

# Numerical Modelling

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*the anatomy of an ocean model*

- **Lesson 1 :**
    - Introduction
    - *Activity 1 [run an ocean model]*
  - **Lesson 2 : [D109]**
    - Equations of motions
    - Subgrid-scale parameterization
    - *Activity 2 [Dynamics of an ocean gyre]*
  - **Lesson 3 : [BBB]**
    - Horizontal Discretization
    - *Activity 2 [Dynamics of an ocean gyre]*
  - **Lesson 4 : [BBB]**
    - Numerical schemes
    - *Activity 3 [Impacts of numerics]*
  - **Lesson 5 : [BBB]**
    - Vertical coordinates
  - Model parameterizations
  - *Activity 4 [Impacts of topography]*
- 
- **Lesson 6 : [BBB]**
    - Boundary Forcings
    - Presentation of the model CROCO
    - *Activity 5 [Design a realistic simulation]*
  - **Lesson 7 : [BBB]**
    - Diagnostics and validation
    - *Activity 6 [Analyze a realistic simulation]*

Presentations and material  
will be available at :

**jgula.fr/ModNum/**

# Useful references

## Extensive courses:

- MIT: <https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-950-atmospheric-and-oceanic-modeling-spring-2004/lecture-notes/>
- Princeton: [https://stephengriffies.github.io/assets/pdfs/GFM\\_lectures.pdf](https://stephengriffies.github.io/assets/pdfs/GFM_lectures.pdf)

## Overview on ocean modelling and current challenges:

- Griffies et al., 2000, Developments in ocean climate modelling, Ocean Modelling. <http://jgula.fr/ModNum/Griffiesetal00.pdf>
- Griffies, 2006, "Some Ocean Model Fundamentals", In "Ocean Weather Forecasting: An Integrated View of Oceanography", 2006, Springer Netherlands. [http://jgula.fr/ModNum/Griffies\\_Chapter.pdf](http://jgula.fr/ModNum/Griffies_Chapter.pdf)
- Fox-Kemper et al, 19, "Challenges and Prospects in Ocean Circulation Models" <http://jgula.fr/ModNum/FoxKemperetal19.pdf>

## ROMS/CROCO:

- <https://www.myroms.org/wiki/>
- Shchepetkin, A., and J. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following- coordinate ocean model. Ocean Modell. <http://jgula.fr/ModNum/ShchepetkinMcWilliams05.pdf>

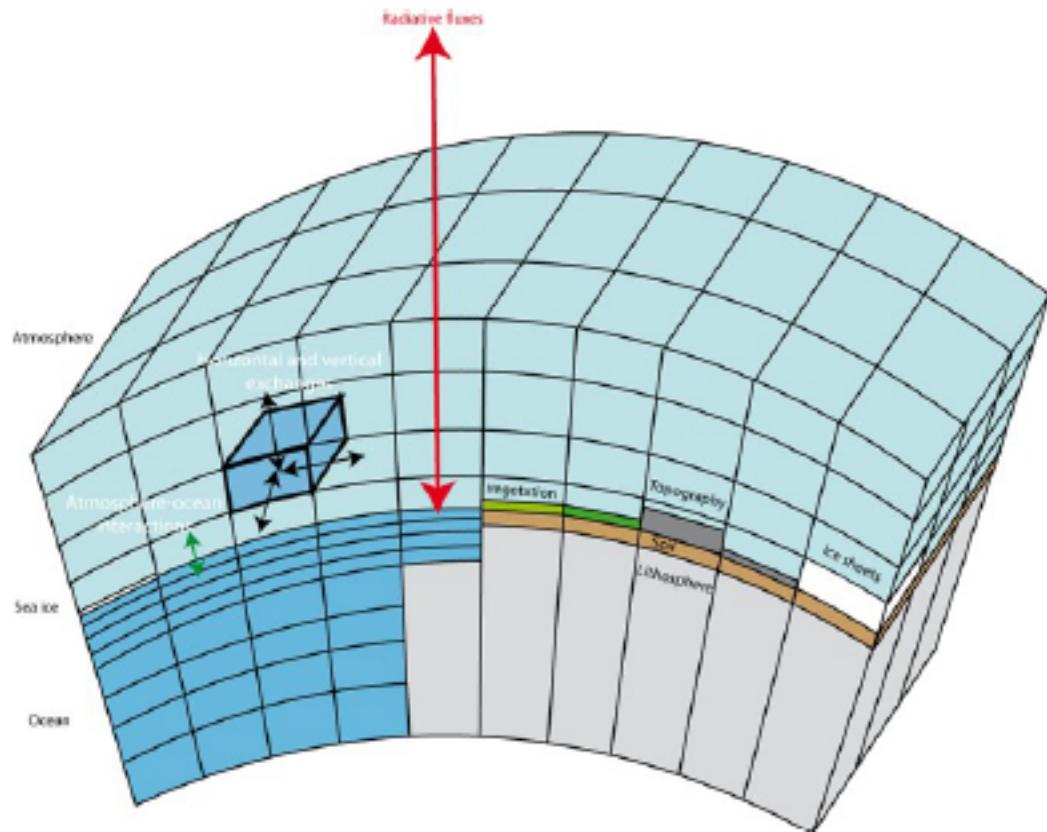
# #3 Discretization

Master's degree 2<sup>nd</sup> year Marine Physics

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# Discretization

The ocean is divided into boxes : discretization

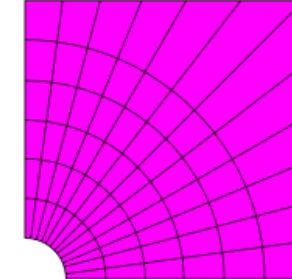


*Example of a finite difference grid*

# Discretization

## Structured grids

Identified by regular connectivity  
= can be addressed by  $(i, j, k)$

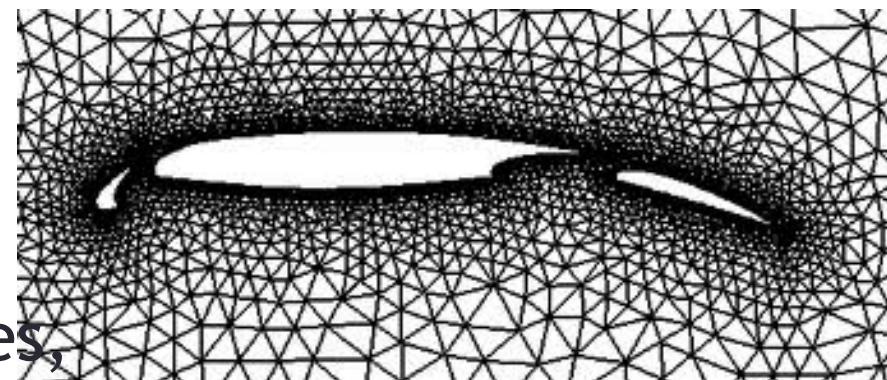


← ROMS

## Unstructured grids

The domain is tiled using more general geometrical shapes (triangles, ...) pieced together to optimally fit details of the geometry.

- ✓ Good for tidal modeling, engineering applications.
- ✓ Problems:  
geostrophic balance accuracy, conservation and positivity properties,



...

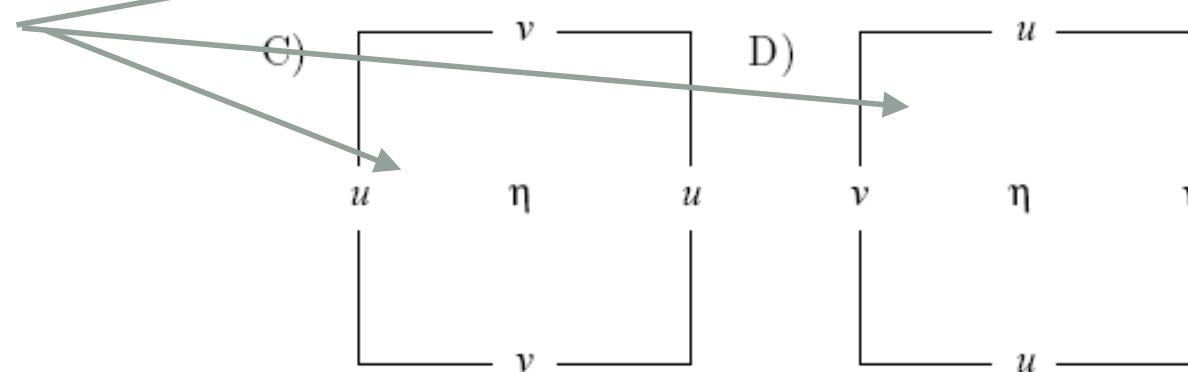
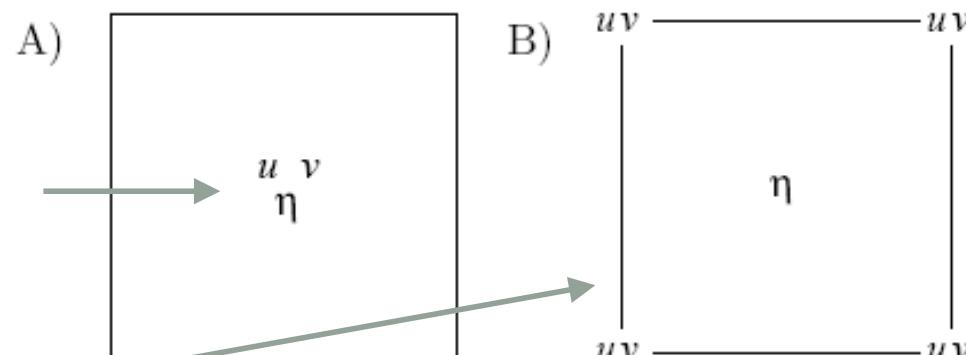
# Horizontal discretization

Different types of Horizontal Grids (Arakawa Grids):

Non-staggered  
(= collocated variables)

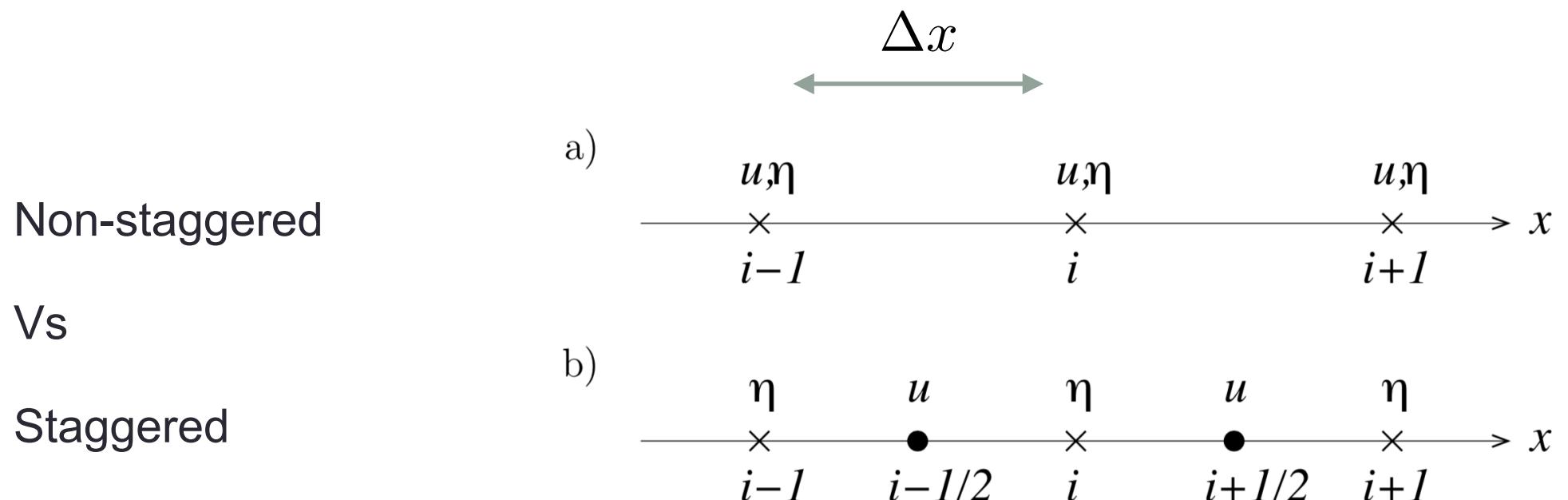
Or

Staggered



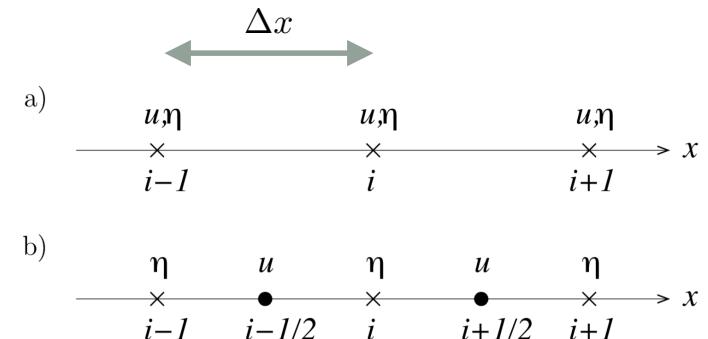
# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



