

# Continuous distance-based skyline queries in road networks

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## ABSTRACT

In recent years, the research community has introduced various methods for processing skyline queries in road networks. A skyline query retrieves the skyline points that are not dominated by others in terms of static and dynamic attributes (i.e., the road distance). This paper addresses the issue of efficiently processing continuous skyline queries in road networks. Two novel and important distance-based skyline queries are presented, namely, the *continuous  $d_c$ -skyline query* ( $Cd_c$ -SQ) and the *continuous  $k$  nearest neighbor-skyline query* ( $Cknn$ -SQ). A grid index is first designed to effectively manage the information of data objects and then two algorithms are proposed, the  $Cd_c$ -SQ algorithm and the  $Cd_c$ -SQ<sup>+</sup> algorithm, which are combined with the grid index to answer the  $Cd_c$ -SQ. Similarly, the  $Cknn$ -SQ algorithm and the  $Cknn$ -SQ<sup>+</sup> algorithm are developed to efficiently process the  $Cknn$ -SQ. Extensive experiments using real road network datasets demonstrate the effectiveness and the efficiency of the proposed algorithms.

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## 1. Introduction

Given a set of multidimensional points  $D$ , a *skyline query* retrieves the data points that are not dominated by any other point. Specifically, point  $p$  *dominates* another point,  $p'$ , if its value in each dimension is as good or better than that of  $p'$  and is better in at least one dimension. The points retrieved by executing the skyline query are termed the *skyline points* ( $SP$  for short). Fig. 1 shows an example of a skyline query, where the data points correspond to hotels with two properties: rank and price (note that a higher rank and a lower price are desired in this example). Hotels  $o_1$ ,  $o_2$ , and  $o_3$  are the  $SP$  among the five hotels. The others (i.e.,  $o_4$  and  $o_5$ ) are dominated by  $o_1$  and  $o_3$  in terms of rank and price. As the skyline query is a powerful tool for preference-based data analysis, and as it has a wide

range of real applications, it has attracted much attention in the database community [1–11].

In recent years, several studies [12–15] have investigated how to process skyline queries over data objects (e.g., restaurants and hotels) in road networks. Different from the traditional skyline query, processing a skyline query in road networks requires taking the location of the query object into consideration. As shown in Fig. 2, the five hotels ( $o_1$  to  $o_5$ ) and the user issuing a skyline query,  $q$ , are located in a road network. In this scenario, the road distance from each hotel to  $q$  (defined as the distance along the shortest path between them) becomes one of the dimensions considered when determining the  $SP$ . The road distance of each hotel needs to be computed first, and then the three properties of distance, rank, and price can be used to find the  $SP$  of  $q$ . As can be seen, hotel  $o_5$ , which is previously dominated by  $o_1$  and  $o_3$  (referring to Fig. 1) is no longer dominated, because  $o_5$  is closer to  $q$  than  $o_1$  and  $o_3$  (i.e.,  $o_5$  is better in the distance dimension), and thus,  $o_5$  is promoted to a  $SP$ . When the query object moves in the road network, the distance between the query object and object  $o_i$  changes as well. Hence, such a dimension is referred to as a *dynamic dimension*.

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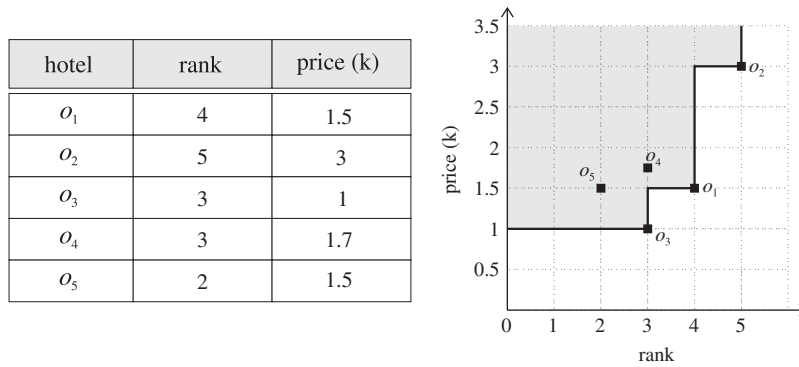


Fig. 1. Traditional skyline query.

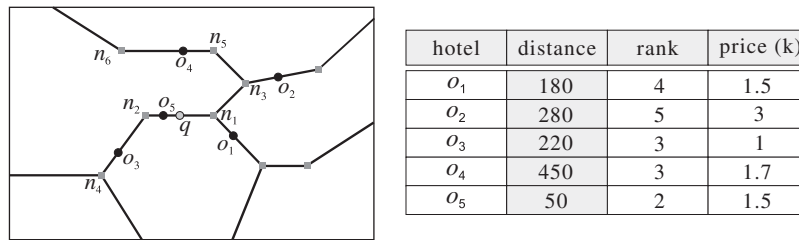


Fig. 2. Skyline query in road networks.

As opposed to the dynamic dimension, the other dimensions that remain unchanged (e.g., price and rank) are called the *static dimensions*. When a dynamic dimension is introduced into skyline query processing, the complexity of processing such a query becomes much higher, as the values in the dynamic dimension (e.g., distance) continue to change.

Previous methods for processing a skyline query in road networks have focused exclusively on determining the *SP* for a *static* query object. That is, these methods can only be used to efficiently process a *snapshot skyline query*. However, in road network applications, the query object *moves with time*. Due to the movement of the query object, the *SP* result will inevitably change. If the techniques for processing a snapshot skyline query are used, they must be repeatedly evaluated to keep producing the new *SP*. This incurs a tremendous query re-evaluation cost, especially in a highly dynamic environment where the query object frequently changes location. In this paper, a query retrieving the *SP* in a dynamic environment is referred to as a *continuous skyline query*. The goal of this paper is to study how to process such a continuous skyline query efficiently in road networks.

In some cases, there can be a large number of *SP* for a skyline query [1,8,16], with most of the *SP* are far away from the query object, and therefore, the query object is usually not interested in them. For instance, in order to find accommodation as soon as possible, a traveler will prefer a closer hotel with good quality to one that is farther away. As a result, retrieving the *SP* that are far away from the query object is simply a waste of time and incurs additional processing costs. For this reason, this paper presents two novel *distance-based* continuous skyline queries that find only the *SP* that are closer to the

query object. The first is the *continuous  $d_e$ -skyline query* ( $Cd_e$ -SQ for short), and the second is the *continuous  $k$  nearest neighbor-skyline query* ( $Cknn$ -SQ for short). The two queries are defined as follows:

- $Cd_e$ -SQ: Given path  $P_q$ , along which query object  $q$  moves, a set of data objects  $S_o$  and distance value  $d_e$ ,  $Cd_e$ -SQ retrieves a set of skyline points,  $SP_p$ , for each point  $p$  on  $P_q$ , such that the road distance from each  $o \in SP_p$  to point  $p$  is less than or equal to  $d_e$ . The *SP* satisfying  $Cd_e$ -SQ (i.e., those within the distance range  $d_e$ ) are the  $d_e$ -*SP*.
- $Cknn$ -SQ: Given query path  $P_q$ , a set of data objects  $S_o$ , and a value of  $k$ ,  $Cknn$ -SQ finds a set of skyline points,  $SP_p$ , for each point  $p \in P_q$ , such that the number of skyline points in  $SP_p$  is equal to  $k$  and the road distance of each  $o \in SP_p$  to  $p$  is less than or equal to that of each *SP* in  $S_o - SP_p$  (that is, objects in  $SP_p$  are the  $k$  nearest *SP* of point  $p$ ). The *SP* retrieved by  $Cknn$ -SQ are the  $knn$ -*SP*.

$Cd_e$ -SQ and  $Cknn$ -SQ are two useful queries that can be found in many fields and application domains. A real-world example is that of a traveler who is planning a trip from city A to city B, who may want to know which hotels are better to stay in en route. In this scenario, the traveler can issue a  $Cd_e$ -SQ to find the nearest hotels (i.e., within distance range  $d_e$ ) with good quality (i.e., higher rank and lower price) during the trip. Another real-world application is collaborative promotion. Consider a set of theaters and a set of restaurants in a road network. An advertisement company can pose a  $Cknn$ -SQ to find the  $k$  skyline theaters that are closer to street C. Also, the company can find the  $k$  nearest skyline restaurants by executing a

*Cknn-SQ*. Then, a collaborative advertisement about the theaters and the restaurants (e.g., a restaurant offering 10% discount for customers watching movies in a theater) is broadcast to the pedestrians on street C so as to attract more customers.

Fig. 3 illustrates an example of processing a  $Cd_e$ -SQ and a  $Cknn$ -SQ, where objects  $o_1$  to  $o_4$  and query object  $q$  are located in a road network, represented as a graph consisting of nodes and edges. Fig. 3(a) shows the static attributes (i.e., rank and price) of the four objects. In this example, query object  $q$  moves from node  $n_1$  to node  $n_2$  (the query path is  $\overline{n_1 n_2}$ ). Assume that a  $Cd_e$ -SQ is issued to find the  $d_e$ -SP whose road distances to  $q$  are within 300 (i.e.,  $d_e = 300$ ). When  $q$  is located at point  $p_1$  (as shown in Fig. 3(b)), only object  $o_2$  is the  $d_e$ -SP for the  $Cd_e$ -SQ. Although  $o_1$  is not dominated by  $o_2$ , it cannot be a  $d_e$ -SP because its distance to  $q$  is greater than  $d_e$ . When  $q$  moves to point  $p_2$  (as shown in Fig. 3(c)), the distance of object  $o_4$  to  $p_2$  is equal to that of object  $o_2$ . That is, when  $q$  is located at the left side of  $p_2$ , object  $o_4$  becomes closer to  $q$  than  $o_2$ . Thus,  $o_4$  is better than  $o_2$  in the distance dimension, and  $o_4$  becomes a SP. As the road distances of  $o_2$  and  $o_4$  are both less than  $d_e$ , they are the  $d_e$ -SP. Consider another example of finding the  $2nn$ -SP of  $q$  from  $p_1$  to  $p_2$  (i.e., processing a  $C2nn$ -SQ). When  $q$  is located at point  $p_1$ , objects  $o_2$  and  $o_1$  are the  $2nn$ -SP of  $q$ . Note that although  $o_4$  is closer to  $q$  than  $o_1$ , it still cannot be a  $2nn$ -SP. This is because  $o_4$  is dominated by  $o_2$  in terms of distance, rank, and price. However, once  $q$  moves to the left side of  $p_2$ ,  $o_4$  is better than  $o_2$  in distance dimension, and thus,  $o_4$  becomes a SP. Since  $o_4$  and  $o_2$  are closer to  $q$  than  $o_1$ , the  $2nn$ -SP are changed to  $o_4$  and  $o_2$ .

To process the  $Cd_e$ -SQ and the  $Cknn$ -SQ in road networks, the road distances between the data objects and the query object need to be computed. Although the road distance can be computed based on Dijkstra's algorithm [17] or the  $A^*$  algorithm [18], which have been shown to be simple and efficient, computing the distances of all data objects will incur extremely high I/O and CPU costs, especially for a road network consisting of a large number of edges and nodes. An even greater challenge comes from the query object

moving along a query path instead of being static. Hence, the distances from data objects to the query object change as the query object moves, resulting in numerous distance re-computations. It is necessary to design effective pruning strategies that can remove data objects that should not be in the query results, so as to significantly reduce the amount of distance re-computations.

Another major problem to be tackled is how to avoid repetitively processing snapshot skyline queries for any point on the query path. Consider again the example in Fig. 3. If the  $Cd_e$ -SQ is processed, object  $o_4$  is added into the query result as  $q$  moves to the left side of point  $p_2$ . That is, after point  $p_2$ , the  $Cd_e$ -SQ result changes from  $\{o_2\}$  to  $\{o_4, o_2\}$ . Similarly, when the  $Cknn$ -SQ is evaluated, the  $Cknn$ -SQ result changes after point  $p_2$  (i.e., from  $\{o_2, o_1\}$  to  $\{o_4, o_2\}$ ). These points, after which the query result changes (e.g., point  $p_2$ ), are the *result turning points*. An important characteristic of such a result turning point is that the query result between two consecutive result turning points remains the same. Based on this characteristic, the problem of repetitively processing snapshot skyline queries can be greatly reduced to finding all result turning points and their corresponding query results.

The major contributions of this paper are summarized as follows:

- Two novel distance-based continuous skyline queries are presented, namely, the  $Cd_e$ -SQ and the  $Cknn$ -SQ, to find the skyline points in road networks. The two queries are important types of skyline queries with many real applications.
- Appropriate data structures are designed to represent a road network consisting of edges and nodes, in order to store information of the data objects in the network. Furthermore, a grid index is used to access only a small proportion of data objects while computing the  $Cd_e$ -SQ and the  $Cknn$ -SQ results, allowing the required I/O and CPU costs to be greatly reduced.
- Two algorithms are proposed, namely, the  $Cd_e$ -SQ algorithm and the  $Cd_e$ -SQ<sup>+</sup> algorithm, which are associated with the grid index to efficiently determine the

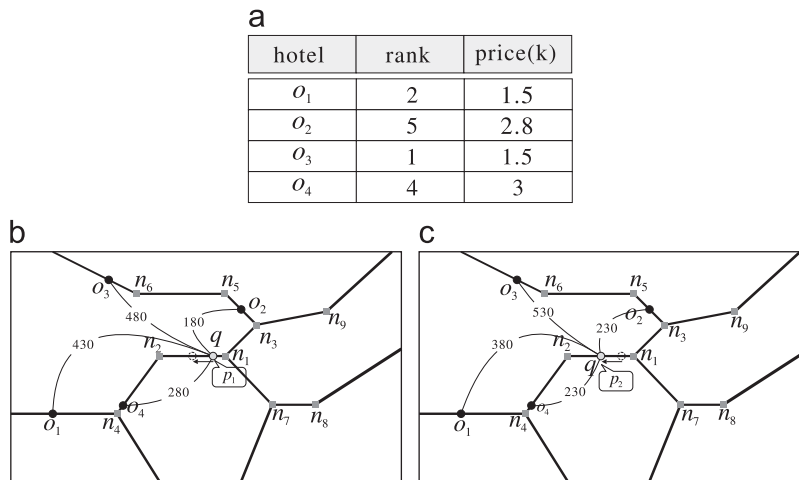


Fig. 3. Example of processing  $Cd_e$ -SQ and  $Cknn$ -SQ. (a) Static attributes of objects. (b) Distances of objects to  $p_1$ . (c) Distances of objects to  $p_2$ .

$d_e$ -SP for any points on the query path. Similarly, two algorithms are developed, the *Cknn-SQ algorithm* and the *Cknn-SQ<sup>+</sup> algorithm*, to find the *knn-SP* for the query path.

- A comprehensive set of experiments is conducted. The performance results demonstrate the efficiency and the usefulness of the proposed *Cd<sub>e</sub>-SQ* and *Cknn-SQ* processing algorithms.

The remainder of this paper is organized as follows. In Section 2, we first review the related work for processing the skyline queries, and then discuss the previous studies on answering the continuous range and *KNN* queries in road networks. Section 3 describes the data structures used to represent a road network and the grid index used to facilitate the query processing. In Sections 4 and 5, we present how the proposed algorithms can be used to answer the *Cd<sub>e</sub>-SQ* and the *Cknn-SQ*, respectively. Section 6 shows extensive experiments on the performance of our methods. In Section 7, we conclude the paper with directions on future work.

## 2. Related work

Processing skyline queries in road networks is an emerging research topic in recent years. In this section, we first review the existing methods for processing the traditional skyline queries. Then, we discuss the related work that investigates how to answer the skyline queries in road networks. Finally, we survey some related work that focuses on answering the continuous range and *KNN* queries in road networks, which are similar to the *Cd<sub>e</sub>-SQ* and the *Cknn-SQ* presented in this paper.

### 2.1. Traditional skyline queries

The skyline query, also known as the Maximal Vector Problem [19,20], was first studied in the area of computational geometry. Several main-memory algorithms had been proposed to solve the maximal vector problem. However, those main-memory algorithms are inefficient for the skyline query, due to the large sizes of data sets. Borzsonyi et al. [1] first introduced the skyline operator into database systems and proposed two algorithms, Block-Nested-Loops (BNL) and Divided-and-Conquer (D&C), to evaluate the skyline result. BNL sequentially scans the entire dataset and compares each new data object to all skyline candidates kept in memory. Only the data objects not dominated by others are kept as skyline candidates. D&C recursively divides the dataset into partitions so as to fit in memory, and evaluates the local skyline points for each of them. Then, the global skyline points are obtained by progressively merging the local ones. However, both BNL and D&C need to process the entire dataset before reporting skyline points. To progressively report skyline points, Chomicki et al. [2] proposed the Sort-Filter-Skyline (SFS) algorithm which is an improvement of BNL. The idea of SFS is to sort the entire dataset in advance according to a monotonic preference function (e.g., sum of dimension values). Sorting the dataset guarantees that each data object cannot be dominated by

ones behind it in the sorted data list. Therefore, a data object not dominated by others in front of it can be directly reported as a skyline point. A main disadvantage of SFS is that all data objects should be scanned at least once after sorting. Bartolini et al. [21] further proposed the Sort and limit skyline algorithm (SaLSa) to avoid scanning the complete set of sorted data objects. Tan et al. [10] developed two progressive methods to improve the skyline query performance. The first method encodes each data object into a bit vector, and then vectors of all objects form a bitmap. Benefiting from the bitmap transformation, a data object can be quickly determined whether it is a skyline point or not by applying a bitwise *and* operation. The second method is to transform high dimensional data objects into a single dimensional space so that a B<sup>+</sup>-tree can be used to index them. Then, the local skyline points are found by traversing the B-tree and to be merged into a global one. Kossmann et al. [5] observed that the nearest object to the origin must be a skyline point. Based on this observation, they first use an R-tree to find the nearest neighbor (NN), and then based on the NN the data space is partitioned into one dominated subspace and two non-determined subspaces. The data objects in the dominated subspace can be pruned, and the next nearest neighbor is iteratively found in each non-determined subspace. The subspace is recursively partitioned until all skyline points are reported. However, a major problem of such a method is that multiple traversals of the R-tree are required for determining the skyline points. To overcome this problem, Papadias et al. [8] proposed a Branch and Bound Skyline (BBS) algorithm to retrieve the skyline points by prioritizing accesses at partially dominated nodes of the R-tree. All of these research mentioned above focused exclusively on processing snapshot skyline queries over data objects with static dimensions.

