

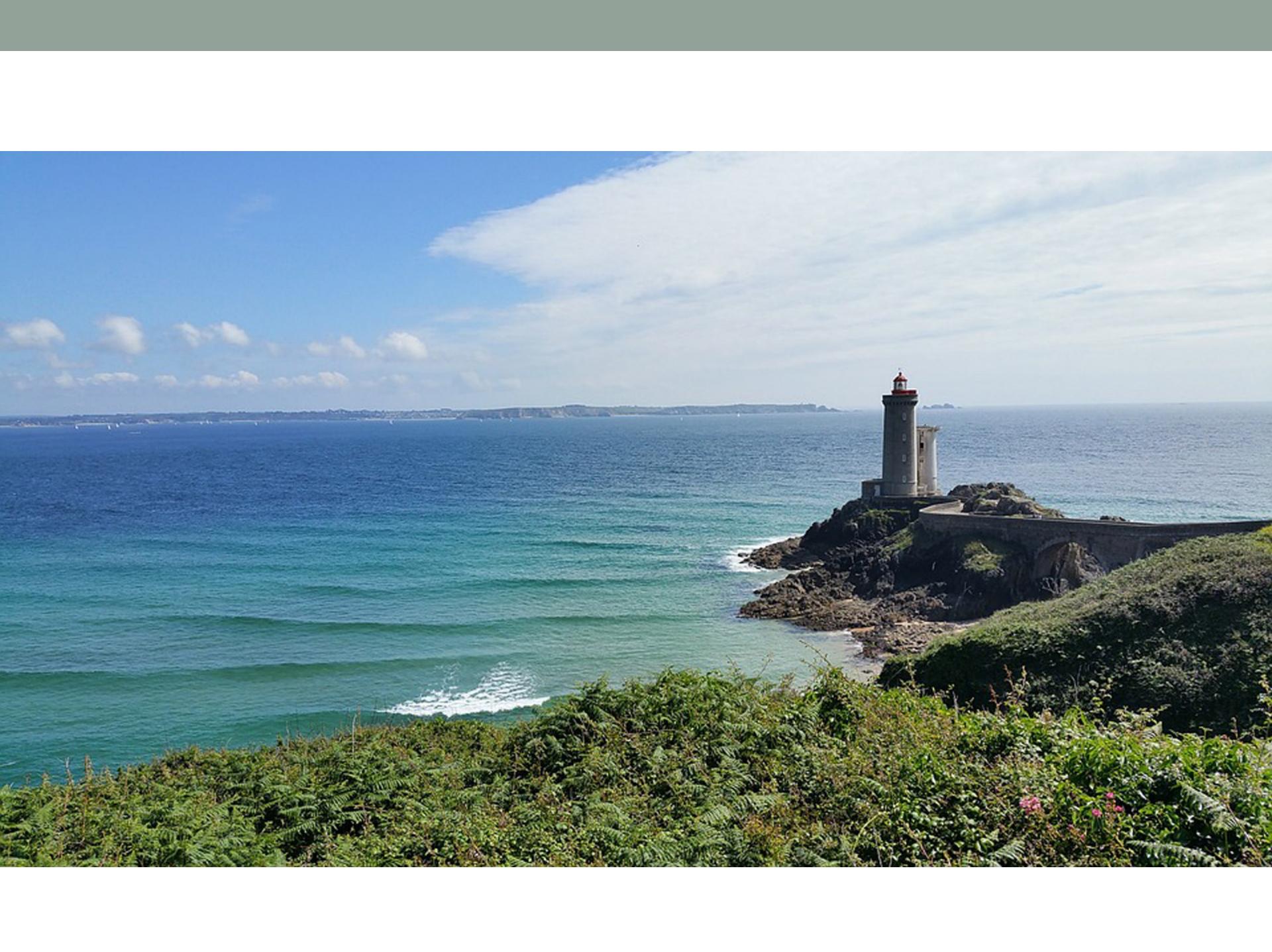
OCEAN WAVES

INTRODUCTION

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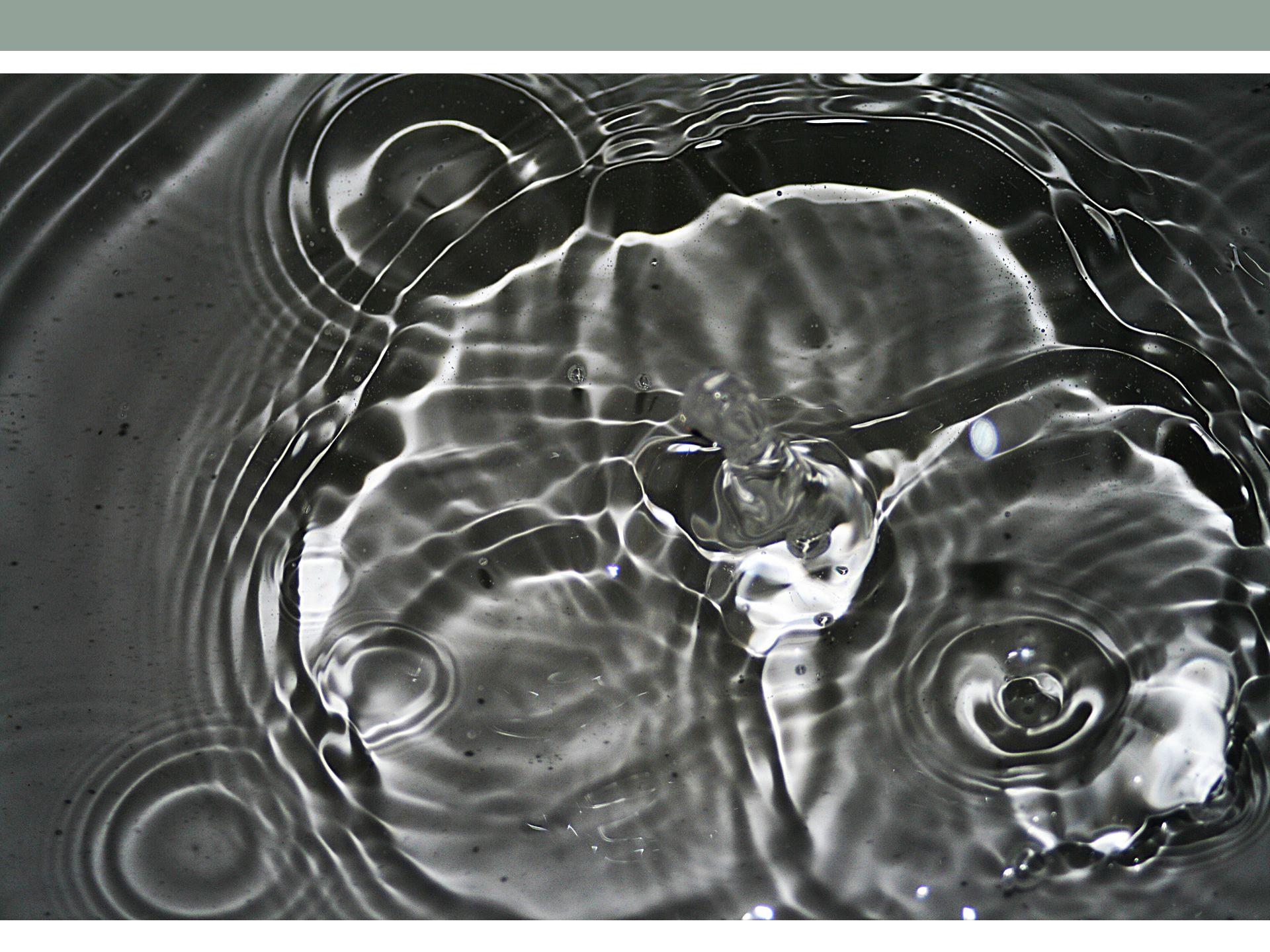
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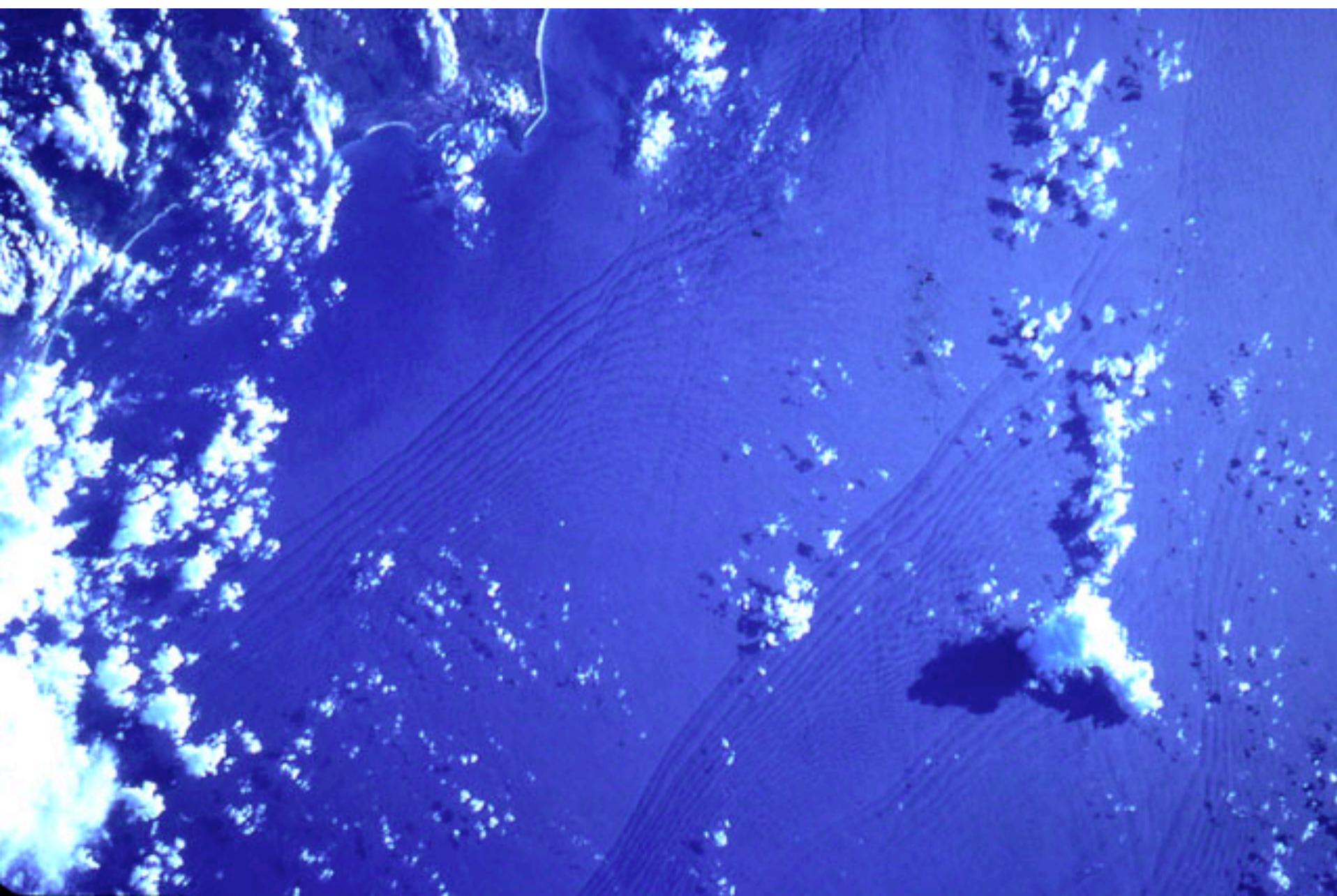