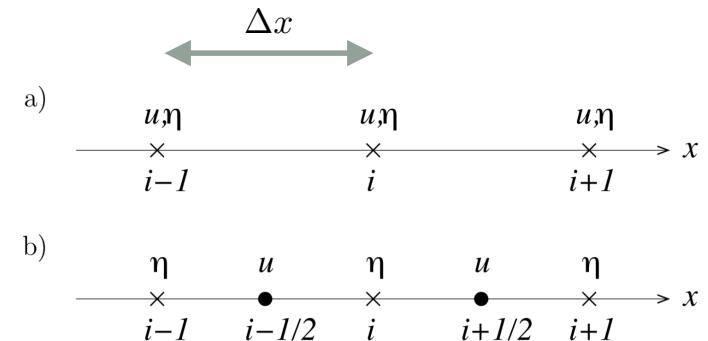
1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

*Solutions of the continuous equations are non-dispersive waves*

$\theta(x, t) = \theta_o e^{i(kx - \omega t)}$  *with dispersion relation*  $\omega = ck$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

Solutions of the **continuous equations** are non-dispersive waves

$$\theta(x, t) = \theta_o e^{i(kx - \omega t)} \text{ with dispersion relation } \omega = ck$$

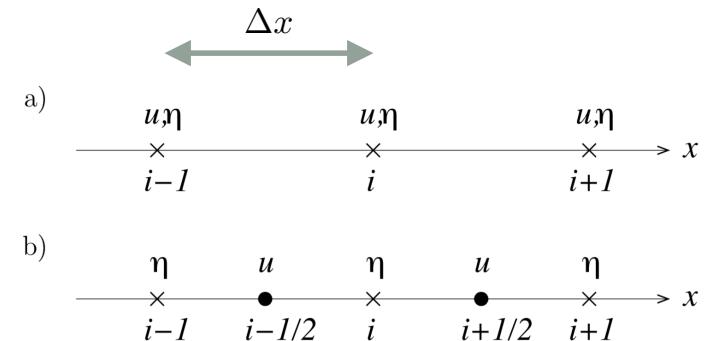
**Discretized equations** with the centered second order derivative are:

$$d_t \theta + \frac{c}{\Delta x} \delta_i \bar{\theta}^i = 0$$

$$d_t \theta_i + \frac{c}{2\Delta x} (\theta_{i+1} - \theta_{i-1}) = 0$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

Substituting in our solution:

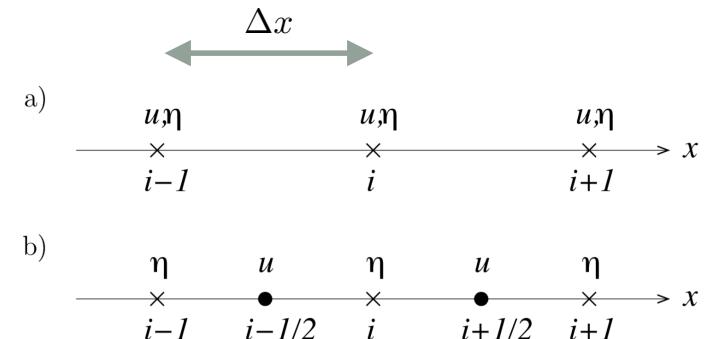
$$\theta_i(x, t) = \theta_0 e^{i(kx - \omega t)}$$

$$\theta_{i-1}(x, t) = \theta_0 e^{i(k(x - \Delta x) - \omega t)}$$

$$\theta_{i+1}(x, t) = \theta_0 e^{i(k(x + \Delta x) - \omega t)}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$

Substituting in our solution gives:

$$\begin{aligned} -i\omega &= -\frac{c}{2\Delta x} (e^{ik\Delta x} - e^{-ik\Delta x}) \\ &= -\frac{ci}{\Delta x} \sin k\Delta x \end{aligned}$$

Now the solution is **dispersive!!!**

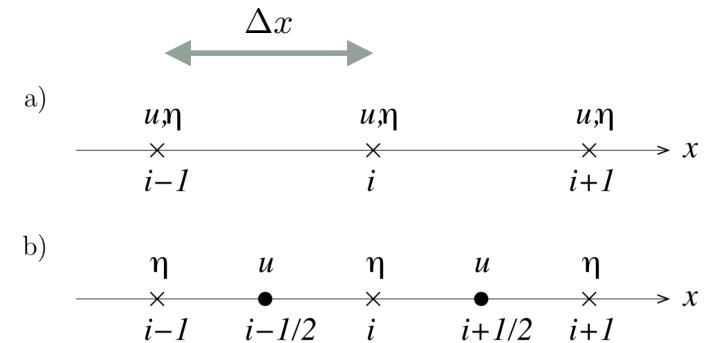
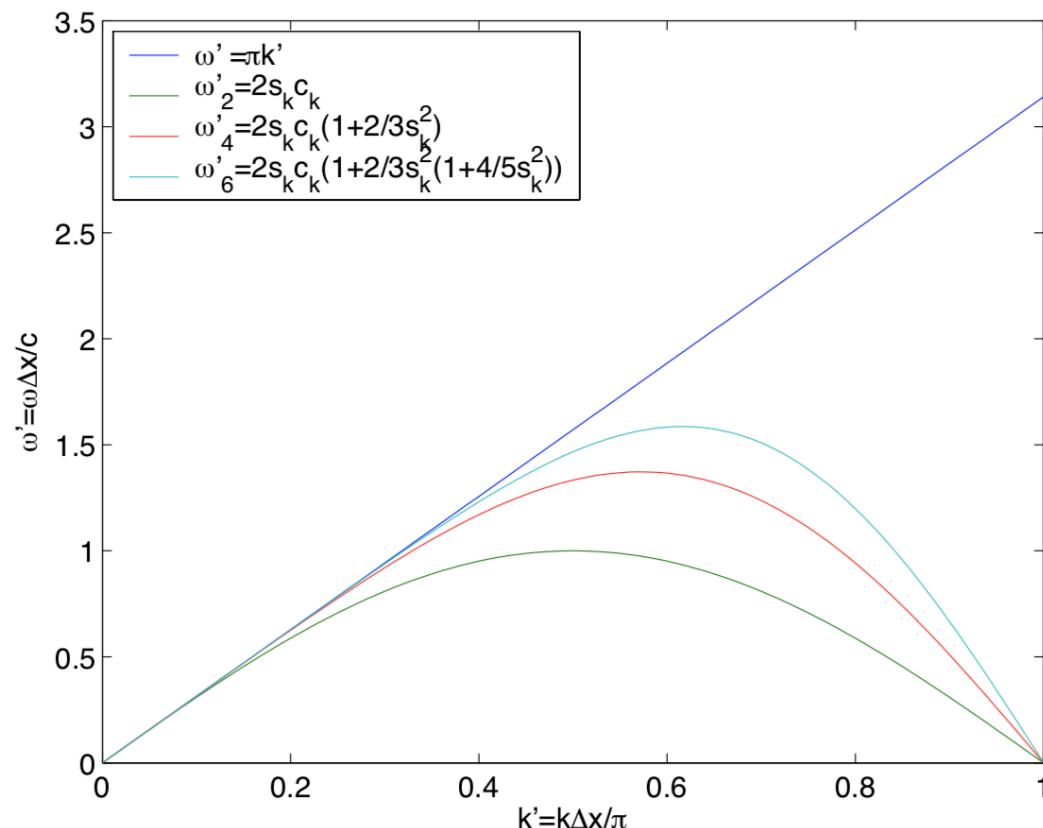
Even if it will converge to the non-dispersive solution in the limit of small  $\Delta x$

$$\omega = \frac{c}{\Delta x} \sin k\Delta x \xrightarrow{\Delta x \rightarrow 0} ck$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

1. The advection equation  $\partial_t \theta + c \partial_x \theta = 0$



Dispersion relations for constant flow advection using second, fourth, and sixth order spatial differences.

# Horizontal discretization

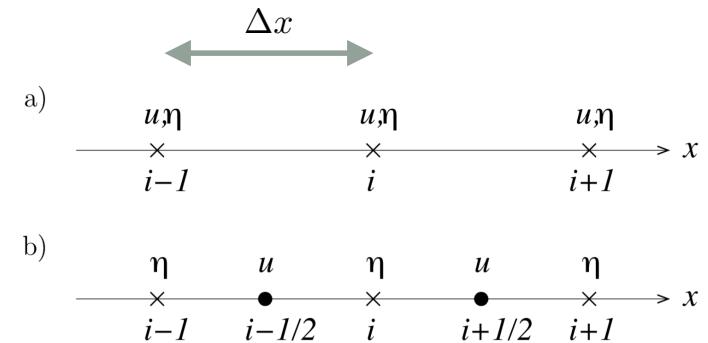
Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

$$\begin{aligned}\partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u\end{aligned} \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta$$

*Solutions of the continuous equations are non-dispersive waves*

$$\eta = \eta_o e^{i(kx - \omega t)} \quad \text{with dispersion relation} \quad \omega = \pm \sqrt{gHk}$$



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

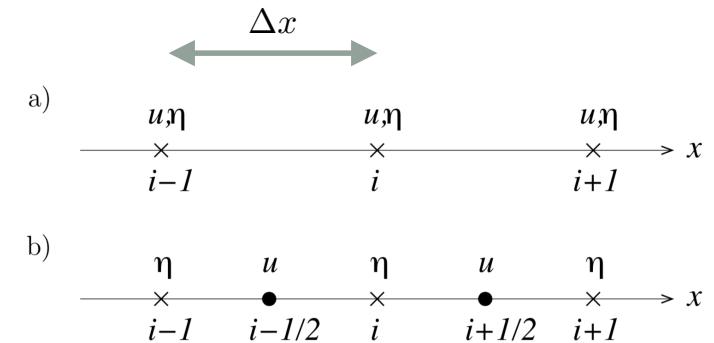
$$\begin{aligned}\partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta\end{aligned}$$

