

Internal Waves in the Ocean

Master 2 – Physique de l’Océan et du Climat

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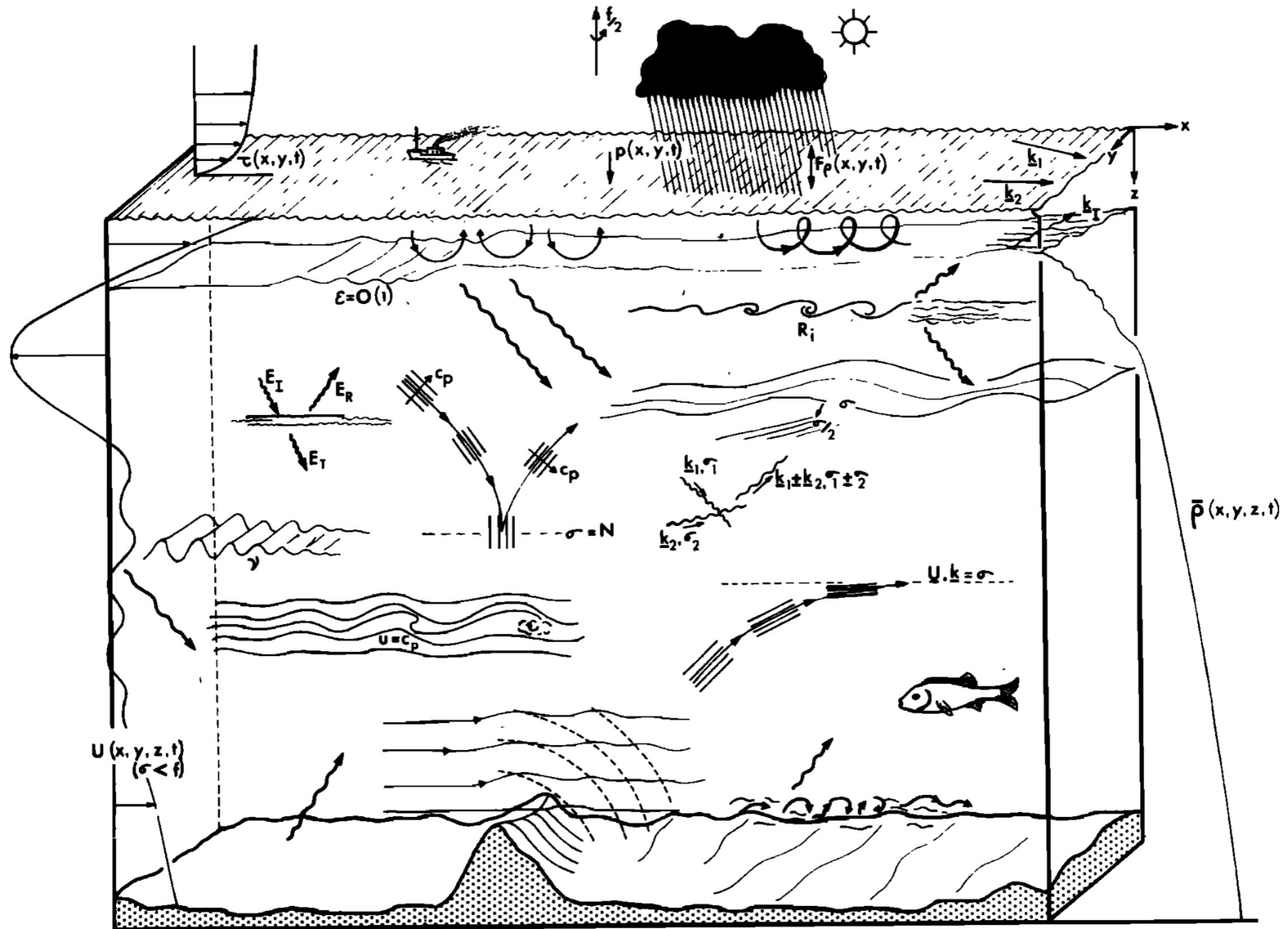
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Outline

1. A general introduction to ocean waves
2. What are internal waves ? Why do we study internal waves ?
3. Internal waves in the two-layer shallow-water model
4. Internal waves in the continuously-stratified model
5. Generation of internal waves
- 6. Propagation of internal waves**
7. Dissipation of internal waves and impacts

6. Propagation of internal waves

Physical processes affecting the propagation and dissipation of internal waves



6. Propagation of internal waves

Lifecycle of internal waves : generation —> propagation —> dissipation

Research topic : predicting the path of the internal wave energy.

Some notions of **ray tracing** : a method coming from geometrical optics where the path of the waves is modelled by assuming that the medium is “slowly varying”, spatially and temporally, compared to the wavelength and period of the waves.
(this corresponds to the WKB (Wentzel–Kramers–Brillouin) approximation.)

6. Propagation of internal waves

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- Let $\vec{U}(\vec{x}, t)$ be the background current and consider a wave with an intrinsic frequency ω_0 and wavenumber k . (“Intrinsic” means that the property is measured in a frame moving with the fluid)
- The frequency measured in a fixed frame (doppler-shifted frequency) is $\omega = \omega_0 + \vec{k} \cdot \vec{U}$ (modified dispersion relation)
Note that it can be written
 $\omega = \vec{k} \cdot (\vec{c}_0 + \vec{U})$, with \vec{c}_0 the intrinsic group velocity of the wave.

6. Propagation of internal waves

General dispersion relation :

$$\omega(\vec{x}, t) = \omega_0[\vec{k}(\vec{x}, t), \lambda(\vec{x}, t)] + \vec{k} \cdot \vec{U}$$

where ω_0 is the intrinsic dispersion relation and λ is a function of the medium ($\lambda = N(\vec{x}, t) + f(\vec{x})$ for internal waves) and \vec{U} is the background velocity.

The ray paths (corresponding to the wave energy), as well as the variations of the wavenumber k and the frequency ω along the ray path are then given by:

$$\frac{dx_i}{dt} = \frac{\partial \omega_0}{\partial k_i} + U_i \quad \text{for } x_i = x, y, z \quad \rightarrow \text{evolution of the position of energy}$$
$$\frac{dk_i}{dt} = - \frac{\partial \omega_0}{\partial \lambda} \frac{\partial \lambda}{\partial x_i} - \sum_j k_j \frac{\partial U_j}{\partial x_i} \quad \rightarrow \text{evolution of the wavenumber}$$
$$\frac{d\omega}{dt} = \frac{\partial \omega_0}{\partial \lambda} \frac{\partial \lambda}{\partial t} + \sum_j k_j \frac{\partial U_j}{\partial t} \quad \rightarrow \text{evolution of the intrinsic frequency}$$

6. Propagation of internal waves

Examples of ray tracing: 2-d horizontal, i.e., barotropic background current.