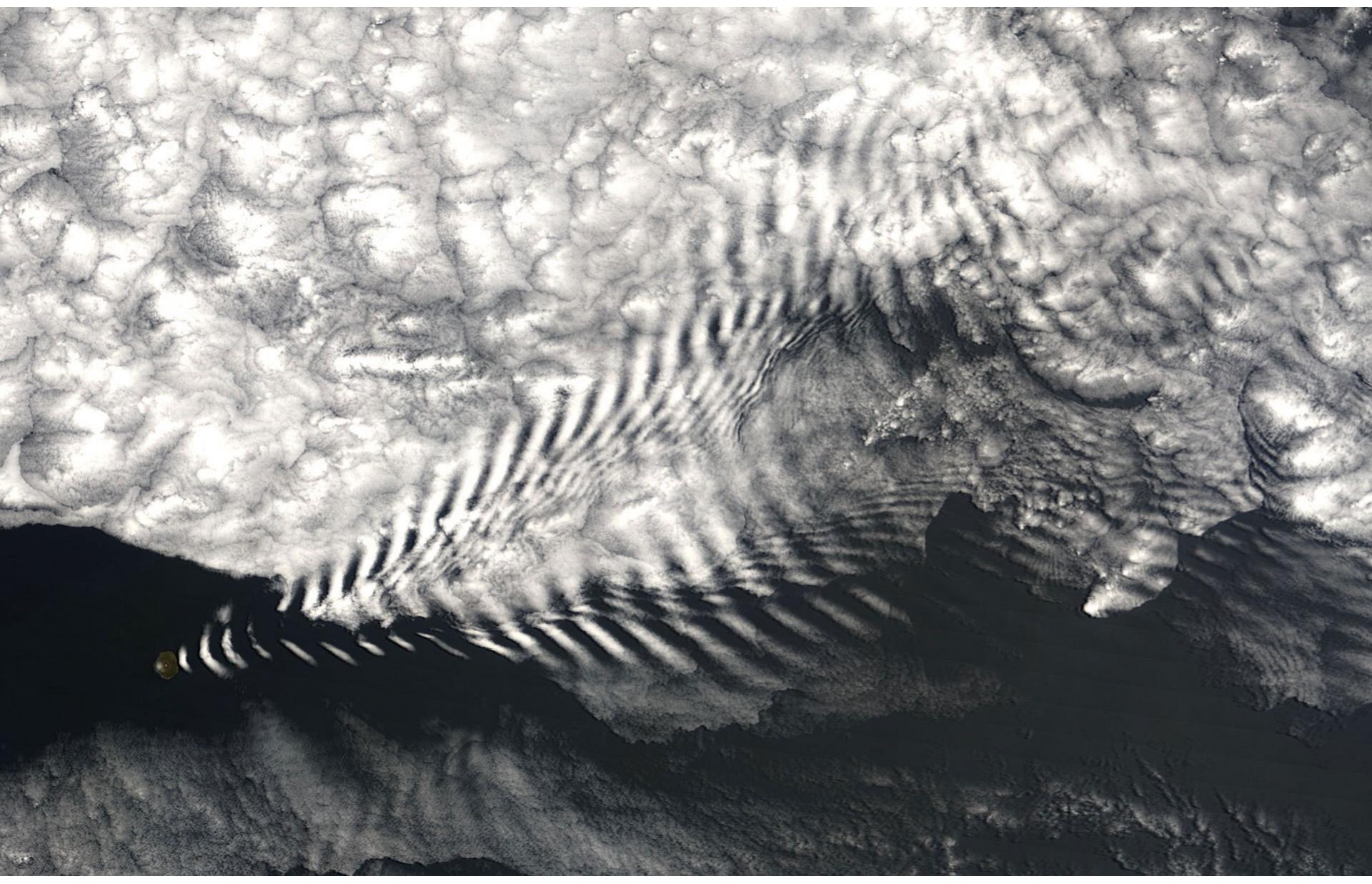






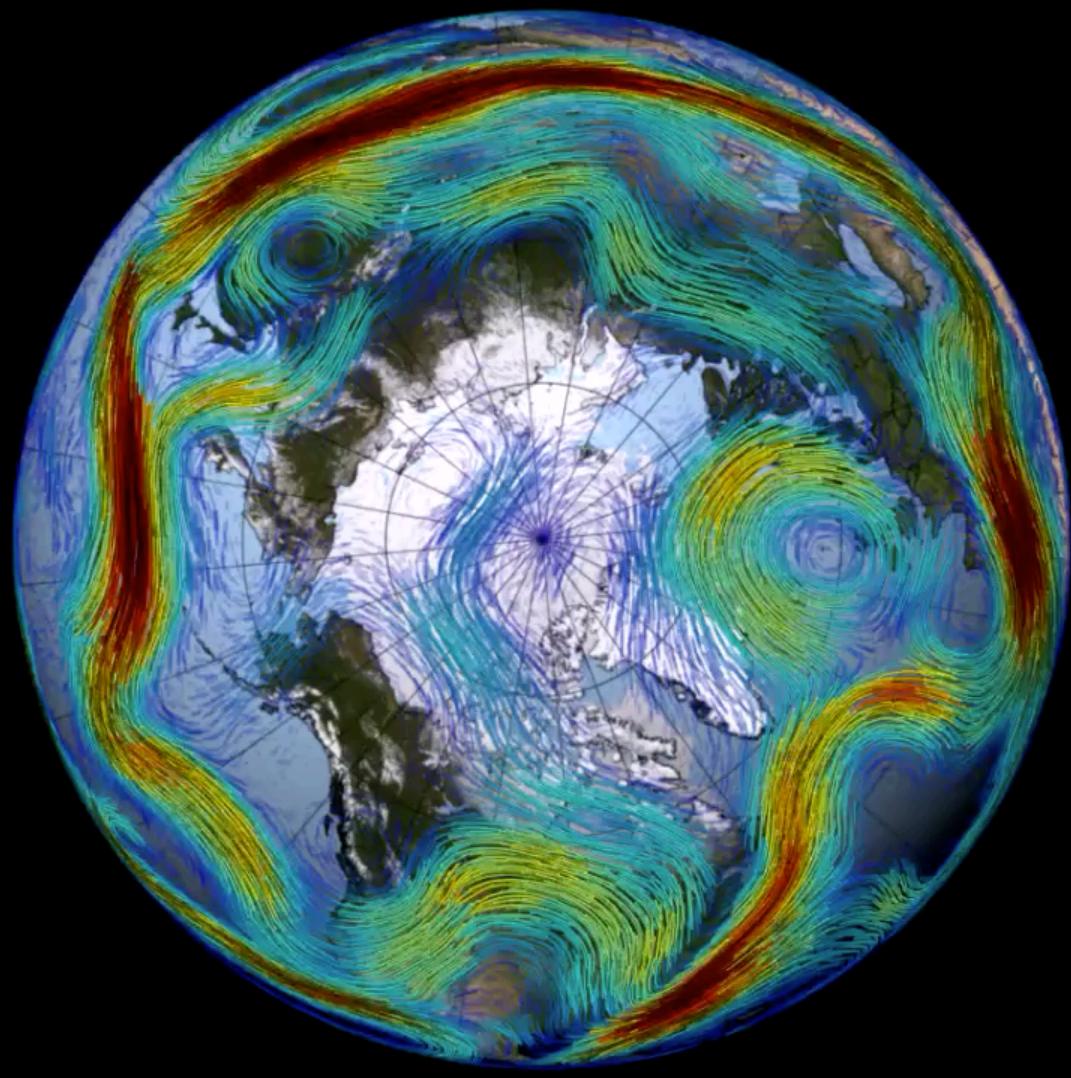


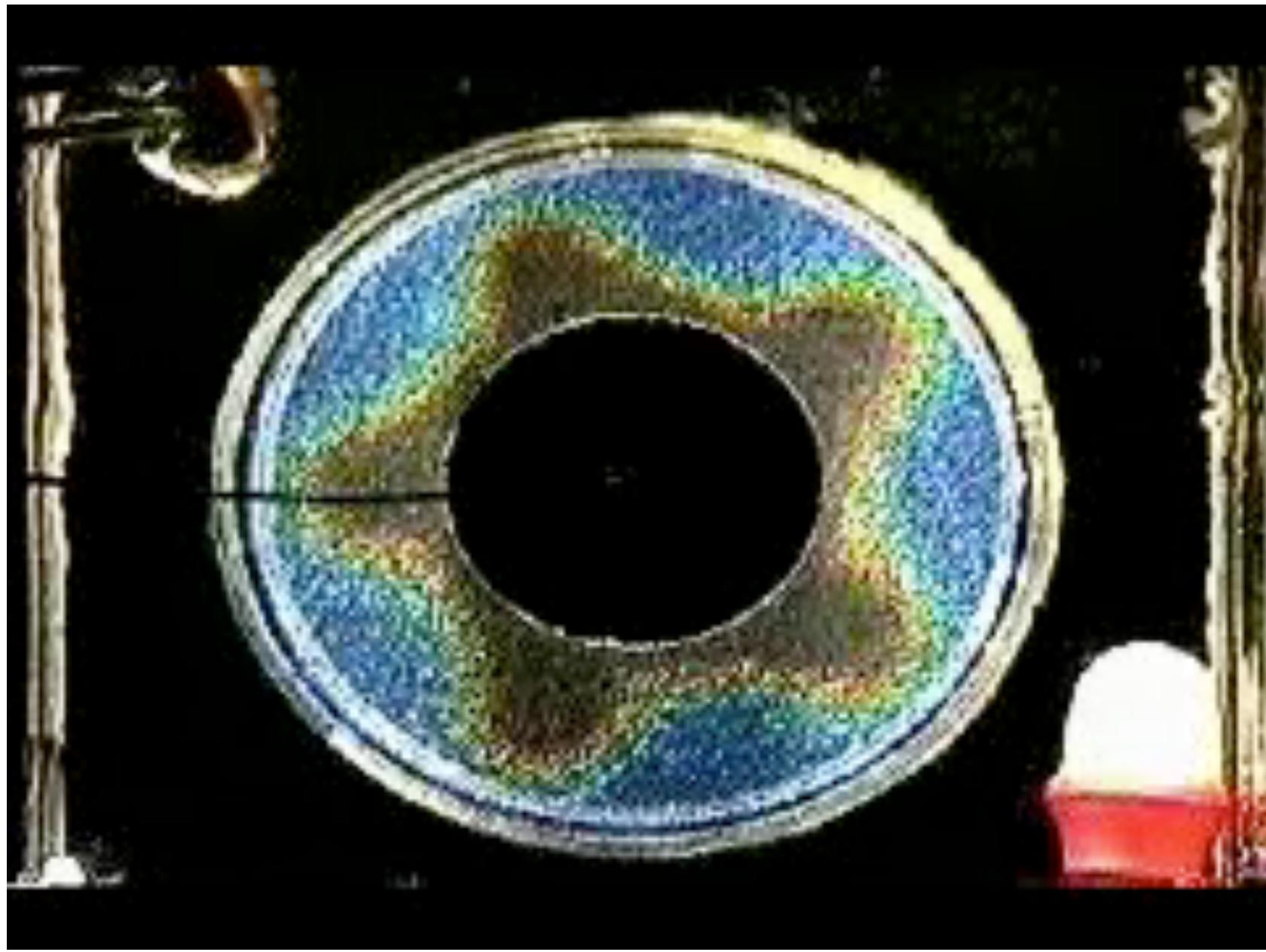


