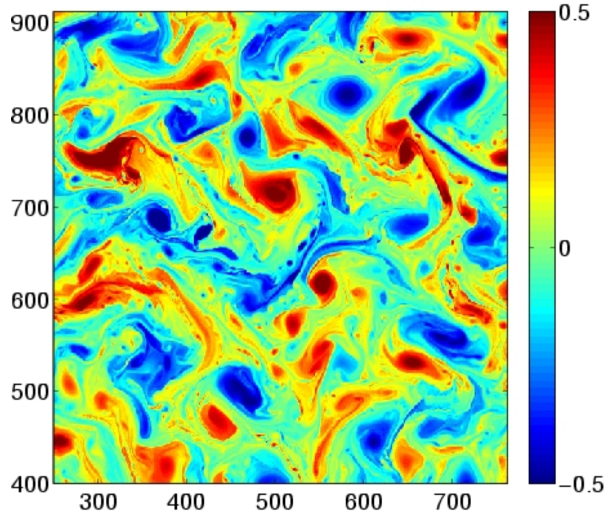
Ocean turbulence and mesoscale eddies

X. Capet, L. Hua

Winter 2011



Potential vorticity:

$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) = Q \xrightarrow{z \rightarrow z^*} \nabla^2\psi = Q \quad (1)$$

where $\boldsymbol{u}_h = \boldsymbol{\nabla}_{h_\perp}\psi$

ψ is essentially pressure

(1) is a Poisson equation

Analogies ???

Potential vorticity:

$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) = Q \xrightarrow{z \rightarrow z^*} \nabla^2\psi = Q \text{ (1)} \quad \text{where} \quad \boldsymbol{u}_h = \boldsymbol{\nabla}_{h_\perp}\psi$$

ψ is essentially pressure

(1) is a Poisson equation

Analogies ???

Electrostatics and Newtownian gravitation

$$\nabla^2\phi = 4\pi G\rho$$

with ρ the density (0 where there is no matter) and Φ the potential

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

with ρ the charge density (0 where there is no charge) and Φ the electric potential

Difference: we care about $\nabla_{h_\perp}\psi$ not $\nabla\phi$

Analogies

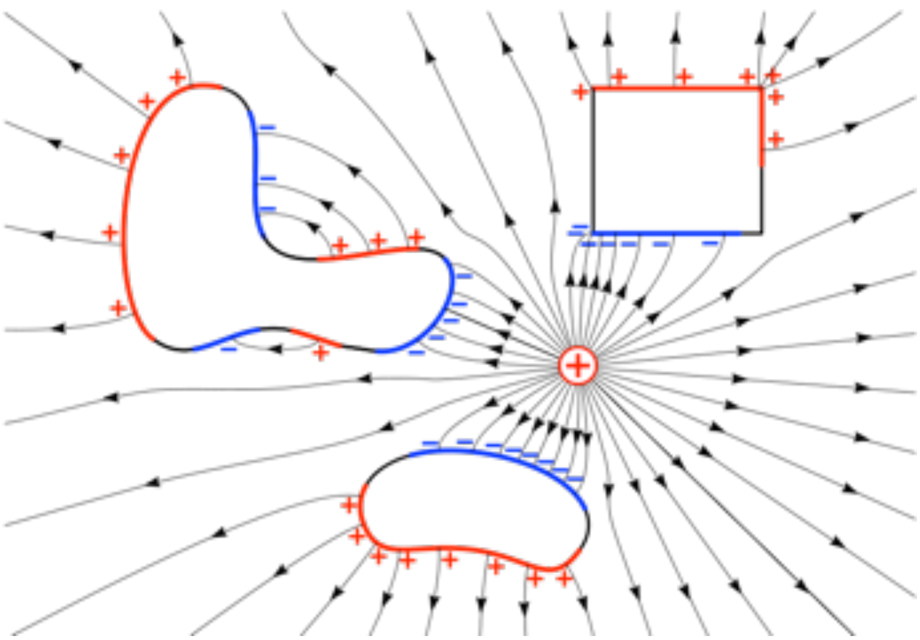
dont forget to mention the fact that there is a f2/N2 factor.

