1.4. Propagation and dissipation of waves

- Review of mechanisms *(see Thorpe75.pdf):*

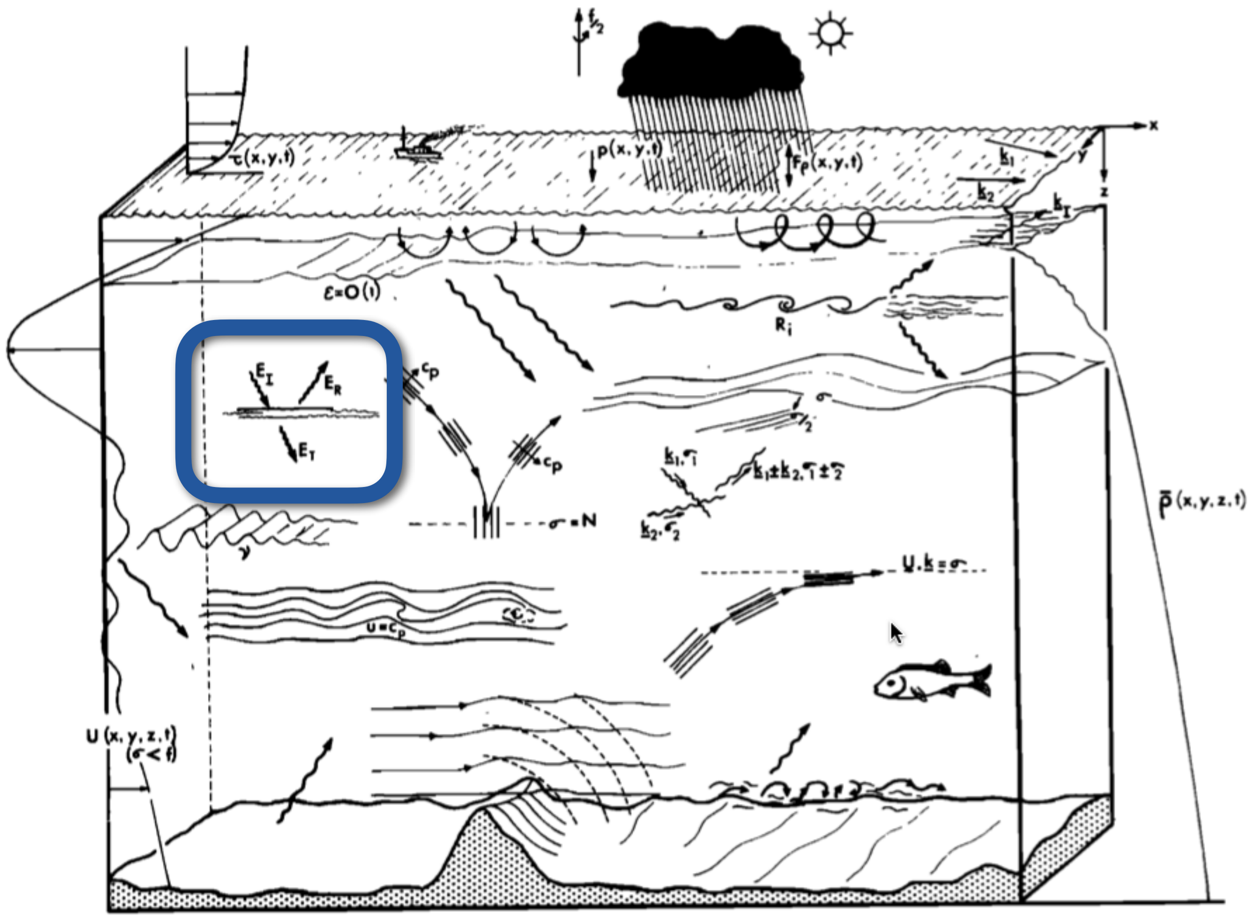


Fig. 5. Physical processes affecting internal waves.