*Solutions of the continuous equations are non-dispersive waves*

$$\eta = \eta_o e^{i(kx - \omega t)} \quad \text{with dispersion relation} \quad \omega = \pm \sqrt{gHk}$$

Discretized equations with the centered second order derivative on the **unstaggered grid** are:

$$\longrightarrow \partial_{tt} \eta = \frac{gH}{\Delta x^2} \delta_{ii} \bar{\eta}^{ii} \quad \text{with} \quad \delta_{ii} \bar{\eta}^{ii} = \frac{1}{4} (\eta_{i-2} - 2\eta_i + \eta_{i+2})$$



# Horizontal discretization

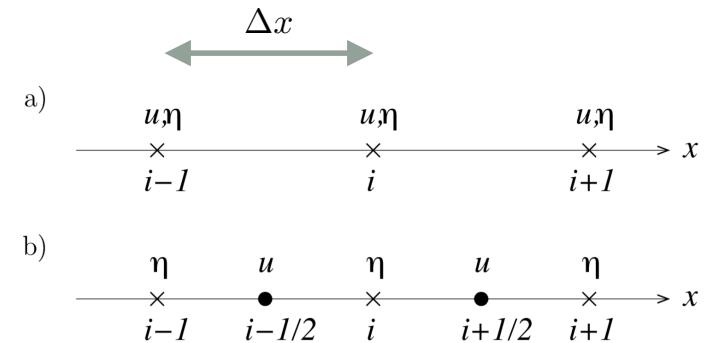
Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

$$\begin{aligned}\partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u\end{aligned} \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta$$

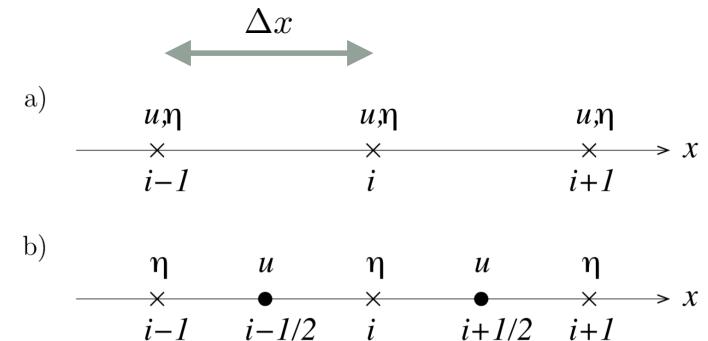
Substituting in our solution on the unstaggered grid gives :

$$\begin{aligned}-\omega^2 &= \frac{gH}{4\Delta x^2} (e^{-i2k\Delta x} - 2 + e^{i2k\Delta x}) \\ &= \frac{gH}{4\Delta x^2} (2 \cos 2k\Delta x - 2) \\ &= -\frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} \cos^2 \frac{k\Delta x}{2}\end{aligned}$$



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem



## 2. Gravity waves

$$\begin{aligned}\partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u\end{aligned} \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta$$

Substituting in our solution on the unstaggered grid gives :

$$\begin{aligned}-\omega^2 &= \frac{gH}{4\Delta x^2} (e^{-i2k\Delta x} - 2 + e^{i2k\Delta x}) \\ &= \frac{gH}{4\Delta x^2} (2 \cos 2k\Delta x - 2) \\ &= -\frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} \cos^2 \frac{k\Delta x}{2}\end{aligned}$$

- Question:
  - What is the dispersion relation on the staggered grid?

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

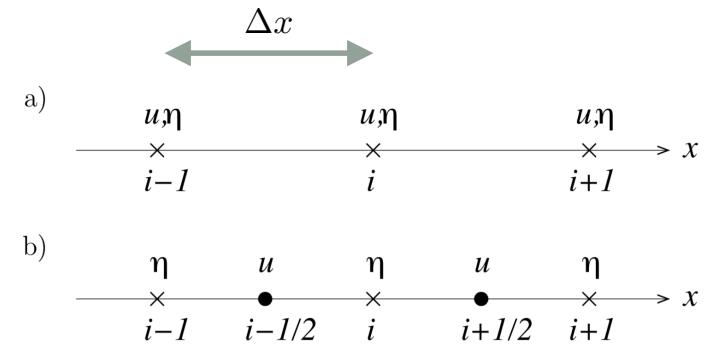
$$\begin{aligned} \partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta \end{aligned}$$

Discretized equations with the centered second order derivative on the **staggered grid** are:

$$\begin{aligned} \partial_t u &= -\frac{g}{\Delta x} \delta_i \eta \\ \partial_t \eta &= -\frac{H}{\Delta x} \delta_i u \end{aligned}$$

This can be written as a system:

$$\begin{pmatrix} \partial_t & \frac{g}{\Delta x} \delta_i \\ \frac{H}{\Delta x} \delta_i & \partial_t \end{pmatrix} \begin{pmatrix} u \\ \eta \end{pmatrix} = 0 \quad \begin{pmatrix} -i\omega & \frac{2ig}{\Delta x} \sin \frac{k\Delta x}{2} \\ \frac{2iH}{\Delta x} \sin \frac{k\Delta x}{2} & -i\omega \end{pmatrix} \begin{pmatrix} u \\ \eta \end{pmatrix} = 0$$

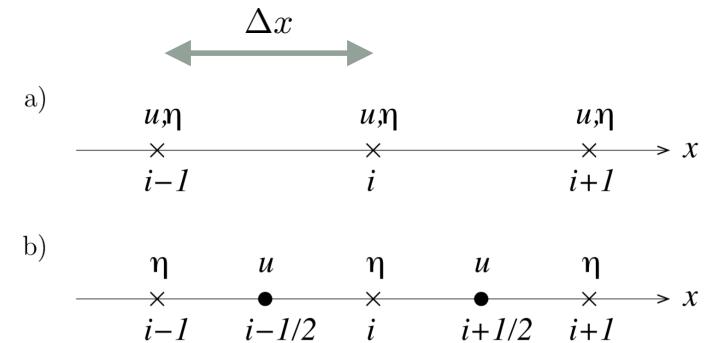


# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

$$\begin{aligned}\partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u\end{aligned} \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta$$



Substituting in our solution on the staggered grid gives :

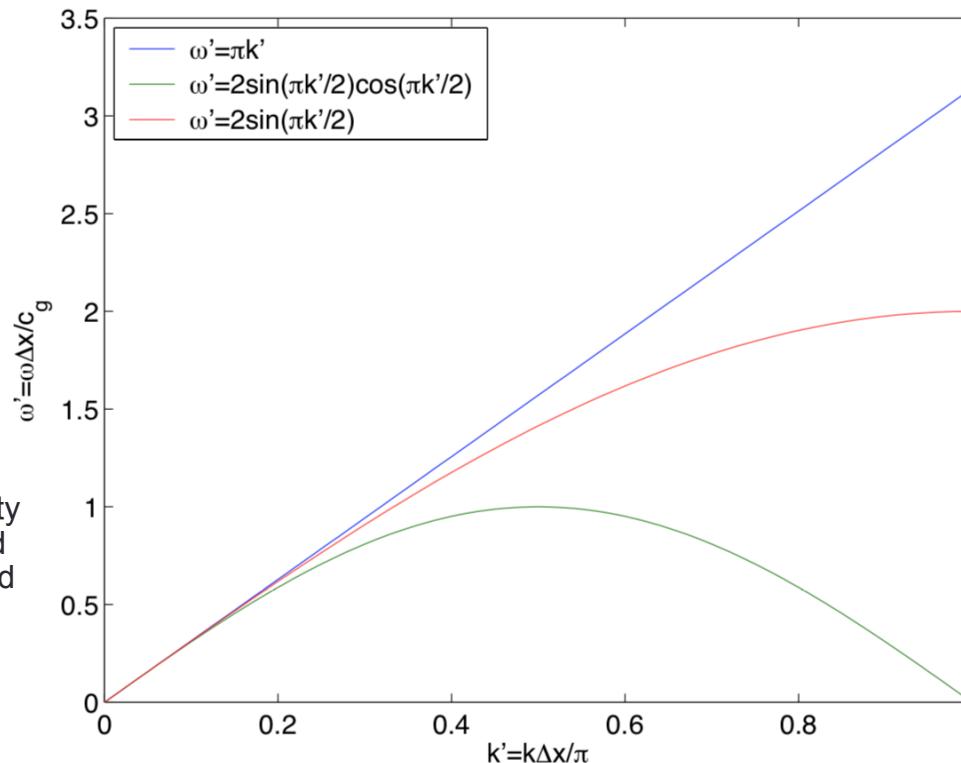
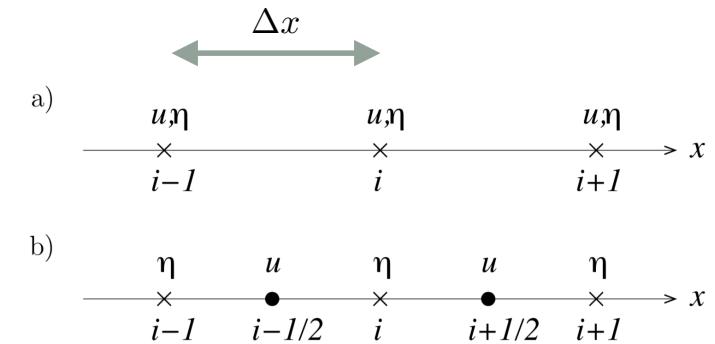
$$\omega^2 - \frac{4gH}{\Delta x^2} \sin^2 \frac{k\Delta x}{2} = 0$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Gravity waves

$$\begin{aligned} \partial_t u &= -g \partial_x \eta \\ \partial_t \eta &= -H \partial_x u \longrightarrow \partial_{tt} \eta = g H \partial_{xx} \eta \end{aligned}$$



Dispersion of numerical gravity wave for the unstaggered grid (green) and the staggered grid (red). The continuum ( $= k$ ) is plotted for comparison (blue).

When compared to the continuum we see that the numerical modes are still dispersive on the staggered grid, but:

there is no false extrema, unlike the non-staggered grid,

the group speed is of the correct sign everywhere, even if reduced.

