

# A Novel Scheme for 3D Face Recognition Based on Partial Differential Equations

Lin Kezheng

Department of Computer Science  
and Technology  
Harbin University of Science and  
Technology  
Harbin, China  
Email: Link@hrbust.edu.cn

Liu Shuai

Department of Computer Science  
and Technology  
Harbin University of Science and  
Technology  
Harbin, China  
Email: yuan\_shuai5566@126.com

Cheng Weiyue

Department of Computer Science  
and Technology  
Harbin University of Science and  
Technology  
Harbin, China  
Email: 308147404@qq.com

**Abstract**—In this paper, we proposed a novel scheme to recognize 3D face based on partial differential equations (PDE). We proposed a novel distance between two boundary curves using the difference of the corresponding Fourier series coefficients. The distance of each adjacent boundary curves is taken as the feature of 3D face for recognition. Moreover, resemblance distance algorithm is applied for recognition. Experiment results and analysis show that the proposed scheme has a high identification rate in neutral 3D face database.

**Keywords:** 3D Face Recognition, Partial Differential Equations, Surface Reconstruction

## I. INTRODUCTION

The research of 3D face modeling and recognition has been one of the most popular topic in computer graphics and vision area in recent years. Especially, parametric 3D face model, has a wide range of applications, such as facial animation[1], facial expression simulation and analysis[2], etc. However, as always been, face recognition is a big challenging task, and it's more difficult than the 2D one obviously. To solve this problem, researchers proposed different algorithms and many kinds of models. One way is the parametric face model, in 1972, Parke used a relative low number of triangles to simulate the facial expression animation [3]. After that, on the basis of this algorithm, some derivative models were built. Until year 2007, another method, Partial Differential Equations (PDE) theory was used into modeling the 3D face [4]. Compared with the Spline-based method, this method outstands in relative small number of parameters, seamless merging of surface patches, intuitive shape generation and manipulation. Therefore, it has been considered as a powerful tool to describe the face surface.

Elyan, etc, created the parametric 3D face model using the PDEs based method. In their model, a fixed 28 boundary curves are extracted from the scanned 3D face, and then nine PDEs are created by taking these 28 curves which treated as boundary conditions. This method is of great efficiency and it provides some kind of flexibility of descriptive accuracy for 3D face model. However, in allusion to different people,

it provides different face shape and surface, so Chuanjun Wang, etc, proposed an error driven 3D face modeling scheme [5], it can create different numbers of curves and PDEs for different people, making the model more flexible. In consideration of 3D face recognition, feature extraction plays the most important roll. In the algorithm of Lu, etc [6], firstly, they make the scanned cloud-points aligned, secondly, they do the resample hierarchically, and finally 94 feature points are selected in a special area, which are treated as the feature face. Thin plate spline is used as the tool for describing the deformation. This method is great of accuracy, but the model is based on cloud-points, there exists a large amount of data, meanwhile, there will be much more coefficients to be handled. Inspired by [7], we use the PDE method to reconstruct the 3D face, for better recognition, we use the distance of every adjacent boundary curves as the face features, for the number of PDEs are fixed and the boundary curves are automatically generated, so the distance for every pair of face is different. To calculate the distance of every adjacent boundary curves, we use the principal component of the order of Fourier series as the index.

The rest of this paper is organized as following. In section II, we introduce the method on Bloor-Wilson PDE based 3D face modeling. In section III, we detail the proposed scheme, and experiment result and analysis are presented in section IV. Conclusion and future work are given in section V.

