Miranda House
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Comparative Study of Flat Λ-CDM and 3rd Order Cosmological Model

A Project Report Submitted for Summer Internship

At

DS KOTHARI RESEARCH CENTRE,
MIRANDA HOUSE COLLEGE

by

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Abstract

This paper presents a comprehensive study of the Flat-ΛCDM model and the 3rd order cosmological model, focusing on a detailed examination of key cosmological parameters such as the Hubble constant (H_0), the absolute magnitude of Type Ia supernovae (M_B), and other relevant parameters. Posterior distributions and correlations among these parameters are analyzed to enhance the understanding of the underlying cosmological models. Additionally, the evolution of $H(z)/(1+z)$ values as a function of redshift is explored, providing insights into the models' behavior over cosmic time.

Conducted as part of a Summer Internship at DS Kothari Research Centre, Miranda House College, this project report, titled "Comparative Study of Flat-ΛCDM and 3rd Order Cosmological Model," presents a detailed application of several key statistical techniques essential for cosmological data analysis. These methodologies include:

1. **Monte Carlo Simulations:** Used for generating random samples and understanding the probabilistic nature of cosmological phenomena.
2. **Bayes' Theorem:** Applied to update the probability estimates of hypotheses based on observational data.
3. **Chi-Square Fitting:** Utilized for assessing the goodness of fit between theoretical models and observed data.
4. **Likelihood Analysis:** Implemented to estimate the parameters of cosmological models by maximizing the likelihood function.
5. **Markov Chain Monte Carlo (MCMC):** Employed to sample from complex probability distributions and perform parameter estimation.

These statistical methods have been implemented using Python, allowing for efficient and effective data analysis. The application of these techniques to cosmological data has provided insights into the underlying physical processes and contributed to the broader understanding of the universe. This report not only demonstrates the practical utility of these methods in cosmology but also underscores their importance in the broader context of scientific research and data analysis.

The findings and methodologies detailed in this report are expected to aid future research in cosmology, offering robust tools and techniques for analyzing complex data sets.

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Introduction

1.1 Purpose of the Project

The primary purpose of this project is to explore, understand, and apply various statistical methods that are crucial for the analysis of cosmological data. Cosmology, the scientific study of the large scale properties of the universe as a whole, relies on vast and complex data sets obtained from observations such as the Cosmic Microwave Background (CMB), galaxy surveys, and other astronomical phenomena. Proper analysis of this data is essential for deriving meaningful scientific insights and advancing our understanding of the universe.

1.2 Key Objectives

1.2.1 Application of Statistical Technique:

- The project aims to delve into specific statistical methods including Monte Carlo simulations, Bayes' theorem, chi-square fitting, likelihood analysis, and Markov Chain Monte Carlo (MCMC). Each of these methods offers unique advantages in handling and interpreting cosmological data.

1.2.2 Improving Data Analysis Skills:

- Through the practical application of these statistical methods, the project seeks to enhance the data analysis skills of the participants. This includes learning how to preprocess data, apply statistical models, and interpret results within the context of cosmology.

1.2.3 Enhancing Understanding of Cosmological Phenomena:

- By analyzing real cosmological data, the project aims to provide insights into the underlying physical processes governing the universe. This includes studying the distribution of galaxies, understanding cosmic background radiation, and investigating dark matter and dark energy.

1.2.4 Implementation Using Python:

- By leveraging Python, a powerful programming language widely used in scientific computing, the project intends to implement these statistical techniques efficiently. Python's extensive libraries and tools facilitate sophisticated data analysis and visualization, making it an ideal choice for this study.

1.2.5 Contributing to Scientific Research:

- The methodologies and findings of this project are expected to contribute to the broader field of cosmological research. By providing robust tools and techniques for data analysis, the project supports ongoing and future research endeavors in cosmology.

1.3 Significance:

The significance of this project lies in its potential to bridge the gap between theoretical statistical methods and practical data analysis in cosmology. By doing so, it not only aids in the accurate interpretation of cosmological data but also fosters a deeper understanding of the universe's structure, origin, and evolution. The skills and knowledge gained through this project are valuable for students and researchers, preparing them for advanced studies and professional work in astrophysics and related fields.

Bayesian Inference

2.1 Introduction

Bayes' Theorem is a fundamental principle in probability theory and statistics that describes how to update the probability of a hypothesis as more evidence or information becomes available. It is named after the Reverend Thomas Bayes, who first provided an equation that allows new evidence to update beliefs.

In the context of cosmology, Bayes' Theorem is used to update the probability of cosmological models or parameters given new observational data. This theorem plays a crucial role in Bayesian inference, a statistical method that incorporates prior knowledge along with new evidence to make probabilistic statements about parameters.

2.2 Formula

Bayes' Theorem is mathematically expressed as:

$$P(H|E) = \frac{P(E|H).P(H)}{P(E)}$$

where:

- $P(H|E)$ is the posterior probability: the probability of the hypothesis H given the evidence E. ;
- $P(E|H)$ is the Likelihood: the probability of evidence H given the hypothesis E.
- P(H) is the prior probability: the initial probability of the hypothesis H before observing the evidence.
- P(E) is the marginal likelihood or evidence: the total probability of the evidence E.

2.3 Application in Cosmology

In cosmology, Bayes' Theorem can be applied to update the probability of different cosmological models or parameters based on new observational data, such as measurements of the Cosmic Microwave Background (CMB), supernovae data, or galaxy surveys.

2.4 Example

Suppose we have two competing cosmological models, $M_1 and M_2$,and we obtain new observational data D. We can use Bayes' Theorem to update our beliefs about the models based on this data.

1. Prior Probability $P(M_1) and P(M_2)$: These are our initial beliefs about the likelihood of each model before seeing the new data.
2. Likelihood $P(D|M_1) and P(D|M_2)$:These represent how probable the new data is under each model.
3. Marginal Likelihood $P(D)$:This is the total probability of observing the data, summed over all possible models.

Using Bayes' Theorem, we update our beliefs to find the posterior probabilities $P(M_1|D) and P(M_2|D)$, which tell us how likely each model is given the new data.

2.5 Practical Implementations

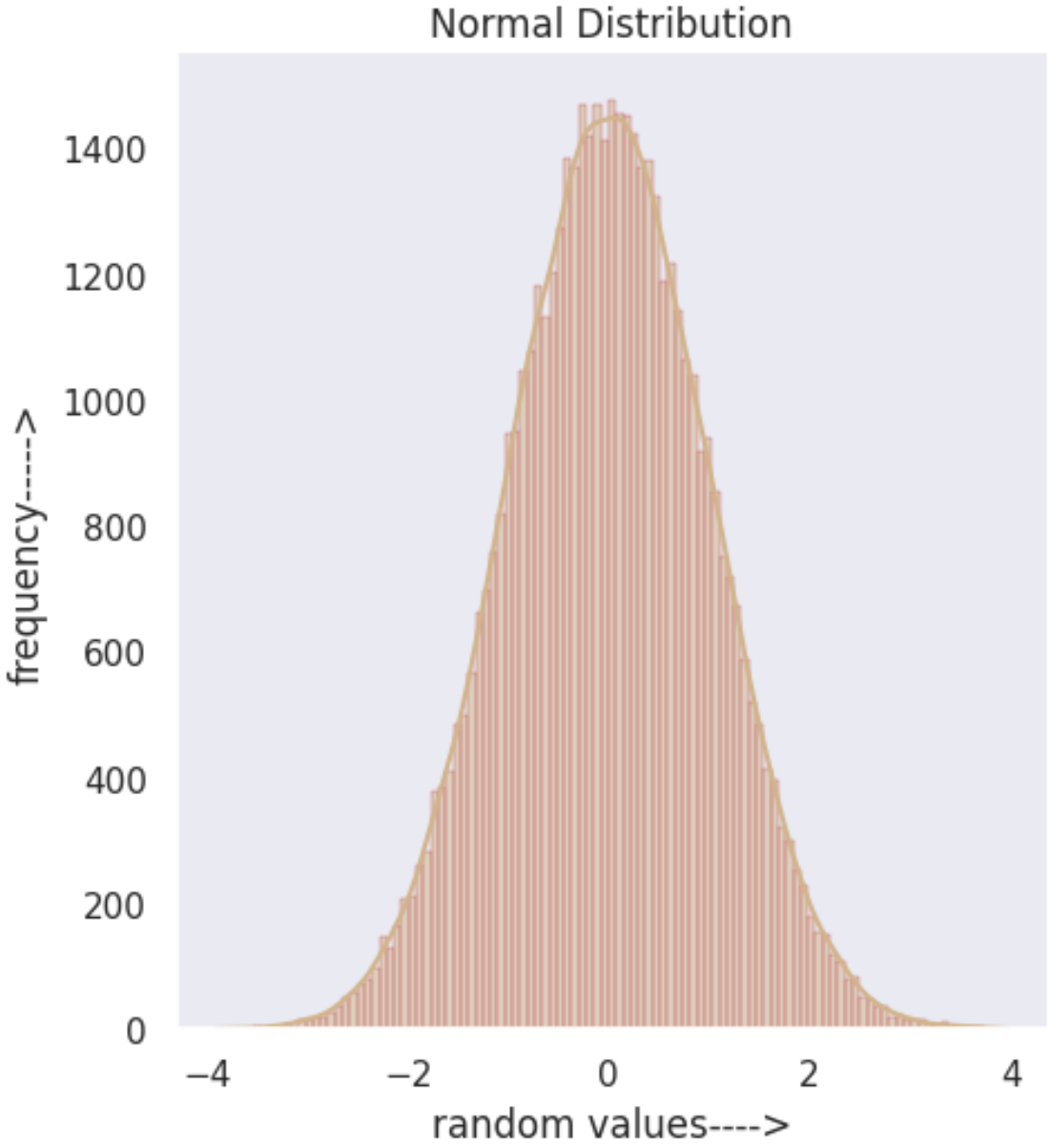
In practice, Bayesian inference in cosmology often involves computational techniques to estimate the posterior distributions, especially when dealing with complex models and large data sets. Markov Chain Monte Carlo (MCMC) methods are frequently used to sample from the posterior distribution.

