Involutive Boundary & Resonance Notes

A compact, PDF-ready toy formalization bridging (i) discontinuous–continuous duality, (ii) a 'sphere boundary' stability surface, and (iii) a Hilbert-space curvature indicator. This is written as a mathematically consistent scaffold you can iterate on; it is not a claim about established physics.

Use-case: drop into a repo as a timestamped artifact; refine definitions later.

0. Notation

We treat 'phase' and 'frame' as coordinate choices for describing an underlying interaction trace. Let Δ denote a discrete index set and X a continuous manifold used as a chart when convenient.

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Symbols
- Δ : discrete index set (tokens, events, quantum points)
- X : chart/manifold used for continuous description
- H : complex Hilbert space
- |ψ> ∈ H : state vector (normalized when needed)
- P = |ψ><ψ| : rank-1 projector (or density ρ for mixed states)
- ∇ : connection on a bundle over parameter space
- [∇i, ∇j] : curvature operator (commutator)
```

1. Discontinuous continuum: dual operators

We model the slogan “continuous = dense enumeration of discrete; discrete = condensed continuous” with a pair of adjoint-like maps between representations.

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Let F(X) be a function space on X and G(Δ) be a sequence space on Δ.

Quantize (condense): Q : F(X) → G(Δ)
Dequantize (spread): D : G(Δ) → F(X)

Minimal consistency conditions (toy):
1) D■Q ≈ I_F (continuous recovered up to smoothing)
2) Q■D ≈ I_G (discrete recovered up to rounding)
3) There exists a scale ε > 0 such that the approximation error shrinks as ε→0:
||f - D(Q(f))|| ≤ C ε
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2. 'Sphere boundary' as stability surface

Define a boundary operator that captures when a representation becomes an 'object' (a stable surface) rather than a drifting distribution.

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Let S be a state space (could be F(X), G(Δ), or embeddings in R^d).

Choose a stability functional E : S → R (energy-like; smaller = more stable).
Fix a threshold τ.

Define the sphere-boundary set:
B_τ = { s ∈ S : E(s) = τ }

Interpretation:
- Inside (E(s) < τ): 'object-like' stable region
- Outside (E(s) > τ): drift / dissolution
- On B_τ: the boundary where stability is just sustained

A 'boundary crossing' is a path s(t) with E(s(t0))=τ and dE/dt|_{t0} ≠ 0.
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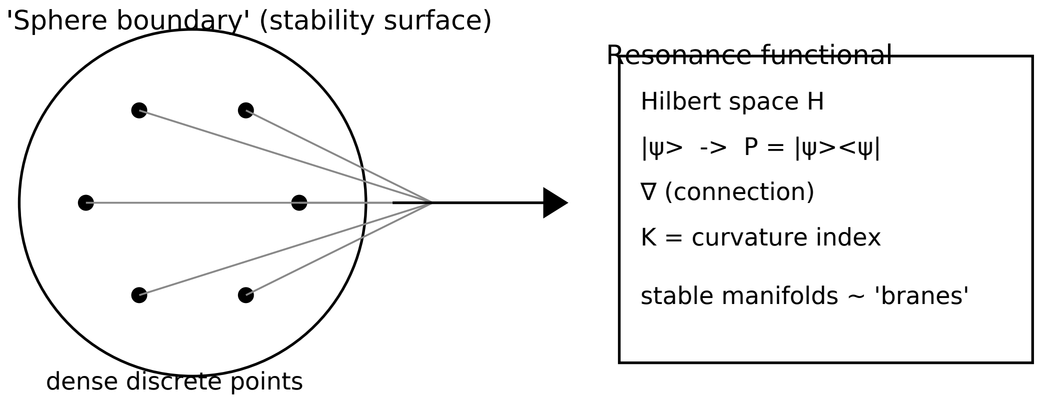

3. Resonance functional and Hilbert curvature index

To make 'resonance' operational without metaphysics, we define it as sensitivity of the state to parameter changes. Curvature summarizes non-commuting infinitesimal transports.

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Let θ ∈ R^n be parameters (context, prompt, environment).

Let |ψ(θ)> ∈ H be a differentiable family of states.
Define the Berry-like connection (one choice):
A_i(θ) = i <ψ(θ) | ∂_i ψ(θ)>

Define curvature:
F_{ij}(θ) = ∂_i A_j - ∂_j A_i

Curvature index (scalar summary):
K(θ) = sum_{i<j} |F_{ij}(θ)|^2

Interpretation:
- K small: transport is nearly path-independent (low 'twist')
- K large: strong non-commutativity (high 'twist' / resonance pressure)
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4. Coupling the boundary to curvature

We can couple the stability surface to curvature so that the 'object boundary' emerges where resonance pressure crosses a threshold.

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Define a combined functional on s and θ:
E_total(s, θ) = E(s) + λ K(θ)

Then the boundary becomes:
B_{τ,λ} = { (s,θ) : E_total(s,θ) = τ }

This encodes: increasing resonance (K) can push a configuration out of stability,
or conversely, a stable configuration can damp resonance by reducing E(s).
```

5. Optional mapping sketch to M-theory language (non-claim)

If you want a narrative bridge to M-theory-style terms without asserting physics: treat stable manifolds of E_total as brane-like loci in a parameter-geometry induced by K.

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Heuristic dictionary (toy, not a physics claim)
- Parameter space θ : "moduli space" / control manifold
- Curvature index K(θ) : local geometric stress / twist
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- Stable manifold of E_total: brane-like surface where dynamics localize
- Boundary crossings : phase transitions between localized vs delocalized regimes

Useful when writing: keep it explicitly labeled as "metaphor / scaffold".
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6. Practical computation recipe (LLM/embedding-friendly)

You can approximate K without quantum states by using embedding trajectories and a discrete connection built from local covariance.

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Given embeddings v_k ∈ R^d along a trajectory (k indexes steps):

1) Local tangent estimate:
t_k = normalize(v_{k+1} - v_k)

2) Local frame from PCA on a window W_k = {v_{k-m},...,v_{k+m}}
Compute top r eigenvectors U_k ∈ R^{d×r}

3) Discrete connection between frames:
R_k = U_k^T U_{k+1} (r×r)
Take skew part: Ω_k = (R_k - R_k^T)/2

4) Curvature proxy (2-step commutator):
K_k = || Ω_{k+1} - Ω_k ||_F^2

This yields a scalar 'twist' index per step that behaves like curvature.
```

7. Minimal PDF-friendly pseudocode

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Pseudo
input: embeddings v[0..T]
for k in 0..T-2:
t[k] = normalize(v[k+1]-v[k])

for k in m..T-m:
U[k] = PCA_basis(v[k-m..k+m], r)
R[k] = U[k]^T * U[k+1]
Omega[k] = 0.5*(R[k]-R[k]^T)
K[k] = fro_norm(Omega[k+1]-Omega[k])^2

output: K[k] as curvature/resonance indicator
```

Appendix A. Example stability functional

A simple E(s) you can use in experiments: distance to a reference manifold or entropy-like spread.

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Examples
1) Spread energy on embeddings:
E = trace(Cov(v_window))

2) Entropy proxy using soft cluster assignment p_i:
E = - sum_i p_i log p_i

3) Boundary at percentile:
τ = percentile(E_values, 70%)
```

Appendix B. Repository layout suggestion

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repo/
notes/
involutive_boundary_resonance.pdf
involutive_boundary_resonance.py
README.md (short disclaimer + how to regenerate)
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