There have been some work [22–24] considering the problem of processing continuous skyline queries in streaming databases, in which the dimension value of each data object changes with time. As the changing dimension values of each object would inevitably result in different skyline points at different time instants, these works mainly investigated how to efficiently and rapidly update the skyline points. Zhang et al. [25] further addressed the issue of processing continuous skyline queries for streaming data with uncertainty. Recently, Huang et al. [26] were the first to take into consideration the distance between data objects in processing the continuous skyline queries. However, their approach aims at the Euclidean space rather than road networks as we considered in this paper. As the road distance needs to be computed based on the connectivity of the network rather than simply using the two objects' locations, efficiently processing continuous skyline queries in road networks is much more complicated.

To process the continuous skyline queries, we design a pruning strategy based on the Euclidean lower-bound property that *the road distance of an object to the query object is always greater than or equal to its Euclidean distance*. If object's road distance is close to its Euclidean distance, the pruning strategy will perform well because much expensive road distance computation can be avoided by considering their Euclidean distances. As such,

our pruning strategy utilizes the Euclidean distance to prune the non-qualifying objects so that the algorithm needs to compute the road distances of candidates only.

## 2.2. Skyline queries in road networks

In recent years, processing skyline queries in road networks has received considerable attention. Deng et al. [12] extended the concept of the spatial skyline [27] to road networks and presented the *multi-source skyline query* (and MSQ for short). Given a set of  $m$  query objects and a set of  $n$  data objects in a road network, each data object  $o$  is mapped to a  $m$ -dimensional point, where the value of the  $i$ th dimension refers to the road distance between  $o$  and the  $i$ th query object. Then, MSQ retrieves the skyline points that are not dominated in terms of the  $m$  dimensions. Deng et al. proposed three algorithms: Euclidean Distance Constraint (EDC), Lower Bound Constraint (LBC), and Collaborative Expansion (CE), to solve the MSQ problem. To improve the search performance, EDC and LBC utilize Euclidean distance as the lower bound of road distance to prune data objects. As for CE, the pruning strategy is to start from the  $m$  query objects to search the road network for candidate skyline objects. Once an object has been visited  $m$  times, those objects that have never been visited can be pruned. However, these algorithms may generate too many candidates and cause unnecessary road distance computation. Hence, Zou et al. [15] proposed the Shared Shortest Path (SSP) algorithm associated with the Shortest Path Tree (SP-Tree) to overcome the problems. The criteria for determining the skyline points in the above methods are based on the road distances between data objects and query objects. Other studies [28,13] considered the skyline problem in *multi-cost transportation networks* (MCN), where each edge (i.e., road segment) is associated with multiple cost values and the skyline points are determined based on these cost values. Huang et al. [29] studied another skyline problem of finding the skyline points that are not dominated in terms of only two attributes: (1) their network distance to a query location  $q$  and (2) the detour distance from  $q$ 's predefined route on the road network.

Jang et al. [30] were the first to address the issue of processing continuous skyline queries in road networks. The idea is to pre-compute a range  $R$  for each data object  $o$  such that if the query object is within  $R$ , then  $o$  must be a skyline point in terms of dynamic dimension (i.e., its road distance to the query object) and static dimension. Then, the skyline points of the query object can be determined based on the pre-computed ranges of all data objects. However, two major problems have limited the applicability of this method. The first problem is that pre-computing all object ranges incurs tremendous processing cost, especially for a road network with large size. The second is that the skyline result cannot provide useful information to the user because the skyline points far away from the query object are also included in the query result. Hence, the method in [30] cannot be applied to  $Cd_k$ -SQ and  $Cknn$ -SQ (in which the skyline result is affected by the user given  $d_k$  and  $k$ ). More appropriate

and efficient methods must be designed to answer these two types of queries.

## 2.3. Continuous range and KNN queries

Jensen et al. [31] proposed a data model and a definition of abstract functionality required for KNN queries in spatial networks databases. To compute network distances, they adopt the Dijkstra's algorithm for online evaluation of the shortest path. Papadias et al. [32] described a framework that integrates network and Euclidean information, and answers static range and KNN queries. Kolahdouzan et al. [33] proposed a solution-based approach for static KNN queries in spatial networks. Their approach, called VN<sup>3</sup>, precalculates the network Voronoi polygons (NVPs) and some network distances, and then processes the KNN queries based on the properties of the Network Voronoi diagrams. Kolahdouzan et al. [34] and Cho et al. [35] developed different effective techniques, UBA and UNICONS, to reduce the number of K NN evaluations by allowing the KNN result to be valid for a time interval. However, the techniques are adequately designed to deal with continuous KNN queries over static objects. Mouratidis et al. [36] first addressed the issue of continuous monitoring KNN on moving objects and proposed an incremental monitoring algorithm (IMA) to re-evaluate query at those time instants at which updates occur. Due to the nature of discrete location updates, IMA would return incorrect KNN result as long as there exist some time points within two consecutive updates at which the KNN set changes. To overcome this limitation, Huang et al. [37] proposed a CK NN method to determine the KNN at each timestamp under the situation that all objects (including the query object) move continuously in a road network.

Although the methods proposed for the above queries can be used to find the objects within a distance range or the  $k$  nearest objects, they cannot directly applied to answering the  $Cd_k$ -SQ and the  $Cknn$ -SQ presented in this paper. For the  $Cd_k$ -SQ (the  $Cknn$ -SQ), the objects within the distance range  $d_k$  (the  $k$  nearest objects) may still be dominated by other objects and cannot be the  $d_k$ -SP (the  $knn$ -SP). Thus, efficient technique for determining the dominance relationship between objects is required in processing the  $Cd_k$ -SQ and the  $Cknn$ -SQ, especially for continuous queries with a long query path. Moreover, finding the result turning points for the  $Cd_k$ -SQ and the  $Cknn$ -SQ is more complex than that for the continuous range and KNN queries. This is because both the road distance and the dominance relationship of objects are the criteria for determining the result turning points (however, only the road distance is considered in processing the continuous range and KNN queries).

## 3. Data structures and grid index

In the proposed system, all data objects are located in a road network, which is represented as an undirected weighted graph consisting of a set of nodes and a set of edges. Since databases contain large amounts of information that need to be maintained (e.g., information of data



objects, nodes, and edges), a grid index is used to efficiently manage such information. **However, it should be noted that the proposed algorithms (which will be discussed later) are generic and can be used together with the existing popular index structures.**

The data space covering the entire road network is first divided into  $N \times N$  grid cells, and then for each grid cell  $g(i,j)$ , information of the network and data objects is stored in the following three tables, respectively:

- The *edge table*  $T_{edge}$  stores the intersection information of each edge  $e$  with cell  $g(i,j)$ . The information contains: (1) the nodes  $n_i$  and  $n_j$  connecting edge  $e$  if  $n_i$  is the node within  $g(i,j)$ ; (2) the length  $len$ ; and (3) cell  $g(m,n)$  enclosing node  $n_j$ .
- The *node table*  $T_{node}$  maintains the coordinate information  $(x,y)$  of each node that is enclosed in  $g(i,j)$ .
- The *object table*  $T_{obj}$  stores the information of each object  $o$  that is located on the edge intersecting cell  $g(i,j)$ . The information includes: (1) the coordinates of  $o$ ; (2) the edge  $e$  containing  $o$ ; (3) the static attributes  $SA$  (e.g., price and rank of a hotel); and (4) a set  $S_{DO}$  of the objects dominated by  $o$  in terms of  $SA$ . Note that the  $S_{DO}$  of object  $o$  is preprocessed by comparing its  $SA$  to those of the other objects. As the  $SA$  of each object is static, the preprocessed  $S_{DO}$  will remain unchanged.

To exemplify the information maintained in the three tables, a road network is shown in Fig. 4(a), which consists of 15 edges and 17 nodes. As shown in Fig. 4(b), the data space covering the road network is

divided into  $4 \times 4$  grid cells, with the cell numbers ranging from  $g(0,0)$  to  $g(3,3)$ . Taking cell  $g(2,1)$  (depicted as a grey rectangle) as an example, the detailed information of  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$  for cell  $g(2,1)$  is shown in Fig. 4(c). As edges  $e_1, e_2, e_4, e_6, e_{10}$ , and  $e_{11}$  intersect cell  $g(2,1)$ ,  $T_{edge}$  stores the information of the six edges.  $T_{node}$  maintains the coordinates of nodes  $n_2$  and  $n_5$ , which are enclosed by  $g(2,1)$ . In  $T_{obj}$ , the information of objects  $o_1, o_2, o_3$ , and  $o_5$  is kept because they are located on the edges intersecting cell  $g(2,1)$  (i.e., edges  $e_6, e_1$ , and  $e_{11}$ ). Note that some grid cells may exist that do not have any information about edges, nodes, and objects, if that information can be found from other cells. For instance, edge  $e_6$  (and object  $o_1$ ) intersects (and is enclosed in) cell  $g(1,1)$ . Hence, its information can be found in the tables of cell  $g(1,0)$  or  $g(2,1)$ , and thus  $g(1,1)$  need not maintain any information. With such a strategy, only the cells enclosing the nodes need to maintain  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$  (rather than all of the  $N \times N$  cells), and the storage cost can therefore be reduced. In the following, we describe how to compute the road distance, denoted as  $d_{o,q}$ , between object  $o$  and query object  $q$ , by taking advantage of this grid index.

For ease of exposition, the road network presented in Fig. 4 is used again to illustrate the process of computing the road distance between objects. Suppose that query object  $q$  is located at node  $n_2$  and that the road distance between objects  $q$  and  $o_4$  needs to be computed (i.e., computing  $d_{n_2,o_4}$ ). To enhance the performance of the computation, a min-heap  $H$  is used to store the nodes visited so far, in which the node with the minimum

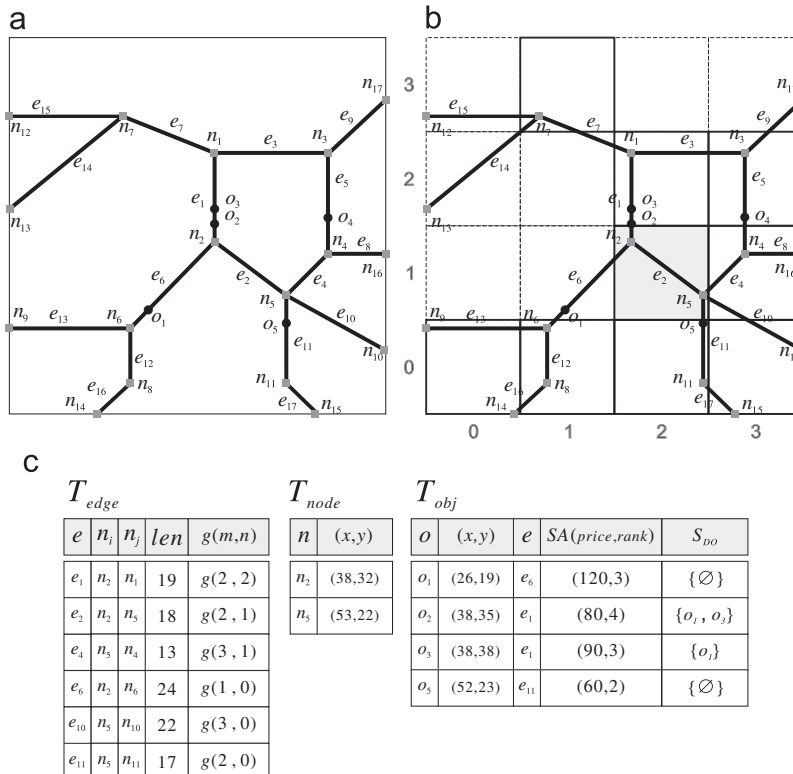


Fig. 4. The grid index structure. (a) A road network. (b) A grid index. (c) Information of  $g(2,1)$ .

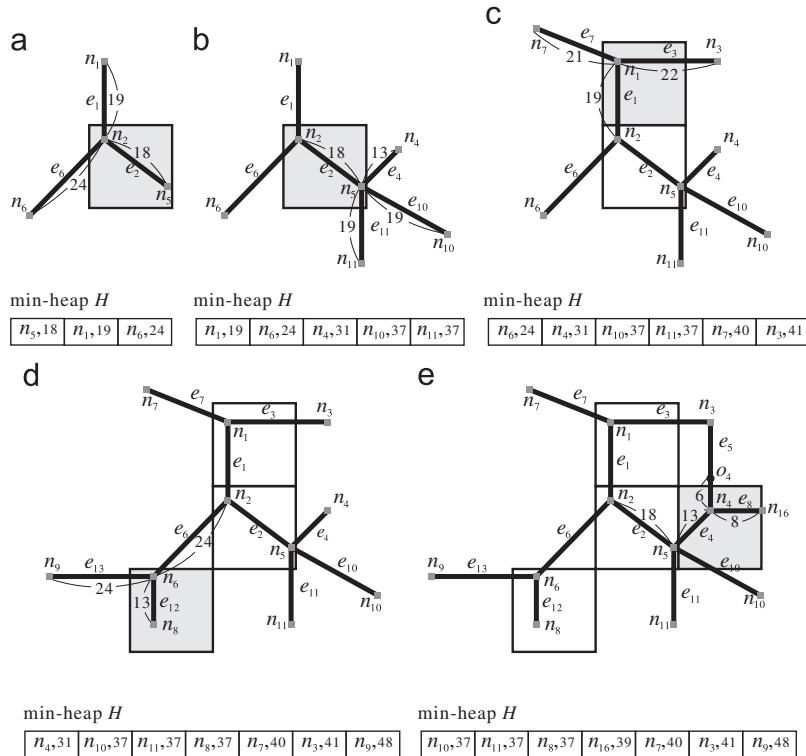
distance to node  $n_2$  is first de-heaped. Initially, the cell  $g(2, 1)$  enclosing node  $n_2$  is accessed to obtain the information of edges  $e_1$ ,  $e_2$ , and  $e_6$  that connect with  $n_2$ . Having obtained the edge information, the distances from nodes  $n_2$  to  $n_1$ ,  $n_5$ , and  $n_6$  are computed. Then, the three nodes with their distances are en-heaped into  $H$ , as shown in Fig. 5(a). As none of the edges ( $e_1$ ,  $e_2$ , and  $e_6$ ) contain object  $o_4$ , it can be checked to see if it is located on the edges connecting nodes  $n_1$ ,  $n_5$ , and  $n_6$ . As node  $n_5$  has the minimum distance to  $n_2$ , it is first de-heaped, and then the distances from  $n_2$  to  $n_4$ ,  $n_{10}$ , and  $n_{11}$  are computed, as shown in Fig. 5(b). Again, as none of the edges contain object  $o_4$ ,  $n_1$  becomes the next node to visit, as its distance to  $n_2$  is the minimum in  $H$ . Since node  $n_1$  is enclosed in cell  $g(2, 2)$ , the cell is accessed to get the information of edges  $e_3$  and  $e_7$  (which connect with  $n_1$ ) and to compute the distances  $d_{n_2, n_3}$  and  $d_{n_2, n_7}$  (as shown in Fig. 5(c)). Subsequently,  $n_6$  becomes the next node with the minimum distance to  $n_2$ , and it is de-heaped from  $H$ . The cell  $g(1, 0)$  enclosing  $n_6$  is accessed to compute the distances of nodes  $n_8$  and  $n_9$  to  $n_2$ , respectively (as shown in Fig. 5(d)). Finally, when cell  $g(3, 1)$  is accessed, a path from  $n_2$  to  $o_4$  can be found, allowing the road distance  $d_{n_2, o_4}$  to be computed, using the formula  $18 + 13 + 6 = 37$  (as shown in Fig. 5(e)). So far, the road distances to  $n_2$  of all nodes in  $H$  are greater than or equal to 37, meaning that the distances along the paths, starting from  $n_2$  and passing through these nodes to  $o_4$ , cannot be less than 37. Therefore, the road distance  $d_{n_2, o_4}$  is directly set to 37 and the remaining cells need not be accessed. Benefiting from

the grid index, the road distance between objects can be efficiently computed, which helps to reduce the cost of query processing.

The following sections present the proposed algorithms associated with the grid index to determine the  $Cd_e$ -SQ and  $Cknn$ -SQ results.

#### 4. Processing techniques for $Cd_e$ -SQ

Given a query path  $P_q$ ,  $Cd_e$ -SQ retrieves the  $d_e$ -SP for each point  $p \in P_q$ , such that the road distance between each  $d_e$ -SP and  $p$  is less than or equal to  $d_e$ . In this section, the  $Cd_e$ -SQ algorithm and the  $Cd_e$ -SQ<sup>+</sup> algorithm are developed to efficiently determine the  $d_e$ -SP. The main idea of the proposed algorithms is to divide the query path  $P_q$  into a set of edges,  $S_e$ , which are then considered sequentially in determining the  $d_e$ -SP. For each edge  $e \in S_e$ , the procedure of the  $Cd_e$ -SQ approaches (i.e., both the  $Cd_e$ -SQ algorithm and the  $Cd_e$ -SQ<sup>+</sup> algorithm) includes (1) a *global SP determination phase*: obtains a set of global SP, denoted as  $GSP$ , by accessing only a few cells of the grid index, such that object  $o \in GSP$  must be a  $d_e$ -SP for point  $p$  on edge  $e$  (i.e.,  $o$  is a SP for point  $p$  and its road distance  $d_{p, o} \leq d_e$ ), and (2) a *result turning point determination phase*: determines the result turning points on edge  $e$ , such that the  $d_e$ -SP between two consecutive result turning points remain the same, and finds the corresponding  $d_e$ -SP of these points.



**Fig. 5.** Computing the road distance between  $n_2$  and  $o_4$ . (a) Accessing  $g(2, 1)$ . (b) Accessing  $g(2, 1)$ . (c) Accessing  $g(2, 2)$ . (d) Accessing  $g(1, 0)$ . (e) Accessing  $g(3, 1)$ .