1.4. Propagation and dissipation of waves

C. Wave reflection/refraction by a density jump

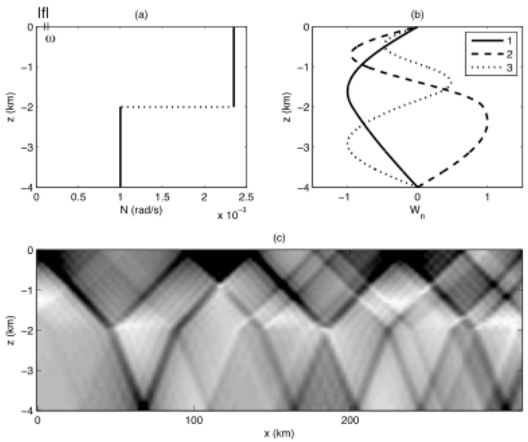
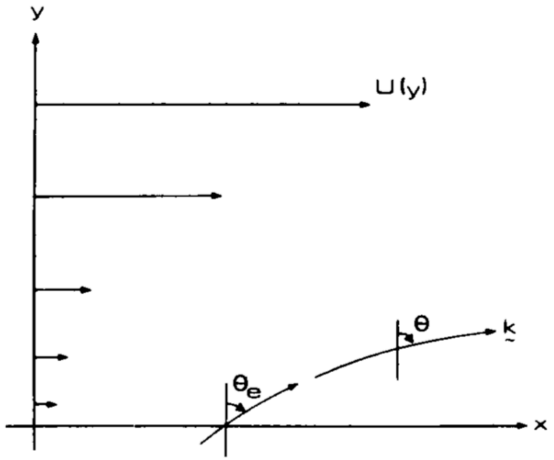
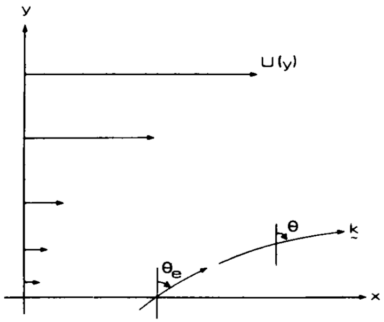


Fig. 5.7: Stratification with two layers of constant N, with $|f| < \omega < N_2 < N_1$ (**a**). Panel **b** shows the first three eigenmodes (5.30), with $C_{1,n}$ chosen such that their amplitudes are one. Modal coefficients are $a_n = 1/n$. The resulting superposition of 20 modes, representing the amplitude of u, is shown in **c**. White denotes zero; black, maximum values.

or by a horizontal shear of mean velocity:



1.4. Propagation and dissipation of waves



Waves in a moving medium:

- Let $\vec{U}(\vec{x},t)$ be the background current.
- The frequency observed by a stationary observer is: $\omega = \vec{k}\cdot(\vec{c_0}+\vec{U})$
- The intrinsic *[doppler-shifted]* frequency
  (= frequency measured in a frame of reference moving with the fluid) $\omega_0 = \omega - \vec{k}\cdot\vec{U}$

1.4. Propagation and dissipation of waves

Notions of ray-tracing

Ray tracing is a method coming from geometrical optics where we model the path of waves by assuming that the medium is varying slowly spatially and temporally compared to the wavelength and period of the waves. *This corresponds to the WKB (Wentzel–Kramers–Brillouin) approximation.*

The dispersion relation of a wave can be written, in the general case:

$$\omega(\overrightarrow{x}, t) = \Omega_0[\overrightarrow{k}(\overrightarrow{x}, t), \lambda(\overrightarrow{x}, t)] + \overrightarrow{k} \cdot \overrightarrow{U}$$

where λ is a property of the medium (like H for long gravity waves, c_s for acoustic waves, N for internal waves) and U is the background velocity.

1.4. Propagation and dissipation of waves

Notions of ray-tracing

The ray paths (corresponding to the motion wave energy), as well as the variation of the wavenumber and the frequency along the ray path are then given by:

.

$$\frac{dx_i}{dt} = \frac{\partial \Omega_0}{\partial k_i} + U_i$$

$$\frac{dk_i}{dt} = -\frac{\partial \Omega_0}{\partial \lambda}\frac{\partial \lambda}{\partial x_i} - k_j \frac{\partial U_j}{\partial x_i}$$

$$\frac{d\omega}{dt} = \frac{\partial \Omega_0}{\partial \lambda}\frac{\partial \lambda}{\partial t} + k_j \frac{\partial U_j}{\partial t}$$

for x_i in x, y, z

1.4. Propagation and dissipation of waves

Notions of ray-tracing

For acoustic waves in a (x,z) plane in a horizontal background flow U(z).

