EE/CSCI 451
Fall 2020
Homework 4

Assigned: September 18, 2020
Due: September 24, 2020, 11:59PM AOE
Total Points: 100

1 [10 points]

Explain the following terms:

1. Diameter of a network
2. Bisection width of a network
3. Multistage network
4. Rearrangeable network
5. Non blocking network
6. Congestion in a network
7. 3 stage CLOS network
8. Butterfly network
9. Shuffle-exchange network
10. Fat tree

2 [10 points]

Figure 1 shows a 4 input/output CLOS network with control setting for each switch box. What permutation does this network realize?
Permutation:
$0 \rightarrow$
$1 \rightarrow$
$2 \rightarrow$
$3 \rightarrow$

3 [20 points]

In the class, we discussed an algorithm to perform routing on a Shuffle-Exchange Network which requires $2k$ routing steps (shuffles and exchanges), where k is the number of bits in the binary representation of source and destination. For a binary number $x_{k-1}x_{k-2}\ldots x_0$, we define suffix as the most significant bits $x_{k-1}x_{k-2}$... of the number and prefix as the least significant bits $\ldots x_1x_0$ of the number.

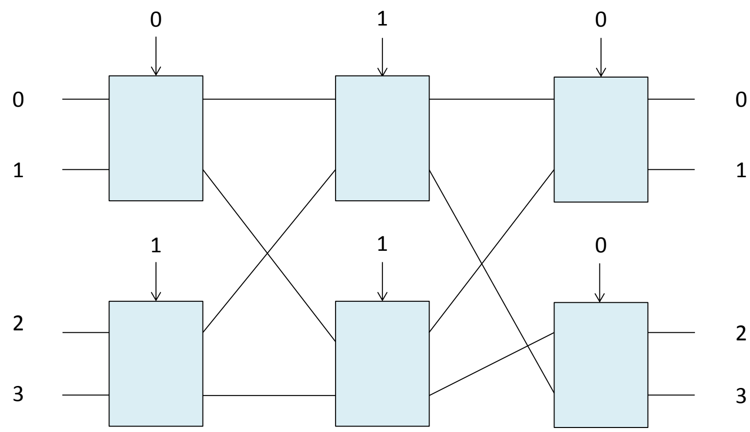

Figure 1: 4 input/output CLOS network

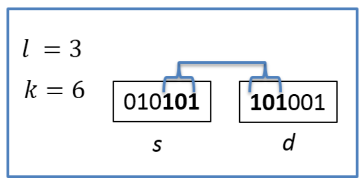

Figure 2: Prefix 101 of s is equal to suffix 101 of d

Let l denote the length of the longest prefix of the source (in binary representation) which is a suffix of the destination (again in binary representation). See Figure 2 for example.

1. Consider a 16 input and output shuffle-exchange network, a source node $s = 1010$ and a destination node $d = 1001$. For this pair of source and destination, find a route which has ≤ 4 links $(2(k - l) = 2(4 - 2) = 4)$. Write all the intermediate nodes and the operation (Shuffle or Exchange) performed at each step. (5 points)
   Hint: Longest prefix of s which is equal to the suffix of d is 10.

2. Modify the algorithm discussed in the lecture so that it runs in at most $2(k - l)$ steps.

4 [5 points]

A mesh of trees is a network that imposes a tree interconnection on a grid of processing nodes. A $\sqrt{p} \times \sqrt{p}$ mesh of trees is constructed as follows. Starting with a $\sqrt{p} \times \sqrt{p}$ grid, a complete binary tree is imposed on each row of the grid. Then a complete binary tree is imposed on each column of the grid. Figure 3 illustrates the construction of a 4×4 mesh of trees. Assume that the nodes at intermediate levels are switching nodes. Determine the diameter of a $\sqrt{p} \times \sqrt{p}$ mesh of trees.

5 [25 points]

In the class, we defined an n input and n output CLOS network as a 3-stage network with $\sqrt{n}$ $\sqrt{n} \times \sqrt{n}$ crossbar switches in each stage.

