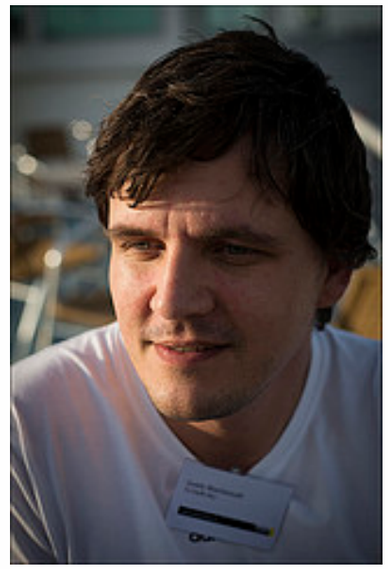
A Theory of Name Resolution

Pierre Neron[1]

Andrew Tolmach[2]



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Guido Wachsmuth[1]

1 TU Delft Delft University of Technology

2 Portland State UNIVERSITY

[ESOP15]

Static Name Resolution

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module A {
    def s = 5
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module B {
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    def y = 3 + s
    def f =
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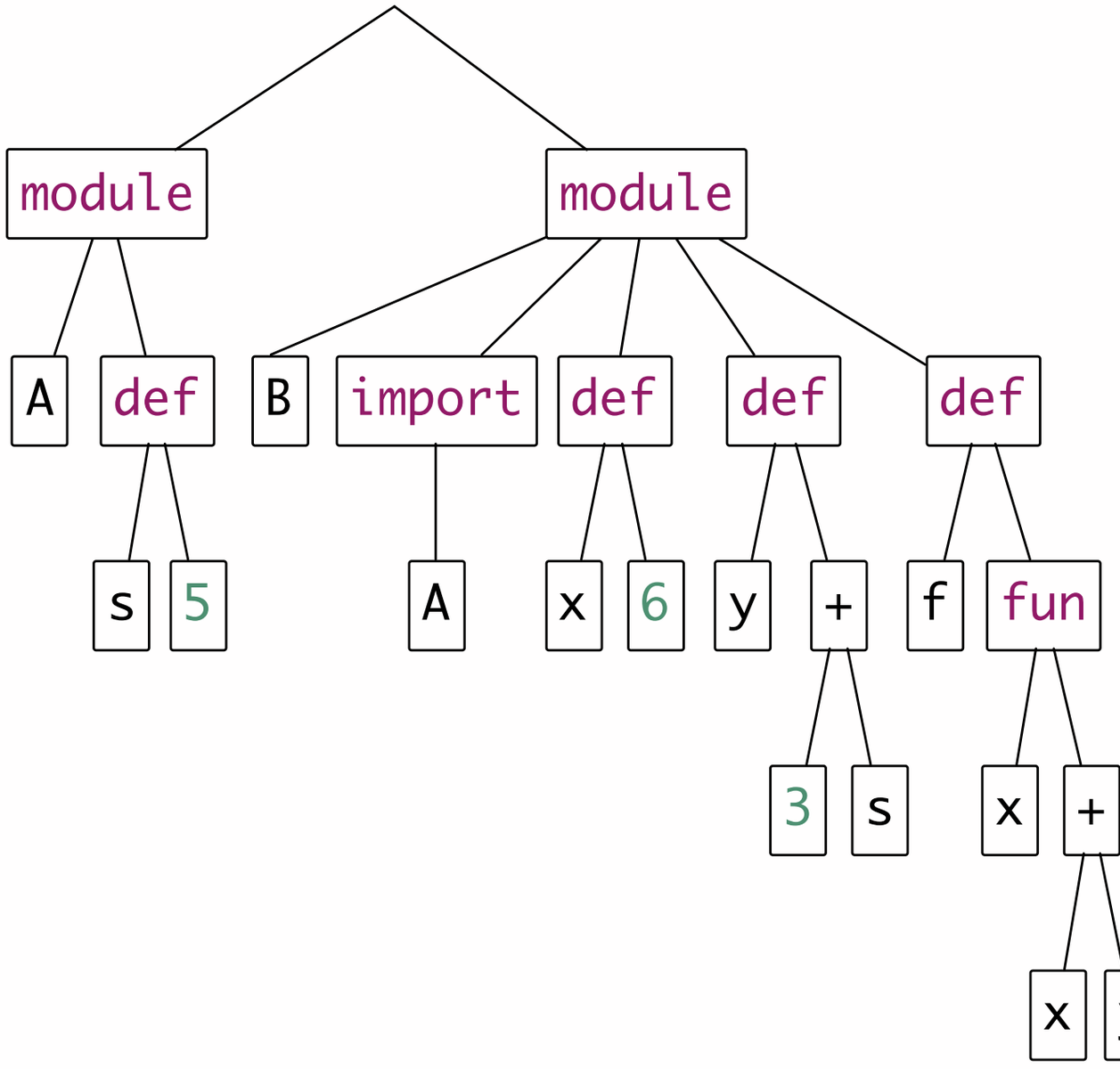

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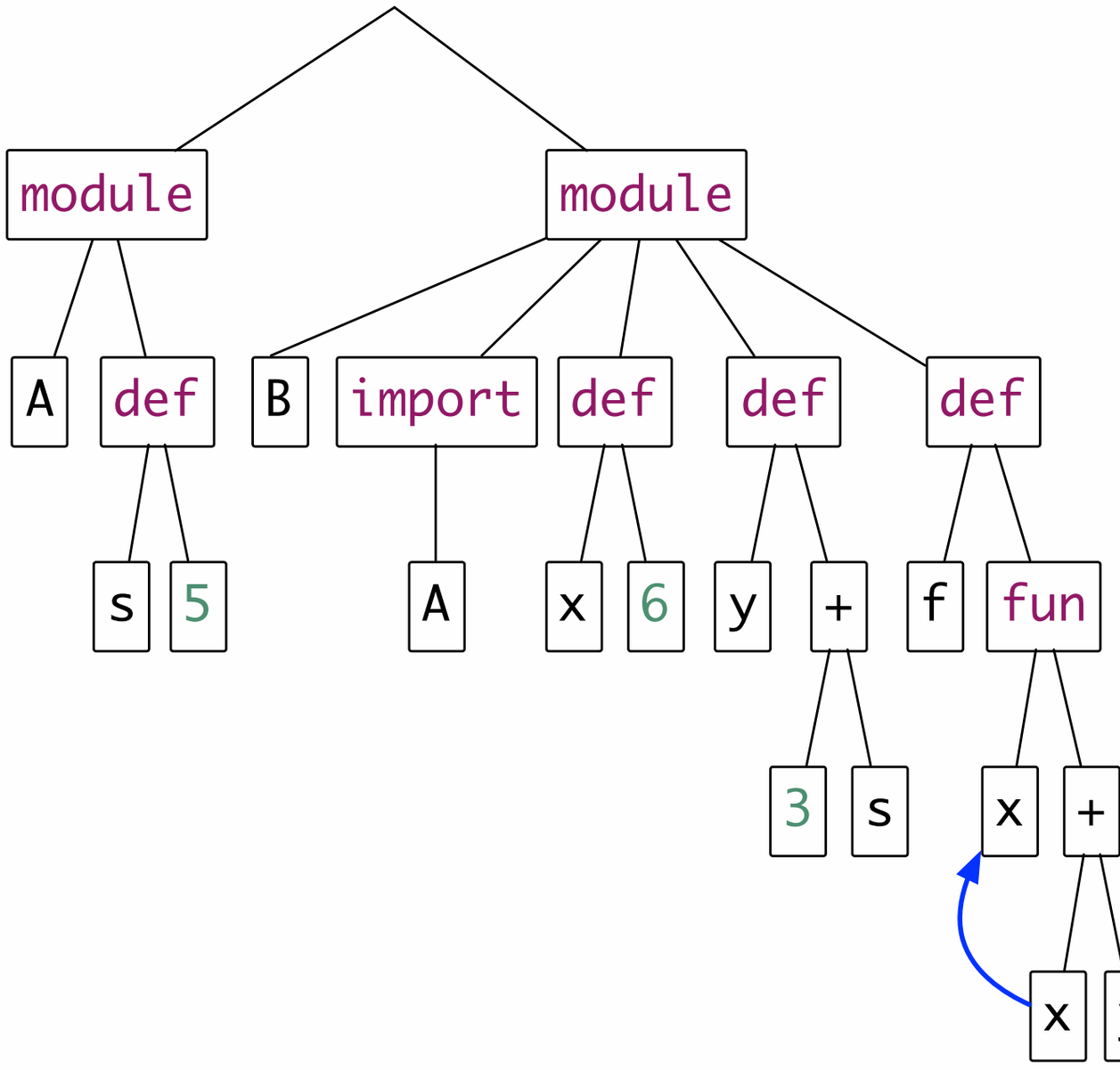