2.6 Examples

Uniform and Normal PDF

2.7 Conclusion

Bayes' Theorem is a powerful tool in cosmology for updating our understanding of the universe as new data becomes available. By incorporating prior knowledge and new evidence, it allows for a more flexible and robust analysis of cosmological models and parameters.


Normal Distribution
1400
1200
1000
800
600
400
200
0
frequency----->
−4
−2
0
2
4
random values---->
Figure 2.1: Normal Random Distribution

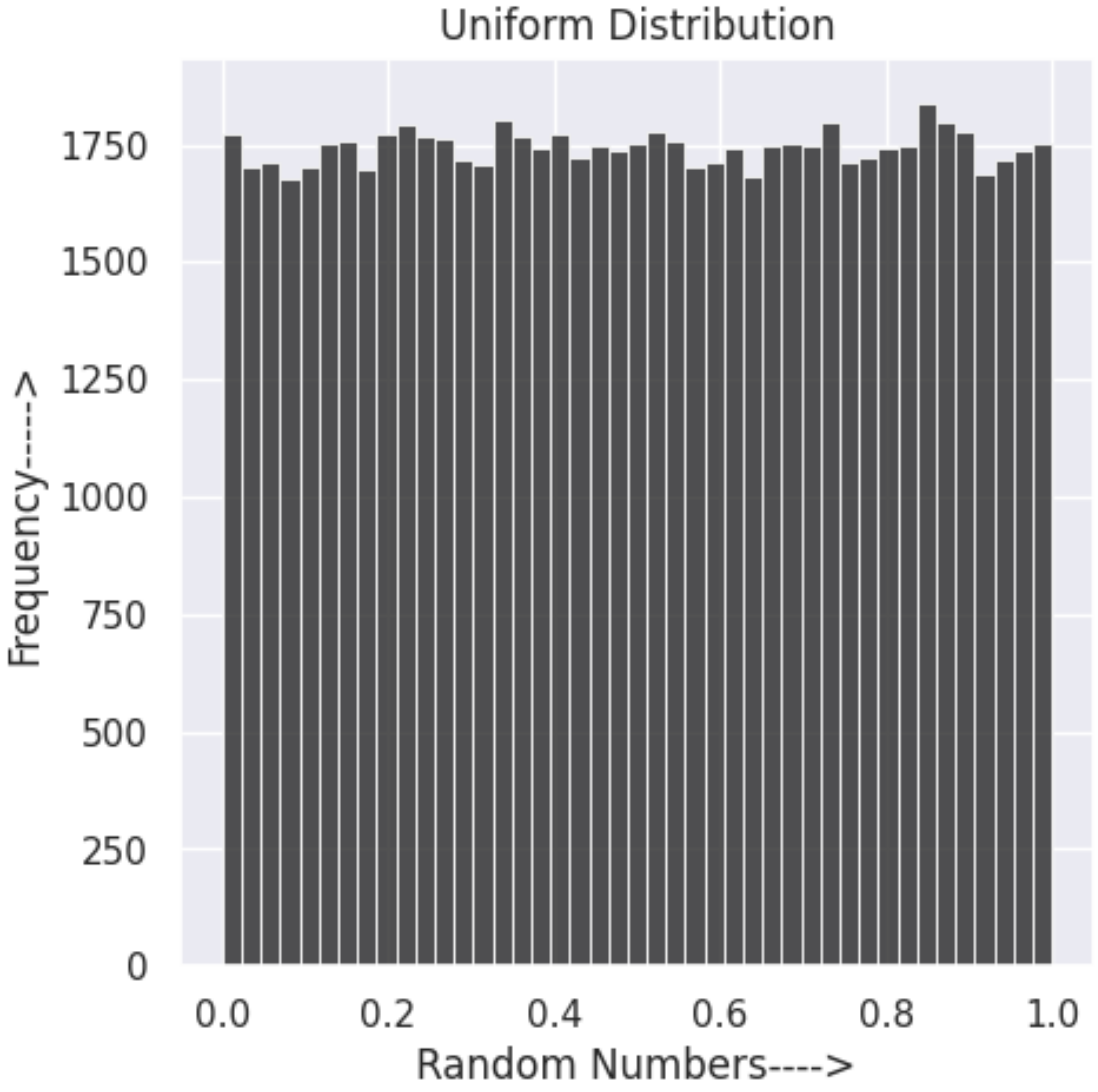


Figure 2.2: Uniform Random Distribution

Monte Carlo

Monte Carlo Simulation Technique

Monte Carlo simulation is a computational technique that uses random sampling to obtain numerical results. It is widely used in various fields, including physics, finance, engineering, and cosmology, to model and analyze complex systems that are analytically intractable.

Principle

The basic principle of Monte Carlo simulation is to use random sampling to explore the behavior of a system. By generating a large number of random samples and calculating the results for each sample, one can approximate the desired quantity with high accuracy.

Steps in Monte Carlo Simulation

1. **Define the Problem**: Identify the system or process to be simulated and the quantity to be estimated.

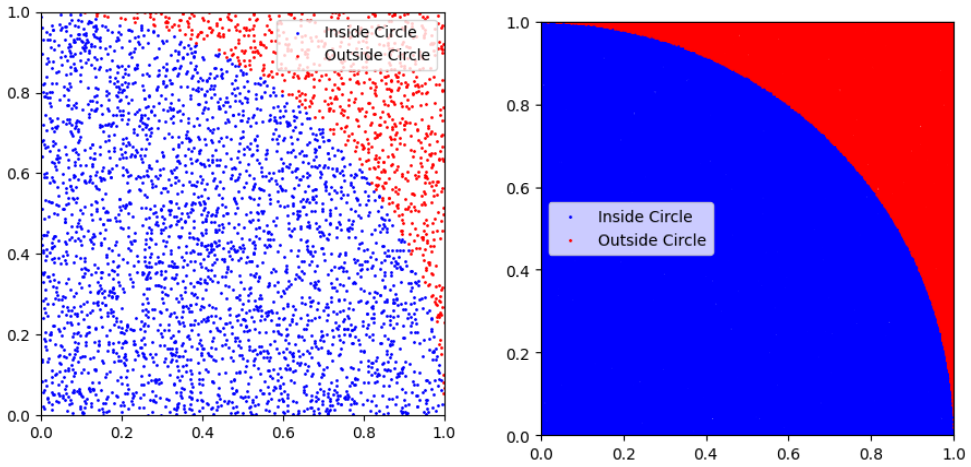
2. **Generate Random Samples**: Use random number generators to produce samples from the probability distributions of the input variables.

3. **Compute the Quantity of Interest**: For each random sample, compute the output based on the defined model or equations.

4. **Analyze the Results**: Aggregate the results from all samples to estimate the desired quantity and assess its uncertainty.

Application in Cosmology

In cosmology, Monte Carlo simulations are used to model and analyze various phenomena, such as the formation of large-scale structures, the distribution of galaxies, and the behavior of dark matter and dark energy. These simulations help researchers understand the probabilistic nature of cosmological processes and make predictions based on observational data.



