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A Partial Differential Equation Algorithm for Image Enhancement*

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***Abstract*—A novel approach for image enhancement is presented in this paper. The enhancement result is achieved by combination the means of contrast enhancement, image smoothing and image sharpening techniques into one partial differential equation (PDE) framework. In the image evolution process, three kinds of operation execute at the same time, using the regularization parameters to adjust the proportion of each operation in the evolution process, ultimately achieve the desired results. The simulations for both gray and color images are given in the experiments. Experimental results demonstrate that the method substantially improves the subjective quality of the enhanced images.**

***Index Terms*—image enhancement, contrast enhancement, image smoothing, image sharpening, PDE.**

I. Introduction

Much effort has been made to enhance low contrast images by improving several factors, such as contrast enhancement, image sharpening, denoising and color accuracy improving [1]. Image enhancement techniques belong to low-level image processes areas which are characterized by the fact both its inputs and outputs are images [2]. According to the mathematical tools used in image enhancement technique, we can get the following classifications: pixel and algebra-based gray scale transformation for image contrast enhancement, fourier transform based frequency domain filtering method for image smoothing, denoising and sharpening, wavelet transform based image denoising, as well as PDE based image restoration, segmentation and smoothing methods.

Image enhancement techniques mainly include the direct manipulation of pixel values based on histogram modification technique and neighborhood pixel gray-scale transformation for image smoothing and edge sharpening. Sometimes, the enhancement object can be completed using a single method, but more usually several methods should be joint together in order to achieve satisfactory results.

In this paper, we will build a PDE based model that integrates a variety of image enhancement methods into one framework, using the regularization parameters in the evolution process to balance the conflicting evolutionary process, thereby obtaining the optimal results. The basic idea of PDE based image processing method is image evolution according to a specified partial differential equation in continuous mathematical models to get desired results [3]. Compared to the traditional techniques, PDEs based digital image processing methods exhibit stronger local adaptability and high flexibility due to the accumulated experience of the traditional image processing techniques and PDE's complete theory system and numerical methods. Using a PDE based framework, it is convenient to combine, modify and extend various enhancement methods simultaneously, thus improving the performance or broader application areas of the method [4].

The rest of the paper is organized as follows: Section II provides an overview of the related techniques. Section III presents the proposed PDE based framework and some improvements. The experimental results are discussed in Section IV, followed by the conclusions in Section V.

II. Related Techniques

The first step of PDE based image processing method is to establish a model using a PDE to meet the processing requirements. A commonly used method of modeling is to establish an energy functional, get the Euler equation function using variational method which is the required PDE. Another method is to establish the corresponding PDE through comparing the expected evolution of the image with some physics process, such as heat conduction or impurity diffusion. In this section, some related works for image enhancement based on PDE are introduced.

A. Contrast enhancement

Among the image enhancement techniques, contrast enhancement is an important quality factor for providing better experience of image perception to viewers. Histogram equalization (HE) is widely used to enhance low-contrast images which is suitable to all kinds of imaging conditions. Details of traditional HE technique can be found in [2]. The HE is an operation on the whole image, the process is difficult to control effectively, thus the visual effect is not soft enough and easily cause the image details loss. A HE method based

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on PDE was introduced in [5]. Suppose an image I with a intensity value range [0,1], establish the following energy functional:

$$E(I) = \frac{1}{2}\int (I(x,y) - \frac{1}{2})^2 dxdy - \frac{1}{4}\int\int |I(x,y) - I(u,v)|dxdydudv \quad (1)$$