## II. BLOOR-WILSON PDE METHOD

The BWPDE method treats surface design as a boundary value problem [8], where surface can be defined using a small number of parameters. The method produces a parametric surface  $S(u, v)$ , defined as the solution to an elliptic PDE, it is described as following Eq. (1).

$$\left( \frac{\partial^2}{\partial u^2} + a^2 \frac{\partial^2}{\partial v^2} \right)^2 X(u, v) = 0 \quad (1)$$

Where  $u$  and  $v$  are the parametric surface independent variables, and  $a \geq 1$  is a parameter that controls the relative rates of smoothing between the  $u$  and  $v$  parameter directions. The partial differential operator in Eq.(1) represents a smoothing process in which the value of the function at any point on the surface is, in a certain sense, a weighted average of the surrounding values. Thus, a surface is obtained as a smooth transition between the boundary conditions. In Fig.1, we can see that the internal curves play an important role in determining the overall shape of the surface for using a four order BWPDE.

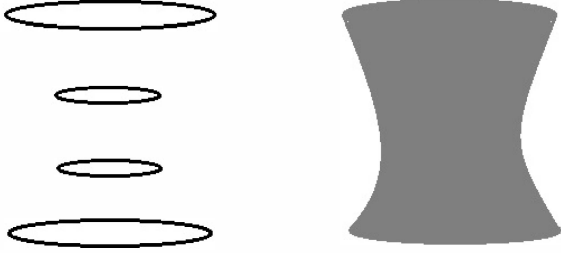


Fig. 1. Example of a PDE surface generated by BWPDE method

Assuming that the effective region in  $uv$  space is restricted to  $0 \leq u \leq 1$  and  $0 \leq v \leq 2\pi$ , using the method of separation of variable, the analytic solution to Eq.(1) can be written as the following way.

$$X(u, v) = A_0(u) + \sum_{n=1}^{\infty} [A_n(u) \cos(nv) + B_n(u) \sin(nv)] \quad (2)$$

where

$$\begin{aligned} A_0(u) &= \alpha_{00} + \alpha_{01}u + \alpha_{02}u^2 + \alpha_{03}u^3 \\ A_n(u) &= \alpha_{n1}e^{anu} + \alpha_{n2}ue^{anu} + \alpha_{n3}e^{-anu} + \alpha_{n4}ue^{-anu} \\ B_n(u) &= \beta_{n1}e^{anu} + \beta_{n2}ue^{anu} + \beta_{n3}e^{-anu} + \beta_{n4}ue^{-anu} \end{aligned}$$

The traditional methods[4][9] truncate the first terms of Eq.(2) to approximate the PDE surface, which means, The left part is defined as Eq.(3).

$$r(u, v) = r_1(u)e^{wu} + r_2(u)e^{-wu} + r_3(u)ue^{wu} + r_4(u)ue^{-wu} \quad (3)$$

Where  $w = a(K+1)$ ,  $r_1, r_2, r_3, r_4$  can be solved by calculating the difference between the original boundary conditions and the following finite Fourier series expansion:

$$F(u, v) = A_0(u) + \sum_{n=1}^K [A_n(u) \cos(nv) + B_n(u) \sin(nv)] \quad (4)$$

### III. THE PROPOSED SCHEME

In the 3D face modeling based on PDE, the parametric 3D face is represented as a series of parameters that represent the PDE solutions. It is convenient to describe the deformation through a set of boundary conditions. For a better recognition of 3D face, we make use of the distance between every adjacent boundary curves as the feature index. Our proposed algorithm mainly includes the following steps, first is the filter of large numbers of cloud-points, we deal it with a median filter, where the window size is 20; Second is the identification of the face landmarks, it includes the nose tip, the nose peaks and the inner eye corner, etc; Then what we should do is the align of neutral face with all directions by calculating the mid-line plane [10]; And next the interpolation is necessary for better extraction of boundary curves; Finally we extract the features, the distance of adjacent boundary curves, meanwhile, resemblance distance algorithm is applied for recognition.