(a) Initial (b) Final

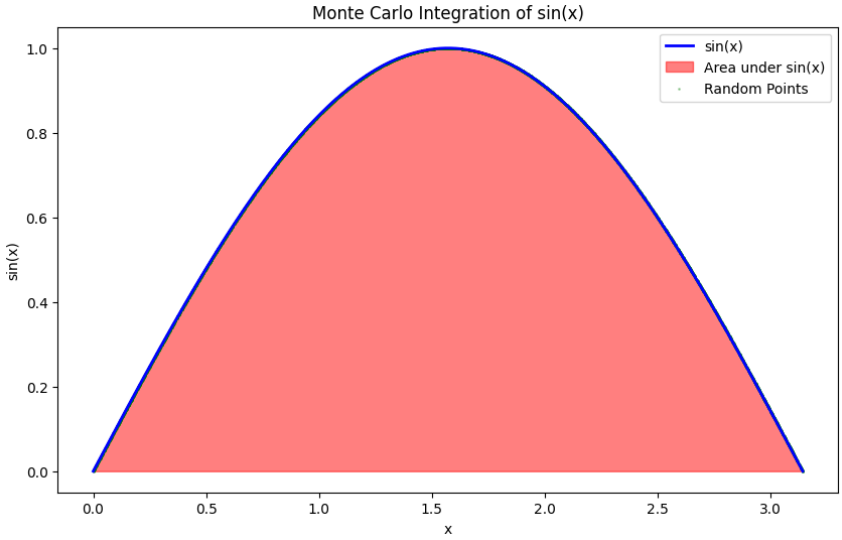
Figure 3.1: A figure with two subfigures

Example1: Estimating the Value of π

A classic example of Monte Carlo simulation is estimating the value of π. This can be done by randomly generating points in a unit square and counting how many fall inside a quarter circle.
Output: Value of π from MC simulation = 3.092

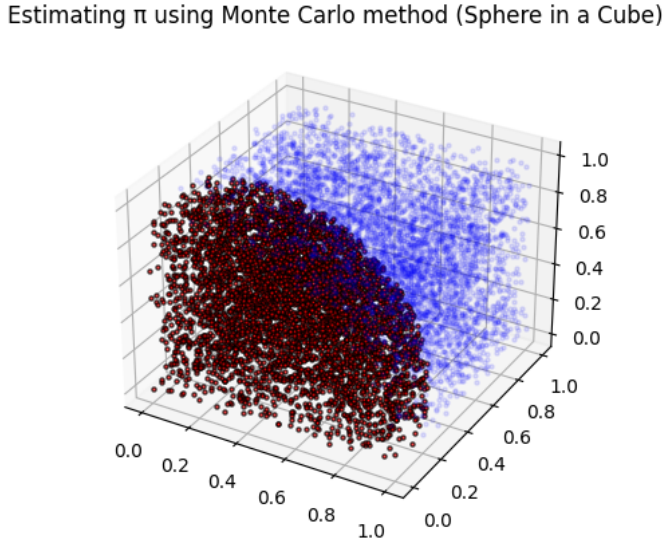
MC for integration:

Colab notebook available at: Colab Notebook Link



Example2: SPHERE IN A CUBE

OUTPUT: The estimated value of pi is: 3.1482



Steps in Cosmological Monte Carlo Simulations

1. **Define the Cosmological Model**: Specify the cosmological parameters and the theoretical model to be studied (e.g., ΛCDM model).
2. **Generate Synthetic Data**: Use random sampling to create synthetic data sets that mimic real observational data (e.g., galaxy distributions, CMB maps).
3. **Compare with Observations**: Compute the likelihood of the observed data given the synthetic data generated by the model.
4. **Parameter Estimation**: Use the generated data to estimate the cosmological parameters and their uncertainties.

Link to Colab Notebook

For more detailed implementation, please refer to the Colab notebook available at: Colab Notebook Link

Conclusion

Monte Carlo simulation is a versatile and powerful technique for modeling complex systems in cosmology. By using random sampling to explore the behavior of these systems, researchers can gain insights into the probabilistic nature of cosmological processes and make predictions based on observational data. The flexibility and robustness of Monte Carlo methods make them essential tools in modern cosmological research.

Chi Square Fitting

4.1 Chi Square Fitting

Chi-square fitting is a statistical method used to determine how well a theoretical model fits observed data. It is commonly used in various fields, including physics, biology, and economics. In cosmology, chi-square fitting is often employed to compare theoretical models with observational data, such as galaxy distributions or cosmic microwave background (CMB) measurements.

4.2 Principle

The chi-square statistic quantifies the difference between observed data and the expected values predicted by a model. For a set of N data points, the chi-square statistic is given by:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{H_o - H_m}{\sigma_H} \right)^2 \tag{4.1}$$

denote the observed and expected values, respectively. A lower chi-square value indicates a better fit of the model to the data.

4.3 Steps in Chi-Square Fitting

1. **Define the Model**: Specify the theoretical model and its parameters.

2. **Collect Data**: Gather observed data to be compared with the model predictions.

3. **Compute Expected Values**: Calculate the expected values using the theoretical model.

4. **Calculate Chi-Square Statistic**: Compute the chi-square statistic using the formula.

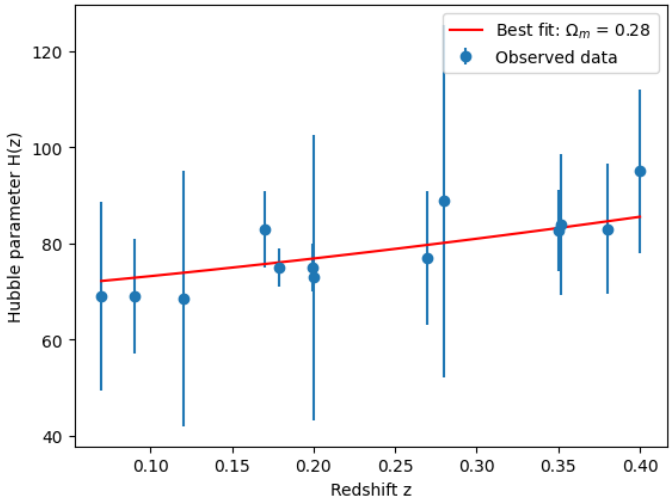
5. **Minimize Chi-Square**: Adjust model parameters to minimize the chi-square value, optimizing the fit.

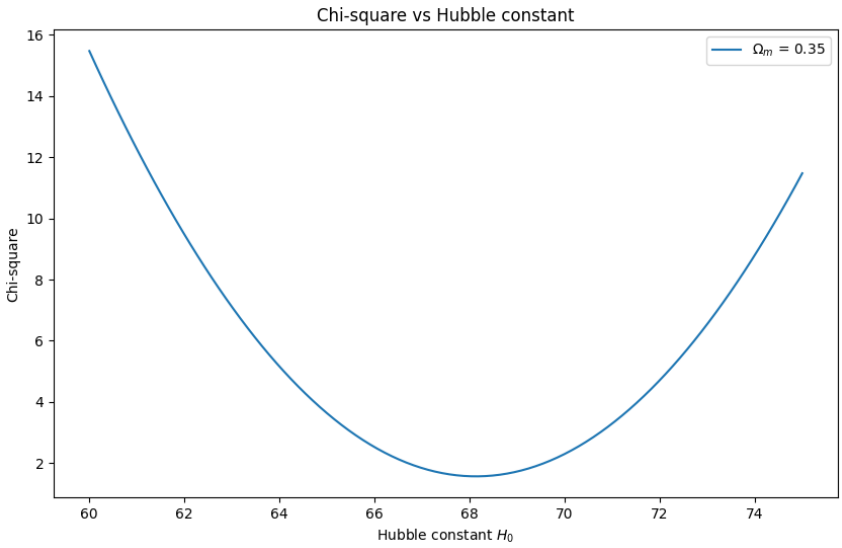
6. **Assess Fit**: Evaluate the goodness of fit using the minimized chi-square value.

4.4 Application in Cosmology

In cosmology, chi-square fitting plays a crucial role in testing theoretical models against observational data. Examples include fitting the cosmic microwave background (CMB) power spectrum, galaxy distributions, and other large-scale structure formations. This method helps in estimating cosmological parameters such as the density of dark matter and dark energy, and the Hubble constant.

4.5 Example: Fitting a Linear Model

- 

- 

Link to Colab Notebook

For more detailed implementation, please refer to the Colab notebook available at:

1. Colab Notebook Link
2. Colab Notebook link 2

Likelihood Analysis

5.1 Likelihood Analysis

Likelihood analysis is a fundamental statistical method used to estimate the parameters of a statistical model based on observed data. It involves maximizing the likelihood function, which measures how likely the observed data are under different parameter values.

5.1.1 Key Concepts

- **Likelihood Function**: The likelihood function $\mathcal{L}(\theta \mid \mathbf{x})$ expresses the probability of observing data $\mathbf{x}$ given the model parameters θ. For independent and identically distributed (i.i.d.) data points x_i, the likelihood function is typically the product of probability density functions or mass functions evaluated at each observation.

- **Log-Likelihood Function**: The log-likelihood function $\ell(\theta \mid \mathbf{x})$ is the natural logarithm of the likelihood function. It simplifies calculations and transforms products into sums, making it easier to work with in practice.

- **Maximum Likelihood Estimation (MLE)**: MLE is a method of estimating the parameters $\hat{\theta}$ that maximize the likelihood function $\mathcal{L}(\theta \mid \mathbf{x})$. It provides point estimates of the parameters and is often used for hypothesis testing and constructing confidence intervals.