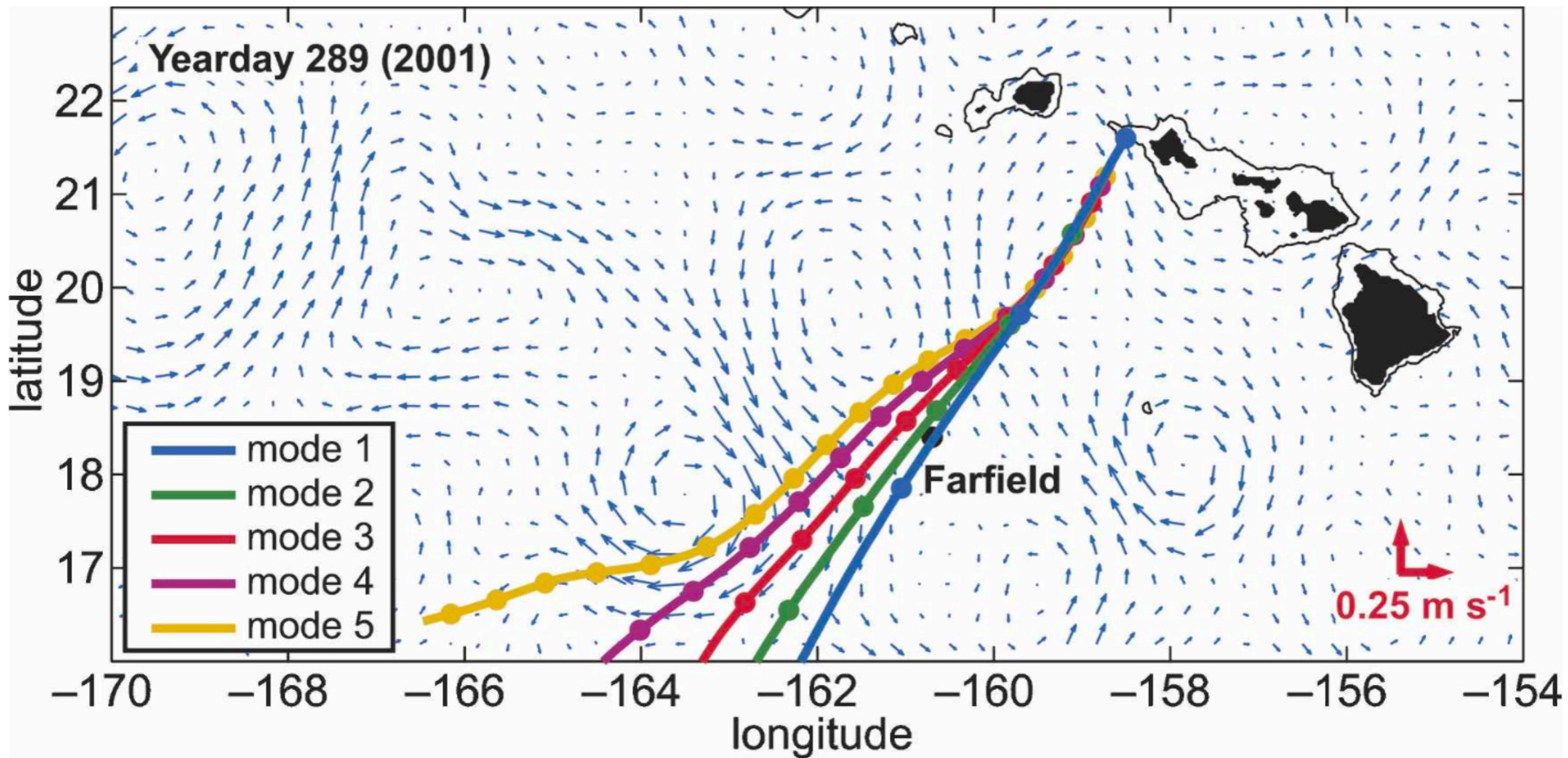


FIG. 10. Propagation paths of modes 1–5 generated in the Kauai Channel and traveling through the mesoscale eddy field seen during the week of 16 Oct 2001. Currents are derived from sea surface height measured by satellite altimeters (adjusted by a factor of $\frac{1}{2}$). Daily positions of the modes are indicated by the solid circles.

6. Propagation of internal waves

Another effect of the background current : shifting the phase of the wave (called incoherence)

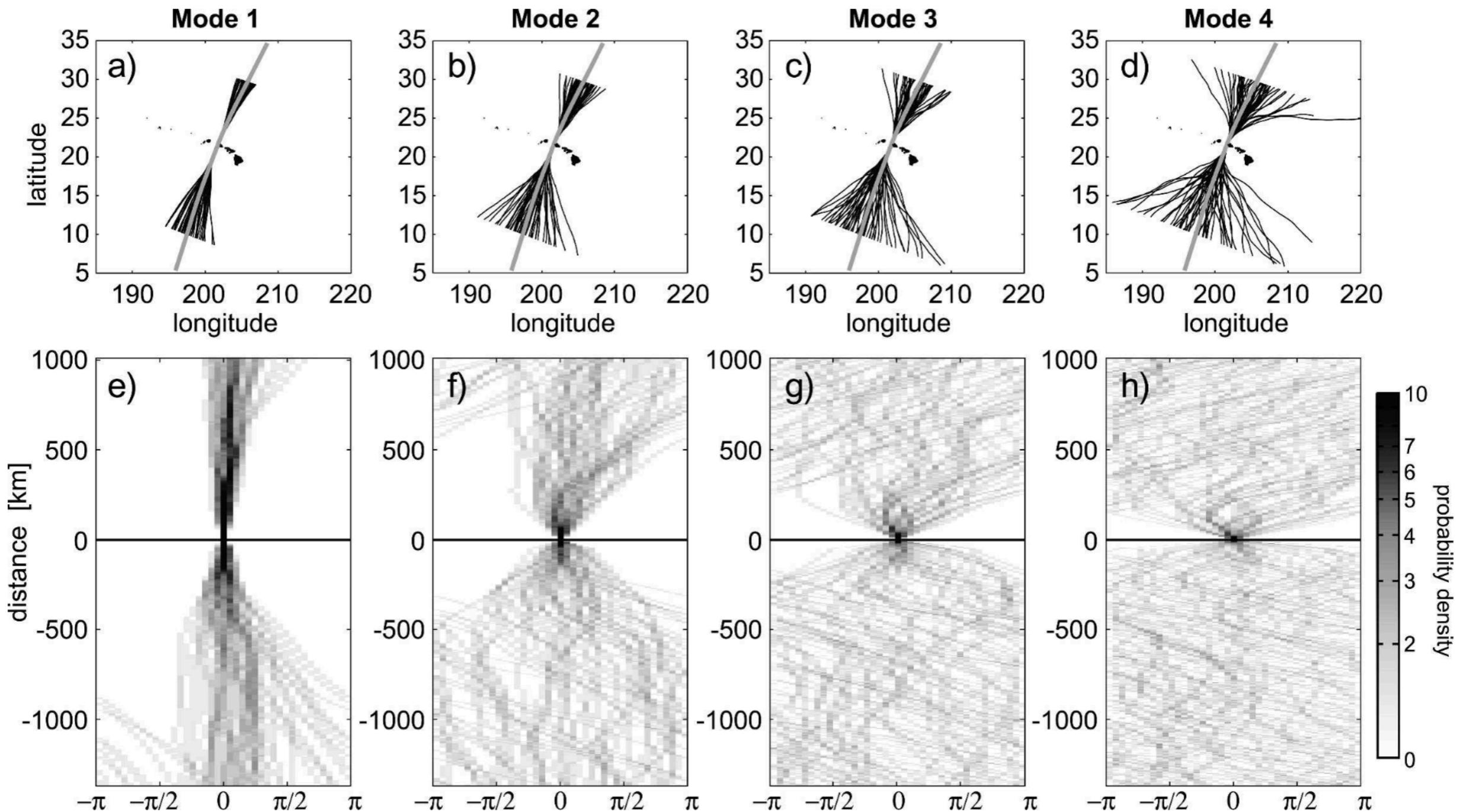


FIG. 13. (a)–(d) Paths of modes 1–4 propagating through the mesoscale eddy field for 52 weeks (September 2001–September 2002). Internal waves are generated in the Kauai Channel and launched along T/P track 112 (gray line). (e)–(h) Phase offsets (relative to no currents) as functions of distance plotted as probability densities.

6. Propagation of internal waves

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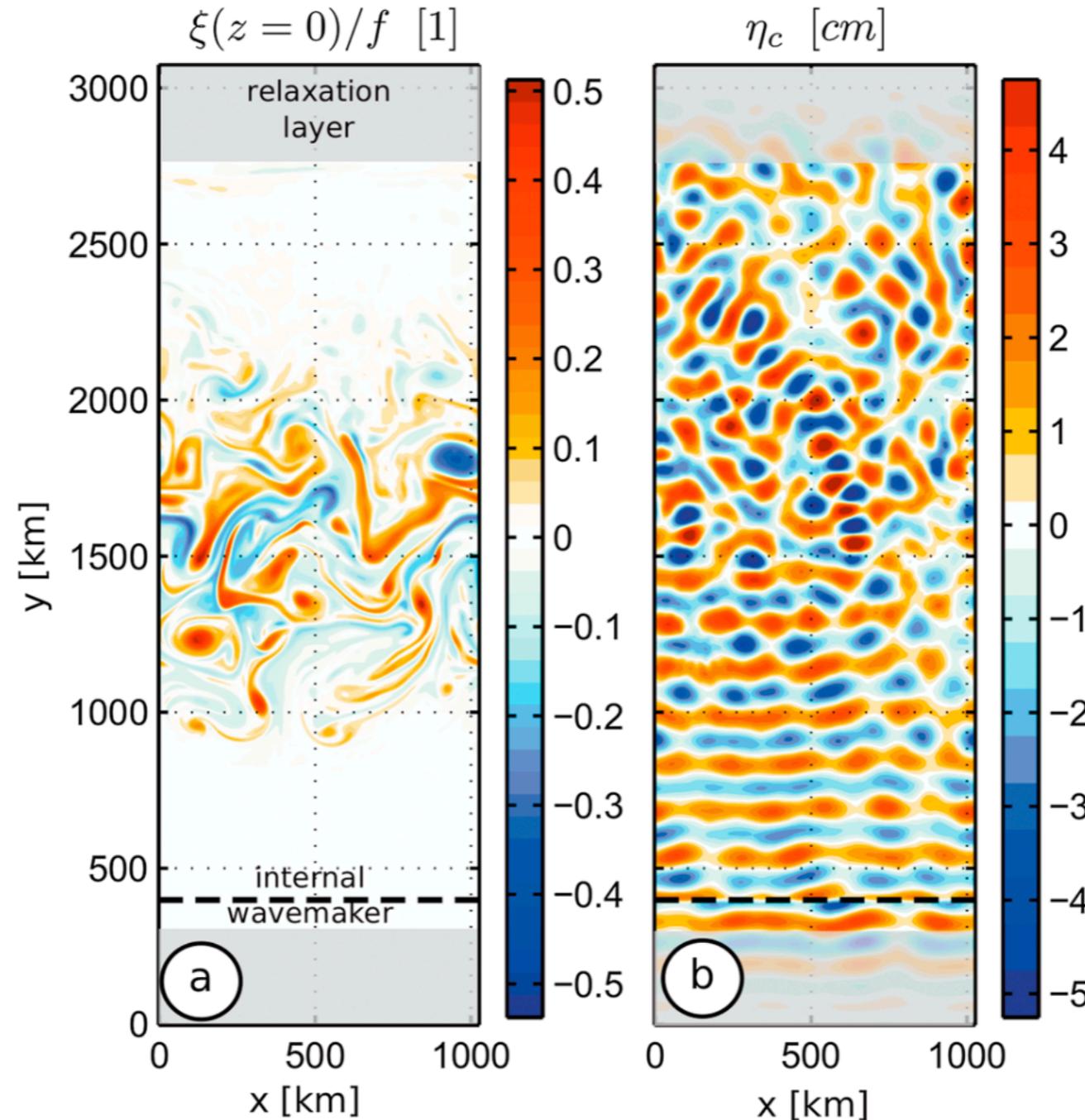