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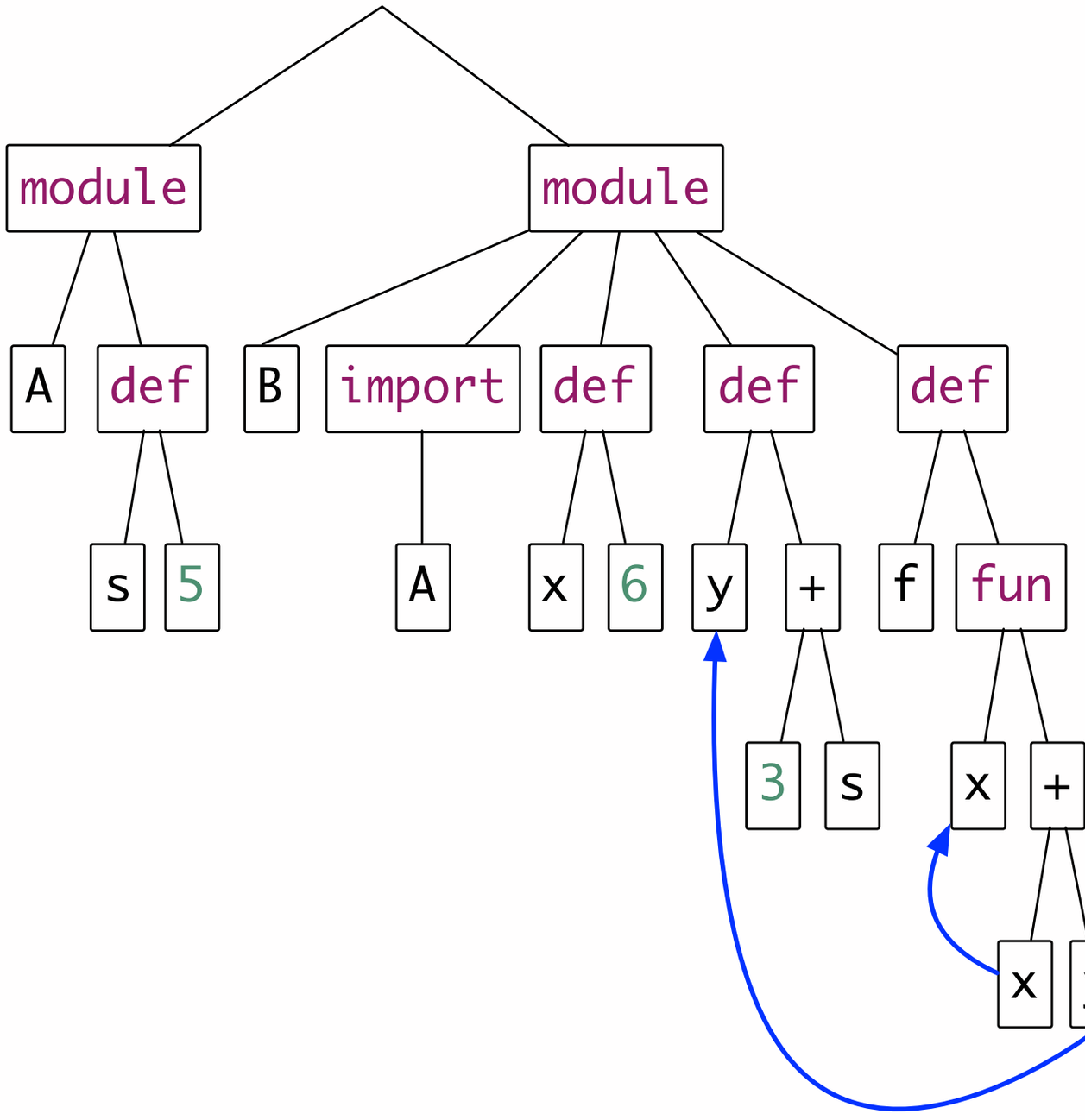

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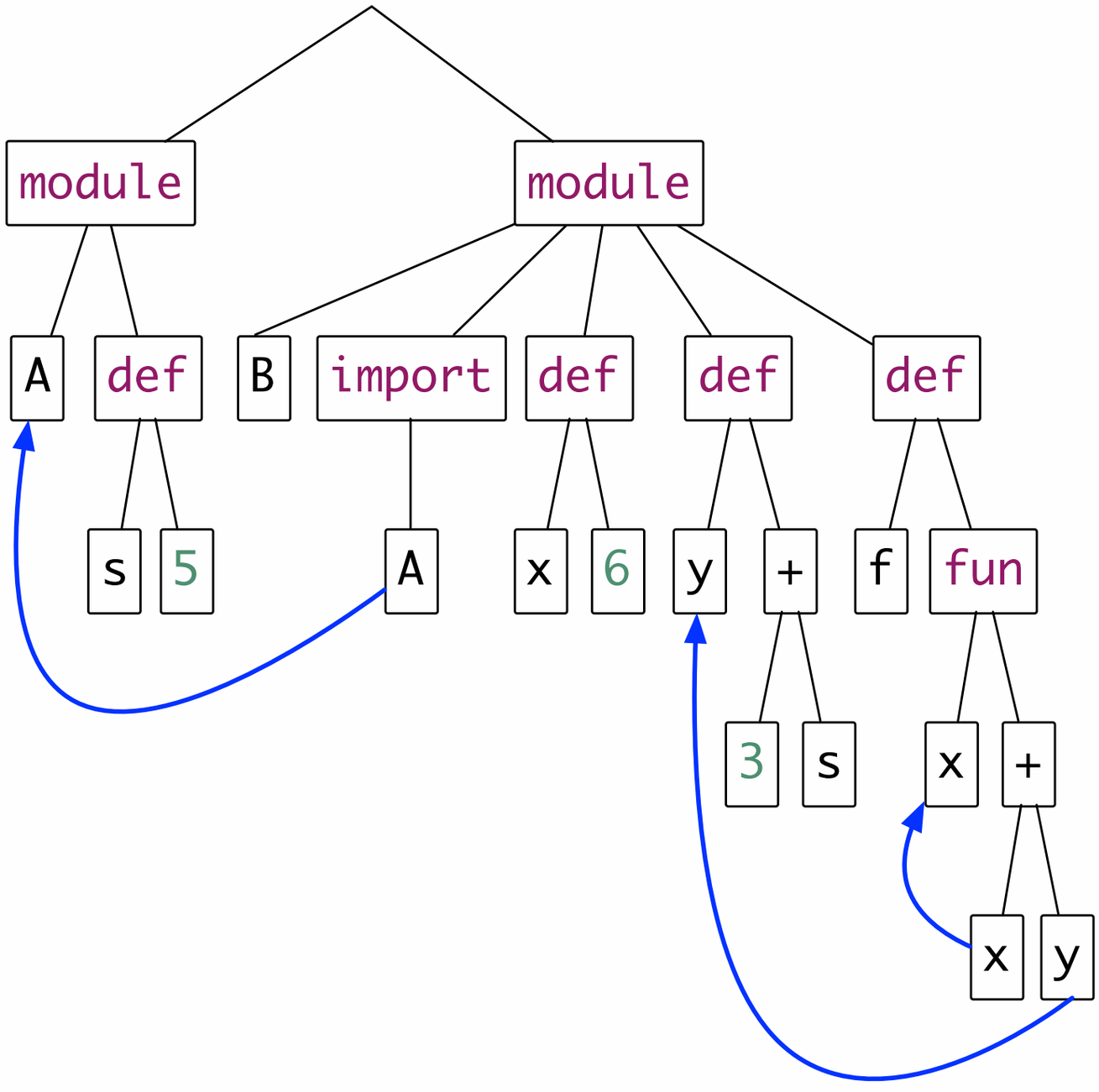