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

$$\begin{aligned}\partial_t u - fv + g\partial_x \eta &= 0 \\ \partial_t v + fu &= 0 \\ \partial_t \eta + H\partial_x u &= 0\end{aligned}$$

*Solutions of the continuous equations are waves following the dispersion relation:*

$$\left| \begin{pmatrix} -i\omega & -f & gik \\ f & -i\omega & 0 \\ Hik & 0 & -i\omega \end{pmatrix} \right| = 0 \Rightarrow \begin{cases} \omega = 0 \\ \omega^2 = f^2 + gHk^2 \end{cases}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

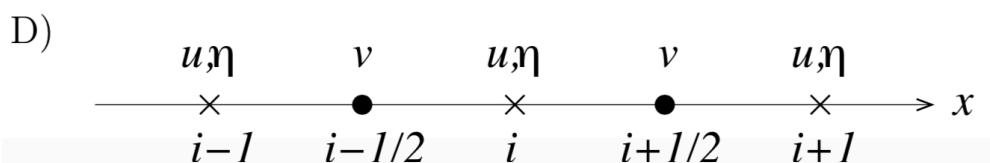
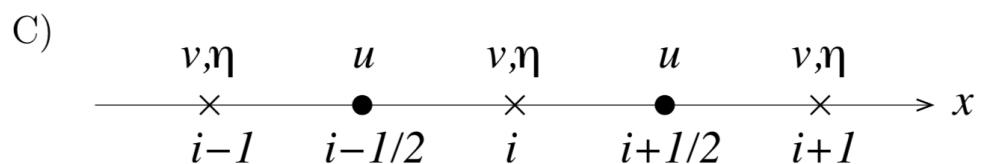
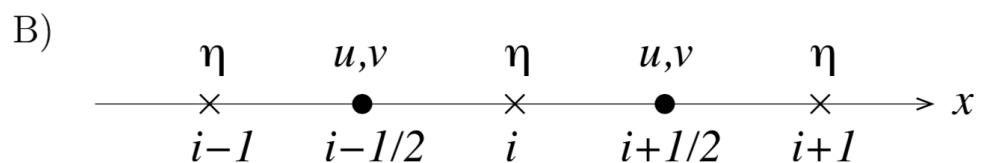
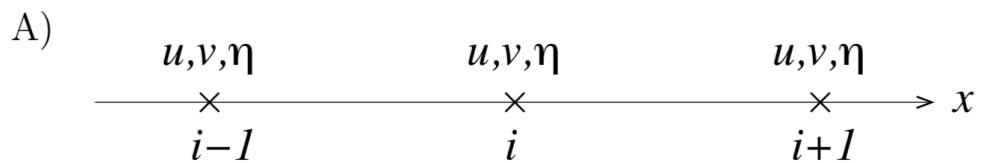
## 2. Inertia-Gravity waves

$$\partial_t u - fv + g \partial_x \eta = 0$$

$$\partial_t v + fu = 0$$

$$\partial_t \eta + H \partial_x u = 0$$

Now, 4 different grids are possible:



# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

- A-grid model

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + f u &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i &= 0\end{aligned}$$

- B-grid model

$$\begin{aligned}\partial_t u - f v + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f u &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u &= 0\end{aligned}$$

- C-grid model

$$\begin{aligned}\partial_t u - f \bar{v}^i + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f \bar{u}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u &= 0\end{aligned}$$

- D-grid model

$$\begin{aligned}\partial_t u - f \bar{v}^i + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + f \bar{u}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i &= 0\end{aligned}$$

# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves

The corresponding dispersion relations are :

A:  $\frac{\omega^2}{f^2} = 1 + \frac{4L_d^2}{\Delta x^2} s_k^2 c_k^2$

B:  $\frac{\omega^2}{f^2} = 1 + \frac{4L_d^2}{\Delta x^2} s_k^2$

C:  $\frac{\omega^2}{f^2} = c_k^2 + \frac{4L_d^2}{\Delta x^2} s_k^2$

D:  $\frac{\omega^2}{f^2} = c_k^2 + \frac{4L_d^2}{\Delta x^2} s_k^2 c_k^2$

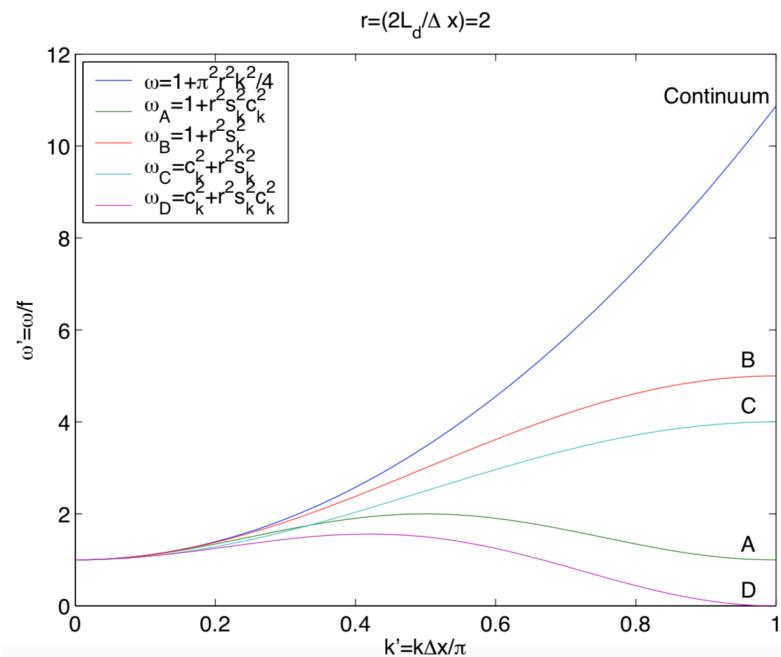
$$s_k = \sin \frac{k\Delta x}{2} \quad c_k = \cos \frac{k\Delta x}{2}$$

$$L_d = \sqrt{gH}/f$$

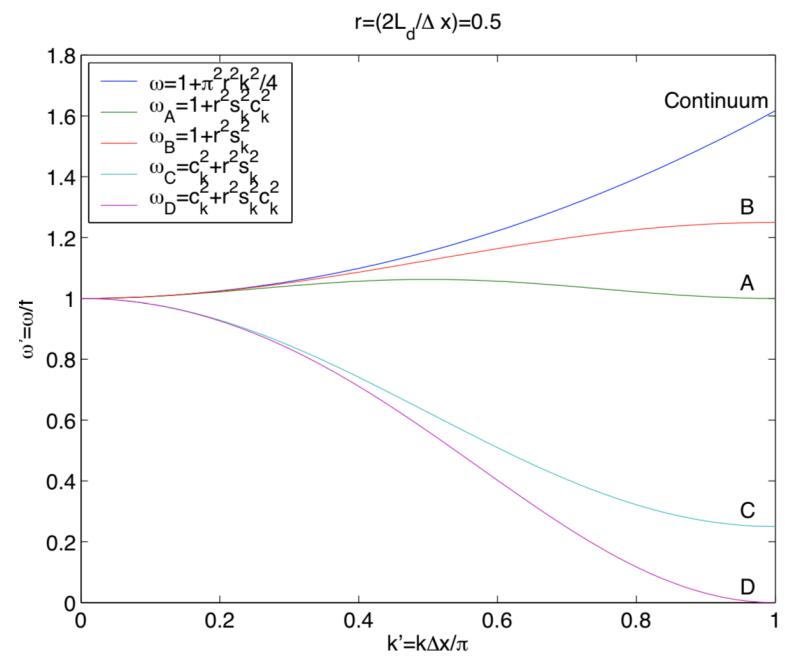
# Horizontal discretization

Staggered Vs unstaggered : the 1D problem

## 2. Inertia-Gravity waves



deformation radius is resolved

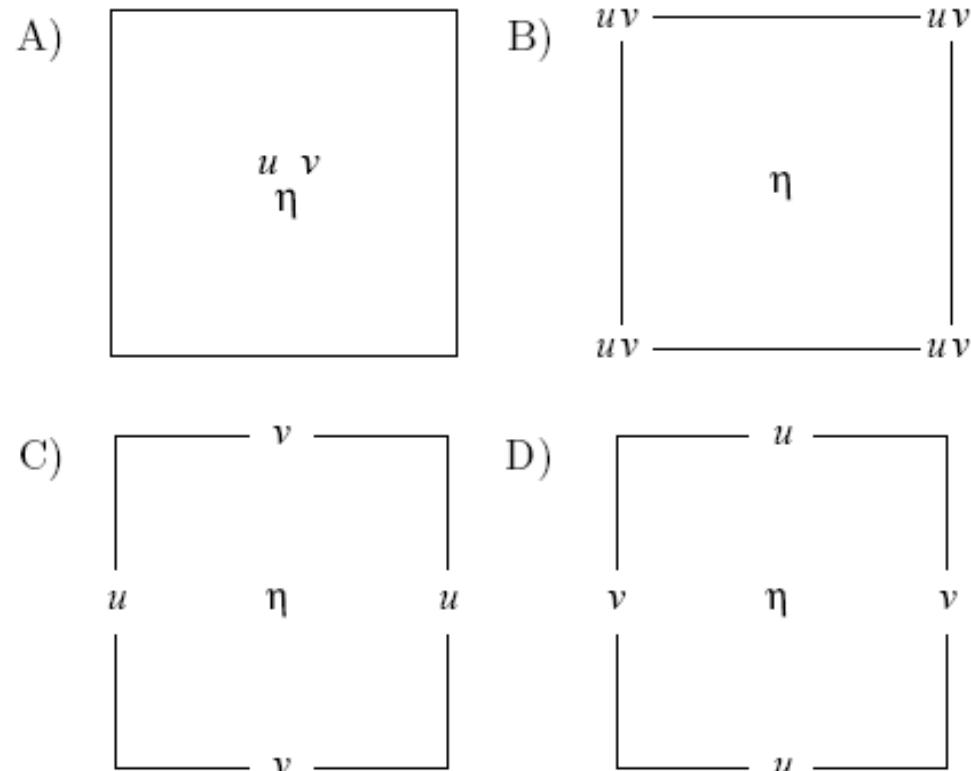


deformation radius is not resolved

Staggering variables in the form of the B grid is most likely to avoid computational modes when solving one-dimensional shallow water equations.

# Horizontal discretization

Horizontal Arakawa Grids:



Linear shallow water equation:

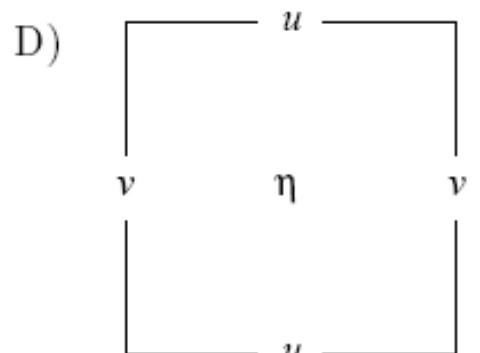
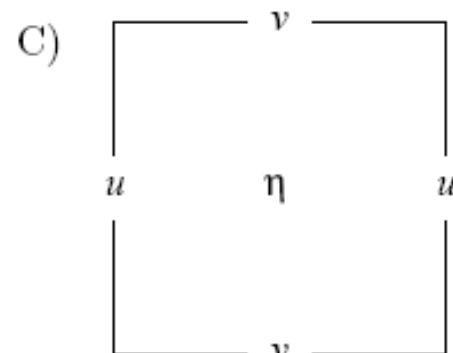
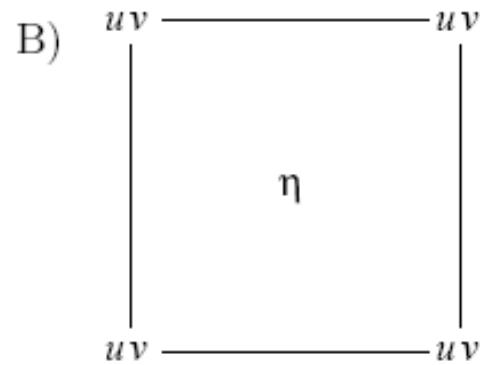
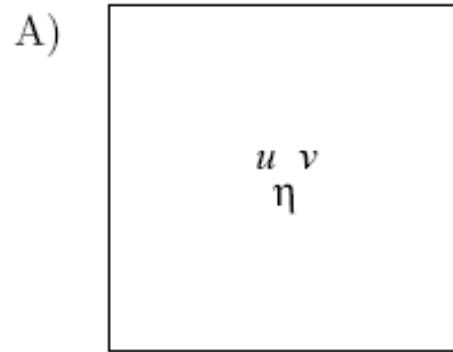
$$\partial_t u - fv + \frac{g}{\Delta x} \delta_i \eta = 0$$

$$\partial_t v + fu + \frac{g}{\Delta y} \delta_j \eta = 0$$

$$\partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_y \eta = 0$$

# Horizontal discretization

Horizontal Arakawa Grids:



Linear shallow water equation:

$$\partial_t u - fv + \frac{g}{\Delta x} \delta_i \eta = 0$$

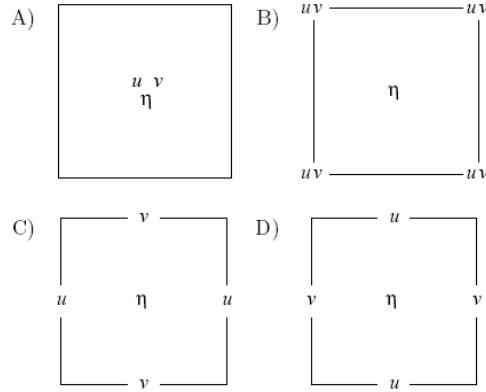
$$\partial_t v + fu + \frac{g}{\Delta y} \delta_j \eta = 0$$

$$\partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_y \eta = 0$$

- Question:

- Which grid minimises the number of averaging between points when solving linear SW equations in 2d?

