All screenshot of excel are attached.

1 Minimize Inscribed Triangle Perimeter

First, we revisit the definition of linear interpolation between two points. As from the Wikipedia page:

> linear interpolation is a method of curve fitting using linear polynomials to construct new data points within the range of a discrete set of known data points.

We define this function

$$I(M, N, t) = tM + (1 - t)N$$

where M and N are coordinates (in this case, in $\mathbb{R}^2$), $t \in [0, 1]$. Function I gives a new coordinate in the straight line between M and N depending on the value of parameter t. Now we shall begin the formulation of the model.

For this problem, we would like to find the location of points D, E, F lies on line AB, BC, AC in acute triangle ABC such that the new triangle DEF has minimal perimeter.

Above definition clearly suggests that the location of D, E, F can be expressed in terms of the linear interpolation of AB, BC, and AC. Hence, for the **decision variables**, we could have parameter t_D, t_E, and t_F, from which the coordinates could be calculated based on function I.

Furthermore, we define the distance function $dist(M, N) = \sqrt{(x_M - x_N)^2 + (y_M - y_N)^2}$. As the problem seeks to minimized the perimeter of inscribed triangle DEF, the **objective function** is

$$\min dist(D, E) + dist(E, F) + dist(D, F)$$

in which the D, E, F can be substituted by the decision variables:

$$\min dist(I(A, B, t_D)) + dist(I(B, C, t_E)) + dist(I(A, C, t_F))$$

Then, we can list all the constraints contained.

- Points on Line

  It is intuitive to see that points D, E, F must in the line between AB, BC, AC. The linear interpolation function I addresses this.

- Parameter Range

  As the our linear interpolation function I specifies, $t \in [0, 1]$.

2 Sunco Oil Problem

For this problem, x_i, the number of barrels sent via arc i as labeled in Figure 1 is the **decision variables** (Unit: 1000 bbl). The **objective function** for this problem would be

$$\min \sum_i C_i x_i$$

where C_i is the unit shipping cost via arc i.

Here we enumerate all constraints

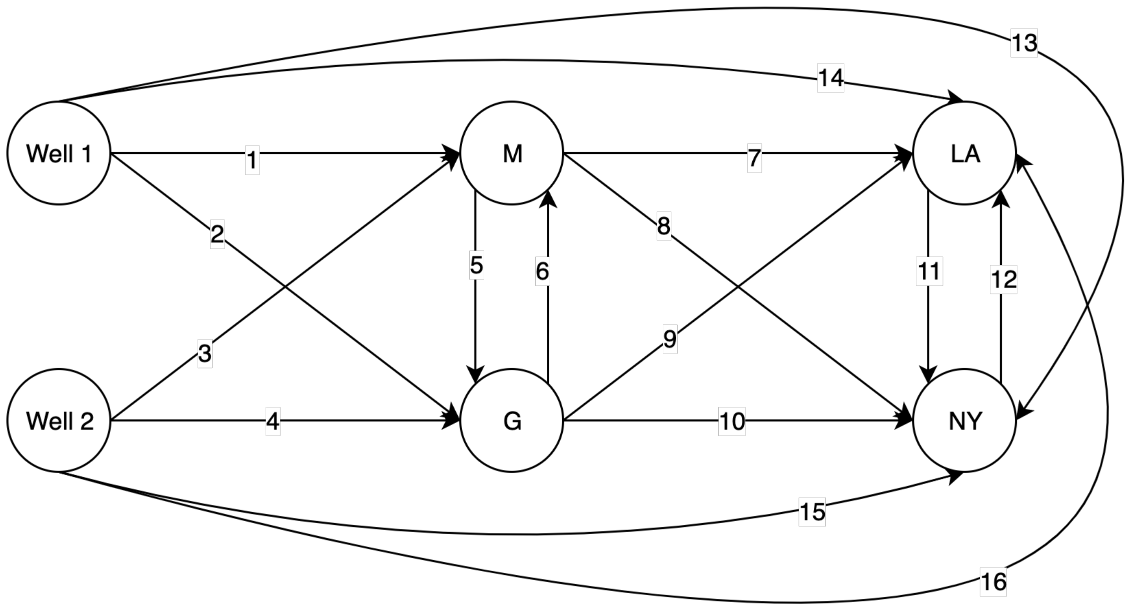

Figure 1: Network Flow of Sunco Oil Problem (labeled)

1. Cleared Ports

   Number of barrels ship into the ports must be all shipped out via outward arcs.

$$x_1 + x_3 + x_6 = x_5 + x_7 + x_8 \quad \text{(Mobile cleared)}$$
$$x_2 + x_4 + x_5 = x_6 + x_9 + x_{10} \quad \text{(Galveston cleared)}$$