Figure 1. Overview of the numerical simulation KE4 (most intense mesoscale turbulence) at $t = 500$ days. (a) Detided surface relative vorticity $\xi(z = 0)$, normalized by the Coriolis frequency. (b) Tidal harmonic of sea level η_{\cos} . The location of the internal tide wavemaker is represented by the horizontal dashed line. Grey shadings represent areas where all fields are strongly relaxed toward initial values.

6. Propagation of internal waves

Examples of ray tracing: 3-d, shooting a wave beam through a mesoscale eddy.

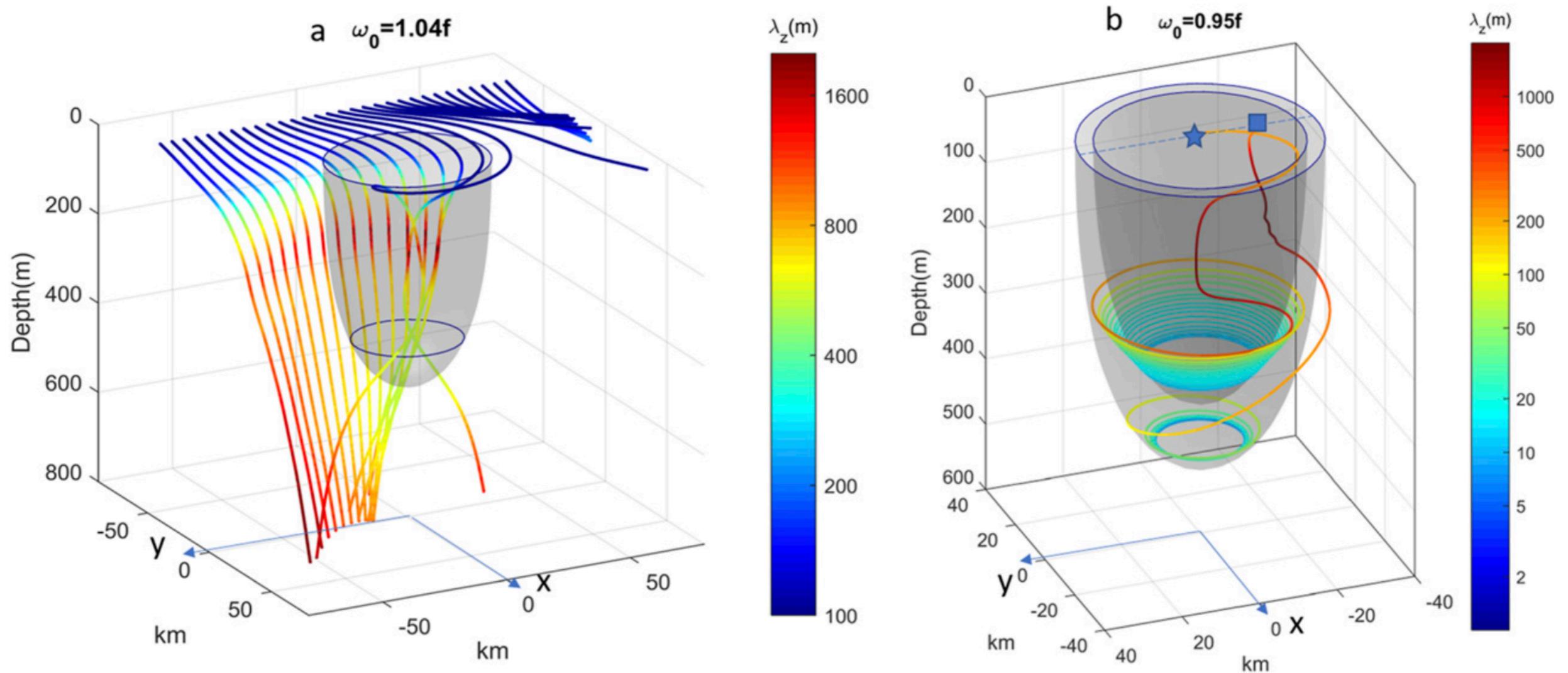
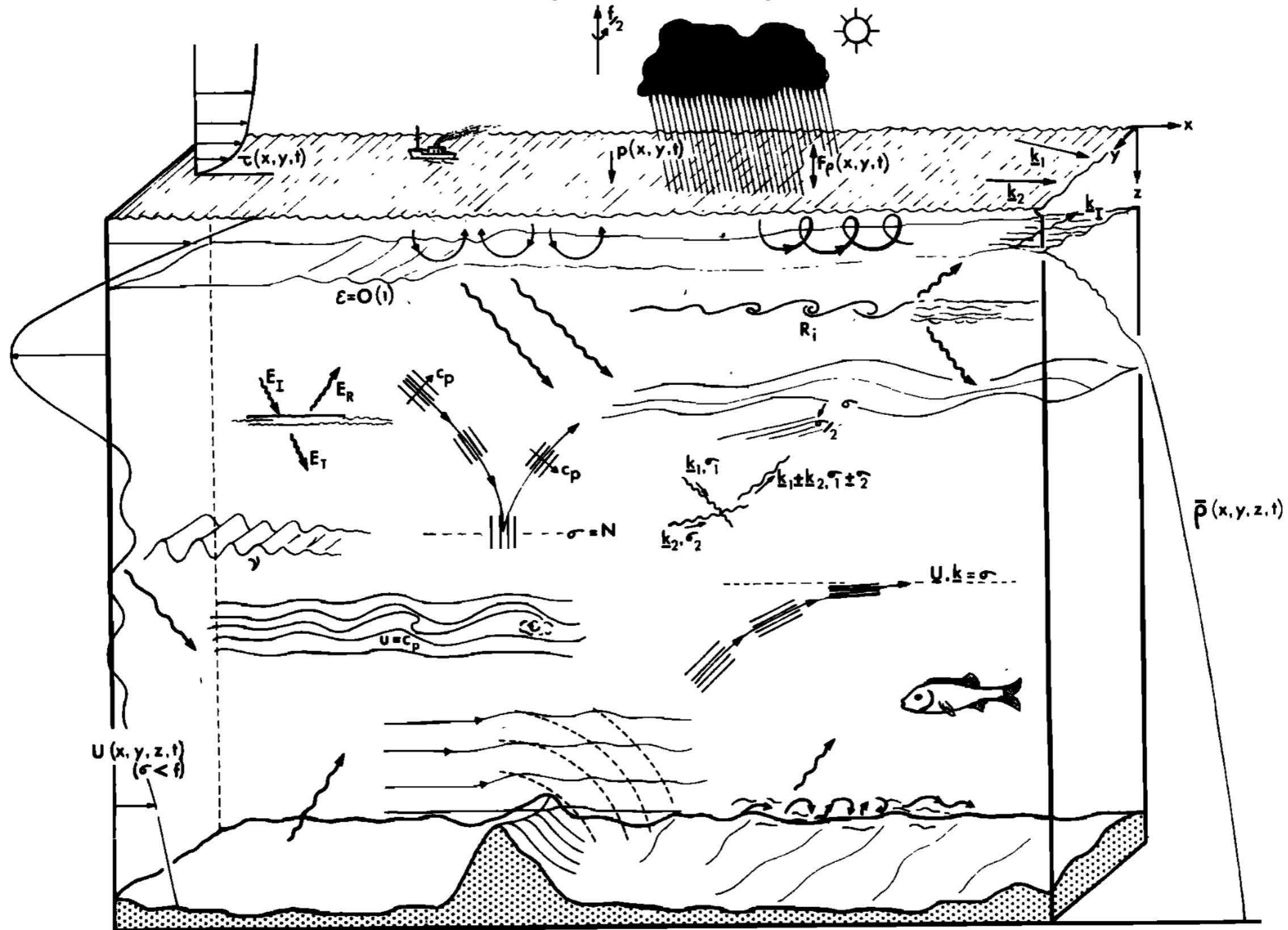