# Horizontal discretization



- A grid:

$$\begin{aligned}\partial_t u - fv + \frac{g}{\Delta x} \delta_i \bar{\eta}^i &= 0 \\ \partial_t v + fu + \frac{g}{\Delta y} \delta_j \bar{\eta}^j &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^i + \frac{H}{\Delta y} \delta_j \bar{v}^j &= 0\end{aligned}$$

- B grid:

$$\begin{aligned}\partial_t u - fv + \frac{g}{\Delta x} \delta_i \bar{\eta}^j &= 0 \\ \partial_t v + fu + \frac{g}{\Delta y} \delta_j \bar{\eta}^i &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^j + \frac{H}{\Delta y} \delta_j \bar{v}^i &= 0\end{aligned}$$

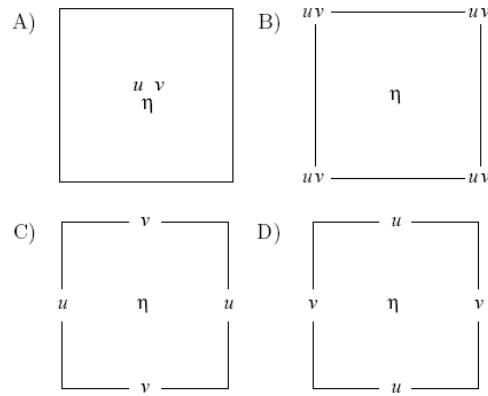
- C grid:

$$\begin{aligned}\partial_t u - f\bar{v}^{ij} + \frac{g}{\Delta x} \delta_i \eta &= 0 \\ \partial_t v + f\bar{u}^{ij} + \frac{g}{\Delta y} \delta_j \eta &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i u + \frac{H}{\Delta y} \delta_j v &= 0\end{aligned}$$

- D grid:

$$\begin{aligned}\partial_t u - f\bar{v}^{ij} + \frac{g}{\Delta x} \delta_i \bar{\eta}^{ij} &= 0 \\ \partial_t v + f\bar{u}^{ij} + \frac{g}{\Delta y} \delta_j \bar{\eta}^{ij} &= 0 \\ \partial_t \eta + \frac{H}{\Delta x} \delta_i \bar{u}^{ij} + \frac{H}{\Delta y} \delta_j \bar{v}^{ij} &= 0\end{aligned}$$

# Horizontal discretization



Response of each operator:

$$\begin{aligned} R(\delta_i \phi) &= 2i \sin \frac{k \Delta x}{2} = 2i s_k \\ R(\delta_j \phi) &= 2i \sin \frac{l \Delta y}{2} = 2i s_l \\ R(\bar{\phi}^i) &= \cos \frac{k \Delta x}{2} = c_k \\ R(\bar{\phi}^j) &= \cos \frac{l \Delta y}{2} = c_l \end{aligned}$$

Dispersion relations:

- A grid:

$$\omega^2 = f^2 + \frac{4gH}{\Delta x^2} s_k^2 c_k^2 + \frac{4gH}{\Delta y^2} s_l^2 c_l^2$$

$$\text{or } \left(\frac{\omega}{f}\right)^2 = 1 + r_x^2 s_k^2 c_k^2 + r_y^2 s_l^2 c_l^2$$

- B grid:

$$\omega^2 = f^2 + \frac{4gH}{\Delta x^2} s_k^2 c_l^2 + \frac{4gH}{\Delta y^2} s_l^2 c_k^2$$

$$\text{or } \left(\frac{\omega}{f}\right)^2 = 1 + r_x^2 s_k^2 c_l^2 + r_y^2 s_l^2 c_k^2$$

- C grid:

$$\omega^2 = f^2 c_k^2 c_l^2 + \frac{4gH}{\Delta x^2} s_k^2 + \frac{4gH}{\Delta y^2} s_l^2$$

$$\text{or } \left(\frac{\omega}{f}\right)^2 = c_k^2 c_l^2 + r_x^2 s_k^2 + r_y^2 s_l^2$$

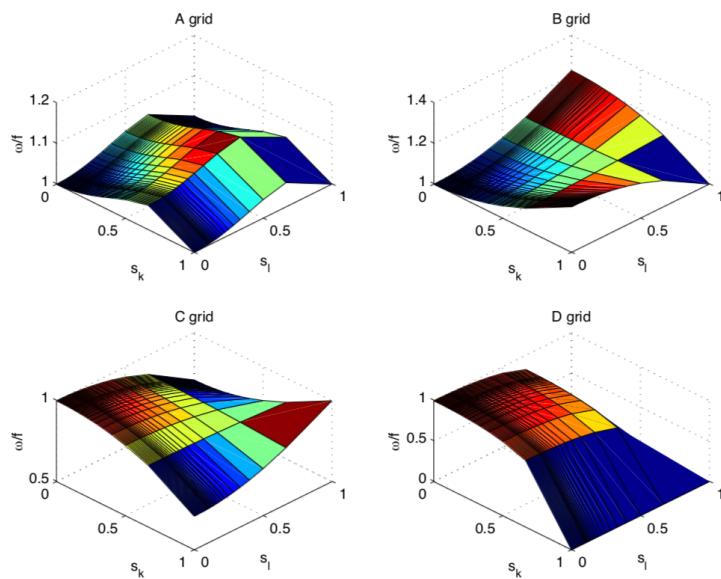
- D grid:

$$\omega^2 = f^2 c_k^2 c_l^2 + \frac{4gH}{\Delta x^2} s_k^2 c_k^2 c_l^2 + \frac{4gH}{\Delta y^2} s_l^2 c_k^2 c_l^2$$

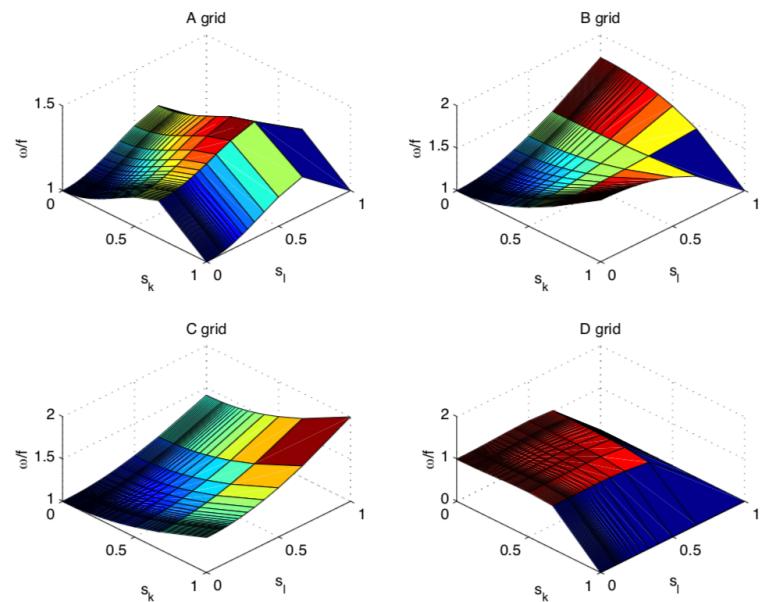
$$\text{or } \left(\frac{\omega}{f}\right)^2 = (1 + r_x^2 s_k^2 + r_y^2 s_l^2) c_k^2 c_l^2$$

# Horizontal discretization

Coarse resolution:



High resolution:



D is always bad.

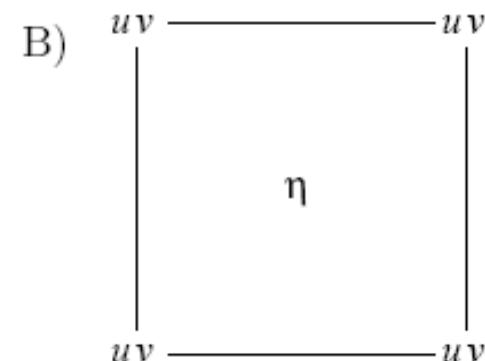
B underestimates frequency for short two-dimensional waves

C is the only grid with monotonically increasing frequency (i.e. right sign of group velocity) at high res.

# Horizontal discretization

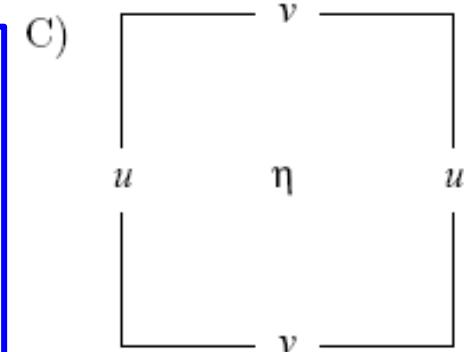
- B grid is preferred at coarse resolution, when Coriolis is important:

- Superior for poorly resolved inertia-gravity waves.
- Good for Rossby waves: collocation of velocity points.
- Bad for gravity waves: computational checkerboard mode



