Qpoland$_classiq_challenge$

Contents

1 Introduction

The harmonic oscillator is a fundamental model in physics described by the differential equation:

$$y'' + \omega^2 y = 0$$

This project solves the harmonic oscillator equation using a quantum algorithm and compares the results against classical solutions. The main objectives include:

- Simulating the oscillator's dynamics using a quantum circuit.
- Evaluating the system's kinetic and potential energy over time.
- Optimizing the quantum circuit for efficiency.

2 Quantum Algorithm Framework

The quantum algorithm used in this project consists of three stages:

1. **Encoding:** Preparing the initial quantum state based on the problem's initial conditions.
2. **Entanglement and Dynamics:** Evolving the system through quantum gates to represent the dynamics of the oscillator.
3. **Decoding:** Reversing parts of the encoding process and extracting results via measurements.

Key features:

- An ancilla register stores computational states.
- A work register facilitates entanglement and measurement.

3 Results

3.1 Comparison with Classical Solutions

The quantum algorithm successfully reproduced the solution $y(t) = \sin(t) + \cos(t)$ for the interval $t \in [0, 1]$, matching the classical results with high accuracy.

3.2 Energy Dynamics

- **Kinetic Energy (KE):** Simulated and matched the classical form $0.5 \times (\cos(t) - \sin(t))^2$.
- **Potential Energy (PE):** Simulated and matched $0.5 \times \cos^2(t)$.

3.3 Parameter Range Observations

Accuracy decreased slightly for extended simulation bounds, highlighting the importance of optimizing parameters for larger intervals.

4 Circuit Optimization

The circuit's complexity grows with the number of qubits:

- **Gate Count:** Scales approximately as $2^n(n-1)$, where n is the number of qubits.
- **Optimization:** By minimizing the depth and adjusting parameters, computational costs were reduced while maintaining accuracy.

5 Key Insights and Future Work

5.1 Insights

1. Quantum algorithms can accurately model physical systems like the harmonic oscillator.
2. Energy evaluations align well between classical and quantum frameworks.
3. Circuit optimization is crucial for balancing resource costs and accuracy.

5.2 Future Directions

- Extend simulations to coupled oscillators or higher-dimensional systems.
- Incorporate error mitigation techniques to improve reliability.
- Explore alternate encoding methods for greater scalability.

6 Conclusion

This project demonstrates the potential of quantum algorithms for solving differential equations. While challenges like circuit complexity persist, the promising results indicate significant opportunities for broader quantum simulations.