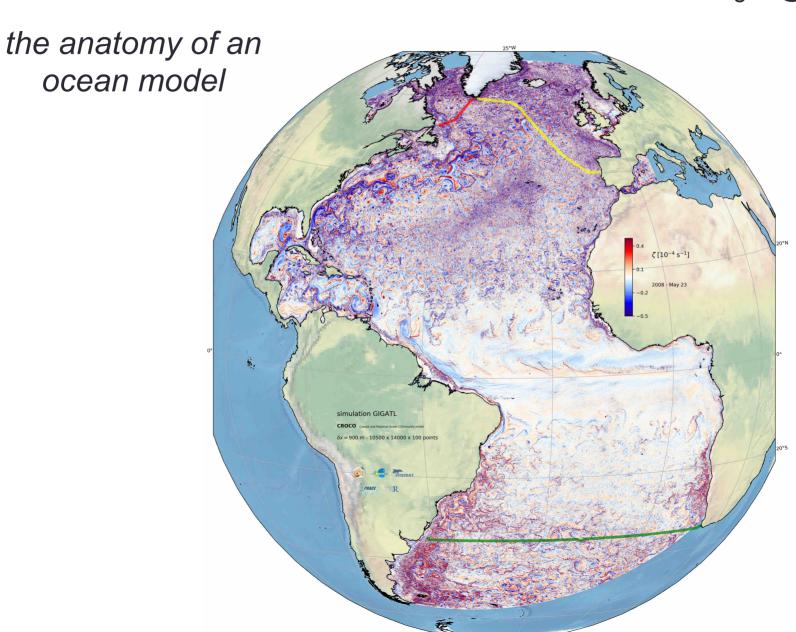
Numerical Modelling

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Lesson 1:

- Introduction
- Activity 1 [run an ocean model]
- Lesson 2 : [B012]
 - Equations of motions
 - Subgrid-scale parameterization
 - Activity 2 [Dynamics of an ocean gyre]
- Lesson 3 : [D109]
 - Horizontal Discretization
 - Numerical schemes
 - Activity 3 [Impacts of numerics]
- Lesson 4 : [D109]
 - Vertical coordinates
 - Model parameterizations
 - Activity 4 [Impact of topography]

- Lesson 5 : [D109]
 - Boundary Forcings
 - Presentation of the model CROCO
 - Activity 4 [Design a realistic simulation]
- Lesson 6: [D109]
 - Diagnostics and validation
 - Activity 5 [Analyze a realistic simulation]
- Lesson 7: [D109]
 - Work on your projet

Presentations and material will be available at:

jgula.fr/ModNum/

Useful references

Extensive courses:

- MIT: https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/
- Princeton: https://stephengriffies.github.io/assets/pdfs/GFM lectures.pdf

Overview on ocean modelling and current challenges:

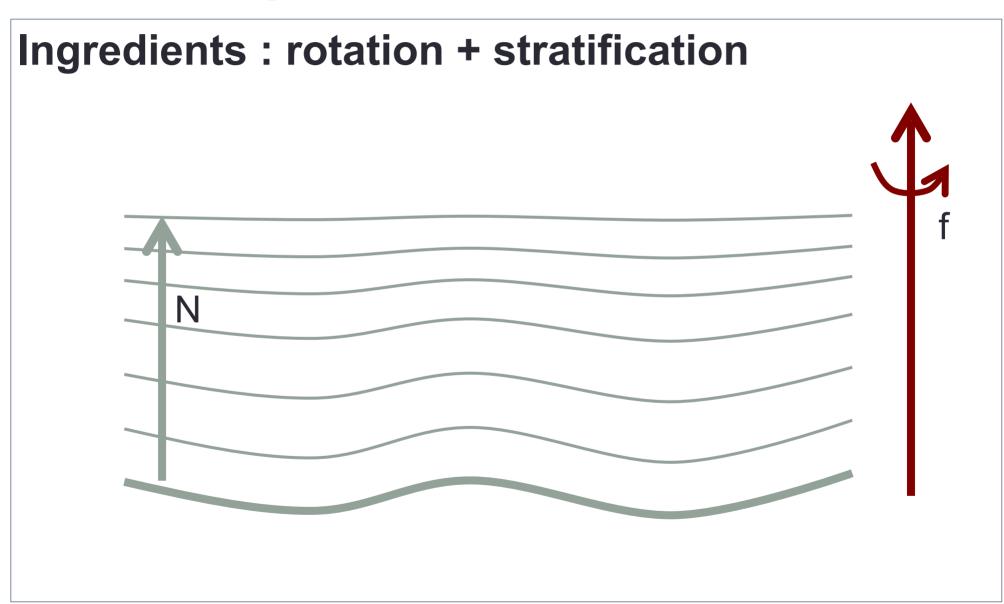
- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling.
 ModNum/Griffiesetal00.pdf
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. http://jgula.fr/ModNum/Griffies Chapter.pdf
- Fox-Kemper et al, 19, "Challenges and Prospects in Ocean Circulation Models" http://jgula.fr/ModNum/FoxKemperetal19.pdf