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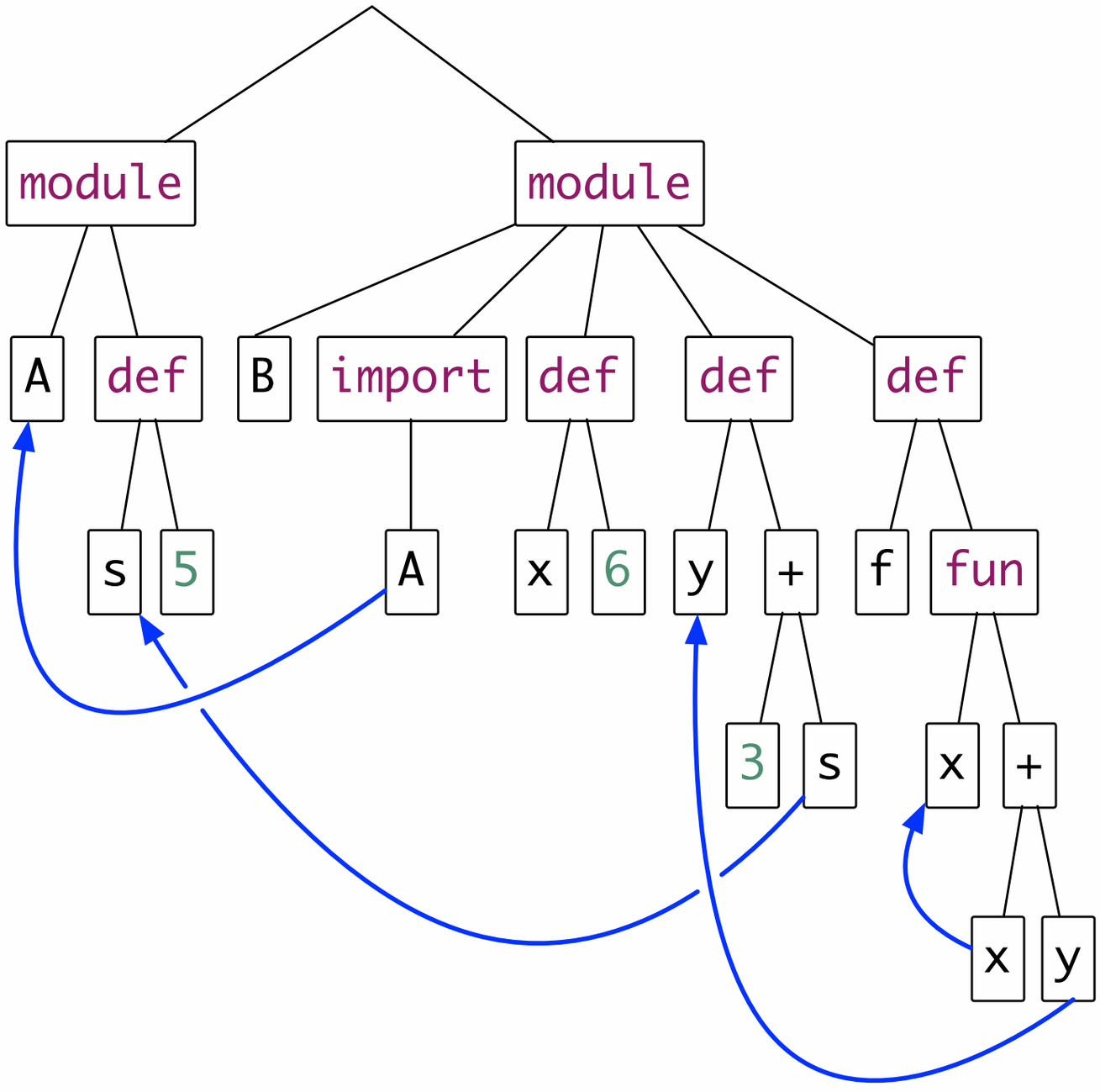

Use Scope Graphs to Describe Name Resolution

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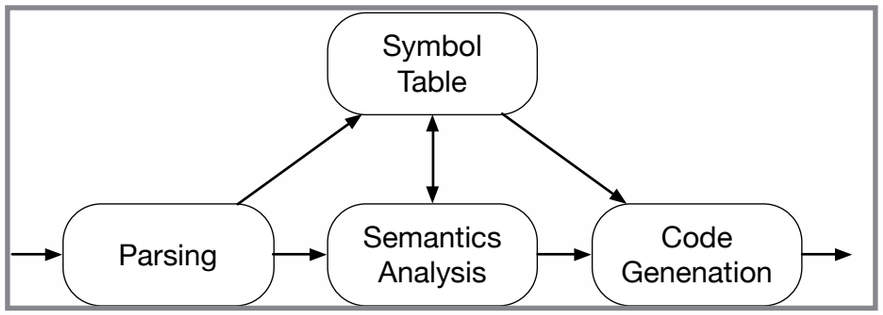

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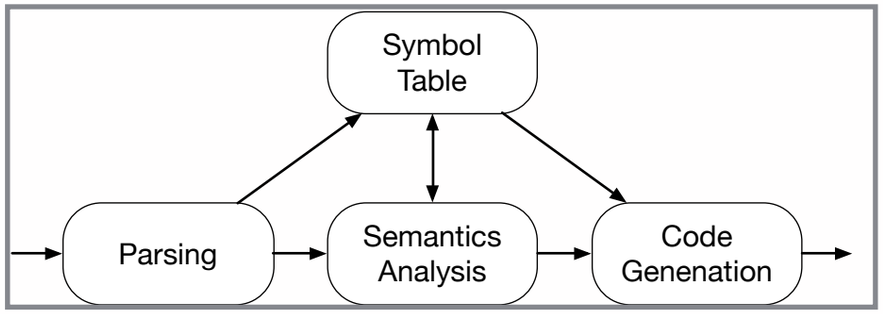

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Semantics

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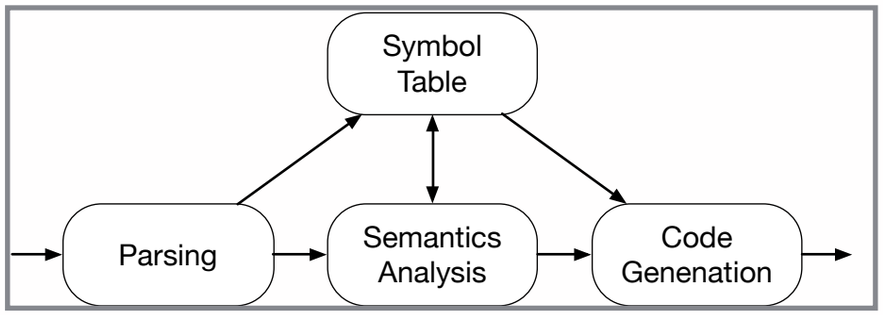

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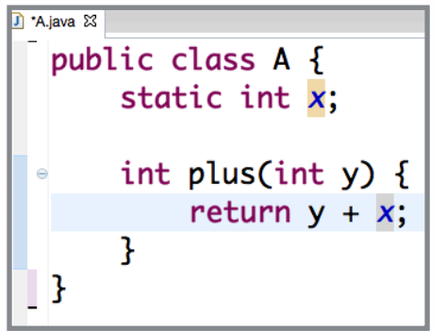

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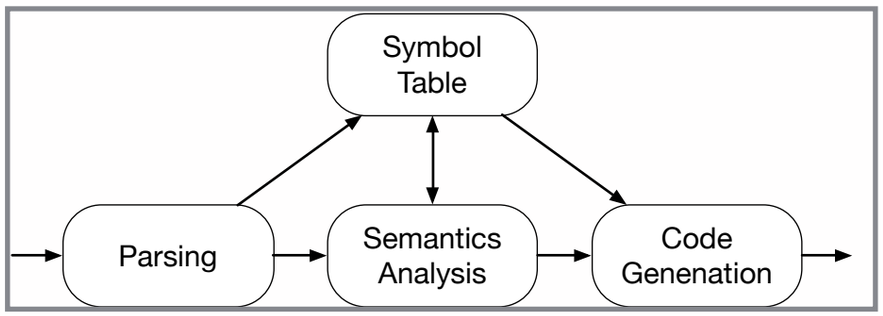

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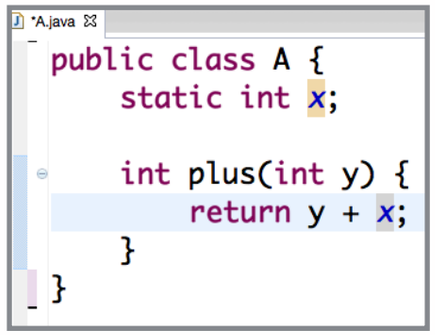

IDE

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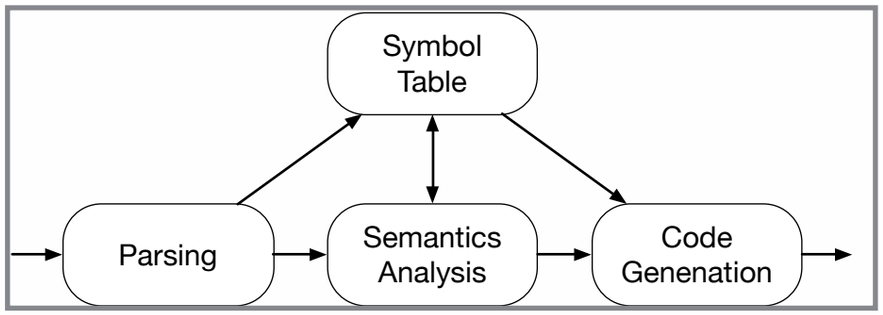

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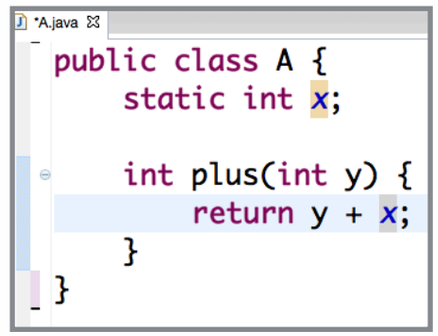