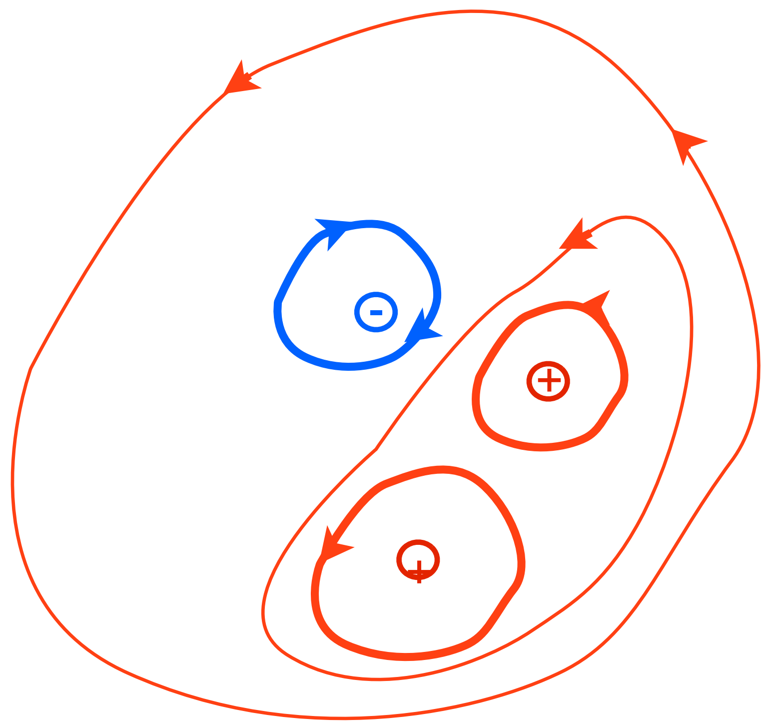
compensated charge (dipole)

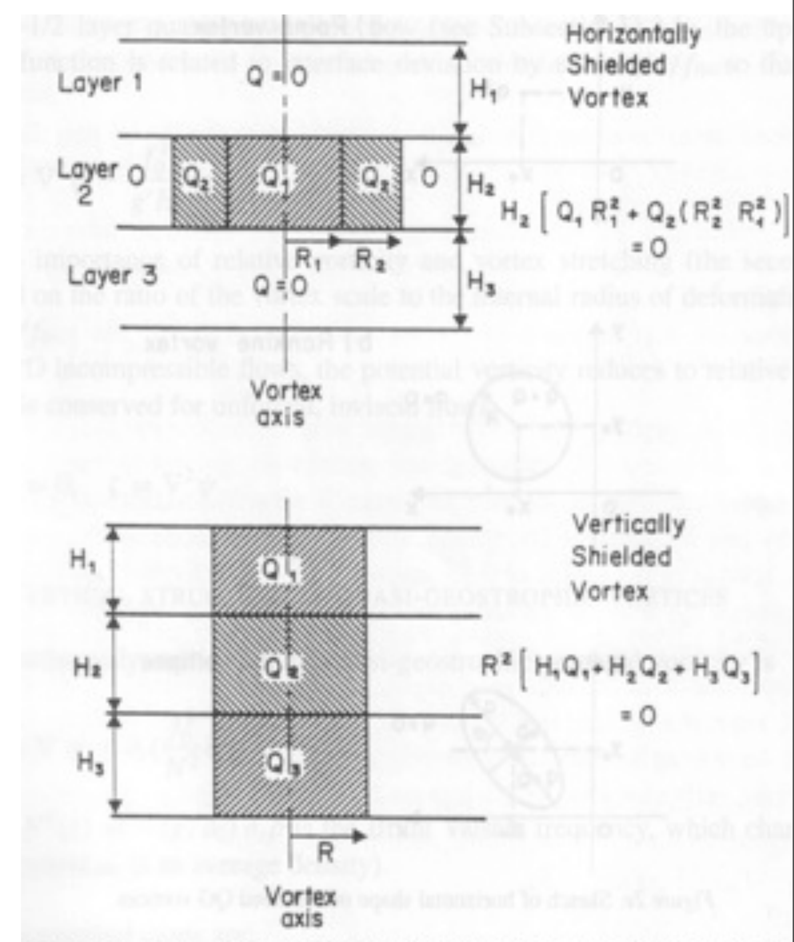
Limits to the analogy:

- we care about $\nabla_{h_\perp}\psi$ not $\nabla\phi$

- $\partial_z\psi \sim \rho \longrightarrow$ complicates the issue of boundary condition slightly.