ROMS/CROCO:

- https://www.myroms.org/wiki/
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell. http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf

#1 Which Equations?

Which Equations?



Which Equations?

Momentum equations (3d)

$$\frac{D\vec{u}}{Dt} = \dots$$

Conservation of mass

$$\frac{D\rho}{Dt} = \mathcal{S}_{\rho}$$

Conservation of heat

$$\frac{DT}{Dt} = \mathcal{S}_T$$

Conservation of salinity

$$\frac{DS}{Dt} = \mathcal{S}_S$$

Equation of state :

$$\rho = \rho(T, S, p)$$

[7 equations for the 7 variables: u,v,w,p,T,S,ρ]

Which Equations?

- Momentum equations (3d)
 - $\frac{D\rho}{Dt} = \mathcal{S}_{\rho}$

Conservation of mass

- Conservation of salinity
- Equation of state :

$$\frac{DT}{Dt} = \mathcal{S}_T$$

$$\frac{DS}{Dt} = \mathcal{S}_S$$

$$\rho = \rho(T, S, p)$$

[7 equations for the 7 variables: u,v,w,p,T,S,ρ]

- Navier-Stokes Equations (NS)
- Non-hydrostatic Primitive Equations (NH)
- Hydrostatic Primitive Equations (PE)
- Shallow-water (SW)
- Quasi-geostrophic (QG)
- 2D Euler equations
- Etc.

Type of models

Navier Stokes DNS = Direct Numerical Simulation

LES = Large Eddy Simulation

CFD

Process studies

Ocean
Circulation
Models

Idealized models

PE = Primitive Equations models

SW = Shallow-Water models

• SQG = Surface Quasi-Geostrophic models

QG = Quasi-Geostrophic models

Etc.

SW

PF

SQG

QG

Navier-Stokes Equations:

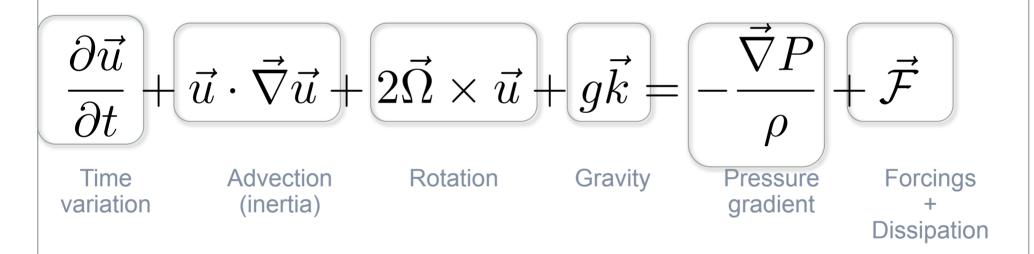
$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + g\vec{k} = -\frac{\vec{\nabla}P}{\rho} + \vec{\mathcal{F}}$$

Momentum equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$

Mass conservation (no source/sink)

Navier-Stokes Equations:



$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$

Mass conservation (no source/sink)

Navier-Stokes Equations:

Linearized momentum equations

- + continuity equation
- + adiabatic motion:
- = Acoustic modes (sound waves)

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla}P
\frac{\partial P}{\partial t} = -\rho_0 c_s^2 \vec{\nabla}P \cdot \vec{u}$$

$$\partial_{tt}P = c_s^2 \nabla^2 P$$

With $c_s \approx 1500\,\mathrm{m\,s^{-1}}$ in water, a model requires a very small time-step to solve these equations.

