_VADV_WENO5
#elif defined TS_VADV_C2
#else
# undef  TS_VADV_SPLINES    /* Splines vertical advection             */
# define TS_VADV_AKIMA      /* 4th-order Akima vertical advection     */
# undef  TS_VADV_WENO5      /* 5th-order WENOZ vertical advection     */
# undef  TS_VADV_C2         /* 2nd-order centered vertical advection */
#endif

#undef  TS_VADV_FCT         /* Flux correction of vertical advection */

#ifdef VADV_ADAPT_IMP
# define  TS_VADV_SPLINES
# undef   TS_VADV_AKIMA
# undef   TS_VADV_WENO5
# undef   TS_VADV_C2
#endif
```

Vertical Advective Schemes

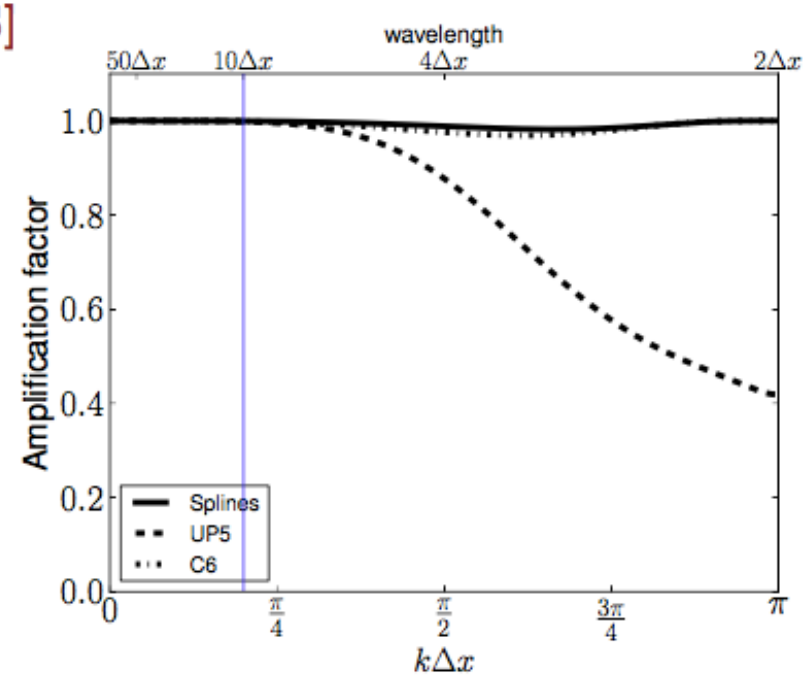
[TS_VADV_SPLINES, UV_VADV_SPLINES]

Les flux sont obtenus comme solution du problème tridiagonal

$$\mathrm{Hz}_{k+1}\widetilde{q}_{k-1/2} + 2(\mathrm{Hz}_k + \mathrm{Hz}_{k+1})\widetilde{q}_{k+1/2} + \mathrm{Hz}_k\widetilde{q}_{k+3/2} = 3(\mathrm{Hz}_k\overline{q}_{k+1} + \mathrm{Hz}_{k+1}\overline{q}_k)$$



$$q(\xi) = \overline{q}_k + \frac{\widetilde{q}_{k+\frac{1}{2}} - \widetilde{q}_{k-\frac{1}{2}}}{h_k}\xi + 6\left(\frac{\widetilde{q}_{k+\frac{1}{2}} + \widetilde{q}_{k-\frac{1}{2}}}{2} - \overline{q}_k\right)\left(\frac{\xi^2}{h_k^2} - \frac{1}{12}\right)$$



Vertical Advective Schemes

[VADV_ADAPT_IMP]

Idée : séparer la vitesse verticale Ω en une contribution explicite et une contribution implicite [Shchepetkin, 2015]

$$\Omega = \mathbf{\Omega^{(e)}} + \mathbf{\Omega^{(i)}}, \qquad \mathbf{\Omega^{(e)}} = \frac{\Omega}{f(\alpha^z_{\text{adv}}, \alpha_{\max})}, \qquad f(\alpha^z_{\text{adv}}, \alpha_{\max}) = \begin{cases} 1, & \alpha^z_{\text{adv}} \leq \alpha_{\max} \\ \alpha/\alpha_{\max}, & \alpha^z_{\text{adv}} > \alpha_{\max} \end{cases}$$

→ $\mathbf{\Omega^{(e)}}$ intégré avec un schéma explicite (avec CFL $\alpha_{\max}$)

→ $\mathbf{\Omega^{(i)}}$ intégré avec une schéma d'Euler upwind implicite

Configuration	Résolution	ancien Δt	nouveau Δt
BENGUELA [Penven et al.]	25 km	6300 s	7140 s
OMAN [Vic et al.]	2 km	160 s	470 s

- Pragmatique mais aucun contrôle sur les erreurs
- Impose l'utilisation d'un schéma linéaire pour la partie explicite

Vertical Advective Schemes

[TS_VADV_AKIMA] (compute_vert_tracer_fluxes.h)

$$\begin{aligned} \widetilde{q}^{C4}_{k-1/2} &= (7/6)\widetilde{q}^{C2}_{k-1/2} - (1/12)(q_{k+1} + q_{k-2}) \\ &= \widetilde{q}^{C2}_{k-1/2} - \frac{1}{6}(d_k - d_{k-1}), \qquad d_k = \frac{\Delta q_{k+1/2} + \Delta q_{k-1/2}}{2} \end{aligned}$$

pour le schéma AKIMA on remplace la moyenne algébrique des pentes par la moyenne harmonique

$$d_k = \begin{cases} \dfrac{2}{\frac{1}{\Delta q_{k+1/2}} + \frac{1}{\Delta q_{k-1/2}}} & \text{si } \Delta q_{k+1/2}\Delta q_{k-1/2} > 0 \\ 0 & \text{sinon} \end{cases}$$

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Activate explicit horizontal viscosity

See uv3dmix.F

UV_VIS2 = Laplacian

UV_VIS4 = bilaplacian

UV_VIS_SMAGO = Smagorinsky parametrization of turbulent viscosity

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Activate explicit horizontal mixing of tracers

See t3dmix.F

TS_DIFF2 = Laplacian

TS_DIFF4 = bilaplacian

TS_MIX_S = mixing along iso-sigma surfaces

TS_MIX_GEO = mixing along geopotential surfaces

TS_MIX_ISO = mixing along isopycnal surfaces

Horizontal viscosity/mixing

[UV_VIS2] Tenseur visqueux (uv3dmix.F)

$$\boldsymbol{\sigma}(\mathbf{u}_h) = \begin{pmatrix} \partial_x u - \partial_y v & \partial_y u + \partial_x v \\ \partial_x v + \partial_y u & -(\partial_x u - \partial_y v) \end{pmatrix}$$

l'opérateur de viscosité est donc donné par

$$-\boldsymbol{\nabla}_h \cdot \left\langle \mathbf{u}'_h \mathbf{u}'_h \right\rangle = \frac{1}{\mathrm{Hz}} \boldsymbol{\nabla}_h \cdot (A_M \ \mathrm{Hz}\ \boldsymbol{\sigma}) \,, \qquad A_M \leftrightarrow \mathrm{visc2}$$

Cette formulation assure

1. La conservation de la quantité de mouvement
2. La conservation du moment angulaire
3. Le terme visqueux est strictement dissipatif

[UV_VIS4] Même logique appliquée 2 fois ($B_M \leftrightarrow \mathrm{visc4}$)

$$-\boldsymbol{\nabla}_h \cdot \left\langle \mathbf{u}'_h \mathbf{u}'_h \right\rangle = -\frac{1}{\mathrm{Hz}} \boldsymbol{\nabla}_h \cdot (B_M \ \mathrm{Hz}\ \boldsymbol{\sigma}') \,, \qquad \boldsymbol{\sigma}' = \boldsymbol{\sigma}(\boldsymbol{\nabla}_h \cdot \boldsymbol{\sigma}(\mathbf{u}_h))$$

Horizontal viscosity/mixing

[UV_VIS_SMAGO, UV_VIS2]

Coefficient de viscosité turbulente

$$A_M = C_M \, (\Delta x \Delta y) \sqrt{(\partial_x u)^2 + (\partial_y v)^2 + 2(\partial_y u + \partial_x v)^2}$$

Par défaut $C_M = 1/10$ (paramétre *horcon* dans la routine hvisc_coef)

[TS_DIF_SMAGO, TS_DIF2]

Coefficient de diffusion turbulente

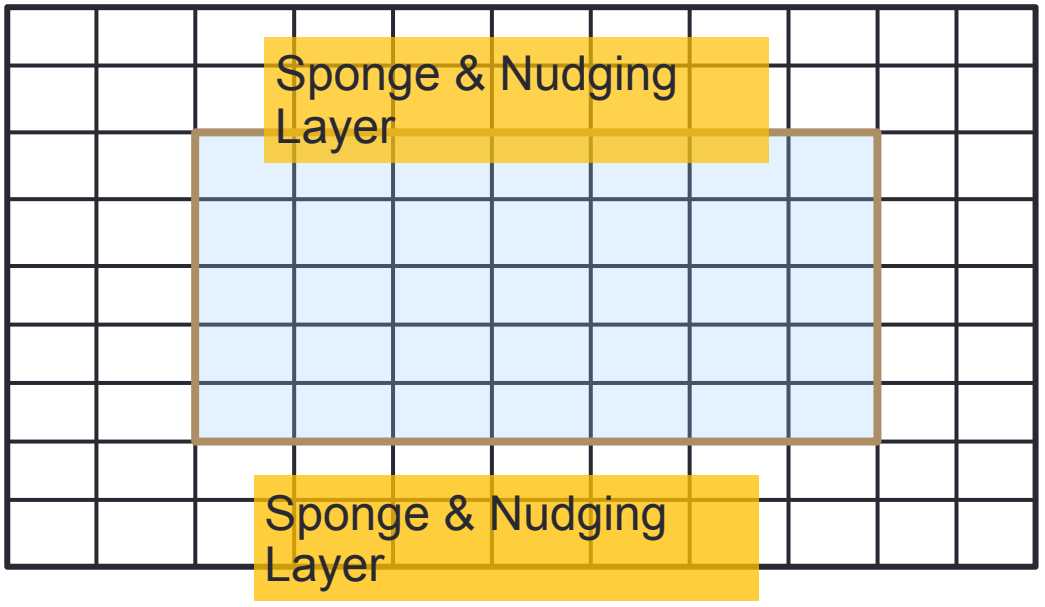
$$A_S = C_S \, (\Delta x \Delta y) \sqrt{(\partial_x u)^2 + (\partial_y v)^2 + 2(\partial_y u + \partial_x v)^2}$$

Par défaut $C_S = 1/12$ (paramétre *horcon* dans la routine hdiff_coef)