Complete QG

Complete QG system includes interior PV equation and boundary conditions necessary for inversion

$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) = Q$$

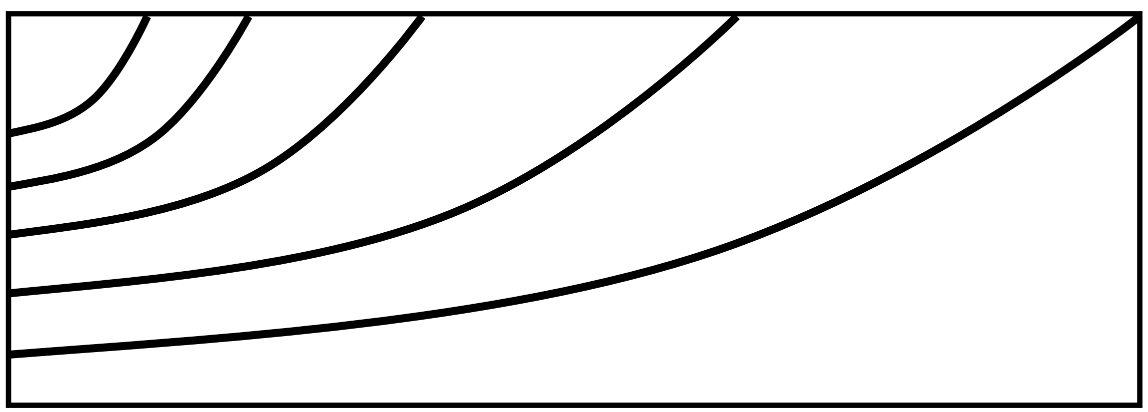
$$\partial_z\psi|_{z=0} = \theta_s^*$$

$$\partial_z\psi|_{z=-H} = \theta_b^*$$

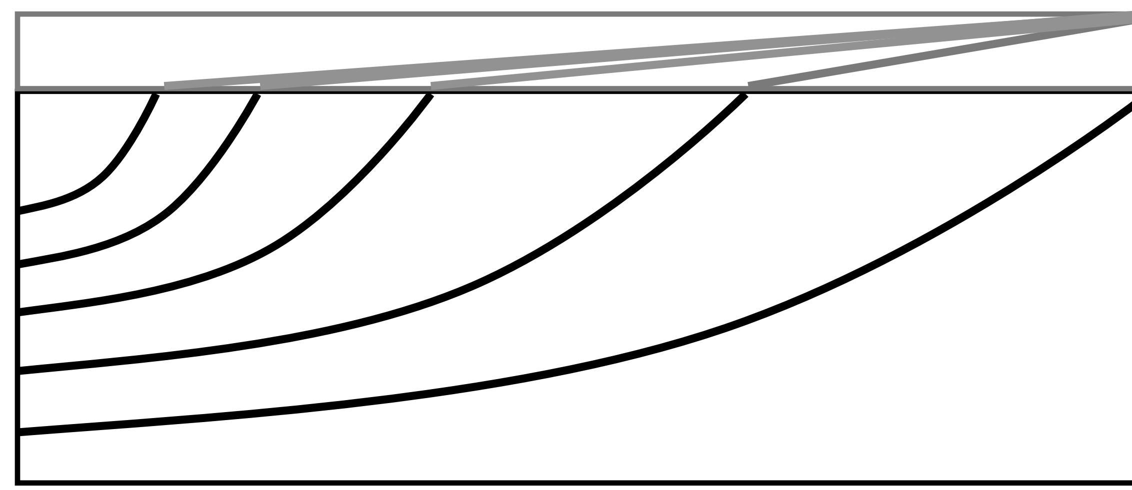
$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) = Q$$

In the dz layer, $\delta z\, Q = \delta z\, \nabla_h^2\psi + \frac{f_0^2}{N^2}\left(\theta^*(dz) - \theta^*(0)\right)$

$$Q = \nabla_h^2\psi - \frac{f_0^2}{N^2}\theta^*(0)/\delta z$$



dz



$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) = Q$$

$$\partial_z\psi|_{z=0} = \theta_s^*$$

$$\longleftrightarrow$$

$$\nabla_h^2\psi + \partial_z\left(\frac{f_0^2}{N^2}\partial_z\psi\right) - \frac{f_0^2}{N^2}\theta^*(0)\delta_0 = Q$$

$$\partial_z\psi|_{z=0} = 0$$

The surface boundary condition for temperature can be replaced by surface PV (Dirac).

Surface quasi-geostrophy

we take $Q_{int} = 0$

and $N^2 = C^{\underline{te}}$

$$\nabla_h^2 \psi = -\frac{f_0^2}{N^2} \partial_z^2 \psi$$

We can apply the separation of variable method

$$\psi(x, y, z) = f(x, y)\, g(z)$$

Surface QG

Surface quasi-geostrophy

we take $Q_{int} = 0$ and $N^2 = C^{te}$ $\longrightarrow$ $\nabla_h^2 \psi = -\dfrac{f_0^2}{N^2}\partial_z^2 \psi$

we also consider a semi-infinite ocean (no bottom)

We can apply the separation of variable method

$$\psi(x,y,z) = f(x,y)\,g(z)$$

$$\rightarrow \begin{cases} \partial_x^2 f + \partial_y^2 f = a\ f \\ g'' = ag \end{cases} \rightarrow \begin{cases} \psi \ \text{ has the form } \ \psi \propto f(x,y)\,e^{kz} \ \text{ (condition z = -∞)} \\ \text{with } \ \nabla_h^2 f = k^2 f \end{cases}$$

$\partial_z \psi|_{z=0} = \theta_s^*$ $\rightarrow$ if $\theta_s(x,y) = \sum \hat{\theta}_s(k_x,k_y)\,e^{i(k_x x + k_y y)}$

$\psi_s(x,y) = \sum \hat{\psi}_s(k_x,k_y)\,e^{i(k_x x + k_y y)}$ with a relationship between $\hat{\psi}$ and $\hat{\theta}_s$

Surface quasi-geostrophy

$$\psi(x,y,z) = \sum \frac{\hat{\theta}_s(k_x,k_y)}{k}\, e^{i(k_x x + k_y y)} e^{k\,z}$$

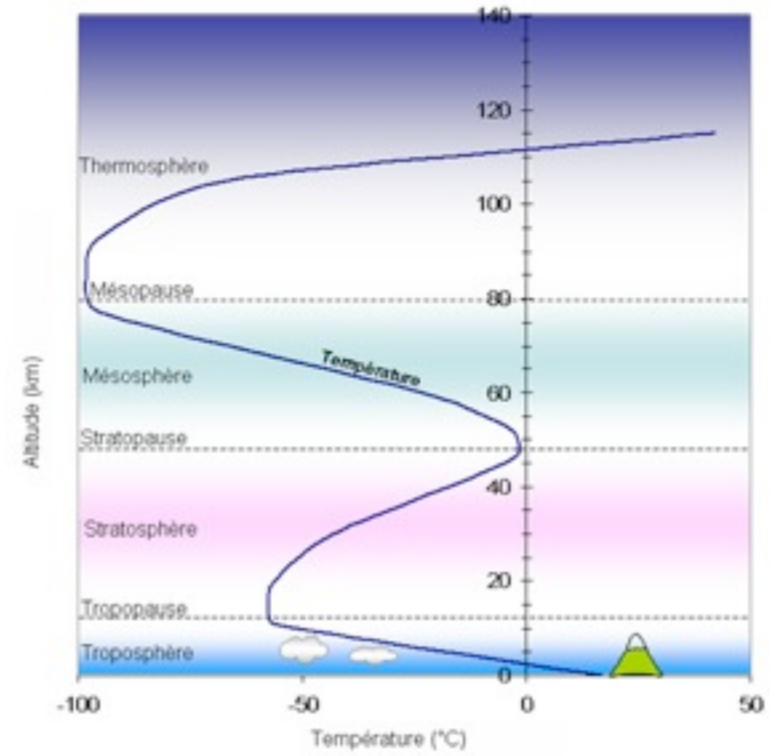
where $$\rho_s(x,y) = \sum \hat{\rho}_s(k_x,k_y)\, e^{i(k_x x + k_y y)}$$

or when rescaling,

$$\psi(x,y,z) = \sum \frac{g\,\alpha}{\rho_0 N_0}\frac{\hat{\theta}_s(k_x,k_y)}{k}\, e^{i(k_x x + k_y y)} e^{k\,z}$$

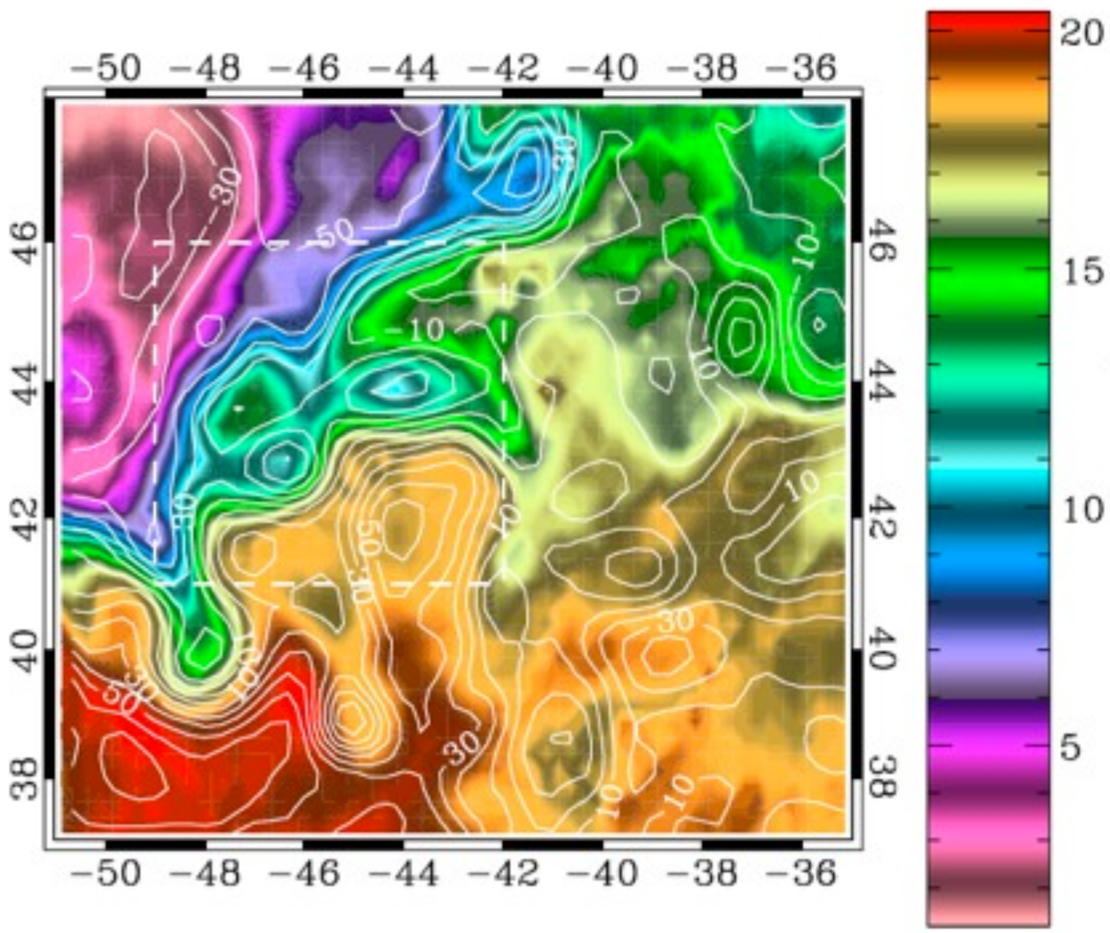
Several remarks:

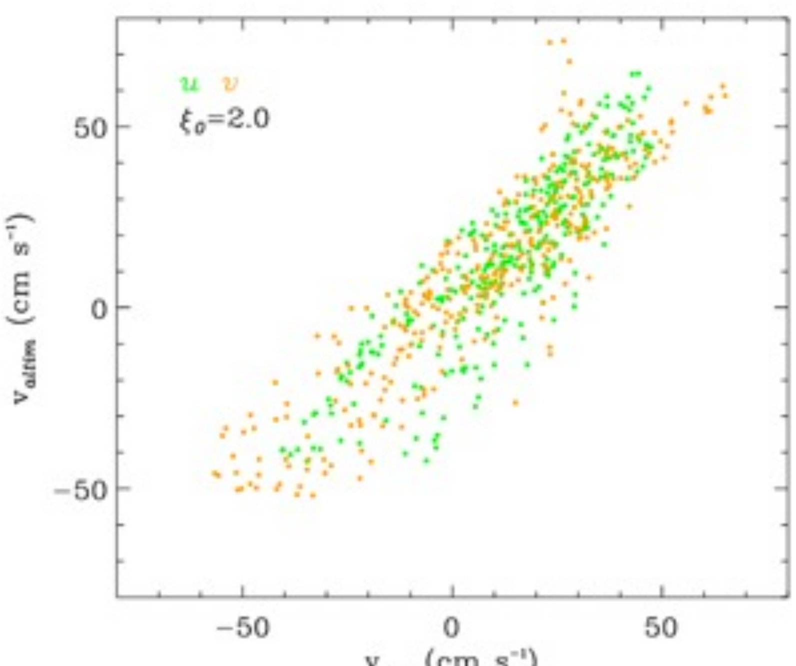
- Every wavenumber has a certain attenuation with depth
- what is the typical length scale for perturbations that reach from top to bottom ?
- Most important: the entire dynamics is driven surface density. If you know surface density you know **u** and w (3d)
- Usually there is interior PV
- Tropopause
- interior QG versus SQG: range of influence (see below)

