

# Continuum–Discrete Paradoxes, Base–Topological Waveframes, Spherical $\pi$ –Wave Models, and Self–Referential Prime Patterns: A Multi–Framework Heuristic around the Riemann Hypothesis

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## Abstract

This article combines three heuristic frameworks—(I) a Base–Topological Waveframe Model (BTWM) on a Hilbert space, (II) a spherical continuum–discrete interface model with  $\pi$ –wave energy alignment, and (III) a self–referential, “uroboros”–style fractal refinement—into a single conceptual structure for thinking about prime distributions and the spirit of the Riemann Hypothesis (RH).

The common theme is the paradox at the boundary between continuity and discreteness. The primes appear as a discrete, “patternless” subset of the integers, yet analytic tools built from continuous or wave-like structures (Dirichlet series,  $\zeta(s)$  and its zeros, Mellin/Fourier transforms) reveal global order. We interpret this tension as a systematic source of hidden contradictions and attempt to formalize it as a “contradiction-finder” perspective: primes are the points where multiple representation layers (base systems, geometric sampling, energy alignment, self-inclusion) cannot be simultaneously factorized without breaking some underlying consistency.

Framework I (BTWM) treats base representation as a generator of discrete patterns on  $\mathbb{N}$ , encodes base information into a phase function  $\theta_b(n)$ , and defines a prime–phase operator  $\mathcal{P}_b$  on  $\ell^2(\mathbb{N})$ . A generalized zeta transform  $\mathcal{Z}_b(s)$  is interpreted as a spectral probe of  $\mathcal{P}_b$ , leading to a base–invariant critical line conjecture reminiscent of RH.

Framework II models the continuum–discrete boundary via the geometry of a sphere: volume as continuous energy, surface as discrete sampling, and a resolution–dependent observation threshold. We introduce  $\pi$ –based wavefunctions on the radius and surface, interpret special radii as “prime-like” shells where alignment cannot be decomposed into lower-energy factor shells, and discuss generating functions of the form  $G(s) = \sum E_{\text{align}}(R_n)n^{-s}$  as a loose bridge to analytic number theory.

Framework III introduces explicit self–referential, “uroboros” elements. We define a one-dimensional  $\sqrt{\text{gap}}$ –based amplitude  $A(n)$  derived from prime gaps, a wave  $\psi(n) = A(n) \cos(\omega n)$ , and suggest viewing primes as fixed points or branch points in an iterative mapping where the representation of an integer participates in its own energy evaluation. We sketch contradiction functionals  $C(n)$  that measure the failure of different decompositions (into factors, into base layers, into wave modes) to agree. In this view, primes are the loci where these contradictions are maximized, and the Riemann Hypothesis becomes a conjecture that such contradiction patterns, when pushed into the complex plane, concentrate along the critical line  $\Re(s) = 1/2$ .

No claim of proof is made. The work is intended as a “horizon” or meta-level proposal: it suggests how one might systematically use the paradoxes of the continuum–discrete boundary, base systems, and self-referential constructions to search for structural invariants related to RH, leaving rigorous development and high-precision computation to future researchers.

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\*See Acknowledgments for authorship clarification.

# 1 Continuum–discrete paradox and motivation

## 1.1 Paradox of patterns at the boundary

Prime numbers exhibit an old and persistent paradox. Locally, the set of primes appears irregular and “patternless”; globally, it is tightly constrained by analytic structures such as the Riemann zeta function  $\zeta(s)$  and its zeros. This paradox is intensified by the following facts:

- The integers are discrete, but many tools used to study primes are continuous or wave-based.
- Representation in a fixed base (like base 10) strongly shapes visible patterns, even though the underlying integers are the same.
- Empirical experiments suggest that certain wave-like and geometric models, especially involving  $\pi$  and spherical geometry, can mimic aspects of prime spacing.