#### A. Preprocessing

The original 3D face data is supposed in the form of point cloud. It can be represented as a set of points  $P_i = \{x_i, y_i, z_i\} i = 1, 2, \dots, N$ , where  $N$  is the total number of points, and  $(x_i, y_i, z_i)$  represents the point. For avoiding the effect of noise, we first use a median filter to reduce it. On the purpose of generating a PDE face and convenient for recognition, we transform it into a depth map [11]. In the depth map, the  $x$  and  $y$  coordinates are regarded as the grid plane and  $z$  coordinate indicates the depth information. In our scheme, we use 1mm as a unit for resampling, the resampled depth information can better describe the 3D face geometric surface. Fig.2 shows a 3D face depth image in 3 directions: front, below and side. After the 3D face images are transformed into the depth map representation, we divide the face image into 5 facial regions with 9 PDEs according to the standard definition of 3D face in MPEG-4 [12].

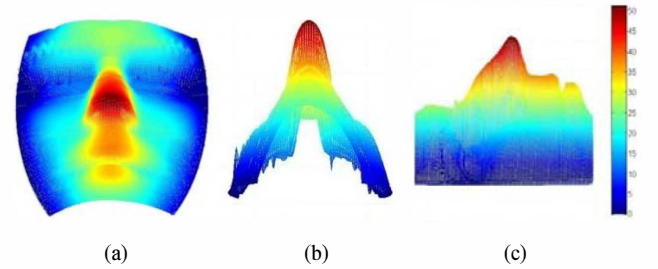


Fig. 2. 3D depth face image in 3 directions.(a) Front (b) below (c) side

The 5 facial regions are named as the forehead area, the eye area, the nose area, the mouse area and the chin area from top to bottom. In consideration of different regions appear different complexities, we use different PDE curves to rebuild them, setting the PDEs rate as 1:2:2:2:2. Fig. 3 shows the 5 segmentation regions of a 3D face.

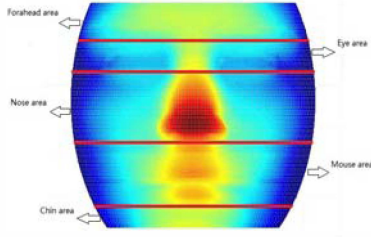


Fig. 3. 3D face segmentation

### B. Adaptive parameterization and reconstruction

The adaptation method can describe the different regions of face adaptively, and there are two mainly reasons for using this method. Firstly, the importance of different regions varies. For example, in facial expression analysis and synthesis, the accuracy of the eye area is much more important than forehead area. And secondly, in order to avoid the over-parameterization problem, it's necessary to use suitable descriptive resources. For example, if too many terms of parameters are used to depict the reconstructed 3D face, the resulting parameters might become unstable, which means small changes of the original input data will result in large fluctuation of the reconstructed parameters.

In the work of Elyan and Yu[4][9], a direct truncation of  $K$  is used to describe the corresponding 3D face surface, and here we use the method of Chuanjun Wang[13] to reconstruct the 3D face surface, where a different function terms of  $A_0(u), A_n(u)\cos(nv), B_n(u)\sin(nv)$  are selected and used for different facial regions, and it is based on the Fourier series representation of the boundary conditions and each boundary condition curve can be decomposed into a sum of sine and cosine components of different frequencies as illustrated in Eq.(5)

$$g = a_0 / 2 + \sum_{n=1}^{\infty} (a_n \cos(n\omega) + b_n \sin(n\omega)) \quad (5)$$

The threshold of the Fourier term of  $a_0, a_n, b_n$  is all set to be 0.4mm, that means  $|a_0|, |a_n|, |b_n|$  above the sensor resolution of 0.4mm will be selected, while the others will be set to 0 and ignored. The boundary curves are selected with an iterative process but rather a linear one. Specifically, based on the segmentation of 3D face, 10 boundary condition curves are first extracted, and the internal boundary condition curves of each surface patch are adaptively extracted.