The  $Cd_e$ -SQ algorithm and the  $Cd_e$ -SQ<sup>+</sup> algorithm are described in Sections 4.1 and 4.2, respectively. Finally, a complexity analysis of the proposed  $Cd_e$ -SQ approaches is given in Section 4.3.

#### 4.1. $Cd_e$ -SQ algorithm

In the global SP determination phase of the  $Cd_e$ -SQ algorithm, the purpose of obtaining the GSP for edge  $e$  is to greatly reduce the number of objects that need to be considered in finding the  $d_e$ -SP. That is, the objects not in the GSP can be pruned immediately. Here, the following theorem is provided to facilitate the determination of the GSP.

**Theorem 4.1.** *Given a set of  $d_e$ -SP, denoted as  $SP_p^e$ , for a point  $p$  on an edge  $e$  connecting two nodes  $n_s$  and  $n_e$ , we have  $SP_p^e \subseteq SP_{n_s}^e \cup SP_{n_e}^e \cup S_e$ , where  $SP_{n_s}^e$  and  $SP_{n_e}^e$  are the  $d_e$ -SP sets for nodes  $n_s$  and  $n_e$ , respectively, and  $S_e$  is the set of objects located on edge  $e$ .*

**Proof.** We prove the theorem by contradiction. Assume that an object  $o \in SP_p^e$  but  $o \notin SP_{n_s}^e \cup SP_{n_e}^e \cup S_e$ . As  $o \notin SP_{n_s}^e \cup SP_{n_e}^e$ , one of the following four conditions holds: (1) both the distance  $d_{n_s,o}$  from  $n_s$  to  $o$  and the distance  $d_{n_e,o}$  from  $n_e$  to  $o$  are greater than  $d_e$ , (2) an object  $o' \in SP_{n_s}^e$  dominates  $o$  in terms of the static attributes and the dynamic attribute (i.e.,  $d_{n_s,o'} \leq d_{n_s,o}$ ), and the road distance  $d_{n_e,o}$  from  $n_e$  to  $o$  is greater than  $d_e$ , (3) the road distance  $d_{n_s,o} > d_e$ , and  $o$  is dominated by an object  $o'' \in SP_{n_e}^e$  whose distance  $d_{n_e,o''} \leq d_{n_e,o}$ , or (4) there exists an object  $o' \in SP_{n_s}^e$  (and  $o'' \in SP_{n_e}^e$ ) dominating  $o$  and the distance  $d_{n_s,o'} \leq d_{n_s,o}$  (and  $d_{n_e,o''} \leq d_{n_e,o}$ ).

- Condition 1 holds: Due to  $o \notin S_e$  (i.e.,  $o$  is not on edge  $e$ ), the distances along the paths passing through  $n_s$  or  $n_e$  to  $p$  must be greater than  $d_e$ . Therefore,  $o$  cannot be a  $d_e$ -SP for point  $p$ .
- Condition 2 holds: As  $o$  is dominated by  $o'$  in terms of static attributes, it can be the  $d_e$ -SP for point  $p$  only if its distance  $d_{p,o}$  (i.e., the length of the path passing through  $n_s$  to  $p$ ) is less than  $d_{p,o'}$ . However,  $o \notin S_e$  indicates that  $d_{p,o} \geq d_{p,o'}$ , and thus  $o$  is not a  $d_e$ -SP.
- Condition 3 holds: Similar to the reason in Condition 2.
- Condition 4 holds: Because both objects  $o'$  and  $o''$  dominate  $o$ , the necessary condition for  $o$  to be a  $d_e$ -SP is that  $d_{p,o} < d_{p,o'}$  and  $d_{p,o} < d_{p,o''}$ . However,  $o \notin S_e$  implies that either  $d_{p,o} \geq d_{p,o'}$  or  $d_{p,o} \geq d_{p,o''}$ , so that  $o$  is not a  $d_e$ -SP.

No matter which condition holds,  $o$  cannot be a  $d_e$ -SP for point  $p$ . This leads to a contradiction that  $o \in SP_p^e$ .  $\square$

The above theorem adequately demonstrates that if the GSP for edge  $e$  is set to  $SP_{n_s}^e \cup SP_{n_e}^e \cup S_e$ , then the  $d_e$ -SP for any point  $p$  on edge  $e$  will be included in the GSP. As such, it is only necessary to retrieve the objects on edge  $e$  and the  $d_e$ -SP for nodes  $n_s$  and  $n_e$  in order to obtain the GSP. Next, we discuss how to efficiently retrieve the  $d_e$ -SP for nodes  $n_s$  and  $n_e$  by accessing only a few cells of the grid index.

Recall that object  $o$  is the  $d_e$ -SP for node  $n$  only if: (1) it cannot be dominated by other objects in terms of static and dynamic attributes; and (2) the road distance  $d_{n,o}$  is less than or equal to  $d_e$ . This means that the objects whose distances to  $n$  are greater than  $d_e$  can be pruned, no matter if they are dominated by others or not. However, computing the road distances of all objects to compare with  $d_e$  will incur significant CPU and I/O costs. In order to filter out those distant objects without computing their road distances, a pruning strategy is designed based on the property of the road distance  $d_{n,o}$  always being greater than or equal to their Euclidean distance  $d_{n,o}^E$ . That is, the objects whose  $d_{n,o}^E > d_e$  are directly pruned because their road distances must be greater than  $d_e$ . The grid cells that intersect the circle centered at node  $n$  with radius  $d_e$  are accessed to retrieve objects. If the Euclidean distance  $d_{n,o}^E$  of a retrieved object  $o$  is greater than  $d_e$ , then its road distance  $d_{n,o}$  need not be computed. Otherwise,  $d_{n,o}$  is computed using the method mentioned in Section 3 and compared with  $d_e$ . For each object  $o$  whose  $d_{n,o} \leq d_e$ , looking up the  $S_{DO}$  information can determine whether  $o$  is dominated by the other objects within the distance range  $d_e$ , in terms of the static attributes. Only those objects not dominated by others in terms of either static attributes or dynamic attribute (i.e., the road distance to  $n$ ) are the  $d_e$ -SP for node  $n$ . By performing the above procedure, the  $d_e$ -SP sets,  $SP_{n_s}^e$  and  $SP_{n_e}^e$ , for nodes  $n_s$  and  $n_e$  can be determined, respectively. Algorithm 1 gives the details of how the GSP for edge  $e$  is obtained.

**Algorithm 1.** The global SP determination phase of the  $Cd_e$ -SQ algorithm.

**Input:** A grid index, a distance  $d_e$ , an edge  $e$  connecting two nodes  $n_s$  and  $n_e$   
**Output:** The GSP for edge  $e$   
 /\* determining  $SP_{n_s}^e$  for node  $n_s$  \*/  
 access the cells that intersect the circle centered at  $n_s$  with radius  $d_e$  to obtain  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$ ;  
**foreach** object  $o_i$  stored in  $T_{obj}$  **do**  
   compute the Euclidean distance  $d_{n_s,o_i}^E$ ;  
   **if**  $d_{n_s,o_i}^E \leq d_e$  **then**  
     compute the road distance  $d_{n_s,o_i}$  by accessing the grid index;  
   **foreach** object  $o_j$  whose  $d_{n_s,o_j} \leq d_e$  **do**  
     /\*  $o_i$  is not dominated by others in terms of static attributes \*/  
     **if**  $o_i \notin S_{DO}$  of the other objects within the distance range  $d_e$  **then** insert  $o_i$  into  $SP_{n_s}^e$ ;  
     /\*  $o_i$  is not dominated by others in terms of dynamic attribute \*/  
     **elseif**  $o_i \in o'.S_{DO}$  and  $d_{n_s,o_i} < d_{n_s,o'}$  **then** insert  $o_i$  into  $SP_{n_s}^e$ ;  
 determine  $SP_{n_e}^e$  for node  $n_e$  in a similar way;  
 retrieve the set  $S_e$  of objects on edge  $e$ ;  
 set GSP to  $SP_{n_s}^e \cup SP_{n_e}^e \cup S_e$ ;

The example in Fig. 6 is used to illustrate how to obtain the GSP for edge  $e_1$  connecting nodes  $n_1$  and  $n_2$ . Initially, the  $d_e$ -SP set for node  $n_1$  (i.e.,  $SP_{n_1}^e$ ) is determined, which is shown in Fig. 6(a). As the circle centered at  $n_1$  with radius  $d_e$  intersects five cells  $g(1,0)$ ,  $g(1,1)$ ,  $g(1,2)$ ,  $g(2,0)$ , and  $g(2,2)$  (cell  $g(2,1)$  does not store any information because no node is contained in it, the objects  $o_1$ ,  $o_2$ ,  $o_3$ , and  $o_6$  stored in the  $T_{obj}$  tables of the five cells will be retrieved. Because the Euclidean distance  $d_{n_1,o_3}^E$  between node  $n_1$



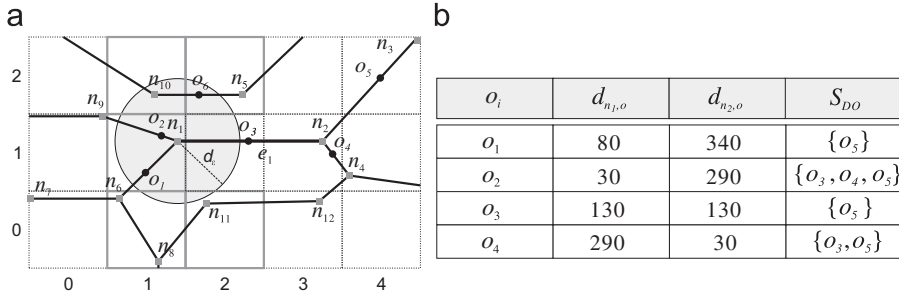


Fig. 6. The global SP determination phase of the  $Cd_e$ -SQ algorithm. (a) Determining the  $SP_{n1}^e$  for node  $n_1$ . (b) Obtaining the GSP for edge  $e_1$ .

and object  $o_3$  is greater than  $d_e$ , the road distance of object  $o_3$  need not be computed. As for object  $o_6$ , its road distance  $d_{n_1,o_6}$  to node  $n_1$  is computed and is found to not be the  $d_e$ -SP, as  $d_{n_1,o_6} > d_e$ . Only the two objects  $o_1$  and  $o_2$  are within the distance  $d_e$  from node  $n_1$  and could be the  $d_e$ -SP. Having looked up the  $S_{DO}$  information, it is known that  $o_1 \notin o_2.S_{DO}$  and  $o_2 \notin o_1.S_{DO}$  (meaning that  $o_1$  and  $o_2$  do not dominate each other in terms of the static attributes), and thus, both objects are the  $d_e$ -SP for node  $n_1$  ( $SP_{n1}^e = \{o_1, o_2\}$ ). The  $SP_{n2}^e$  for node  $n_2$  can be obtained in a similar way, which is equal to  $\{o_4\}$ . As object  $o_3$  is located on edge  $e_1$  (i.e.,  $S_{e_1} = \{o_3\}$ ), the GSP for edge  $e_1$  is finally set to  $\{o_1, o_2, o_3, o_4\}$  as shown in Fig. 6(b). The four objects will be considered in the result turning point determination phase to find the result turning points with the corresponding  $d_e$ -SP.

The main idea of the result turning point determination phase is to set up landmarks on edge  $e$  and then find the result turning points. Only the landmarks are possible result turning points. There are four types of landmarks, which are defined as follows:

- $L_o^+$ : Assume that the road distance  $d_{q,o}$  between object  $o$  and query object  $q$  is greater than  $d_e$ , and that  $q$  moves toward  $o$ .  $L_o^+$  is the landmark on edge  $e$  at which  $d_{q,o} = d_e$ . That is, object  $o$  cannot be a  $d_e$ -SP of  $q$  until  $q$  passes through  $L_o^+$ . For example, in Fig. 7(a) where  $d_e = 100$ , the landmark  $L_{o_3}^+$  of object  $o_3$  refers to the point at which the road distance to  $n_1$  is equal to 30. Before  $q$  reaches  $L_{o_3}^+$ , the road distance  $d_{q,o_3} > 100$ , therefore,  $o_3$  cannot be the  $d_e$ -SP.
- $L_o^-$ : Assume that  $d_{q,o} < d_e$ , and that  $q$  moves away from  $o$ . Once  $q$  passes through the landmark  $L_o^-$ , the road distance  $d_{q,o}$  between  $o$  and  $q$  is greater than  $d_e$ , therefore,  $o$  is not the  $d_e$ -SP of  $q$ . For instance, object  $o_3$ , shown in Fig. 7(a), cannot be the  $d_e$ -SP when  $q$  moves to the right of  $L_{o_3}^-$ , as the distance to  $n_1$  is equal to 230.
- $L_{o'o'}^+$ : Assume that object  $o$  dominates another object, say  $o'$ , in terms of the static attributes (i.e.,  $o' \in o.S_{DO}$ ). The landmark  $L_{o'o'}^+$ , at which  $d_{q,o'} = d_{q,o}$ , means that if  $q$  is on the left of  $L_{o'o'}^+$ , then  $o$  cannot dominate  $o'$  in terms of the dynamic attribute (i.e., the road distance  $d_{q,o'} < d_{q,o}$ ). On the other hand,  $o$  dominates  $o'$  when  $q$  is on the right of  $L_{o'o'}^+$ . Consider objects  $o_3$  and  $o_4$  in Fig. 7(a). As object  $o_3 \in o_4.S_{DO}$  (referring to Fig. 6(b)),

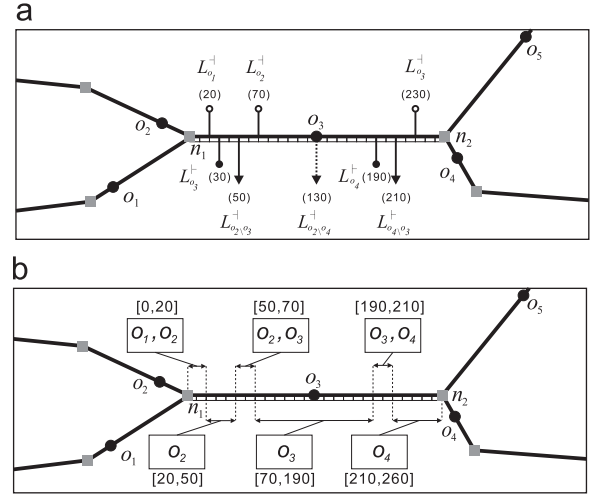


Fig. 7. The result turning point determination phase of the  $Cd_e$ -SQ algorithm. (a) Landmarks of GSP. (b) Determining the result turning points.

there exists  $L_{o_4'o_3}^+$  on the point whose distance to  $n_1$  is equal to 210. Note that if  $L_{o'o'}^+$  is between  $L_o^-$  and  $L_o^+$  (i.e., within the range  $[L_o^-, L_o^+]$ ), then  $L_{o'o'}^+$  can be ignored, because both  $o$  and  $o'$  are not  $d_e$ -SP when  $q$  moves within  $[L_o^-, L_o^+]$ .

- $L_{o'o'}^-$ : Assume that  $o' \in o.S_{DO}$ . Before  $q$  reaches the landmark  $L_{o'o'}^-$ ,  $o$  can dominate  $o'$  in terms of the static and dynamic attributes. However, once  $q$  passes  $L_{o'o'}^-$ ,  $o$  cannot dominate  $o'$  because  $d_{q,o'} < d_{q,o}$ . Similarly, when  $L_{o'o'}^-$  is within  $[L_o^-, L_o^+]$  (e.g.,  $L_{o_2'o_4}^-$  in Fig. 7(a)), it is ignored.

Having obtained the GSP for edge  $e$ , the four landmarks of each object in the GSP are determined on-the-fly (note that only the objects in the GSP need to be considered in finding the landmarks). First, we describe how to obtain the landmarks  $L_o^+$  and  $L_o^-$  of each object  $o \in GSP$ . If object  $o \in S_e$  (i.e.,  $o$  is on edge  $e$ ),  $L_o^+$  and  $L_o^-$  are set to the points on edge  $e$  at which the distances to  $n_s$  and  $n_e$  are equal to  $d_{n_s,o} - d_e$  and  $d_{n_e,o} - d_e$ , respectively. Otherwise, it is either  $o \in SP_{n_s}^e$  for node  $n_s$  or  $o \in SP_{n_e}^e$  for node  $n_e$ . In the case where  $o \in SP_{n_s}^e$  (or  $o \in SP_{n_e}^e$ ), landmark  $L_o^+$  (or  $L_o^-$ ) on edge  $e$  is the only one whose distance to  $n_s$  (or  $n_e$ ) is equal to

$d_e - d_{n_s, o}$  (or  $d_e - d_{n_e, o}$ ). The number of landmarks  $L_o^+$  and  $L_o^-$  is at most  $2 \cdot |S_e| + |SP_{n_s}^e| + |SP_{n_e}^e|$ , where  $|S_e|$ ,  $|SP_{n_s}^e|$ , and  $|SP_{n_e}^e|$  are the numbers of objects in  $S_e$ ,  $SP_{n_s}^e$ , and  $SP_{n_e}^e$ , respectively. Next, the determination of the landmark  $L_{o, o'}^+$  is divided into two cases: (1) object  $o' \in SP_{n_s}^e$  but  $o' \notin S_e$  and (2) object  $o' \in S_e$ . In the first case, if there exists the landmark  $L_{o, o'}^+$  on edge  $e$ , then  $L_{o, o'}^+$  is set to the point at which the distance to  $n_s$  is equal to  $(d_{n_s, o} - d_{n_s, o'})/2$ . In the later case, as object  $o'$  is on edge  $e$ ,  $L_{o, o'}^+$  is the point where the distance to  $n_s$  is equal to  $(d_{n_s, o} + d_{n_s, o'})/2$ . Let  $|SP_{n_s}^e - S_e|$  (and  $|SP_{n_e}^e - S_e|$ ) be the number of objects in  $SP_{n_s}^e$  (and  $SP_{n_e}^e$ ) but not in  $S_e$ . The number of landmark  $L_{o, o'}^+$  is at most  $|SP_{n_s}^e - S_e| \cdot (|GSP| - |SP_{n_s}^e - S_e|) + |S_e| \cdot |SP_{n_e}^e - S_e|$ . Finally, the landmark  $L_{o, o'}^-$  is obtained according to the following cases: (1) object  $o \in SP_{n_s}^e$  but  $o \notin S_e$  and (2) object  $o \in S_e$ . In the first (or second) case, the landmark  $L_{o, o'}^-$  is set to the point at which the distance to  $n_s$  is equal to  $(d_{n_s, o} - d_{n_s, o})/2$  (or  $(d_{n_s, o} + d_{n_s, o})/2$ ). The maximum number of  $L_{o, o'}^-$  is similar to that of  $L_{o, o'}^+$ . Therefore, the total number of the determined landmarks is at most  $2 \cdot |S_e| + |SP_{n_s}^e| + |SP_{n_e}^e| + 2 \cdot (|SP_{n_s}^e - S_e| \cdot (|GSP| - |SP_{n_s}^e - S_e|) + |S_e| \cdot |SP_{n_e}^e - S_e|)$ .