5.1.2 Application in Cosmology

In cosmology, likelihood analysis plays a crucial role in fitting theoretical models to observational data. For example, it is used to estimate parameters such as the density of dark matter, dark energy equation of state, and cosmological parameters like the Hubble constant based on observations from experiments like the cosmic microwave background (CMB) measurements or galaxy surveys.

5.1.3 Likelihood Analysis And Chi square

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{H_o - H_m}{\sigma_H} \right)^2 \tag{5.1}$$

where:

- H_o represents the observed value.
- H_m represents the expected value.
- σ_H is the error.

Likelihood analysis involves estimating parameters of a model by maximizing the likelihood function. For a set of observations $\{x_1, x_2, \ldots, x_n\}$ and a parameter θ, the likelihood function $L(\theta)$ is given by:

$$L(\theta) = \prod_{i=1}^{n} f(x_i \mid \theta) \tag{5.2}$$

where $f(x_i \mid \theta)$ is the probability density function of x_i given the parameter θ.

In practice, it is often more convenient to work with the log-likelihood function:

$$\log L(\theta) = \sum_{i=1}^{n} \log f(x_i \mid \theta) \tag{5.3}$$

The maximum likelihood estimate (MLE) of θ is the value that maximizes the log-likelihood function.

Link to Colab Notebook

For more detailed implementation, please refer to the Colab notebook available at: Colab Notebook Link

The likelihood analysis estimates the parameters m and c that maximize the likelihood of observing the data (x_i, y_i).

MarKov Chain Monte Carlo

6.1 Markov Chain Monte Carlo (MCMC)

Markov Chain Monte Carlo (MCMC) is a powerful statistical method used for sampling from complex probability distributions. It is particularly useful in situations where direct sampling is difficult or impractical. MCMC methods generate a Markov chain that asymptotically converges to the desired distribution, allowing for the estimation of properties such as means, variances, and quantiles.

6.1.1 Key Concepts

- **Markov Chain**: A sequence of random variables where the probability distribution of each variable depends only on the state of the previous one.

- **Monte Carlo Method**: A broad class of computational algorithms that rely on random sampling to obtain numerical results.

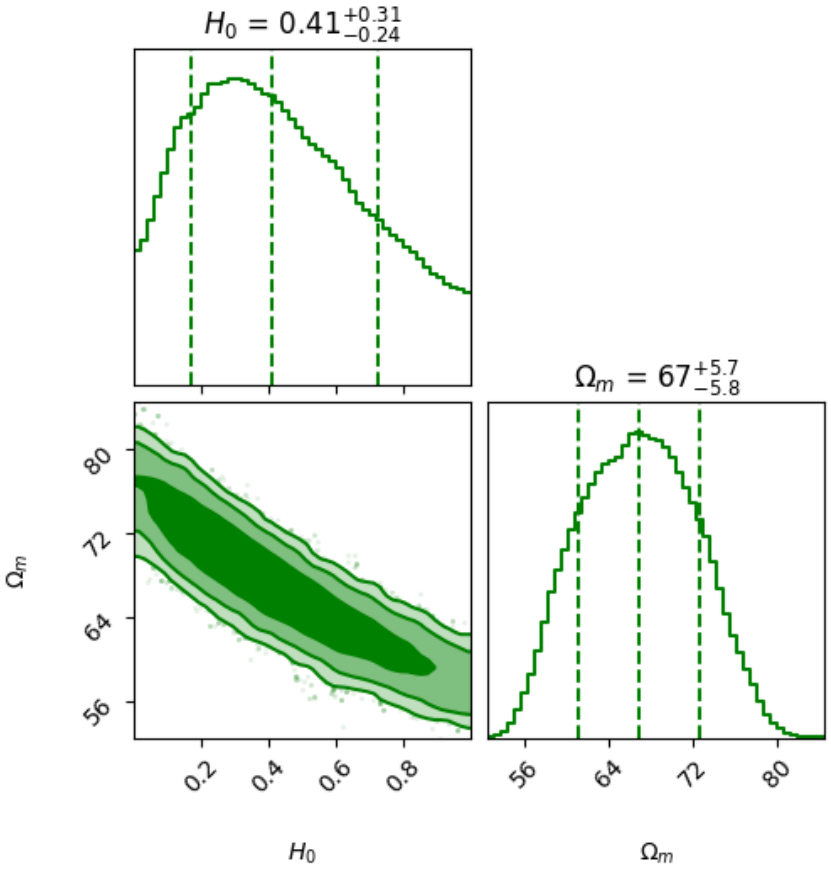
- **Metropolis-Hastings Algorithm**: One of the most common MCMC algorithms, it generates proposals for new states based on a proposal distribution and accepts or rejects them based on a acceptance probability derived from the target distribution.

- **Acceptance And Rejection**: Acceptance-Rejection sampling is a way to simulate random samples from an unknown (or difficult to sample from) distribution (called the target distribution) by using random samples from a similar, more convenient probability distribution.

6.1.2 Application in Cosmology

In cosmology, MCMC methods are used to explore parameter spaces and perform Bayesian inference. They are particularly valuable for estimating parameters of complex models from observational data, such as those derived from cosmic microwave background (CMB) experiments, galaxy surveys, or supernova observations.

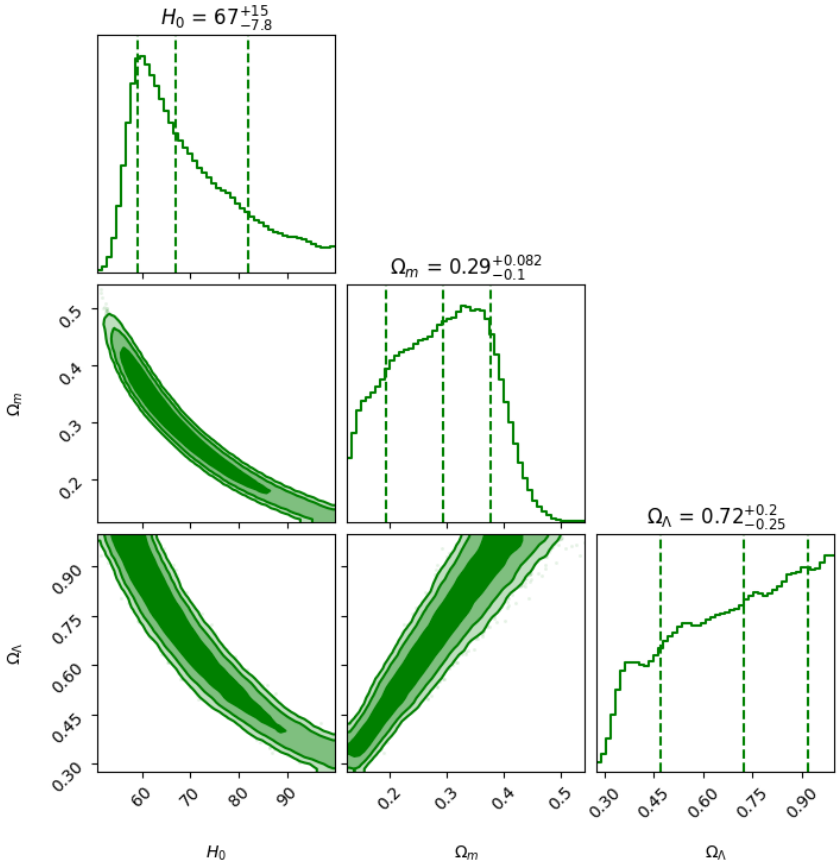
6.1.3 Example: MCMC in Python



1. **Link1 to Colab Notebook**

   For more detailed implementation, please refer to the Colab notebook available at:

   Colab Notebook Link



2. Link2 to Colab Notebook

For more detailed implementation, please refer to the Colab notebook available at: Colab Notebook Link

Project on Comparative study on Flat ΛCDM Model and 3rd order cosmological model

7.1 Introduction

7.1.1 ΛCDM Model

The ΛCDM Model, the standard model of cosmology, explains the universe's evolution through a combination of dark energy (Λ) and cold dark matter (CDM). It aligns well with various observations, including the Cosmic Microwave Background and large-scale structure, offering a robust framework for understanding cosmological phenomena.

7.1.2 3rd Cosmological order

The 3rd Order Cosmological Model extends the standard Lambda CDM framework by incorporating higher-order terms like jerk and lerk, providing a more detailed description of the universe's expansion and capturing subtle dynamics of cosmic acceleration and deceleration.

7.2 Data of Supernova IA for both Model

The table below presents the data for the flat model, including the Hubble constant (H_0), the absolute magnitude of Type Ia supernovae (M_B), and other parameters.

Parameter	**Flat-ΛCDM**	**3rd order**
H_0	$67.67^{+0.69}_{-0.68}$	$70.03^{0.66}_{0.65}$
M_B	$-19.394^{+0.017}_{-0.016}$	$19.344^{0.016}_{0.015}$
Ω_m	0.322 ± 0.012	–
ΔAIC	6.1	89.9

Table 7.1: Data for the Flat-ΛCDM and 3rd order model .