1. Draw the network for $n = 16$.

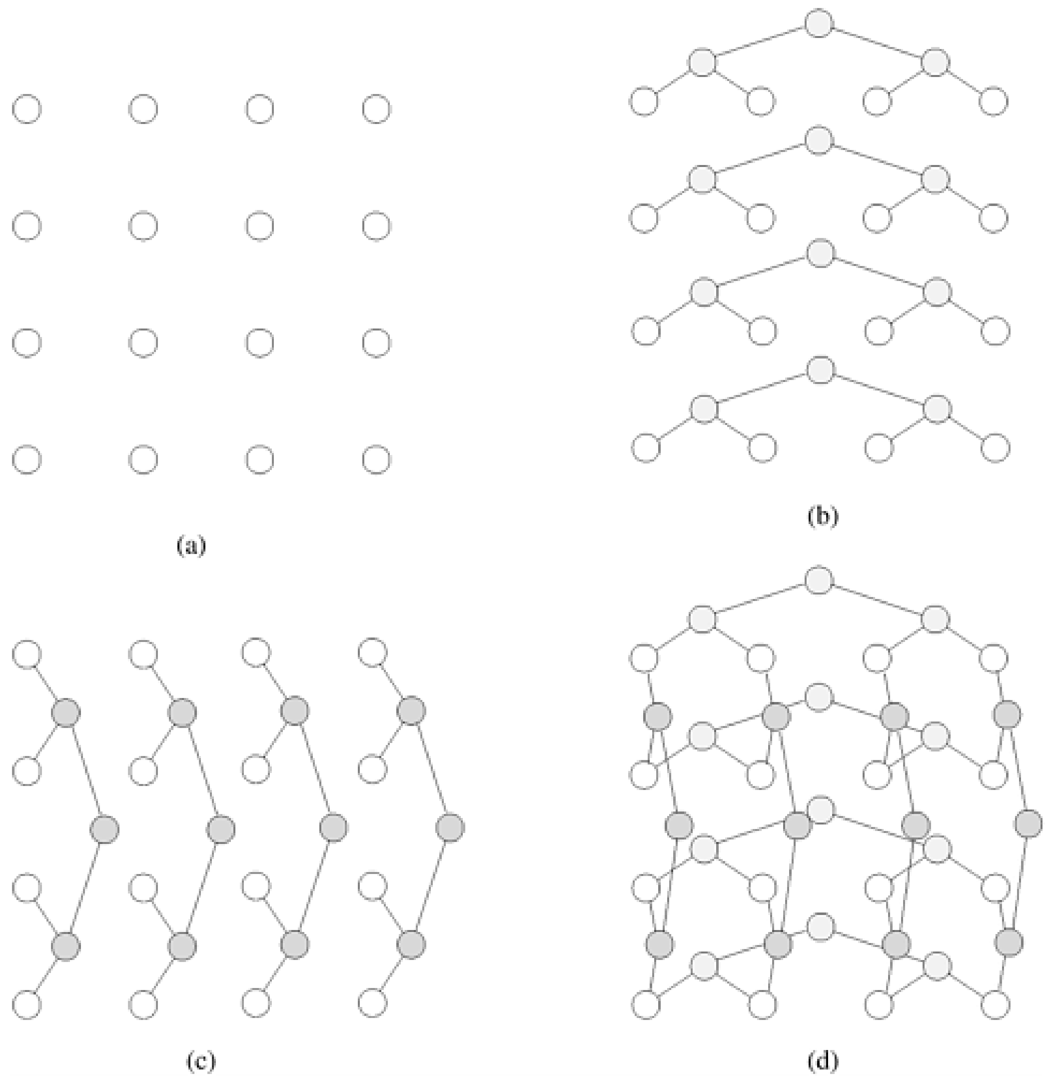

Figure 3: The construction of a 4×4 mesh of trees: (a) a 4×4 grid, (b) complete binary trees imposed over individual rows, (c) complete binary trees imposed over each column, and (d) the complete 4×4 mesh of trees.

2. We can extend the 3-stage CLOS network as follows: A p input and p output CLOS network can be defined as a 3-stage network where Stage 0 and Stage 2 consist of $\frac{p}{2}$ 2×2 switches while Stage 1 consists of two $\frac{p}{2} \times \frac{p}{2}$ switches. The definition can be applied recursively to Stage 1 until the network consists only of 2×2 switches.
   Using the above definition, each $\sqrt{n} \times \sqrt{n}$ crossbar switch of the network in part a can be implemented using only 2×2 switches. Draw a $\sqrt{16} \times \sqrt{16}$ crossbar switch and define the connectivity requirement from Stage 0 to Stage 1.