**Algorithm 2** illustrates how to determine the result turning points with the corresponding  $d_e$ -SP by using the landmarks of objects in the GSP. Initially, queue  $Q$  is utilized to keep the information of the object landmarks. If the landmark  $L_o^+$  (or  $L_o^-$ ) of object  $o \in GSP$  is located on edge  $e$ , it will be inserted into  $Q$  so as to know the location where  $o$  will be within (or out of) distance range  $d_e$ . Also, object  $o$ 's landmark  $L_{o, o'}^+$  (or  $L_{o, o'}^-$ ) on edge  $e$  is kept in  $Q$  so that the point through which  $o$  can (or cannot) dominate  $o'$  is determined. Then, the landmarks in  $Q$  are sorted in ascending order of their distances to node  $n_s$ , and will be considered sequentially to determine which are the result turning points:

- When the landmark  $L_o^+$  of object  $o$  is considered: if there exists a landmark  $L_{o', o}^+$  of object  $o'$  in  $Q$ , then  $L_o^+$  is not a result turning point. This is because even though the road distance of  $o$  to  $q$  is less than  $d_e$  between  $[L_o^+, L_{o', o}^+]$ ,  $o$  is still dominated by  $o'$  before  $q$  reaches  $L_{o', o}^+$ . Otherwise (i.e., no  $L_{o', o}^+$  exists),  $o$  becomes a  $d_e$ -SP after  $L_o^+$ , and thus  $L_o^+$  is a result turning point.
- When the landmark  $L_o^-$  of object  $o$  is considered: if object  $o$  is currently in the  $d_e$ -SP set, its road distance  $d_{q, o}$  to  $q$  must be less than or equal to  $d_e$ . Once  $q$  passes through  $L_o^-$ ,  $d_{q, o}$  is greater than  $d_e$ , and  $o$  needs to be removed from the  $d_e$ -SP set. That is,  $L_o^-$  is a result turning point.
- When the landmark  $L_{o, o'}^+$  of object  $o$  is considered: as query object  $q$  reaches the landmark  $L_{o, o'}^+$ , object  $o$  will dominate object  $o'$  in terms of the static and dynamic attributes. If both objects  $o$  and  $o'$  are currently in the  $d_e$ -SP set, then after  $L_{o, o'}^+$ ,  $o'$  is no longer the  $d_e$ -SP because it is dominated by  $o$ . Therefore,  $L_{o, o'}^+$  is a result turning point.
- When the landmark  $L_{o, o'}^-$  of object  $o$  is considered: the landmark  $L_{o, o'}^-$  indicates that object  $o$  cannot dominate  $o'$  when  $q$  reaches  $L_{o, o'}^-$ . If object  $o$  is currently in the  $d_e$ -SP set and no landmark  $L_{o', o}^+$  of object  $o'$  can be found in  $Q$ , object  $o'$  is promoted to a  $d_e$ -SP at  $L_{o, o'}^-$  and a result turning point occurs.

**Algorithm 2.** The result turning point determination phase of the  $Cd_e$ -SQ algorithm.

**Input:** A distance  $d_e$ , a set GSP, an edge  $e$  connecting two nodes  $n_s$  and  $n_e$

**Output:** A set of tuples in form of  $\langle [n_i, n_j], SP^e \rangle$ ,

where  $SP^e$  is the  $d_e$ -SP set between  $[n_i, n_j]$

create an empty queue  $Q$ ;

/\* determining the landmarks of each object in GSP \*/

**foreach** object  $o \in GSP$  **do**

    determine  $o$ 's landmarks  $L_o^+$  and  $L_o^-$ ;

**if**  $L_o^+$  is on  $e$  **then** insert  $L_o^+$  into  $Q$ ;

**if**  $L_o^-$  is on  $e$  **then** insert  $L_o^-$  into  $Q$ ;

**foreach** object  $o' \in GSP - \{o\} \cap o.S_{DO}$  **do**

        determine  $o$ 's landmarks  $L_{o, o'}^+$  or  $L_{o, o'}^-$ ;

**if**  $L_{o, o'}^+$  is on  $e$  and  $L_o^+$  is closer to  $n_s$  than  $L_{o, o'}^+$

**then** insert  $L_{o, o'}^+$  into  $Q$ ;

**if**  $L_{o, o'}^-$  is on  $e$  and  $L_o^-$  is closer to  $n_s$  than  $L_{o, o'}^-$

**then** insert  $L_{o, o'}^-$  into  $Q$ ;

sort the landmarks in  $Q$  in ascending order of their distances to node  $n_s$ ;

set  $n_i$  and  $SP^e$  to  $n_s$  and the  $d_e$ -SP for  $n_s$ , respectively;

/\* determining the result turning points with corresponding  $d_e$ -SP \*/

**while**  $Q$  is not empty **do**

    de – queue  $o.L$ ;

**switch**  $o.L$  **do**

**case**  $L_o^+$

**if** there is no  $L_{o', o}^+ \in Q$  **then** return  $\langle [n_i, L_o^+], SP^e \rangle$ ,  $n_i = L_o^+$ ,

            and add  $o$  into  $SP^e$ ;

**case**  $L_o^-$

**if**  $o \in SP^e$  **then** return  $\langle [n_i, L_o^-], SP^e \rangle$ ,  $n_i = L_o^-$ ,

            and remove  $o$  from  $SP^e$ ;

**case**  $L_{o, o'}^+$

**if**  $o \in SP^e$  and  $o' \in SP^e$  **then** return  $\langle [n_i, L_{o, o'}^+], SP^e \rangle$ ,  $n_i = L_{o, o'}^+$ ,

            and remove  $o'$  from  $SP^e$ ;

**otherwise**

            /\*  $o.L$  corresponds to  $L_{o, o'}^-$

**if**  $o \in SP^e$  and there is no  $L_{o', o}^+ \in Q$

**then** return  $\langle [n_i, L_{o, o'}^-], SP^e \rangle$ ,

$n_i = L_{o, o'}^-$ , and add  $o'$  into  $SP^e$ ;

return  $\langle [n_i, n_e], SP^e \rangle$ ;

Consider again the example in Fig. 7, where the initial  $d_e$ -SP set is  $\{o_1, o_2\}$  (i.e., the  $SP_{n_1}^e$  of node  $n_1$ ) and seven landmarks from  $L_{o_1}^+$  to  $L_{o_3}^-$  are possible result turning points. (1) Considering  $L_{o_1}^+$ : object  $o_1$  is removed from the  $d_e$ -SP set as its road distance  $d_{q, o_1} > d_e$  after  $L_{o_1}^+$ . Thus,  $L_{o_1}^+$  is a result turning point and the  $d_e$ -SP set changes from  $\{o_1, o_2\}$  to  $\{o_2\}$ . (2) Considering  $L_{o_3}^+$ : even though the road distance  $d_{q, o_3} < d_e$ , object  $o_3$  cannot be the  $d_e$ -SP because there exists a landmark  $L_{o_2, o_3}^+$  behind  $L_{o_3}^+$  (meaning that  $o_3$  is dominated by  $o_2$  between  $[L_{o_3}^+, L_{o_2, o_3}^+]$ ). (3) Considering  $L_{o_2, o_3}^-$ : as no object can dominate  $o_3$ , it becomes the  $d_e$ -SP, and  $L_{o_2, o_3}^-$  is a result turning point where the  $d_e$ -SP set is  $\{o_2, o_3\}$ . (4) Considering  $L_{o_2}^+$ : object  $o_2$  is removed from the  $d_e$ -SP set since its distance  $d_{q, o_2} > d_e$ . Therefore,  $L_{o_2}^+$  is a result turning point. (5) Considering  $L_{o_4}^+$ : object  $o_4$  is added into the  $d_e$ -SP set and thus  $L_{o_4}^+$  is a result turning point. (6) Considering  $L_{o_4, o_3}^-$ : object  $o_3$  will be dominated by  $o_4$  in terms of the static and dynamic attributes. As such,  $o_3$  needs to be removed from the  $d_e$ -SP set

(changing from  $\{o_3, o_4\}$  to  $\{o_4\}$ ). (7) Considering  $L_{o_3}^{-1}$ : because object  $o_3$  is currently not in the  $d_e$ -SP set, its landmark  $L_{o_3}^{-1}$  will not affect the  $d_e$ -SP set (i.e., not a result turning point). The complete  $d_e$ -SP results from nodes  $n_1$  to  $n_2$  are shown in Fig. 7(b).

#### 4.2. $Cd_e$ -SQ<sup>+</sup> algorithm

The  $Cd_e$ -SQ<sup>+</sup> algorithm is proposed to further improve the query performance of the  $Cd_e$ -SQ algorithm by applying a different pruning strategy in the global SP determination phase. Recall that the  $Cd_e$ -SQ algorithm utilizes the Euclidean distance between objects as a lower bound of the road distance in order to prune the non-qualifying objects. Although the Euclidean distance can be easily computed with respect to the objects' coordinates, it cannot properly approximate the road distance. This will result in: (1) more grid cells needing to be accessed (e.g., cell  $g(2,0)$  in Fig. 6(a) which slightly intersects the circle with radius  $d_e$ ); and (2) more candidates whose Euclidean distances are less than or equal to  $d_e$  but with road distances that are greater than  $d_e$  (e.g., object  $o_6$ ).

**Algorithm 3.** The global SP determination phase of the  $Cd_e$ -SQ<sup>+</sup> algorithm.

**Input:** A grid index, a distance  $d_e$ , an edge  $e$  connecting two nodes  $n_s$  and  $n_e$   
**Output:** The GSP for edge  $e$   
 /\* determining  $SP_{n_s}^e$  for node  $n_s$  \*/  
 create an empty queue  $Q$ ;  
 insert cell  $g(i,j)$  enclosing node  $n_s$  into  $Q$ ;  
**while**  $Q$  is not empty **do**  
   de-queue  $g(i,j)$  and access its  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$ ;  
   **foreach** edge  $e$  stored in  $T_{edge}$  **do**  
     **if** node  $n_i$  has been visited **then**  
       **foreach** object  $o$  on edge  $e$  **do**  
         compute the road distance  $d_{n_s,o}$  of  $o$  to  $n_s$ ;  
         **if**  $d_{n_s,o} \leq d_e$  **then** insert  $o$  into a candidate set  $S_{candidate}$ ;  
         compute the road distance  $d_{n_s,n_j}$  from nodes  $n_s$  to  $n_j$ ;  
         **if**  $d_{n_s,n_j} < d_e$  **then** insert cell  $g(m,n)$  enclosing  $n_j$  into  $Q$ ;  
     **foreach** object  $o \in S_{candidate}$  **do**  
       **if**  $o \notin S_{DO}$  of the other objects in  $S_{candidate}$  **then** insert  $o$  into  $SP_{n_s}^e$ ;  
       **elseif**  $o \in o'$ ,  $S_{DO}$  and  $d_{n_s,o} < d_{n_s,o'}$  **then** insert  $o$  into  $SP_{n_s}^e$ ;  
   determine  $SP_{n_e}^e$  for node  $n_e$  in a similar way;  
   retrieve the set  $S_e$  of objects on edge  $e$ ;  
   set GSP to  $SP_{n_s}^e \cup SP_{n_e}^e \cup S_e$ ;

To remedy this problem, the  $Cd_e$ -SQ<sup>+</sup> algorithm conducts the Breadth First Search on the road network (starting from nodes  $n_s$  and  $n_e$ ), and examines objects in the order they are visited. Suppose that the  $d_e$ -SP set of node  $n_s$  (i.e.,  $SP_{n_s}^e$ ) needs to be determined. During the course of determining the  $SP_{n_s}^e$ , queue  $Q$  is used to keep the accessed cells of the grid index, and initially  $Q$  only contains the cell enclosing node  $n_s$ . In each iteration, an element of  $Q$  (i.e., a cell  $g(i,j)$ ) will be retrieved, so as to obtain its corresponding  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$  information (referring to Fig. 4). For each edge  $e$  stored in  $T_{edge}$ , once its node  $n_i$  has been visited, the road distance  $d_{n_s,n_j}$  from  $n_s$  to its node  $n_j$  will be computed. If  $d_{n_s,n_j} < d_e$ , the road distances of those objects located on edge  $e$  must be less than  $d_e$ . Thus, they are kept for dominance test and the

cell  $g(m,n)$  enclosing node  $n_j$  is inserted into  $Q$ . Otherwise, the cell  $g(m,n)$  enclosing  $n_j$  need not be accessed (i.e.,  $g(m,n)$  is not inserted into  $Q$ ). This is because all the road distances of the unvisited objects stored in  $T_{obj}$  of  $g(m,n)$  are greater than  $d_e$ . In this case, only the objects on edge  $e$  whose road distances to  $n_s$  are less than or equal to  $d_e$  will be kept. The above process proceeds until  $Q$  is empty. Finally,  $SP_{n_s}^e$  is determined by performing the dominance tests among the kept objects. Also,  $SP_{n_e}^e$  of node  $n_e$  can be obtained in a similar way. The pseudo code of the detailed steps for the global SP determination phase of the  $Cd_e$ -SQ<sup>+</sup> algorithm is given in Algorithm 3.

Consider again the example in Fig. 6. If the  $SP_{n_1}^e$  of node  $n_1$  is obtained by accessing the cells that intersect the circle with radius  $d_e$  centered at  $n_1$ , five cells  $g(1,0)$ ,  $g(1,1)$ ,  $g(1,2)$ ,  $g(2,0)$ , and  $g(2,2)$  need to be accessed. However, only the cell  $g(1,1)$  enclosing node  $n_1$  will be accessed, as the determination of  $SP_{n_1}^e$  is based on the Breadth First Search (the road distances from nodes  $n_1$  to  $n_2$ ,  $n_6$ , and  $n_9$  are all greater than  $d_e$ ). By applying the Breadth First Search on the grid index, the cells that do not need to be accessed and the road distance computations of those distant objects can be effectively reduced so as to improve the performance of the global SP determination phase. As the result turning point determination phase of the  $Cd_e$ -SQ<sup>+</sup> algorithm is similar to that of the  $Cd_e$ -SQ algorithm, it is omitted from presentation.

#### 4.3. Time complexity analysis of $Cd_e$ -SQ approaches

Since the  $Cd_e$ -SQ approaches (i.e., the  $Cd_e$ -SQ and  $Cd_e$ -SQ<sup>+</sup> algorithms) include the global SP determination phase and the result turning point determination phase, we consider the two phases individually:

- In the global SP determination phase,  $SP_{n_s}^e$  and  $SP_{n_e}^e$  for nodes  $n_s$  and  $n_e$ , respectively, need to be determined so as to obtain the GSP. Consider first the time complexity of determining  $SP_{n_s}^e$ . Let  $|o_{n_s}|$  be the number of objects whose road distances to  $n_s$  have to be computed and  $|o'_{n_s}|$  be the number of objects whose road distances to  $n_s$  are less than or equal to  $d_e$ , and  $|o'_{n_s}| \leq |o_{n_s}|$ . Note that for the  $Cd_e$ -SQ<sup>+</sup> algorithm, the size of  $|o_{n_s}|$  is equal to that of  $|o'_{n_s}|$ . Then, the time complexity of determining  $SP_{n_s}^e$  is  $O(|o_{n_s}| \cdot C_{dist} + |o'_{n_s}|^2)$ , where  $C_{dist}$  refers to the cost of computing road distance by accessing the grid index. Similarly, the time complexity of determining  $SP_{n_e}^e$  is  $O(|o_{n_e}| \cdot C_{dist} + |o'_{n_e}|^2)$ . Thus, we have the time complexity of the global SP determination phase which is  $O((|o_{n_s}| + |o_{n_e}|) \cdot C_{dist} + |o'_{n_s}|^2 + |o'_{n_e}|^2)$ .
- The result turning point determination phase consists of two steps (referring to Algorithm 2). The first step is to determine the landmarks of each object in GSP and maintain a queue  $Q$  to store these landmarks. The second step is to find the result turning points with corresponding  $d_e$ -SP. Let  $|GSP|$  and  $|Q|$  be the sizes of GSP and queue  $Q$ , respectively. The time complexity of the first step is  $O(|GSP|^2 + |Q| \cdot \log|Q|)$ , where  $O(|GSP|)$  refers to the cost for determining the landmarks and

$O(|Q| \cdot \log|Q|)$  represents the cost for sorting the landmarks. The complexity of second step is  $O(|Q|)$ . Therefore, the total time complexity of the result turning point determination phase is  $O(|GSP|^2 + |Q| \cdot (\log|Q| + 1))$ .

## 5. Processing techniques for Cknn-SQ

A Cknn-SQ can be utilized to find the  $k$  nearest SP (i.e., the  $knn$ -SP) for each point  $p$  of query path  $P_q$ . The Cknn-SQ algorithm and the Cknn-SQ<sup>+</sup> algorithm are proposed to efficiently process such a Cknn-SQ, both of which divide  $P_q$  into a set of edges and then consider these edges sequentially to find the  $knn$ -SP. Similar to the Cd<sub>e</sub>-SQ approaches presented in Section 4, the Cknn-SQ and Cknn-SQ<sup>+</sup> algorithms consist of two phases: (1) the global SP determination phase that accesses the grid index to obtain the GSP for each edge  $e$ ; and (2) the result turning point determination phase that determines the result turning points with their corresponding  $knn$ -SP by taking into consideration the objects in the GSP only.