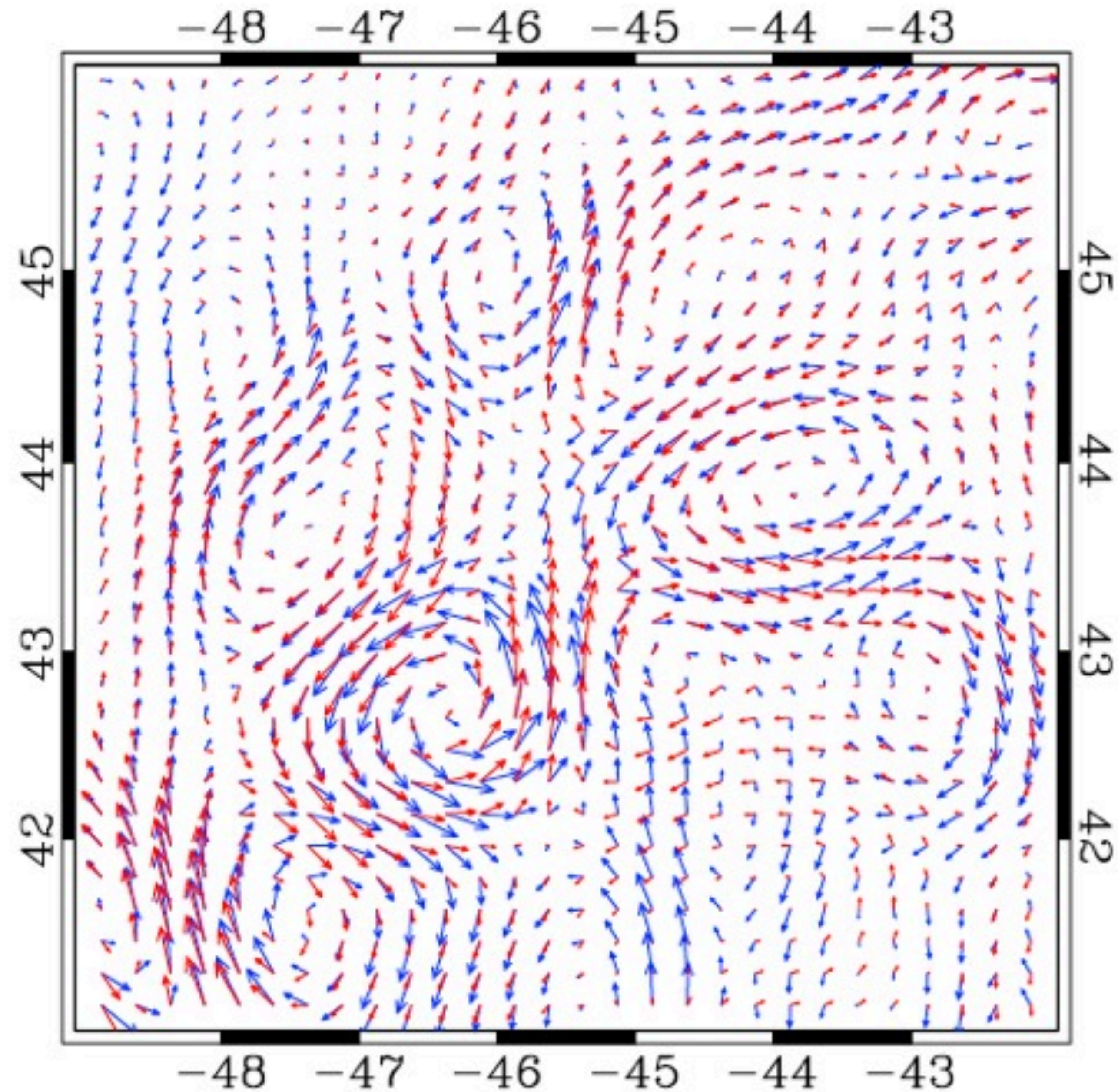


Applications of SQG theory

Surface quasi-geostrophy







Potential use of microwave sea surface temperatures for the estimation of ocean currents, 2006W. J. Isern-Fontanet, B. Chapron, G. Lapeyre, P. Klein

There are some conditions in which ocean currents can be deduced from temperature, not only from altimetry

2d

$$\frac{dq}{dt} = D_q$$

$$q = \nabla^2 \psi$$

$$\hat{\psi} = \frac{1}{k^2}\,\hat{q}$$

SQG

$$\frac{d\theta}{dt} = D_\theta$$

$$\theta = \partial_z \psi$$

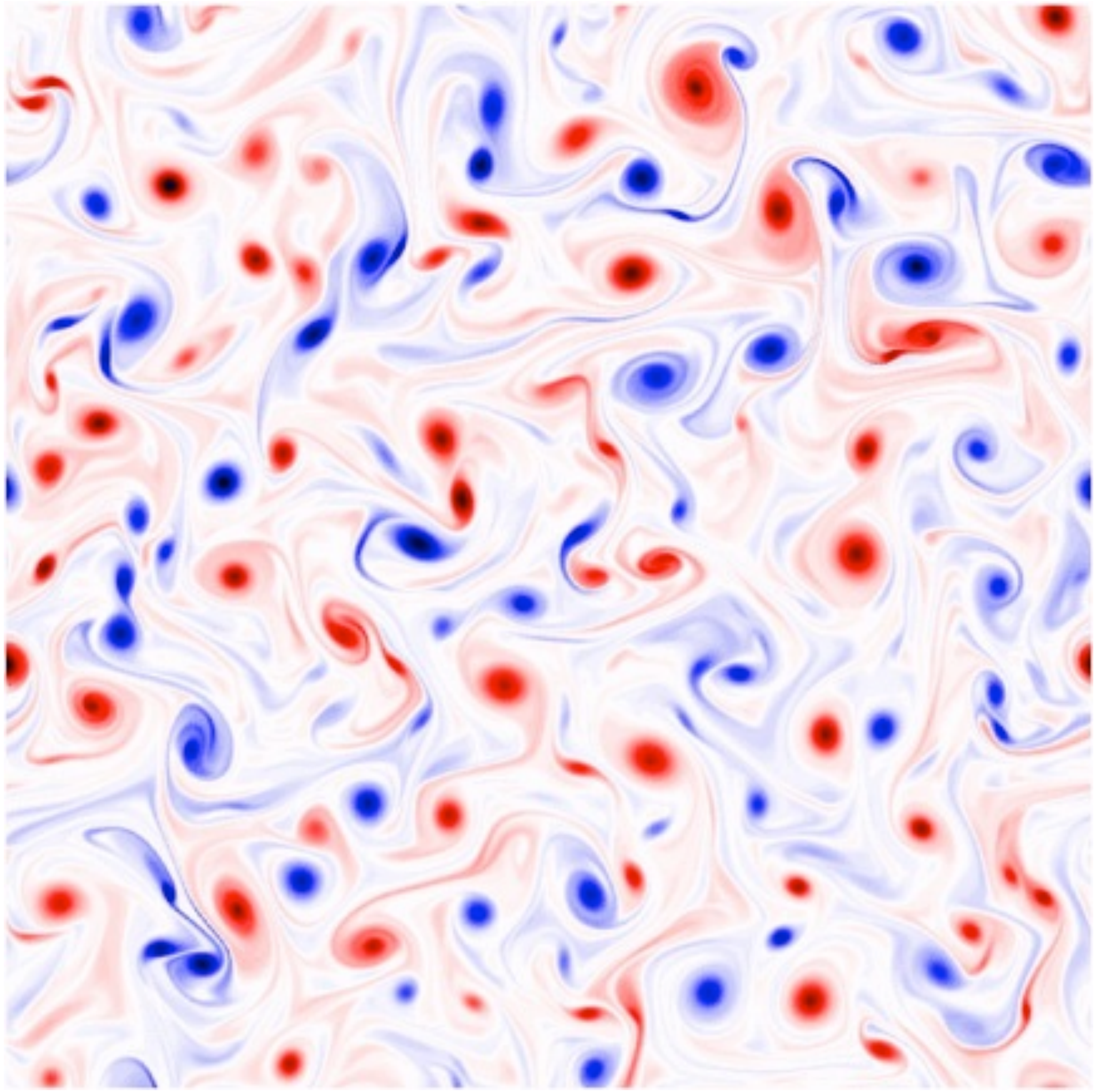
$$\hat{\psi} = \frac{1}{k}\,\hat{\theta}$$

