Laboratory Exercise 2: Interpolation

OBJECTIVES: To implement some interpolation schemes and understand some of their properties.

Work through the exercises on this sheet. Try to complete as much as possible. If you finish early, work through the optional material at the end.

1 Visualizing the interpolating function

Browse the python script `linterp.py` (supplied) and make sure you understand what it does. Compile and run the procedure. Is the output what you expect.

Modify the procedure so that it also displays a cubic Lagrange fit to the data and the original function $\sin(x)$ on the same plot. For the cubic fit you will need to think carefully about what to do near the ends of the data—try assuming that the function is periodic.

2 Convergence of the interpolation

Write a procedure that will estimate $\cos(x_0)$ using linear interpolation between $x_0 - \Delta x/2$ and $x_0 + \Delta x/2$. Take $x_0 = \pi/4$ and start with $\Delta x = 1$. Compare the interpolated value with the true value. Try successively halving the value of Δx, and evaluate the error (absolute value of the interpolated value minus the true value) as Δx varies. Try plotting the error versus Δx on a log-log plot, and hence confirm that linear interpolation is second-order accurate.

Repeat the exercise for cubic Lagrange interpolation. Do your results agree with the theoretical order of accuracy?

3 Optional

Repeat the cubic Lagrange visualization of section 1 but using the function `f = round(sin(x))`. What do you notice about the cubic Lagrange fit to the data? Does the problem improve if you increase the number of data points?

Modify the procedure of section 1 so that it uses the values of $\sin(x)$ and its derivative at the given data points to construct a Hermite cubic fit. Is it more or less accurate than cubic Lagrange?