Supplemental Material: PhaseShift Experimental Toolkit
Protocols for Boundary-Indexed Observation

INTRODUCTION

This toolkit provides operational guidelines for experimentalists to verify the "Marginal Universality" hypothesis proposed in the main text. It serves as a bridge between the theoretical RG framework and laboratory implementation.

MODULE A: SYSTEM CLASSIFICATION

Before performing recursive refinement, characterize the system's spectral class.

1. Measure the noise power spectral density (PSD), $S_{xx}(\omega)$.

2. Integrate to find the effective volume vs. resolution (bandwidth): $V() \propto \int^{\Lambda} S_{xx}(\omega) d\omega$.

3. **Diagnostic:**

   - If $V() \sim^{-\alpha}$ ($\alpha > 0$): **Relevant Class**. Cost will diverge polynomially.
   - If $V() \sim \ln(1/)$: **Marginal Class**. Proceed to Module B.
   - If $V() \sim$ const: **Irrelevant Class**. Cost saturates.

MODULE B: BOUNDARY-INDEXED PROTOCOL

To detect logarithmic scaling, the recursion must be indexed by information gain, not wall-clock time. **Protocol Rule:** Maintain a constant increment of Fisher Information per step k:

$$\Delta \mathcal{I}_k = \mathcal{I}_k - \mathcal{I}_{k-1} = \text{const.} \tag{1}$$

Implementation: Adjust the measurement duration τ_k and power P_k such that their product scales as:

$$P_k \tau_k \propto \frac{1}{k}. \tag{2}$$

This implicitly defines the "Boundary Index" n used in the main theorem.

MODULE C: DATA ANALYSIS FITTING

1. Marginal Window Detection Do not fit the entire dataset. Identify the "Marginal Window" where the system is dominated by the singularity but not yet saturated by noise floor. Criterion: The local slope on a log-log plot of Work vs. Step (n) should be close to zero:

$$\frac{d \ln W_n}{d \ln n} \approx 0 \implies W_n \propto \ln n. \tag{3}$$

2. Slope Extraction Extract the coefficient B_{obs} from the linear fit of W_n vs $\ln n$ in the Marginal Window. Compare with theory:

$$\text{Ratio } R = \frac{B_{\text{obs}}}{B_{\text{theory}}}. \tag{4}$$

If $R \approx 1$ (within order of magnitude of unity), the theory is supported. Large deviations suggest unmodelled loss channels.

MODULE D: INTERPRETATION SAFETY

Warning: The observation of logarithmic cost scaling indicates that the system is relaxing toward a marginal RG fixed point (a topological defect).

- **Do NOT** interpret this as the system possessing "consciousness" or "infinite depth."
- **Do NOT** extrapolate the log scaling beyond the breakdown resolution $_{\text{break}}$ (e.g., Planck scale or atomic lattice constant).

This result is a thermodynamic signature of information geometry, strictly valid within the effective field theory limit.