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Activate areas of enhanced viscosity and diffusivity near lateral open boundaries.



cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Apply surface and bottom stresses as body-forces

cppdefs.h

```
                        /* Lateral Momentum Advection (default UP3) */
# define UV_HADV_UP3
# undef  UV_HADV_C4
# undef  UV_HADV_C2
                        /* Lateral Tracer Advection (default UP3) */
# undef  TS_HADV_UP3
# define TS_HADV_RSUP3
# undef  TS_HADV_UP5
# undef  TS_HADV_C4
# undef  TS_HADV_WENO5
                        /* Lateral Explicit Momentum Mixing */
# undef  UV_VIS2
# ifdef UV_VIS2
#  define UV_VIS_SMAGO
# endif
                        /* Lateral Explicit Tracer Mixing */
# undef  TS_DIF2
# undef  TS_DIF4
# undef  TS_MIX_S
                        /* Sponge layers for UV and TS */
# define SPONGE
                        /* Semi-implicit Vertical Tracer/Mom Advection */
# undef  VADV_ADAPT_IMP
                        /* Vertical Mixing */
# undef  BODYFORCE
# undef  BVF_MIXING
# define LMD_MIXING
# undef  GLS_MIXING
# ifdef LMD_MIXING
#  define LMD_SKPP
#  define LMD_BKPP
#  define LMD_RIMIX
#  define LMD_CONVEC
#  undef  LMD_DDMIX
#  define LMD_NONLOCAL
# endif
# ifdef GLS_MIXING
#  define GLS_KKL
#  undef  GLS_KOMEGA
#  undef  GLS_KEPSILON
#  undef  GLS_GEN
#  undef  KANTHA_CLAYSON
#  undef  CRAIG_BANNER
#  undef  CANUTO_A
#  undef  ZOS_HSIG
# endif
```

Vertical Mixing Parameterization

BVF_MIXING = Simple mixing scheme based on the Brunt-Väisälä frequency

LMD_MIXING = Large/McWilliams/Doney mixing (turbulent closure for interior and planetary boundary layers) = KPP

GLS_MIXING = Generic Length Scale scheme

Vertical Mixing Parameterization

- Equations:

$$\begin{aligned}
\frac{D\langle \mathbf{u}_h \rangle}{Dt} + f\mathbf{k} \times \langle \mathbf{u}_h \rangle &= \frac{\boldsymbol{\nabla}_h p}{\rho_0} - \boldsymbol{\nabla} \cdot \left\langle \mathbf{u}_h' \mathbf{u}_h' \right\rangle - \partial_z \left\langle w' \mathbf{u}_h' \right\rangle \\
\partial_z p &= -g\rho' \\
\boldsymbol{\nabla} \cdot \langle \mathbf{u} \rangle &= 0 \\
\frac{D\langle T \rangle}{Dt} &= -\frac{\partial_z Q_s}{\rho_0 C_{p,o}} - \boldsymbol{\nabla}_h \cdot \left\langle \mathbf{u}_h' T' \right\rangle - \partial_z \left\langle w' T' \right\rangle \\
\frac{D\langle S \rangle}{Dt} &= -\boldsymbol{\nabla}_h \cdot \left\langle \mathbf{u}_h' S' \right\rangle - \partial_z \left\langle w' S' \right\rangle \\