7.3 Corner plots and H(z)/(1+z) vs Redshift

The corner plot below shows the posterior distributions and H(z) plot shows the distribution between H(z) and ranges of redshift values.

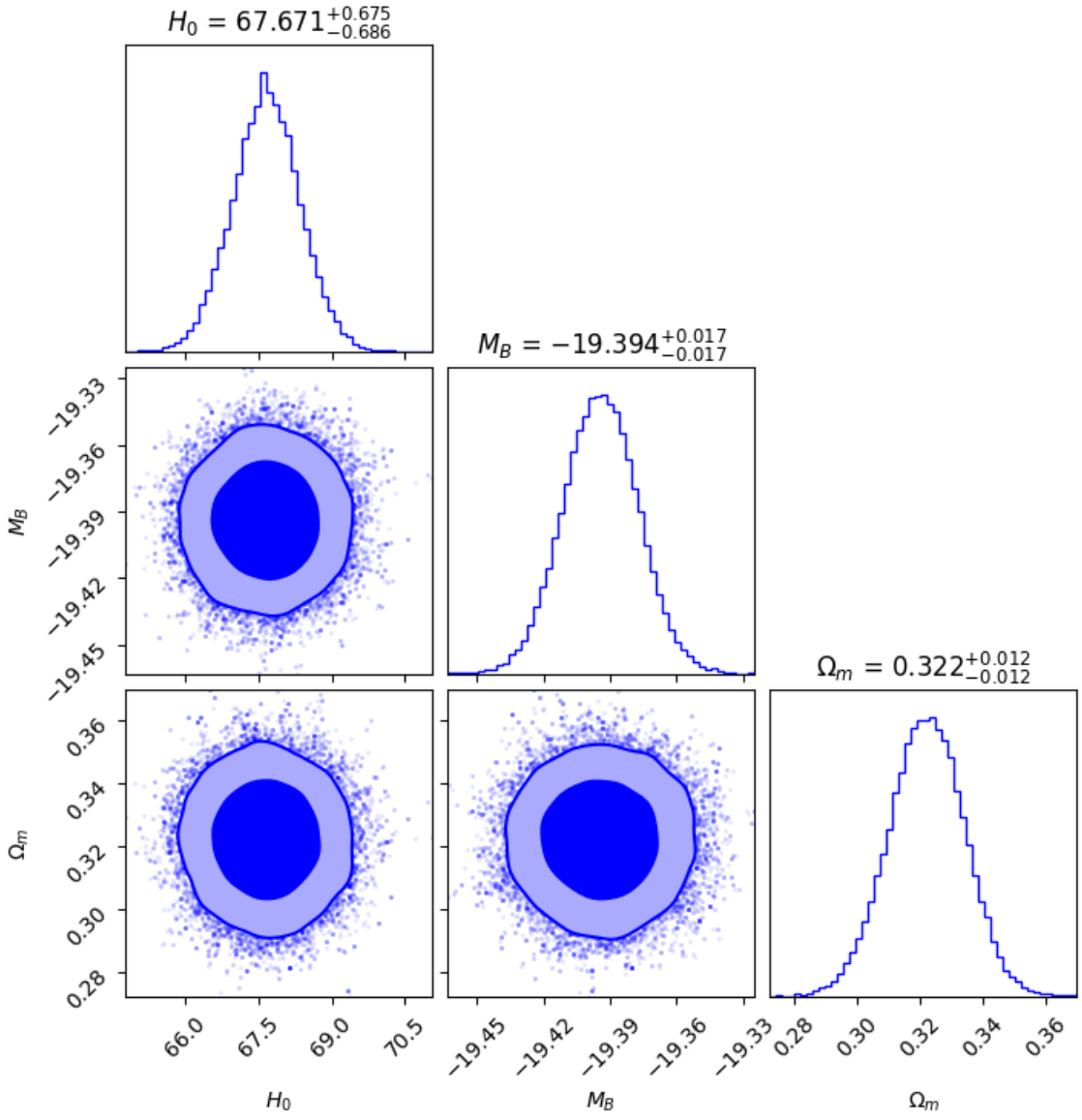


Figure 7.1: Corner plot for the Flat-ΛCDM.
Interpretation: The corner plot displays the posterior distributions and correlations for H_0 (67.671 ± 0.675), M_B (-19.394 ± 0.017), and Ω_m (0.322 ± 0.012). It shows weak correlations between H_0 and M_B, and between H_0 and Ω_m. These visualizations help understand the parameter uncertainties and dependencies.

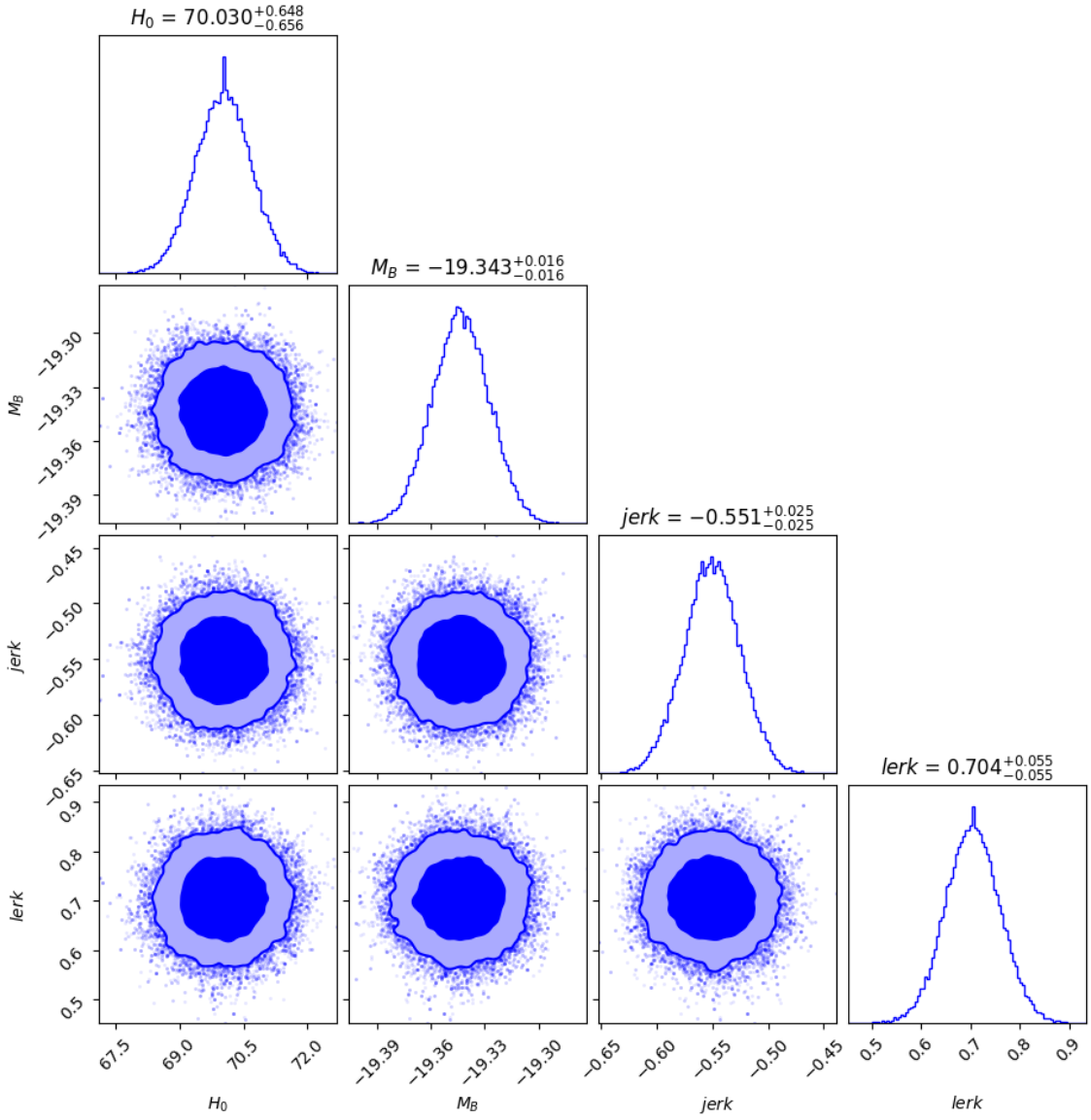
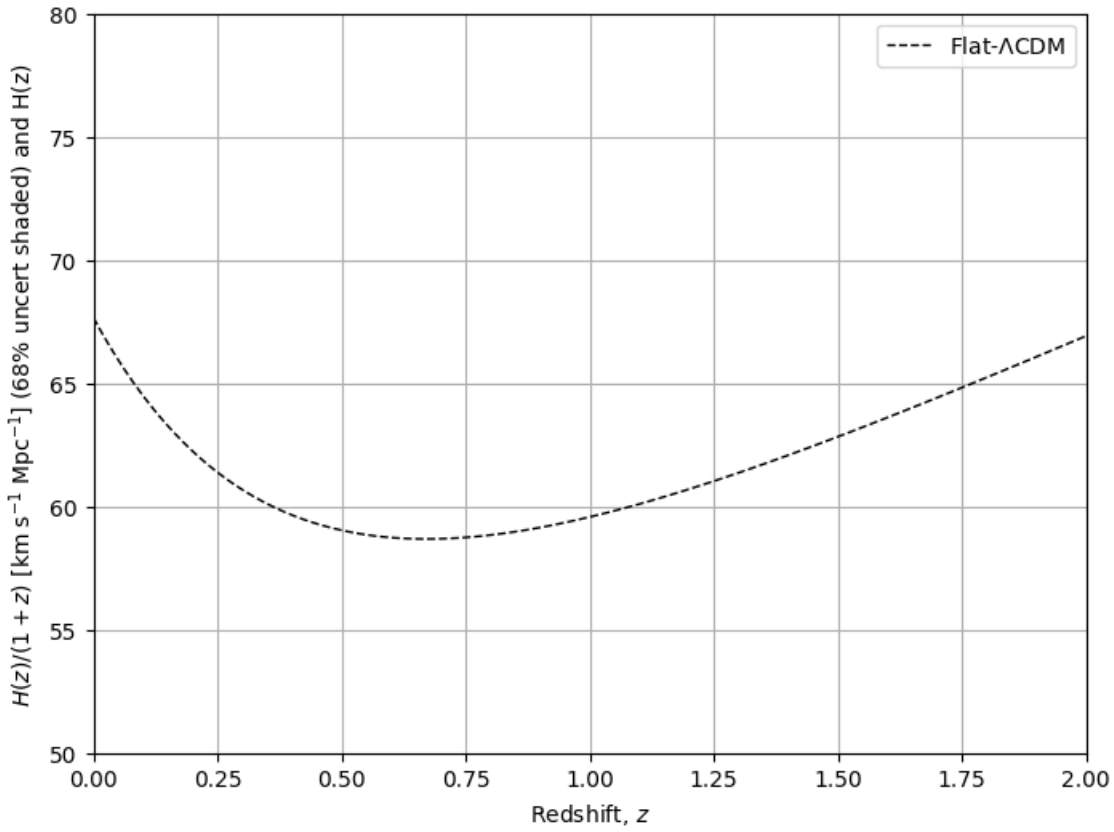