### C. Feature extraction of 3D face

We know that 3D face geometrical surface based on PDE algorithm can be presented by a series of discretization boundary curves, when face expression deformation occurs, it reflects in PDE surface, and it also reflects in boundary curves. For these curves can be presented by a series of Fourier series, it also reflects in these Fourier series. For this reason, we can model the deformation of 3D face in

boundary curves Fourier series space. In terms of a set of discretization PDEs, it can be written in the following form (6).

$$F_p = \{X_1(u, v), X_2(u, v) \dots X_N(u, v)\} \quad (6)$$

$F_p$  is a PDE face, and  $X_i(u, v)$  presents the  $i$ th PDE surface, for each PDE surface, it can be presented by a set of Fourier series, like the expression in (7).

$$F_{fs} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ \dots \\ C_M \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1k} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2k} \\ & & & \dots & \\ c_{M1} & c_{M2} & c_{M3} & \dots & c_{MK} \end{pmatrix} = (B_1 \ B_2 \ \dots \ B_K) \quad (7)$$

Where  $M$  is the total number of PDE surfaces,  $M = 3 * N + 1$ , presents the highest order of the Fourier series. What if the  $K$ th was not selected, then  $C_{jk} = 0$ ,  $B_K = (b_K^{(1)} \ b_K^{(2)} \ \dots \ b_K^{(M)})^T$ , it describes the number of  $M$  coefficients of the  $K$ th order of Fourier series

For every 3D face, we use the fixed 28 curves to reconstruct it automatically using the method of adaptive parameterization, and the reconstruction error is very low. For better recognition of 3D face, we propose a novel method by using the distance between two adjacent curves as the index for recognition. In order to extract the mentioned distance features, we define a novel distance between adjacent boundary curves. Suppose each curve is presented by Fourier series, then the Fourier series coefficients of a boundary curve can be expressed as the following form, shown in Eq.(8).

$$C_i = (c_i^1 \ c_i^2 \ \dots \ c_i^k), 1 \leq i \leq 27 \quad (8)$$

Thus, the distance between every two adjacent curves can be presented as the form in Eq.(9).

$$D_i = (|c_i^1 - c_{i+1}^1| \ |c_i^2 - c_{i+1}^2| \ \dots \ |c_i^k - c_{i+1}^k|) \quad (9)$$

where  $k$  presents the highest order of Fourier series. For the arbitrary two adjacent curves, if there doesn't exist the distance of the  $k$ th order of Fourier series, set  $c_i^k$  to be 0, we say that the dissimilarity of every order of Fourier series reflects the information of two adjacent curves.

For 3D face recognition, we use the resemblance distance method to deal with it. It is defined as the following form in Eq. (10).

$$S_i = \sum_{i=1}^{27} (|D_i - D|) \quad (10)$$

Where  $D_i$ , from the neutral database, presents the 3D face features set, and  $D$  is a 3D face to be matched. It is like a classifier, our purpose is to look for a smallest  $S_i$ , then we say, the 3D face  $D$  belongs to the  $i$ th in 3D face database. Fig. 4 illustrates the method of boundary curve distance features of one PDE in eye area.

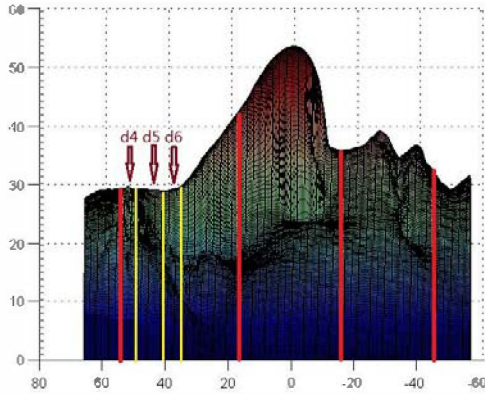


Fig. 4. Distance of eye area boundary curves

#### IV. EXPERIMENTAL RESULTS AND ANALYSIS

In the process of experiments, we mainly use the Bosphorus 3D face database for research [14]. This database features a rich set of facial expression deformations and it has been widely used for evaluation of face recognition algorithm as well as facial expressions analysis. The expression variations in this database contain not only six universal emotional expressions (happiness, surprise, fear, sadness, anger and disgust) but also face action units of the facial action coding system. In our experiments, 1740 3D facial scans of 105 subjects are used, where each subject includes 10-16 facial scans with different facial expression. The experiment is conducted with Matlab 2010b.

