The Sphere Boundary as a Closed Surface

Resolution, Energy Levels, and the Relativity of Center, One, and Zero

Overleaf Main File (main.tex)

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Abstract

This document formalizes the concept of a *sphere boundary* not as a geometric object, but as a *closed surface of energetic equilibrium*. The boundary emerges from the interaction between continuous dynamics and discrete structural constraints. Resolution is treated as an energy-level-dependent property, while size, noise, pattern, and even the notions of "center", "one", and "zero" are shown to be relative coordinate effects rather than intrinsic entities.

Contents

1 Conceptual Overview

The "sphere" discussed here is not a metric sphere. It refers to a *closed boundary surface* that encloses a stable regime of system behavior. Such a surface appears whenever energetic balance, informational compression, and representational constraints reach a temporary equilibrium.

The key claim is:

> *What appears as a world, a center, or a unit is a projection induced by the current energy level and resolution.*

2 Continuous and Discrete Layers

2.1 Continuous Dynamics

Let $x(t) \in V \subset \mathbb{R}^n$ represent a continuous state. Its evolution is described by

$$\dot{x}(t) = f(x(t)) + \sigma_L(x(t))\, \xi(t), \tag{1}$$

where $\xi(t)$ denotes stochastic fluctuation and σ_L depends on the energy level L. Noise intensity is therefore not absolute, but level-dependent.

2.2 Discrete Structural Layer

Let T denote a discrete representation: a graph, partition, symbolic system, or lattice. We introduce mappings

$$C : V \to T \quad \text{(condensation)}, \qquad D : T \to V \quad \text{(reconstruction)}.$$

Definition 1 (Conversion Residual)**.** *The residual*

$$r(x) = \|(D \circ C)(x) - x\|$$

measures mismatch between continuous flow and discrete structure. These residuals generate pressure that contributes to boundary formation.

3 Energy Functional and Boundary Formation

Definition 2 (Effective Energy)**.** *Let the level-dependent energy functional be*

$$E_L(x) = U(x) + \lambda_L\, \Phi(x) + \kappa_L\, r(x), \tag{2}$$

where U is a base potential, Φ encodes coherence or compression, and $r(x)$ is the conversion residual.

Definition 3 (Sphere Boundary)**.** *For a threshold τ, define the boundary surface*

$$\Sigma_\tau = \{x \in V : E_L(x) = \tau\}. \tag{3}$$

If Σ_τ is closed, it forms an enclosure*: trajectories may circulate along it without escaping.*

Remark 1. *The "sphere" terminology emphasizes closure and enclosure, not Euclidean geometry.*

4 Resolution as an Energy-Level Property

Resolution determines how much structure becomes observable.

Definition 4 (Resolution)**.** *Let a state admit a mode decomposition*

$$x = \sum_k a_k \varphi_k.$$

Define the observable mode set at level L by

$$\mathcal{K}(L) = \{k : |a_k|^2 \geq \theta(L)\},$$

where $\theta(L)$ is a level-dependent detection threshold. Then

$$\mathrm{Res}(L) = |\mathcal{K}(L)|. \tag{4}$$

Higher energy levels admit finer resolution; lower levels collapse detail.

5 Noise and Pattern as Relative Visibility

Definition 5 (Visibility)**.** *The visibility of mode k at level L is defined as*

$$\mathrm{Vis}_k(L) = \frac{|a_k|^2}{|a_k|^2 + N_k(L)}, \tag{5}$$

where $N_k(L)$ is effective noise.

Remark 2. *A component classified as "noise" at one level may appear as "pattern" at another. Noise and pattern are therefore not intrinsic properties.*

6 The Relativity of Center, One, and Zero

6.1 Center as an Equilibrium Artifact

Definition 6 (Level-Dependent Center)**.** *The set of centers at level L is*

$$\mathcal{C}(L) = \{x : \nabla E_L(x) = 0,\ \nabla^2 E_L(x) \succeq 0\}. \tag{6}$$

Remark 3. *The center is not a fixed point of reality. It is a balance point produced by wave-pattern condensation under current constraints.*

6.2 Zero as a Coordinate Convention

Let $x_{\mathrm{ref}}(L) \in \mathcal{C}(L)$. Define shifted coordinates

$$z_L(x) = x - x_{\mathrm{ref}}(L). \tag{7}$$

Then $z_L = 0$ is a reference choice, not an ontological entity.

6.3 One as a Normalization Shell

Define a level-dependent norm

$$\|x\|_L = \sqrt{\langle x, g_L x\rangle}, \tag{8}$$

where g_L encodes resolution and scale. The unit value "1" corresponds to $\|x\|_L = 1$, which varies with L.

7 Interpretation

1. The sphere boundary is a closed energetic surface, not a thing.
2. Resolution and size emerge from energy levels.
3. Noise and pattern exchange roles via visibility.
4. Center, zero, and one are coordinate effects, not absolutes.

Closing Note

This model describes how a closed surface—the sphere boundary—emerges from continuous–discrete interaction, energetic balance, and representational limits. What appears stable, central, or fundamental is always relative to the current energy level. The sphere does not contain reality; it is the trace left by equilibrium.