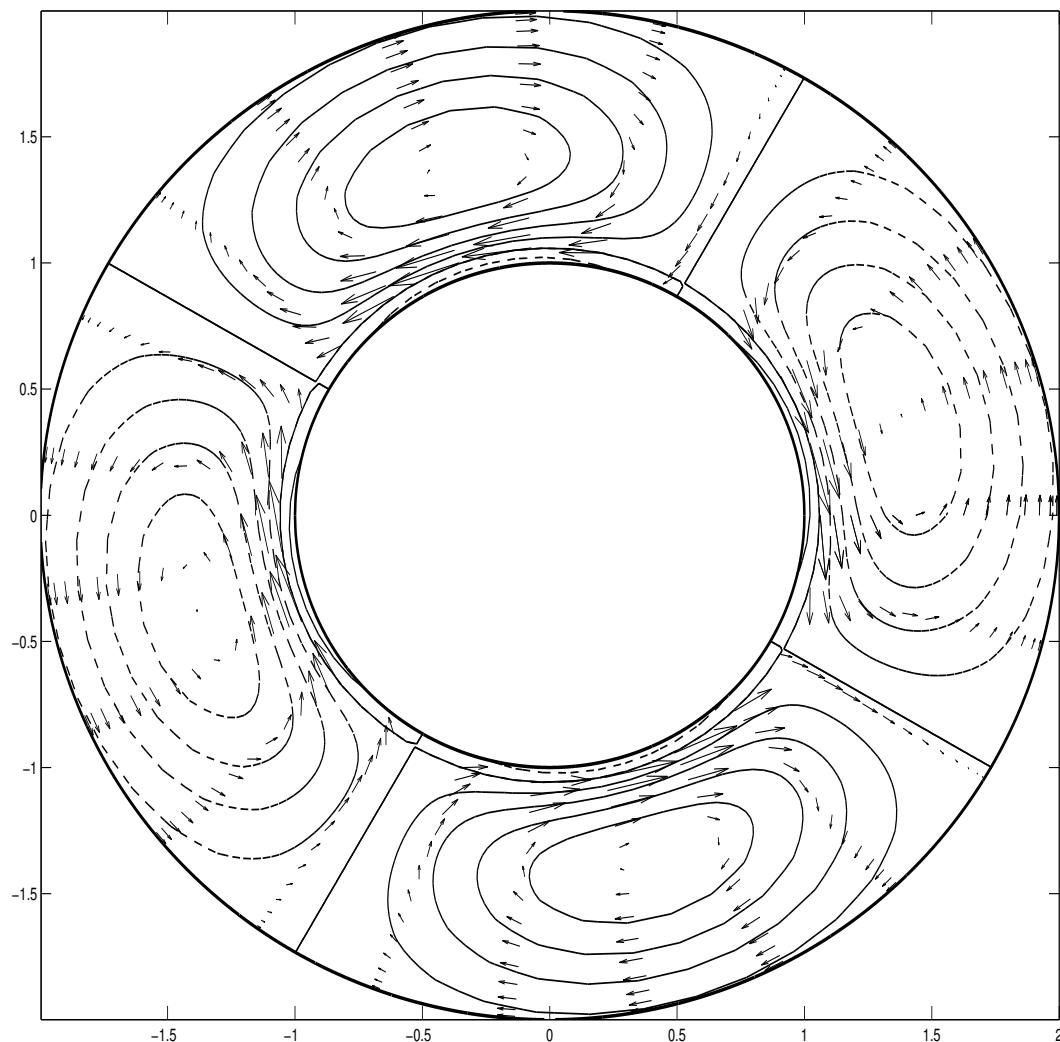


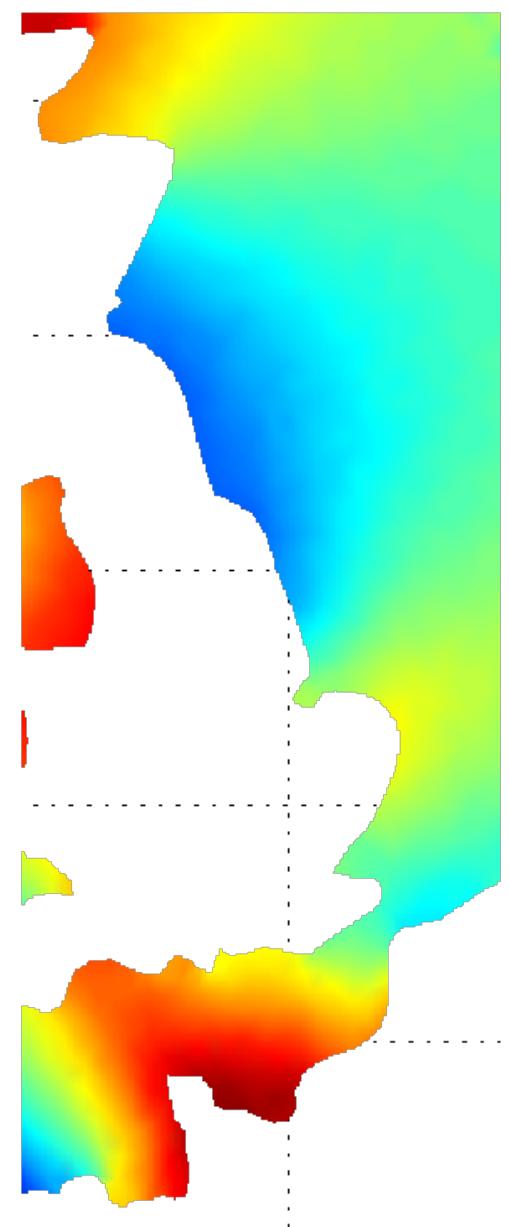
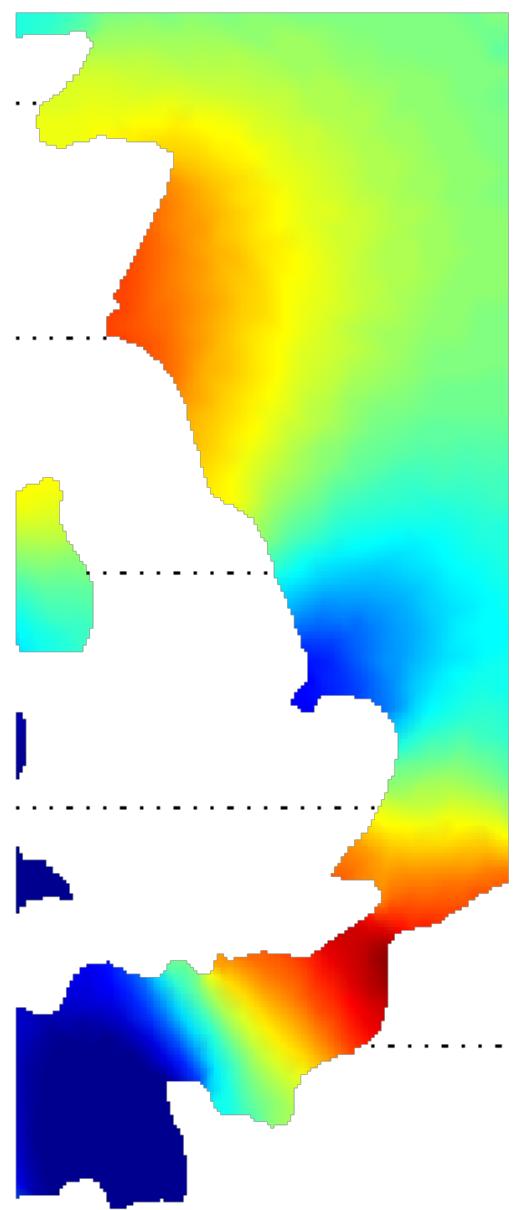
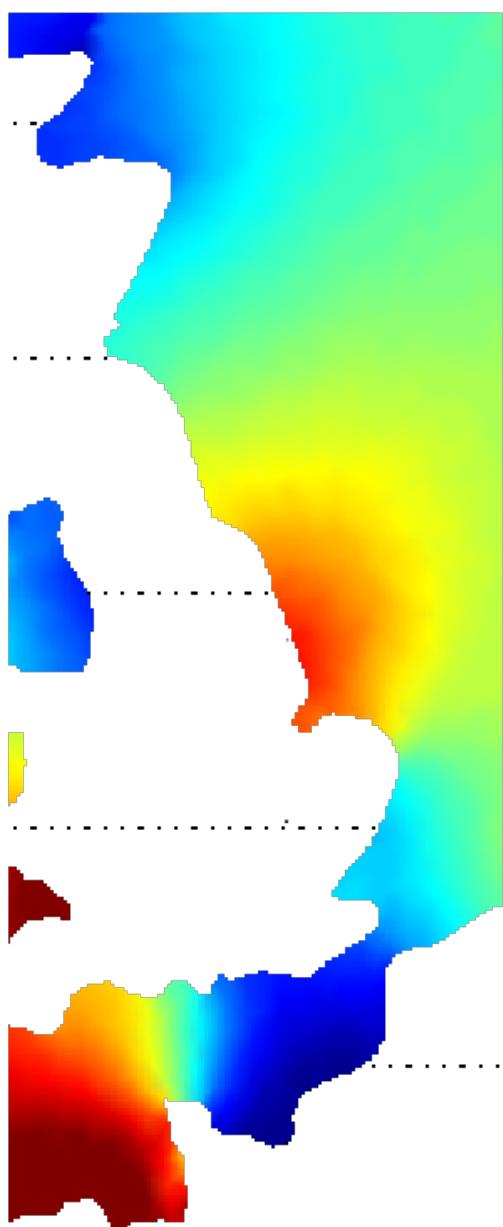


