HOMEWORK #8

1. Mediscan, Inc., leases specialized x-ray equipment to hospitals. There are currently 3 machines located in New York, 2 in Chicago, and 3 in Los Angeles. Hospitals in Dallas require 4 machines. Those in Denver and Phoenix each need 2 machines. The cost of shipping one machine from each city to each hospital is given in the following table:

	Shipping cost per machine		
From/To	**Dallas**	**Denver**	**Phoenix**
New York	1600	1800	2500
Chicago	900	1000	1800
Los Angeles	1400	1000	400

   (a) Draw a distribution network indicating the appropriate supplies, demands, and other relevant data (where appropriate, add dummy nodes and arcs to have a balanced problem). (3 points)

   (b) Formulate a mathematical model to determine how many machines should be shipped from each city to each hospital so as to incur the least cost. (5 points)

   (c) Use STORM to report the optimal solution and associated cost. (3 points)

   (d) Mediscan, Inc., has just received an additional request for two of their machines in Miami. It costs $1300 to ship one machine to Miami from New York, $1400 from Chicago, and $2700 from Los Angeles. Use STORM to solve the modified problem. How does this change affect the optimal solution and optimal objective function value in part (c)? (4 points)

2. TPine Trees, Inc., has harvested 400 tons of lumber from pine forests located outside of Vancouver, 200 tons from forests near Seattle, and 150 tons outside of Portland. Japan has placed an order for 200 tons at a price of $1200 per ton, Taiwan needs 300 tons and will pay $1100 per ton, and Singapore wants 250 tons at $1000 per ton. It costs Pine Trees, Inc., $500 to get each ton from the lumber fields to the port in Vancouver, $400 per ton to the port in Seattle, and $300 per ton to Portland. The following table provides the cost for shipping each ton by sea from these ports to the respective countries:

	Cost (in $) for shipping a ton of lumber by sea		
From/To	**Japan**	**Taiwan**	**Singapore**
Vancouver	250	250	200
Seattle	250	200	200
Portland	200	150	150

(a) Draw a *balanced* distribution network indicating the appropriate supplies, demands, and other relevant data. (5 points)

(b) Use STORM to determine a distribution plan that maximizes corporate profits. Report the optimal distribution plan and associated proit. (3 points)

(c) After harvesting all of the lumber in their fields, Pine Trees, Inc., has been told that Singapore must cancel their order. All the lumber must still be trucked from the fields to the ports in Vancouver, Seattle, and Portland at the given costs. Lumber not shipped from those ports must be stored at a cost of \$25, \$20, and \$15 per ton, respectively. Use STORM to solve the modified problem. What is the optimal plan and total profit? (7 points)

3. Cole Chemicals makes six different compounds. Each compound is produced once during a 24-hour day. The profit obtained depends not only on which compound is produced, but also on the next compound produced. This occurs because each compound leaves some impurities that affect the quality of the next compound produced. Profit (in dollars) are indicated in the following table:

	FOLLOWED BY					
COMPOUND	**A**	**B**	**C**	**D**	**E**	**F**
A	—	250	300	100	—	170
B	250	—	160	—	270	150
C	300	160	—	230	—	140
D	100	—	230	—	220	—
E	—	270	—	220	—	100
F	170	150	140	—	100	—

(a) Report the order in which to produce the compounds and the total profit, as determined by STORM. Is this the optimal solution? (5 points)

(b) The Manager of Cole Chemicals has been told that she also needs to produce a new compound, G, each day. This new compound cannot be produced immediately before or after A. The profits associated with compounds following G are the same as when those compounds follow A. Appropriately modify the problem and use STORM to report the new sequence in which to produce the compounds that maximize the daily profit. (5 points)

4. A clique in a network G with vertices V and edges E is a set S of vertices such that each pair of vertices in S is connected by an edge. For a given network, consider the problem of finding a clique having the most number of vertices.

   (a) Identify the five components of this combinatorial optimization problem. (5 points)

   (b) Develop and state the steps of either a greedy algorithm or a finite improvement algorithm for attempting to solve the maximum-clique. What concerns do you have about your algorithm? (10 points)

5. For the shortest-path problem that was formulated in class:

   (a) Use STORM to find the cost of the shortest route from every city to every other city for the problem of Texas Airways from the class lecture notes. Present your answer in a table in alphabetical order of city name. (5 points)

   (b) Formulate an integer programming problem to find a shortest path from a given starting node s to an ending node t in a general network with given distances d_{ij} between nodes i and j connected by an arc. Then use the file sp.xls in *Canvas* and OpenSolver in EXCEL to find the least-cost route from Houston to El Paso for the problem of Texas Airways. (**Hint:** Let x_{ij} be 1 if you use arc ij in the shortest path and 0, otherwise. To be sure you have a path, you need constraints stating that the total number of arcs leaving node s should be 1 and the total number of arcs entering node t should be 1. For all other nodes, the total number of arcs in the path going into that node should be equal to the total number of arcs leaving that node. You can also add constraints that set all variables x_{ij} for which there is no arc ij to 0.) (20 points)

   (c) Describe how a finite-improvement algorithm would attempt to solve the general shortest-path problem (you need not provide details of how the movement mechanism works). What concerns do you have about your algorithm? (5 points)