Project # 1: The Blending Problem of Hexxon Oil
Part I: Formulate a Mathematical Model

Mr. Sam Barton, production manager at the Hexxon Oil Company, has been asked by Mr. James Arden, the vice president of production, to formulate a new daily production plan for the three brands of gasoline the company sells: Regular (90 octane), Unleaded (96 octane), and Supreme (100 octane). In their meeting, Mr. Arden brought with him the data in Table 1 consisting of the projected minimum daily demands for these three gasolines and their respective selling prices, prepared by Ms. Jean Farrow. When Mr. Arden expressed the desire to achieve the highest possible daily profit, Mr. Barton said that he would have to meet with Mr. Allen, the production supervisor, to discuss the availability and costs of the constituents used in making the three brands of gasoline and would then get back to the vice president with a production plan.

TABLE 1: Selling Prices and Demands for Gasoline

Brand of Gasoline	Minimum Octane Rating	Selling Price ($/bbl)	Demand (bbl/day)
Regular	90	16.50	2000
Unleaded	96	18.00	4000
Supreme	100	22.50	3000

When Mr. Barton discussed the problem with the production supervisor, Mr. Allen said he would obtain the necessary information pertaining to the four constituents used in making the three brands of gasoline. At their meeting the next day, he brought the data in Table 2 that includes, for each constituent: (a) the octane rating, (b) the cost in dollars per barrel, and (c) the maximum supply available per day.

TABLE 2: Data on the Constituents for Blending Gasoline

Blending Constituent	Octane Rating	Cost ($/bbl)	Supply (bbl/day)
1	102	15.00	2500
2	96	12.00	3000
3	93	9.00	3500
4	110	24.00	2000

Mr. Allen reminded Mr. Barton that each of the three brands of gasoline must meet the minimum standard for octane rating (see Table 1). When asked precisely how this is accomplished, Mr. Allen explained that the octane rating of a mixture consisting of x_1, x_2, x_3, and x_4 barrels of the four constituents is the ratio of $102x_1 + 96x_2 + 93x_3 + 110x_4$ to the total number of barrels of the mixture, namely, $x_1 + x_2 + x_3 + x_4$.

(a) Formulate a production model for Mr. Barton to maximize the daily profits that satisfies all constraints. (35 points)

(b) Ms. Jean Farrow in the Accounting Department has discovered that the costs of the constituents given in Table 2 were not exactly correct. Formulate the new objective function on the basis of the following memo she sent to Mr. Barton: (5 points)

TO: Mr. S. Barton, Production Manager
FROM: Ms. J. Farrow, Head of Accounting
RE: Cost of Blending Constituent 1 to Make Supreme Gasoline

The cost of blending x barrels of Constituent 1 when making Supreme gas should be $13.50 + 0.001x$ per barrel, instead of $15.00 per barrel.