Firstly, we consider the situation of using different PDE boundary curves for reconstruction in different 3D face areas. Because different areas presents different complexity, using less PDEs to reconstruct the 3D face will have a relatively higher error, and it's not necessary to use more PDEs on the contrary. So we first find the suitable PDEs to reconstruct the 3D face, it is clearly shown in the following Fig. 5-9. We set the accuracy of axis resampling to be 0.4mm, from [4] we know that using 28 boundary curves can perfectly reconstruct the face, the ROC curves is shown in Fig. 10. For the forehead area, select one PDE, and for the other areas, we use two PDEs for each one. Every curve is presented by the form of Fourier series, the order is truncated to 10. Thus, we can get a high accuracy and simply expressed 3D face, from every 3D face, we can get 27 adjacent distance features for recognition.

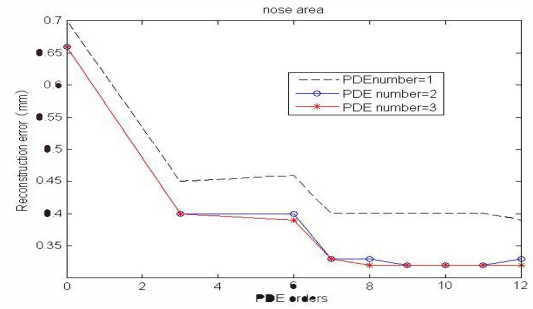


Fig. 5. Errors from different number of PDEs in Nose area

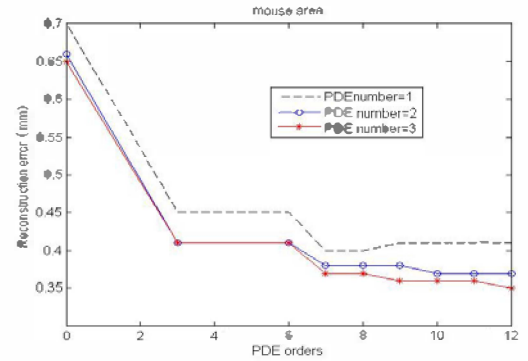


Fig. 6. Errors from different number of PDEs in mouse area

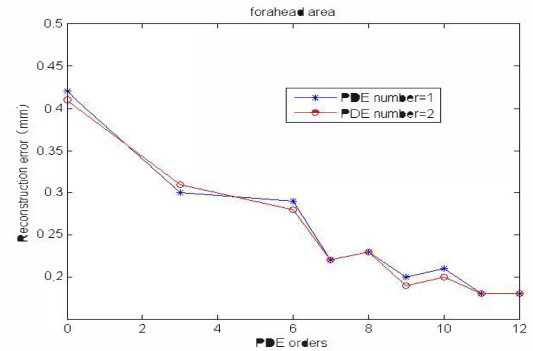


Fig. 7. Errors from different number of PDEs in forehead area

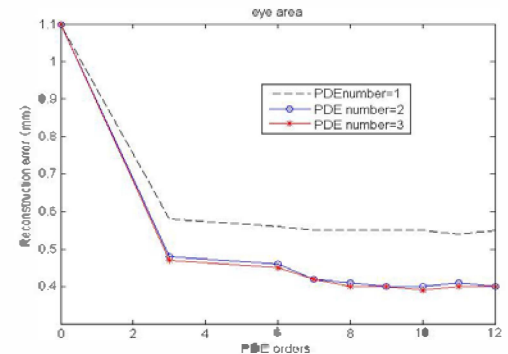


Fig. 8. Errors from different number of PDEs in eye area

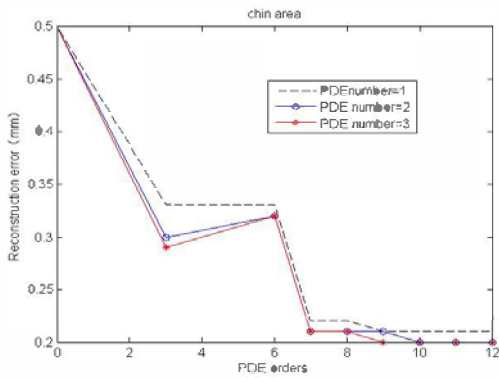


Fig. 9. Errors from different number of PDEs in chin area

For 3D face recognition, there exists 105 3D faces in the database, and we mainly use the identification rate as the evaluation index. In our experiment, we respectively choose 105 neutral 3D faces, 428 lower facial action units and 428 upper facial action units, and also 16 emotional expression pictures for each 3D face. Table 1 shows the testing results.

