GARCH Model Analysis on Financial Time Series: A Step-by-Step Guide

1. Data Retrieval for Time Series Analysis

To analyze financial time series data, we start by collecting historical price data for a financial instrument, represented by its closing prices over a specified period. We calculate the daily returns from this data, as they provide insight into daily fluctuations and volatility, which are essential for volatility modeling.

- **Daily Return Calculation**:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

  where P_t is the closing price on day t and P_{t-1} is the closing price on the previous day. The daily return series R_t helps us understand the asset's volatility and forms the basis for more advanced analysis like GARCH modeling.

2. Plotting Closing Prices and Daily Returns

Visual representations are critical for observing trends, volatility, and patterns over time.

- **Closing Price Plot**: Shows the overall price movement of the asset over time, allowing us to see trends or shifts in price levels.
- **Daily Return Plot**: By examining daily returns, we observe the day-to-day variability in returns, which often exhibit periods of higher and lower volatility (a phenomenon called "volatility clustering").

3. Descriptive Statistics for Price and Return Series

To summarize the data, we calculate a variety of descriptive statistics that capture both central tendency and dispersion. This includes mean, median, maximum, minimum, standard deviation, and additional metrics to assess normality and distributional characteristics.

- **Key Metrics**:
  - **Mean**: Indicates the average value over the period.
  - **Standard Deviation**: Measures the spread of the values around the mean, reflecting volatility.
  - **Jarque-Bera Test**: A statistical test for normality, testing whether skewness and kurtosis are consistent with a normal distribution.

4. Histogram Plots with Normal Distribution Fit

Histograms allow us to visually inspect the distribution of both closing prices and daily returns. Overlaying a normal distribution on the histogram enables us to assess how closely the actual distribution matches the normal.

- **Histogram Interpretation**:
  - For closing prices, a deviation from normality may indicate price skewness or trends.
  - For returns, deviations from the normal distribution are common in financial data, often showing fat tails (leptokurtosis), which suggests a higher probability of extreme returns.

5. ARCH-LM Test for Conditional Heteroskedasticity

The ARCH-LM (Lagrange Multiplier) test is used to detect the presence of autoregressive conditional heteroskedasticity (ARCH) effects in the return series, a necessary condition before applying a GARCH model.

- **Theory**:
  - If returns exhibit periods of clustering volatility, they may follow an ARCH process, indicating that today's volatility depends on past periods' squared returns.
  - The null hypothesis for the ARCH-LM test states that there are no ARCH effects, i.e., volatility is constant over time.

6. Fitting GARCH Models

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is designed to capture and model volatility clustering in financial time series. We fit multiple GARCH models to the return series to determine which best captures its volatility dynamics.

- **GARCH Model Structure**:
  - GARCH(p, q) models use past return shocks (ARCH terms) and past volatility (GARCH terms) to predict current volatility.
  - The GARCH(p, q) model can be represented as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

    where σ_t^2 is the conditional variance, ω is a constant, α_i represents the ARCH parameters, and β_j represents the GARCH parameters.

- **Model Evaluation**:
  - **Akaike Information Criterion (AIC)**: Used to select the optimal model by balancing fit and model complexity. Lower AIC values indicate better model fit.
  - **Q Test and ARCH-LM Test on Residuals**:
    - **Q Test (Ljung-Box Test)**: Assesses whether residuals (standardized errors) exhibit autocorrelation. For a well-fitted model, the residuals should not show significant autocorrelation.
    - **ARCH-LM Test on Residuals**: Verifies that no additional ARCH effects remain in the residuals after fitting. A well-specified GARCH model should remove ARCH effects from the residuals.

7. Ranking Models by AIC, Q Test, and ARCH-LM Test Results

After fitting multiple GARCH models, we rank them based on the AIC value, with lower values indicating a better model. For each model, we also check whether the residuals meet the requirements of no autocorrelation (Q Test) and absence of ARCH effects (ARCH-LM Test). This approach ensures that our selected model not only fits well according to AIC but also satisfies diagnostic checks for model adequacy.

- **Model Selection**:
  - By comparing models, we can select the one with the lowest AIC, non-significant Q Test, and non-significant ARCH-LM Test, which implies both a good fit and appropriate model assumptions.