Particle Filter

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1 Introduction

In this Assignment, the Estate estimation using the particle filter is being investigated. Particle filters update their prediction in an approximate (statistical) manner. The samples from the distribution are represented by a set of particles; each particle has a likelihood weight assigned to it that represents the probability of that particle being sampled from the probability density function.

2 Particle filter

Particle filter has three main steps:

1. Drawing M samples
2. Generating a prior posterior
3. Correct the posterior PDF

In the first step m points are being sampled around the initial position. After that for each of the sampled points, the motion model is computed by using following equation.

$$\hat{x}_{n+1,n} = F\hat{x}_{n,n} + Gu_n + w_n \tag{1}$$

In the equation 1:
$x_{n+1,n}$ is the predicted system state vector at time step $n+1$.
$x_{n,n}$ is the predicted system state vector at time step n.
u_n is the control variable or input variable.
w_n is the process noise or distrubance.
F is the state transition matrix.
G is a control matrix or input transition matrix.
If we consider the system a 2D system, the state matrix, $\hat{x}_n$, can be shown as follow:

There exists a measurement as well. That can be computed as follow:

$$Z_n = H\hat{x}_n + v_n \tag{2}$$

$$\begin{bmatrix} \hat{x} \\ \hat{\dot{x}} \\ \hat{y} \\ \hat{\dot{y}} \end{bmatrix}$$

By using the equation 2 the actual from sensors arrive.

In the equation 2 v_n is called the measurement noise and H is called measurement matrix.
After that the position of each particle is computed. After that by utilizing the measurement position, the weight of each particle is computed.

$$p(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-(\frac{(x-\mu_x)^2}{2\sigma_x^2}) + \frac{(y-\mu_y)^2}{2\sigma_y^2})} \quad (3)$$

After that the new position is the weighted average of all the particles positions and their weights. Then the resmapling is being performed.
If we consider that for M samples we have a weight. Then we create bins from them by utilizing the following equation:

$$\beta_m = \frac{\sum_{n=1}^{m} w_n}{\sum_{l=1}^{M} w_l} \quad (4)$$

where we note that we will have $\beta_M = 1$. We then select a random number, ρ, sampled from a uniform density on [0, 1). For M iterations we add to the new list of samples, the sample whose bin contains ρ.

2.1 Implementation

The implementation is done in Python by using Pygame. At top left corner the covariance matrix is plotted as an ellipse, and blue, green and red rectangles show predicted position, Ground Truth and measured position respectively. In the implementation, the number of particles is 100.

Figure 1: covariance values over iterations

2.2 Question1

In this question the prediction step is 8 times of the measurement step. The Transition matrix, F, is as follow:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The state matrix is $\hat{x}_k = \begin{bmatrix} \hat{x}_k \\ \hat{y}_k \end{bmatrix}$ .The control matrix G= $\begin{bmatrix} r\Delta t/2 & r\Delta t/2 \\ r\Delta t/2 & r\Delta t/2 \end{bmatrix}$. Process noise is $Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.15 \end{bmatrix}$. The car is supposed to move from left top corner that is the point(0,0) to the right and the state matrix is supposed to be $p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. The car is moving from left to right 1 meter. It should be mentioned that the car speed is the same over all iterations. The observation matrix is of the form $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.
The final formulation is:

$$\begin{bmatrix} \hat{x}_{n+1,n} \\ \hat{y}_{n+1,n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_n \\ \hat{y}_n \end{bmatrix} + \begin{bmatrix} r\Delta t/2 & r\Delta t/2 \\ r\Delta t/2 & r\Delta t/2 \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} + w_n \Delta t \tag{5}$$

The most important observation of this experiment is that, by coming the observation the state covariance matrix is decreased and as expected the covariance matrix of particles get smaller. As can been in figure 2, particle filter can predict the measurement with a good accuracy.

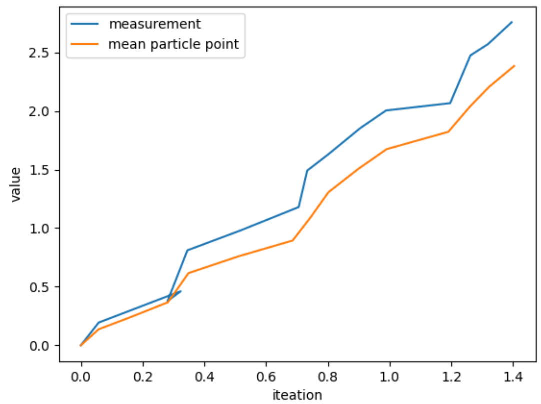

Figure 2: Comparison with particle filter and measurement

2.3 Question2

In the question 2, the system is supposed to rotate around the point $M = [10, 10]$ and $L = 0.3$ is the distance between wheels. The velocity in the x and y and angle direction can be computed by using the following equations:

$$\dot{x} = r/2(u_r + u_l)cos(\theta) + w_x \tag{6}$$

$$\dot{x} = r/2(u_r + u_l)sin(\theta) + w_y \tag{7}$$

$$\dot{\theta} = r(u_r - u_l)/L \tag{8}$$

In order get the car rotate along the landmark point, the control signals must be applied to wheels, in a manner that if the distance is being greater, the left wheel signal will be applied only (in a left circular rotation along the point). The rest of formulation is the same. Like previous question, by combining the observation the state covariance is being decreased and estimated positions move more smooth..The overall formulation is similar to 9

$$\begin{bmatrix} \hat{x}_{n+1,n} \\ \hat{y}_{n+1,n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_n \\ \hat{y}_n \\ \hat{\theta_n} \end{bmatrix} + \begin{bmatrix} r\Delta t cos(\theta) & 0 \\ r\Delta t sin(\theta) & 0 \\ 0 & \Delta tr/l \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} + w_n \Delta t \tag{9}$$

The state matrix will be $P_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

As can be seen in figure 3, The covariance has increased by coming a measurement and prediction can predict the measurement in a good manner.

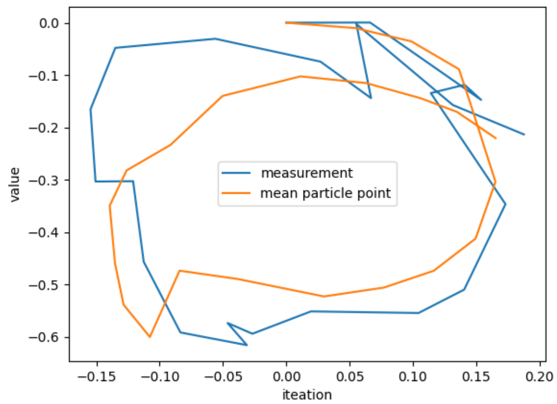

Figure 3: Comparison with particle filter and measurement

2.4 Question 3

We can formulate Cartesian coordinate system in polar as $\begin{bmatrix} \sqrt{(p_x - x)^2 + (p_y - y)^2} \\ \arctan{(p_y - y)/(p_x - x)} - \theta \end{bmatrix}$

where (p_x, p_y) is the landmark and (x,y) are the position of the vehicle in the landmark coordinate. x and y are positions in Lidar system. By converting the System to polar.The H matrix also must be modified and follow the Jacobian of the above transform which can be written as $\begin{bmatrix} \frac{-p_x+x}{\sqrt{x^2+y^2}} & \frac{-p_y+y}{\sqrt{x^2+y^2}} & 0 \\ \frac{y-p_y}{\sqrt{(p_x-x)^2+(p_y-y)^2}} & \frac{x-p_x}{\sqrt{(p_x-x)^2+(p_y-y)^2}} & -1 \end{bmatrix}$.

The samples importance is then computed via distance and bearing. Thre result is shown in figure 4. The covariance area decreases by coming the measurement.

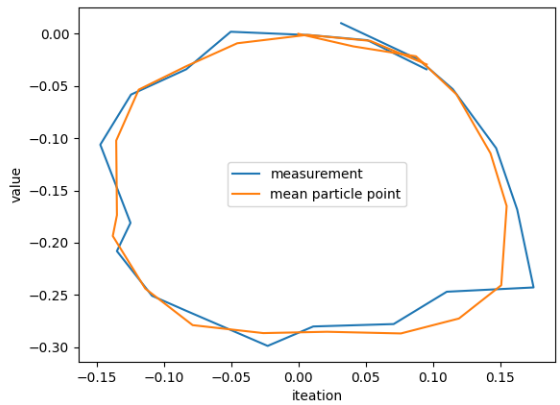

Figure 4: Measurement values with noise over iterations