- C grid is preferred at fine resolution, when Coriolis is less important

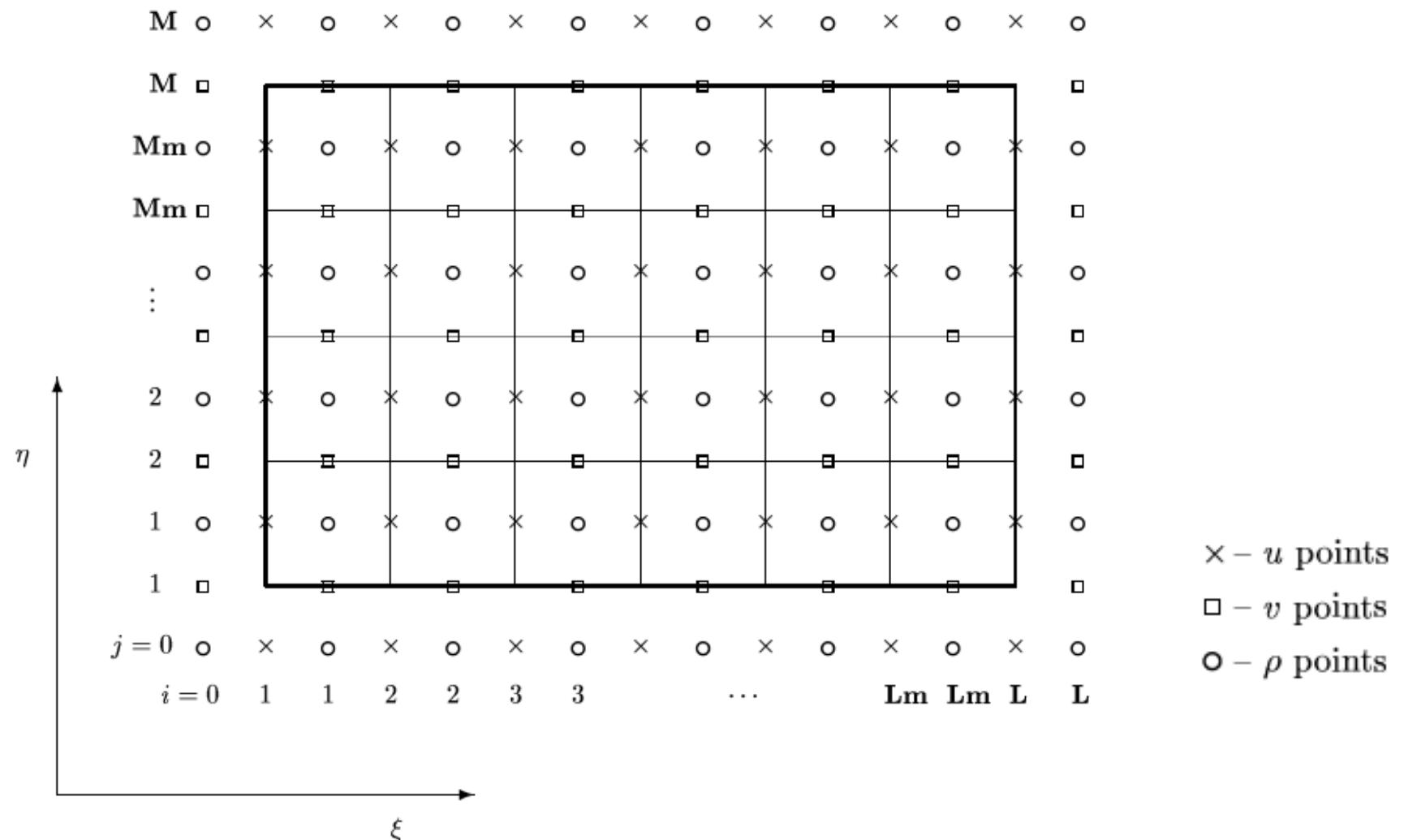
- Superior for gravity waves.
- Good for well resolved inertia-gravity waves.
- Bad for poorly resolved waves: Rossby waves (computational checkerboard mode) and inertia-gravity waves due to averaging the Coriolis force.



ROMS

# Horizontal discretization

**ROMS:** Arakawa C-grid



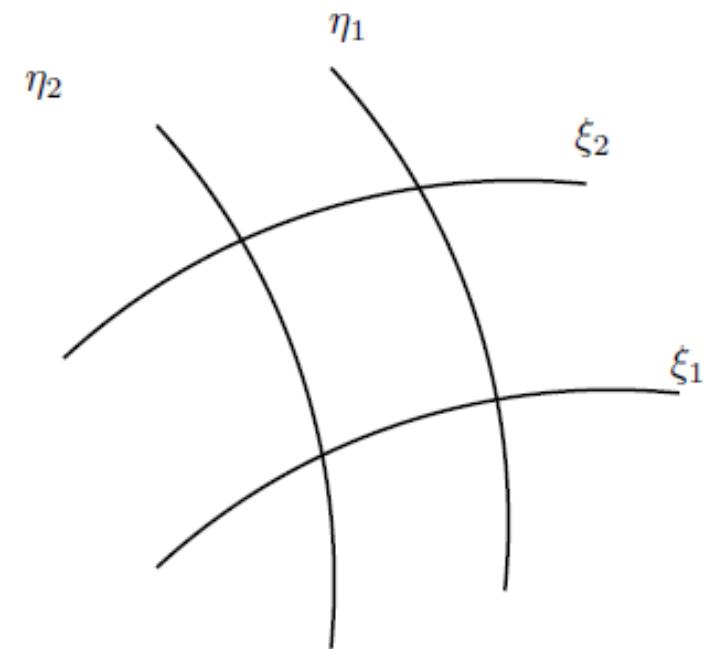
# Horizontal curvilinear coordinates

- **ROMS:** is formulated in general horizontal curvilinear coordinates:

$$(ds)_\xi = \left( \frac{1}{m} \right) d\xi$$

$$(ds)_\eta = \left( \frac{1}{n} \right) d\eta$$

$m, n$ : scale factors relating the differential distances to the physical arc lengths



$$\vec{v} \cdot \hat{\xi} = u$$

$$\vec{v} \cdot \hat{\eta} = v$$

# Horizontal curvilinear grid

- **ROMS:** is formulated in general horizontal curvilinear coordinates:

$$(ds)_\xi = \left( \frac{1}{m} \right) d\xi$$

$$(ds)_\eta = \left( \frac{1}{n} \right) d\eta$$

*With classical formulas for div, grad, curl and lap in curvilinear coordinates:*

$$\nabla \phi = \hat{\xi} m \frac{\partial \phi}{\partial \xi} + \hat{\eta} n \frac{\partial \phi}{\partial \eta}$$

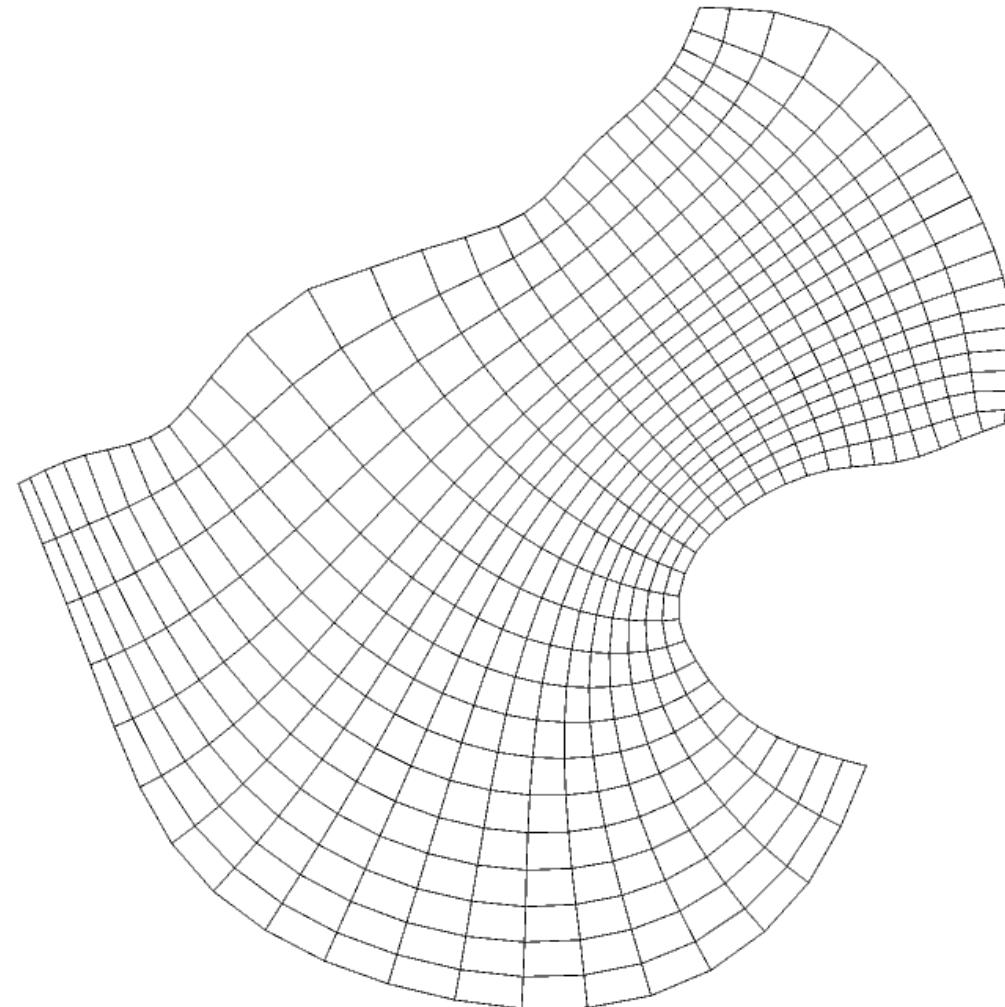
$$\nabla \cdot \vec{a} = mn \left[ \frac{\partial}{\partial \xi} \left( \frac{a}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{b}{m} \right) \right]$$

$$\nabla \times \vec{a} = mn \begin{vmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \hat{k} \\ \frac{\partial}{\partial \xi} & \frac{\partial}{\partial \eta} & \frac{\partial}{\partial z} \\ \frac{a}{m} & \frac{b}{n} & c \end{vmatrix}$$

$$\nabla^2 \phi = \nabla \cdot \nabla \phi = mn \left[ \frac{\partial}{\partial \xi} \left( \frac{m}{n} \frac{\partial \phi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{n}{m} \frac{\partial \phi}{\partial \eta} \right) \right]$$

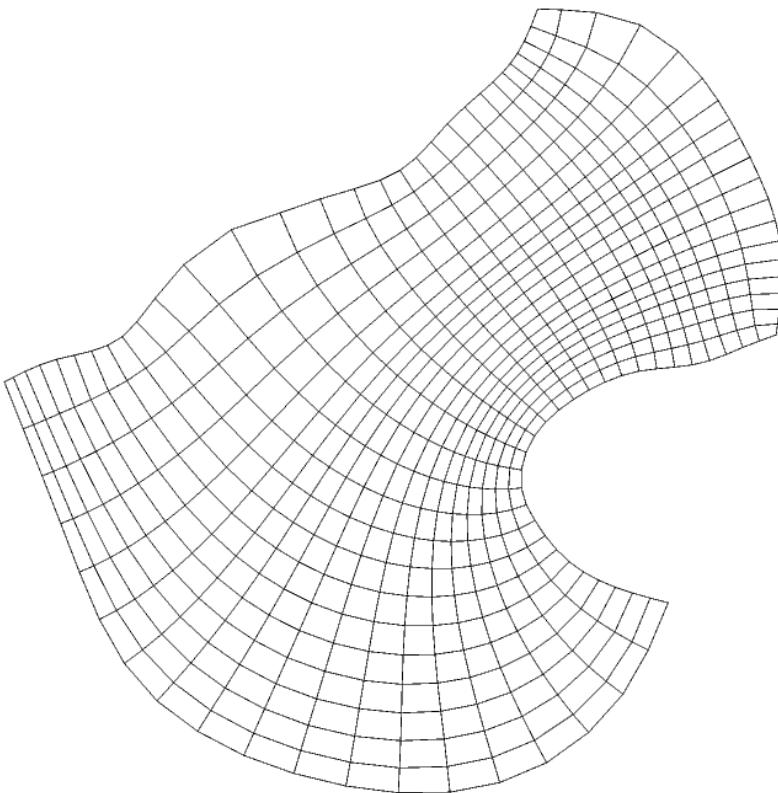
# Horizontal curvilinear grid

- This is a possible grid:



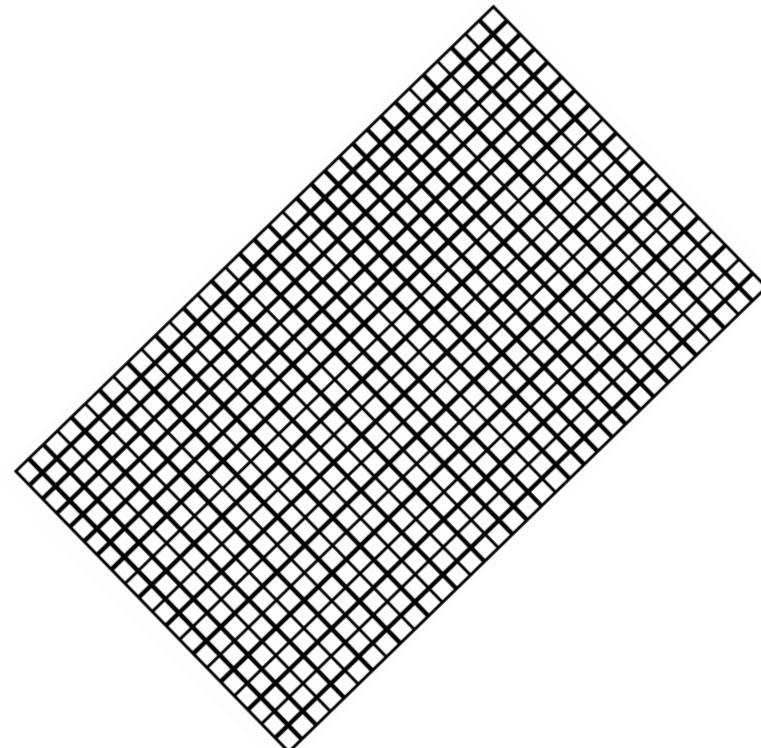
# Horizontal curvilinear grid

- This is a possible grid:



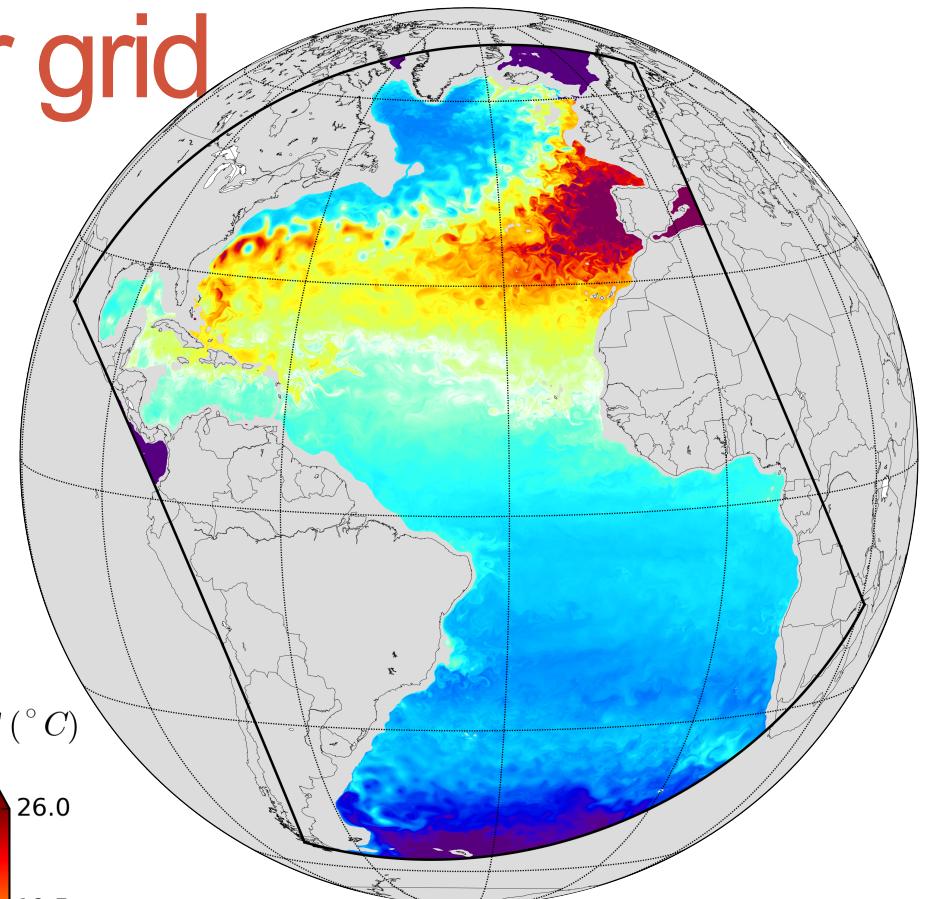
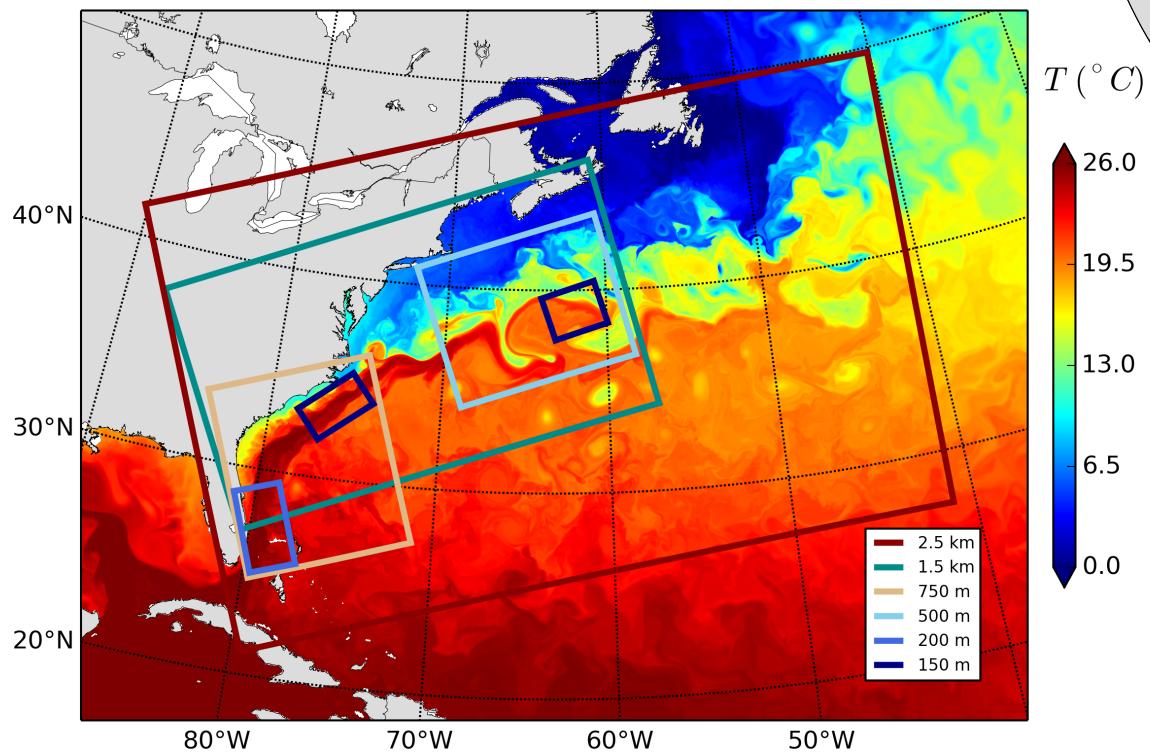
In practice variations in  $dx$  and  $dy$  should be minimized to minimize errors and optimize computation time.

**So avoir extreme distortions and be as close as rectangular grids as possible (+ use land masks)**



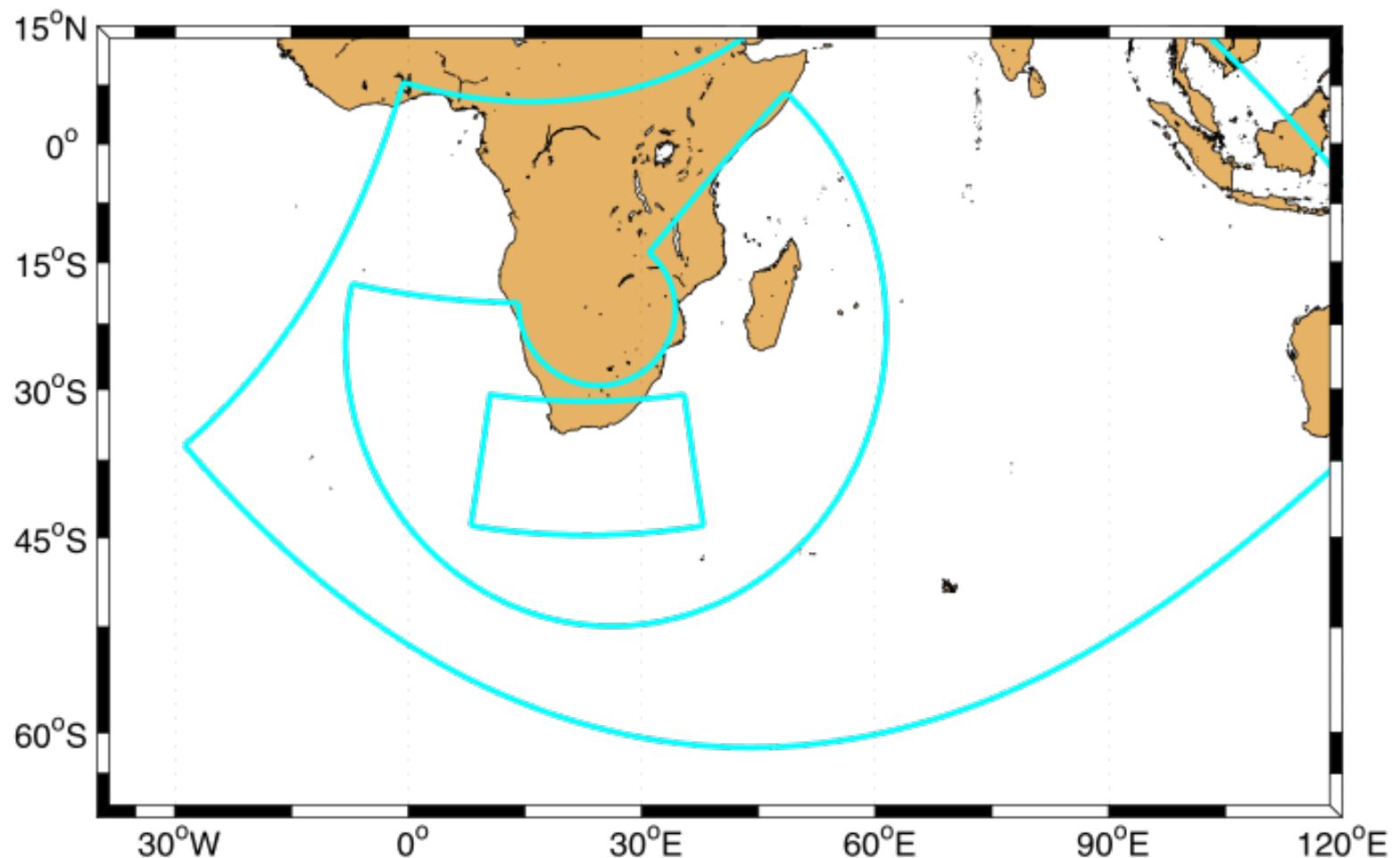
# Horizontal curvilinear grid

- Example of realistic domains:



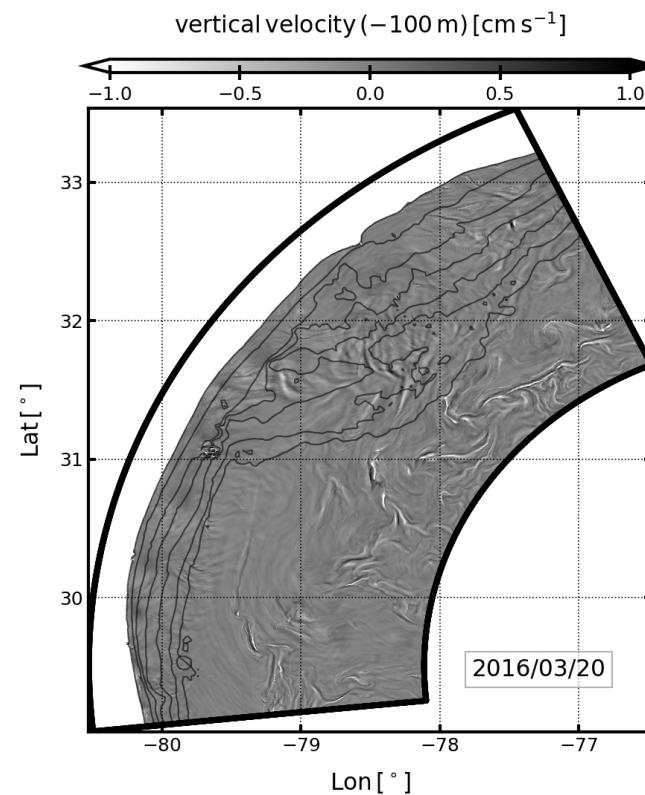
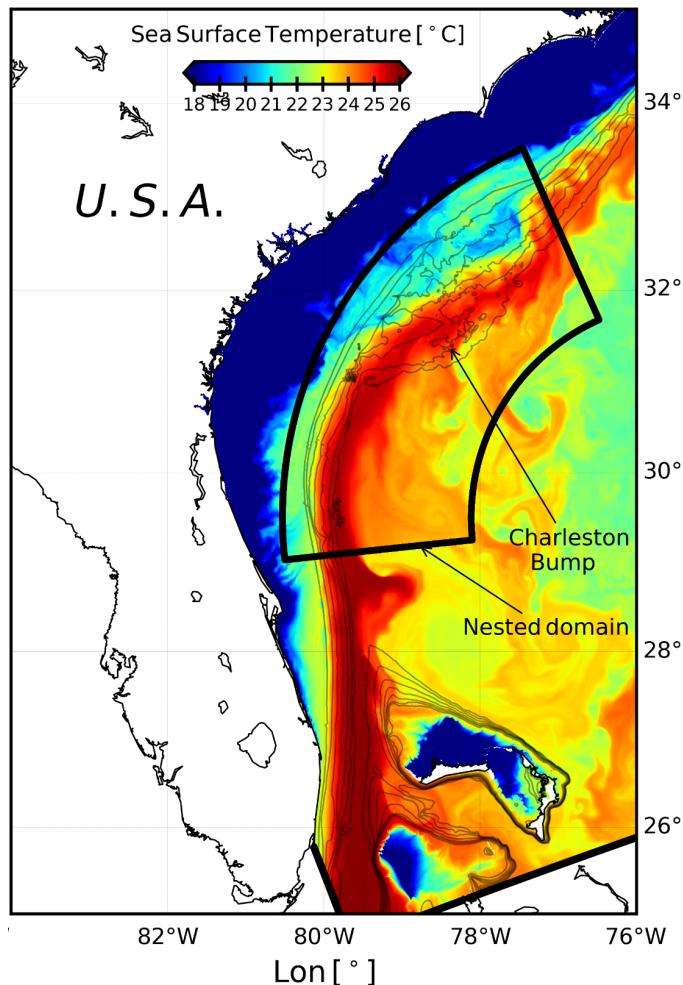
# Horizontal curvilinear grid

- Example of realistic domains (with gentle bendings):



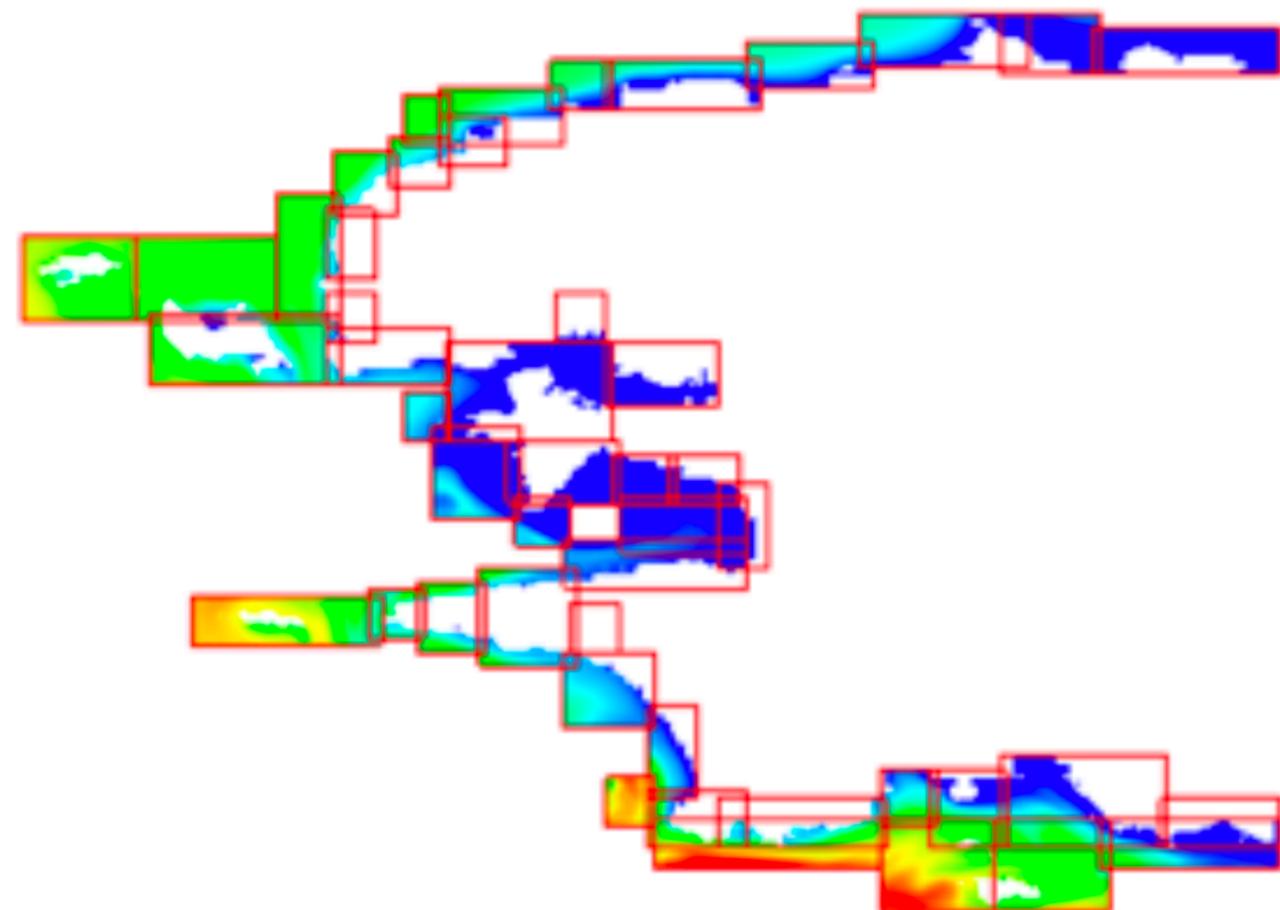
# Horizontal curvilinear grid

- Example of realistic domains (with gentle bendings):



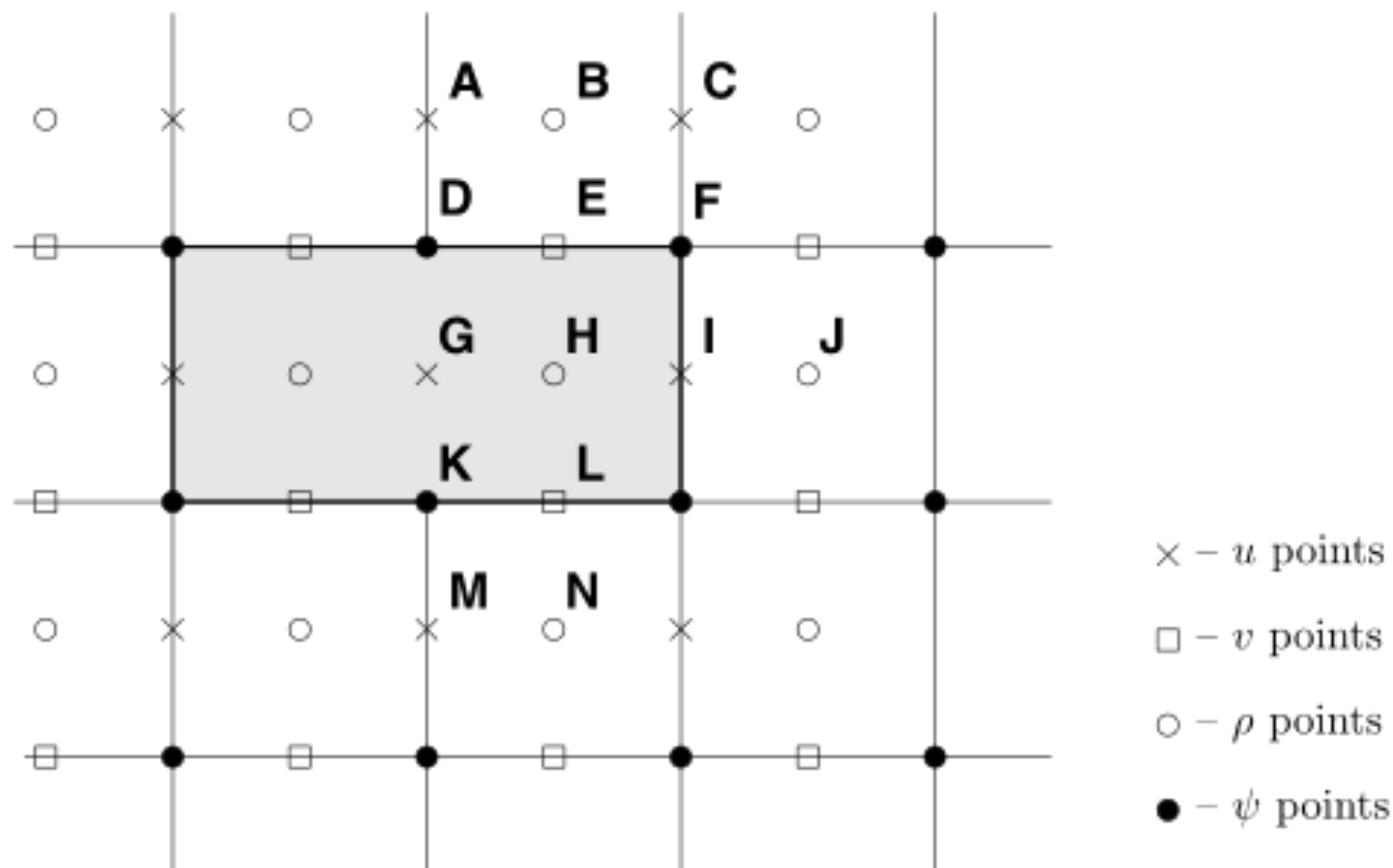
# Horizontal curvilinear grid

- Another method = massive multigrain



# Land/sea Mask

Variables within the masked region are set to zero by multiplying by the mask for either the u, v or rho points :



# Land/sea Mask

Variables within the masked region are set to zero by multiplying by the mask for either the u, v or rho points :



# Land/sea Mask

See the code routines:

```
#ifdef MASKING
# define SWITCH *
#else
# define SWITCH !
#endif

!#####
do k=1,N
  do i=IstrU,Iend
    u(i,j,k,nnew)=(DC(i,k)-DC(i,0)) SWITCH umask(i,j)
```