### 5.1. Cknn-SQ algorithm

In the global SP determination phase of the Cknn-SQ algorithm, the GSP for edge  $e$  is determined based on the following corollary, which demonstrates that the  $knn$ -SP for any point on edge  $e$  belong to the union of the objects on  $e$  and the  $knn$ -SP for the two endpoints of edge  $e$ .

**Corollary 5.1.** Given a set of  $knn$ -SP, denoted as  $SP_p^k$ , for a point  $p$  on an edge  $e$  connecting two nodes  $n_s$  and  $n_e$ , we have  $SP_p^k \subseteq SP_{n_s}^k \cup SP_{n_e}^k \cup S_e$ , where  $SP_{n_s}^k$  and  $SP_{n_e}^k$  are the  $knn$ -SP sets for nodes  $n_s$  and  $n_e$ , respectively, and  $S_e$  is the set of objects located on edge  $e$ .

The main idea of finding the  $knn$ -SP set for node  $n$  is to determine a search region, denoted as  $SR_n$ , to ensure that at least  $k$  skyline points of node  $n$  can be found within this region. That is, an object could be the  $knn$ -SP for node  $n$  only if it is within  $SR_n$ . In order to determine the region  $SR_n$ , we first find the SP in terms of the static attributes (termed the *static SP*), and then compute the road distances of the  $k$  static SP whose Euclidean distances to node  $n$  are the smallest, so as to obtain the  $k$ th smallest road distance, denoted as  $d_{kth}$ . The region  $SR_n$  can be represented as a circle centered at node  $n$  with radius  $d_{kth}$ , which guarantees that  $k$  static SP must be inside  $SR_n$ . As such, each object outside  $SR_n$  is either farther from  $n$  than the  $k$  static SP or is dominated by one of them so that it cannot be the  $knn$ -SP. Using the above approach, the search regions  $SR_{n_s}$  and  $SR_{n_e}$  of nodes  $n_s$  and  $n_e$  can be, respectively, determined to prune the non-qualifying objects. Then, Algorithm 1 mentioned in Section 4 can be applied to find the GSP for edge  $e$  by changing the circle centered at  $n_s$  (and  $n_e$ ) with radius  $d_e$  to the region  $SR_{n_s}$  (and  $SR_{n_e}$ ).

Continue the example shown in Fig. 6, where objects  $o_1$ ,  $o_2$ , and  $o_6$  are the static SP. Assume that a Cknn-SQ is issued to find the  $knn$ -SP for edge  $e_1$ , and that  $k=2$ . During

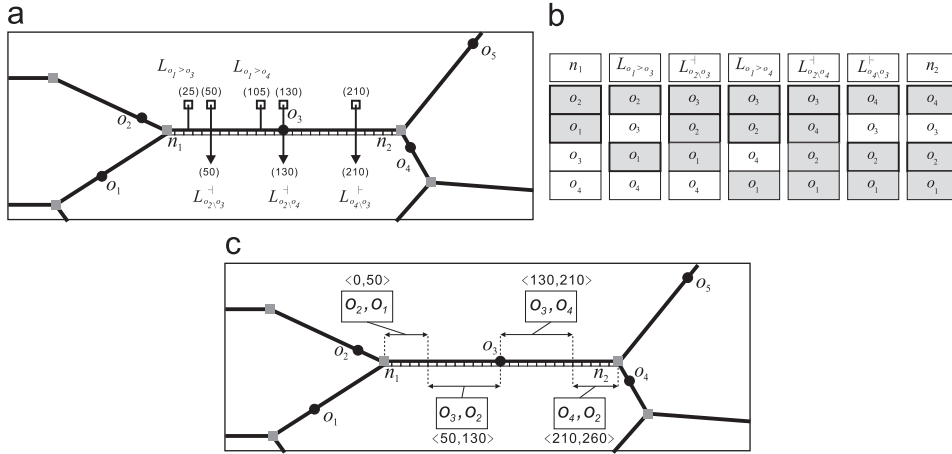
the course of finding the  $knn$ -SP set for node  $n_1$  (i.e.,  $SP_{n_1}^k$ ), the road distances  $d_{n_1,o_1}$  and  $d_{n_1,o_2}$  of objects  $o_1$  and  $o_2$  will be computed because they have the smallest Euclidean distances to  $n_1$ . Then, the search region  $SR_{n_1}$  is represented as a circle centered at  $n_1$  with a radius  $d_{n_1,o_1}$ . As  $SR_{n_1}$  intersects cells  $g(1, 1)$ ,  $g(1, 2)$ , and  $g(2, 2)$ , the three cells are accessed to retrieve the information of objects  $o_1$ ,  $o_2$ ,  $o_3$ , and  $o_6$ . Referring to Fig. 6(b),  $SP_{n_1}^k = \{o_2, o_1\}$ . Similarly,  $SP_{n_2}^k$  can be obtained as  $\{o_4, o_2\}$  (note that objects  $o_3$  and  $o_5$  are dominated by  $o_4$  in terms of static and dynamic attributes). Finally, as  $S_{e_1} = \{o_3\}$  (i.e.,  $o_3$  is on edge  $e_1$ ), the GSP for edge  $e_1$  is set to  $\{o_2, o_1, o_4, o_3\}$ .

In the result turning point determination phase, three types of landmarks are set up on edge  $e$  by using the dominance relationships between objects in the GSP and each is individually checked to find the result turning points. The first type of landmark is defined as  $L_{o>o'}$ , which refers to the point at which the road distance of object  $o$  to  $L_{o>o'}$  is equal to that of object  $o'$ . Note that objects  $o$  and  $o'$  cannot dominate each other in terms of the static attributes (that is,  $o \notin o'.S_{DO}$  and  $o' \notin o.S_{DO}$ ). Once query object  $q$  passes  $L_{o>o'}$ , object  $o'$  is closer to  $q$  than  $o$ . For instance, in Fig. 8(a), object  $o_3$  starts to get closer to  $q$  than  $o_1$  after the landmark  $L_{o_1>o_3}$  (i.e., the point where the road distance to  $n_1$  is equal to 25). The determination of the landmark  $L_{o>o'}$  is divided into two cases: (1) object  $o \in SP_{n_s}^k$  but  $o \notin S_e$  and (2) object  $o \in S_e$ . In the first (or second) case, the landmark  $L_{o>o'}$  is set to the point at which the distance to  $n_s$  is equal to  $(d_{n_s,o'} - d_{n_s,o})/2$  (or  $(d_{n_s,o'} + d_{n_s,o})/2$ ). The number of landmark  $L_{o>o'}$  on edge  $e$  is at most  $|SP_{n_s}^k - S_e| \cdot (|GSP| - |SP_{n_s}^k - S_e|) + |S_e| \cdot |SP_{n_e}^k - S_e|$  (refer to Section 4.1). The other two types of landmarks are the same as  $L_{o'o'}$  and  $L_{o'o'}$  defined in Section 4.1, and hence the total number of landmarks is at most  $3 \cdot (|SP_{n_s}^k - S_e| \cdot (|GSP| - |SP_{n_s}^k - S_e|) + |S_e| \cdot |SP_{n_e}^k - S_e|)$ .

The procedure of the result turning point determination phase is detailed in Algorithm 4. During the process, the objects in the GSP are sorted in ascending order of their road distances to query object  $q$ , and the first  $k$  objects not dominated by others are kept in set  $SP^k$  (i.e., the  $knn$ -SP set). Initially,  $q$  is located on node  $n_s$ , and thus, the  $SP^k$  is set to  $SP_{n_s}^k$ . Then, the landmarks  $L_{o>o'}$ ,  $L_{o'o'}$ , and  $L_{o'o'}$  of each object  $o$  in the GSP are determined and inserted into queue  $Q$ , in which the landmarks closer to  $n_s$  will be considered first:

- When the landmark  $L_{o>o'}$  of object  $o$  is considered: it implies that objects  $o$  and  $o'$  cannot dominate each other in terms of the static attributes, and that  $o'$  is closer to  $q$  than  $o$  after  $L_{o>o'}$ . That is, the order of objects  $o$  and  $o'$  in the GSP exchanges once  $q$  reaches  $L_{o>o'}$ . If  $o \notin SP^k$ , then  $SP^k$  remains unchanged no matter whether  $o' \in SP^k$  or not (i.e.,  $L_{o>o'}$  is not a result turning point). In the case where  $o \in SP^k$  and  $o' \in SP^k$ , as  $o$  and  $o'$  cannot dominate each other, at  $L_{o>o'}$  the change of their order will not affect  $SP^k$ . The only case where  $L_{o>o'}$  could be a result turning point is when  $o \in SP^k$  but  $o' \notin SP^k$ . Due to  $o' \notin SP^k$ , object  $o'$  is either: (1) dominated by another object  $o'' \in SP^k$ ; or (2) a skyline point but not a  $knn$ -SP. If the first condition holds, at  $L_{o>o'}$  object  $o'$  is still dominated by  $o''$  and it





**Fig. 8.** The result turning point determination phase of the Cknn-SQ algorithm. (a) Landmarks of GSP. (b) knn-SP for each landmark. (c) Determining the result turning points.

cannot be inserted into  $SP^k$  (that is,  $SP^k$  remains the same). Otherwise, the second condition holds. Because object  $o'$  replaces  $o$  as the  $k$ th nearest SP at  $L_{o > o'}$ , object  $o$  (or  $o'$ ) needs to be removed from (or added into)  $SP^k$ . Hence,  $L_{o > o'}$  is a result turning point.

- When the landmark  $L_{o, o'}^+$  of object  $o$  is considered: it indicates that object  $o$  dominates  $o'$  after  $L_{o, o'}^+$ . Also, the order of objects  $o$  and  $o'$  in the GSP exchanges after  $L_{o, o'}^+$ . If object  $o' \notin SP^k$ , then  $L_{o, o'}^+$  must not be a result turning point regardless of whether  $o$  is in  $SP^k$ . Otherwise (i.e.,  $o' \in SP^k$ ),  $o'$  will be removed from  $SP^k$  because  $o$  begins to dominate  $o'$  after  $L_{o, o'}^+$ , and hence  $L_{o, o'}^+$  becomes a result turning point. In such a case, if  $o \notin SP^k$  (the reason is that  $o$  is not one of the  $k$  nearest SP), then  $o$  replaces  $o'$  as the  $k$ th nearest SP and is inserted into  $SP^k$ . If  $o \in SP^k$ , the number of objects in  $SP^k$  will be less than  $k$  because  $o'$  is removed from  $SP^k$ , and thus, the  $k$ th SP in the GSP needs to be inserted into  $SP^k$ .
- When the landmark  $L_{o, o'}^-$  of object  $o$  is considered: it implies that after  $L_{o, o'}^-$ , object  $o'$  is no longer dominated by  $o$  (i.e.,  $o' \notin SP^k$  before  $L_{o, o'}^-$ ), and that  $o'$  precedes  $o$  in GSP. In the case where  $o \notin SP^k$  and  $o' \notin SP^k$ , there still exists object  $o'' \in SP^k$  dominating  $o$  after  $L_{o, o'}^-$ , therefore,  $SP^k$  remains unchanged. In the case where  $o \in SP^k$  but  $o' \notin SP^k$ ,  $o'$  has a chance to be a knn-SP (i.e.,  $L_{o, o'}^-$  could be a result turning point). If no object in  $SP^k$  can dominate  $o'$ , then  $o'$  becomes a new knn-SP and the  $k$ th nearest SP in  $SP^k$  needs to be removed.

**Algorithm 4.** The result turning point determination phase of the Cknn-SQ algorithm.

**Input:** A number  $k$ , a set GSP, an edge  $e$  connecting two nodes  $n_s$  and  $n_e$

**Output:** A set of tuples in form of  $\langle [n_i, n_j], SP^k \rangle$ , where  $SP^k$  is the knn-SP set between  $[n_i, n_j]$

sort the objects in GSP in ascending order of their distances to node

$n_s$  and set  $SP^k$  to  $SP_{n_s}^k$ ;

create an empty queue  $Q$ ;

/\* determining the landmarks of each object in GSP \*/

**foreach** object  $o \in GSP$  **do**

```

foreach object  $o' \in GSP - \{o\}$  do
  if  $o' \notin SP_{n_s}$  then
    determine  $o'$ 's landmark  $L_{o > o'}$ ;
    if  $L_{o > o'}$  is on  $e$  then insert  $L_{o > o'}$  into  $Q$ ;
  else
    determine  $o'$ 's landmarks  $L_{o, o'}^+$  or  $L_{o, o'}^-$ ;
    if  $L_{o, o'}^+$  is on  $e$  then insert  $L_{o, o'}^+$  into  $Q$ ;
    if  $L_{o, o'}^-$  is on  $e$  then insert  $L_{o, o'}^-$  into  $Q$ ;

```

sort the landmarks in  $Q$  in ascending order of their distances to node  $n_s$ ;

set  $n_i$  to  $n_s$ ;

/\* determining the result turning points with corresponding knn-SP \*/

**while**  $Q$  is not empty **do**

de – queue  $o.L$ ;

**switch**  $o.L$  **do**

```

  case  $L_{o > o'}$ 
    if  $o \in SP^k$  and  $o' \notin SP^k$  then
      if there is no object  $o'' \in SP^k$  dominating  $o'$ 
        then return  $\langle [n_i, L_{o > o'}], SP^k \rangle$ ,  $n_i = L_{o > o'}$ ,
        remove  $o$  from  $SP^k$ , and insert  $o'$  into  $SP^k$ ;
    case  $L_{o, o'}^+$ 
      if  $o' \in SP^k$  then
        return  $\langle [n_i, L_{o > o'}], SP^k \rangle$ ,  $n_i = L_{o > o'}$ , remove  $o'$  from  $SP^k$ ;
      if  $o \notin SP^k$  then insert  $o$  into  $SP^k$ ;
      else insert the  $k$ -th SP in GSP into  $SP^k$ ;
    otherwise
      /*  $o.L$  corresponds to  $L_{o, o'}^-$  */
      if  $o \in SP^k$  and  $o' \notin SP^k$  then
        if there is no object  $o'' \in SP^k$  dominating  $o'$  then
          return  $\langle [n_i, L_{o, o'}^-], SP^k \rangle$ ,  $n_i = L_{o, o'}^-$ ,
          remove the  $k$ -th SP from  $SP^k$ , and insert  $o'$  into  $SP^k$ ;

```

return  $\langle [n_i, n_e], SP^k \rangle$ ;

Fig. 8 continues the previous example of Cknn-SQ in Fig. 6, where the objects in the GSP are sorted in ascending order of their distances to node  $n_1$  and are represented as  $\{o_2, o_1, o_3, o_4\}$ . The initial result of  $SP^k$  (i.e.,  $SP_{n_1}^k$ ) is set to  $\{o_2, o_1\}$ . As shown in Fig. 8(a), five landmarks are considered sequentially, from landmarks  $L_{o_1 > o_3}$  to  $L_{o_3 > o_4}$ , so as to find the result turning points with corresponding knn-SP. (1)



Considering  $L_{o_1 > o_3}$ : the order of objects  $o_1$  and  $o_3$  in the GSP exchanges, and  $GSP = \{o_2, o_3, o_1, o_4\}$ . Because object  $o_3 \in o_2.S_{DO}$  (referring to Fig. 6(b)),  $o_3$  is still dominated by  $o_2$  at  $L_{o_1 > o_3}$ , and hence  $SP^k$  remains unchanged. (2) Considering  $L_{o_2 > o_3}^-$ : object  $o_2$  cannot dominate  $o_3$  and the GSP changes to  $\{o_3, o_2, o_1, o_4\}$ . As no object dominates  $o_3$ , it becomes a  $knn$ -SP and the second  $knn$ -SP  $o_1$  in  $SP^k$  is removed. That is,  $L_{o_2 > o_3}^-$  is a result turning point and  $SP^k$  is updated to  $\{o_3, o_2\}$ . (3) Considering  $L_{o_1 > o_4}$ : the GSP changes to  $\{o_3, o_2, o_4, o_1\}$ . Because both objects  $o_1$  and  $o_4$  are not in  $SP^k$ , the  $knn$ -SP remains the same. (4) Considering  $L_{o_2 > o_4}^-$ : the GSP changes to  $\{o_3, o_4, o_2, o_1\}$ . As object  $o_3$  cannot dominate  $o_4$ ,  $SP^k$  is updated from  $\{o_3, o_2\}$  to  $\{o_3, o_4\}$ . (5) Considering  $L_{o_4 > o_3}^+$ : object  $o_4$  begins to dominate  $o_3$  and the GSP changes to  $\{o_4, o_3, o_2, o_1\}$ . After removing  $o_3$  from  $SP^k$ , the second SP in the GSP (i.e., object  $o_2$ ) needs to be added into  $SP^k$ , and thus  $SP^k$  is updated to  $\{o_4, o_2\}$ . Finally, the  $knn$ -SP result for each result turning point is obtained and shown in Fig. 8(c).