FIG. 13. (a) Ray tracing for slightly super inertial waves, $\omega_0 = 1.04f$, out of the negative vorticity core of the eddy with initial positions along the line $x = -60$ km. The colors represent the vertical wavelength, and the gray-shaded area is the $\zeta = -0.02f$, $f_e = 0.98f$ isosurface; (b) ray tracing for slightly subinertial waves $\omega_0 = 0.95f$ out of the negative vorticity core of the eddy with initial position along the line $x = 0$ km. The colors represent the vertical wavelength, and the gray-shaded areas are the $\zeta = -0.022f$, $f_e = 0.978f$ and $\zeta = -0.045f$, $f_e = 0.955f$ isosurfaces. For the sake of clarity, only two rays are plotted; the square and the star represent the rays' initial positions, respectively, at $x = -15$ km and $x = 1$ km.

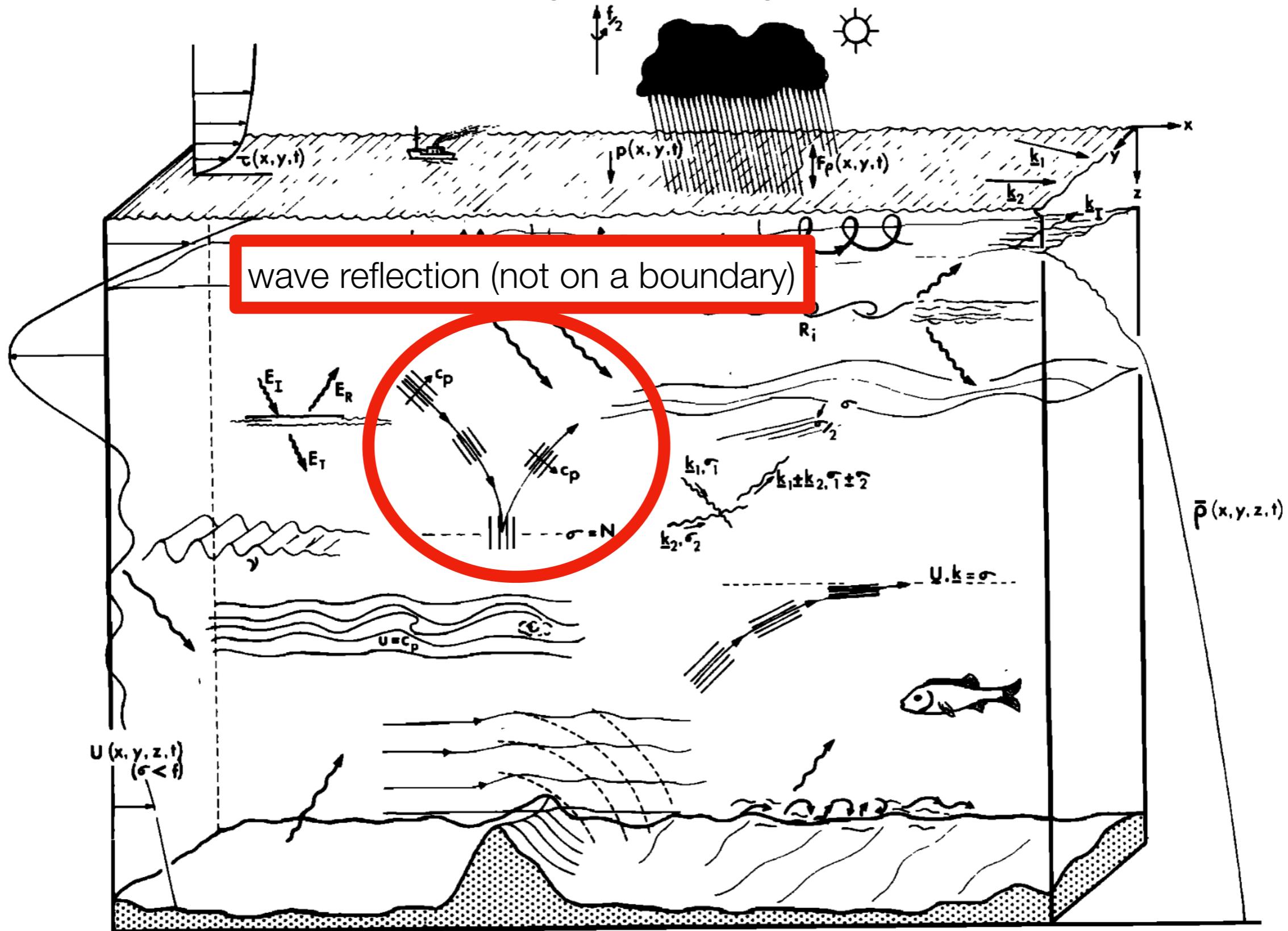
6. Propagation of internal waves

Illustration of some mechanisms affecting wave propagation



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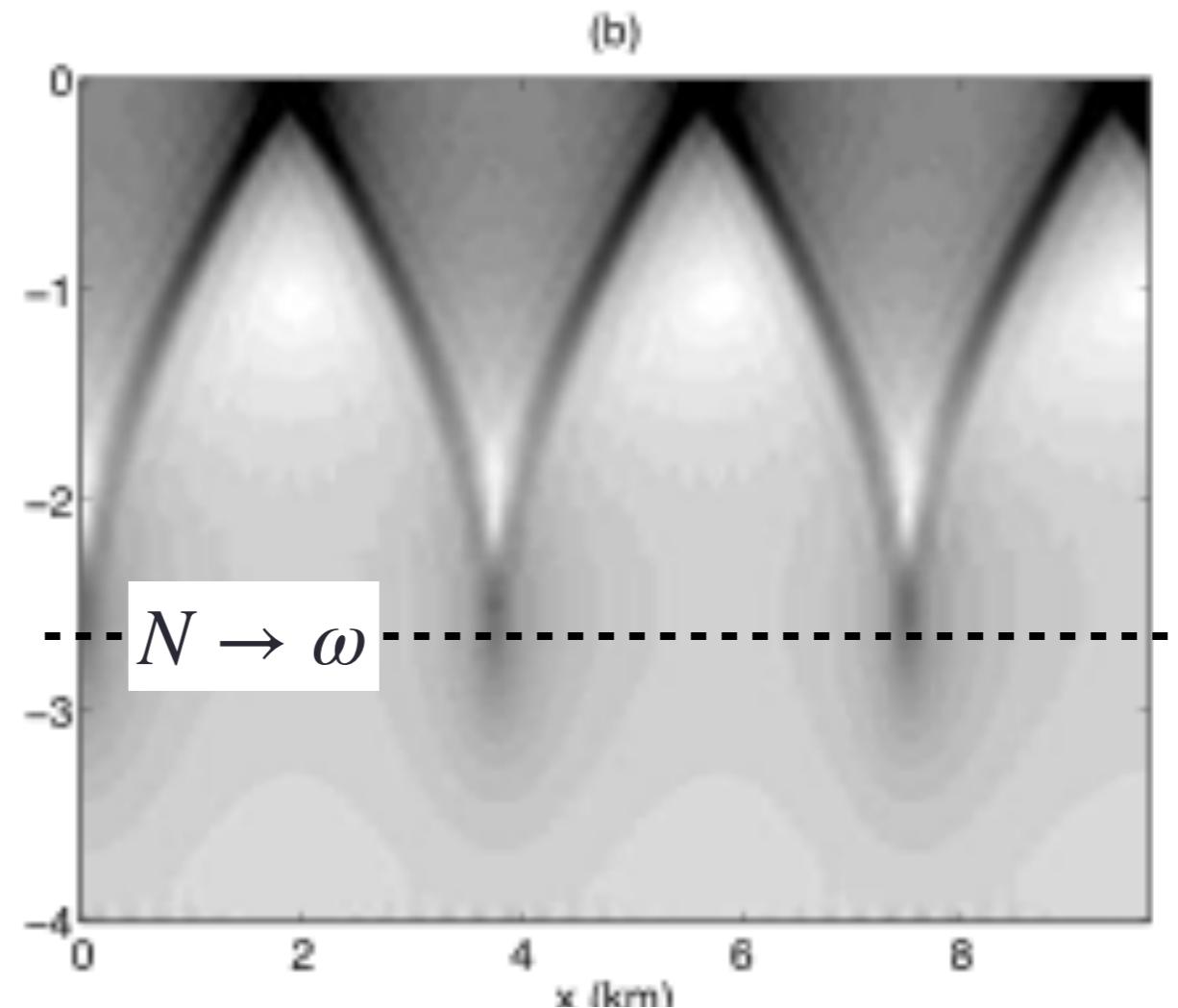
6. Propagation of internal waves

