

```
syms x1 x2 alpha k
```

```
f = x1^2 + 2*x2^2
```

$$f = x_1^2 + 2x_2^2$$

```
g = gradient(f)
```

$$g = \begin{pmatrix} 2x_1 \\ 4x_2 \end{pmatrix}$$

```
x = [x1;x2]
```

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

```
x_val = [2;1]
```

$$x_{\text{val}} = \begin{matrix} 2 \times 1 \\ 2 \\ 1 \end{matrix}$$

```
size = 100
```

```
size = 100
```

```
g_val = subs(g, x, x_val)
```

$$g_{\text{val}} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

```
all_x = zeros(2, size + 1)
```

$$\text{all\_x} = \begin{matrix} 2 \times 101 \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

```
all_f = zeros(1, size + 1)
```

$$\text{all\_f} = \begin{matrix} 1 \times 101 \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{matrix} \end{matrix}$$

```
all_x(:, 1) = x_val
```

$$\text{all\_x} = \begin{matrix} 2 \times 101 \\ \begin{matrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \dots \end{matrix} \end{matrix}$$

```
1 0 0 0 0 0 0 0 0 0 0 0 0
```

```
all_f(1) = subs(f, x, x_val)
```

```
all_f = 1x101
```

```
6 0 0 0 0 0 0 0 0 0 0 0 0...
```

```
threshold = 0.001
```

```
threshold = 1.0000e-03
```

```
echo off;
for i = 2:size+1
    d = - g_val;
    x_new = x_val + alpha * d;

    phi = subs(f, x, x_new);

    alpha_vals = solve([gradient(phi) == 0, alpha >= 0], alpha);
    phi_vals = double(subs(phi, alpha, alpha_vals));
    [phi_vals, alpha_idx] = sort(phi_vals);

    alpha_val = alpha_vals(alpha_idx(1));

    x_val = subs(x_new, alpha, alpha_val);
    g_val = subs(g, x, x_val);

    all_x(:, i) = x_val;
    all_f(i) = subs(f, x, x_val);
    % norm(g_val), threshold *
end
echo on;
```

```
syms k
```

```
xk = 1/3^k * [2; (-1)^k]
```

```
xk =
```

$$\begin{pmatrix} \frac{2}{3^k} \\ \frac{(-1)^k}{3^k} \end{pmatrix}$$

```
xks = double(subs(xk, 0:size))
```

```
xks = 2x101
```

```
2.0000 0.6667 0.2222 0.0741 0.0247 0.0082 0.0027 0.0009 ...
1.0000 -0.3333 0.1111 -0.0370 0.0123 -0.0041 0.0014 -0.0005
```

```
sym(xks(:, 1:5))
```

```
ans =
```

$$\begin{pmatrix} 2 & 0.6667 & 0.2222 & 0.0741 & 0.0247 \\ 1 & -0.3333 & 0.1111 & -0.0370 & 0.0123 \end{pmatrix}$$

```
sym(all_x(:, 1:5))
```

```
ans =
```

$$\begin{pmatrix} 2 & 0.6667 & 0.2222 & 0.0741 & 0.0247 \\ 1 & -0.3333 & 0.1111 & -0.0370 & 0.0123 \end{pmatrix}$$

```
% compare the computed x's from sd and the equation
all(all(all_x - xks < 0.0001))
```

```
ans = logical
      1
```

```
sym(all_f(1:4) / 9)
```

```
ans = (0.6667 0.0741 0.0082 9.1111e-04)
```

```
sym(all_f(2:5))
```

```
ans = (0.6667 0.0741 0.0082 9.1449e-04)
```

```
% compare the recurrence relation of f
all(all_f(1:size-1)/9 - all_f(2:size) < 0.0001)
```

```
ans = logical
      1
```