## 5.2. Cknn-SQ<sup>+</sup> algorithm

Similar to the  $Cd_e$ -SQ<sup>+</sup> algorithm, the goal of the  $Cknn$ -SQ<sup>+</sup> algorithm is to further improve the performance of the global SP determination phase by performing the Breadth First Search on the road network. Algorithm 5 illustrates how to determine the GSP for edge  $e$  based on the Breadth First Search. To determine the  $knn$ -SP set  $SP_{n_s}^k$  of node  $n_s$ , we use queue  $Q$  (and list  $\mathcal{L}$ ) to keep the accessed cells of the grid index (and the objects that have been visited so far and could be the SP). Note that the objects in  $\mathcal{L}$  are sorted in ascending order of their distances to node  $n_s$ , and that initially the cell enclosing node  $n_s$  is inserted into  $Q$ . In each iteration, an element of  $Q$  (i.e., a cell  $g(i, j)$ ) will be retrieved so as to obtain its corresponding  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$  information. For each edge  $e$  stored in  $T_{edge}$ , when its node  $n_i$  has been visited, the road distances  $d_{n_s, o}$  from  $n_s$  to each object  $o$  on edge  $e$  will be first computed and then  $o$  is inserted into  $\mathcal{L}$ . If all of the objects in front of  $o$  in  $\mathcal{L}$  cannot dominate  $o$ , then  $o$  needs to be kept in  $\mathcal{L}$ , because it not only could be a SP but could also dominate the objects behind it (that is, the objects behind  $o$  will be removed from  $\mathcal{L}$  if they are dominated by  $o$ ). Otherwise,  $o$  is removed from  $\mathcal{L}$  because it cannot be a SP. When the number of objects in  $\mathcal{L}$  is greater than or equal to  $k$ , distance  $d_{kth}$ , which refers to the road distance of the  $k$ th object to  $n_s$ , can be used to determine whether it is necessary to examine the rest of the grid cells not yet accessed. If the road distance  $d_{n_s, n_j}$  from  $n_s$  to node  $n_j$  is less than  $d_{kth}$ , the cell  $g(m, n)$  enclosing node  $n_j$  needs to be accessed (i.e.,  $g(m, n)$  is inserted into  $Q$ ). Otherwise, the cell  $g(m, n)$  is not accessed. This is because all of the road distances of the unvisited objects stored in  $T_{obj}$  of  $g(m, n)$  must be greater than those of the first  $k$  objects in  $\mathcal{L}$ , and thus, those objects cannot be the  $knn$ -SP. The grid cells are repeatedly accessed until  $Q$  is empty, and finally the first  $k$  objects in  $\mathcal{L}$  are the  $knn$ -SP of  $n_s$ . Similarly, the  $knn$ -SP set  $SP_{n_e}^k$  of node  $n_e$  can be obtained using the method mentioned above. After retrieving the set  $S_e$  of the objects on edge  $e$ , the GSP is set to  $SP_{n_s}^k \cup SP_{n_e}^k \cup S_e$ .

## 5.3. Time complexity analysis of Cknn-SQ approaches

Similar to the analysis of the  $Cd_e$ -SQ approaches, we analyze the time complexities of the global SP determination phase and the result turning point determination phase separately:

- In order to find the GSP, the  $knn$ -SP sets for nodes  $n_s$  and  $n_e$  (i.e.,  $SP_{n_s}^k$  and  $SP_{n_e}^k$ ) should be determined. The time complexity of determining  $SP_{n_s}^k$  is  $O(|o_{n_s}| \cdot C_{dist} + |o'_{n_s}|^2)$ . For the  $Cknn$ -SQ algorithm,  $|o_{n_s}|$  (and  $|o'_{n_s}|$ ) refers to the number of objects within the search region  $SR_{n_s}$  (and whose road distances to  $n_s$  are less than or equal to that of the  $k$ th nearest static SP), and  $C_{dist}$  is the cost of computing the object's road distance. As for the  $Cknn$ -SQ<sup>+</sup> algorithm,  $|o_{n_s}|$  is the same as  $|o'_{n_s}|$  and refers to the number of objects whose road distances are less than or equal to that of the  $k$ th nearest  $knn$ -SP. As the time complexity for determining  $SP_{n_e}^k$  is  $O(|o_{n_e}| \cdot C_{dist} + |o'_{n_e}|^2)$ , the total complexity is  $O((|o_{n_s}| + |o_{n_e}|) \cdot C_{dist} + |o'_{n_s}|^2 + |o'_{n_e}|^2)$ .
- The result turning point determination phase consists of two steps (referring to Algorithm 4). The first step is to determine the object landmarks and maintain a queue  $Q$  to store these landmarks. Its time complexity is  $O(|GSP|^2 + |Q| \cdot \log|Q|)$ , where  $|GSP|$  and  $|Q|$  are the sizes of GSP and queue  $Q$ , respectively. The second is to find the result turning points and their  $knn$ -SP, and the complexity is  $|Q|$ . As such, the time complexity of the result turning point determination phase is estimated as  $O(|GSP|^2 + |Q| \cdot (\log|Q| + 1))$ .

**Algorithm 5.** The global SP determination phase of the  $Cknn$ -SQ<sup>+</sup> algorithm.

**Input:** A grid index, a number  $k$ , an edge  $e$  connecting two nodes  $n_s$  and  $n_e$   
**Output:** The GSP for edge  $e$   
 /\* determining  $SP_{n_s}^k$  for node  $n_s$  \*/  
 create an empty queue  $Q$  and a sorted list  $\mathcal{L}$ , and set  $d_{kth} = \infty$ ;  
 insert cell  $g(i, j)$  enclosing node  $n_s$  into  $Q$ ;  
**while**  $Q$  is not empty **do**  
   de-queue  $g(i, j)$  and access its  $T_{edge}$ ,  $T_{node}$ , and  $T_{obj}$ ;  
   **foreach** edge  $e$  stored in  $T_{edge}$  **do**  
     **if** node  $n_i$  has been visited **then**  
       **foreach** object  $o$  on edge  $e$  **do**  
         compute the road distance  $d_{n_s, o}$  of  $o$  to  $n_s$   
         and insert  $o$  into  $\mathcal{L}$ ;  
         **if**  $o \notin o'.S_{DO}$  of each object  $o'$  in front of it in  $\mathcal{L}$  **then**  
           **foreach** object  $o''$  behind  $o$  **do**  
             **if**  $o'' \in o.S_{DO}$  **then** remove  $o''$  from  $\mathcal{L}$ ;  
           **else** remove  $o$  from  $\mathcal{L}$ ;  
         **if**  $|\mathcal{L}| < k$  **then**  $d_{kth} = \infty$ ;  
         **else**  $d_{kth} = d_{n_s, o_{kth}}$  of the  $k$ -th object  $o_{kth}$  in  $\mathcal{L}$ ;  
         compute the road distance  $d_{n_s, n_j}$  from nodes  $n_s$  to  $n_j$ ;  
         **if**  $d_{n_s, n_j} < d_{kth}$  **then** insert cell  $g(m, n)$  enclosing  $n_j$  into  $Q$ ;  
 insert the first  $k$  objects in  $\mathcal{L}$  into  $SP_{n_s}^k$ ;  
 determine  $SP_{n_e}^k$  for node  $n_e$  in a similar way;  
 retrieve the set  $S_e$  of objects on edge  $e$ ;  
 set GSP to  $SP_{n_s}^k \cup SP_{n_e}^k \cup S_e$ ;

## 6. Performance evaluation

We conduct four sets of experiments for the  $Cd_e$ -SQ and the  $Cknn$ -SQ approaches in this section. The first one is evaluated to study the effect of the number of grid cells on the performance of processing  $Cd_e$ -SQ and  $Cknn$ -SQ. We decide an appropriate cell number from these experiments and use it to build the grid index for query processing in the rest of the experiments. The second one investigates the performance of the  $Cd_e$ -SQ and  $Cd_e$ -SQ<sup>+</sup> algorithms by measuring the CPU time and the number of accessed cells for processing a  $Cd_e$ -SQ. The third set is conducted to demonstrate the efficiency of the  $Cknn$ -SQ and  $Cknn$ -SQ<sup>+</sup> algorithms in terms of CPU time and number of accessed cells. Finally, the last set of experiments studies how well our approaches work for larger road networks.

### 6.1. Experimental settings

All experiments are performed on a PC with AMD Athlon 64 X2 5200 CPU and 2 GB RAM. The algorithms are implemented in C++. We use three road maps in our simulation and generate objects (varying from 0.1 K to 200 K) using the generator proposed in [38]. The first road map is the Oldenburg (a city in Germany) consisting of about 6000 nodes and 7000 edges, the second is the San Joaquin County with about 18,200 nodes and 23,800 edges, and the third is the California Road Network which contains 21,048 nodes and 21,693 edges [39–43]. Each generated object has several static attributes (ranging from 2 to 6) whose values are normalized in the range [0, 1]. In the experimental space, we also generate 30 query paths and each of them consists of multiple edges (ranging from 1 to 16). The start edge  $e$  of each query path is randomly chosen, and once  $e$  has been considered, the next edge is randomly selected from the edges connecting  $e$ . For each query path, we perform a  $Cd_e$ -SQ and a  $Cknn$ -SQ (i.e., find the  $d_e$ -SP and the  $knn$ -SP for each point within the query path), where the default values of  $d_e$  and  $k$  are set to 1% of the entire space and 10, respectively. The performance is measured by the average CPU time and the average number of accessed cells in performing workloads of the 30 queries. Table 1 summarizes the parameters under investigation, along with their default values and ranges.

We compare the proposed  $Cd_e$ -SQ approaches with a naive approach, in which the road distances of all objects are first computed and then the BBS algorithm [8] is applied to finding the skyline points with respect to the computed distances and static attributes. Only the skyline points within the distance range  $d_e$  can be the query result. Similarly, we

compare the performance of the  $Cknn$ -SQ approaches against a naive approach, where the skyline points are first determined using the BBS algorithm and then sorted according to their road distances so as to retrieve the  $k$  nearest skyline points.

### 6.2. Effect of number of grid cells

Fig. 9 studies the effect of the number of grid cells on the performance of processing  $Cd_e$ -SQ and  $Cknn$ -SQ. In the experiments, we vary grid cells from  $32 \times 32$  to  $512 \times 512$  and measures the CPU cost and the number of accessed cells for the proposed algorithms. As shown in Fig. 9(a) and (b), the CPU costs for the  $Cd_e$ -SQ and the  $Cknn$ -SQ approaches decrease with the increasing number of grid cells. This is mainly because each cell would store fewer object as the number of grid cells increases (i.e., the cell size is smaller), so that the time required for finding the GSP in the global SP determination phase can be reduced. On the other hand, a larger number of grid cells would result in the fact that more cells are accessed to find the GSP, which is illustrated in Fig. 9(c) and (d). Because the number of cells decreases the CPU time but increases the number of accessed cells, we need to decide an appropriate cell number used to build the grid index for query processing. As we can see from Fig. 9, when the cell number is greater than  $128 \times 128$ , the improvement of CPU cost for all algorithms becomes insignificant, while the number of cells need to be accessed increases sharply. The experimental result reveals that  $128 \times 128$  is a better choice than the others. Hence, we will use it as the default cell number in all the rest experiments.

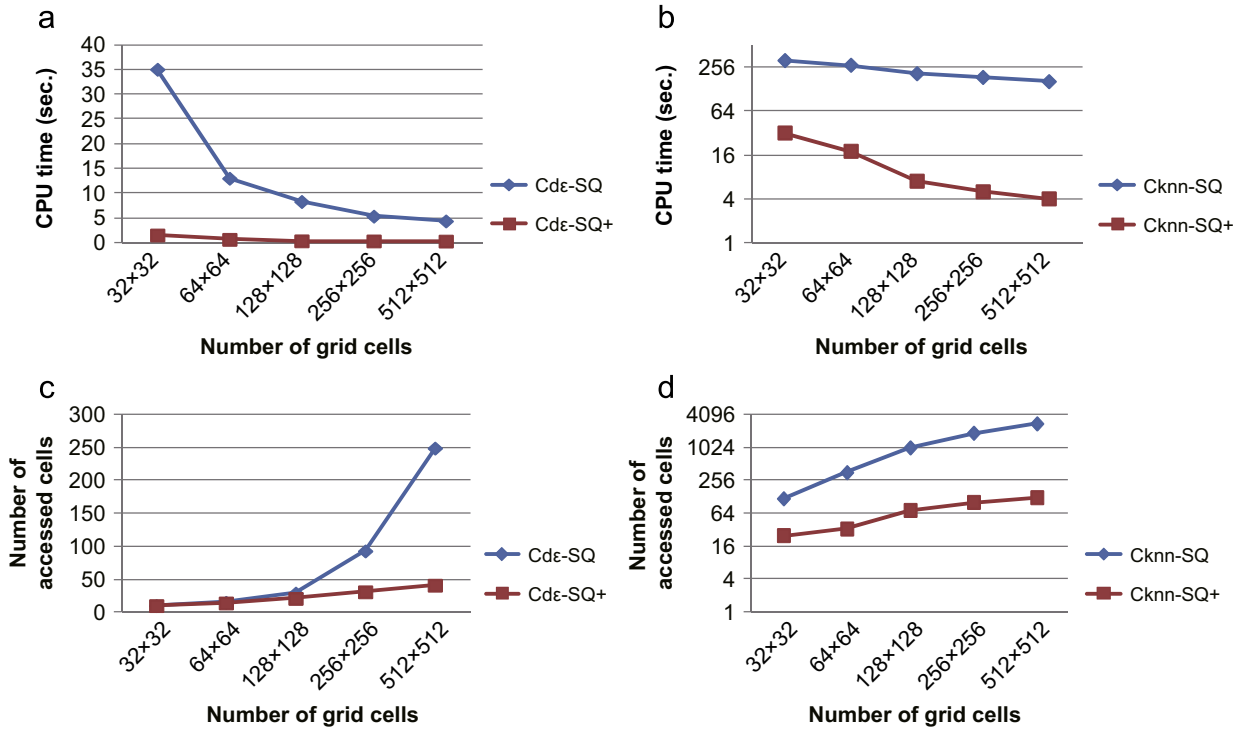
### 6.3. Performance of $Cd_e$ -SQ and $Cd_e$ -SQ<sup>+</sup> algorithms

In this subsection, we compare the  $Cd_e$ -SQ and  $Cd_e$ -SQ<sup>+</sup> algorithms with a naive approach in terms of the CPU time and the number of accessed cells. Four experiments are conducted to investigate the effects of five important factors on the performance of processing  $Cd_e$ -SQ. These important factors are the number of objects, the number of static attributes, the length of query path, the size of query range (i.e., the value of  $d_e$ ), and the distribution of objects inside each grid cell.

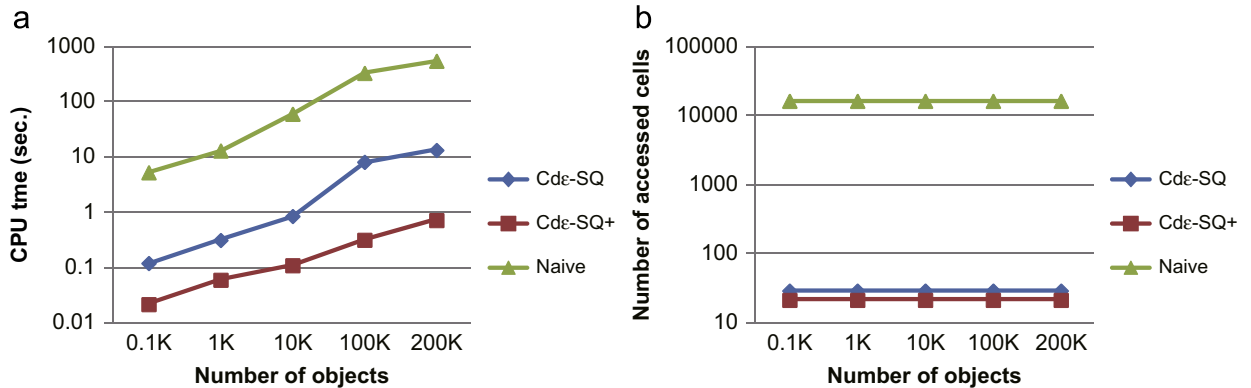
Fig. 10 illustrates the performance of the  $Cd_e$ -SQ approaches as a function of the number of objects (ranging from 0.1 K to 200 K). Note that hereafter all figures use a logarithmic scale for the y-axis. As shown in Fig. 10(a), the  $Cd_e$ -SQ<sup>+</sup> algorithm significantly outperforms the naive approach in the CPU time, even for a smaller number of objects (e.g., 0.1 K) which corresponds to the number of gas stations or hotels. This is mainly because for the naive approach the road distances of all objects have to be computed which incurs high computation cost as the object number increases. Similarly, the performance gap between the  $Cd_e$ -SQ<sup>+</sup> algorithm and  $Cd_e$ -SQ algorithm increases with the increasing number of objects. The reason is that in the  $Cd_e$ -SQ algorithm some objects whose road distances are greater than  $d_e$  have to be considered because their Euclidean distances are less than or equal to  $d_e$ , whereas in the  $Cd_e$ -SQ<sup>+</sup> algorithm the

**Table 1**  
System parameters.

Parameter	Default	Range
Number of objects (K)	100	0.1, 1, 10, 100, 200
Number of static attributes	4	2, 3, 4, 5, 6
Number of grid cells	128 <sup>2</sup>	32 <sup>2</sup> , 64 <sup>2</sup> , 128 <sup>2</sup> , 256 <sup>2</sup> , 512 <sup>2</sup>
Length of query path	4	1, 2, 4, 8, 16
$d_e$ (%)	1	0.1, 0.5, 1, 2, 3
$k$	10	1, 5, 10, 15, 20



**Fig. 9.** Effect of the number of grid cells. (a) CPU cost for the  $Cd_{\epsilon}$ -SQ. (b) CPU cost for the  $Cknn$ -SQ. (c) Number of accessed cells for the  $Cd_{\epsilon}$ -SQ. (d) Number of accessed cells for the  $Cknn$ -SQ.



**Fig. 10.**  $Cd_{\epsilon}$ -SQ performance vs. number of objects. (a) CPU cost for the  $Cd_{\epsilon}$ -SQ. (b) Number of accessed cells for the  $Cd_{\epsilon}$ -SQ.

unnecessary distance computation of these objects can be avoided by applying the Breadth First Search on the road network. The experimental result shown in Fig. 10(b) illustrates that the number of accessed cells for the  $Cd_{\epsilon}$ -SQ algorithm, the  $Cd_{\epsilon}$ -SQ+ algorithm, and the naive approach remains constant regardless of the number of objects. This is because no matter what the object quantity is, only the cells storing objects within distance range  $d_{\epsilon}$  would be accessed in the  $Cd_{\epsilon}$ -SQ and  $Cd_{\epsilon}$ -SQ+ algorithms (for the naive approach all cells need to be accessed). Hence, the number of accessed cells in the proposed algorithms is dominated by  $d_{\epsilon}$  and insensitive to the amount of objects.