Boussinesq Approximation:

Density perturbations small compared to mean background value:

$$\rho = \rho_0 + \rho' \qquad \qquad \rho' << \rho_0$$

$$\rho' << \rho_0$$

Linearize all terms involving a product with density, except the gravity term which is already linear:

$$\rho \vec{u} \rightarrow \rho_0 \vec{u}$$

$$\rho g \rightarrow \rho g$$

Boussinesq Approximation:

[+ incompressibility or adiabatic]

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0$$



Continuity equation

$$\vec{\nabla} \cdot \vec{u} = 0$$

Non hydrostatic boussinesq (NH):

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + \frac{\rho}{\rho_0} g \vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \frac{\vec{\mathcal{F}}}{\rho_0} + \frac{\vec{\mathcal{D}}}{\rho_0}$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

Easier to solve than Navier-Stokes, but still requires to invert a 3d elliptic equation for P (computationally expansive)

Hydrostatic balance:

The vertical component of the Boussinesq momentum equations is

$$\frac{\partial w}{\partial t} + \vec{u} \cdot \vec{\nabla}w + 2\Omega \cos\phi v + \left[\frac{\rho}{\rho_0}g = -\frac{\partial_z P}{\rho_0}\right] + \frac{\mathcal{F}_w}{\rho_0} + \frac{\mathcal{D}_w}{\rho_0}$$

For long horizontal motions (L >> H) the dominant balance is

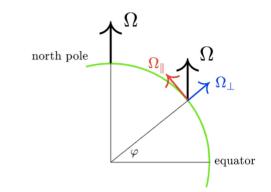
$$\frac{\partial P}{\partial z} = -\rho g$$

Such that pressure is just a vertical integral: $P = \int^{\prime\prime} g \rho dz$

$$P = \int_{z}^{\eta} g\rho dz$$

Traditional approximation

= neglect horizontal Coriolis term



$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + 2\vec{\Omega} \times \vec{u} + \frac{\rho}{\rho_0} g \vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \vec{\mathcal{F}} + \vec{\mathcal{D}}$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + \frac{\rho}{\rho_0} g \vec{k} = -\frac{\vec{\nabla} P}{\rho_0} + \vec{\mathcal{F}} + \vec{\mathcal{D}}$$

Hydrostatic Primitive Equations (PE)

2d momentum with Boussinesq approximation:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \vec{\nabla}_H v + w \frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$
Hydrostatic:
$$\frac{\partial P}{\partial z} = -\rho g$$

• Continuity equation for an incompressible fluid: $\vec{\nabla} \cdot \vec{u} = 0$

Hydrostatic Primitive Equations (PE)

2d momentum with Boussinesq approximation:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

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$$\frac{\partial P}{\partial z} = -\rho g$$

Hydrostatic:

Continuity equation for an incompressible fluid:
$$ec{
abla}\cdotec{u}=0$$

• Conservation of heat and salinity $\frac{DT}{Dt} = \mathcal{S}_T$ $\frac{DS}{Dt} = \mathcal{S}_S$

Equation of state :

$$\rho = \rho(T, S, z)$$

Hydrostatic Primitive Equations (PE)

- 4 prognostics equations for u, v, T, S
- 3 diagnostics equations for w, ρ, P

Hydrostatic Primitive Equations (PE)

2d momentum with Boussinesq approximation:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla}_H u + w \frac{\partial u}{\partial z} - fv = -\frac{\partial_x P}{\rho_0} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \vec{\nabla}_H v + w \frac{\partial v}{\partial z} + fu = -\frac{\partial_y P}{\rho_0} + \mathcal{F}_v + \mathcal{D}_v$$

Hydrostatic:

$$\frac{\partial P}{\partial z} = -\rho g$$

Continuity equation for an incompressible fluid: $\vec{\nabla} \cdot \vec{n} = 0$

$$\vec{\nabla} \cdot \vec{u} = 0$$

Conservation of heat and salinity

$$\frac{DT}{Dt} = \mathcal{S}_T$$
 $\frac{DS}{Dt}$

Equation of state:

$$\rho = \rho(T, S, z)$$

Hydrostatic Primitive Equations (PE)

- 4 prognostics equations for u, v, T, S
- 3 diagnostics equations for w, ρ, P

- + Forcings (wind, heat flux)
- + sub-grid scale parameterizations (bottom drag, mixing, etc.)

