EE/CSCI 451
Fall 2020
Homework 4 solution
Total Points: 100

1 [10 points]

Please refer to slides and textbook.

2 [10 points]

As shown in Figure 1, the permutation is:
$0 \rightarrow 3$
$1 \rightarrow 2$
$2 \rightarrow 1$
$3 \rightarrow 0$

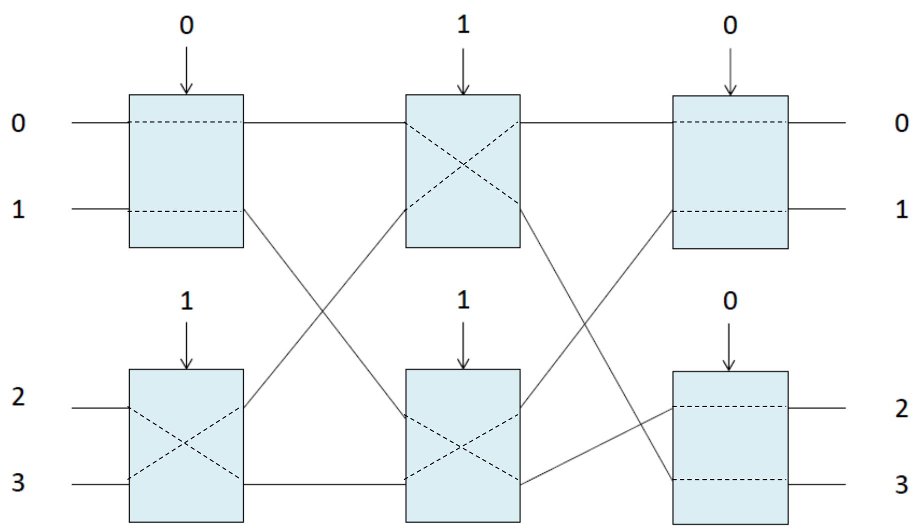

Figure 1: Problem 2

3 [20 points]

1. The intermediates nodes and the operations are as follows

Step	Operation	Intermediate Node
0	-	$s = 1010$
1	Shuffle	0101
2	Exchange	0100
3	Shuffle	1000
4	Exchange	$1001 = d$

2. The following pseudo-code implements the solution:

```
y ← s
  Do i = (k − l − 1) to 0 :
      Shuffle y
      Compare LSB of y with bit i of d
      if not equal Exchange
  end
```

4 [5 points]

As shown in Figure 2, the processors at the corners of the mesh of trees (colored in red) require the largest number of communication links to communicate. This is given by $2\log\sqrt{p}+2\log\sqrt{p}$ or $2\log p$.

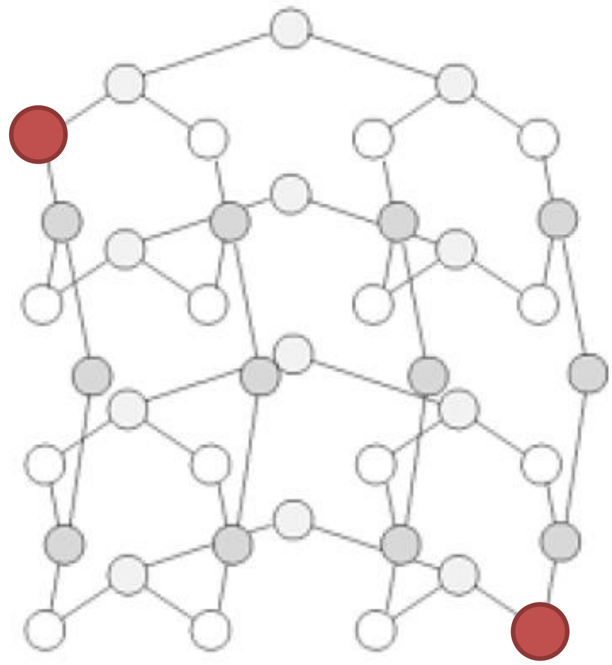

Figure 2: Mesh of trees

5 [25 points]

1. Figure 3 shows the network for $n = 16$.

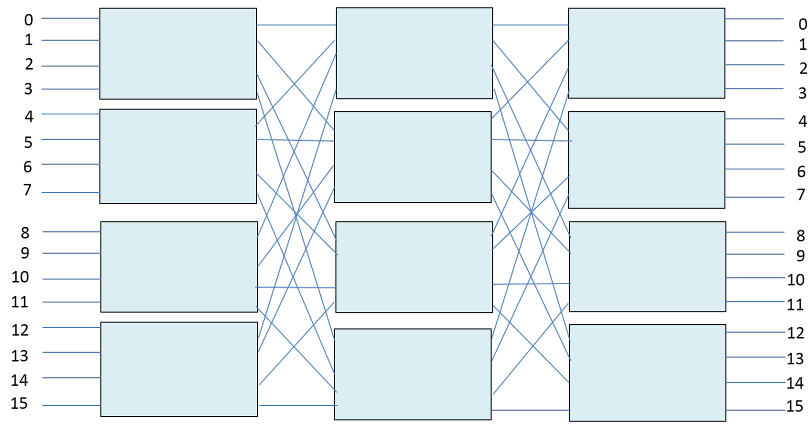

Figure 3: CLOS network for $n = 16$

2. Figure 4 shows a $\sqrt{16} \times \sqrt{16}$ crossbar switch. Each of the 2 switches in Stage 0 is connected to every switch in Stage 1.

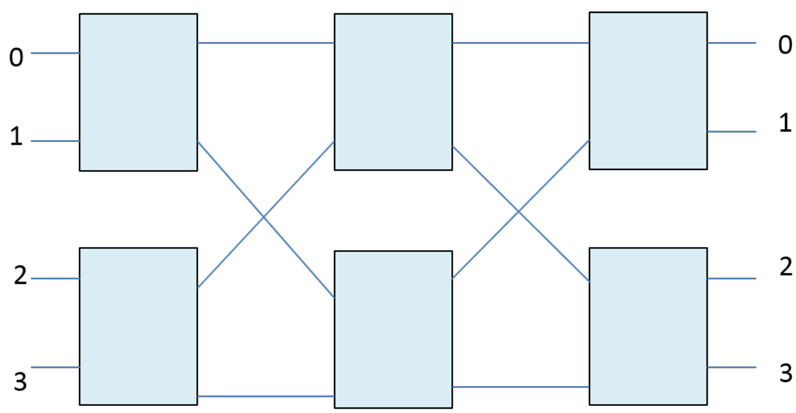

Figure 4: $\sqrt{16} \times \sqrt{16}$ crossbar

3. Each $\sqrt{n} \times \sqrt{n}$ crossbar switch consists of $\log n - 1$ stages and requires $\frac{\sqrt{n}}{2}(\log n - 1)$ 2×2 switches.

   So the total number of switches are: $\frac{3n}{2}(\log n - 1)$ and the total delay is $3(\log n - 1)$.

   However, if you directly apply the recursive definition of the crossbar switch, the total number of switches are $n \log n - \frac{1}{2}n$ and the total delay is $2 \log n - 1$.

6 [20 points]

1. Shown in Figure 5
2. Shown in Figure 6

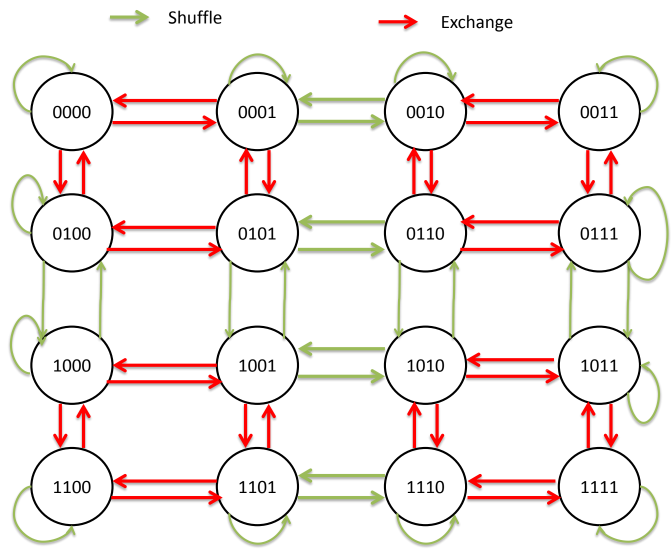

Figure 5: The network for $k = 4$

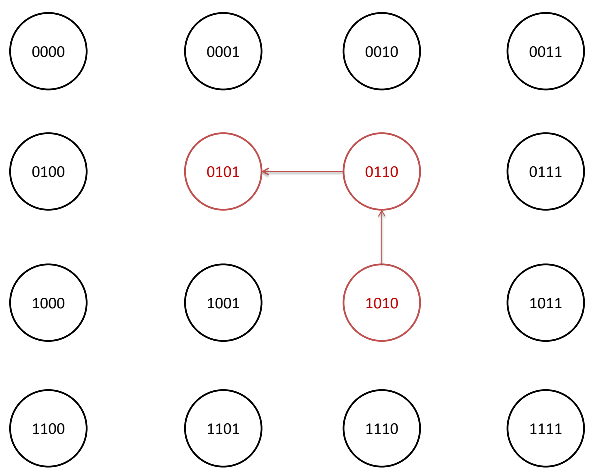

Figure 6: The network for $k = 4$

7 [10 points]

The nodes on the diagonal of the 2-D mesh (i.e., PE(i, i) $0 \leq i < n$) have the maximum congestion. This is because all the paths to transpose the elements in row i pass PE(i, i). The congestion of the network is n.
(Note that if you do not count the path from PE(i, i) to PE(i, i) when computing the congestion, the congestion of the network is $n - 1$.)