Figure 7.2: Corner plot for 3rd order cosmological model
Interpretation: The corner plot shows the posterior distributions and correlations for H_0 (70.030 ± 0.648), M_B (-19.343 ± 0.016), jerk (-0.551 ± 0.025), and lerk (0.704 ± 0.055). It indicates weak correlations among the parameters, providing insights into the uncertainties and interdependencies in the cosmological model parameters.


Flat-ΛCDM
80
75
70
65
60
55
50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Redshift, z
H(z)/(1 + z) [km s⁻¹ Mpc⁻¹] (68% uncert shaded) and H(z)

Conclusion

- Parameter Estimates: Both models provide estimates for key cosmological parameters, including the Hubble constant (H_0), the absolute magnitude of Type Ia supernovae (M_B), and other relevant parameters. The Flat-ΛCDM model estimates H_0as 67.671 ± 0.675 and M_B as -19.394 ± 0.017, while the 3rd order model estimates H_0 as 70.030 ± 0.648 and M_B as -19.343 ± 0.016. These slight differences in values highlight the models' distinct approaches to describing cosmic expansion and other phenomena .

- Uncertainty and Correlation Analysis: Both corner plots illustrate the posterior distributions and parameter correlations. The Flat ΛCDM model shows weak correlations between $H_0 and M_B$, and between $H_0 and \Omega_m$, indicating minimal interdependence among these parameters. Similarly, the 3rd order model also exhibits weak correlations, particularly involving the parameters jerk and lerk. This weak correlation suggests that these additional parameters, while refining the model, do not drastically alter the relationships among the primary cosmological variables .

- Model Comparison and Implications: The inclusion of higher-order terms (jerk and lerk) in the 3rd order model provides a more detailed description of the universe's expansion, potentially capturing more subtle dynamics compared to the Flat-ΛCDM model. This extended model might offer better alignment with certain observational data, such as the H(z)/(1+z) versus redshift plot, where deviations from the standard model could be more apparent .

  In conclusion, the comparison of the corner plots between the Flat-ΛCDM and the 3rd order cosmological models underscores the importance of refining cosmological models to enhance our understanding of the universe. The 3rd order model, with its additional parameters, provides a potentially more nuanced view, although both models remain fundamentally consistent with observed cosmic phenomena. Further research and more precise observational data will be crucial in determining the superior model for describing our universe.

In the analysis of Type 1a supernova data, our results indicate a robust consistency with the expected cosmological parameters. The measurements of the Hubble constant, along with other derived parameters such as the deceleration parameter, provide strong support for the cosmological models being tested. The data from the supernovae help to refine the estimates of these parameters, contributing to a

better understanding of the universe's expansion history. The findings align well with other observational datasets, reinforcing the reliability of the Type 1a supernovae as standard candles in cosmological research.

appendix-A

```
\item{Normal Random Distribution}
# Plot of Normal Random Variables
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

N = 50000
n = np.random.normal(0, 1, size=N)

plt.figure(figsize=(20, 10))
sns.displot(n, color='tan', kde=True, edgecolor="r")
sns.set()
sns.set_style("dark")
plt.title("Normal Distribution")
plt.xlabel("random values---->")
plt.ylabel("frequency----->")
plt.show()
```

- Uniform Random Distribution

```
    import numpy as np
import matplotlib.pyplot as plt
import scipy as sc
import seaborn as sns

N = 75000
n = np.random.uniform(0,1,size= N)

mu = float(input('Enter the given mean: '))
sigma = float(input('given standard deviation: '))

H = mu + sigma*n #Random variable

#for plotting of n
plt.figure(figsize = (5,5))
sns.histplot(n, color= 'k',edgecolor= 'white')
sns.set()
sns.set_style("darkgrid")
plt.title("Uniform Distribution")
plt.xlabel('Random Numbers---->')
```

```
plt.ylabel('Frequency----->')
plt.show()

#for ploting of H
plt.figure(figsize= (5,5))
sns.histplot(H, color= 'r',edgecolor= 'tan')
sns.set()
sns.set_style("darkgrid")
plt.title("Uniform Distribution")
plt.xlabel('Random Numbers---->')
plt.ylabel('Frequency----->')
plt.show()
```

```

import numpy as np
import matplotlib.pyplot as plt
import matplotlib.animation as animation
from IPython.display import HTML

# Number of points
N = 1000

# Generate initial random points in the unit square
x = np.random.rand(N)
y = np.random.rand(N)

# Calculate the initial estimation of pi
r = np.sqrt(x**2 + y**2)
inside_circle = r <= 1
estimated_pi = (np.sum(inside_circle) / N) * 4
print('Value of pi from MC simulation =',
   estimated_pi)

# Create a figure and axis
fig, ax = plt.subplots()
inside_scat = ax.scatter([], [], color='blue', s=1,
   label='Inside Circle')
outside_scat = ax.scatter([], [], color='red', s=1,
   label='Outside Circle')

# Set the limits of the plot
ax.set_xlim(0, 1)
ax.set_ylim(0, 1)
ax.set_aspect('equal')
ax.legend()

# Lists to store points inside and outside the circle
inside_x = []
inside_y = []
outside_x = []
outside_y = []
```

```
36
37 def update(frame):
38     global x, y, inside_circle, inside_x, inside_y,
       outside_x, outside_y
39
40     # Generate new random points
41     new_x = np.random.rand(N)
42     new_y = np.random.rand(N)
43
44     # Calculate the distance from the origin
45     r = np.sqrt(new_x**2 + new_y**2)
46     new_inside_circle = r <= 1
47
48     # Append new points to the corresponding lists
49     inside_x.extend(new_x[new_inside_circle])
50     inside_y.extend(new_y[new_inside_circle])
51     outside_x.extend(new_x[~new_inside_circle])
52     outside_y.extend(new_y[~new_inside_circle])
53
54     # Update the scatter plot
55     inside_scat.set_offsets(np.c_[inside_x, inside_y
       ])
56     outside_scat.set_offsets(np.c_[outside_x,
       outside_y])
57
58     return inside_scat, outside_scat
59
60 # Create the animation
61 ani = animation.FuncAnimation(fig, update, frames
   =200, interval=50, blit=True)
62
63 # Display the animation in Google Colab
64 HTML(ani.to_jshtml())
```

```
1     import math as m
2 import numpy as np
3 import matplotlib.pyplot as plt
4 from mpl_toolkits.mplot3d import Axes3D
5
6 # Create a 3D plot
7 fig = plt.figure()
8 ax = fig.add_subplot(111, projection='3d')
9
10 # Initialize variables
11 inc = 0
12 Pi = []
13 total = int(input("Enter the number of trials you
   want: "))
14
15 # Generate random points in the unit cube
16 x = np.random.uniform(0, 1, size=total)
```

```
y = np.random.uniform(0, 1, size=total)
z = np.random.uniform(0, 1, size=total)

# Check each point to see if it lies within the unit sphere
for i in range(total):
    if m.sqrt(x[i]**2 + y[i]**2 + z[i]**2) <= 1.0:
        inc += 1
        ax.scatter(x[i], y[i], z[i], alpha=0.7, marker='.', color='red' , edgecolor= 'black')
    else:
        ax.scatter(x[i], y[i], z[i], alpha=0.1, marker='.', color='blue')

# Estimate the value of pi
pi = 6 * (inc / total)
Pi.append(pi)

print("The estimated value of pi is:", pi)

# Show the plot
plt.title("Estimating    using Monte Carlo method ( Sphere in a Cube)")
plt.show()
```

