MSOR 411: Homework 1: Due by 4:30pm Monday, September 09, 2019

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1. **Answer to question 1**

   For the following problems, I used the following guideline in the trial-and-error approach.

   - Set all the variable to their minimum requirement.
   - Select the most constrained variable and increase it until it cannot be increased any more (i.e. one or more constraints at the limits).
   - Try increase other variables that does not affect the constraints that are on the limits.

   The following section displays the EXCEL screeshots for the models.

   (a) Blubbermaid

			The Problem of BlubberMaid				
Data							
			Ingredients (oz/lb of product)				
		Polymer A	Polymer B	Polymer C	Base	Demand (lbs)	Profit ($/lb)
	Airtex	4	2	4	6	1000	7
	Extendex	3	2	2	9	500	7
	Resistex	6	3	5	2	400	6
	Inventory (lbs)	500	425	650	1100		
Model		Pounds	Produced	Demand	Profit		
	Airtex	1025	1025	1000	13075		
	Extendex	500	500	500			
	Resistex	400	400	400			
		Polymer A	Polymer B	Polymer C	Base		
	Resource Used	8000	4250	7100	11450		
	Resource Available	8000	6800	10400	17600		

   (b) MakeOrBuy

			The Problem of MTV Steel				
Data	**Tube**	**Selling**	**Demand**	**Time**	**Weld.**	**Prod.**	**Purch.**
	Type	**Price**	**(ft)**	**(min/ft)**	**Material**	**Cost**	**Cost**
	A	10	2000	0.5	1	3	6
	B	12	4000	0.45	1	4	6
	C	9	5000	0.6	1	4	7
	Amt.			40	5500		
	Avail.			(hrs)	(oz)		
Model		Produced	Purchased	Sold	Demand	Profit	
	A	2000	0	2000	2000	14000	
	B	0	4000	4000	4000	24000	
	C	2333	2667	5000	5000	16999	
		Time	Weld. Mat.				
	Used	2399.8	4333				
	Available	2400	5500				
		(min)	(oz)				

(c) Energy problem of Lilliput

		Sulfur Dioxide	**Carbon Monoxide**	**Dust Particles**	**Solid Waste**	**Min. Gen. Capacities**	**Max. Gen. Capacities**	**Cost**		**Total Demand**	
	Coal	1.5	1.2	0.7	0.4	36000	45000	6		125000	
	Natural Gas	0.2	0.5			0	15000	5.5			
	Nuclear	0.5	0.2	0.4	0.5	0	45000	4.5			
	Hydroelectric					0	24000	5			
	Petroleum	0.4	0.8	0.5	0.1	0	48000	7			
	Max. Allowed	75000	60000	30000	25000						
Model		**Min. Gen. Capacities**	**Amount**	**Max. Gen. Capacities**		**Cost**			**lhs**	**Op**	**rhs**
	Coal	36000	36000	45000		216000		**Demand**	128617	>=	125000
	Natural Gas	0	22080	15000		121440		**Nuclear**	5537	<	25723.4
	Nuclear	0	5537	45000		24916.5		**Nat. Gas**	22080	>=	1500
	Hydroelectric	0	60000	24000		300000					
	Petroleum	0	5000	48000		35000					
		Sulfur Dioxide	**Carbon Monoxide**	**Dust Particles**	**Solid Waste**						
	Amount Produced	63184.5	59347.4	29914.8	17668.5						
	Max. Allowed	75000	60000	30000	25000						

2. **Answer to question 2**

1 Unsure where to stop

When increasing the variables, it is hard to tell when to stop. The only solution is to keep trying until the specific combination of variable satisfies all constraints.

2 Does not scale

The trial-and-error approach might only work for small problems that could come with answers with only small number of iterations. However, for large problems, even slightly larger, it is hard for human to manually do the computation.

3 Hard to "Debug" model

If a false / erroneous model was established at the beginning, it is hard to tell until some obvious errors are visible for humans.

3. **Answer to question 3: Fresh Diary Farms**

For this question, we follow the procedure introduced in class to build the model for the Fresh Diary Farms problem.

1 Identify the variables

The values that constitutes the solution to the Fresh Diary Farms problem is the production plan, which are the production quantity low-fat milk, butter, and cheese. We denote these three variables as:

- L: the total quantity of low-fat milk produced daily as gal
- B: the total quantity of butter produced daily as in lb
- C: the total quantity of cheese produced daily as in lb

2 Identify the objective function

The goal for the problem is to maximize the profits of the farm with an production plan, which were discussed in the first step. The profits consists of the sale of low-fat milk, butter, and cheese produced, respectively. At the same time, the unit profit of each product is provided by the problem. Therefore, we could formulate the following equation as the objective function of the Diary Farm Problem:

$$\max_{L,B,C} 0.22L + 0.38B + 0.72C$$

3 Identify the constraints

The constraints for this problem can be categorized into 3 different types: resource constraints, demand constraints, and logical constraints. We discuss these types accordingly.

- Resource constraints
  - Machine Utilization

    Fresh Diary Farms has two different machines that both could process raw milk into the three products: low-fat milk, butter, and cheese. However, the processing speed differs by products and machines. There is an upper limit for the machine utilization of 8 hours (or 480 minutes) per day. Therefore, the production plan must cope with the limitations. Moreover, we introduce a subscript i for the aforementioned symbols for each of the products to denote the machine-independence for each products:

$$L = \sum_i L_i$$
$$B = \sum_i B_i$$
$$C = \sum_i C_i$$

    where $i \in [1..2]$. Hence, we denote the machine utilization constraint as:

$$0.2L_1 + 0.5B_1 + 1.5C_1 \leq 480$$
$$0.3L_2 + 0.7B_2 + 1.2C_2 \leq 480$$

- Demand constraints

– Minimum Required

There is a lower limit for the amounts in the production plan, which can be expressed as

$$L = \sum_i L_i \geq 300$$
$$B = \sum_i B_i \geq 200$$
$$C = \sum_i C_i \geq 100$$

- Logical Constraints

$$\text{All Variables} \geq 0$$

4 Is this a linear programming problem?

With the above three steps, we could formulate the optimization model for the Diary Farm Problem as follows:

$$\begin{aligned}
\max_{L_i, B_i, C_i, \forall i \in [1..2]} \quad & 0.22 \sum_i L_i + 0.38 \sum_i B_i + 0.72 \sum_i C_i \\
s.t. \quad & 0.2L_1 + 0.5B_1 + 1.5C_1 \leq 480 \\
& 0.3L_2 + 0.7B_2 + 1.2C_2 \leq 480 \\
& \sum_i L_i \geq 300 \\
& \sum_i B_i \geq 200 \\
& \sum_i C_i \geq 100 \\
& \forall i \in [1..2], L_i, B_i, C_i \geq 0
\end{aligned}$$

The above model is a linear model, since the objective function, all decision variables and all constraints are all linear

4. **Answer to question 4: Leather Company**

a Identify the variables

In the Leather Company problem, the manager would like to decide a production plan for each of the products, which are baseball gloves, footballs and leather straps, respectively. Therefore, the decision variable in the production plan would be the quantity of each type of the products. Here, we denote use the following symbols to denote the production plan:

- G: the total number of pairs of baseball gloves produced weekly.
- F: the total number of footballs produced weekly.
- S: the total number of leather straps produced weekly.

b Identify additional data

It is easy to notice that the data provide by the problem body is not sufficient for us to build a mathematical model. Additional data is needed for us to formulate the model for the Leather Company problem. We present the following list of symbols to denote the additional data needed and we use the subscript to denote the attribute for each specific variables.