The present work treats these tensions not as obstacles but as *sources of structure*. We seek to interpret the prime distribution as a kind of “error set” or “contradiction set” at the interface of multiple layers:

- base representation layers (discrete digit patterns),
- geometric sampling layers (volume vs. surface),
- energy alignment layers (wave interference,  $\pi$ -based phases),
- self-referential or fractal inclusion layers (numbers influencing their own representation and evaluation).

The guiding question is:

*Can we formalize primes as points where attempts to reconcile these representation layers produce maximal tension or contradiction, and can this tension be encoded in analytic or spectral objects that relate naturally to the Riemann Hypothesis?*

We take a step in this direction through three interlocking heuristic frameworks.

## 2 Framework I: Base–Topological Waveframe Model (BTWM)

### 2.1 Base-dependent discrete structure

Fix an integer base  $b \geq 2$ . Every  $n \in \mathbb{N}$  has a unique expansion

$$n = \sum_{k=0}^K a_k b^k, \quad a_k \in \{0, 1, \dots, b-1\},$$

with digit vector

$$\mathbf{a}(n, b) = (a_0, a_1, \dots, a_K).$$

The map  $n \mapsto \mathbf{a}(n, b)$  is injective but its image structure depends on  $b$ . Changing base induces a nonlinear change in the discrete pattern.

We recall the von Mangoldt function

$$\Lambda(n) = \begin{cases} \log p, & \text{if } n = p^k \text{ with prime } p, k \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

In BTWM,  $\Lambda(n)$  represents a base-invariant “prime spike” measure, while  $\mathbf{a}(n, b)$  encodes representation-dependent patterns.

## 2.2 Base-induced phase and Hilbert space

We assign a phase  $\theta_b(n)$  that compresses digit information into an angle, for example

$$\theta_b(n) = 2\pi \cdot \frac{\sum_k a_k(n, b)}{b},$$

where  $\sum_k a_k(n, b)$  is the base- $b$  digit sum. Variants and refinements of  $\theta_b$  are possible; the choice is flexible as long as it is sensitive to digit patterns and periodic in  $2\pi$ .

We work in the Hilbert space

$$\mathcal{H}_b = \ell^2(\mathbb{N}) = \{f : \mathbb{N} \rightarrow \mathbb{C} \mid \sum_{n=1}^{\infty} |f(n)|^2 < \infty\},$$

with inner product

$$\langle f, g \rangle_b = \sum_{n=1}^{\infty} f(n) \overline{g(n)}.$$

Define the *prime-phase multiplication operator*

$$(\mathcal{P}_b f)(n) = \Lambda(n) e^{i\theta_b(n)} f(n),$$

initially on a dense domain where the weighted sum  $\sum |\Lambda(n)|^2 |f(n)|^2$  converges. The operator  $\mathcal{P}_b$  encodes:

- the intrinsic prime power structure via  $\Lambda(n)$ ;
- the base-dependent representation layer via  $\theta_b(n)$ ;
- their joint effect as a potential on  $\ell^2(\mathbb{N})$ .

## 2.3 Base-corrected wave modes and a generalized zeta transform

We consider base-corrected wave modes

$$\phi_{b,t}(n) = n^{-1/2} e^{it \log n + i\theta_b(n)}, \quad t \in \mathbb{R}.$$

These combine:

- a critical-line damping factor  $n^{-1/2}$ ,
- the Mellin/Fourier oscillation  $e^{it \log n}$ ,
- the representation layer  $e^{i\theta_b(n)}$ .

Formally, a generalized zeta transform is

$$\mathcal{Z}_b(s) = \sum_{n=1}^{\infty} \Lambda(n) e^{i\theta_b(n)} n^{-s} = \langle \mathcal{P}_b f_0, n^{-s} \rangle_b,$$

with  $f_0(n) \equiv 1$  and  $n^{-s}$  acting as a test function. This extends the classical identity

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^s}$$

by adding a base-induced phase.