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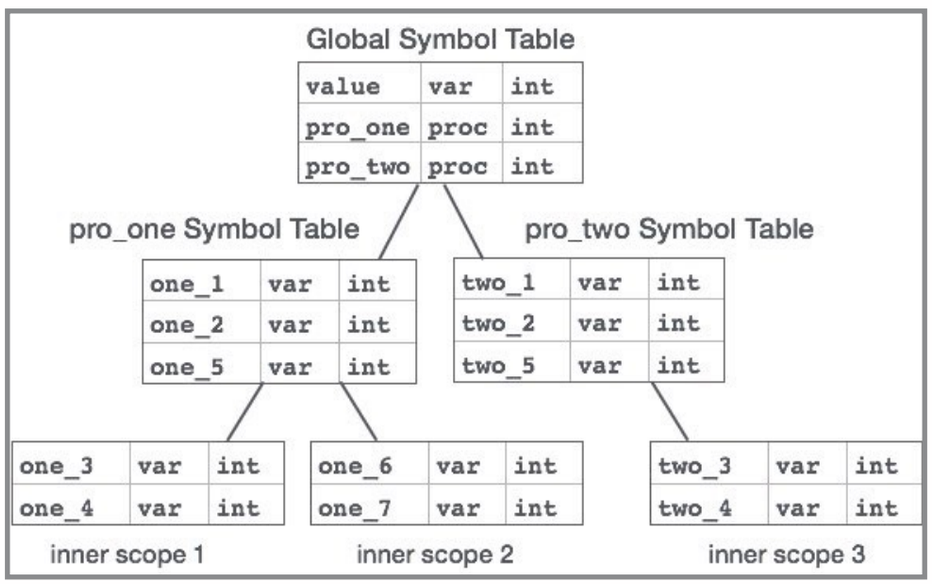

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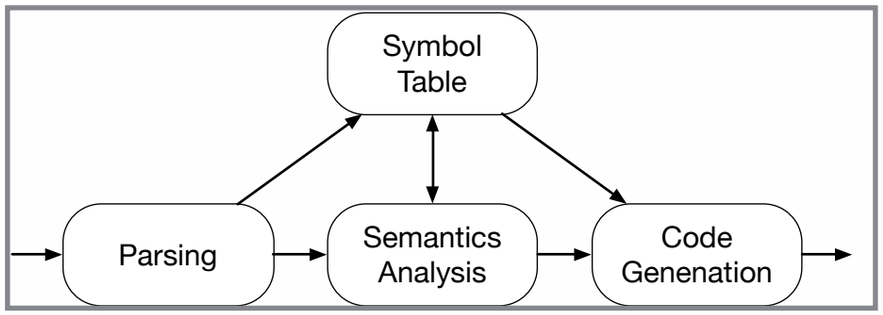

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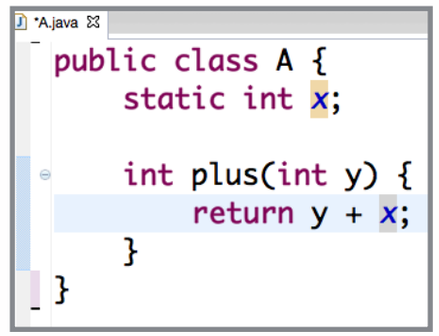

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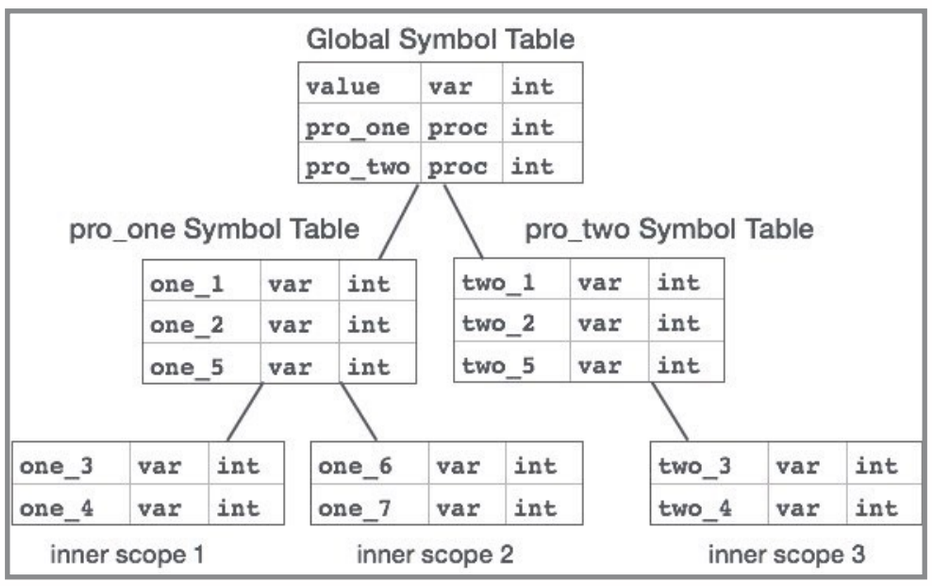

`x:int, Γ`

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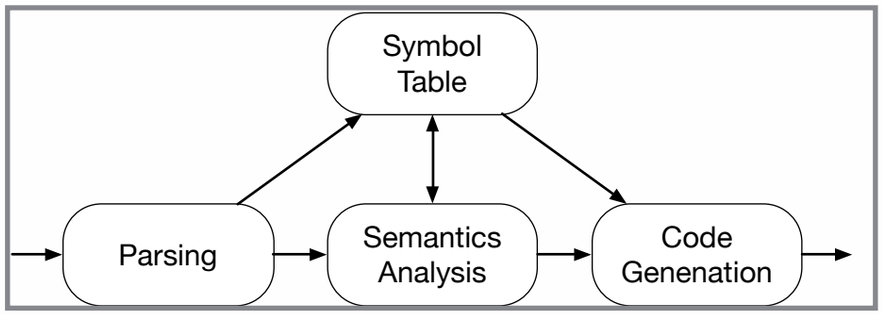

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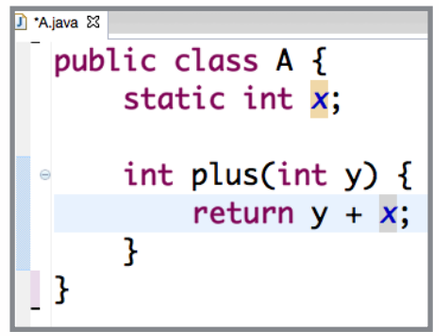

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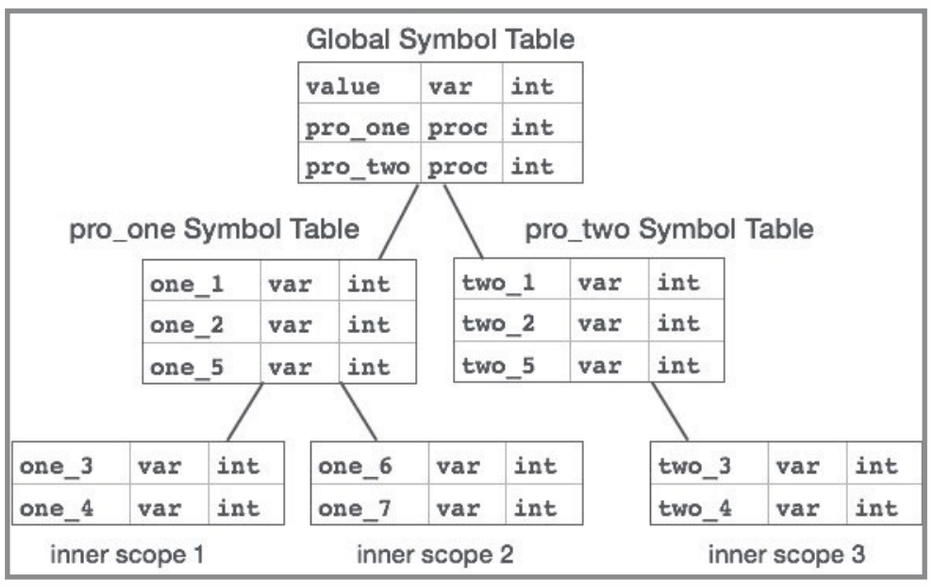

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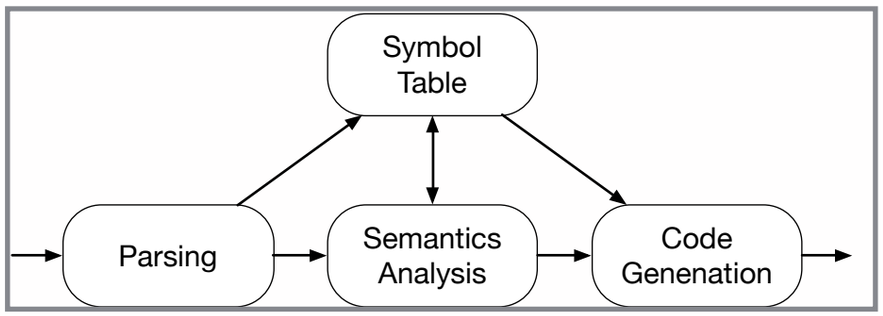

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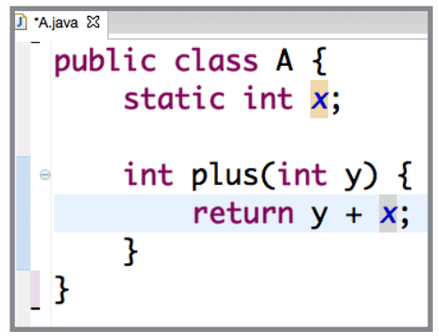

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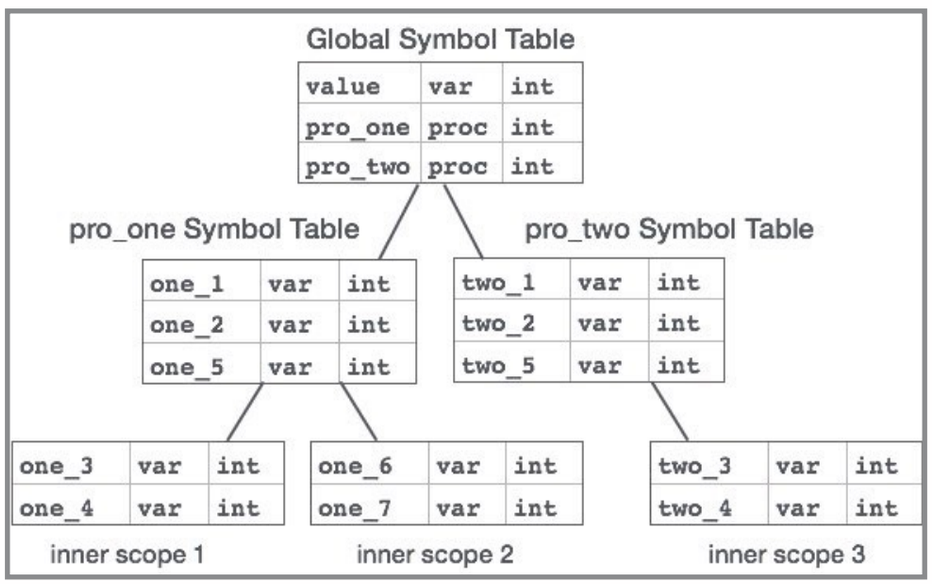

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No standard approach, no re-use

Contrast with Syntax

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A standard formalism

Context-Free Grammars

Contrast with Syntax

A unique definition

A standard formalism

```
program  =  decl*
   decl  =  module id { decl* }
         |  import qid
         |  def id = exp
    exp  =  qid
         |  fun id { exp }
         |  fix id { exp }
         |  let bind* in exp
         |  letrec bind* in exp
         |  letpar bind* in exp
         |  exp exp
         |  exp ⊕ exp
         |  int
    qid  =  id
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Context-Free Grammars

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A standard formalism

Context-Free Grammars

Supports

- Parser
- AST
- Pretty-Printing
- Highlighting

Representing Bound Programs

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- Many approaches to representing the results of name resolution within an (extended) AST, e.g.
  - numeric indexing [deBruijn72]
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- Good support for binding-sensitive AST manipulation
- But: Do not say how to resolve identifiers in the first place!
  - Also: Can’t represent ill-bound programs
  - And: Tend to be biased towards lambda-like bindings

Binding Specification Languages

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- Many proposals for domain-specific languages (DSLs) for specifying binding structure of a (target) language, e.g.
  - Ott [Sewell+10]
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- Generate code to do resolution and record results
- But: what are the **semantics** of such a language?

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- Answer: the meaning of a binding specification for language L should be given by a function from L programs to their **“resolution structures”**

- So we need a (uniform, language-independent) method for describing such resolution structures...

- ...that can be used to compute the resolution of each program identifier

  - (or to verify that a claimed resolution is valid)

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- ...using minimal number of constructs
- Make resolution structure language-independent
  - Handle named collections of names (e.g. modules, classes, etc.) within the theory
  - Allow description of programs with resolution errors

A Theory of Name Resolution

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*For **statically lexically scoped** languages*

A Theory of Name Resolution

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A standard formalism

Scope Graphs

A Theory of Name Resolution

*For **statically lexically scoped** languages*

A unique representation

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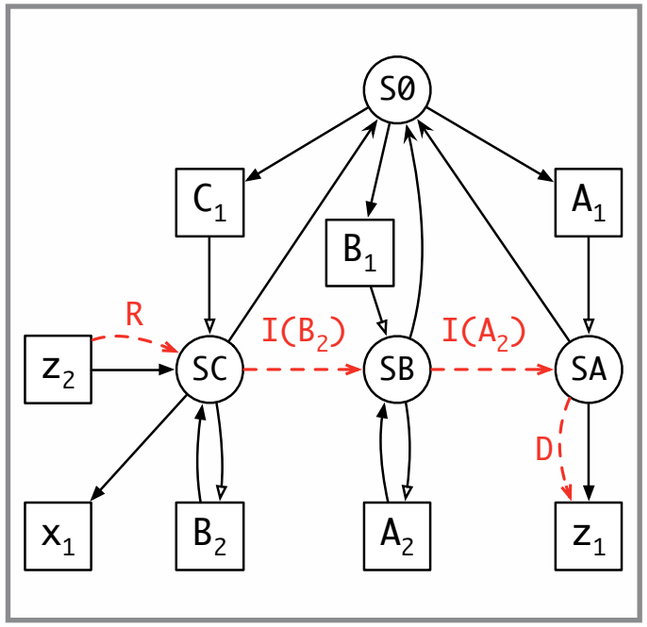

Scope Graphs

A Theory of Name Resolution

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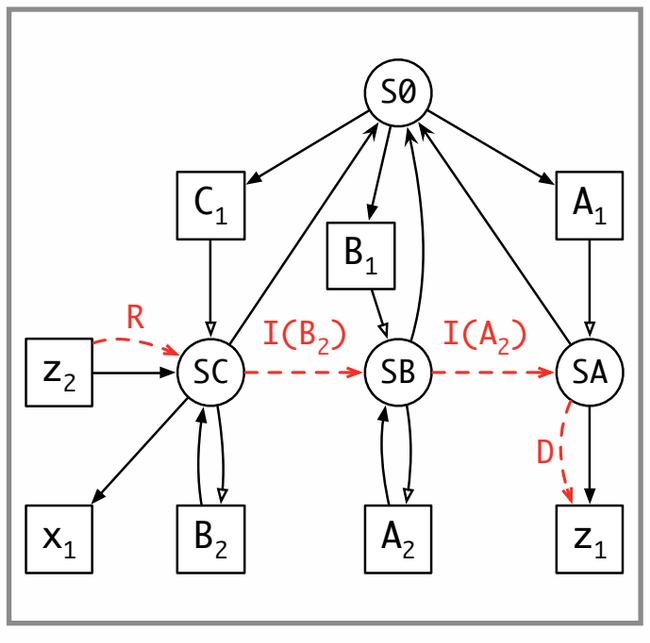
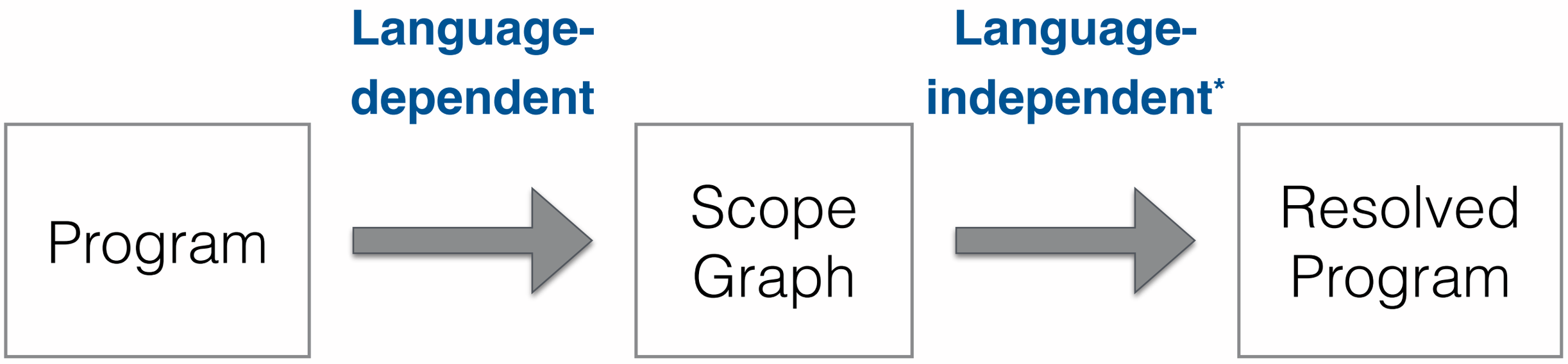
Supports



Scope Graphs

- Resolution
- α-equivalence
- IDE Navigation
- Refactoring tools
- Reasoning tools

Resolution Scheme



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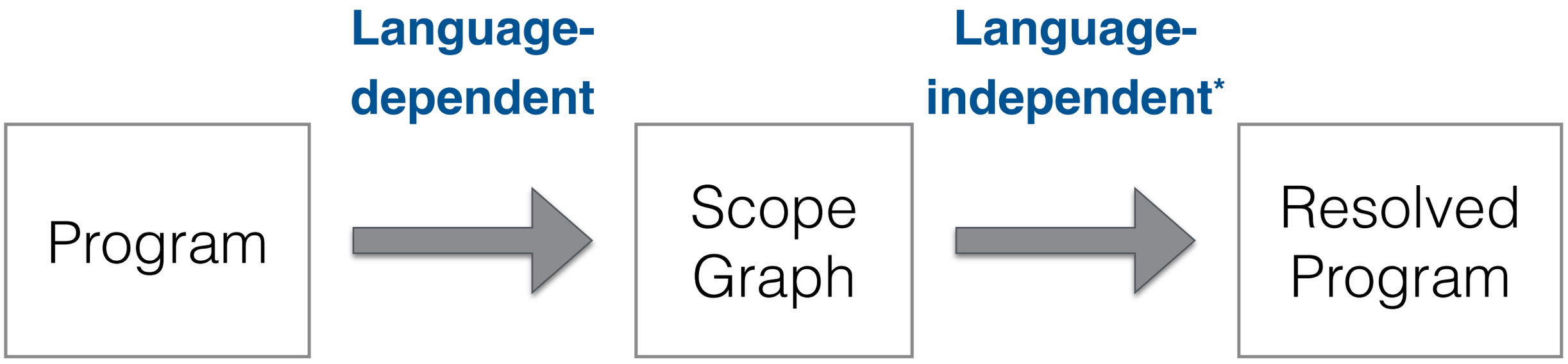

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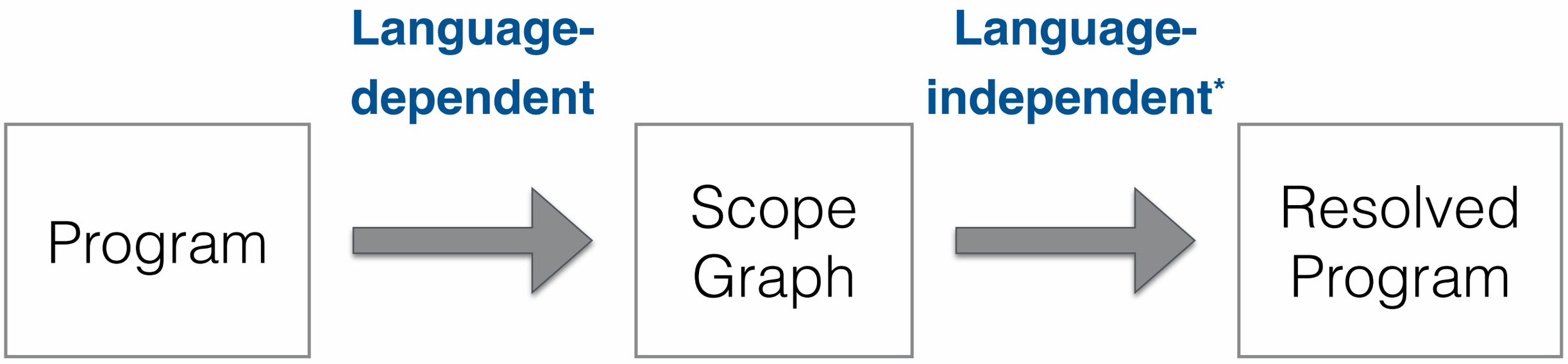

Resolution of a reference in a scope graph:

Building a **path**
from a **reference** node
to a **declaration** node
following path construction **rules**

*Parameterized by notions of path **well-formedness** and **ordering**

Scope Graphs by Example

Simple Scopes