Remember the dispersion relation in the continuously-stratified case :

$$\omega^2 = \frac{N^2 k^2 + f^2 m^2}{k^2 + m^2}$$

That is valid in the cases :

$$N \leq \omega \leq |f| \text{ or } |f| \leq \omega \leq N$$

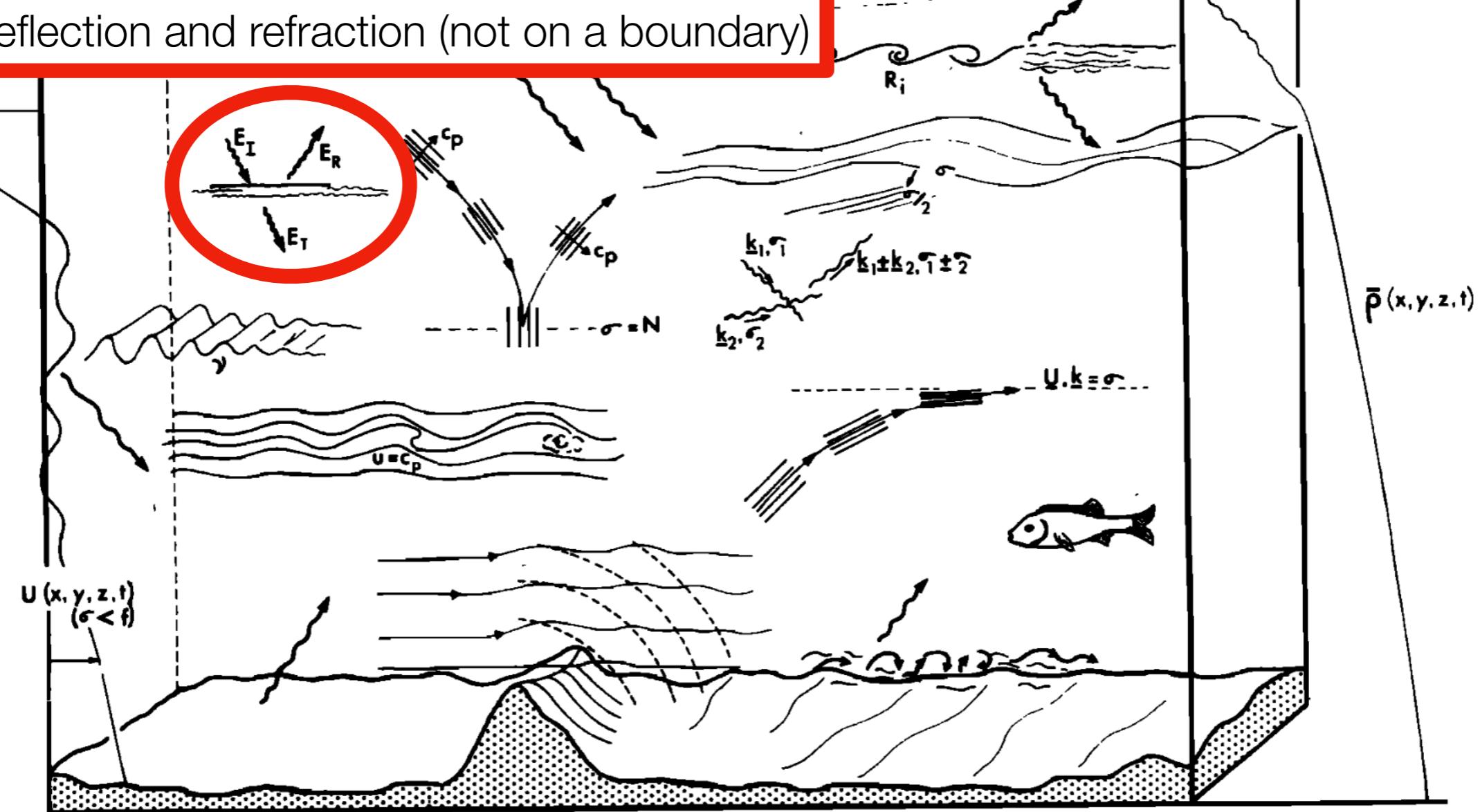
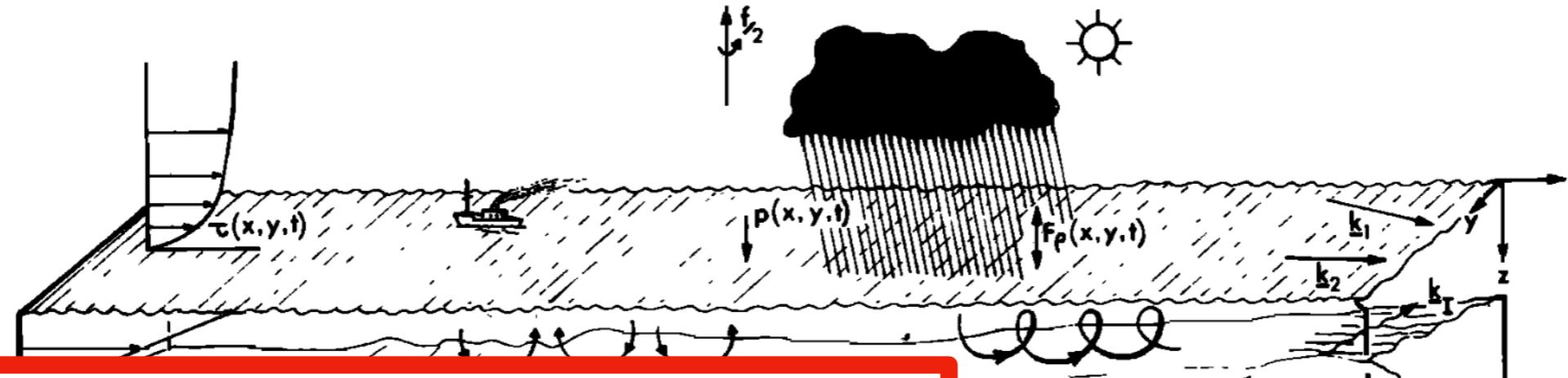


If a wave propagates into a region where $N \rightarrow \omega$ (frequency of the waves), it is reflected and its group velocity becomes vertical near this surface.

Note: they may be trapped into a waveguide if this happens below and above.

6. Propagation of internal waves

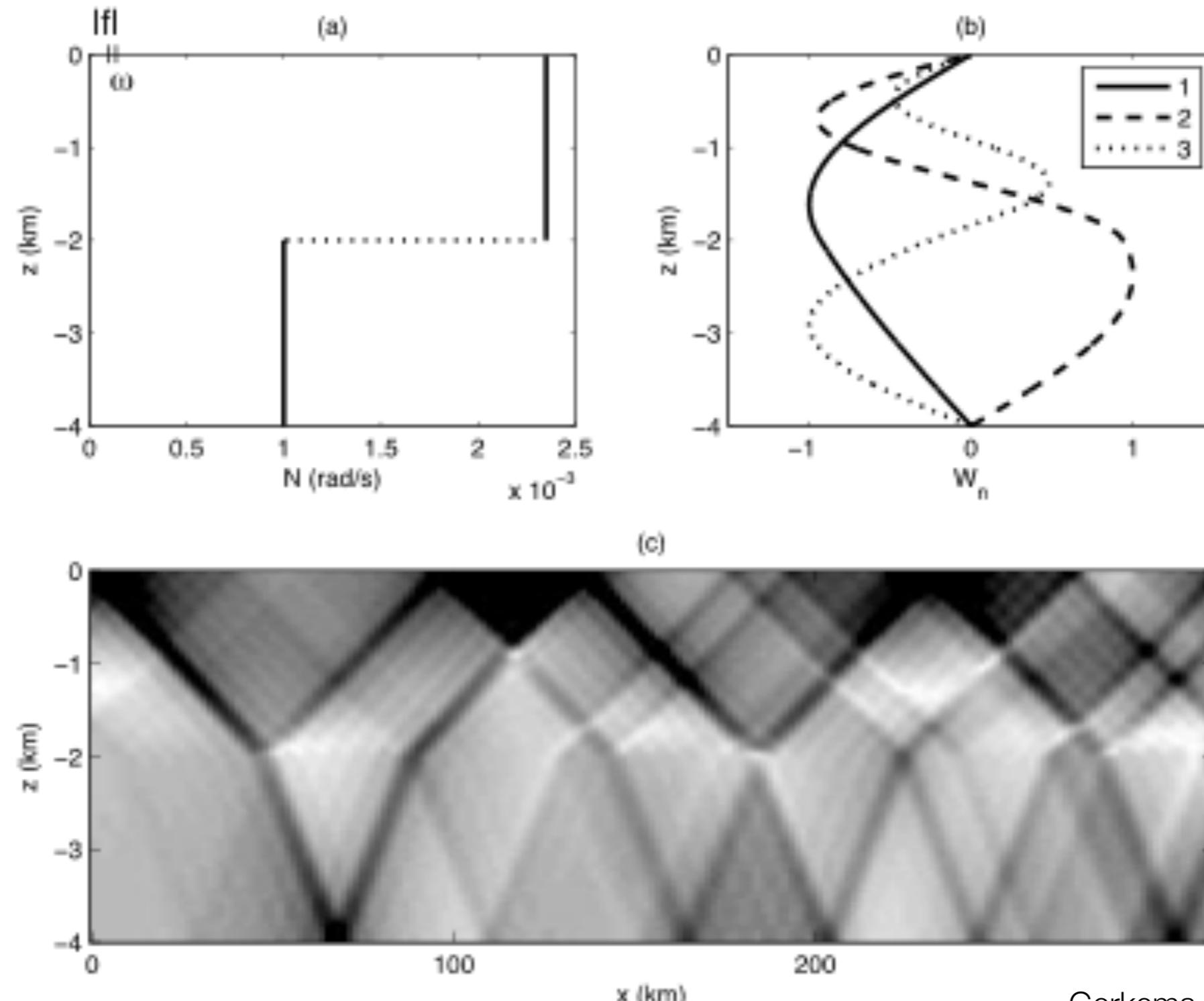
Illustration of some mechanisms affecting wave propagation



6. Propagation of internal waves

wave reflection and refraction (= change of direction of propagation)

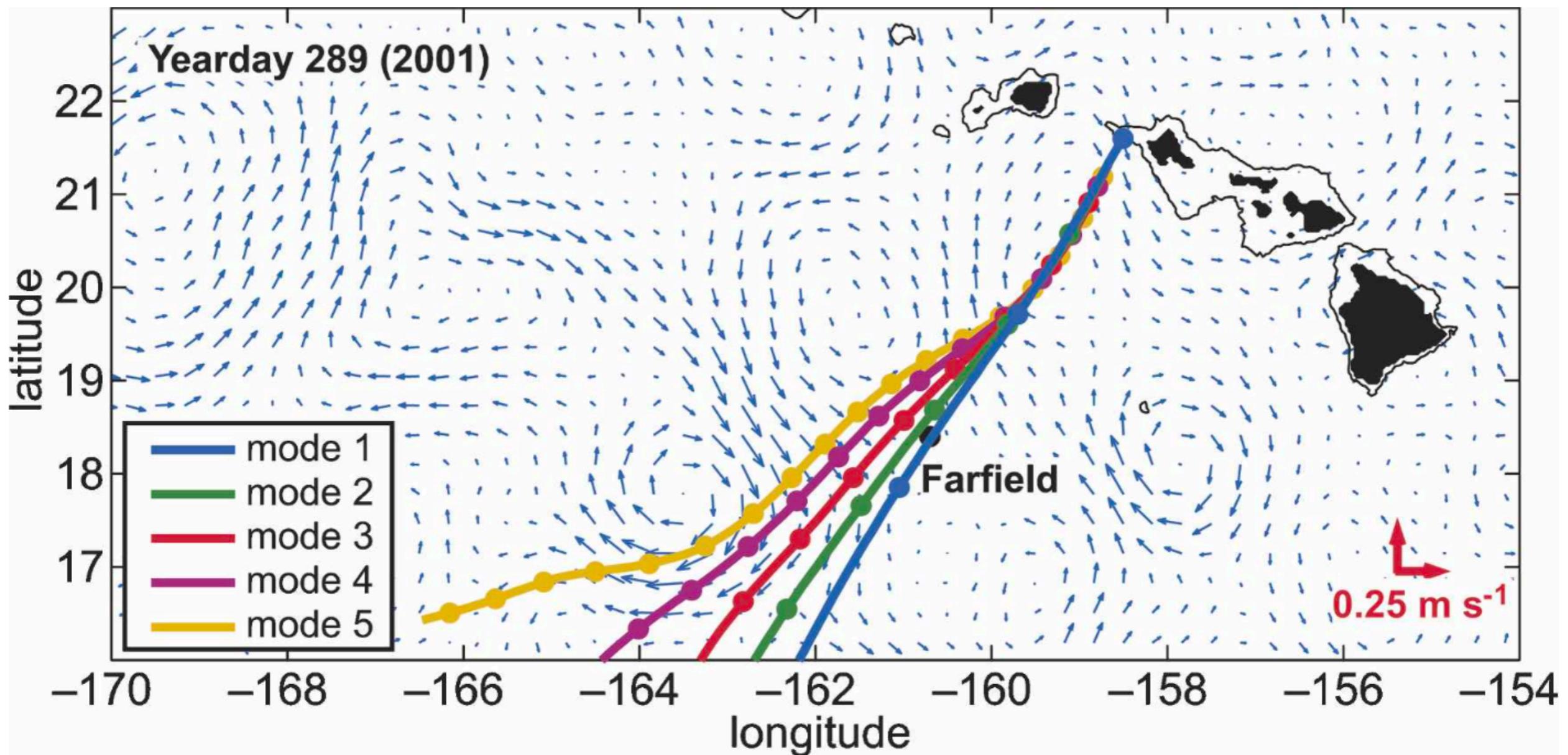
—> on the vertical plane, by a density jump



6. Propagation of internal waves

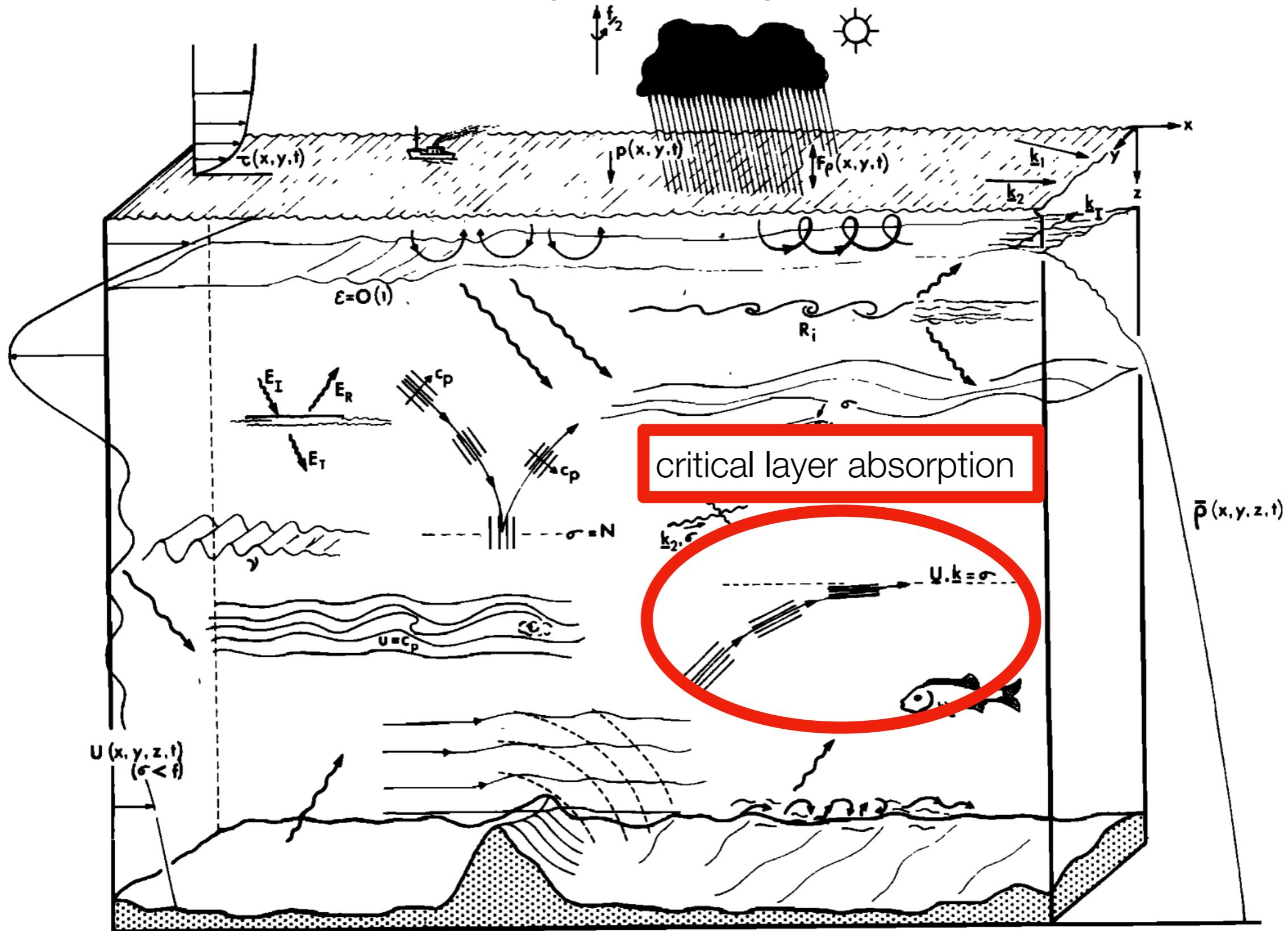
wave reflection and refraction (= change of direction of propagation)