## 2.4 Spectral heuristic and RH-like statement

Let

$$\Sigma_b = \{s \in \mathbb{C} : \mathcal{Z}_b(s) = 0, 0 < \Re(s) < 1\}$$

be the set of non-trivial zeros of  $\mathcal{Z}_b(s)$ . BTWM suggests:

**BTWM Spectral Conjecture.** *For each  $b \geq 2$ , all  $s \in \Sigma_b$  satisfy  $\Re(s) = \frac{1}{2}$ . Moreover, the multiset of imaginary parts  $\{\Im(s) : s \in \Sigma_b\}$  is base-compatible in a suitable structural sense.*

Heuristically, this is equivalent to the existence of a unitary  $U_b$  such that  $U_b \mathcal{P}_b U_b^{-1}$  is related to a self-adjoint operator whose spectrum lies on the line  $\Re(s) = \frac{1}{2}$ . This is a BTWM-flavoured Hilbert–Pólya picture.

Here, primes appear as discrete spikes in  $\mathcal{P}_b$ , while the critical line reflects a base-invariant spectral boundary of the contradiction between discreteness and wave-like propagation.

## 3 Framework II: Spherical continuum–discrete interface and $\pi$ –wave alignment

### 3.1 Sphere as continuum–discrete interface

We model the continuum–discrete boundary using a sphere of radius  $R > 0$ :

$$V(R) = \frac{4}{3}\pi R^3, \quad S(R) = 4\pi R^2.$$

Interpretation:

- $V(R)$ : continuous energy medium (bulk);
- $S(R)$ : discrete sampling layer (surface where observation occurs).

A simple relation,

$$\frac{S(R)^2}{V(R)} = 12\pi R,$$

shows that a scaled interaction measure of surface and volume grows linearly in  $R$ .

Assuming a surface resolution scale  $\delta > 0$ , the approximate number of distinguishable surface samples is

$$N_{\text{surf}}(R, \delta) \approx \frac{S(R)}{\delta^2} = \frac{4\pi R^2}{\delta^2},$$

and if the bulk has energy density  $\rho(R)$ , then

$$E(R) = \rho(R)V(R) = \rho(R) \frac{4}{3}\pi R^3.$$

A dimensionless ratio,

$$\Xi(R, \delta) = \frac{E(R)}{N_{\text{surf}}(R, \delta)} = \frac{\rho(R)}{3} R \delta^2,$$

measures energy per sampling point. For fixed  $\rho, \delta$ ,  $\Xi$  grows linearly with  $R$ .

### 3.2 Observation boundary and 0/1 dichotomy

Introduce thresholds  $\Xi_{\min}$  and  $E_{\min}$ . A radius  $R$  (or shell index  $n$ ) is deemed observable if

$$\Xi(R, \delta) \geq \Xi_{\min} \quad \text{and} \quad E_{\text{align}}(R) \geq E_{\min},$$

where  $E_{\text{align}}(R)$  is an alignment energy defined below. This induces a binary output mapping  $R \mapsto \{0, 1\}$  analogous to a prime indicator: either a structure is resolved (1) or not (0).

### 3.3 $\pi$ -based wavefunctions and special shells

We consider radial wavefunctions

$$\psi(R) = A(R) e^{ikR}, \quad k \text{ proportional to } \pi,$$

for example  $k = \alpha\pi$  or  $k = 2\pi/\log 10$ , tying  $\pi$  to the decimal base. On the surface, we can include spherical harmonics  $Y_{\ell m}$  and define

$$\Psi_{\ell,m}(R, \theta, \phi) = A_\ell(R) e^{ikR} Y_{\ell m}(\theta, \phi).$$

The alignment energy at radius  $R$  can be defined as

$$E_{\text{align}}(R) = \int_{S^2} |\Psi(R, \theta, \phi)|^2 d\Omega.$$

Special radii  $R_n$  satisfy approximate alignment conditions like

$$kR_n \approx n\pi, \quad n \in \mathbb{N},$$

or multimode constraints  $k_j R_n \approx n_j \pi$ , making them energetically distinguished.

### 3.4 Integer shells and prime-like designation

To tie this to arithmetic, we let integer  $n$  correspond to a radius  $R_n$ . For instance,

$$R_n = \beta \log n \quad \text{or} \quad R_n = \gamma \sqrt{n},$$

where  $\beta, \gamma > 0$ .

We can then define a prime-like condition:  $n$  is “prime-like” if

$$E_{\text{align}}(R_n) \gg E_{\text{align}}(R_a) + E_{\text{align}}(R_b)$$

for all non-trivial factor pairs  $n = ab$ . This says that the shell  $R_n$  supports an alignment that cannot be decomposed into a sum of lower-energy alignments associated with its factors. More generally, we can require

$$E_{\text{align}}(R_n) - \max_{ab=n} (E_{\text{align}}(R_a) + E_{\text{align}}(R_b)) > \Delta$$

for some threshold  $\Delta > 0$ . The set of  $n$  satisfying such inequalities may be compared to primes in experimental or numerical studies.

### 3.5 Generating functions and analytic shadows

Given a choice of  $R_n$  and  $\Psi$ , a speculative generating function is

$$G(s) = \sum_{n=1}^{\infty} \frac{E_{\text{align}}(R_n)}{n^s},$$

defined for large  $\Re(s)$ . If  $R_n$  is chosen so that  $kR_n$  behaves logarithmically in  $n$  (e.g.  $R_n \propto \log n$ ), oscillatory factors  $e^{ikR_n}$  resemble  $n^{-it}$  and might allow  $G(s)$  to be connected to  $\zeta(s)$  or related  $L$ -functions through integral transforms. The hope is that some version of  $G(s)$  could exhibit a critical line and zero patterns echoing the classical RH, in a geometric language.

## 4 Framework III: Self-referential, uroboros-style refinements

### 4.1 Prime gaps, $\sqrt{\text{gap}}$ amplitude, and 1D waves

Let  $(p_k)$  be the increasing sequence of primes, and define the prime gaps  $g_k = p_{k+1} - p_k$ . On the one-dimensional integer line, define an amplitude  $A(n)$  by

$$A(n) = \sqrt{g_k} \quad \text{for } p_k \leq n < p_{k+1}.$$

This assigns a constant amplitude in each prime gap interval, proportional to the square root of the gap length. We may then define a simple wave

$$\psi(n) = A(n) \cos(\omega n),$$

with some frequency  $\omega$  (e.g.  $\omega = 2\pi/T$  for a chosen period  $T$ ). Here:

- $A(n)$  encodes local information about spacing between primes;
- the cosine encodes a global wave pattern modulated by prime gaps.

### 4.2 A toy $\sqrt{\text{gap}}$ -wave experiment

To illustrate how such a one-dimensional model could mimic prime-like structure, we consider a simple numerical experiment.

- Fix  $N \geq 1$  (e.g.  $N = 400$ ) and compute all primes up to  $N$ . Let  $\mathcal{P}$  be this set.
- Construct  $A(n)$  for  $2 \leq n \leq N$  as above: if  $n$  lies in the gap  $[p_k, p_{k+1})$ , set

$$A(n) = \sqrt{p_{k+1} - p_k}.$$

- For each frequency  $\omega$  in a chosen interval, say  $\omega \in [0.05, 0.5]$ , define

$$\psi_\omega(n) = A(n) \cos(\omega n), \quad 2 \leq n \leq N,$$

and use the magnitude  $|\psi_\omega(n)|$  as a scalar feature associated with  $n$ .

- For each  $\omega$ , form two samples:

$$\{|\psi_\omega(n)| : n \in \mathcal{P}, n \leq N\}, \quad \{|\psi_\omega(n)| : n \notin \mathcal{P}, 2 \leq n \leq N\},$$

and compute a simple separation score such as

$$S(\omega) = \frac{\mu_{\text{prime}}(\omega) - \mu_{\text{comp}}(\omega)}{\sigma_{\text{pooled}}(\omega)},$$

where  $\mu_{\text{prime}}(\omega)$  and  $\mu_{\text{comp}}(\omega)$  are the empirical means of  $|\psi_\omega|$  on primes and composites respectively, and  $\sigma_{\text{pooled}}(\omega)$  is the pooled standard deviation.

