$$\pi = \left(\frac{R}{p_{00}}\rho\theta_\rho\right)^{R/c_v}$$

$$\Rightarrow \quad \rho = \frac{p_{00}}{R}\frac{\pi^{c_v/R}}{\theta_\rho}$$

$$\because \quad \pi^{c_v/R} = (\pi_0+\pi')^{c_v/R} \simeq \pi_0^{c_v/R}\left(1+\frac{c_v}{R}\frac{\pi'}{\pi_0}\right)$$

and $$\theta_\rho^{-1} = (\theta_{\rho 0}+\theta_\rho')^{-1} \simeq \theta_{\rho 0}^{-1}\left(1-\frac{\theta_\rho'}{\theta_{\rho 0}}\right)$$

$$\therefore \quad \rho \simeq \frac{p_{00}}{R}\frac{\pi_0^{c_v/R}}{\theta_{\rho 0}}\left(1+\frac{c_v}{R}\frac{\pi'}{\pi_0}\right)\left(1-\frac{\theta_\rho'}{\theta_{\rho 0}}\right)$$

$$\simeq \underbrace{\frac{p_{00}}{R}\frac{\pi_0^{c_v/R}}{\theta_{\rho 0}}}\left(1+\frac{c_v}{R}\frac{\pi'}{\pi_0}-\frac{\theta_\rho'}{\theta_{\rho 0}}\right)$$

$$= \rho_0\left(1+\frac{c_v}{R}\frac{\pi'}{\pi_0}-\frac{\theta_\rho'}{\theta_{\rho 0}}\right)$$

Denote "now" by 2 and "previous" step by 1,
Denote the correction constant by C, we should have

$$\iiint \rho_0\left(1+\frac{c_v}{R}\frac{\pi_2'+C}{\pi_0}-\frac{\theta_{\rho 2}'}{\theta_{\rho 0}}\right)$$

$$= \iiint \rho_0\left(1-\frac{c_v}{R}\frac{\pi_1'}{\pi_0}-\frac{\theta_{\rho 1}'}{\theta_{\rho 0}}\right)$$

Thus

$$\iiint \rho_0 \frac{c_v}{R}\frac{C}{\pi_0} = \iiint \rho_0 \frac{c_v}{R}\frac{\pi_1'-\pi_2'}{\pi_0} + \iiint \rho_0 \frac{\theta_{\rho 2}'-\theta_{\rho 1}'}{\theta_{\rho 0}}$$

$$C = \frac{\iiint \rho_0 \frac{c_v}{R}\frac{\pi_1'-\pi_2'}{\pi_0} + \iiint \rho_0 \frac{\theta_{\rho 2}'-\theta_{\rho 1}'}{\theta_{\rho 0}}}{\iiint \frac{c_v}{R}\frac{\rho_0}{\pi_0}}$$

Using $\rho_0 = \frac{P_{00}}{R}\frac{\pi_0^{c_v/R}}{\theta_{\rho 0}}$, it can also be written as

$$C = \frac{\iiint \beta(\pi_1'-\pi_2') + \iiint \beta \frac{R}{c_v}\frac{\pi_0}{\theta_{\rho 0}}(\theta_{\rho 2}'-\theta_{\rho 1}')}{\iiint \beta}$$

where $$\beta = \frac{c_v}{R}\frac{\rho_0}{\pi_0} = P_{00}\frac{c_v}{R^2}\frac{\pi_0^{c_v/R-1}}{\theta_{\rho 0}}$$