Vehicle routing for the communication of time-dependent information

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Presentation Roadef

- The Vehicle Routing Problem (VRP) [1] is one of the most extensively studied problems in operations research due to its methodological interest and practical relevance in many fields such as transportation, logistic, telecommunications, and production.
- This paper presents a vehicle handles the physical information gathering at each node and with technological advances comes the need to incorporate wireless delivery, such that the vehicle must choose whether to go to a node to collect or receive information in the form wireless.

Description of the Problem

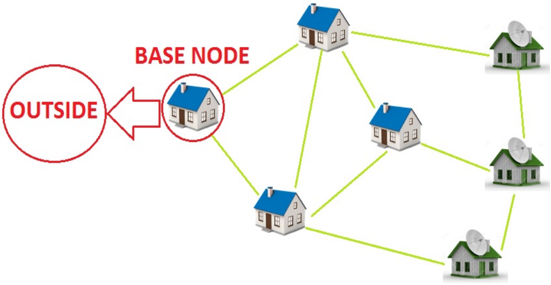


Figure: stations

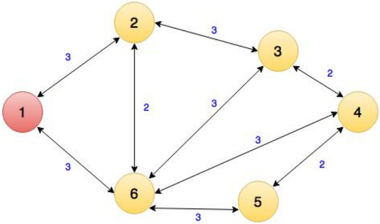
Description of the Problem

(v) The problem is to define routes for the vehicle and also to decide how to collect the information in a way that the amount of information collected to a finite time T is maximized. on the other hand it is important to guarantee that from time to time the information collected will be sent to the outside.

Preliminary Model

(i) Consider a directed graph $D = (V, A)$. We have the set V consisting of n nodes representing n stations and a certain amount of m paths connecting stations represented by the set of arcs A.

(ii) Each road $(i, j) \in A$ has a certain weight t_{ij} representing the time it takes to travel from i to j.



$$t = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 3 & 0 & 0 & 2 \\ 0 & 3 & 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 0 & 2 & 3 \\ 0 & 0 & 0 & 2 & 0 & 3 \\ 3 & 2 & 3 & 3 & 3 & 0 \end{pmatrix}$$

Preliminary Model

We suppose that the amount of information in node j in time k, q_{jk}, is proportional to the elapsed time from the last extraction.

$$q_{jk} = (k - t_{last})r_j \quad (r_j \text{ is the constant of proportional})$$

$$q_{j1} = C_j \qquad (C_j \text{is the amount of initial information})$$

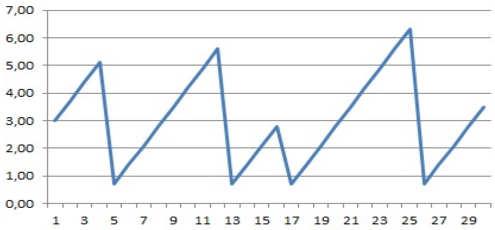


Figure: q_{jk} with $r_j = 0.4, C_j = 3$, collecting or sending wireless in $k = 4, 12, 16, 25$

Wireless

The time for transmission depends:

- on the amount of information transmitted.
- the square of the distance between nodes
- other factors for example:the equipment installed in the nodes, obstacles between stations,etc

Wireless delivery time information

$$\tau_{ijk} = \alpha_{ij} d_{ij}^2 q_{jk} \tag{1}$$

also we can make (M) multiple sending to the same node and the total information time corresponds to the highest.

Preliminary Model

For each $i, j \in V$ We define the following decision variables

$$x_{ijk} = \begin{cases} 1 & \text{If the vehicle goes from } i \text{ to } j \text{ in time } k, (i,j) \in A \\ 0 & c.c. \end{cases}$$

$$w_{jikl} = \begin{cases} 1 & \text{if the information will be sent from node } j \\ & \text{ to node } i \text{ at time } k\text{,and it will take a while } l \\ 0 & c.c. \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{the vehicle is ready to go from the node i to a neighboring} \\ & \text{ in time k, after receiving wireless information} \\ 0 & c.c. \end{cases}$$

Model

$$minimize \left\{ \sum_{k=1}^{T} \max_{j \in V} \{q_{jk}\} \right\}$$

The objective function is to sum every moment the maximum amount information in the nodes

Subject to:

$$\sum_{(1,s)\in A} x_{1st_{1s}} = 1 \tag{2}$$

$$\sum_{(s,1)\in A} x_{s1T} = 1 \tag{3}$$

(1)the vehicle starts from node 1 to a neighboring node.
(2)the vehicle ends at node 1

Model

$$q_{j1} = C_j,\ j \in V \tag{4}$$

$$q_{jk+1} = q_{jk}(1 - \sum_{i \in V(j)} x_{ijk} - \sum_{p=1}^{r} \sum_{i \in V/\{j\}} w_{jikp}) + r_j, j \in V, \tag{5}$$

the constraint (3) amount of initial information in the node j and (4) means that $q_{jk+1} = r_j$ if the vehicle has gone through j or if sent information from j (wireless) at time k, in otherwise $q_{jk+1} = q_{jk} + r_j$

Model

$$x_{ijk} \leq \sum_{(j,p)\in A}^{k+t_{jp}\leq T} x_{jp(k+t_{jp})} + \sum_{p=1}^{S} \sum_{u\in V}^{u\neq j} w_{uj(k+1)p}, \ \forall (i,j) \in A, \tag{6}$$

$$\sum_{l=1}^{S} \sum_{j\in V}^{j\neq i} w_{jikl} \leq M \ \ \forall i \in V \ , \ \forall k \tag{7}$$

$$1 \geq \sum_{(i,j)\in A} x_{ijk} + \left(\frac{1}{M}\right) \sum_{p=1}^{S} \sum_{(j,i)\in V\times V}^{j\neq i} w_{jikp}, \ \text{for each } k, \tag{8}$$

Model

$$\alpha_{ji} d_{ij}^2 q_{jk} w_{jikl} - l \leq 1, \quad (j,i) \in V \times V (j \neq i) \forall k, \forall l \tag{9}$$

$$y_{ik} \leq \sum_{(i,j) \in A}^{k+t_{ij} \leq T} x_{ij(k+t_{ij})}, \quad \forall i \in V, \forall k; \tag{10}$$

$$w_{jikl} \leq \sum_{(p,i) \in A} x_{pi(k-1)}, \forall (j,i), \forall k \tag{11}$$

$$(1/M) \sum_{l=1}^{S} \sum_{j \in V}^{j \neq i} w_{jikl} <= \sum_{s=1}^{T} y_{i(k+s-1)}; \tag{12}$$

$$x_{ijk}, y_{ik}, w_{jikl} \in \{0,1\}, \tag{13}$$

Numerical Results

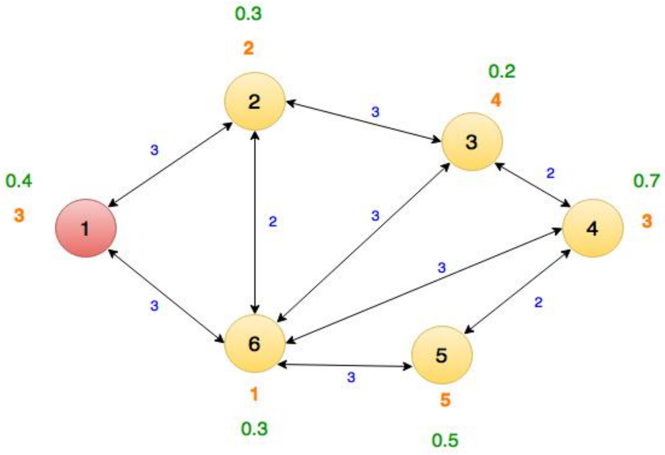
$$V = \{1,2,3,4,5,6\}, \quad C = \{3,2,4,3,1,5\}$$

$$r = \{0.4, 0.3, 0.2, 0.7, 0.5, 0.3\} \quad S = 3, M = 2, T = 25$$

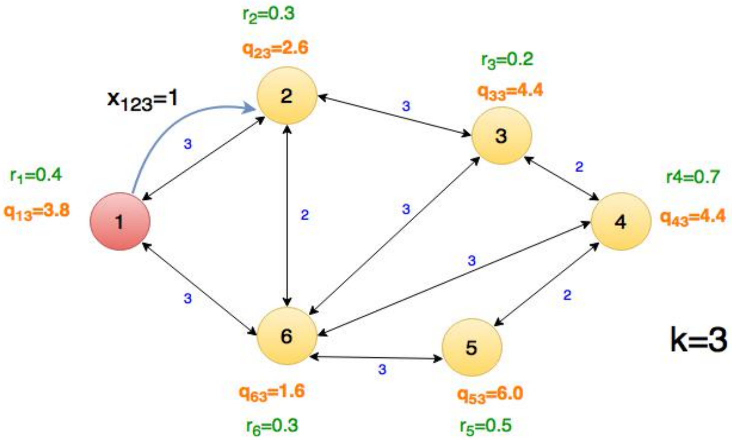
$$t = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 3 & 0 & 0 & 2 \\ 0 & 3 & 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 0 & 2 & 3 \\ 0 & 0 & 0 & 2 & 0 & 3 \\ 3 & 2 & 3 & 3 & 3 & 0 \end{pmatrix} d = \begin{pmatrix} 100 & 300 & 200 & 400 & 400 & 400 \\ 300 & 0 & 3 & 5 & 6 & 7 \\ 200 & 3 & 0 & 2 & 3 & 3 \\ 400 & 5 & 2 & 0 & 2 & 3 \\ 400 & 6 & 3 & 2 & 0 & 3 \\ 400 & 7 & 3 & 3 & 3 & 0 \end{pmatrix}$$

$$\alpha = \begin{pmatrix} 0 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0 & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0 & 0.01 & 0.01 & 0.01 \\ 0.1 & 0.1 & 0.1 & 0 & 0.1 & 0.1 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0 \end{pmatrix}$$

Numerical Results

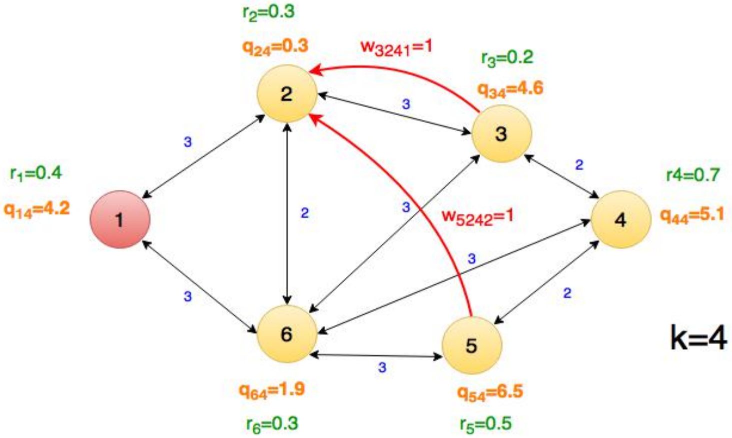


Experiment



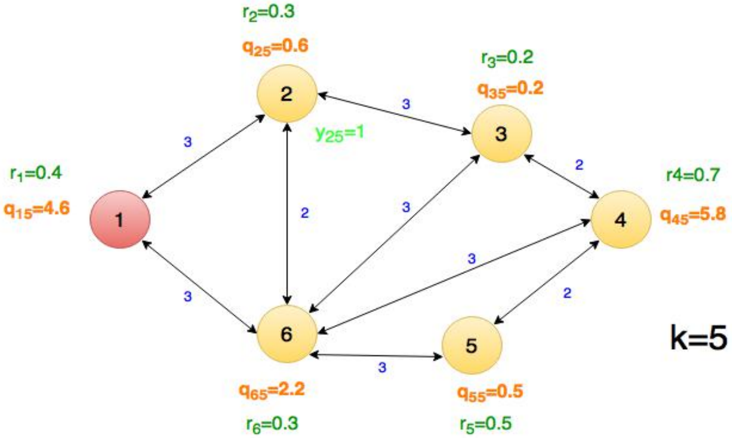
r_2=0.3
q_{23}=2.6
r_3=0.2
q_{33}=4.4
x_{123}=1
2
3
3
3
2
r_1=0.4
q_{13}=3.8
1
2
3
4
r4=0.7
q_{43}=4.4
3
3
2
6
5
3
k=3
q_{63}=1.6
r_6=0.3
q_{53}=6.0
r_5=0.5

Experiment

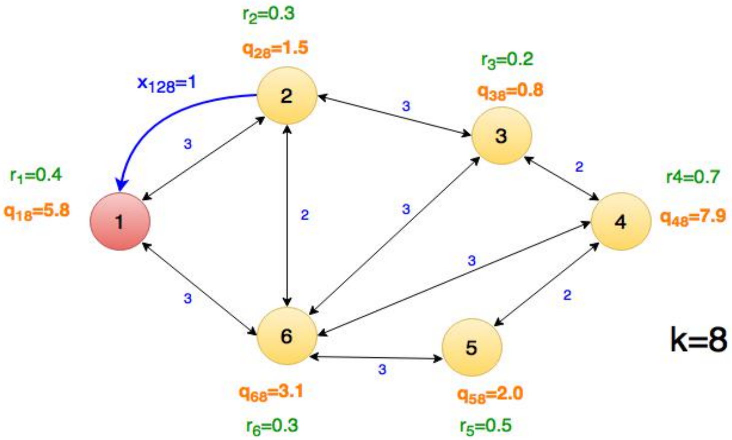


r_2=0.3
q_{24}=0.3
w_{3241}=1
r_3=0.2
q_{34}=4.6
r_1=0.4
q_{14}=4.2
r4=0.7
q_{44}=5.1
w_{5242}=1
2
3
1
4
6
5
3
3
2
2
3
3
3
3
2
3
q_{64}=1.9
r_6=0.3
q_{54}=6.5
r_5=0.5
k=4

Experiment

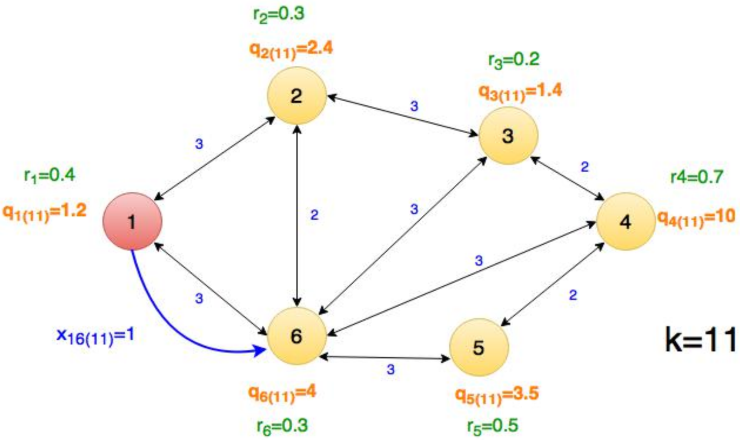

r_2=0.3
q_{25}=0.6
r_3=0.2
q_{35}=0.2
y_{25}=1
r_1=0.4
q_{15}=4.6
r4=0.7
q_{45}=5.8
1
2
3
4
5
6
3
3
2
3
2
3
3
2
3
3
k=5
q_{65}=2.2
r_6=0.3
q_{55}=0.5
r_5=0.5

Experiment



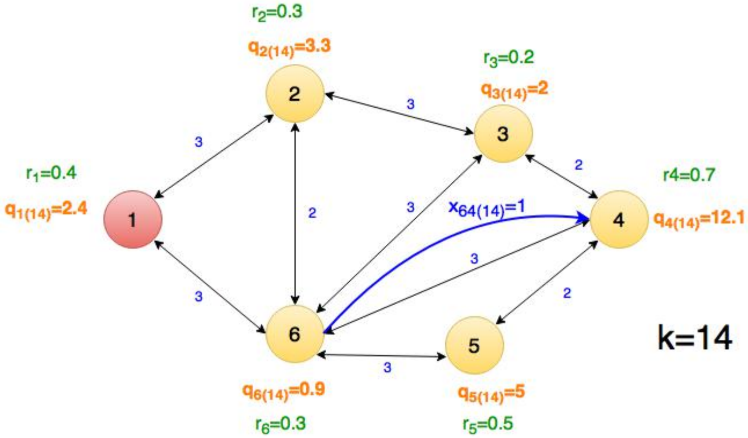
r2=0.3
q28=1.5
r3=0.2
q38=0.8
x128=1
2
3
3
3
2
r1=0.4
q18=5.8
1
2
3
r4=0.7
4
q48=7.9
3
3
2
6
5
k=8
3
q68=3.1
r6=0.3
q58=2.0
r5=0.5

Experiment



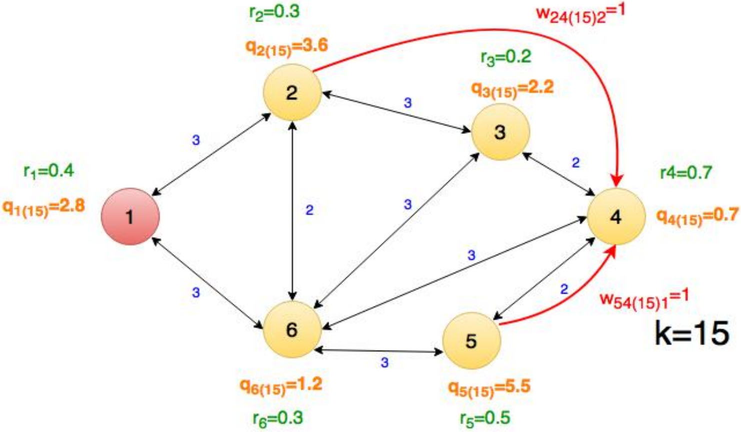
r2=0.3
q2(11)=2.4
r3=0.2
q3(11)=1.4
r1=0.4
q1(11)=1.2
r4=0.7
q4(11)=10
1
2
3
4
5
6
3
3
2
2
3
3
3
2
3
x16(11)=1
q6(11)=4
r6=0.3
q5(11)=3.5
r5=0.5
k=11

Experiment



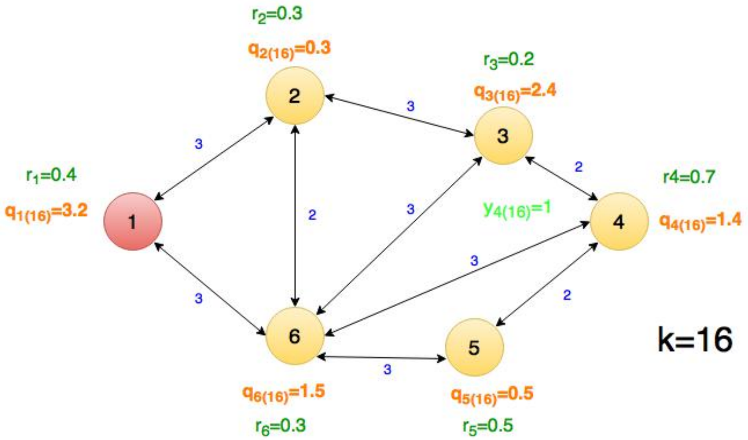
r_2=0.3
$q_{2(14)}$=3.3
r_3=0.2
$q_{3(14)}$=2
r_1=0.4
$q_{1(14)}$=2.4
r4=0.7
$q_{4(14)}$=12.1
$x_{64(14)}$=1
$q_{6(14)}$=0.9
r_6=0.3
$q_{5(14)}$=5
r_5=0.5
1
2
3
4
5
6
3
3
3
2
2
3
3
3
2
3
k=14

Experiment



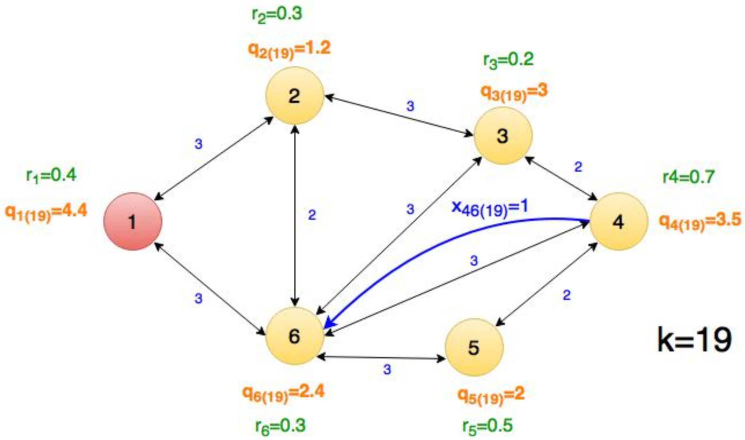
r2=0.3
q2(15)=3.6
w24(15)2=1
r3=0.2
q3(15)=2.2
r1=0.4
q1(15)=2.8
r4=0.7
q4(15)=0.7
w54(15)1=1
k=15
q6(15)=1.2
r6=0.3
q5(15)=5.5
r5=0.5
1
2
3
4
5
6

Experiment



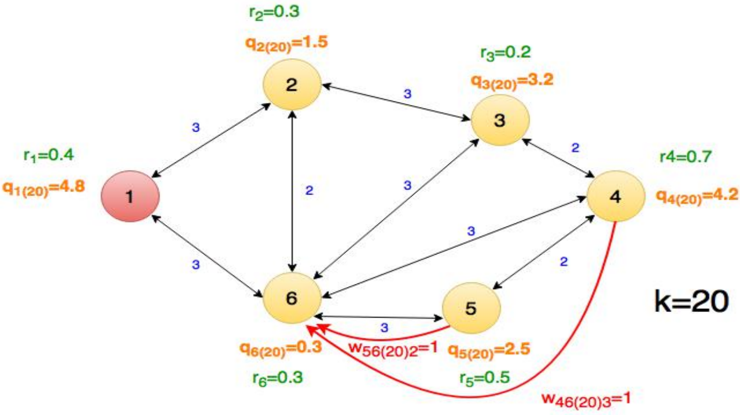
r_2=0.3
$q_{2(16)}$=0.3
r_3=0.2
$q_{3(16)}$=2.4
r_1=0.4
$q_{1(16)}$=3.2
r4=0.7
$q_{4(16)}$=1.4
$y_{4(16)}$=1
1
2
3
4
5
6
3
3
3
2
2
3
3
2
3
3
$q_{6(16)}$=1.5
r_6=0.3
$q_{5(16)}$=0.5
r_5=0.5
k=16

Experiment



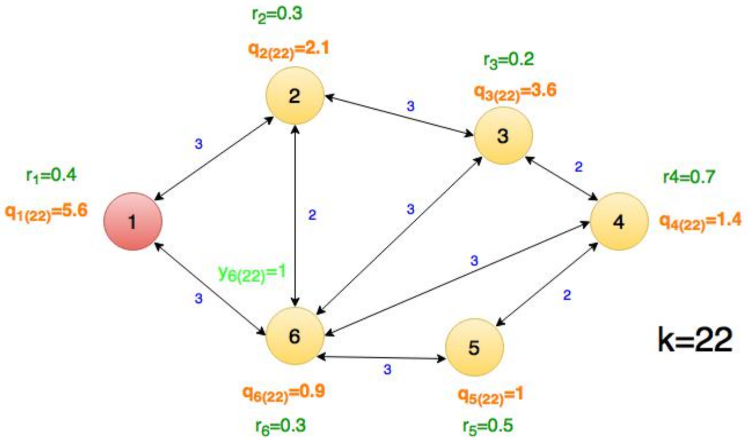
r_2=0.3
$q_{2(19)}$=1.2
r_3=0.2
$q_{3(19)}$=3
r_1=0.4
$q_{1(19)}$=4.4
r4=0.7
$q_{4(19)}$=3.5
$x_{46(19)}$=1
$q_{6(19)}$=2.4
r_6=0.3
$q_{5(19)}$=2
r_5=0.5
1
2
3
4
5
6
3
3
3
2
2
3
3
3
2
3
k=19

Experiment



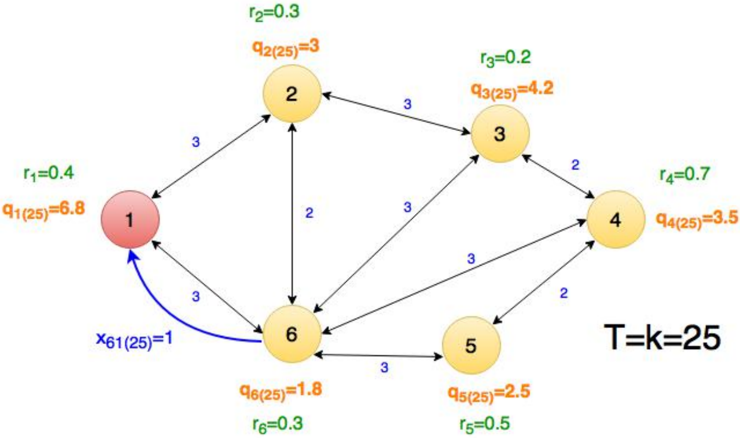
r2=0.3
q2(20)=1.5
r3=0.2
q3(20)=3.2
r1=0.4
q1(20)=4.8
r4=0.7
q4(20)=4.2
1
2
3
4
5
6
3
3
3
2
2
3
3
2
3
k=20
q6(20)=0.3
w56(20)2=1
q5(20)=2.5
r6=0.3
r5=0.5
w46(20)3=1

Experiment



r_2=0.3
$q_{2(22)}$=2.1
r_3=0.2
$q_{3(22)}$=3.6
r_1=0.4
$q_{1(22)}$=5.6
r4=0.7
$q_{4(22)}$=1.4
$y_{6(22)}$=1
1
2
3
4
5
6
3
3
2
3
2
3
3
2
3
$q_{6(22)}$=0.9
r_6=0.3
$q_{5(22)}$=1
r_5=0.5
k=22

Experiment



r_2=0.3
$q_{2(25)}$=3
r_3=0.2
$q_{3(25)}$=4.2
r_1=0.4
$q_{1(25)}$=6.8
r_4=0.7
$q_{4(25)}$=3.5
1
2
3
4
5
6
3
3
2
2
3
3
3
2
3
$x_{61(25)}$=1
T=k=25
$q_{6(25)}$=1.8
r_6=0.3
$q_{5(25)}$=2.5
r_5=0.5

Comparison of objective functions

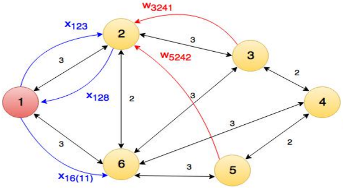
$$minimize \left\{ \sum_{k=1}^{T} \max_{j \in V} \{q_{jk}\} \right\}$$

