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Section 1

HW 3

Question 1:

a) $\left\lceil \frac{size}{2} \right\rceil$

b)

```
HeapItemType MaxHeap::find_min_element() {  
    if (size > 0) {  
        int min = items[size/2].key;  
        int index = size/2;  
        for (int i = size/2 + 1; i < size; i++) {  
            if (min > items[i].key) {  
                min = items[i].key;  
                index = i;  
            }  
        }  
        return items[index];  
    }  
    return NULL;  
}
```

Question 2:

a)

Hypothesis: Index of the left child of a node that is in the k th location in the array is $2k+1$.

Basis case: If $k = 0$, then left child of the root will be 1, which is $2k+1$;

Inductive step:

Assume that $k = n$ and its left child is in the index $2n+1$.

Between left child of the n th node and left child of the $(n+1)^{\text{st}}$ node, there is only right child of the n th node. Therefore, index of left child of the $(n+1)^{\text{st}}$ node is $2n+1+2$, which is $2(n+1)+1$.

Thus; by the mathematical induction, index of the left child of a node that is in the k th location in the array is $2k+1$.

b)

Hypothesis: For a given preorder traversal of size n , there is only one way to create a heap of size n , which has that preorder traversal.

Basis case: For $n=1$, given preorder traversal gives the root of the heap and since $\text{size}=1$, there is only one way to create heap from that preorder traversal.

Inductive step:

Assume that for every integer 1 to $n-1$, hypothesis is correct, which means that for every size 1 to $n-1$, given preorder traversal determines only one heap of given size.

Let us think about a heap of size n . It is obvious that 1st element of the traversal is root and 2nd is left child of the root. Moreover, since the size is known, index of the right child of the root in the traversal can be determined easily. Consider the traversal from 2nd element to the index of the right child of the root (excluding the last one). It is the preorder traversal of the heap whose root is left child of the original root. Now, consider the traversal from index of the right child of the root to last of the given traversal. It is the preorder traversal whose root is the right child of the original root. Since left and right sub-trees' sizes smaller than $n-1$, they can be determined by our assumption. In addition, since we know the root of the heap, from given traversal the heap can be determined.

Therefore; by the mathematical induction any given preorder traversal determines only one heap of given size.

By the lemma proved above, if the preorder traversals are the same then those heaps are the same as well.

Question 4: