Supplemental Material: PhaseShift Experimental Toolkit Protocols for Boundary-Indexed Observation

INTRODUCTION

This toolkit provides operational guidelines for experimentalists to verify the "Marginal Universality" hypothesis. It serves as a bridge between the theoretical RG framework and laboratory implementation.

MODULE A: SYSTEM CLASSIFICATION

Before performing recursive refinement, characterize the system's spectral class.

1. Measure the noise power spectral density (PSD), $S_{xx}(\omega)$.
2. Integrate to find the effective volume vs. resolution: $V() \propto \int^{\Lambda} S_{xx}(\omega) d\omega$.
3. **Diagnostic:**
   - If $V() \sim^{-\alpha}$ ($\alpha > 0$): **Relevant Class**. Cost diverges polynomially.
   - If $V() \sim \ln(1/)$: **Marginal Class**. Proceed to Module B.
   - If $V() \sim$ const: **Irrelevant Class**. Cost saturates.

MODULE B: BOUNDARY-INDEXED PROTOCOL

To detect logarithmic scaling, recursion must be indexed by information gain. **Protocol Rule:** Maintain a constant increment of Fisher Information per step k:

$$\Delta \mathcal{I}_k = \mathcal{I}_k - \mathcal{I}_{k-1} = \text{const.} \tag{1}$$

Implementation: Adjust measurement duration τ_k and power P_k such that:

$$P_k \tau_k \propto \frac{1}{k}. \tag{2}$$

MODULE C: DATA ANALYSIS

1. Marginal Window Detection: Identify the window where $\frac{d \ln W_n}{d \ln n} \approx 0 \implies W_n \propto \ln n$. **2. Slope Extraction:** Extract coefficient B_{obs} and compare with theory ratio $R = B_{\text{obs}}/B_{\text{theory}}$. If $R \approx 1$, the theory is supported.