3. In general, derive an expression for the total number of switches and the total delay from an input to an output for an n input and n output CLOS network with $\sqrt{n}$ $\sqrt{n} \times \sqrt{n}$ crossbar switches each of which is implemented using the definition of CLOS network used in Part 2. (Assume the delay of each 2×2 switch is equal to 1 unit)

6 [20 points]

In this problem, we will design a network for $n \times n = 2^k$ nodes. The nodes will be laid out in 2 dimensions with n rows and n columns. Each row is a Shuffle-Exchange network of size $n = 2^{\frac{k}{2}}$. Similarly, each column is a Shuffle-Exchange network of size $n = 2^{\frac{k}{2}}$. To perform routing on this network, we split the k bits into two chunks of $\frac{k}{2}$ bits each. The most significant $\frac{k}{2}$ bits (left chunk) will be used to perform Shuffle-Exchange Routing in vertical direction (across columns) and the least significant $\frac{k}{2}$ bits (right chunk) will be used to perform Shuffle-Exchange Routing in horizontal direction (along a row).

1. Fill the connections for $k = 4$ in the network shown in Figure 4.

2. Show the intermediate nodes while routing from source $s = 1010$ to destination $d = 0101$ in the network shown in Figure 4.

7 [10 points]

Suppose we want to perform modified matrix transpose of an $n \times n$ matrix using an $n \times n$ 2-D mesh. Matrix(i, j) is initially stored in PE(i, j) of the 2-D mesh. In modified matrix transpose, Matrix(i, j) is to be routed to PE$((n-1) - j, i)$ $(0 \leq i, j < n)$. The permutation is performed as follows: Matrix(i, j) is first routed along row i to PE(i, i), then routed along column i to PE$((n-1) - j, i)$. What is the congestion in the network? Specify all nodes where this congestion occurs in performing this permutation.

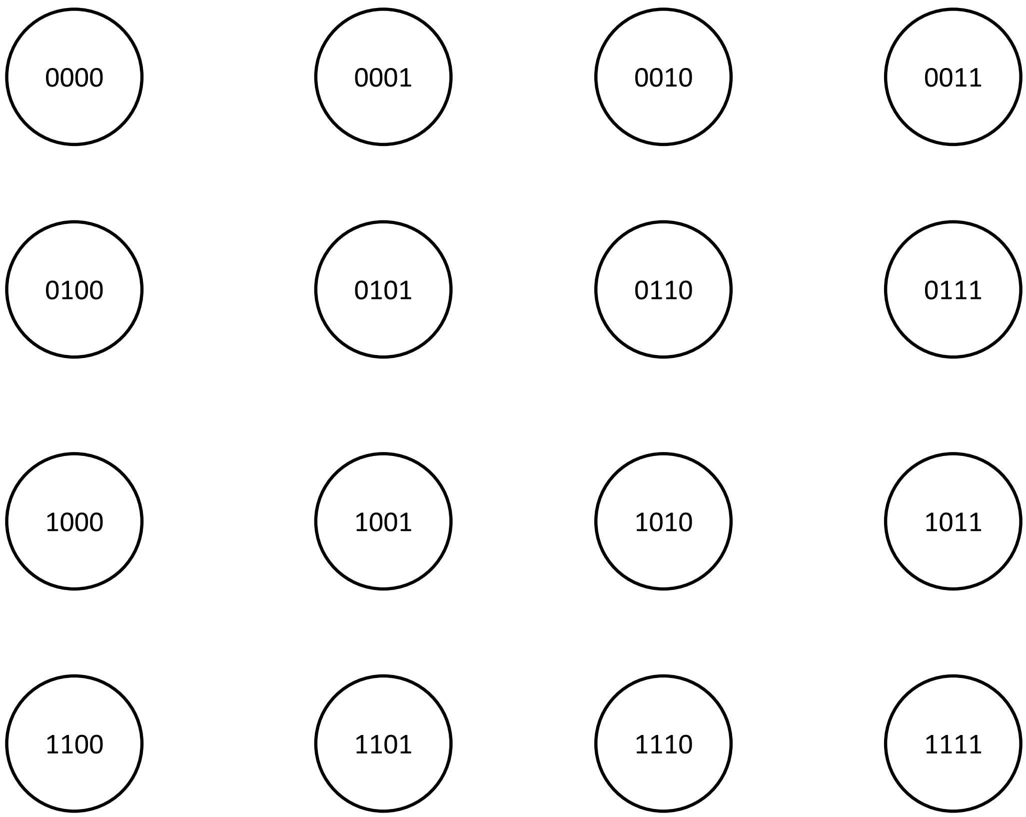

Figure 4: The network for $k = 4$