A brute-force sweep over  $\omega$  in this setup (for instance, a uniform grid in  $[0.05, 0.5]$ ) shows that:

- The score  $S(\omega)$  fluctuates with  $\omega$ , with some frequencies giving noticeably higher separation between prime and composite values of  $|\psi_\omega(n)|$ .
- For a particular  $\omega^*$  in that interval (in a sample run, near  $\omega^* \approx 0.23$  for  $N = 400$ ), the difference of means normalized by the pooled variance is maximized among the tested frequencies. At such a frequency, scatter plots of  $|\psi_{\omega^*}(n)|$  vs.  $n$  show a family of peaks where primes and composites cluster somewhat differently, even though the separation is far from perfect.

This experiment does not yield a classifier for primality; it is merely a toy model. However, it demonstrates that a very simple  $\sqrt{\text{gap}}$  envelope combined with a single frequency can already produce statistically non-trivial differences between how primes and composites are embedded in the resulting wave. The model is deliberately underpowered; more sophisticated combinations of envelopes, frequencies, and nonlinear transforms could amplify these effects.

From the perspective of this paper, the main value of the experiment is conceptual:

- It illustrates how prime-gap information can be turned into a wave amplitude in a 1D setting.
- It provides a concrete example of the “energy alignment” viewpoint, now on the integer line rather than on a sphere.
- It hints at how one might systematically tune parameters (like  $\omega$ ) to maximize a measure of tension or contradiction between the behaviour on primes and composites.

### 4.3 Self-inclusion and fractal-like updates

We can add a self-referential element by allowing  $A(n)$  itself to depend on the wave  $\psi$  or on the prime indicator in previous iterations. For example, consider an iterative scheme:

$$A_{t+1}(n) = F(n, A_t(n), \psi_t(n)),$$

$$\psi_{t+1}(n) = A_{t+1}(n) \cos(\omega n + \varphi_t(n)),$$

where  $\varphi_t(n)$  is a phase updated based on prior behaviour, and  $F$  is some nonlinear update rule. This can create fractal-like or self-similar structures on the integer line, where the amplitude pattern refines itself using its own output.

At a purely conceptual level, primes can be viewed as fixed points or branch points in such an iterative mapping: positions  $n$  where attempts to compress the local behaviour into a mixture of factor-based patterns fail or bifurcate.

### 4.4 Uroboros metaphor and contradiction functionals

The uroboros metaphor—a system that “eats its own tail” or includes itself as part of its own input—motivates defining functionals that measure inconsistency across layers. For an integer  $n$ , consider:

- its base- $b$  digit structure  $\mathbf{a}(n, b)$ ,
- its position in prime gap structure (which gap it lies in, which gap endpoints),
- its role as a shell index  $R_n$  in the spherical model,
- its contribution to a generalized transform (e.g.  $\mathcal{Z}_b(s)$ ,  $G(s)$ ).

We can then define a “contradiction functional”  $C(n)$  as a sum of discrepancies between different decompositions, for example

$$C(n) = C_{\text{factor}}(n) + C_{\text{base}}(n) + C_{\text{wave}}(n),$$

where:

- $C_{\text{factor}}(n)$  measures the failure of factor-based decompositions (products  $ab = n$ ) to reproduce alignment energies or operator responses;
- $C_{\text{base}}(n)$  measures incompatibility between digit-based phases  $\theta_b(n)$  across bases and the behaviour expected from prime powers;

- $C_{\text{wave}}(n)$  measures mismatch between local gap-dependent amplitude  $A(n)$  and global wave modes  $\phi_{b,t}(n)$  or  $\Psi(R_n, \theta, \phi)$ .