TABLE 1 IDENTIFICATION RATE OF DIFFERENT TRAINING SET

Test	Testing set	Number of test	Correct matched	Identification rate
$T_1$	Neutral	105	105	100%
$T_2$	Upper	428	395	92.3%
$T_3$	Lower	428	388	90.8%
$T_4$	Expression	1740	1522	87.5%

From Table 1, we know that for neutral 3D face, it can be matched absolutely right, and even for the upper and lower facial action units, the identification rate 92.3%, 90.8% is higher than 82.9% which comes from the paper of Alyuz, etc, [15].

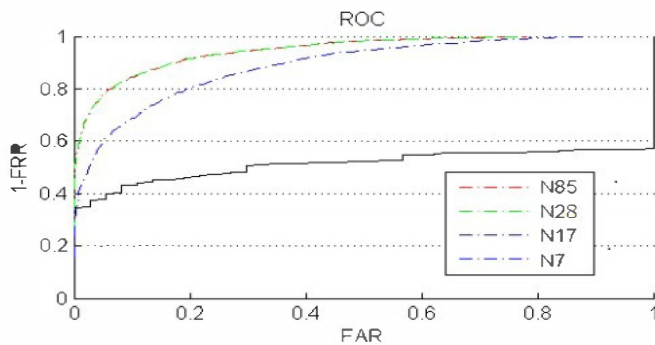


Fig. 10. Effects from different PDE curves for recognition rate

For emotion expressions recognition, the identification rate is 87.5%. Fig. 11 shows the rough identification rate between our method and the method in [15], we can know that our algorithm Fourier Series based Expression Deformation Model (FSEDM) has higher recognition rate than Alyuz's method Expression Deformation Model (EDM).

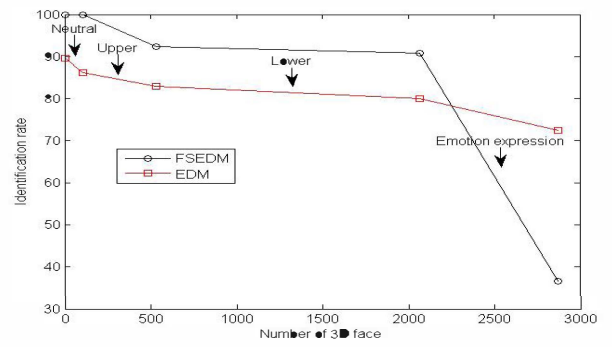


Fig. 11. Identification rate between our method and Alyuz's method

## V. CONCLUSION

In this paper, we have proposed a novel scheme for 3D face recognition using the distance of adjacent PDE boundary curves as face features. In the new scheme, we calculated the distance by using the difference of Fourier series instead, getting 27 distance features of each 3D face for recognition through resemblance distance method. Results show that this new scheme has higher identification rate than the other algorithm in neutral 3D face database. But for emotion expressions recognition, we need to do more research.

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