—> on the horizontal plane, by the horizontal shear of a background current



6. Propagation of internal waves

Illustration of some mechanisms affecting wave propagation

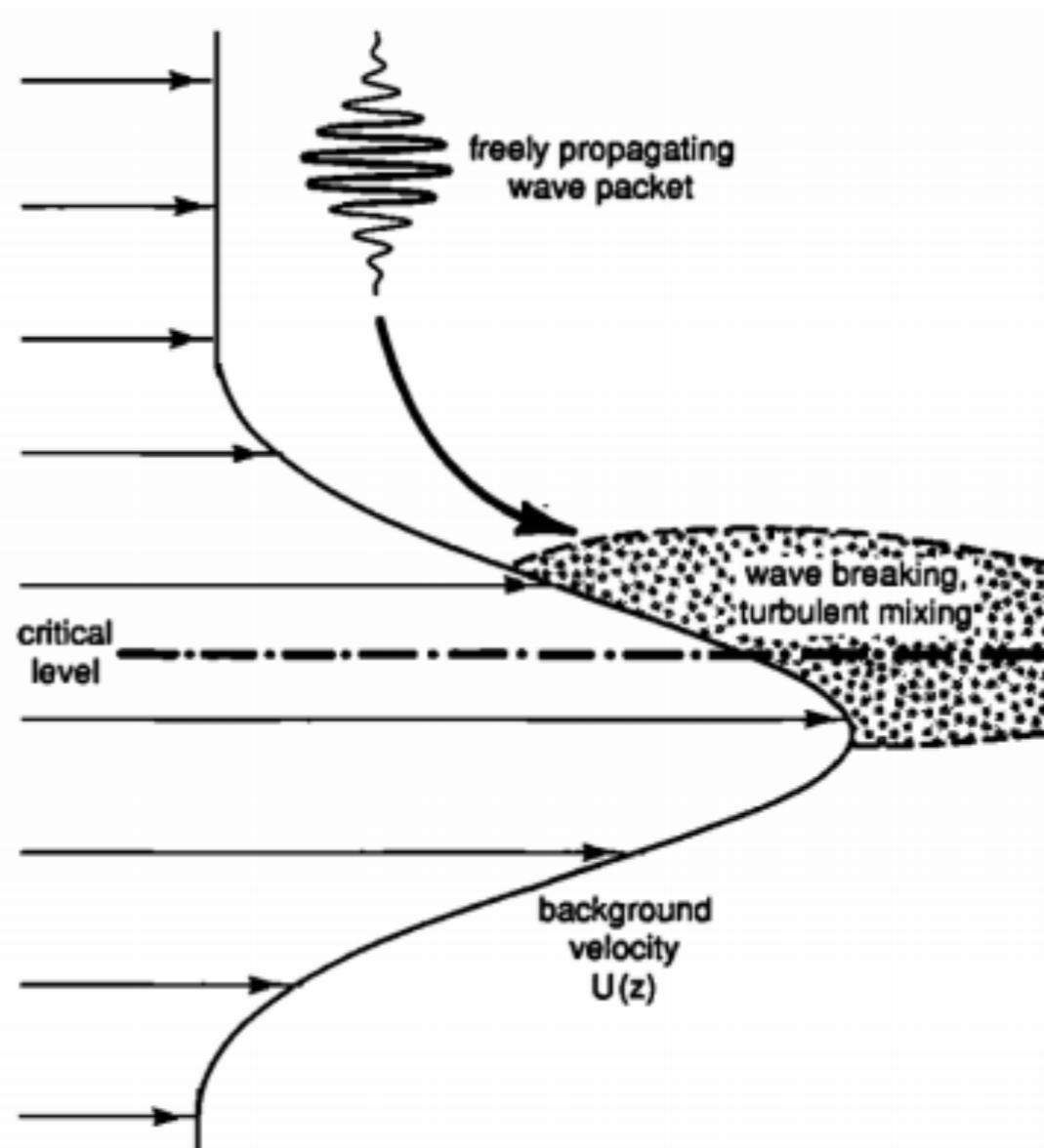


6. Propagation of internal waves

A critical layer is a region where the velocity of the flow is equal to the phase speed of the waves [Bretherton & Garrett, 1968]

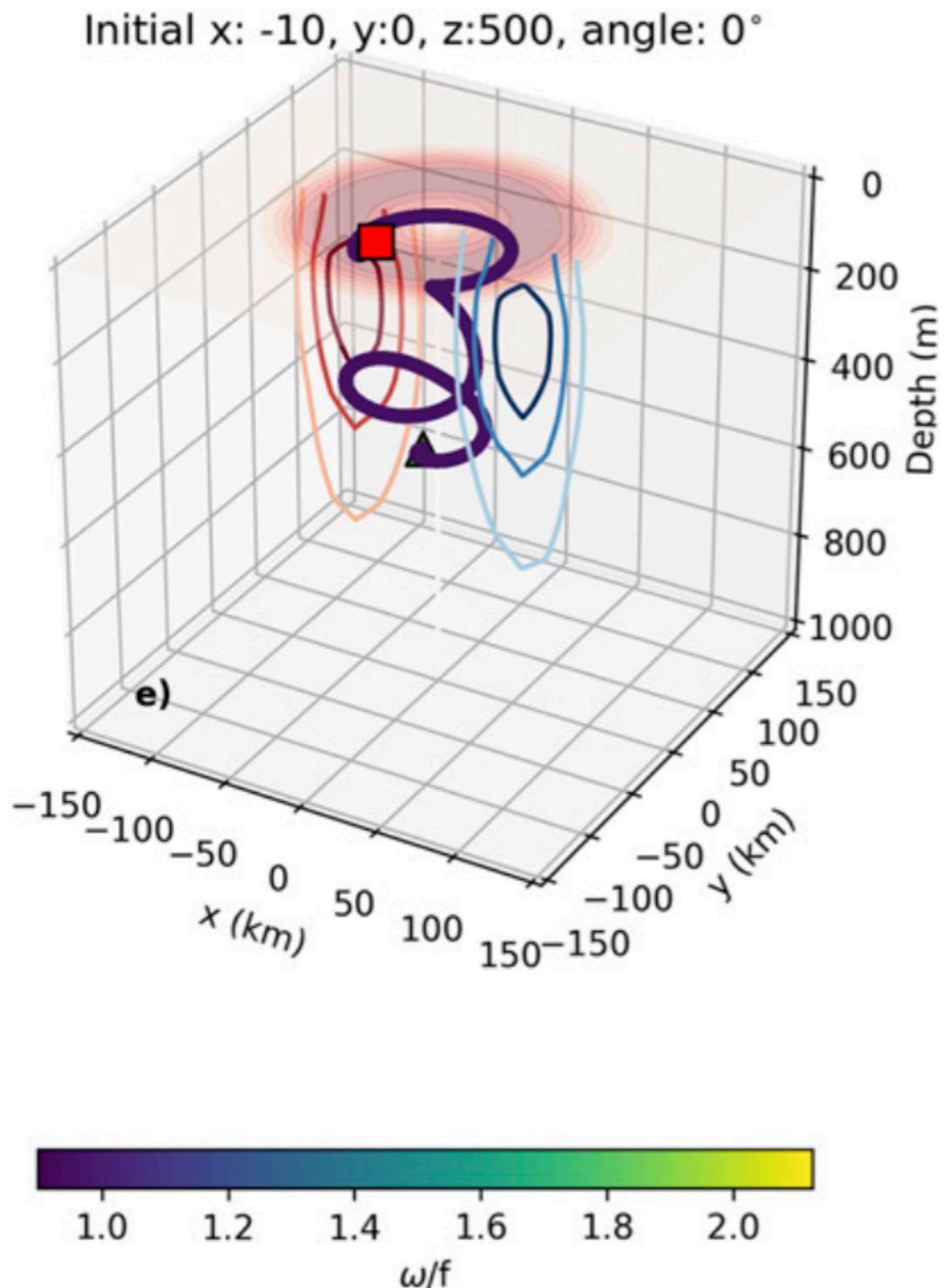
$$\omega = \vec{k} \cdot \vec{U}$$

At such level, energy is transferred to the mean flow, turbulence is generated, and the wave is strongly attenuated.