The frequency is $\Omega_0(k_x, k_z, c_s) = c_s\sqrt{k_x^2 + k_z^2} = |k|c_s$

And the equations are:

.

$$
\begin{aligned}
\frac{dx}{dt} &= \frac{\partial \Omega_0}{\partial k_x} + U = c_s \frac{k_x}{|k|} + U \\
\frac{dz}{dt} &= \frac{\partial \Omega_0}{\partial k_z} = c_s \frac{k_z}{|k|} \\
\frac{dk_x}{dt} &= -\frac{\partial \Omega_0}{\partial c_s}\frac{\partial c_s}{\partial x} = -k\frac{\partial c_s}{\partial x} \\
\frac{dk_z}{dt} &= -\frac{\partial \Omega_0}{\partial c_s}\frac{\partial c_s}{\partial z} - k_x\frac{\partial U}{\partial z} = -k\frac{\partial c_s}{\partial z} - k_x\frac{\partial U}{\partial z} \\
\frac{d\omega}{dt} &= \frac{\partial \Omega_0}{\partial c_s}\frac{\partial c_s}{\partial t} = k\frac{\partial c_s}{\partial t}
\end{aligned}
$$

1.4. Propagation and dissipation of waves

Notions of ray-tracing

For long gravity waves in a horizontal plane in a horizontal background flow U(y) with constant depth H:

The frequency is $\Omega_0(k_x, k_y, H) = k\sqrt{gH}$

And the equations are:

.

$$
\begin{aligned}
\frac{dx}{dt} &= \frac{\partial \Omega_0}{\partial k_x} + U = \sqrt{gH}\frac{k_x}{|k|} + U \\
\frac{dy}{dt} &= \frac{\partial \Omega_0}{\partial k_y} = \sqrt{gH}\frac{k_y}{|k|} \\
\frac{dk_x}{dt} &= 0 \\
\frac{dk_y}{dt} &= -k_x \frac{\partial U}{\partial y} \\
\frac{d\omega}{dt} &= 0
\end{aligned}
$$

1.4. Propagation and dissipation of waves

- Activity:

We consider a long surface gravity wave ($c_0 = \sqrt{gH}$, $\vec{k} = [k_x, k_y, 0]$)

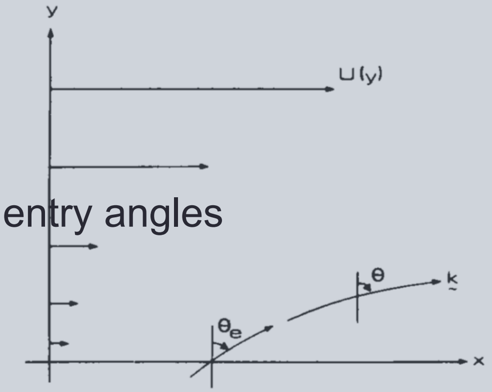
in a domain with constant depth, in a horizontal flow $\vec{U} = [U(y), 0, 0]$.

At the entry point the velocity is $U_e = 0$

1. Write the expression of θ (angle between the y direction and the wavenumber) as a function of $U(y), c_0$ θ_e

2. Draw (qualitatively) the wave paths for positive and negative entry angles

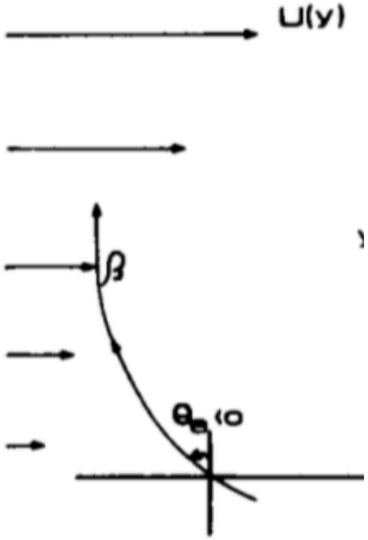
3. Can the wave be reflected?



1.4. Propagation and dissipation of waves

- Negative angle: $\theta_e < 0$

  The ray is bent towards the normal to the current



- Positive angle: Internal reflection: