## Euler Discretization

Euler discretization is one of the simplest and most common methods used in the numerical solution of differential equations. It is especially used to solve ordinary differential equations involving initial value problems. The method is named after the Swiss mathematician Leonhard Euler.

The Euler method obtains an approximate solution by solving a differential equation step by step. In this method, the solution interval is divided into small steps, and at each step, the next point is calculated using the slope.

## Mathematical Basis

Let's say the differential equation we want to solve is:

$$\frac{dy}{dt} = f(t, y)$$

and the initial value $y(t_0) = y_0$. Euler's method solves this equation as follows:

1. First, the step size $h$ is determined.
2. Then, at each step, the new value is calculated:
   $y_{n+1} = y_n + hf(t_n, y_n)$

Here:

- $t_n = t_0 + nh$
- $y_n$ is *the approximate solution value at the time* $t_n$.

## Application Steps

1. Determine the initial value and step size.
2. Starting from the initial value, calculate the new values step by step:
   - $t_{n+1} = t_n + h$
   - $y_{n+1} = y_n + hf(t_n, y_n)$

## Example

Let's say $dy / dt = y$ and the initial value *is* $y(0) = 1$. Let's follow the step-by-step steps to solve this equation using Euler's method:

1. Let's determine *the step size* as h = 0.1.
2. Initially $t_0 = 0$ and $y_0 = 1$.

Let's calculate the first step:

$y_1 = y_0 + hf(t_0, y_0) = 1 + 0.1 \cdot 1 = 1.1$

Let's calculate the second step:

$y_2 = y_1 + hf(t_1, y_1) = 1.1 + 0.1 \cdot 1.1 = 1.21$

Continuing in this way, we get the approximate solution step by step.

Although the Euler method is a simple and fast method, the error rate can increase at large step magnitudes. For this reason, smaller step sizes or more advanced numerical methods are often preferred for more precise solutions.

Let us explain the numerical solution of a differential equation with graphs by the Euler method. For example, *let* $dy / dt = y$ be the differential equation and the initial value $y(0) = 1$.

## 1. Step: Starting Point

The starting point *is* $t_0 = 0$ and $y_0 = 1$.

## 2. Step: Determining the Step Size

Let's determine *the step size* as $h = 0.1$.

## 3. Step: Calculation by Euler Method

Let's calculate the new values step by step using the Euler method.

**Calculation Steps:**

1. $t_1 = t_0 + h = 0 + 0.1 = 0.1$
   $y_1 = y_0 + h \cdot f(t_0, y_0) = 1 + 0.1 \cdot 1 = 1.1$
2. $t_2 = t_1 + h = 0.1 + 0.1 = 0.2$
   $y_2 = y_1 + h \cdot f(t_1, y_1) = 1.1 + 0.1 \cdot 1.1 = 1.21$
3. $t_3 = t_2 + h = 0.2 + 0.1 = 0.3$
   $y_3 = y_2 + h \cdot f(t_2, y_2) = 1.21 + 0.1 \cdot 1.21 = 1.331$

Let's repeat these steps several times and find the $y$ values at each step.

## 4. Step: Graphical Representation

At this point, let's show the values we have calculated on a graph. The graph allows us to compare the actual solution curve and the approximate solution of Euler's method.

### Charting with Python Code

The following code creates the chart by calculating these steps:

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```
import numpy as np
import matplotlib.pyplot as plt
```

```python
# Differential equation: dy/dt = y
def f(t, y):
        return y

# Starting value
t0, y0 = 0, 1
# Step size
h = 0.1
# Number of steps
n_steps = 20

# Calculation with Euler method
t_values = [t0]
y_values = [y0]

t, y = t0, y0
for _ in range(n_steps):
        y = y + h * f(t, y)
        t = t + h
        t_values.append(t)
        y_values.append(y)

# The aactual solution
t_real = np.linspace(0, 2, 100)
y_real = np.exp(t_real)

# Graphic drawing
plt.plot(t_values, y_values, 'b-o', label=' Euler Method')
plt.plot(t_real, y_real, 'r-', label=' Actual Solution')
plt.xlabel('t')
plt.ylabel('y')
plt.legend()
plt.title(' Numerical Solution with Euler Method')
plt.show()
```

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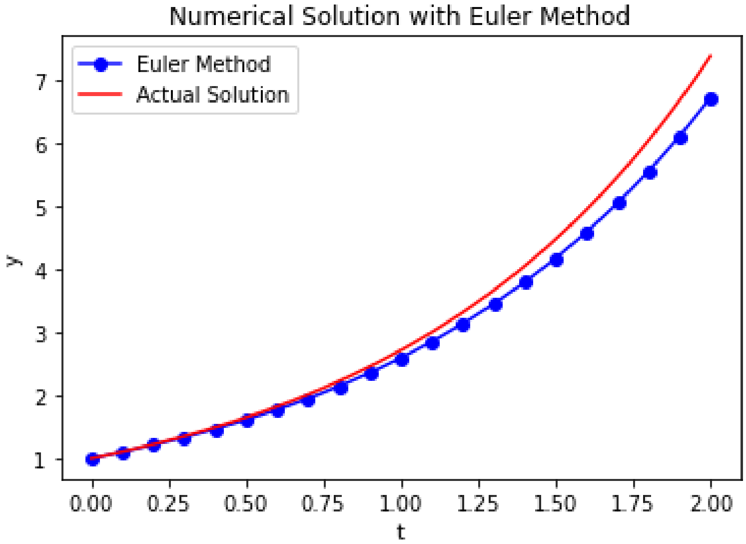
## 5. Step: Interpretation of the Chart

In the graph, the blue dots show the approximate solutions obtained by Euler's method, and the red curve shows the actual solution of the differential equation (exponential function). Euler's method approximates the correct solution at each step, but some error accumulates at each step. By using smaller step sizes, this error is reduced, and the approximate solution is closer to the actual solution.

The graph of Euler's method will be:

1. **Blue Dots and Lines**: Approximate solution points calculated by Euler's method.
2. **Red Curve**: The real solution of the differential equation ($y = e^t$).

This graph helps us visually understand how the Euler method works step by step and how it differs from the real solution.




In the graph, the blue dots and lines show the approximate solutions calculated by Euler's method, and the red curve shows the actual solution of the differential equation ($y = e^t$). While the Euler method gives approximate solutions by stepping by, you can visually understand the difference from the real solution.