\rho &= \rho_{\mathrm{eos}}(\langle T \rangle, \langle S \rangle, z)
\end{aligned}$$

- Vertical mixing parameterization gives:

$$\begin{aligned}
-\left\langle \mathbf{u}_h' w' \right\rangle (z) &= K_m(z) \partial_z \langle \mathbf{u}_h \rangle \\
-\left\langle T' w' \right\rangle (z) &= K_s(z) \partial_z \langle T \rangle
\end{aligned}$$

Vertical Mixing Parameterization

[BVF_MIXING]

- Si $N^2(z) < 0$:

$$K_{m,s}(z) = 0.1 \text{ m}^2 \text{ s}^{-1}$$

- Si $N^2(z) > 0$:

$$K_{m,s}(z) = 10^{-7}/\sqrt{N^2(z)}, \qquad K_{m,s}^{\min} \leq K_{m,s}(z) \leq K_{m,s}^{\max}$$

Bornes par défaut très restrictives :

$$K_{m,s}^{\min} = 3 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}, \qquad K_{m,s}^{\max} = 4 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

[ANA_VMIX]

Possibilité de spécifier analytiquement $K_{m,s}(z)$
(subroutine ana_vmix appelée à chaque pas de temps)

Vertical Mixing Parameterization

LMD_MIXING at surface

[LMD_SKPP1994, LMD_SKPP_MONOB, HBL_SMOOTH]

1. Calcul de la profondeur de couche limite h_{bl} $(z_r \to z_N)$

$$\mathrm{Ri}_b(z) = \frac{g(z_r - z)\,(\rho(z) - \rho_r)\,/\rho_0}{|\mathbf{u}(z) - (\mathbf{u}_h)_r|^2 + V_t^2(z)}, \qquad \mathrm{Ri}_b(-h_{bl}) = \mathrm{Ri}_{cr}$$

2. Dans le cas stable $(B_f > 0)$: $h_{bl} = \min(h_{bl}, h_{ek}, h_{mo})$

$$h_{ek} = 0.7u_\star/f, \qquad h_{mo} = u_\star^3/(\kappa B_f).$$

3. Calcul des viscosités et diffusivités turbulentes

$$K_{m,s}(z) = w_{m,s}\, h_{bl}\, G(z/h_{bl}), \qquad w_{m,s} = \kappa\, u_\star\, \psi_{m,s}(zB_f/u_\star^3)$$

Choix de Ri_{cr} ? $\qquad \mathrm{Ri}_{cr} \in [0.15, 0.45]$

Vertical Mixing Parameterization

LMD_MIXING at surface

[LMD_SKPP2005, LIMIT_UNSTABLE_ONLY, HBL_SMOOTH]

- Critère pour h_{bl} : couche intégrale où la production de turbulence par cisaillement équilibre la dissipation due à la stratification

$$\mathrm{Cr}(z) = \int_z^{\zeta} \mathcal{K}(z') \left\{ |\partial_{z'} \mathbf{u}_h|^2 - \frac{N^2}{\mathrm{Ri}_{cr}} - C_{Ek}\, f^2 \right\} dz' + \frac{V_t^2(z)}{(\zeta - z)}, \; \mathrm{Cr}(-h_{bl}) = 0$$

- Consistant avec la physique originelle de KPP

$$\mathrm{Cr}(-h_{bl}) = 0 \Rightarrow \frac{(\zeta - z) \int_z^{\zeta} \mathcal{K}(z') N^2(z') dz'}{(\zeta - z) \int_z^{\zeta} \mathcal{K}(z') \left\{ |\partial_z \mathbf{u}_h|^2 - C_{Ek}\, f^2 \right\} dz' + V_t^2(z)} = \mathrm{Ri}_{cr}$$

→ cohérent avec problème d'Ekman

→ tend à produire des couches limites plus profondes