The steepest descent with an auxiliary variable t is used to solve the above equation as follows:

$$\frac{\partial I(x,y,t)}{\partial t} = [1 - \frac{I(x,y,t)}{I_{max}}]A_\Omega - A(I(x,y,t)) \quad (2)$$

where A_Ω is the area of the image and $A(\cdot)$ is the area function. Note that the above equation and all other evolutionary equations in this paper have initial conditions:

$$I(x,y,0) = I_0(x,y)$$

and Neumann boundary conditions:

$$\frac{\partial I}{\partial n} = 0$$

where n is the orientation perpendicular to the boundary. Equation (2) has a unique steady-state solution:

$$I(x,y,\infty) = I_{max} \times H(I)$$

where $H(I)$ is the cumulative distribution histogram of image I. This solution is the result of traditional HE method for the image. More generally, the equation (2) can be rewritten as follows [5]:

$$\frac{\partial I(x,y,t)}{\partial t} = f(I(x,y,t)) - I(x,y,t) \quad (3)$$

where $f(I(x,y,t))$ is an arbitrary image enhancement transformation.

B. *Image smoothing by PDEs*

The traditional image smoothing filters include linear smoothing filter such as average filters, Gaussian filters and Order-Statistics filters such as median filter. It is a classical result that the gaussian filter smoothing for image is second order linear diffusion process:

$$\frac{\partial I(x,y,t)}{\partial t} = \triangle I \quad (4)$$

and the median filter is a simple directed diffusion for the image. Traditional image smoothing methods are unique and easy to handle, but they have several drawbacks such as it does not only smooth noise, but also blurs important features of image such as edge information. Traditional smoothing methods cannot comprise any additional information on structures which are need being preserved.

Adaptive smoothing methods are based on the idea to apply a process which depends on local properties of the image itself. A corresponding PDE formulation was first given by Perona and Malik (PM) in 1987 [6]. They apply a nonuniform process which is named anisotropic that reduces the diffusivity at locations which having a larger likelihood to be edges. This likelihood can be measured by $|\nabla I|^2$. Regularized PM model is defined as following equation [7]:

$$\frac{\partial I(x,y,t)}{\partial t} = div[g(|\nabla G_\sigma * I|^2)\nabla I] \quad (5)$$

where g is the diffusion coefficient. If g is a constant, the above equation is reduced to the isotropic diffusion equation. It is then equivalent to the image convolving with a Gaussian function. The idea of anisotropic diffusion is to adaptively choose g such that intra-regions become smooth while edges of inter-regions are preserved. The diffusion coefficient g is generally selected to be a nonnegative function of gradient magnitude so that small variations in intensity such as noise or shading can be well smoothed, and edges with large intensity transition are retained. Among the diffusivities they propose is:

$$g(s^2) = \frac{1}{1 + s^2/\lambda^2} \qquad \lambda > 0$$

The experiments of PM were visually impressive: edges remained stable over a very long time. It was demonstrated that edge detection based on this process clearly outperforms the linear canny detector, even without applying non-maximum suppression and hysteresis thresholding. This is due to the fact that diffusion and edge detection interact in one single process instead of being treated as two independent processes which are to be applied subsequently [8].

Another method for measure likelihood of edges is $|\nabla I|$ which is well known as the total variation (TV) regularization approaches [9]. TV is widely used in image restoration technique for image denoising, which is correspond exactly with image smoothing in image enhancement technologies. Based on TV minimization framework, edges can be preserved by not over-penalizing discontinuities. Improved TV denoising model was proposed in [10] to avoid the defects of the image piecewise constant which has the following form:

$$\frac{\partial I(x,y,t)}{\partial t} = |\nabla I| div(\frac{\nabla I}{|\nabla I|}) \quad (6)$$

C. *Image sharpening by PDEs*

A classical linear sharpening operator is the negative Laplacian of the image which is sensitive to noise and having close relations to the ill-posed inverse diffusion equation. Unsharp masking is another common technique, which is somewhat similar, where the input image is being blurred and its difference from the input image is added back to the input image, increasing contrast. Gabor [11] proposed an anisotropic operator that sharpens edges by subtracting the second directional derivative in the gradient direction and adding the second directional derivative in the perpendicular level-set direction:

$$\frac{\partial I(x,y,t)}{\partial t} = -\triangle I \tag{7}$$

Osher and Rudin [12] proposed a hyperbolic equation which is similar as Kramer filter [13] proposed in 1970s, called shock filter, that can serve as a stable deblurring algorithm which behaves similarly to deconvolution. The formulation of the shock filter equation is:

$$\frac{\partial I(x,y,t)}{\partial t} = -sign(\triangle I)|\nabla I| \tag{8}$$

where

$$\triangle I = I_{xx}(I_x)^2 + 2I_{xy}I_xI_y + I_{yy}(I_y)^2$$

and

$$I_{xx} = \frac{\partial^2 I}{\partial x^2}$$

$$I_{yy} = \frac{\partial^2 I}{\partial y^2}$$

$$I_{xy} = \frac{\partial^2 I}{\partial x \partial y}$$

As noted in the original paper [12], all kinds of noise in the blurred image will also be enhanced. As a matter of fact, this process is extremely sensitive to noise and does not enhance image contrast.

III. Image Enhancement in PDE Framework

In practical work, we often need to use several enhancement methods together at the same time to achieve the better image enhancement performance. For example, in a poor contrast and noisy image, the contrast enhancement operation will lead to noise amplification, and the following smooth operation can not completely remove the noise. On the contrary, use the image filter for image smoothing first will lead to the edge information in the low contrast region loss, nested relations of the level set are changed which does not meet the requirements of shape preserving.

To solve this problem, in this section, various image enhancement methods based on PDEs are combined into one PDE based framework. The regularization parameters are used in the image evolution process to balance the conflicting evolutionary to obtain the optimal treatment effect.

A. PDE modeling

Based on the above analysis, we propose a PDE based framework which expresses image contrast enhancement, edge sharpening and image smoothing operations in one evolution equation :

$$\frac{\partial I(x,y,t)}{\partial t} = \alpha f_1(I) + \beta f_2(I) + \gamma f_3(I) \tag{9}$$

where f_1 is image contrast enhancement operation, f_2 is image smoothing operation and f_3 is image sharpening operation, α, β and γ are regularization parameters.

To alleviate the 'fade' and 'speckle' effect of global HE method, we use the piecewise linear stretch HE method as f_1. The original histogram $[D_{amin}, D_{amax}]$ is first decomposed into N parts. The number of pixels in each section is approximately equal. Then the dynamic range of the target image value range $[D_{bmin}, D_{bmax}]$ is evenly divided into N segments. Make linear interpolation in each section in the cartesian coordinate system of (D_a, D_b):

$$\begin{aligned} f^n(k) = D_{b1}(n) + [D_{b2}(n) - D_{b1}(n) + 1] \\ \times [k - D_{a1}(n)]/[D_{a2}(n) - D_{a1}(n) + 1] \end{aligned} \tag{10}$$

where,

$$k = D_{a1}(n), D_{a1}(n) + 1, ..., D_{a2}(n)$$

is pixel value range in each sub intensity range, $n = 1, 2, ..., N$.

As the ability of preserving edges using the improved TV method [10] was weakened, we add the edge preserving function g to stop the diffusion rate near the edge area of image, so as to achieve the effect of edge protection. Then TV smoothing operation can be rewritten as follows(fidelity term omitted):

$$\frac{\partial I(x,y,t)}{\partial t} = g(|\nabla I|)|\nabla I| div(\frac{\nabla I}{|\nabla I|}) \tag{11}$$

The above equation is the forward diffusion along the tangential direction of the isophote line. For the normal direction, we use the same diffusion. Therefore, the smoothing operation f_2 is:

$$f_2(I) = \frac{\partial I(x,y,t)}{\partial t} = g(|\nabla I|)(I_{NN} + I_{TT}) \tag{12}$$

where I_{NN} and I_{TT} are second derivatives of image I in the normal and tangential directions, respectively. In this paper, $g(|\nabla I|)$ is defined as follows:

$$g(\nabla I) = 1 - exp(-3.315/(\nabla(G_\sigma * I)/m)^8)$$

where G_σ denotes for Gaussian of standard deviation σ and m is a contrast parameter [14]. g is the diffusion coefficient which has a good protective effect for local details of the image. More exactly speaking, gI_{TT} can protect the edge information and gI_{NN} can effectively protected other local details such as corner and T-type region.

The shock filter is used as f_3 for edge sharpening. The diffusion for smoothing will stop which will lead to the noise beside the edge can not be effectively removed. Therefore, we using the improved shock filter in the tangential direction to remove the noise near the edge. Improved shock filter is defined as follows:

$$f_3(I) = \frac{\partial I(x,y,t)}{\partial t} = -wsign(\triangle G_\sigma * I)|\nabla I| \quad (13)$$

where,

$$w = \begin{cases} 1, & \text{if} \quad |\nabla(G_\sigma * I)| > T \\ 0, & \text{if} \quad others \end{cases}$$

where, k is a constant, T is thresholds, other function also can be used as w to improve the performance of edge sharpening.

B. Numerical Implementation

As the forward and backward diffusions are reverse processes, we use a coupled bidirectional flow iteration process with the Neumann boundary conditions as follows:

$$\begin{cases} u^0 = v^0, \quad u_G = G_\sigma * u \\ u^{n+1} = v^n + \tau(\alpha(I_{HE} - v^n) - \gamma wsign(\triangle v_G)|\nabla v|) \\ v^{n+1} = u^{n+1} + \tau\beta g \times (u^{n+1}_{NN} + u^{n+1}_{TT}) \end{cases}$$

where u^0 is the original image, τ is the time step. The idea is to calculate u first and then solve v, repeatedly, and progress this numerical process alternatively in the following simulations until maximum number of iterations reached and get the final image enhancement result.

IV. Performance Evaluation

In order to assess the validity of the proposed technique, a quantities of experiments are simulated. Experiments are carried on low-contrast image, synthesized noisy image and real color image to verify the performance of proposed method. The parameters in this model depend on different images.

To demonstrate the effectiveness of the controllable contrast enhancement operation, we select a low-contrast color image as shown in Fig. 1(a), Fig. 1(b) is the result of directly using HE operation on the intensity (I) channel, Fig. 1(c) is the result of multi-scale retinex (MSR) for color image enhancement [15], Fig. 1(d) is the result of proposed method. $\alpha = 1$, $\beta = 0$ and $\gamma = 0$, respectively for this image.

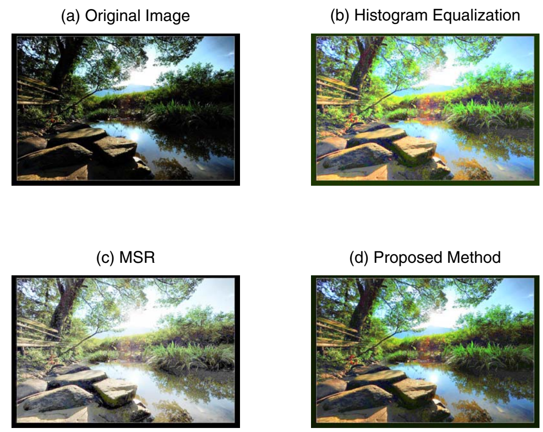

Fig. 1: Contrast enhancement results for color image

The proposed contrast enhancement operation need 75 iterations, $\tau = 0.01$. In Fig. 1, we can observe that the PDE based contrast enhancement operation achieve the best visual effects comparing with the other two methods. The traditional HE method and MSR method both lead to color distortion. Traditional method is based on the conclusion that an image whose pixels tend to occupy the entire range of possible gray levels and tend to be distributed uniformly will shows better performance. For color image, this conclusion may not feasible as the visual performance is not only depends on the intensity but also associate with hue and saturation. Controllable operation may achieve better results.