6. Propagation of internal waves

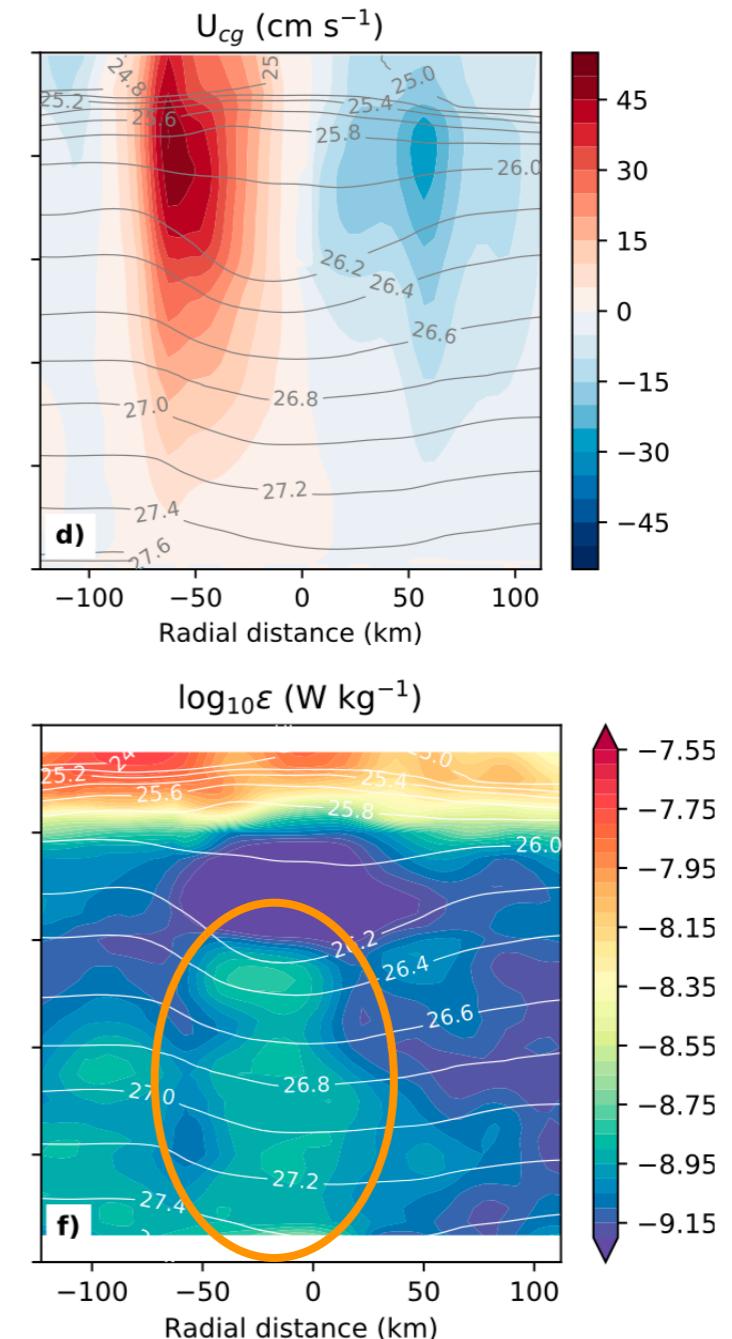
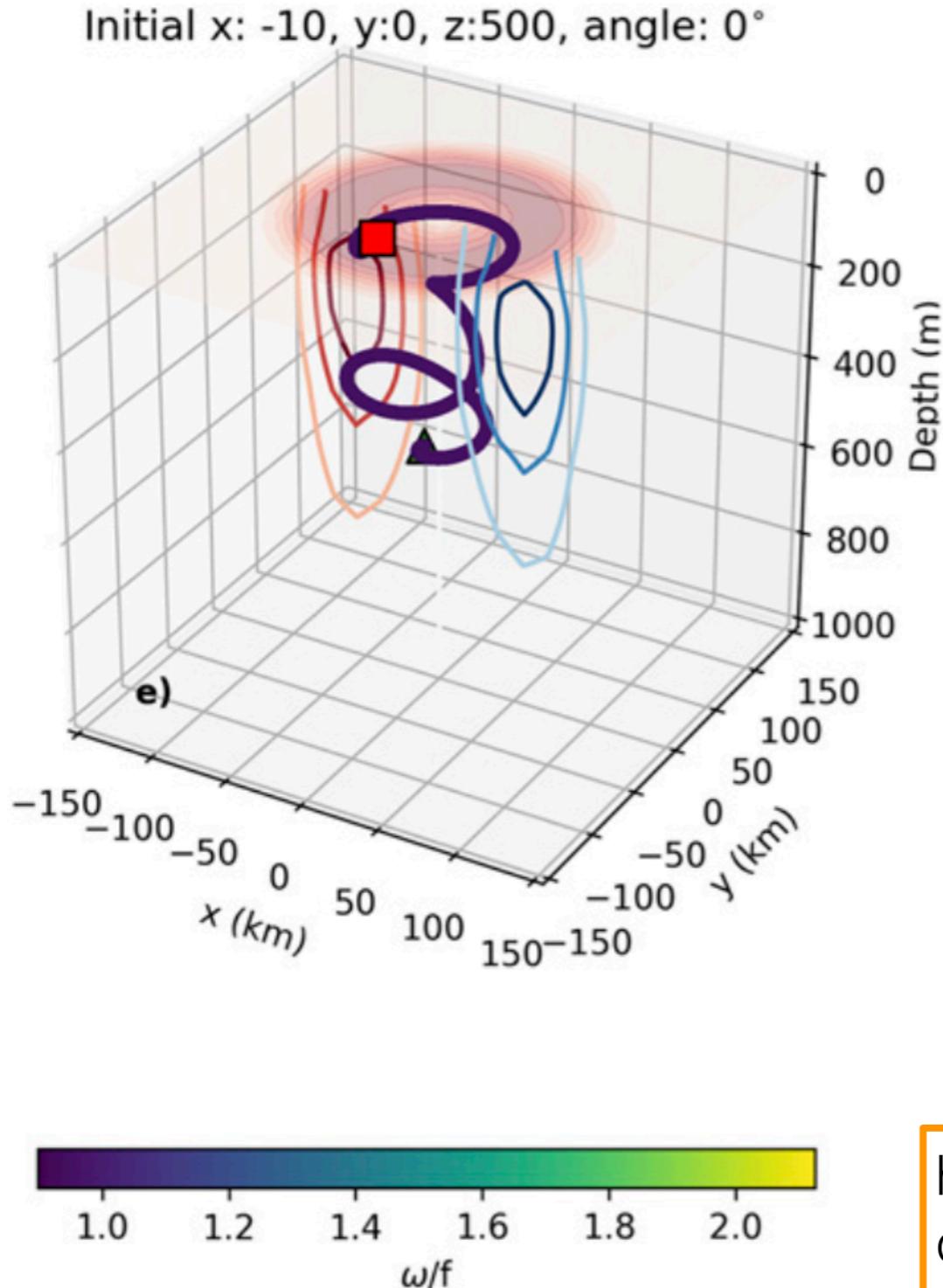
Example of a wave trapped in an anticyclonic eddy and entering a critical layer



backtracking of an internal wave with an initial frequency close to the Coriolis frequency. The wave propagates downward inside the anticyclone.

6. Propagation of internal waves

Example of a wave trapped in an anticyclonic eddy and entering a critical layer



high level of energy
dissipation and mixing
below an anticyclone

6. Propagation of internal waves

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