Markowitz Portfolio Theory

Markowitz Portfolio Theory, also known as Modern Portfolio Theory (MPT), was introduced by Harry Markowitz in 1952. It provides a framework for constructing a portfolio of assets that maximizes expected return for a given level of risk, or equivalently minimizes risk for a given level of expected return.

Key Concepts

1. **Expected Return (E[R])**: The mean of the probability distribution of possible returns for a portfolio.
2. **Risk (σ)**: Measured as the standard deviation of portfolio returns.
3. **Covariance and Correlation**: Measures how two assets move in relation to each other.

Mathematical Formulation

Expected Return of a Portfolio

The expected return of a portfolio $E(R_p)$ is the weighted sum of the expected returns of the individual assets in the portfolio. If the portfolio consists of n assets, the expected return is:

$$E(R_P) = \sum_{i=1}^{n} w_i E(R_i)$$

where:

- w_i is the weight of asset i in the portfolio,
- $E(R_i)$ is the expected return of asset i.

In matrix notation:

$$E(R_p) = \mathbf{w}^T \mathbf{E}(R)$$

where:

- $\mathbf{w}$ is the vector of weights $[w_1, w_2, \ldots, w_n]^T$,
- $\mathbf{E}(R)$ is the vector of expected returns $[E(R_1), E(R_2), \ldots, E(R_n)]^T$.

Portfolio Variance and Standard Deviation

The risk (variance) of the portfolio σ_p^2 is given by:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

where:

- σ_{ij} is the covariance between the returns of asset i and asset j.

In matrix notation:

$$\sigma_p^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$$

where Σ is the covariance matrix of asset returns. The standard deviation (risk) of the portfolio is:

$$\sigma_p = \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}$$

Efficient Frontier

The Efficient Frontier is the set of optimal portfolios that offer the highest expected return for a defined level of risk. These portfolios are represented by a curve in the return-risk space.

Graphical Representation

- **Efficient Frontier**: This is a curve that shows the portfolios with the highest expected return for each level of risk.
- **Capital Market Line (CML)**: This is a line that represents the risk-return trade-off of a portfolio with the inclusion of a risk-free asset.

Backtesting a Portfolio

Backtesting involves testing a portfolio strategy on historical data to evaluate its performance. Here's how you can do it:

1. **Data Collection**: Gather historical price data for the assets in your portfolio.
2. **Portfolio Construction**: Use the historical data to compute expected returns, covariances, and construct the portfolio according to Markowitz's theory.
3. **Simulation**: Simulate the portfolio's performance over the historical period by calculating the portfolio's return and risk for each period.
4. **Performance Metrics**: Evaluate performance using metrics such as cumulative return, annualized return, volatility, Sharpe ratio, and maximum drawdown.