Fig. 2(a) is a synthesized noisy image, Fig. 2(b) is the result after directly histogram equalizing, image sharpening and image smoothing operations, Fig. 2(c) is using HE and traditional PM diffusion in a PDE framework, Fig. 2(d) is the result of proposed method.

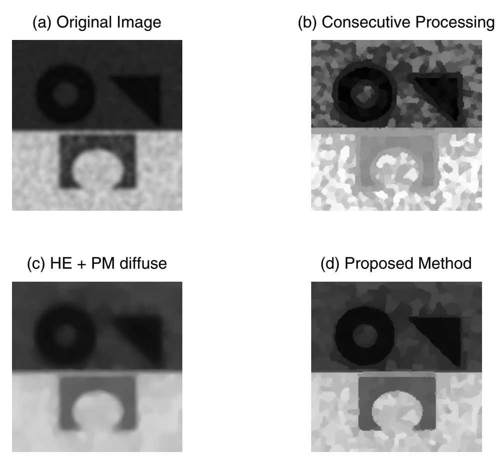

Fig. 2: Image enhancement for synthesized noisy image

In this experiment, $\tau = 0.01$, $\alpha = 1$, $\beta = 5$ and $\gamma = 5$ for our proposed method, the number of iterations is 50. Correspondingly, for the sequential method shown in Fig. 2(b), the time step is 0.01, the number of iterations for smoothing and sharpening operation is 250. According the result, we can conclude that directly using contrast enhancement, smoothing, sharpening operation will enlarge the image noise and blur the edges. Under the PDE frame, proposed method and traditional PM diffusion method both achieve better performance than the sequential method. This may due to the fact that diffusion and contrast enhancement interact in one single process instead of being treated as independent processes which are to be applied subsequently.

Fig. 3(a) is a true color image with 2% pepper and salt noise on intensity channel, Fig. 3(b) is the result after the HE, image sharpening and smoothing operations, Fig. 3(c) is using HE and traditional TV denoising [10] in a PDE framework, Fig. 3(d) is the result of the proposed method.

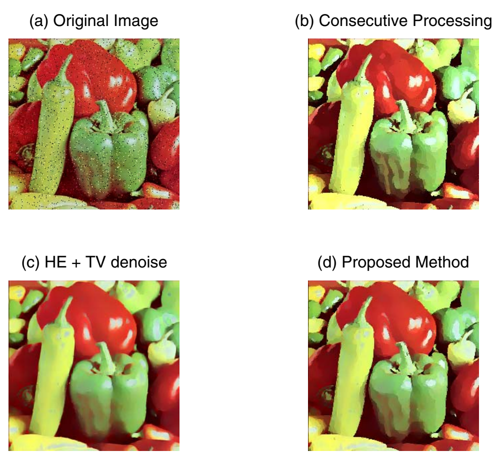

Fig. 3: Image enhancement for color image

In this experiment, $\tau = 0.01$, $\alpha = 1$, $\beta = 5$ and $\gamma = 1$ for our proposed method, the number of iterations is 50. Correspondingly, for the sequential operations, the time step is 0.01, the number of iterations for smoothing operation is 250 and for sharpening operation is 50. The results show that directly using contrast enhancement, smoothing, sharpening operation cannot remove the noise completely. Under the PDE framework, the proposed method achieve better result than traditional TV denoising operation which lead to some details loss.

V. Conclusions

In this paper, a framework based on PDE for image enhancement is proposed. Traditional HE method is evolved in a PDE model that the process of evolution can be freely controlled. Improved image smoothing and sharpening methods are presented and interact in one single process with HE method instead of being treated as independent processes which are to be applied subsequently. The experimental results show that the proposed framework can simultaneously perform contrast enhancement, image sharpening and smoothing operation. Compared with the traditional sequential processing methods, the proposed PDE based framework can achieve better visual effect.

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