Fig. 11 studies the impact of the number of static attributes on the performance of processing  $Cd_{\epsilon}$ -SQ. Fig. 11(a) measures the CPU time for the  $Cd_{\epsilon}$ -SQ algorithm, the  $Cd_{\epsilon}$ -SQ+ algorithm, and the naive approach by varying the number of static attributes from 2 to 6. When this number increases, the CPU overhead for all algorithms increases slightly because more attributes would result in more skyline points so that more dominance tests are performed. As for the number of accessed cells (shown in Fig. 11(b)), for the  $Cd_{\epsilon}$ -SQ and  $Cd_{\epsilon}$ -SQ+ algorithms it is constant and under 30 in spite of the number of attributes (for the naive approach the number of accessed cells is equal to  $128 \times 128$ ). The reason is the

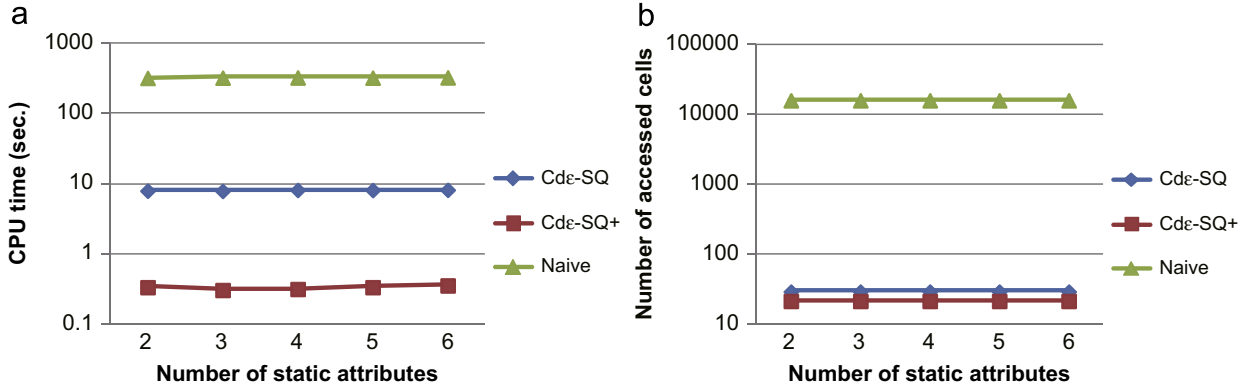


Fig. 11.  $Cd_e$ -SQ performance vs. number of static attributes. (a) CPU cost for the  $Cd_e$ -SQ. (b) Number of accessed cells for the  $Cd_e$ -SQ.

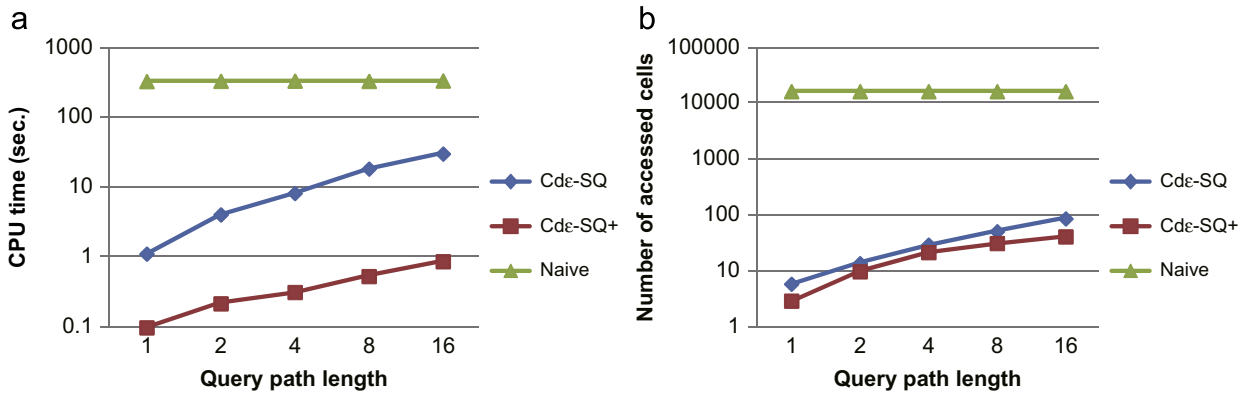


Fig. 12.  $Cd_e$ -SQ performance vs. query path length. (a) CPU cost for the  $Cd_e$ -SQ. (b) Number of accessed cells for the  $Cd_e$ -SQ.

same as that for Fig. 10(b). The experimental results show that the  $Cd_e$ -SQ<sup>+</sup> algorithm outperforms its competitors significantly in all cases, which confirms again that applying the Breadth First Search to prune the non-qualifying objects can greatly improve the performance of processing  $Cd_e$ -SQ.

Fig. 12 evaluates the CPU cost and the number of accessed cells for the  $Cd_e$ -SQ algorithm, the  $Cd_e$ -SQ<sup>+</sup> algorithm, and the naive approach under various query path lengths (varying from 1 to 16). As we can see, the naive approach gives the worst performance in all cases because no pruning strategy is adopted to reduce the distance computations of objects. As for the  $Cd_e$ -SQ and  $Cd_e$ -SQ<sup>+</sup> algorithms, when the query path length is equal to 1 (that is, only the  $d_e$ -SP for one edge need to be determined), the performance of the  $Cd_e$ -SQ<sup>+</sup> algorithm is as good as that of the  $Cd_e$ -SQ algorithm. However, when the query length is greater than 1, the  $Cd_e$ -SQ<sup>+</sup> algorithm outperforms the  $Cd_e$ -SQ algorithm by a factor of 20 to 40 in terms of the CPU cost. In addition, the  $Cd_e$ -SQ<sup>+</sup> algorithm reduces the number of accessed cells by almost a factor of 2 compared to the  $Cd_e$ -SQ algorithm. The improvement is due to the fact that the  $Cd_e$ -SQ<sup>+</sup> algorithm retrieves only the objects whose road distances to one of the nodes within the query path are less than or equal to  $d_e$ . However, in the  $Cd_e$ -SQ algorithm the objects

would be retrieved once their Euclidean distances are less than or equal to  $d_e$ .

In Fig. 13, we measure the CPU time and the number of accessed cells under different values of  $d_e$ . For the  $Cd_e$ -SQ<sup>+</sup> algorithm, as the non-qualifying objects (i.e., the objects outside the distance range  $d_e$ ) can be precisely pruned by applying the Breadth First Search, both the required CPU time and the number of accessed cells increase slightly with the increasing  $d_e$ . For the  $Cd_e$ -SQ algorithm, a smaller  $d_e$  is favorable because less computation of road distances of the qualifying objects is performed. On the other hand, the performance (i.e., both the CPU time and the accessed cell number) of the  $Cd_e$ -SQ algorithm significantly degrades as the value of  $d_e$  increases. For the naive approach, the effect of increasing  $d_e$  is not significant because computing the road distances of all objects dominates the overall performance.

Finally in this subsection, we study how the distribution of objects inside a grid cell affects the performance of the  $Cd_e$ -SQ algorithm, the  $Cd_e$ -SQ<sup>+</sup> algorithm, and the naive approach, by considering three types of object distributions. These object distributions are the *uniform distribution*, the *Gaussian distribution* (with variance 0.05 and a mean being the coordinate of the center of grid cell), and the *Zipf distribution* (with skew coefficient 0.8). As shown in Fig. 14(a), the CPU time of all approaches

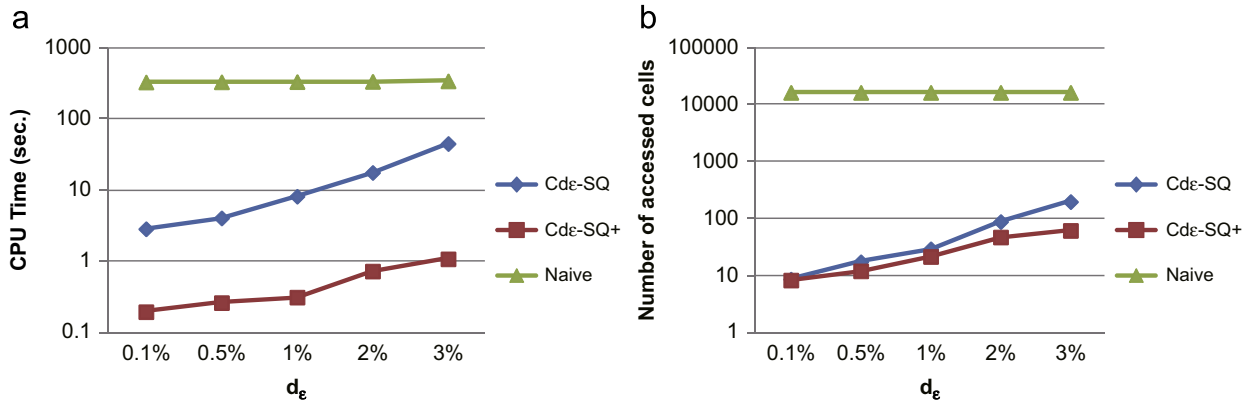


Fig. 13.  $Cd_\epsilon$ -SQ performance vs.  $d_\epsilon$ . (a) CPU cost for the  $Cd_\epsilon$ -SQ. (b) Number of accessed cells for the  $Cd_\epsilon$ -SQ.

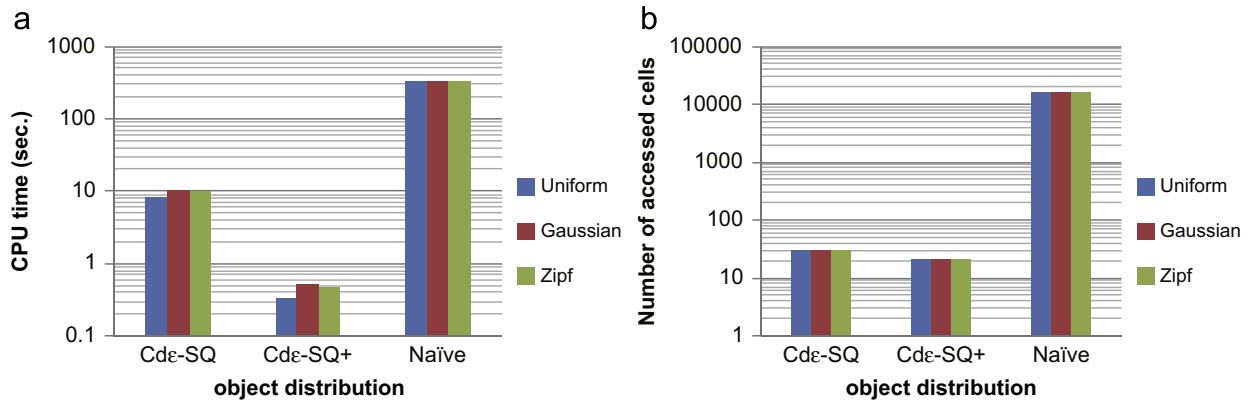


Fig. 14.  $Cd_\epsilon$ -SQ performance vs. object distribution. (a) CPU cost for the  $Cd_\epsilon$ -SQ. (b) Number of accessed cells for the  $Cd_\epsilon$ -SQ.

for the uniform distribution is slightly lower than that for the Gaussian and Zipf distributions. This is because for the Gaussian and Zipf distributions, the objects inside a grid cell are closer to each other (comparing to the uniform distribution). Therefore, more landmarks need to be examined in the result turning point determination phase (no matter which approach is adopted), so that it takes more CPU time to find the result turning points. From the experimental result, we also find that the CPU performance is insensitive to the object distribution. As for the number of accessed cells, Fig. 14(b) indicates that all approaches have the constant number of accessed cells regardless of object distribution, because for our  $Cd_\epsilon$ -SQ and  $Cd_\epsilon$ -SQ+ algorithms the number of accessed cells is dominated by  $d_\epsilon$ , and for the naive approach all grid cells have to be accessed to compute the road distances of objects.

#### 6.4. Performance of Cknn-SQ and Cknn-SQ+ algorithms

To exploit the efficiency of the proposed algorithms in processing a  $Cknn$ -SQ, we compare them in terms of the CPU cost and the number of accessed cells. We study the impacts of different factors on the performance of processing the  $Cknn$ -SQ. These factors are the number of objects,

the number of static attributes, the query path length, the number of nearest neighbors (i.e.,  $k$ ), and the object distribution within a grid cell.

Fig. 15 demonstrates the effect of various numbers of objects on the performance of the  $Cknn$ -SQ algorithm, the  $Cknn$ -SQ+ algorithm, and the naive approach. Fig. 15(a) and (b) investigates the CPU cost and the number of accessed cells, respectively, by varying object number from 0.1 K to 200 K. As shown in Fig. 15(a), the CPU overhead for all algorithms increases with the increase of number of objects because more distance computations of objects are required for finding the GSP. Nevertheless, the  $Cknn$ -SQ+ algorithm significantly outperforms the  $Cknn$ -SQ algorithm and the naive approach in all cases. An interesting observation from Fig. 15(b) is that a larger number of objects results in a less number of cells that need to be accessed in processing the  $Cknn$ -SQ. This is mainly because when the road network is sparse (e.g., object number is 0.1 K or 1 K), more cells have to be accessed so as to find the  $k$ th nearest SP, whereas a dense network (e.g., number of objects is 200 K) may lead to early termination of finding the  $k$ th nearest SP so that less cells are accessed.

In Fig. 16, we study how the number of static attributes affects the CPU time and the number of accessed



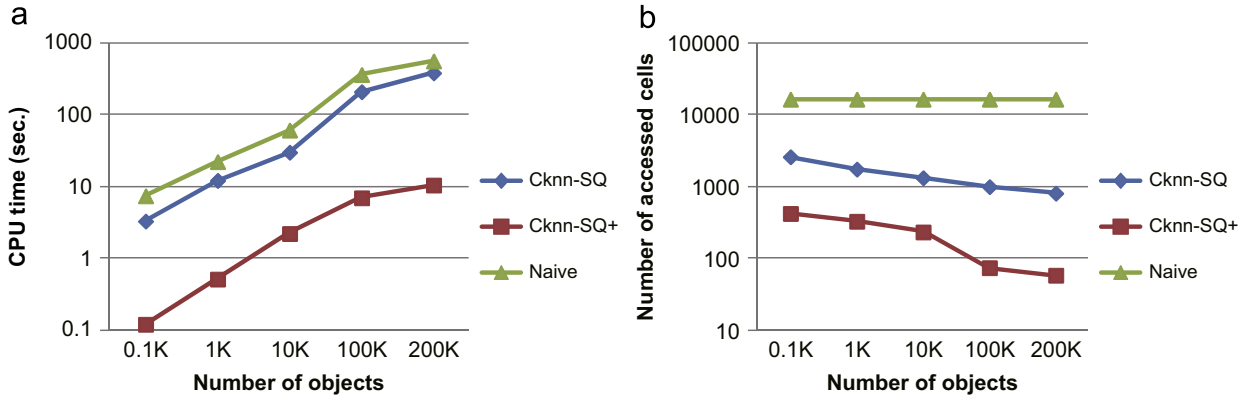


Fig. 15. *Cknn-SQ* performance vs. number of objects. (a) CPU cost for the *Cknn-SQ*. (b) Number of accessed cells for the *Cknn-SQ*.

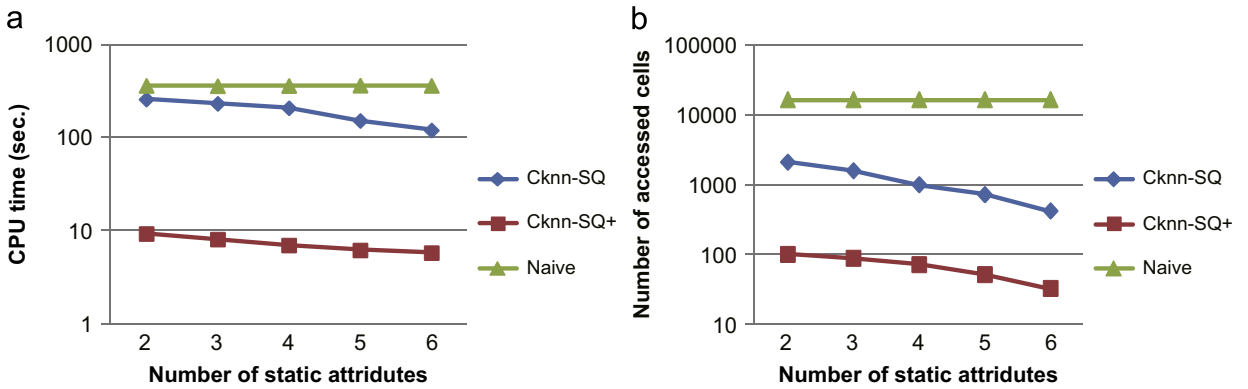


Fig. 16. *Cknn-SQ* performance vs. number of static attributes. (a) CPU cost for the *Cknn-SQ*. (b) Number of accessed cells for the *Cknn-SQ*.

cells for the *Cknn-SQ* algorithm, the *Cknn-SQ*<sup>+</sup> algorithm, and the naive approach. Different from the previous experimental results, the curves for both the *Cknn-SQ* algorithm and the *Cknn-SQ*<sup>+</sup> algorithm shown in Fig. 16(a) and (b) have the decreasing trends. That is, the CPU cost and the accessed cells can be reduced as the number of static attributes increases. The reason for this improvement is that for a small number of static attributes, most of the objects are dominated by fewer objects and thus more cells are accessed to retrieve objects and more distance computations of these objects are required in finding the *k*th nearest *SP*. Conversely, for a large number of attributes, the *k*th nearest *SP* could be determined early by only accessing the cells closer to the query path because they may already contain the *k* *SP*. As for the naive approach, it is insensitive to the number of static attributes so that the CPU time and the number of accessed cells remain about constant under various static attribute numbers.

Fig. 17 shows the CPU cost and the number of accessed cells for the proposed algorithms and the naive approach under various lengths of query path. Similar to the experiments in Fig. 12, we vary the query length from 1 to 16 to investigate its effect on the performance of processing the *Cknn-SQ*. As we can see, the *Cknn-SQ*<sup>+</sup> algorithm has a significantly better performance on both the CPU cost and the number of accessed cells compared

to its competitors. In terms of the CPU overhead, the *Cknn-SQ*<sup>+</sup> algorithm outperforms the *Cknn-SQ* algorithm (and the naive approach) by a factor of up to 25 (and 35) in all cases. In addition, the *Cknn-SQ*<sup>+</sup> algorithm reduces the number of accessed cells by almost two (and three) orders of magnitude compared to the *Cknn-SQ* algorithm (and the naive approach).