#2 Subgrid-scale parameterization

Incompressible Navier-Stokes Equations:

Dissipation of energy/momentum in the NS equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

Viscosity

Type of models

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Etc.

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Ocean
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Idealized models

Incompressible Navier-Stokes Equations:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

Non-linear terms

Viscosity

 Importance of NL terms and viscosity = Reynolds Number

$$Re = \frac{UL}{\nu}$$

Where *U* is a typical velocity of the flow and *L* is a typical length describing the flow.

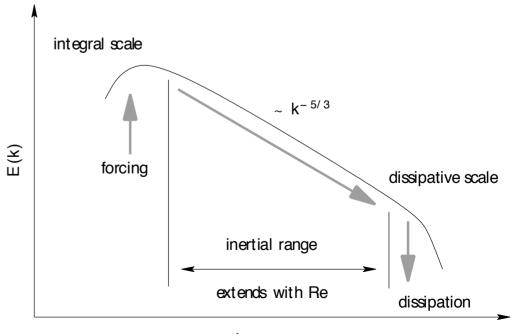
Direct numerical simulation (DNS)

DNS resolves the entire range of turbulent length scales down to the smallest dissipative scales (Kolmogorov scale):

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \approx \left(\frac{\nu^3 L}{U^3}\right)^{1/4} = Re^{-3/4}L$$

where $\, \mathcal{U} \!$ is the kinematic viscosity

And *E* the rate of kinetic energy dissipation



Direct numerical simulation (DNS)

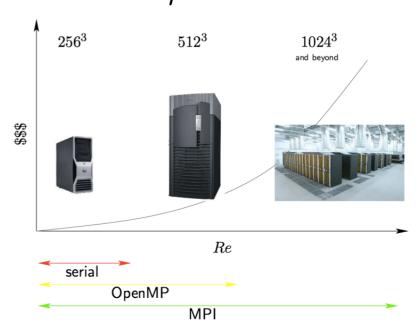
The number of floating-point operations required to complete the simulation is proportional to the number of mesh points:

$$N_x = \frac{L}{\eta} = Re^{3/4}$$

and the number of time steps:

It is extremely expensive as the computational cost scales as Re^3

$$\frac{T}{\Delta t} = \frac{TU}{\eta} = \frac{TU}{L} Re^{3/4}$$



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Reynolds averaging

To reduce the computational cost, one need to reduce the range of time- and length-scales that are being solved for.

The idea is based on separation of mean and turbulent component:

$$u = \overline{u} + u'$$

Where $\overline{u} = \frac{1}{T} \int_0^T u \ dt \text{ or } \overline{u} = \frac{1}{X} \int_0^X u \ dx$

With by definition
$$\overline{u'} = 0$$

Reynolds averaging

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + f \vec{k} \times \vec{u} + g \vec{k} = -\frac{\vec{\nabla} P}{\rho} + \nu \vec{\nabla}^2 \vec{u}$$

Activity:

Adapt the momentum equation:

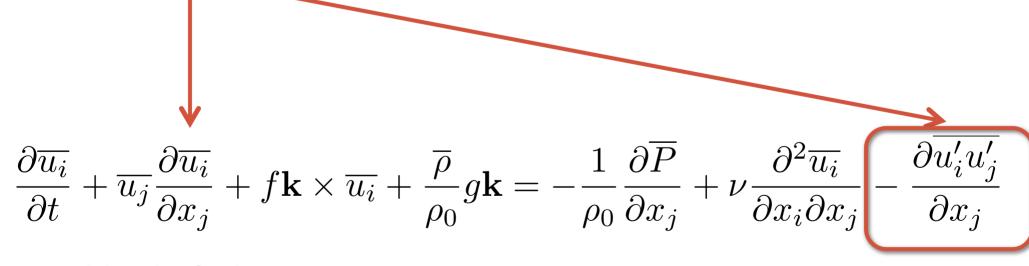
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + f \mathbf{k} \times u_i + \frac{\rho}{\rho_0} g \mathbf{k} = -\frac{1}{\rho_0} \frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}$$

For the mean velocity: $\frac{\partial \overline{u_i}}{\partial t} = ?$

Reynolds averaging

So we resolve only the equations for the mean variables:

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j} \frac{\partial u_i}{\partial x_j} + f \mathbf{k} \times \overline{u_i} + \frac{\overline{\rho}}{\rho_0} g \mathbf{k} = -\frac{1}{\rho_0} \frac{\partial \overline{P}}{\partial x_j} + \nu \frac{\partial^2 \overline{u_i}}{\partial x_i \partial x_j}$$



Advection for the averaged flow

The Closure Problem:

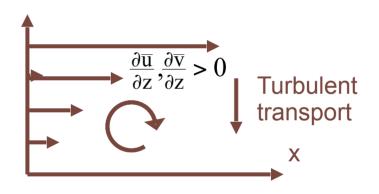
- The number of unknowns is larger than the number of equations
- When we start deriving equations for unknowns, more variables appear.

Progn. Eq. for:	Moment	Equation	No. Eqs.	No. Unknowns
$\overline{U_i}$	First	$\frac{\partial \overline{U_i}}{\partial t} = \dots - \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$	3	6
$\overline{u_i'u_j'}$	First	$\frac{\partial \overline{u_i'u_j'}}{\partial t} = \dots - \frac{\partial \overline{u_k'u_i'u_j'}}{\partial x_k}$	6	10
$\overline{u_i'u_j'u_k'}$	First	$\frac{\partial \overline{u_i'u_j'u_k'}}{\partial t} = \dots - \frac{\partial \overline{u_k'u_i'u_j'u_m'}}{\partial x_m}$	10	15

 In PE models the equations are closed by parameterizing the Reynolds stresses as:

$$\overline{u'w'} = -K_{Mv} \frac{\partial u}{\partial z}$$

$$\overline{v'w'} = -K_{Mv} \frac{\partial v}{\partial z}$$



In ROMS:

$$\mathcal{F}_{u} = \left(\frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial u}{\partial z} \right) + \nabla_{h} (K_{Mh} \cdot \nabla_{h} u) \right) \\
\mathcal{F}_{v} = \left(\frac{\partial}{\partial z} \left(K_{Mv} \frac{\partial v}{\partial z} \right) + \nabla_{h} (K_{Mh} \cdot \nabla_{h} v) \right) \\
\mathcal{S}_{T} = \left(\frac{\partial}{\partial z} \left(K_{Tv} \frac{\partial T}{\partial z} \right) + \nabla_{h} (K_{Th} \cdot \nabla_{h} T) \right) \\
\mathcal{S}_{S} = \left(\frac{\partial}{\partial z} \left(K_{Sv} \frac{\partial S}{\partial z} \right) + \nabla_{h} (K_{Sh} \cdot \nabla_{h} S) \right)$$

Vertical mixing

Horizontal diffusion

Vertical Mixing:

$$K_{Mv}, K_{Tv}, K_{Sv}$$

- Local Turbulence closure: GLS, k-kl (MY2.5), k-ε, κ-ω, etc. [e.g. Warner et al, 2005, Ocean Modelling]
- Non local K-profile parameterization (KPP) [Large et al, 1994, Rev. of Geophysics]

Horizontal diffusion:

Explicit diffusion

$$K_{Mh}, K_{Th}, K_{Sh}$$

Implicit (comes from the advective scheme)

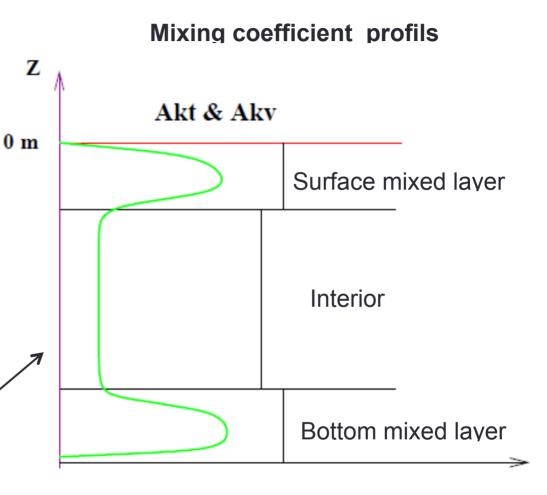
Non local K-profile parameterization

☐ Mixed layer schemes are often based on one-dimensional« column physics »

☐ Boundary layer parameterizations are based either on:

 Turbulent closure (Mellor-Yamada, TKE)