- P: the unit profit for selling one unit of product in dollar.
  $\{P_G, P_F, P_S\}$
- CL: the leather consumption for producing one unit of product in square meters.
  $\{CL_G, CL_F, CL_S\}$
- TM: the machine time usage for producing one unit of product in minutes.
  $\{TM_G, TM_F, TM_S\}$

c Formulate a model.

1 Identify objective function
The goal for the problem is maximize the profit gained from selling the product, which could be written as

$$max_{G,F,S} P_G G + P_F F + P_S S$$

2 Identify constraints

- Resource constraints
  - Material Availability
    The total number of leather availability is limited to 1000 square meters.

$$CL_G G + CL_F F + CL_S S \leq 1000$$

  - Machine Utilization
    The machine utilization could not exceed 40 hours (2400 minutes) every week.

$$TM_G G + TM_F F + TM_S S \leq 2400$$

- Demand constraints
  Unless otherwise specified, there is no demand constraints under the problem setting.
- Logical constraints
  All variables must be greater than zero.

$$\text{All variable} \geq 0$$

3 Is the model a linear model? Gather all the information above, we formulate the mathematical model for this problem as follows:

$$\begin{aligned}
\max_{G,F,S} \quad & P_G G + P_F F + P_S S \\
s.t. \quad & CL_G G + CL_F F + CL_S S \leq 1000 \\
& TM_G G + TM_F F + TM_S S \leq 2400 \\
& G, F, S \geq 0
\end{aligned}$$

This model is not a linear model because the decision variables (G, F, S) is not continuous, as in the sense that the Lether Company cannot sell 0.5 football.

5. **Answer to question 5: Florida Citrus Inc.**

Following the standardized procedure for obtaining a mathematics model, we build a model for the Florida Citrus Inc. to maximize the profit.

1 Identify the variables
The central problem for the Florida Citrus Inc. is to maximize the profit for selling orange and grapefruit concentrate. Their profit solely depends on the sales of these two concentrate. Thus, we identify the following decision variable:

- CT: the volume of the concentrate produced by Florida Citrus Inc. in gallon. $\{CT_O$: volume of orange concentrate produced, CT_G : volume of grapefruit concentrate produced$\}$

2 Identify the objective function

The sales of concentrates is the only source of the profit and at the same time, the raw materials, grapefruit and orange juice, incur costs to produce the final product. Following the notation of the problem, we obtain the following objective function:

$$\max_{CT_O, CT_G} 6CT_O + 8CT_G - 1.5OJ - 2GJ$$

Before we proceed to identify the constraints, we also identify the following equality constraints for juice and concentrates:

$$0.7OJ = CT_O$$
$$0.75GJ = CT_G$$

Thus, the objective function further simplifies to

$$\max_{OJ, GJ} 2.7OJ + 4GJ$$

3 Identify the constraints

- Resource constraints
  - Machine Utilization
    The distillation machine has a limited utilization of 80 hours per week. The production plan must not over-utilize the machine to avoid potential problems that might cause.
    $$\frac{1}{25}OJ + \frac{1}{20}GJ \leq 80$$
- Demand constraints
  - Storage capacity
    The storage capacity for each type of concentrate are limited to 1000 gallons.Therefore, the storage capacity constraints could be written as:
    $$CT_O = 0.7OJ \leq 1000$$
    $$CT_G = 0.75GJ \leq 1000$$
- Logical constraints
  All variables must be greater than zero.
  $$\text{All variable} \geq 0$$

4 Formulate the model

Based on all the information we obtained above, we could formulate a mathematical model for the Florida Citrus Inc. to maximize their profit.

$$\begin{aligned}
\max_{OJ, GJ} \quad & 2.7OJ + 4GJ \\
s.t. \quad & \frac{1}{25}OJ + \frac{1}{20}GJ \leq 80 \\
& 0.7OJ \leq 1000 \\
& 0.75GJ \leq 1000 \\
& OJ, GJ \geq 0
\end{aligned}$$