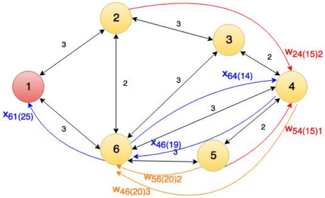
$$maximize \left\{ \sum_{k=1}^{T} \sum_{(i,j) \in A} x_{ijk} q_{jk} + \sum_{k=1}^{T} \sum_{l=1}^{S} \sum_{(j,i) \in V \times V} w_{jikl} q_{jk} \right\}$$

$$minimize \left\{ \max_{j,k} \{q_{jk}\} \right\}$$

route

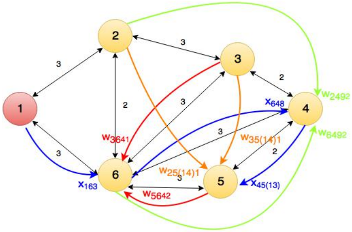
$$o.f1 : minimize \left\{ \sum_{k=1}^{T} \max_{j \in V} \{q_{jk}\} \right\}$$

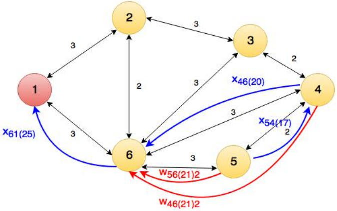




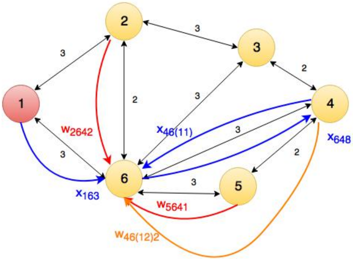
route

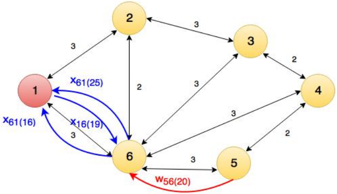
$$o.f2 : maximize \left\{ \sum_{k=1}^{T} \sum_{(i,j)\in A} x_{ijk} q_{jk} + \sum_{k=1}^{T} \sum_{l=1}^{S} \sum_{(j,i)\in V\times V} w_{jikl} q_{jk} \right\}$$



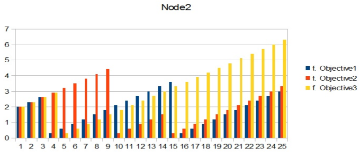


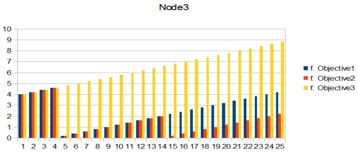
$$o.f3 : minimize \left\{ \max_{j,k}\{q_{jk}\} \right\}$$

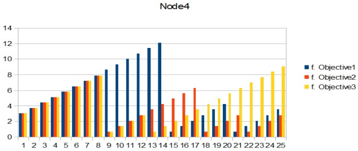


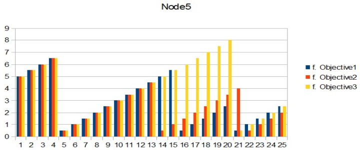


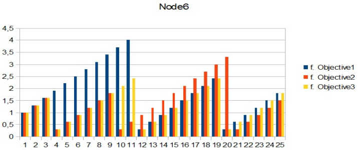
Amount of information in the nodes



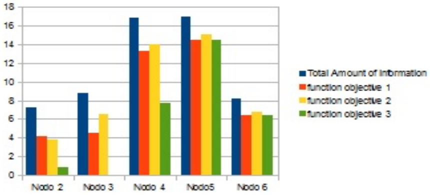


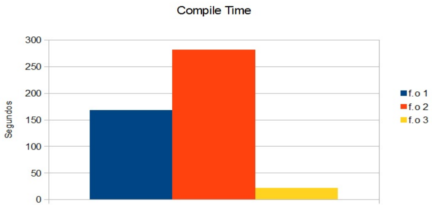






Amount of information in the nodes





Improvements

- Including constraints that allow the wireless transmission from nodes in a given neighborhood and not from primary nodes in the network.
- Compare the solutions obtained with other objective function.
- Eliminate some non necessary nodes in V'.

Thank you for
your attention