```
def y = x + 1
def x = 5
```

Simple Scopes

```
def y1 = x2 + 1
def x1 = 5
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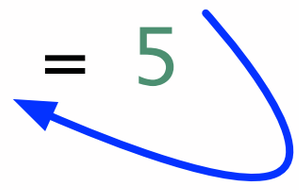

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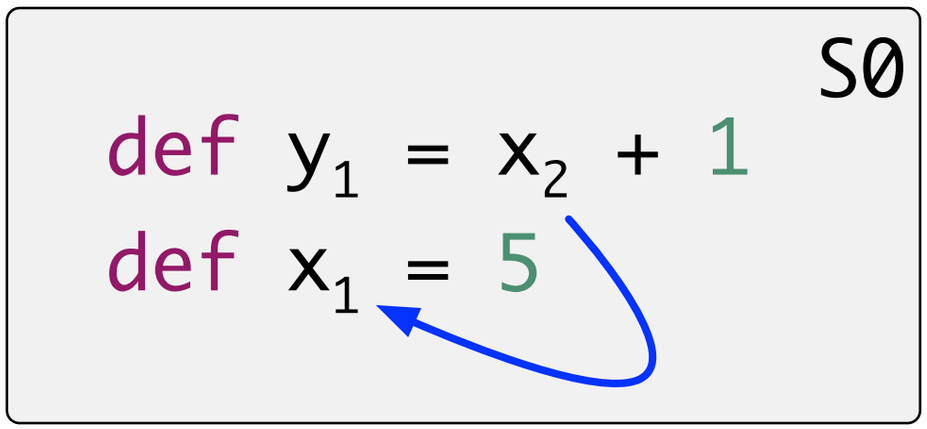
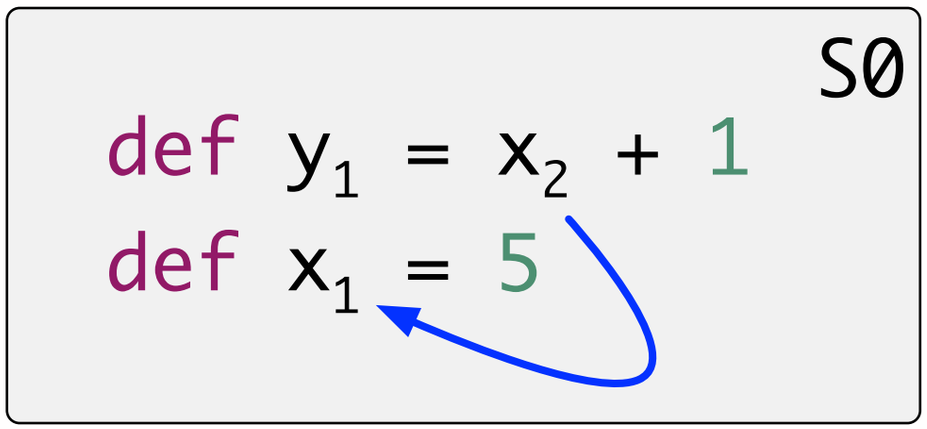
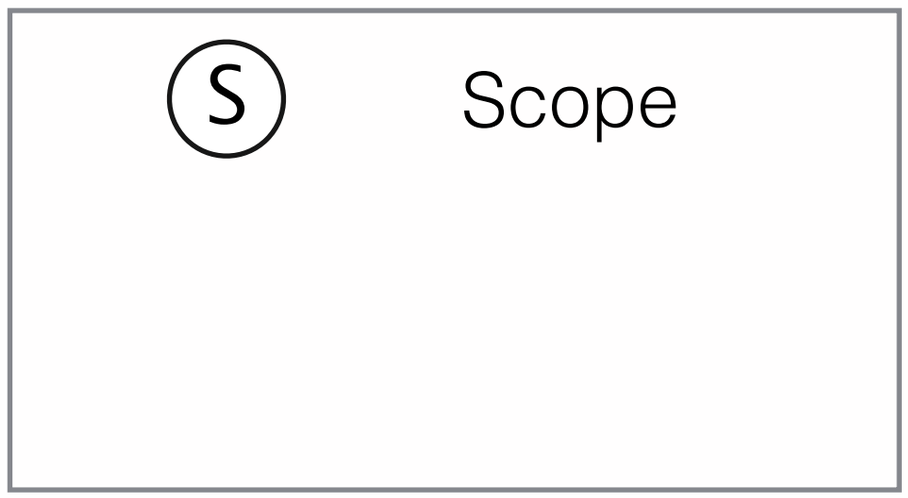