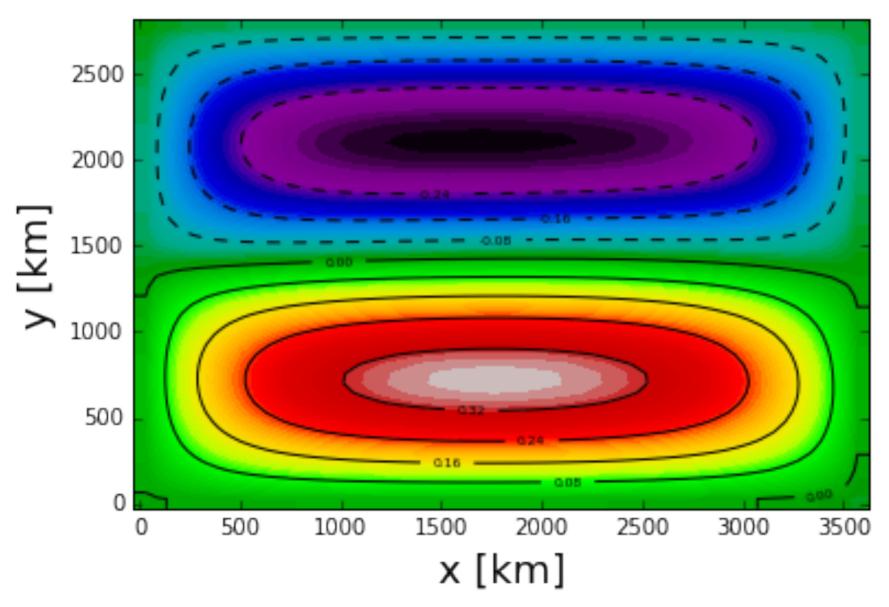
K profile (KPP)



Principle scheme of KPP turbulent

Activity 1 - Run an idealized ocean basin





Activity 1 - Run an idealized ocean basin

- Jobcomp (compilation)
- · cppdefs.h (Numerical/physical options)
- param.h (gris size/ parallelisation)
- <u>croco.in</u> (choice of variables, parameter values, etc.)

1) Preparing and compiling the model

For that use the the jobcomp bash file ./jobcomp

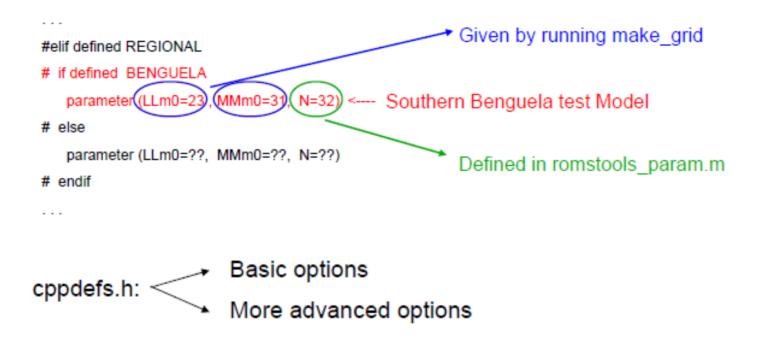
- 1. Set library path
- 2. Automatic selection of option accordingly the platform used
- 3. Use of makefile
 - C-preprocessing step : .F → .f using the CPP keys definitions (in cppdefs.h file, customization of the code)
 - Compilation step : .f → .o (object) using Fortran compiler
 - Linking step: link all the .o file and the librairy (Netcdf, MPI, AGRIF)

--> produce the executable roms

1) Preparing and compiling the model

Edit the param.h and cppdefs.h file to set-up the model

param.h defines the size of the arrays in ROMS:



- Define CPP keys used by the C-preprocessor when compiling the model.
- Reduce the code to its minimal size: fast compilation.
- Avoid FORTRAN logical statements: efficient coding.