2. Demand The number of barrels in customer location must satisfy the demand

$$x_7 + x_9 - x_{11} + x_{12} + x_{14} + x_{16} \geq 170 \quad \text{(LA demand)}$$
$$x_8 + x_{10} + x_{11} - x_{12} + x_{13} + x_{15} \geq 150 \quad \text{(NY demand)}$$

3. Supply Wells has limited supply per day.

$$x_1 + x_2 + x_{13} + x_{14} \leq 180 \quad \text{(Well 1 supply)}$$
$$x_3 + x_4 + x_{15} + x_{16} \leq 210 \quad \text{(Well 2 supply)}$$

4. Non-negativity The shipping of each arc is greater or equal to zero.

$$\forall i \in [1, ..., 16], x_i \geq 0$$

3 Parameter Estimation

3.1 A Ball

Decision Variable is parameter g, as we are doing parameter estimation. **Objective function** is

$$\min \sum_{t}^{n} \left\| s(t:g) - \hat{y}_t \right\|^2$$

Constraint is only non-negativity $g \geq 0$

3.2 Stock

Decision Variables are parameters a and b. **Objective function** is

$$\min \sum_{t}^{n} \left\| P_X(P_Y, P_Z, t : a, b) - \hat{P_X}(t) \right\|$$

No **Constraints** as it is only parameters.

4 Manufacturing Process

Decomposing the problem, we could get a general sense that three processing sequences are going on in this manufacturing process, two of which produces product A and one of which produces product B (sequence 1 for A: [1,2,4,2], sequence 2 for A, [1,2,4,3], sequence 3 for B: [1,3,4]). We define x_i, the input raw material to the i-th processing sequence, as the **decision variable**. Here we present a series of definitions to consolidate our model building.

1. Facility Sequence $sf(i, j)$ denotes the j-th facility in i-th processing sequence. For example $sf(1, 3)$ means the 3rd facility in sequence 1, which is facility 4.

2. Sequence Running Cost $cost(i)$

$$cost(i) = \sum_{j} c_{sf(i,j)} \frac{x_i \prod_{k}^{j-1} r_{sf(i,k)}}{G_{sf(i,j)}}$$

   where c is the hourly cost of facility, r is the recovery rate after passing a facility, G is the input gallons per hour to each facility. The second term basically denotes the hours at each facility, which is the actual input gallons divided by the theoretical inputs per hour.

   As an example, calculating the cost of running procedure A through process 2 is

$$cost(2) = 150\frac{x_2}{300} + 200\frac{(0.9)x_2}{450} + 180\frac{(0.9)(0.95)x_2}{250} + 250\frac{(0.9)(0.95)(0.85)x_2}{350}$$

3. Raw material cost $raw(i)$ gives the raw material cost for each sequence depending on the types of product they produce.

4. Facility hours $hours(f)$ is hours that f-th facility runs in all the processes. It is a bit complicated to express in mathematical terms, as some of the facility presents more than once in processes, and therefore omitted.

5. Sequence Production $prod(i)$

$$prod(i) = x_3 \prod_{j} r_{sf(i,k)}$$

   As an example, the end product of B out of sequence 3 is

$$prod(3) = (0.9)(0.85)(0.8)x_3$$

6. Sequence Output Price $price(i)$ gives the price of A or B depending on the sequence number.

7. **Objective Function** shows the

$$\max \sum_i price(i)prod(i) - cost(i) - raw(i)x_i$$

Here we list all the **constraints** related

1. Maximum Daily Sales

$$\begin{aligned} prod(1) + prod(2) &\leq 1700 && \text{(Max daily sales A)} \\ prod(3) &\leq 1500 && \text{(Max daily sales B)} \end{aligned}$$

2. Shipping Facility Limit

$$prod(1) + prod(2) + prod(3) \leq 2500 \qquad \text{(Total shipping limit)}$$

3. Facility Daily Hours

$$\begin{aligned} hours(1) &\leq 16 && \text{(Facility 1 daily hours)} \\ hours(2) &\leq 12 && \text{(Facility 2 daily hours)} \\ hours(3) &\leq 12 && \text{(Facility 3 daily hours)} \\ hours(4) &\leq 16 && \text{(Facility 4 daily hours)} \end{aligned}$$

4. Non-negativity All variables greater than zero.

5 Excel Model Attachments

Minimize Inscribed Triangle Perimeter							
Data		N/A	x	y			
	A		0	0			
	B		40	0			
	C		15	25			
Decision Variables		t	x	y			
	D	0.3750006	15.000025	0	AB		
	E	0.8000005	19.999988	20.000012	BC		
	F	0.7058831	10.588247	17.647078	AC		
Objective Function							
		Edge distances					
	DE	20.615531					
	EF	9.7014003					
	DF	18.190194					
	Perimeter	48.507125					
Constraints							
	Parameter Range						
		D	0	<=	0.3750006	<=	1
		E	0	<=	0.8000005	<=	1
		F	0	<=	0.7058831	<=	1

Figure 2: Spreadsheet Model for Problem 1

Sunco Oil Problem						
Data	**Unit Cost**	To ($/1000 bbl)				
	From	Mobile	Galveston	NY	LA	
	Well 1	10	13	25	28	
	Well 2	16	11	24	26	
	Mobil	0	7	15	18	
	Galveston	6	0	12	15	
	NY	1.00E+10	1.00E+10	0	20	
	LA	1.00E+10	1.00E+10	20	0	
Decision Variables		* Here we put 1e10 to simplify our calculation				
	Count	To (1000 bbl)				
	From	Mobile	Galveston	NY	LA	
	Well 1	0	0	110	0	
	Well 2	0	40	0	170	
	Mobil	0	0	0	0	
	Galveston	0	0	40	0	
	NY	0	0	0	0	
	LA	0	0	0	0	
Objective Function						
	Cost Per Route	To ($/1000 bbl)				
	From	Mobile	Galveston	NY	LA	
	Well 1	0	0	2750	0	
	Well 2	0	440	0	4420	
	Mobil	0	0	0	0	
	Galveston	0	0	480	0	
	NY	0	0	0	0	
	LA	0	0	0	0	
					Total Cost	8090
Constraints						
Demand	LA	170	>=	170		
	NY	150	>=	150		
Supply	Well 1	110	<=	180		
	Well 2	210	<=	210		
Clear port	Mobil	0	=	0		
	Galveston	0	=	0		
Nonnegativity		All decision var	>=	0		

Figure 3: Spreadsheet Model for Problem 2

A Ball						
Data						
	time	1	2	3	4	
	distance	5	19.5	44	78.5	
Decision Variables						
	g	9.80225989				
Objective Function						
	time	1	2	3	4	
	estimated	4.90112994	19.6045198	44.1101695	78.4180791	
	error	0.00977529	0.01092438	0.01213732	0.00671103	**0.03954802**
Constraints						
non-negativity	g	9.80225989	>=	0		

Figure 4: Spreadsheet Model for Problem 3.1

Predict Stock					
Data	Day	1	2	3	
	X	6	4	5	
	Y	1	1	3	
	Z	2	1	2	
Decision Variables					
	a	-0.5000003			
	b	3.38888889			
Objective Function					
	Day	1	2	3	
	Predicted_X	6.27777751	2.88888862	5.27777698	
	Error	0.07716035	1.2345685	0.07716005	**1.38888889**
Constraints					
	None				

Figure 5: Spreadsheet Model for Problem 3.2

Manufacturing Process									
Data									
	Process		Facility Sequence			Input (gal / hr)			
	1	1	2	4	2	300	450	250	400
	2	1	2	4	3	300	450	250	350
	3	1	3	4		500	480	400	
	Process		% Recovery			Running Cost ($/hr)			
	1	0.9	0.95	0.85	0.8	150	200	180	220
	2	0.9	0.95	0.85	0.75	150	200	180	250
	3	0.9	0.85	0.8		300	250	240	
	Process	Product	% Total Recovery	Raw Material Unit Cc	Selling Price ($)	Cumulated Product of % Recovery			
	1	A	0.5814	5	20	1	0.9	0.855	0.72675
	2	A	0.5450625	5	20	1	0.9	0.855	0.72675
	3	B	0.612	6	18	1	0.9	0.765	
Variables	Process	**Input (gal)**	Ouput (gal)	Gain ($)	Raw Material Cost($)	Summarized Process Running Cost ($)	Process Profit ($)		
and	1	**2923.976608**	1700	34000	14619.88304	5600.328947	13779.78801		
Objective Function	2	**0**	0	0	0	0	0		
	3	**1307.189542**	800	14400	7843.137255	1997.058824	4559.803922		
			Step Cost				**18339.59193**		
	Process	Step 1	Step 2	Step3	Step 4				
	1	1461.988304	1169.590643	1800	1168.75				
	2	0	0	0	0				
	3	784.3137255	612.745098	600					
Constraints									
Maximum Daily Sales	A	1700	<=	1700					
	B	800	<=	1500					
Shipping Limit	A+B	2500	<=	2500					
Facility Hours			Process						
	Facility	1	2	3	Total Hours				
	1	9.746588694	0	2.614379085	12.36096778	<=	16		
	2	5.847953216	0	0	5.847953216	<=	12		
	3	0	0	2.450980392	2.450980392	<=	12		
	4	10	0	2.5	12.5	<=	16		
Non-negativity	All vars	All vars	>=	0					

Figure 6: Spreadsheet Model for Problem 4