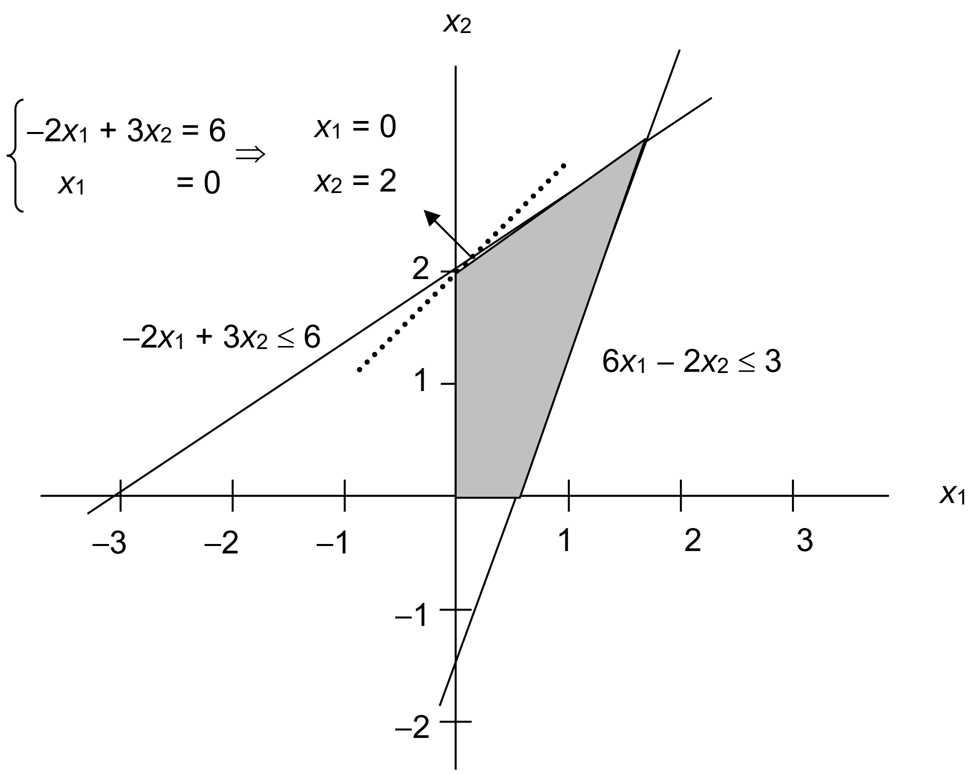
Solutions to Homework #4

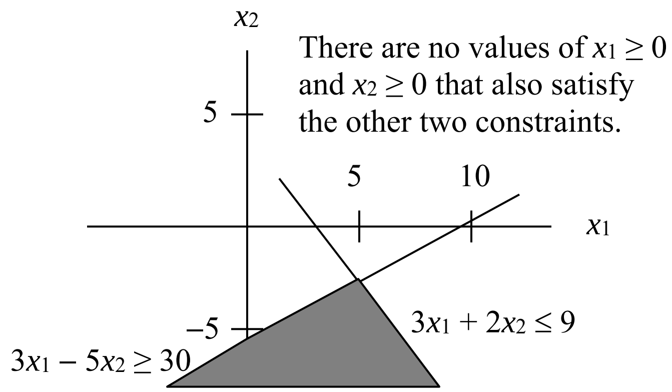
1. The constraint $2x_1 + 3x_2 \le 24$ is redundant and is not shown in the following graph:



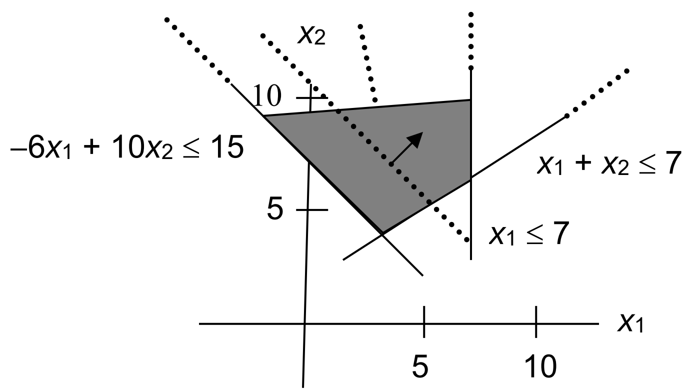
2. The four extreme points of the feasible region in Exercise 1 are listed in the following table:

Extreme Point	x_1	x_2	Obtained by Solving
A	0	0	$x_1 = 0$ $x_2 = 0$
B	0.5	0	$6x_1 - 2x_2 = 3$ $x_2 = 0$
C	1.5	3	$6x_1 - 2x_2 = 3$ $-2x_1 + 3x_2 = 6$
D	0	2	$-2x_1 + 3x_2 = 6$ $x_1 = 0$

3. The LP is infeasible because of the following:



4. The LP is unbounded because of the following graphical solution:



5. (a)

$$B = [A_{*1}, A_{*2}, A_{*4}, A_{*5}] = \begin{bmatrix} 1 & -3 & -8 & 1 \\ 0 & -5 & -2 & 0 \\ -1 & -4 & -10 & 0 \end{bmatrix} \qquad N = [A_{*3}, A_{*6}, A_{*7}] = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

(b)

$$Ax = Bx_B + Nx_N = Bx_B + N(0) = \begin{bmatrix} 1 & -3 & -8 & 1 \\ 0 & -5 & -2 & 0 \\ -1 & -4 & -10 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 \\ -12 \\ -19 \end{bmatrix} = b.$$

(c)

$$cx = c_B x_B + c_N x_N = c_B x_B + c_N(0) = c_B x_B = (4,5,-2,3)\,(1,2,1,3) = 21.$$

6. (a) Clearly, the vector $\mathbf{d} = (0, 1, 1, 1) \geq \mathbf{0}$ and also

$$Ad = \begin{bmatrix} 3 & -2 & -2 & 4 \\ 0 & 2 & 2 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$cd = (4,-2,-1,1)\,(0,1,1,1) = -2 < 0.$$

(b) This LP is not unbounded because the LP is not even feasible. You can see this because, if you add the two constraints, you obtain $3x_1 \leq -3$, that is, $x_1 \leq -1$, but you also know that $x_1 \geq 0$. That is, there is no value for x_1 that satisfies both $x_1 \leq -1$ and $x_1 \geq 0$, and so the LP is infeasible.

7. The standard-form LP is:

$$
\begin{array}{lrrrrl}
\min & 3\bar{x}_1 & + 2x_2 & & & \\
\text{s.t.} & -6\bar{x}_1 & + \; x_2 & - s_1 & & = 4 \\
 & -3\bar{x}_1 & - 2x_2 & & - s_2 & = 10 \\
 & \bar{x}_1, & x_2, & s_1, & s_2 & \geq 0
\end{array}
$$