1) Preparing and compiling the model

View cppdef.h file



```
BASIC OPTIONS
                Configuration Name */
# define BENGUELA
                Parallelization */
#undef OPENMP
# undef MPI
                Embedding */
# undef AGRIF
                Open Boundary Conditions */
#undef TIDES
# define OBC EAST
# undef OBC WEST
# define OBC NORTH
# define OBC SOUTH
                 Embedding conditions */
# ifdef AGRIF
# undef AGRIF_OBC_EAST
# define AGRIF OBC WEST
# define AGRIF_OBC_NORTH
# define AGRIF OBC SOUTH
# endif
               Applications */
# undef BIOLOGY
#undef FLOATS
# undef STATIONS
#undef PASSIVE TRACER
#undef SEDIMENTS
#undef BBL
```

```
MORE ADVANCED OPTIONS
                 Model dynamics */
# define SOLVE3D
# define UV COR
# define UV ADV
# ifdef TIDES
# define SSH TIDES
# define UV TIDES
# define TIDERAMP
# endif
                 Grid configuration */
# define CURVGRID
# define SPHERICAL
# define MASKING
                      Input/Output & Diagnostics */
# define AVERAGES
# define AVERAGES K
# define DIAGNOSTICS TS
# define DIAGNOSTICS UV
                       Equation of State */ ...
                      Surface Forcing */ ...
                      Lateral Forcing */ ...
                      Input/Output & Diagnostics */ ...
                      Bottom Forcing */ ...
                       Point Sources - Rivers */ ...
                      Lateral Mixing */ ...
                      Vertical Mixing */ ...
                      Open Boundary Conditions */ ...
/*
                      Embedding conditions */ ...
/*
```

2) Running the model

The namelist roms.in

roms.in provides the run time parameters for ROMS:

```
title:
    Southern Benguela
time stepping: NTIMES dt[sec] NDTFAST NINFO
              5400 60
        480
                         1
S-coord: THETA S, THETA B, Hc (m)
                    10.0d0
      6.0d0
              0.0d0
grid: filename
                                          Warning! These
              ROMS FILES/roms grd.nc
                                        should be identical to
forcing: filename
                                             the ones in
              ROMS FILES/roms frc.nc
bulk forcing: filename
                                        romstools_param.m
              ROMS FILES/roms blk.nc
climatology: filename
              ROMS_FILES/roms_clm.nc
boundary: filename
              ROMS FILES/roms bry.nc
initial: NRRFC_filename
              ROMS FILES/roms ini.nc
           NRST, NRPFRST / filename
restart:
          480 -1
              ROMS FILES/roms rst.nc
```

```
history: LDEFHIS, NWRT, NRPFHIS / filename
      T 480 0
             ROMS FILES/roms his.nc
averages: NTSAVG, NAVG, NRPFAVG / filename
          48 0
             ROMS FILES/roms avg.nc
primary history fields: zeta UBAR VBAR U V wrtT(1:NT)
             T F F F F 10*F
auxiliary history fields: rho Omega W Akv Akt Aks HBL Bostr
              FFFFFFF
primary averages: zeta UBAR VBAR U V wrtT(1:NT)
         T T T T T 10*T
auxiliary averages: rho Omega W Akv Akt Aks HBL Bostr
          FTTFTFTT
rho0:
   1025.d0
lateral_visc: VISC2, VISC4 [m^2/sec for all]
             0.
tracer diff2: TNU2(1:NT)
                         [m^2/sec for all]
       10*0.d0
bottom drag: RDRG [m/s], RDRG2, Zob [m], Cdb min, Cdb max
        0.0d-04  0.d-3  1.d-2  1.d-4  1.d-1
gamma2:
        1.d0
           X SPONGE [m], V SPONGE [m^2/sec]
sponge:
                     800
         100.e3
nudg_cof: TauT_in, TauT_out, TauM_in, TauM_out [days for all]
             360
                    10
                         360
```

Activity 1 - Run an idealized ocean basin

```
param.h
      parameter (LLm0=60,
                                    MMm0=50.
                                                   N=10
· cppdefs.h
 define UV_COR
  define UV_VIS2
  define TS_DIF2
  define ANA_GRID
  define ANA_INITIAL
· ana_grid.F
f0=1.E-
beta=0.
· croco.in
bottom_drag:
                       RDRG2, Zob [m], Cdb_min, Cdb_max
            RDRG(m/s),
            3.e-4
gamma2:
lin_EOS_cff: R0 [kg/m3], T0 [Celsius], S0 [PSU], TCOEF [1/Celsius], SCOEF [1/PSU]
lateral_visc: VISC2 [m^2/sec ]
tracer_diff2: TNU2
                   [m^2/sec]
```

Homework

- For next time:
 - Read https://www.jgula.fr/ModNum/Stommel48.pdf
 - Read https://www.jgula.fr/ModNum/Munk50.pdf

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