This experiment evaluates the effect of the number of nearest neighbors (i.e., *k*) on the CPU time and the accessed cells of the *Cknn-SQ* algorithm, the *Cknn-SQ*<sup>+</sup> algorithm, and the naive approach. As shown in Fig. 18(a), when *k* increases, the CPU overhead for the *Cknn-SQ* and *Cknn-SQ*<sup>+</sup> algorithms grows. The reason is that as *k* becomes greater, the number of qualifying objects inevitably increases so that more distance computations of these objects and more dominance tests between them are required. As for the effect of *k* on the number of accessed cells shown in Fig. 18(b), the curve for both algorithms has the increasing trend. This is because as *k* increases, the *k*th nearest *SP* becomes farther from the query path, and hence more cells would be accessed. Nevertheless, benefiting from the Breadth First Search, the *Cknn-SQ*<sup>+</sup> algorithm is more insensitive to *k* compared to the *Cknn-SQ* algorithm. The experimental result shows that the naive approach yields the worst performance no matter in the CPU cost or the number of accessed cells.

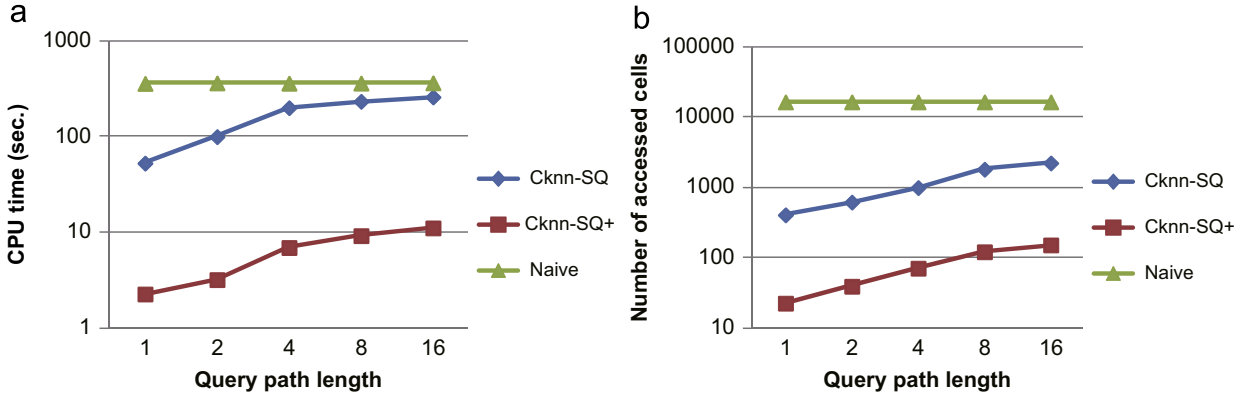


Fig. 17. *Cknn-SQ* performance vs. query path length. (a) CPU cost for the *Cknn-SQ*. (b) Number of accessed cells for the *Cknn-SQ*.

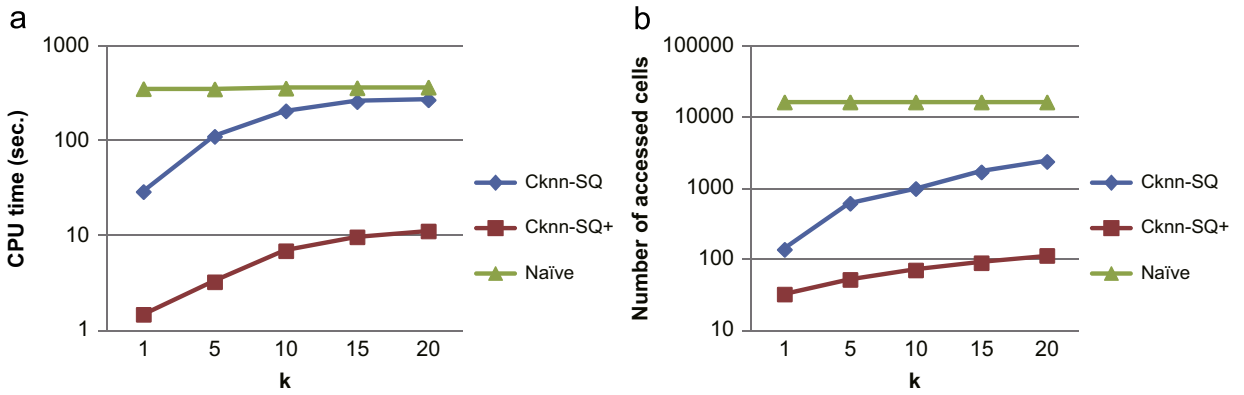


Fig. 18. *Cknn-SQ* performance vs. *k*. (a) CPU cost for the *Cknn-SQ*. (b) Number of accessed cells for the *Cknn-SQ*.

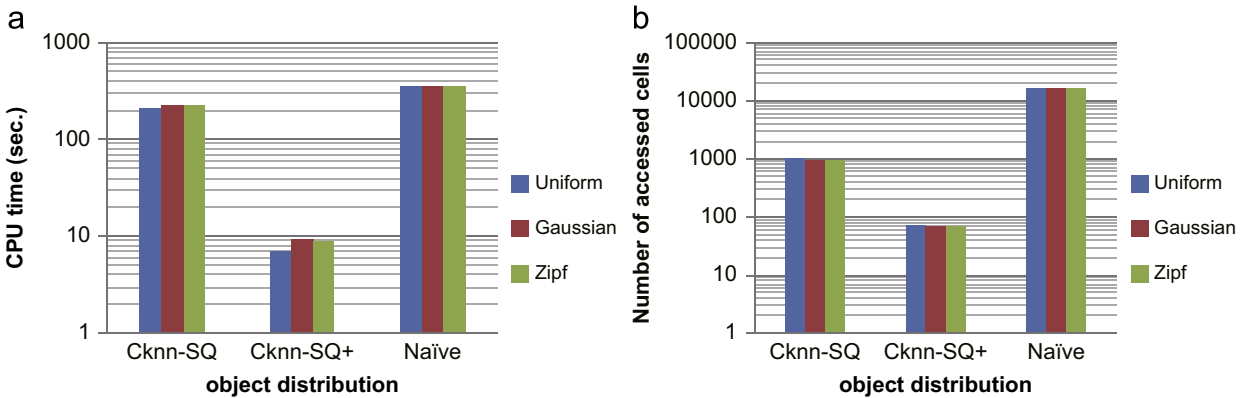
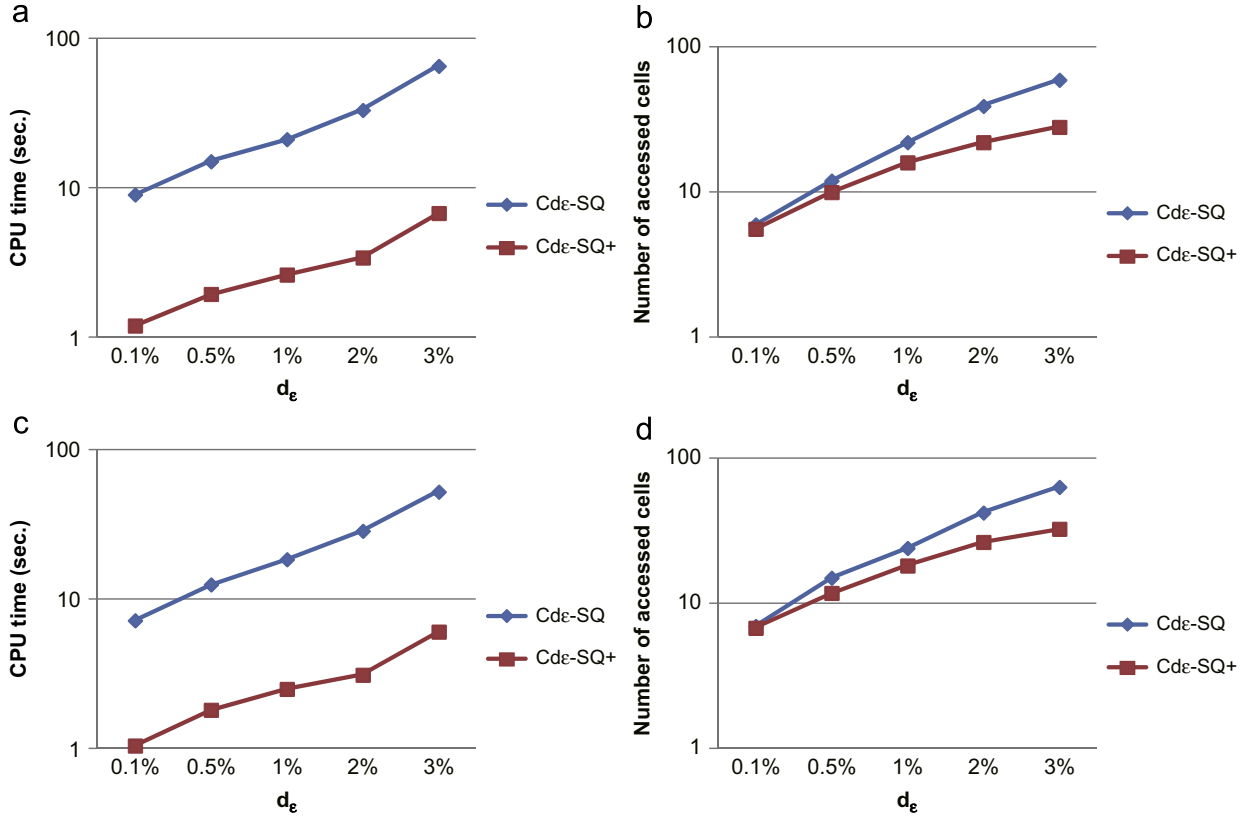


Fig. 19. *Cknn-SQ* performance vs. object distribution. (a) CPU cost for the *Cknn-SQ*. (b) Number of accessed cells for the *Cknn-SQ*.

The last experiment measures the CPU overhead and the number of accessed cells for the *Cknn-SQ* algorithm, the *Cknn-SQ*<sup>+</sup> algorithm, and the naive approach, using different object distributions (including the uniform distribution, the Gaussian distribution, and the Zipf distribution). As shown in Fig. 19(a), each *Cknn-SQ* approach achieves a slightly better CPU performance for the uniform distribution than for the Gaussian and Zipf distributions. The reason is that for the uniform

distribution fewer object landmarks are considered in the result turning point determination phase. However, Fig. 19(b) shows that when the objects inside each grid cell follow the uniform distribution, the *Cknn-SQ* algorithm and the *Cknn-SQ*<sup>+</sup> algorithm require a little more accessed cells (comparing to the Gaussian and Zipf distributions). This is because the Gaussian and Zipf distributions could result in a higher chance of determining the *k*th nearest *SP* early in the global *SP* determination phase.



**Fig. 20.**  $Cd_{\epsilon}$ -SQ performance for the larger road networks. (a) CPU cost for San Joaquin County. (b) Number of accessed cells for San Joaquin County. (c) CPU cost for California Road Network. (d) Number of accessed cells for California Road Network.

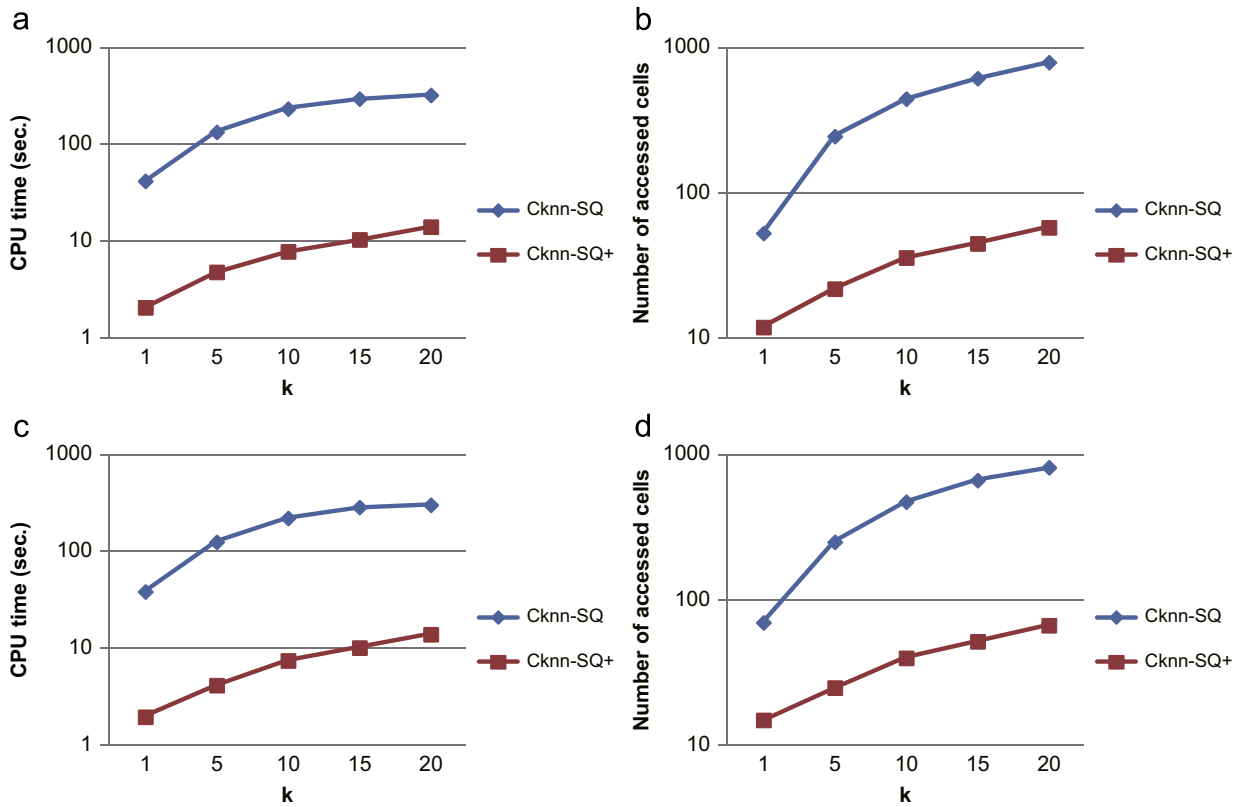
### 6.5. Performance for larger road networks

In this subsection, we conduct two sets of experiments to study how well the proposed  $Cd_{\epsilon}$ -SQ and  $Cknn$ -SQ approaches work for two larger road networks, the San Joaquin County with about 18,200 nodes and 23,800 edges and the California Road Network consisting of 21,048 nodes and 21,693 edges. Fig. 20(a) and (b) measures the CPU time and the number of accessed cells for our  $Cd_{\epsilon}$ -SQ approaches using the San Joaquin County, respectively, as a function of  $d_{\epsilon}$  (varying from 0.1% to 3%). Fig. 20(c) and (d) studies the performance of our  $Cd_{\epsilon}$ -SQ approaches for the California Road Network under various values of  $d_{\epsilon}$ . Compared to the experimental result in Fig. 13 (which shows the performance for a small road network, Oldenburg), the CPU overhead for both the  $Cd_{\epsilon}$ -SQ algorithm and the  $Cd_{\epsilon}$ -SQ+ algorithm increases, while the number of accessed cells decreases. The reason for the higher CPU time is that the road connectivity of the larger road networks is more complicated so that more time is required for computing the road distances of objects within the distance range  $d_{\epsilon}$ . Nevertheless, the CPU overhead still increases within a slow rate with a fairly acceptable range. On the other hand, the decrease of accessed cells for the larger road networks is mainly because each grid cell stores more information of objects and hence the  $Cd_{\epsilon}$ -SQ can be terminated by accessing fewer cells closer to the query path.

The last set of experiments investigates the performance of the proposed  $Cknn$ -SQ approaches for the San Joaquin County and the California Road Network by measuring the CPU time and the number of accessed cells under different values of  $k$ . As shown in Fig. 21(a) and (c), the CPU time of the  $Cknn$ -SQ and  $Cknn$ -SQ+ algorithms for the San Joaquin County and the California Road Network is higher than that for the Oldenburg (shown in Fig. 18(a)). Conversely, as we can see from Fig. 21(b) and (d), the number of accessed cells for the San Joaquin County and the California Road Network is much less than that in the Oldenburg case (shown in Fig. 18(b)). Similar to the reasons mentioned in the above set of experiments (i.e., Fig. 20), the higher CPU time (the less number of accessed cells) is owing to the complicated road connectivity (more object information storing in each grid cell) of the larger road networks. From the experimental results, we know that the proposed  $Cd_{\epsilon}$ -SQ and  $Cknn$ -SQ approaches are also suitable for a road network with large number of nodes and edges.

## 7. Conclusions and future work

This paper focused on processing two novel and important distance-based skyline queries in road networks. The first was the continuous  $d_{\epsilon}$ -skyline query ( $Cd_{\epsilon}$ -SQ) and the second was the continuous  $k$  nearest neighbor-skyline query ( $Cknn$ -SQ). A grid index was designed to manage the information of data objects



**Fig. 21.** *Cknn-SQ* performance for the larger road networks. (a) CPU cost for San Joaquin County. (b) Number of accessed cells for San Joaquin County. (c) CPU cost for California Road Network. (d) Number of accessed cells for California Road Network.

located in a road network consisting of edges and nodes, and made use of the grid index to access only a small proportion of data objects while computing the  $Cd_k$ -SQ and *Cknn-SQ* results. The  $Cd_k$ -SQ algorithm and the  $Cd_k$ -SQ<sup>+</sup> algorithm were developed, both of which consist of a global *SP* determination phase and a result turning point determination phase, to efficiently process the  $Cd_k$ -SQ. Also, the *Cknn-SQ* algorithm and the *Cknn-SQ*<sup>+</sup> algorithm were proposed to find the *Cknn-SQ* result. Comprehensive experiments demonstrated the effectiveness and the efficiency of the proposed approaches.

There are several interesting avenues for the future extensions of this work. One important avenue is to extend the  $Cd_k$ -SQ and *Cknn-SQ* approaches to be suitable for dynamic environment where the objects move with time. Another extension is to address the issue of efficiently handling the updates of objects (such as updates of static and dynamic attributes). A further extension is how to make proper use of data structures so as to enhance the query performance.

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