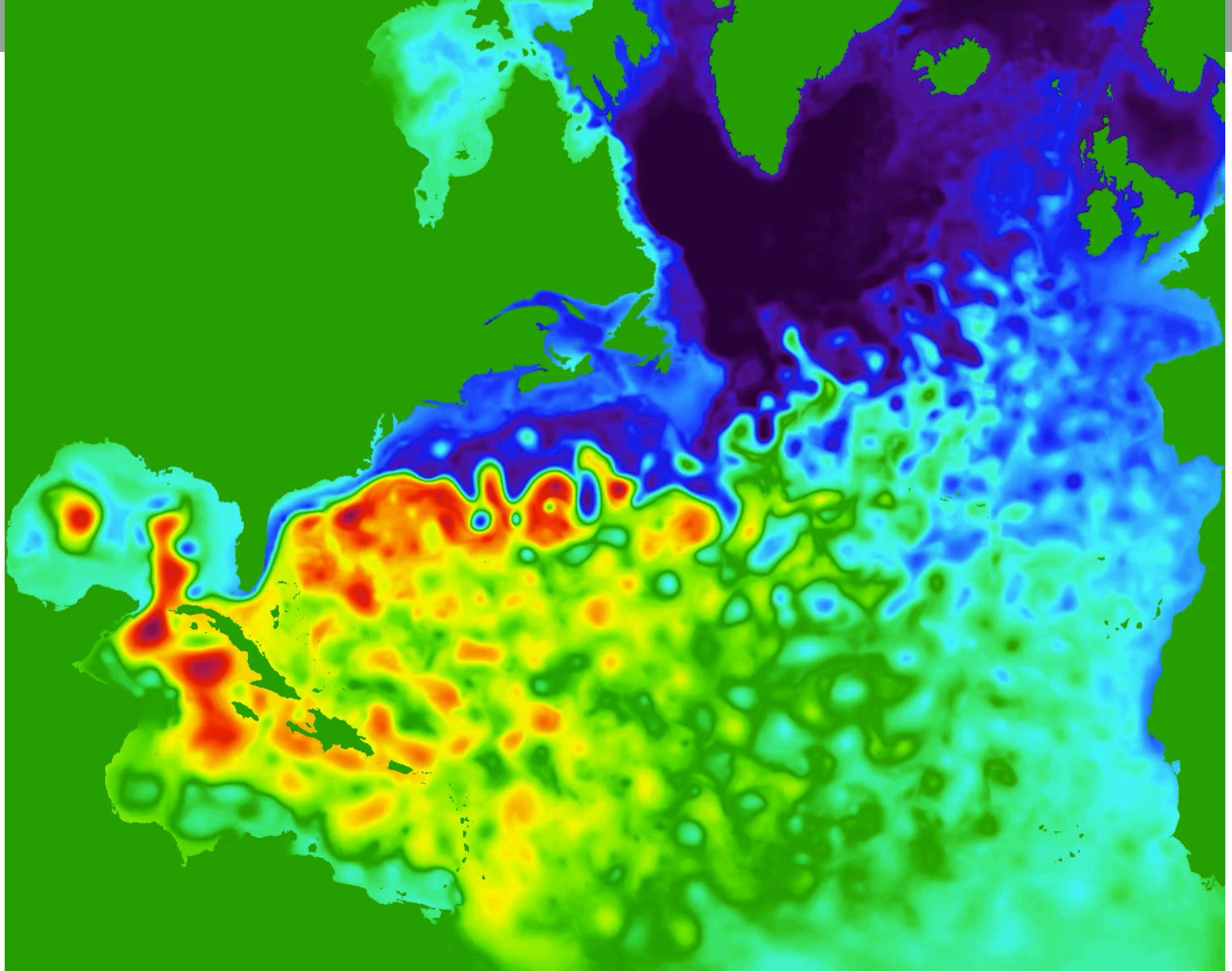
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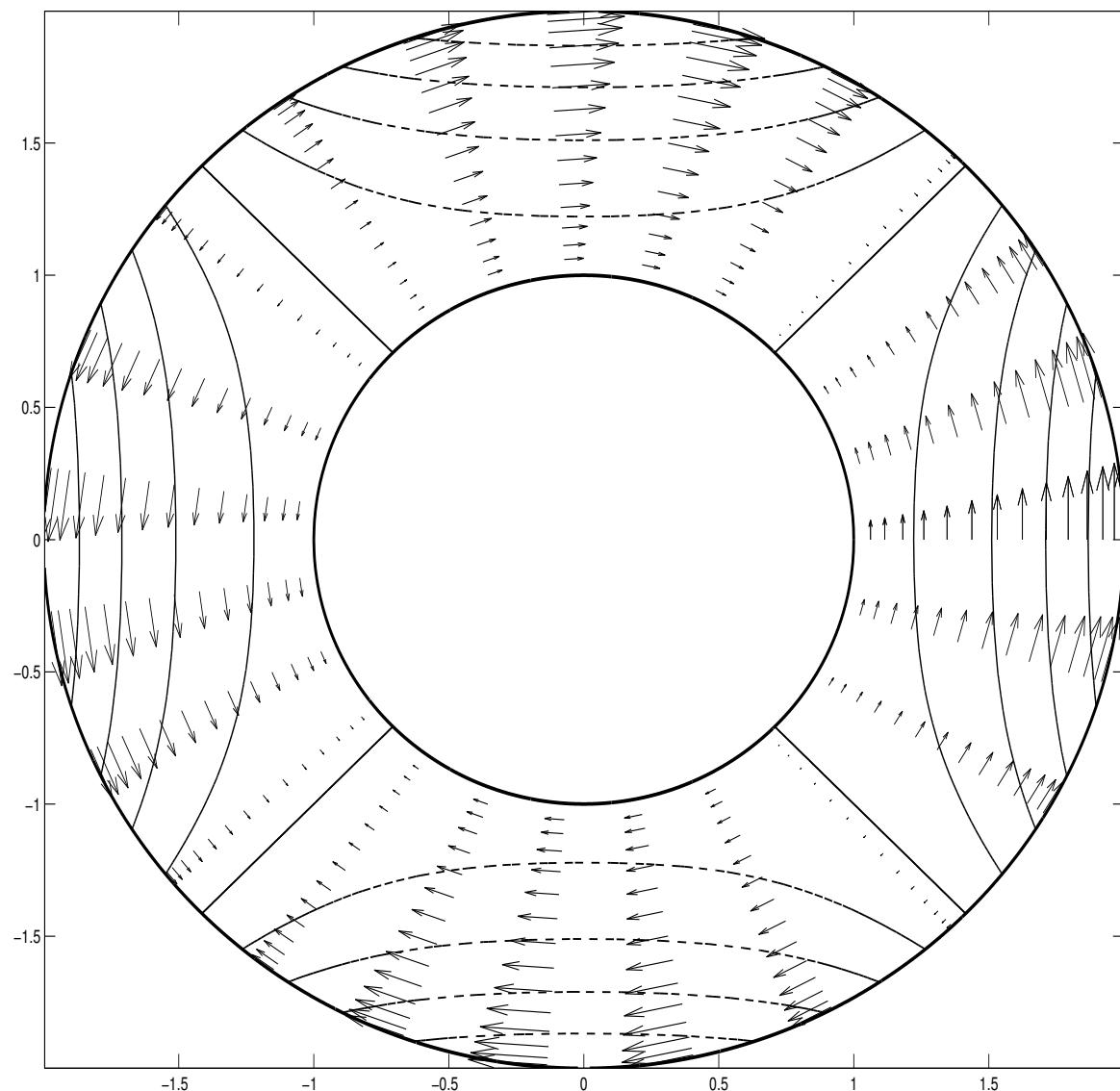


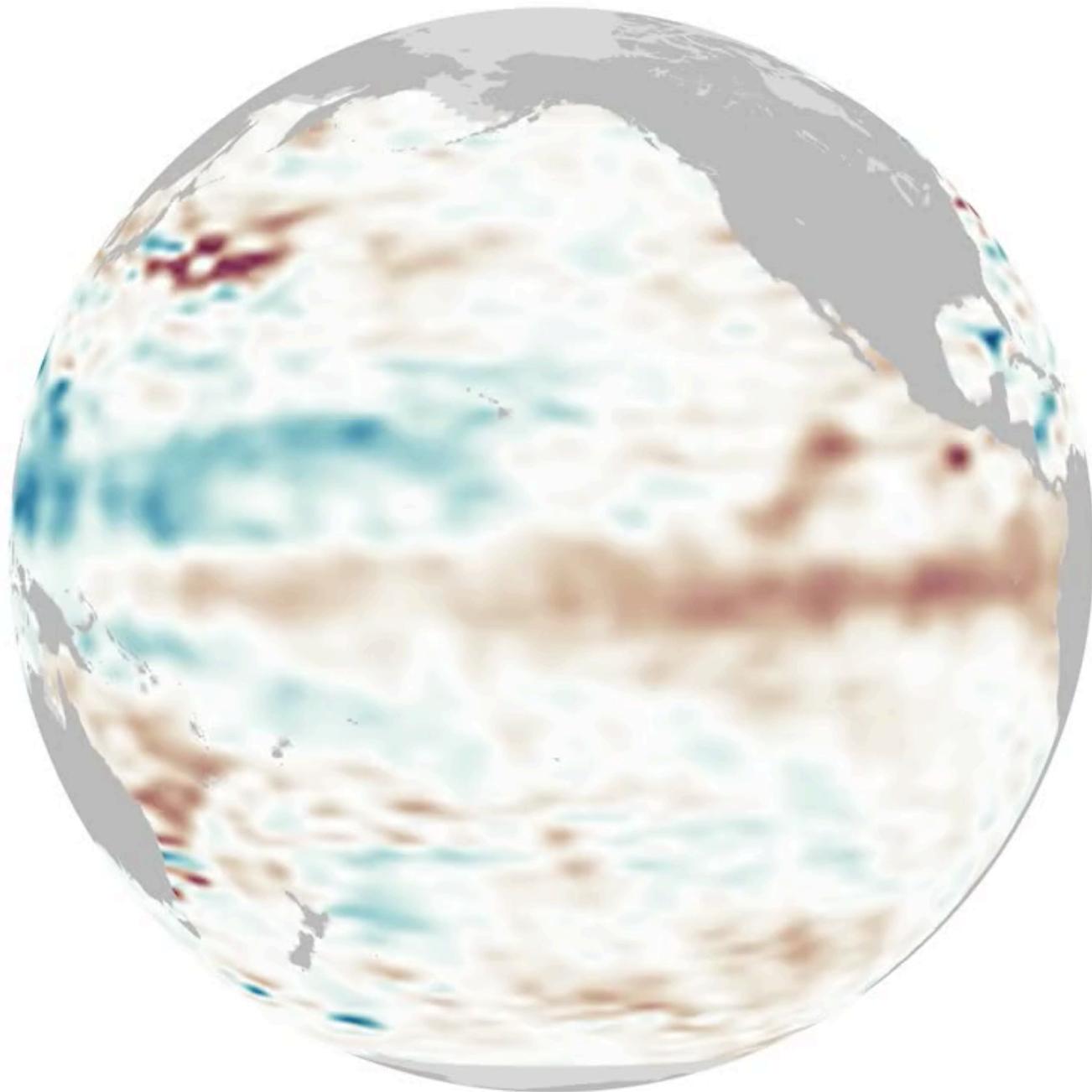












Sea Surface Height Anomaly (mm)