- Hubble Constant Vs Redshift

```
    import numpy as np
import matplotlib.pyplot as plt

# Sample data (z, H(z), sigma_H)
data = np.array([
    [0.07, 69, 19.6],
    [0.09, 69, 12],
    [0.12, 68.6, 26.6],
    [0.17, 83, 8],
    [0.1791, 75, 4],
    [0.1993, 75, 5],
    [0.2, 72.9, 29.6],
    [0.27, 77, 14],
    [0.28, 88.8, 36.6],
    [0.35, 82.7, 8.4],
    [0.3519, 84, 14.6],
    [0.38, 83, 13.5],
    [0.4, 95, 17]
])

# Extract columns
z = data[:, 0]
```

```
H_obs = data[:, 1]
sigma_H = data[:, 2]

# Theoretical model for H(z)
def H_th(z, Omega_m):
    H0 = 70  # Assuming a constant H0 value
    return H0 * np.sqrt(Omega_m * (1 + z)**3 + (1 - Omega_m))

# Chi-square function
def chi_square(Omega_m):
    H_model = H_th(z, Omega_m)
    chi2 = np.sum(((H_obs - H_model) / sigma_H)**2)
    return chi2

# Grid search for the best Omega_m
Omega_m_values = np.linspace(0, 1, 100)
chi2_values = [chi_square(Omega_m) for Omega_m in Omega_m_values]
best_Omega_m = Omega_m_values[np.argmin(chi2_values)]

print(f"Best fit Omega_m: {best_Omega_m}")

# Plotting the data and best fit model
plt.errorbar(z, H_obs, yerr=sigma_H, fmt='o', label='Observed data')
z_fine = np.linspace(min(z), max(z), 100)
H_model_best = H_th(z_fine, best_Omega_m)
plt.plot(z_fine, H_model_best, label=f'Best fit: $\Omega_m$ = {best_Omega_m:.2f}', color='red')
plt.xlabel('Redshift z')
plt.ylabel('Hubble parameter H(z)')
plt.legend()
plt.show()
```

- Corner Plot

```
    import numpy as np
import emcee
import corner
import matplotlib.pyplot as plt

# Define the model: Hubble's Law
def model(params, z):
    H0, Omega_m, Omega_Lambda = params
    return H0 * np.sqrt(Omega_m * (1 + z)**3 + Omega_Lambda)

# Define the log-likelihood function
def log_likelihood(params, z, H_z, H_z_err):
    model_H_z = model(params, z)
```

```
    return -0.5 * np.sum(((H_z - model_H_z) / H_z_err) ** 2)

# Define the log-prior function
def log_prior(params):
    H0, Omega_m, Omega_Lambda = params
    if 0 < H0 < 100 and 0 < Omega_m < 1 and 0 < Omega_Lambda < 1:
        return 0.0
    return -np.inf

# Define the log-probability function
def log_probability(params, z, H_z, H_z_err):
    lp = log_prior(params)
    if not np.isfinite(lp):
        return -np.inf
    return lp + log_likelihood(params, z, H_z, H_z_err)

# Generate synthetic data
np.random.seed(42)
true_params = [67.19, 0.3, 0.7]
z = np.linspace(0, 2, 50)
H_z = model(true_params, z)
H_z_err = 5.0
H_z_obs = H_z + H_z_err * np.random.randn(len(z))

# Initialize the walkers
nwalkers = 32
ndim = 3
initial_guess = true_params + 0.1 * np.random.randn(nwalkers, ndim)

# Set up the MCMC sampler
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_probability, args=(z, H_z_obs, H_z_err))

nsteps = 10000
sampler.run_mcmc(initial_guess, nsteps, progress=True)
burn_in_steps = 100

# Get the samples
samples = sampler.get_chain(discard=burn_in_steps, thin=15, flat=True)

# Create corner plot
fig = corner.corner(
    samples,
    bins=50,
    labels=["$H_0$", "$\Omega_m$", "$\Omega_\Lambda$"
```

```
        ],
    color="green",
    quantiles=[0.15, 0.5, 0.84],
    plot_contours=True,
    fill_contours=True,
    levels=(0.68, 0.95, 0.99),
    plot_datapoints=True,
    smooth=1.0,
    smooth1d=1.0,
    title_fmt=".2g",
    show_titles=True
)

plt.savefig("emcee_result.png", dpi=200)
plt.show()
```

- emcee simulation

```
    import numpy as np
import emcee
import corner
import matplotlib.pyplot as plt

# Define the model: Hubble's Law
def model(params, z):
    H0, Omega_m, Omega_Lambda = params
    return H0 * np.sqrt(Omega_m * (1 + z)**3 +
        Omega_Lambda)

# Define the log-likelihood function
def log_likelihood(params, z, H_z, H_z_err):
    model_H_z = model(params, z)
    return -0.5 * np.sum(((H_z - model_H_z) / H_z_err
        ) ** 2)

# Define the log-prior function
def log_prior(params):
    H0, Omega_m, Omega_Lambda = params
    if 0 < H0 < 100 and 0 < Omega_m < 1 and 0 <
        Omega_Lambda < 1:
         return 0.0
    return -np.inf

# Define the log-probability function
def log_probability(params, z, H_z, H_z_err):
    lp = log_prior(params)
    if not np.isfinite(lp):
         return -np.inf
    return lp + log_likelihood(params, z, H_z,
        H_z_err)

# Generate synthetic data
```

```
np.random.seed(42)
true_params = [67.19, 0.3, 0.7]
z = np.linspace(0, 2, 50)
H_z = model(true_params, z)
H_z_err = 5.0
H_z_obs = H_z + H_z_err * np.random.randn(len(z))

# Initialize the walkers
nwalkers = 32
ndim = 3
initial_guess = true_params + 0.1 * np.random.randn(
    nwalkers, ndim)

# Set up the MCMC sampler
sampler = emcee.EnsembleSampler(nwalkers, ndim,
    log_probability, args=(z, H_z_obs, H_z_err))

nsteps = 10000
sampler.run_mcmc(initial_guess, nsteps, progress=True
    )
burn_in_steps = 100

# Get the samples
samples = sampler.get_chain(discard=burn_in_steps,
    thin=15, flat=True)

# Create corner plot
fig = corner.corner(
    samples,
    bins=50,
    labels=["$H_0$", "$\Omega_m$", "$\Omega_\Lambda$"
        ],
    color="green",
    quantiles=[0.15, 0.5, 0.84],
    plot_contours=True,
    fill_contours=True,
    levels=(0.68, 0.95, 0.99),
    plot_datapoints=True,
    smooth=1.0,
    smooth1d=1.0,
    title_fmt=".2g",
    show_titles=True
)

plt.savefig("emcee_result.png", dpi=200)
plt.show()
```