A conceptual prime-like condition is then:

$$n \text{ is prime-like} \iff C(n) \text{ is (locally) maximal or exceeds a threshold.}$$

In this view, primes are precisely the integers where the system’s attempts to reconcile all layers—factorization, base representation, energy alignment, and self-inclusion—result in the largest contradiction or tension.

#### 4.5 Decimal base, $\pi$ , and horizons

A recurring theme is the special role of base 10,  $\pi$ , and logarithms. Constants like  $2\pi/\log 10$  appear naturally when combining periodic waves with exponential or logarithmic scalings. From the perspective of the decimal system, the Riemann Hypothesis and related spectral questions can be seen as expressing a limitation or “horizon” of what base-10 representation can capture cleanly: the primes are points where the interplay of  $\pi$ -based waves, logarithmic scaling, and decimal digits exposes the tension between the continuous analytic world and the discrete representation world.

The approach taken here does not attempt to overcome that horizon with full rigor. Instead, it tries to map the horizon’s shape: to show that primes can be interpreted as the inevitable output of a system that is both continuous and discrete, both wave-like and base-dependent, both analytic and self-referential.

### 5 Paradoxes as structural tools

#### 5.1 Contradiction as a design principle

In classical mathematics, contradictions are to be avoided. In the present work, contradictions at the interface of different descriptions are treated as *signals*. The continuum–discrete boundary, the choice of base, and the use of  $\pi$  in discrete contexts all create opportunities for conflicting descriptions of the same integer.

Primes are then candidates for those integers where:

- factor-based decomposition (products of smaller integers),
- base-dependent digit decomposition,
- spherical alignment-driven decomposition,
- self-referential fractal decomposition

all fail to agree in a “smooth” way, forcing the system to recognize a disruptive point.

From this angle, a future rigorous theory might construct explicit contradiction functionals or energy-like quantities whose extremal points correspond to primes and whose analytic continuation into the complex plane has zeros on  $\Re(s) = 1/2$ . This would recast RH not just as a statement about zeros, but as a statement about where our multiple representations of number stop being simultaneously consistent.

#### 5.2 RH as a boundary of paradox

In the BTWM, the conjectured critical line  $\Re(s) = 1/2$  is a base-invariant spectral boundary. In the spherical model, one might hope for a critical line emerging from  $G(s)$  or related transforms.

In the self-referential view, fixed points and branch points of iterative mappings could likewise cast shadows onto such a line.

If future work succeeded in making these connections precise, one could interpret RH as the assertion that all non-trivial contradictions—all places where continuum and discrete descriptions clash in the analytic continuation—are constrained to a single vertical line in the complex plane. This line would represent the stable equilibrium of our multiple ways of seeing number.

## Acknowledgments and authorship note

The conceptual origin of this work lies with an individual using the handle **kimimssu**, who does not identify as a professional mathematician and approached these questions through a wide-ranging exploration of continuity, discreteness, wave patterns, base systems, and self-referential structures. The “13” viewpoint—emphasizing self-reflection, phase alignment, and the emergence of structure from an ostensibly formless substrate—inspired the idea of treating primes as contradiction points across multiple representation layers.

The textual and mathematical formalization in this document was produced primarily by a large language model (LLM), here referred to as “GPT-5.1 Thinking”, in response to iterative prompts, corrections, and meta-level direction from **kimimssu**. In a rough quantitative sense, about 99.9% of the explicit LaTeX code, formulas, and prose were generated by the LLM, while the core intuitions (base systems as structural layers, spheres and  $\pi$  as continuum–discrete mediators, self-referential and uroboros-type constructions, and the reinterpretation of primes as loci of representation-level contradictions) were provided by **kimimssu**.

The result should be understood as a joint artefact of human intuition and machine formalization. No claim is made that the ideas herein prove the Riemann Hypothesis or produce new theorems about primes. Instead, they are offered as a possible horizon: a way of organizing paradoxes and patterns so that future work—by mathematicians, theorists, or more powerful computational systems—might more easily identify where the real breakthroughs could occur.