Brownian Motion

Brownian motion describes the random motion of small particles in a liquid or gas. This motion was first observed by botanist Robert Brown in 1827, and is named after him. Brown noticed that pollen grains in water were moving continuously and chaotically under a microscope.

Key features

1. **Random Movement**: Brownian motion occurs in a completely random and unpredictable manner. The next position of a particle depends on its current position, but in which direction and how much it will move is completely random.
2. **Perpetual Motion**: Particles move non-stop. This occurs as a result of the fact that molecules constantly collide with each other.
3. **Molecular Collisions**: The main reason for this random motion is the constant collision of liquid or gas molecules into particles. Particles constantly change direction as a result of these collisions.

Mathematical Model

Several methods have been developed to mathematically model Brownian motion. One of the most common patterns is known as the Wiener process. This model is one of the cornerstones of the theory of stochastic (random) processes.

The Wiener process $W(t)$ has the following properties:

1. $W(0) = 0$.
2. The process constantly increases over time.
3. For any $t_1 < t_2$, $W(t_2) - W(t_1)$ is normally distributed and has a mean of zero, with a variance of $t_2 - t_1$.
4. The process has independent increments. That is, the increments in different time intervals are independent of each other.

The Brownian Movement and Finance

In financial markets, the Brownian motion is used to model the random fluctuations of the prices of stocks and other assets over time. This model is a key component in financial models, especially the Black-Scholes option pricing model.

The Black-Scholes model assumes that the price of an asset has a log-normal distribution and that this price is followed by a continuous time Wiener process. This is used to understand how asset prices change over time and the probabilistic nature of these changes.

Summary

Brownian motion is a fundamental physical and mathematical phenomenon that describes the random motion of particles in a liquid or gas. This concept has wide applications not only in physical systems, but also in finance, biology and many other fields.

Let us explain the Brownian motion with graphs. First, let's simulate a Brownian motion and then show the results with graphs.

Brownian Motion Simulation

To determine the position of a particle at ttt time, a random small motion is added at each time step. This move is usually performed using a random number with a normal distribution.

Simulation with Python Code

The following code simulates the Brownian motion of a particle in a two-dimensional plane and graphically displays the results:

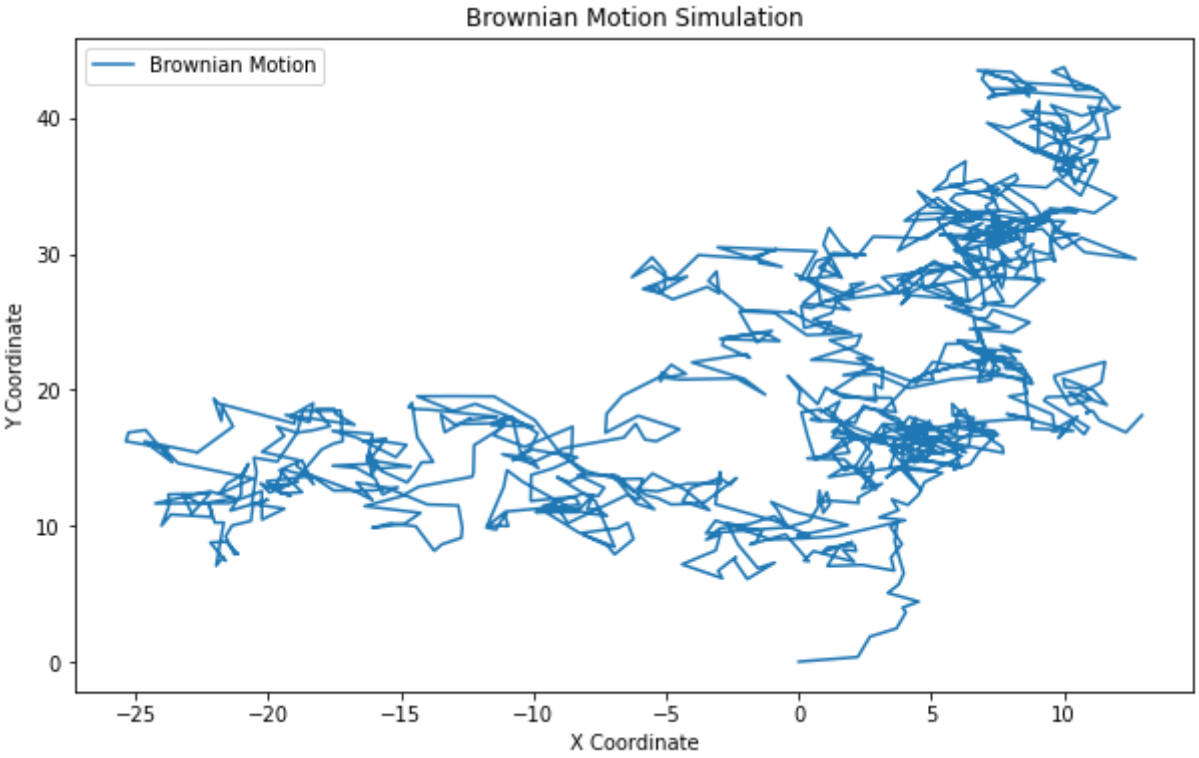
```
import numpy as np
import matplotlib.pyplot as plt

# Number of steps and step size
n_steps = 1000
delta_t = 1
sigma = np.sqrt(delta_t) #The standard deviation is usually taken as√(delta_t)

# Starting point
x = np.zeros(n_steps)
y = np.zeros(n_steps)

# Brownian motion
for i in range(1, n_steps):
    x[i] = x[i-1] + np.random.normal(0, sigma)
    y[i] = y[i-1] + np.random.normal(0, sigma)

# Graphic drawing
plt.figure(figsize=(10, 6))
plt.plot(x, y, label='Brownian Motion')
plt.title('Brownian Motion Simulation')
plt.xlabel('X Coordinate')
plt.ylabel('Y Coordinate')
plt.legend()
plt.show()
```

This code simulates the random motion of a particle in a two-dimensional plane and displays it graphically. At each step, the *x* and *y* coordinates of the particle are updated with random numbers with normal distribution.

Details in the Chart

1. **Starting Point**: The particle starts from the point (0, 0).
2. **Random Movement**: At each step, the *x* and *y* coordinates are changed by a random amount.
3. **Path:** The path taken by the particle is shown as a constantly changing line.

This simulation of Brownian motion helps us visually understand how particles move randomly in liquid or gas. Now let's run this code and create the visual.



In the graph, you can see how a particle moves randomly in the two-dimensional plane with Brownian motion. The particle follows this random path, making small movements in a random direction at each step, starting from the starting point. This movement is the result of constant collisions of liquid or gas molecules.