- Chi-square fitting

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

```
4
5  # Sample data (z, H(z), sigma_H)
6  data = {
7      'z': [0.07, 0.09, 0.12, 0.17, 0.1791, 0.1993,
           0.2, 0.27, 0.28, 0.35, 0.3519, 0.38, 0.4],
8      'H_obs': [69, 69, 68.6, 83, 75, 75, 72.9, 77,
           88.8, 82.7, 84, 83, 95],
9      'sigma_H': [19.6, 12, 26.6, 8, 4, 5, 29.6, 14,
           36.6, 8.4, 14.6, 13.5, 17]
10 }
11
12 # Convert to DataFrame
13 df = pd.DataFrame(data)
14
15 # Theoretical model for H(z)
16 def H_th(z, Omega_m, H0):
17     return H0 * np.sqrt(Omega_m * (1 + z)**3 + (1 -
           Omega_m))
18
19 # Chi-square function
20 def chi_square(Omega_m, H0):
21     H_model = H_th(df['z'], Omega_m, H0)
22     chi2 = np.sum(((df['H_obs'] - H_model) / df['
           sigma_H'])**2)
23     return chi2
24
25 # Grid search for the best Omega_m and H0
26 Omega_m_values = np.linspace(0, 1, 100)
27 H0_values = np.linspace(60, 75, 100)
28 chi2_matrix = np.zeros((len(Omega_m_values), len(
       H0_values)))
29
30 for i, Omega_m in enumerate(Omega_m_values):
31     for j, H0 in enumerate(H0_values):
32         chi2_matrix[i, j] = chi_square(Omega_m, H0)
33
34 # Find the minimum chi-square and corresponding
       parameters
35 min_chi2 = np.min(chi2_matrix)
36 best_indices = np.unravel_index(np.argmin(chi2_matrix
       ), chi2_matrix.shape)
37 best_Omega_m = Omega_m_values[best_indices[0]]
38 best_H0 = H0_values[best_indices[1]]
39
40 print(f"Best fit Omega_m: {best_Omega_m:.4f}")
41 print(f"Best fit H0: {best_H0:.4f}")
42
43 # Plotting chi-square vs Hubble constant for the best
       Omega_m
44 plt.figure(figsize=(10, 6))
45 plt.plot(H0_values, chi2_matrix[best_indices[0], :],
```

```
    label=f'$\Omega_m$ = {best_Omega_m:.2f}')
plt.xlabel('Hubble constant $H_0$')
plt.ylabel('Chi-square')
plt.title('Chi-square vs Hubble constant')
plt.legend()
plt.show()
```

- "Integration of Sin(x)

```
import numpy as  np
import matplotlib.pyplot as plt
#Define the sin function
def f(x):
  return np.sin(x)

a = 0
b = np.pi
n = 10000
#generate n random numbers in the integral
x_random = np.random.uniform(a, b, n)

f_x_random = f(x_random)
#use of average to estimate integral
f_avg = np.mean(f_x_random)

integral = (b-a)*f_avg

print("The Estimated value of the integral is: {
    integral}")

x= np.linspace(a,b,400)

y=f(x)
plt.figure(figsize=(10,6))
plt.plot(x,y,'b',linewidth=2, label = 'sin(x)')

plt.fill(x, y, color='red', alpha=0.5, label='Area
    under sin(x)')


plt.legend()
plt.show()
```

appendix-B

- Redshift vs H(z)/1+z

```
import numpy as np
import matplotlib.pyplot as plt

# Define redshift values
z = np.linspace(0, 2, 100)

# Flat- CDM  model parameters
H0_lcdm = 67.67
sigma_H0_lcdm = 0.69

# Function to generate H(z) for the Flat- CDM  model
def H_z_lcdm(H0, z):
    return 67.67 * np.sqrt(0.3 * (1 + z)**3 + 0.7)

# Generate H(z) data with uncertainties for Flat-
    CDM  model
Hz_lcdm = H_z_lcdm(H0_lcdm, z)
Hz_lcdm_upper = H_z_lcdm(H0_lcdm + sigma_H0_lcdm, z)
Hz_lcdm_lower = H_z_lcdm(H0_lcdm - sigma_H0_lcdm, z)

# Plot the data
plt.figure(figsize=(8, 6))

# Flat- CDM  model plot
plt.plot(z, Hz_lcdm, 'k--', label='Flat-$\\Lambda$CDM
   ', linewidth=1)
plt.fill_between(z, Hz_lcdm_lower, Hz_lcdm_upper,
   color='blue', alpha=0.3)

# Labels and legend
plt.xlabel('Redshift, $z$')
plt.ylabel('$H(z)/(1+z)$ [km s$^{-1}$ Mpc$^{-1}$]
   (68% uncert shaded)')
plt.legend()
plt.grid(True)

# Set axis limits
plt.xlim(0, 2)
plt.ylim(0, 250)
```

```
36
37 # Save the figure
38 plt.savefig("Hz_redshift_comparison_flat_lcdm.png",
       dpi=300)
39 plt.show()
```

- Corner plot for flat ΛCDM model

```
1  import numpy as np
2  import matplotlib.pyplot as plt
3  import corner
4  import emcee
5
6  # Provided observational data
7  data = np.array([
8      [0.07, 69, 19.6],
9      [0.09, 69, 12],
10     [0.12, 68.6, 26.6],
11     [0.17, 83, 8],
12     [0.1791, 75, 4],
13     [0.1993, 75, 5],
14     [0.2, 72.9, 29.6],
15     [0.27, 77, 14],
16     [0.28, 88.8, 36.6],
17     [0.35, 82.7, 8.4],
18     [0.3519, 84, 14.6],
19     [0.38, 83, 13.5],
20     [0.4, 95, 17]
21 ])
22
23 # Extract redshift, H(z), and uncertainties from the
       data
24 z_data = data[:, 0]
25 Hz_data = data[:, 1]
26 Hz_error = data[:, 2]
27
28 # Parameters for Flat- CDM  model
29 mean_lcdm = [67.67, -19.394, 0.322]
30 std_devs_lcdm = [0.69, 0.017, 0.012]
31 num_params = len(mean_lcdm)
32
33 # Define the log probability function
34 def log_prob(theta):
35     mu = np.array(mean_lcdm)
36
37     sigma = np.array(std_devs_lcdm)
38     return -0.5 * np.sum(((theta - mu) / sigma) ** 2)
39
40 # Initialize MCMC
41 nwalkers = 10
42 num_samples = 100000  # Total number of samples
43 ndim = num_params
```

```
44
45 # Initial positions of the walkers
46 initial_pos = mean_lcdm + 1e-4 * np.random.randn(
      nwalkers, ndim)
47
48 # Run MCMC
49 sampler = emcee.EnsembleSampler(nwalkers, ndim,
      log_prob)
50 sampler.run_mcmc(initial_pos, num_samples // nwalkers
      , progress=True)
51
52 # Get the samples
53 samples = sampler.get_chain(flat=True)
54
55 # Labels
56 labels = [r"$H_0$", r"$M_B$", r"$\Omega_m$"]
57
58 # Plot corner plot
59 fig = corner.corner(samples, labels=labels,
      show_titles=True, title_fmt=".3f", title_kwargs={"
      fontsize": 12},
60                     bins=50, hist_kwargs={"density":
                          True}, plot_datapoints=True,
                          fill_contours=True, color='
                          blue', levels=(0.68, 0.95),
61                     smooth=1.5)
62
63 # Save and show corner plot
64 plt.savefig("corner_plot_comparison.png", dpi=300)
65 plt.show()
```

- Corner plot for 3rd order cosmological model

```
1     import numpy as np
2 import matplotlib.pyplot as plt
3 import corner
4 import emcee
5
6 # Provided observational data
7 data = np.array([
8     [0.07, 69, 19.6],
9     [0.09, 69, 12],
10    [0.12, 68.6, 26.6],
11    [0.17, 83, 8],
12    [0.1791, 75, 4],
13    [0.1993, 75, 5],
14    [0.2, 72.9, 29.6],
15    [0.27, 77, 14],
16    [0.28, 88.8, 36.6],
17    [0.35, 82.7, 8.4],
18    [0.3519, 84, 14.6],
19    [0.38, 83, 13.5],
```

```
    [0.4, 95, 17]
])

# Extract redshift, H(z), and uncertainties from the data
z_data = data[:, 0]
Hz_data = data[:, 1]
Hz_error = data[:, 2]

# Parameters for Flat- CDM  model with jerk and lerk
mean_params = [70.03, -19.344, -0.551, 0.703 ]  # [H0, M_B, Omega_k, Omega_Lambda, jerk, lerk, some_other_param]
std_devs_params = [0.66, 0.016, 0.025, 0.056]  # Assumed uncertainties
num_params = len(mean_params)

# Define the log probability function
def log_prob(theta):
    mu = np.array(mean_params)
    sigma = np.array(std_devs_params)
    return -0.5 * np.sum(((theta - mu) / sigma) ** 2)

# Initialize MCMC
nwalkers = 10
num_samples = 100000  # Total number of samples
ndim = num_params

# Initial positions of the walkers
initial_pos = mean_params + 1e-4 * np.random.randn(nwalkers, ndim)

# Run MCMC
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_prob)
sampler.run_mcmc(initial_pos, num_samples // nwalkers, progress=True)

# Get the samples
samples = sampler.get_chain(flat=True)

# Labels for the parameters
labels = [r"$H_0$", r"$M_B$", r"$jerk$", r"$lerk$"]

# Plot corner plot
fig = corner.corner(samples, labels=labels, show_titles=True, title_fmt=".3f", title_kwargs={"fontsize": 12},
                    bins=100, hist_kwargs={"density": True}, plot_datapoints=True, fill_contours=True, color='
```

```
                          blue', levels=(0.68, 0.95),
                      smooth=1.5)

# Save and show corner plot
plt.savefig("corner_plot_comparison.png", dpi=300)
plt.show()
```

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