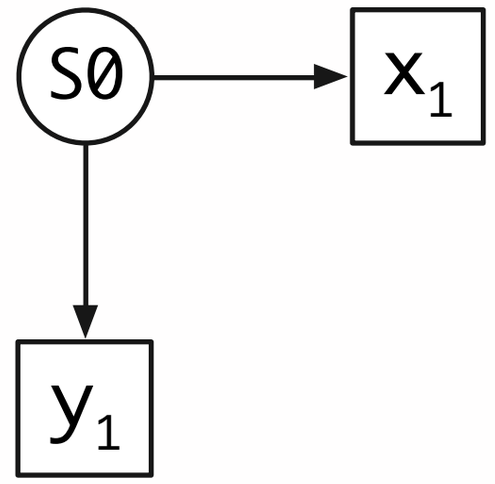
Simple Scopes

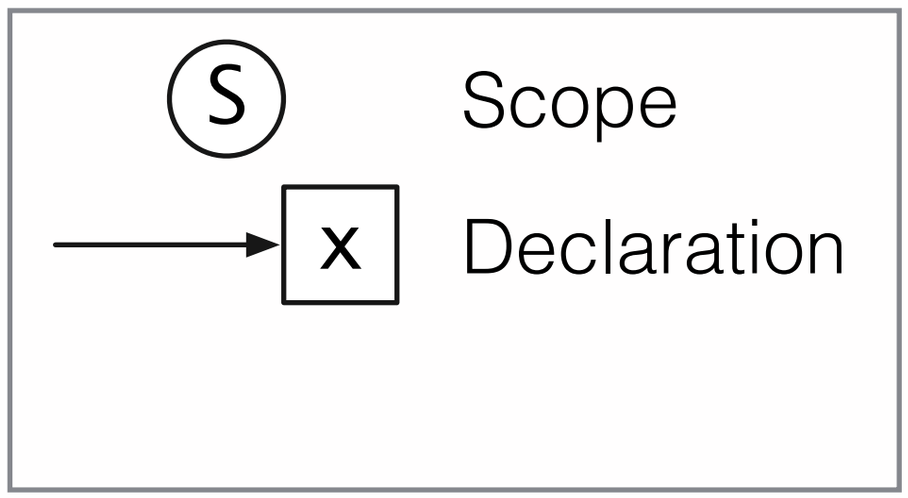




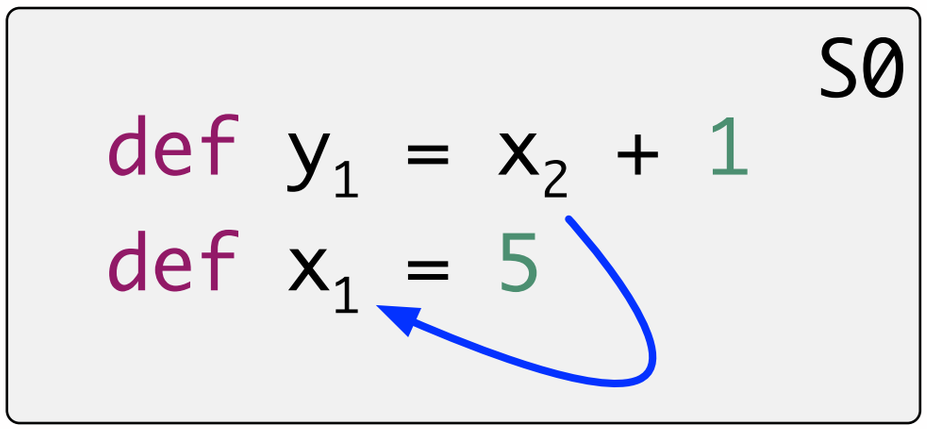
Simple Scopes

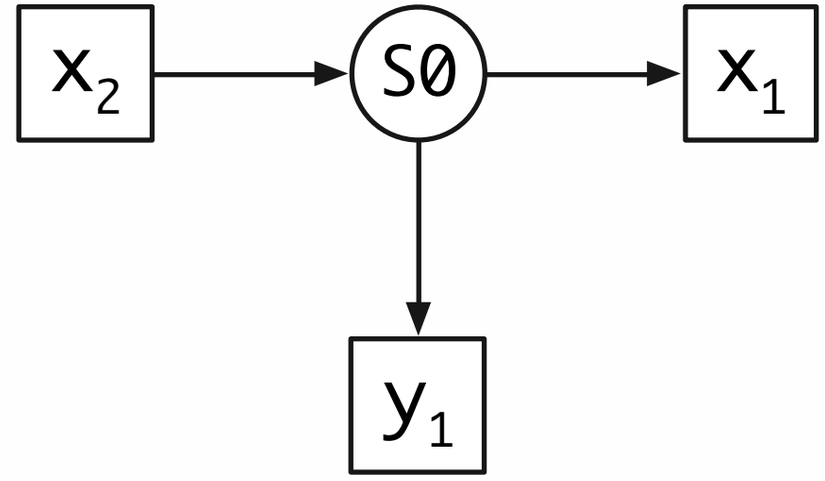
```
                                    S0
def y1 = x2 + 1
def x1 = 5
```