$(\zeta - z) \int_z^{\zeta} |\partial_{z'} \mathbf{u}_h|^2 \, dz' \geq |\mathbf{u}_h(z) - \mathbf{u}_h(\zeta)|^2.$

Vertical Mixing Parameterization

LMD_MIXING in the interior

$$K_{m,s}(z) = K^{\mathrm{sh}}_{m,s}(z) + K^{\mathrm{iw}}_{m,s}(z) + K^{\mathrm{dd}}_{m,s}(z)$$

- **[LMD_RIMIX, RI_(H − V)SMOOTH]** (Large et al., 1994)

$$\mathrm{Ri}_g = N^2 / \left[(\partial_z u)^2 + (\partial_z v)^2\right]$$

$$K^{\mathrm{sh}}_{m,s}(z) = \begin{cases} K_{0,c} & \mathrm{Ri}_g < 0 \quad \leftarrow \text{[LMD_CONVEC]} \\ K_0 \left[1 - (\mathrm{Ri}_g/\mathrm{Ri}_0)^3\right] & 0 < \mathrm{Ri}_g < \mathrm{Ri}_0 \\ 0 & \mathrm{Ri}_0 < \mathrm{Ri}_g \end{cases}$$

$K_0 = 5 \times 10^{-3}\ \mathrm{m^2\ s^{-1}}$, $\mathrm{Ri}_0 = 0.7$.

- **[LMD_NUW_GARGETT]** (Gargett & Holloway)

$$K^{\mathrm{iw}}_m(z) = {}^{10^{-6}}\!/\sqrt{\max(N^2(z), 10^{-7})}, \qquad K^{\mathrm{iw}}_s(z) = {}^{10^{-7}}\!/\sqrt{\max(N^2(z), 10^{-7})}$$

- **[LMD_DDMIX]** (cf Large et al., 1994, eqns (31))

Vertical Mixing Parameterization

LMD_MIXING at the bottom

[LMD_BKPP, HBBL_SMOOTH]

→ Même principe que pour le KPP de surface mais cette fois-ci on recherche la valeur critique $\mathrm{Ri}_{\mathrm{cr}}$ (≈ 0.3) en partant du fond

$$h_{\mathrm{bbl}} = \min\left(h_{\mathrm{bbl}}, \frac{0.7 u_{\star,b}}{|f|}\right)$$

$$K_{m,s}(z) = \kappa\, u_{\star,b}\, h_{bbl}\, G(\sigma), \qquad \sigma = {}^{(z-h)}/_{h_{bbl}}$$

Vertical Mixing Parameterization

[GLS_MIXING]

$$\begin{aligned} \partial_t e &= \partial_z \left(K_e \partial_z e \right) + P + B - \varepsilon, & K_e &= K_M / \mathrm{Sc}_e \\ \partial_t \psi &= \partial_z \left(K_\psi \partial_z \psi \right) + \psi e^{-1} \left(\beta_1 P + \beta_3^{\pm} B - \beta_2 \varepsilon \right), & K_\psi &= K_M / \mathrm{Sc}_\psi \end{aligned}$$

avec

$$\psi = \left(c_\mu^0 \right)^p e^m l^n, \qquad P = K_m \left[(\partial_z u)^2 + (\partial_z v)^2 \right], \qquad B = -K_s N^2$$

et

$$\varepsilon = \left(c_\mu^0 \right)^{3+p/n} e^{3/2+m/n} \psi^{-1/n}, \qquad l = \left(c_\mu^0 \right)^3 e^{3/2} \varepsilon^{-1}$$

$$K_m = \boldsymbol{c_\mu} \left(e^2 / \varepsilon \right), \qquad K_s = \boldsymbol{c_\mu}' \left(e^2 / \varepsilon \right)$$

Une instance de GLS se caractérise par :

- Un choix pour les exposants m, n et p (e.g. $k-\varepsilon$, $k-\omega$, $k-kl$, gen)
- Une valeur pour les coefficients β_1, β_2 et $\beta_3^{\pm}$
- Une valeur pour les nombres de Schmidt Sc_e et Sc_ψ