-220

0

220

Date

Dec

Jan

Feb

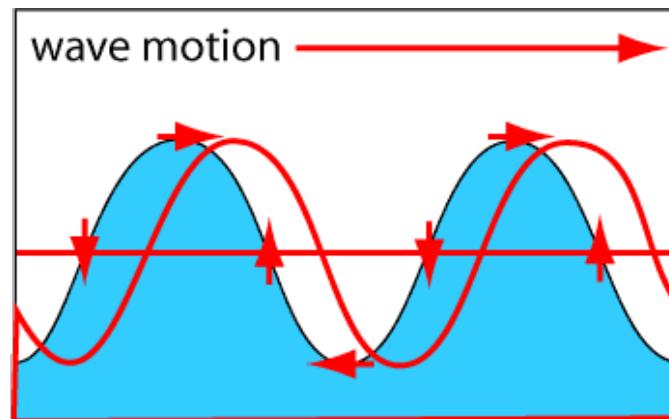
Ocean Waves

Definition of a wave:

- *A wave is a recognizable signal that is transferred from one part of the medium to another with a recognizable velocity of propagation. The signal may be any feature of the disturbance, such as a maximum or an abrupt change in some quantity, provided that it can be clearly recognized and its location at any time can be determined. [Whitham: « Linear and nonlinear waves »]*
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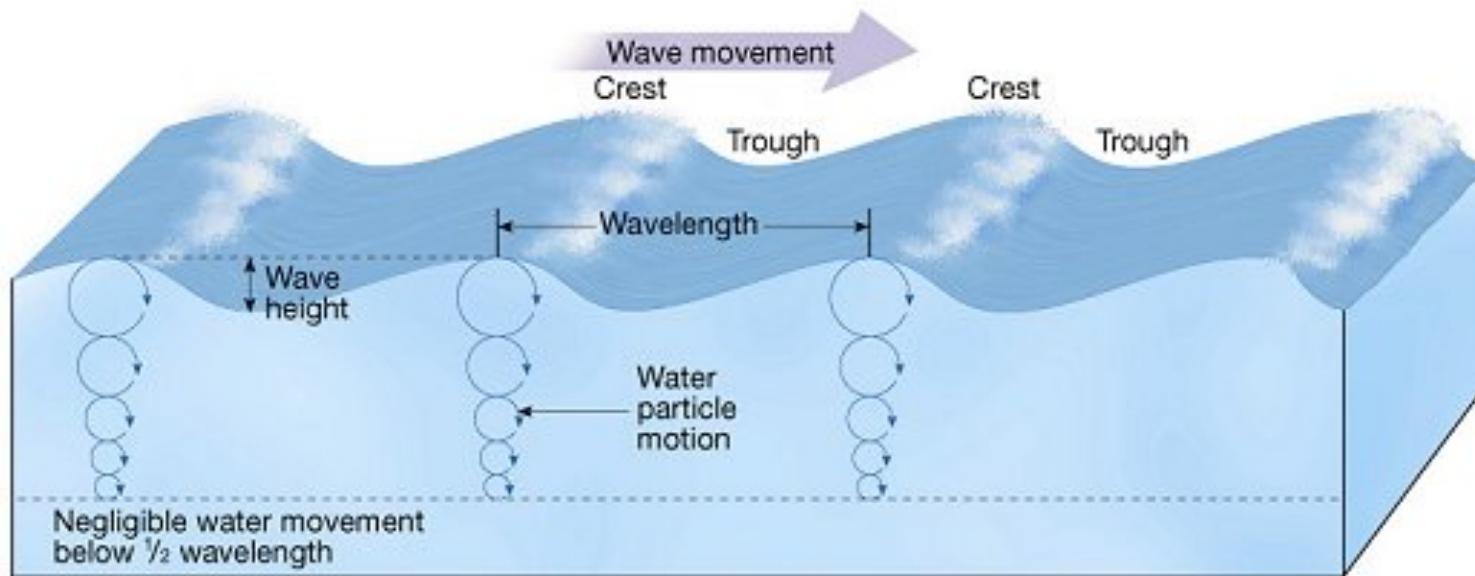
Ocean Waves

- A wave results when fluid is displaced from a position of equilibrium.
- The restoration of the fluid to equilibrium will produce a movement of the fluid back and forth, called a wave orbit.



Ocean Waves

- Waves propagates energy but do not transport water



Ocean Waves

The different type of waves can be classified on the basis of:

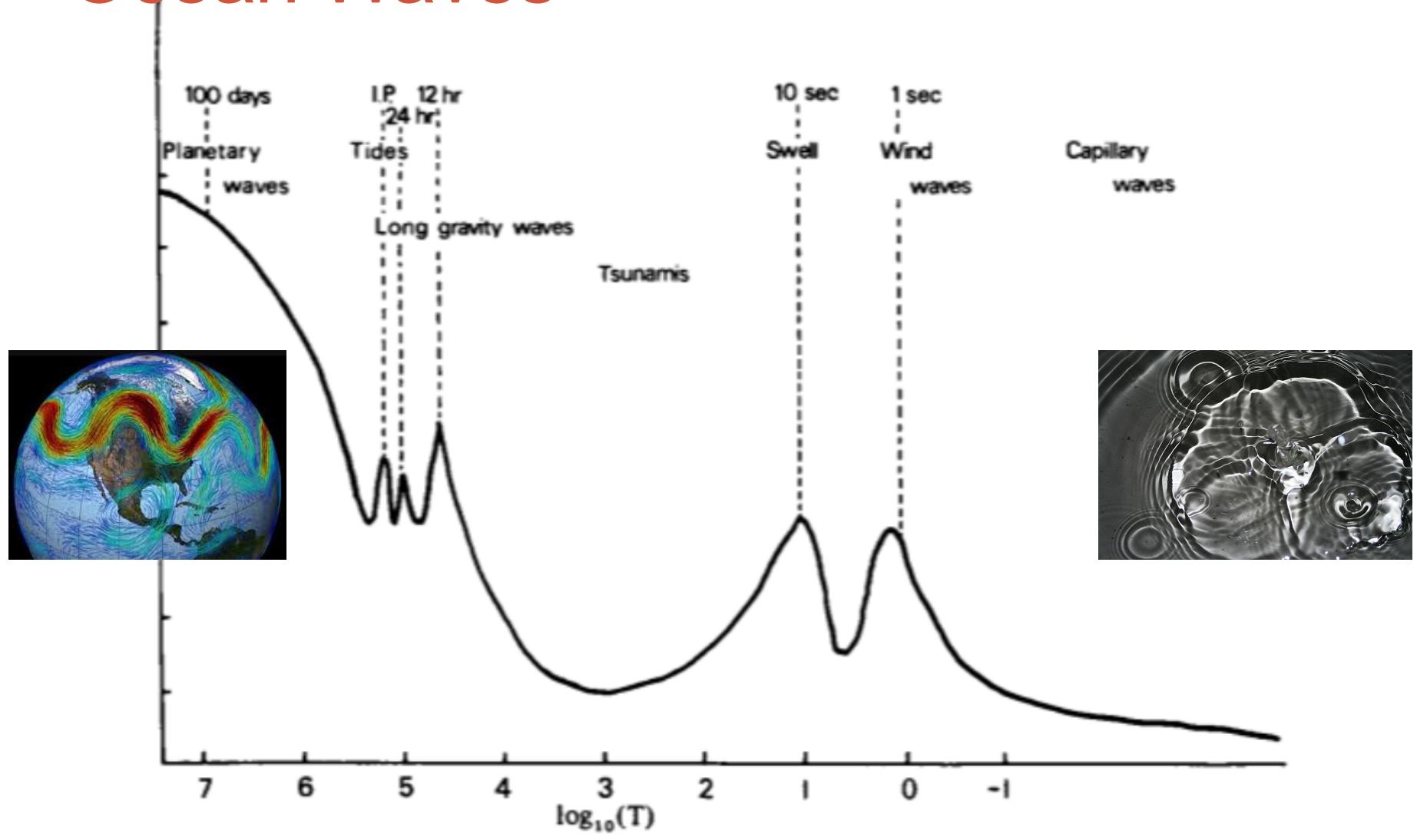
- Restoring force
- Disturbing force
- Wavelength
- Free wave Vs forced wave