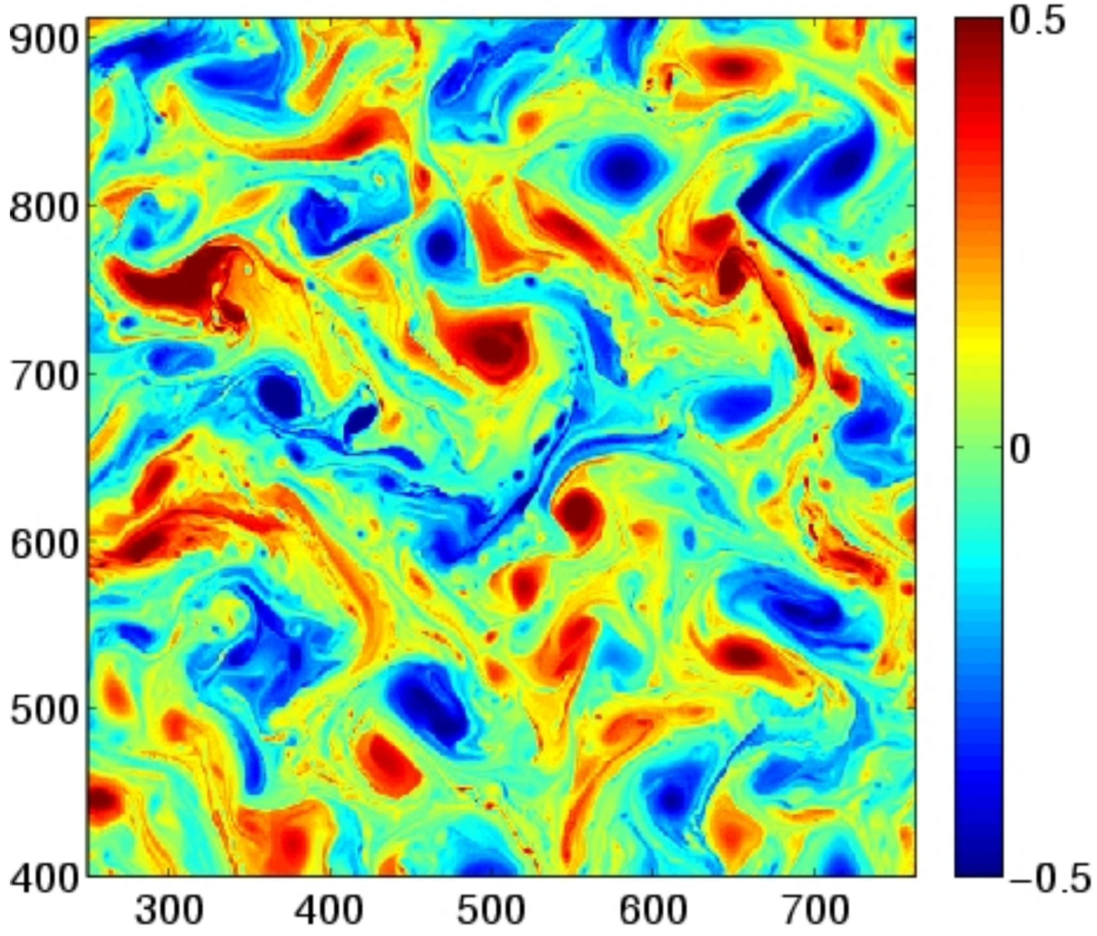
What is the effect of these differences on on the flow (i.e. ψ of a coherent structure - q or Θ patch) ?

The larger the structure the more intense the effect on the flow but this is more pronounced for 2d flows. This will have some consequences on the turbulent behavior of the flow. Small structures tend to be wiped out in 2d and be more resistant in SQG.

This is one way to interpret energy spectra in 2D/QG versus SQG

2d versus SQG





The larger the structure the more intense the effect on the flow but this is more pronounced for 2d flows. This will have some consequences on the turbulent behavior of the flow. Small structures tend to be wiped out in 2d and be more resistant in SQG.

This is one way to interpret energy spectra in 2D/QG versus SQG (with more energy at scales below the mesoscale in SQG)