Ocean Waves

Ocean Waves

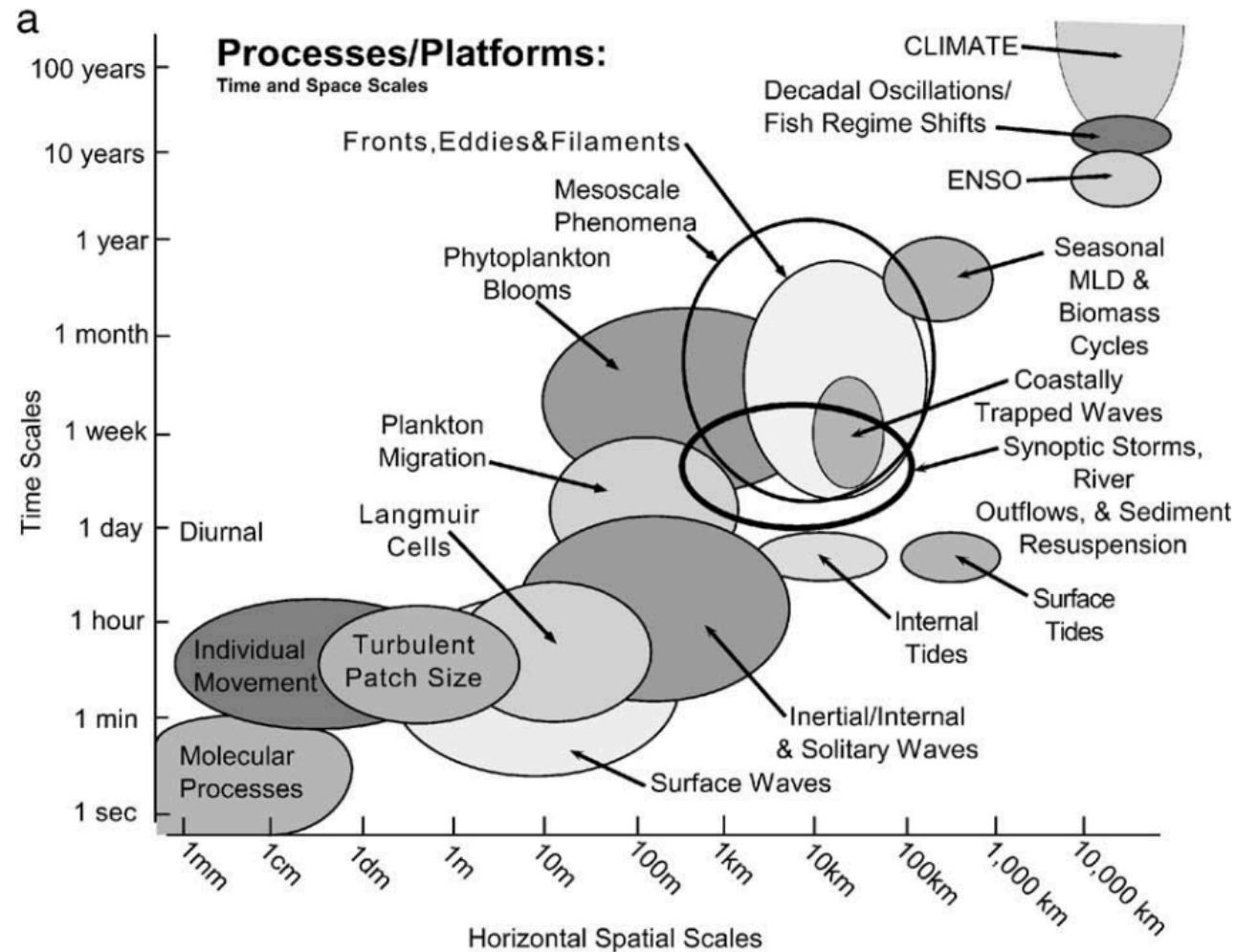
	Disturbing Force	Restoring Force	Wavelength	Periods
Acoustic waves		Pressure (compressibility)	1 mm - 10 km	< 1 s
Capillary waves		Surface tension	1 mm - 1 cm	< 1 s
Surface gravity waves	Wind, boat, earthquake, etc.	Gravity	1 cm - 100 km	1 s - 1 day
Internal waves	Tides, Wind, Topography, etc.	Gravity (stratification) + Coriolis	1 m - 100 km	1 s - 1 day
Rossby waves		PV (variation of Coriolis with latitude)	100 km - 1000 km	Days - Months
Kelvin wave		Pressure gradient + Coriolis	10 km - 100 km	Days - Months
Equatorial Waves		Pressure gradient + Coriolis	10 km - 1000 km	Days - Years

Ocean Waves

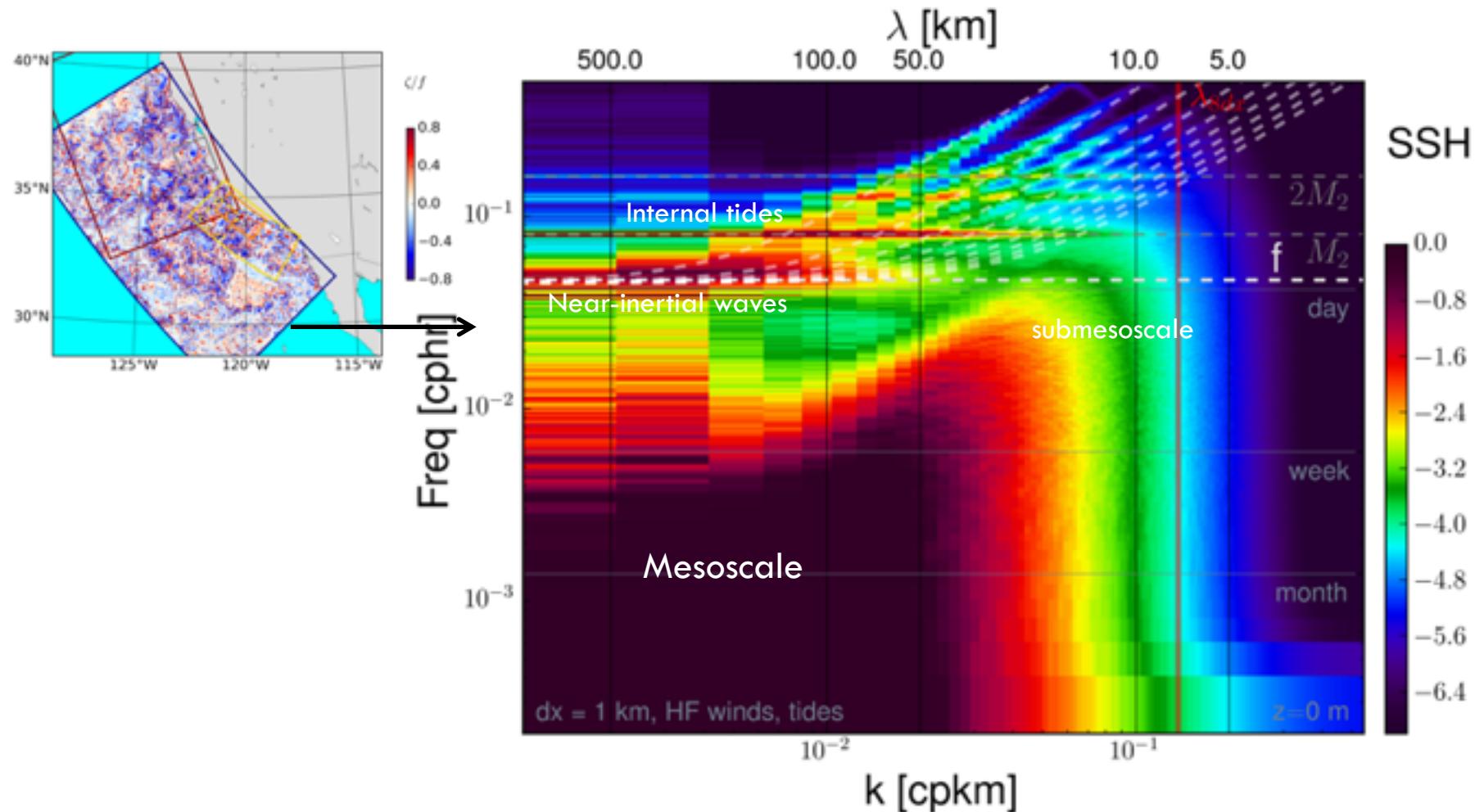


Schematic energy spectrum of ocean variability [Leblond & Mysak]

Ocean Waves



Ocean Waves



Azimuthally-averaged 2D frequency-wavenumber spectra for SSH in California Current

- Mathematically two main classes of waves:

Hyperbolic waves and dispersive waves

Hyperbolic waves

1. **Hyperbolic waves** are formulated in terms of hyperbolic partial differential equations, for example:

$$\eta_t + c\nabla\eta = 0$$

$$\eta_{tt} - c^2\nabla^2\eta = 0$$

Hyperbolic waves

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$$\eta_t + c\nabla\eta = 0$$

$$\eta_{tt} - c^2\nabla^2\eta = 0$$

- With general solutions in the form:

$$\eta = f(x - ct)$$

$$\eta = f(x - ct) + g(x + ct)$$

- Very frequent in acoustics, elasticity, electromagnetism, etc.

Hyperbolic waves

1. Examples of **Hyperbolic waves**

- Flood wave, tidal bores



- Shock wave



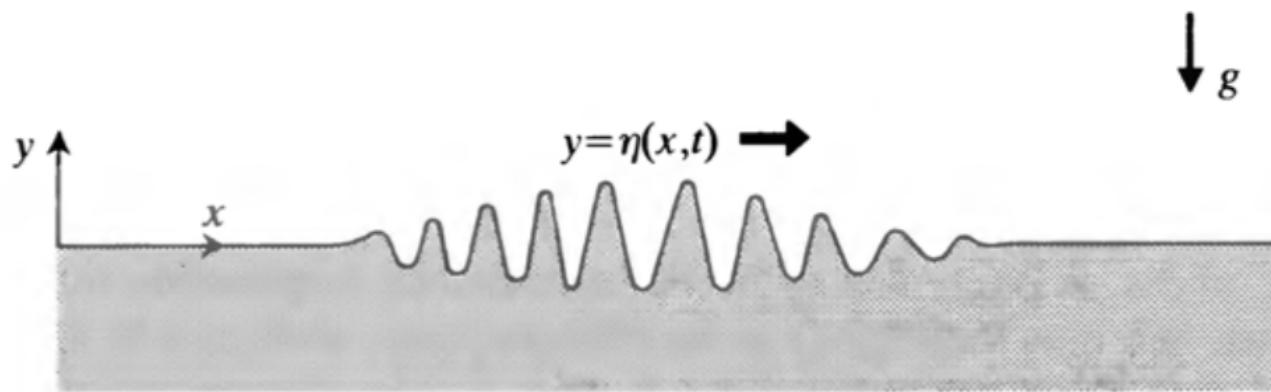
Dispersive waves

2. Dispersive waves come from a variety of partial differential equations, they are characterized principally by their dispersion relation:

$$\omega = f(k)$$

Connecting the frequency and the wave number.

They are visualized as a group of waves where the different Fourier components propagate at different speeds



Dispersive waves

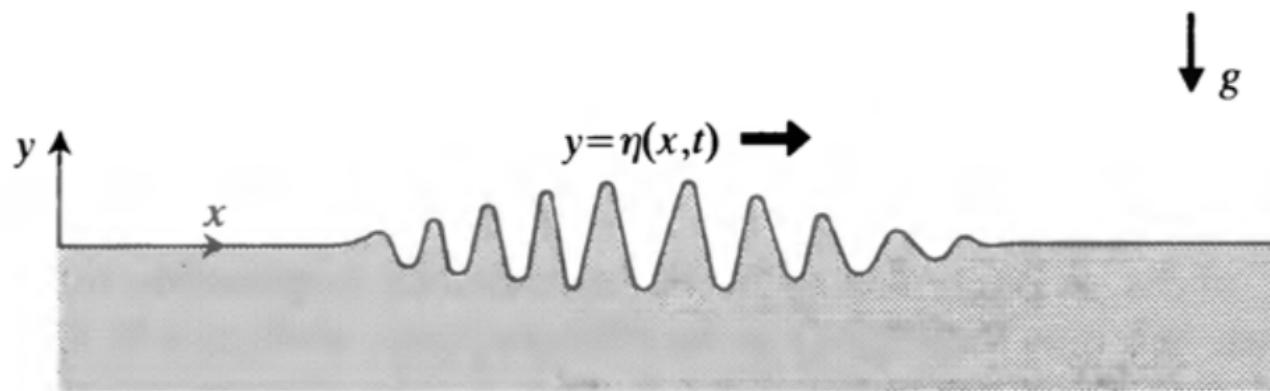
2. Dispersive waves:

$$\omega = f(k)$$

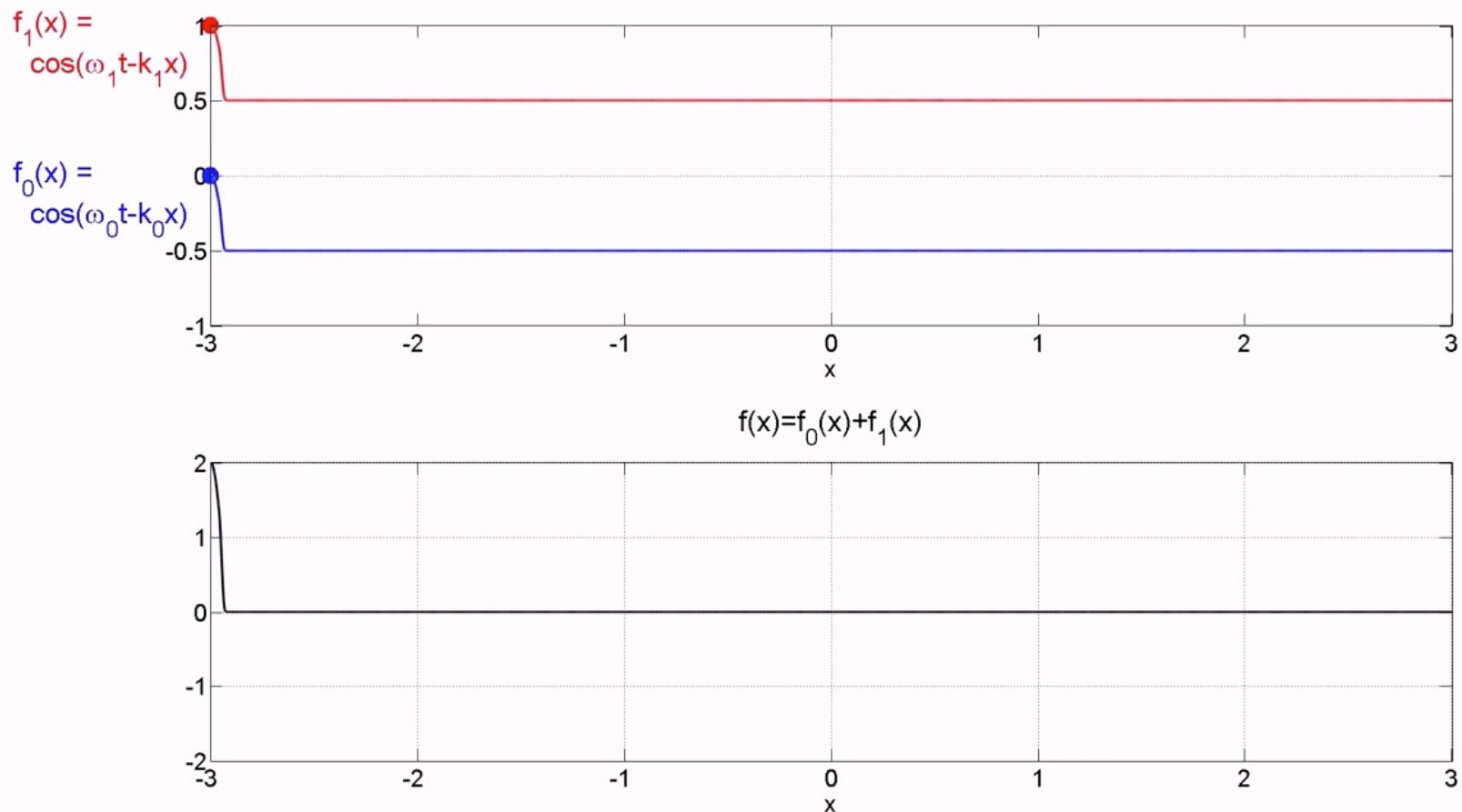
The wave speed is

$$c = \frac{\omega}{k}$$

Energy propagates with the group velocity: $c_g = \frac{\partial \omega}{\partial k}$



Dispersive waves

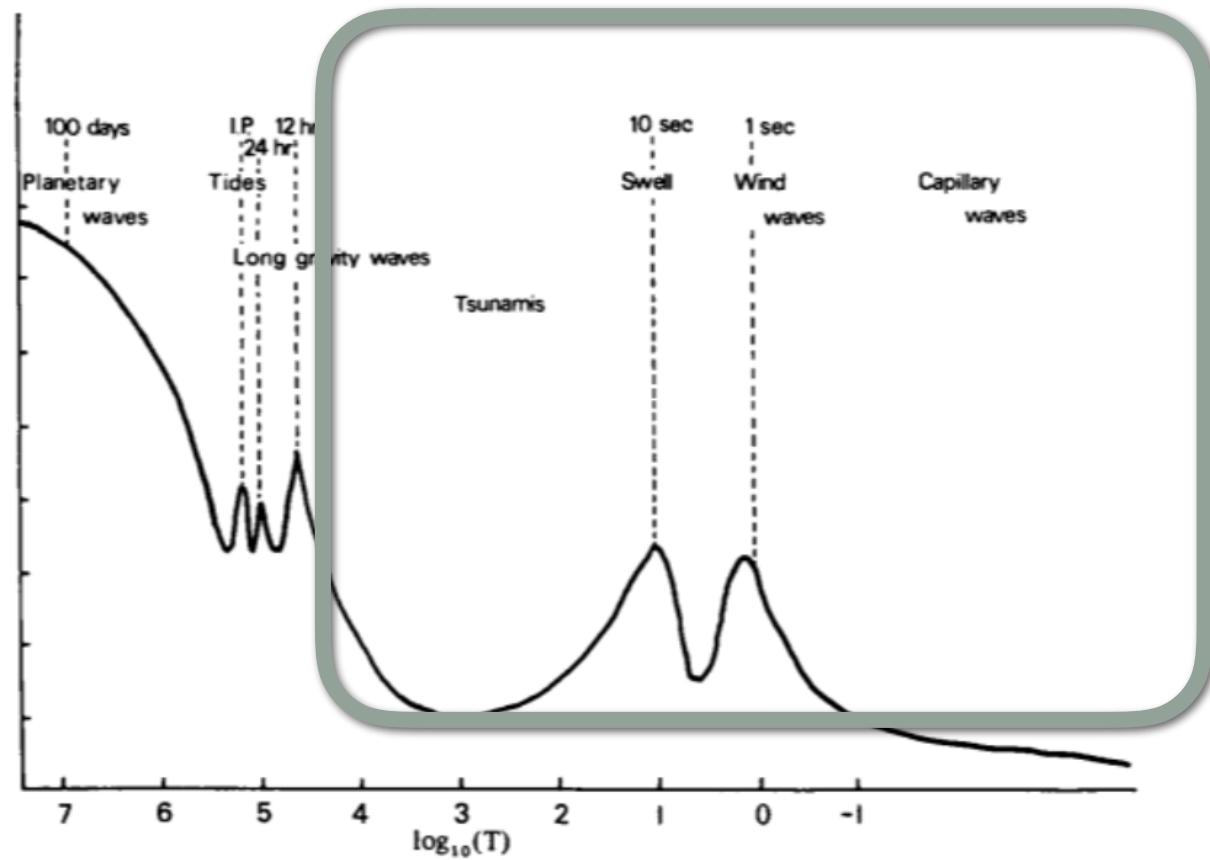


Source: https://www.youtube.com/watch?v=uui9clp_DSg

OCEAN WAVES

1. Surface waves [very brief reminder]
2. Internal waves
3. Long waves
 - a. Rossby waves [Rossby, Poincare, etc.]
 - b. Coastal trapped waves [Kelvin waves, etc.]
 - c. Equatorial waves [Rossby, Kelvin, Yanai]

1. Surface waves



Schematic energy spectrum of ocean variability [Leblond & Mysak]

1. Surface waves

Solving the linearized incompressible Euler equations:

Incompressible Euler equations (convective or Lagrangian form)

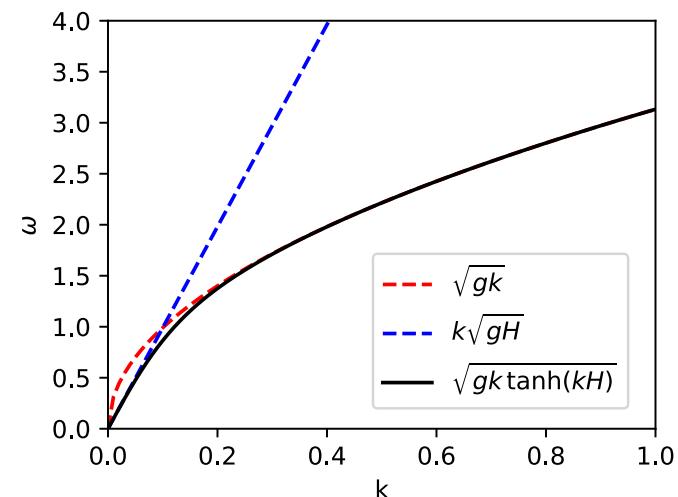
$$\begin{cases} \frac{D\rho}{Dt} = 0 \\ \frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

$$\begin{aligned} u_t &= -p_x \\ v_t &= -p_y \\ w_t &= -p_z - g \\ 0 &= u_x + v_y + w_z, \end{aligned}$$

Looking for a wave solution:

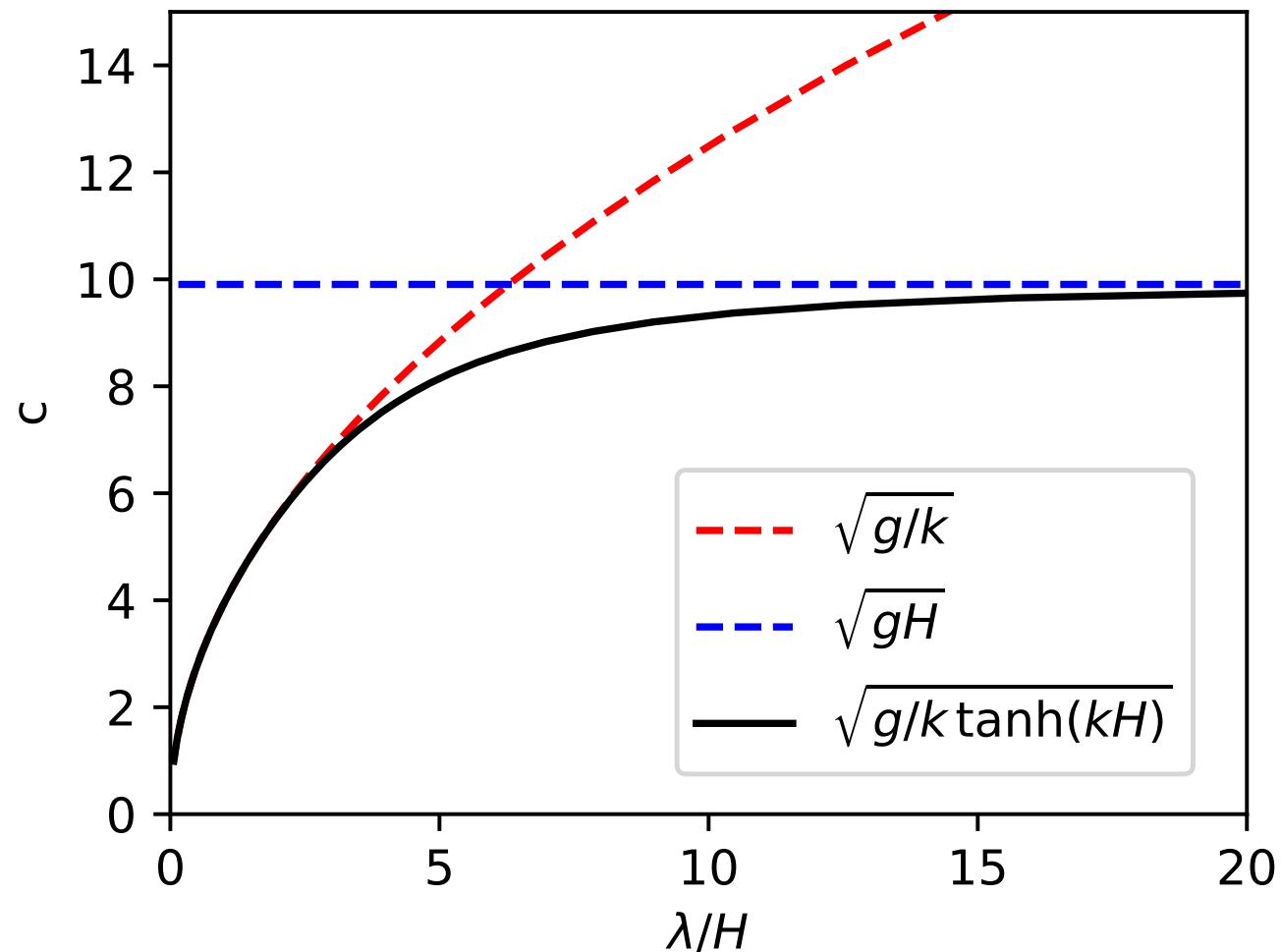
$$\Phi = F(z) e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

We get the dispersion relation :

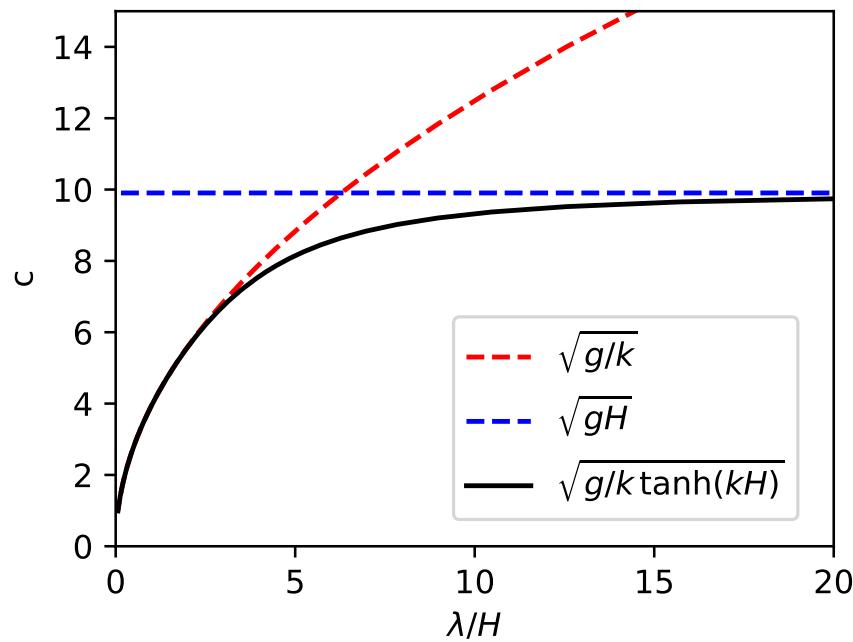


1. Surface waves

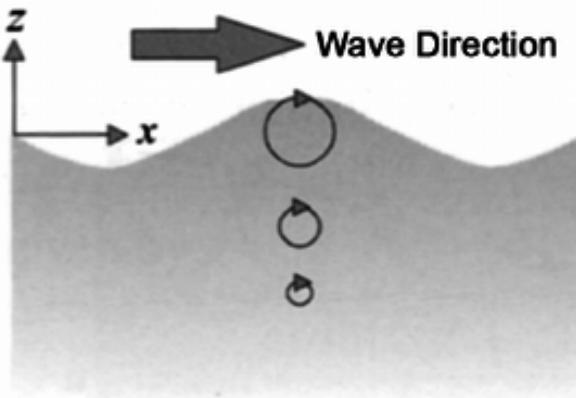
Phase speed: $c = \frac{\omega}{k}$



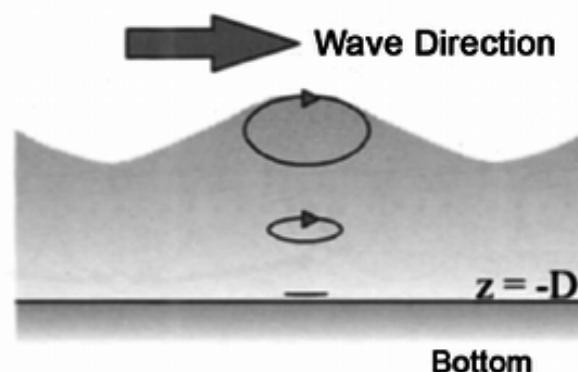
1. Surface waves



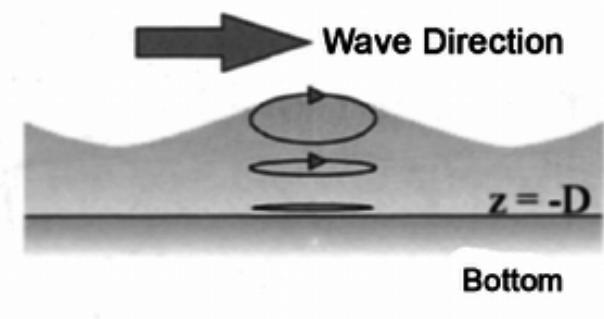
Deep Water



Intermediate Depth

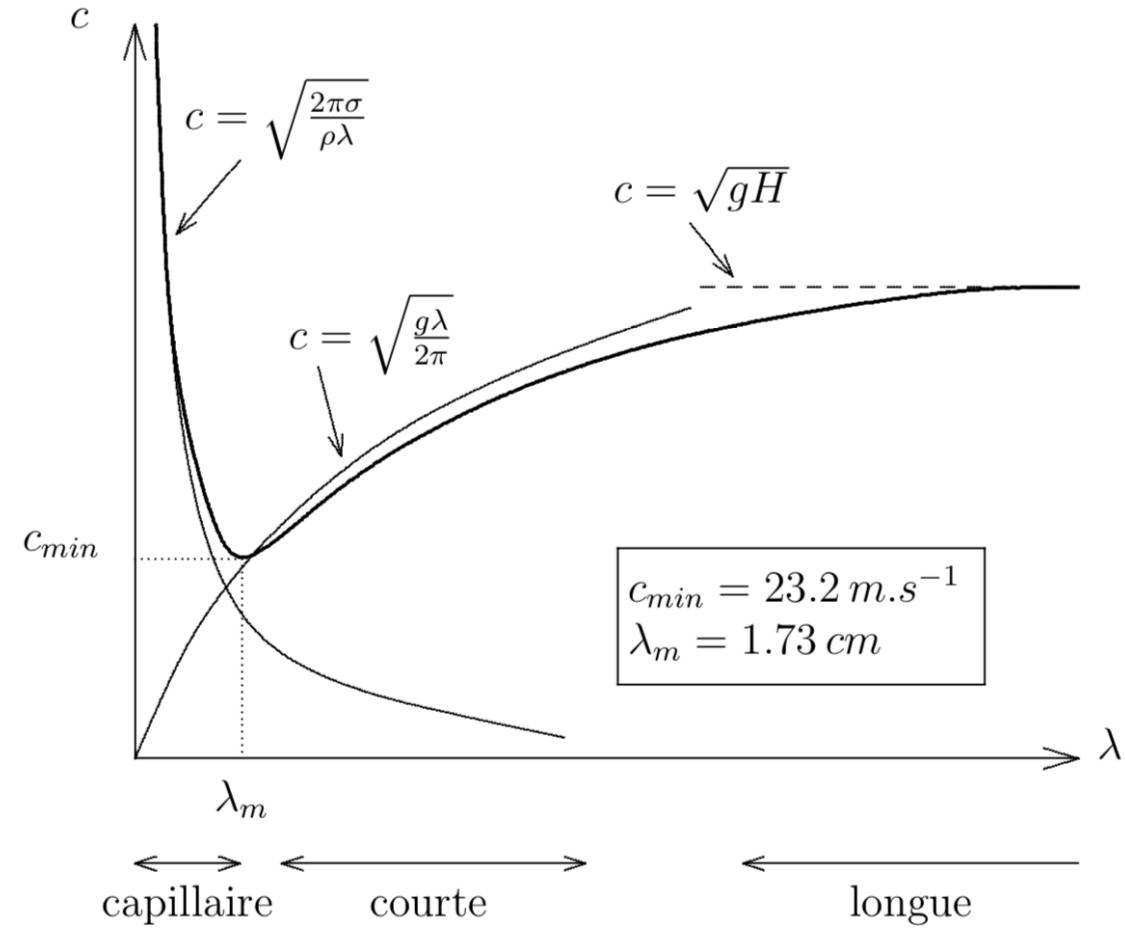


Very Shallow Water



1. Surface waves

With surface tension effects:



OCEAN WAVES

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