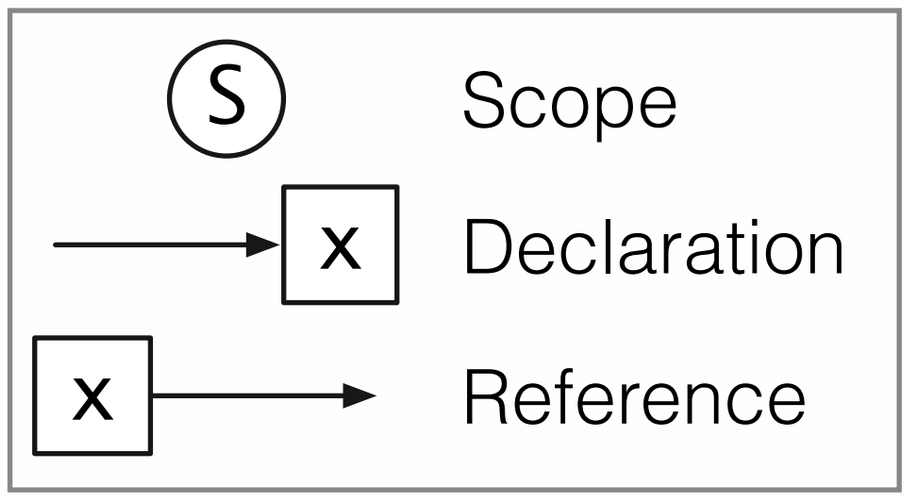




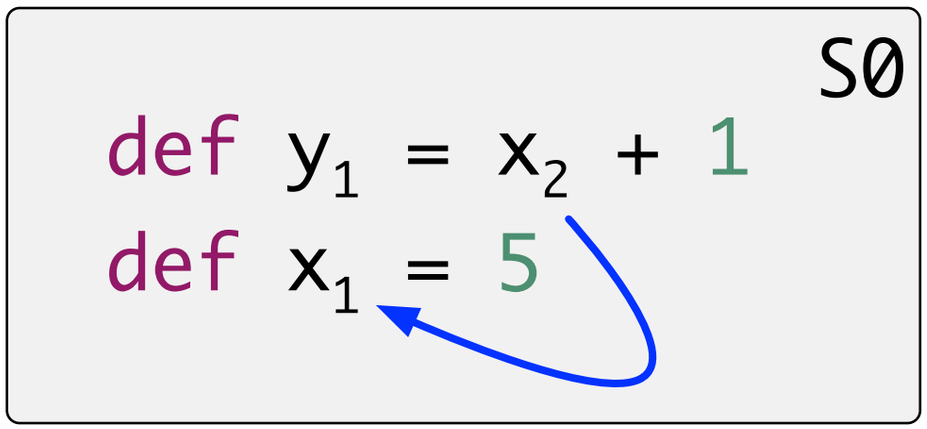
Simple Scopes

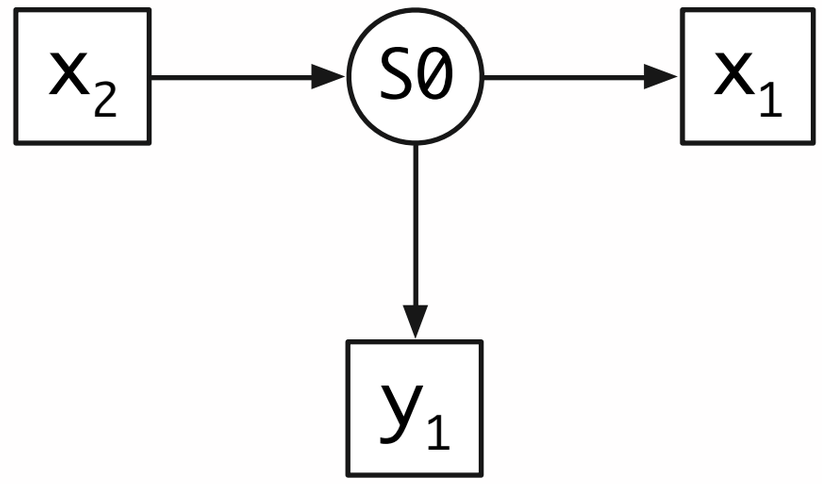


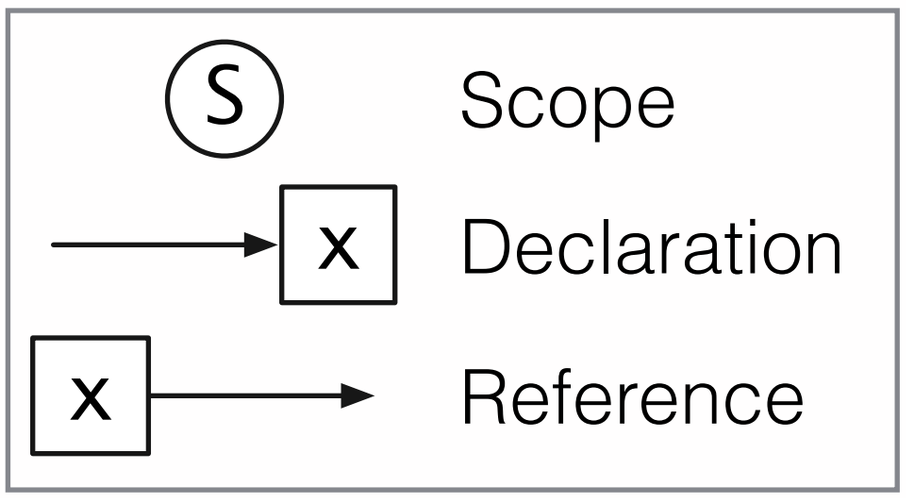


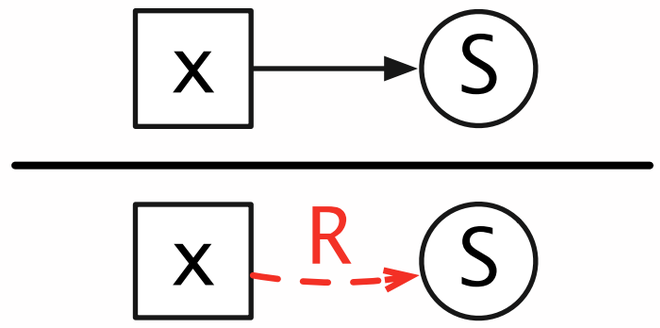


Simple Scopes



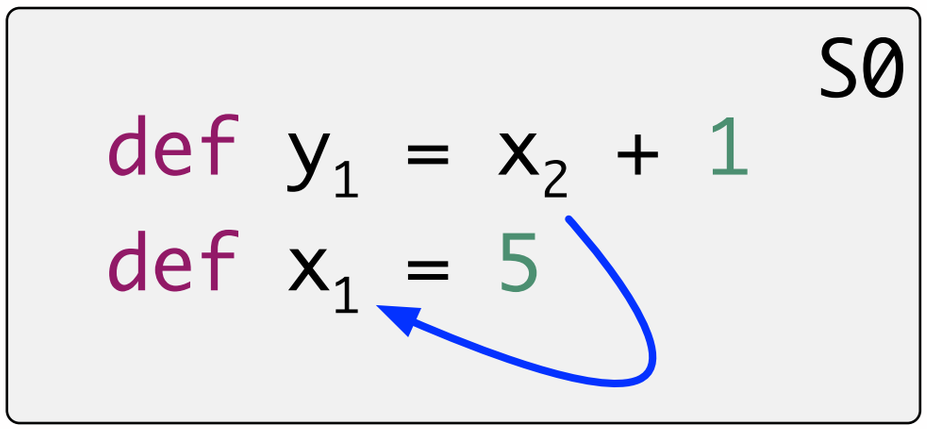


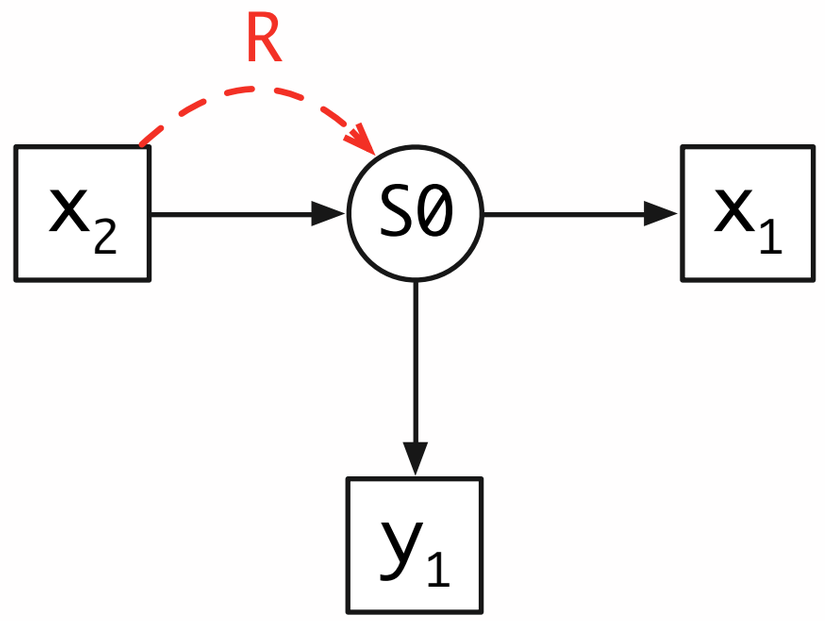


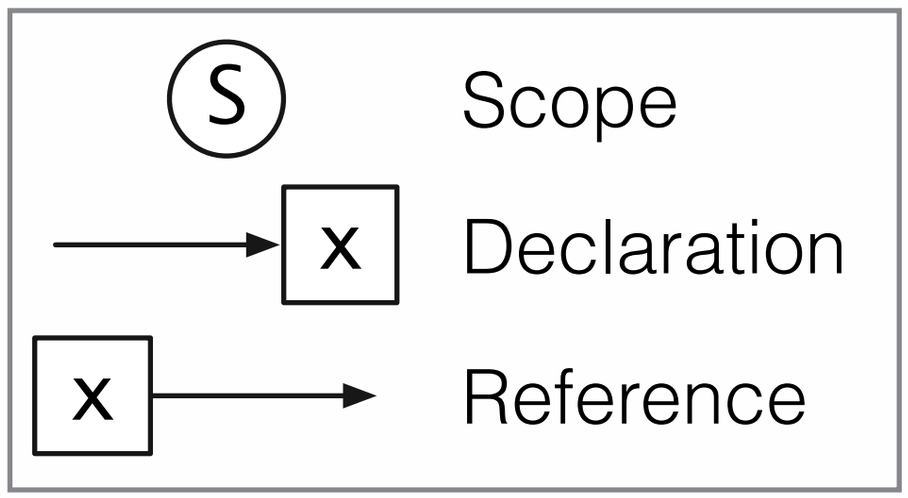


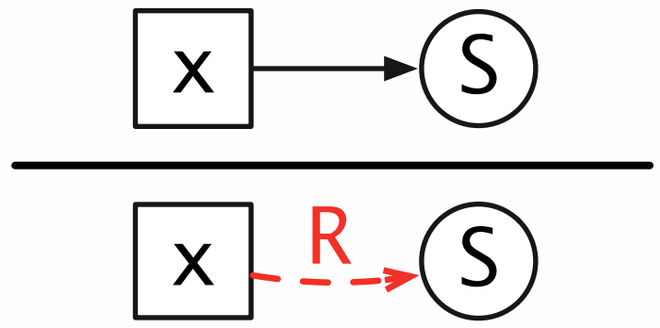
Reference Step

Simple Scopes



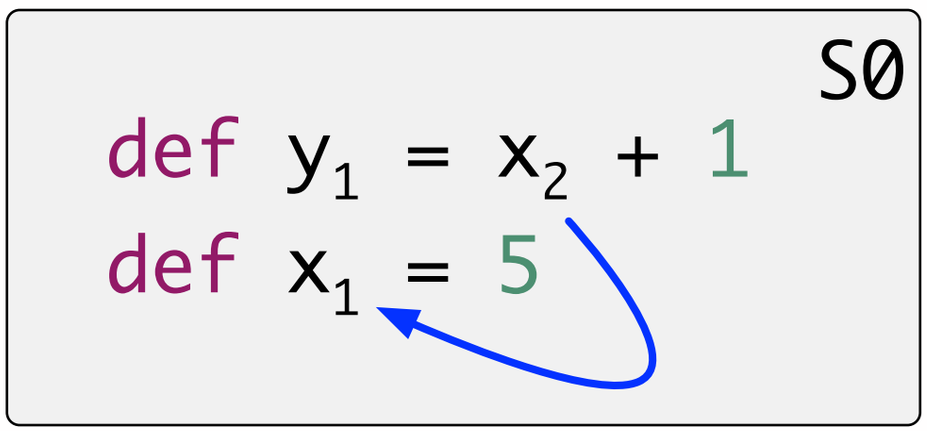


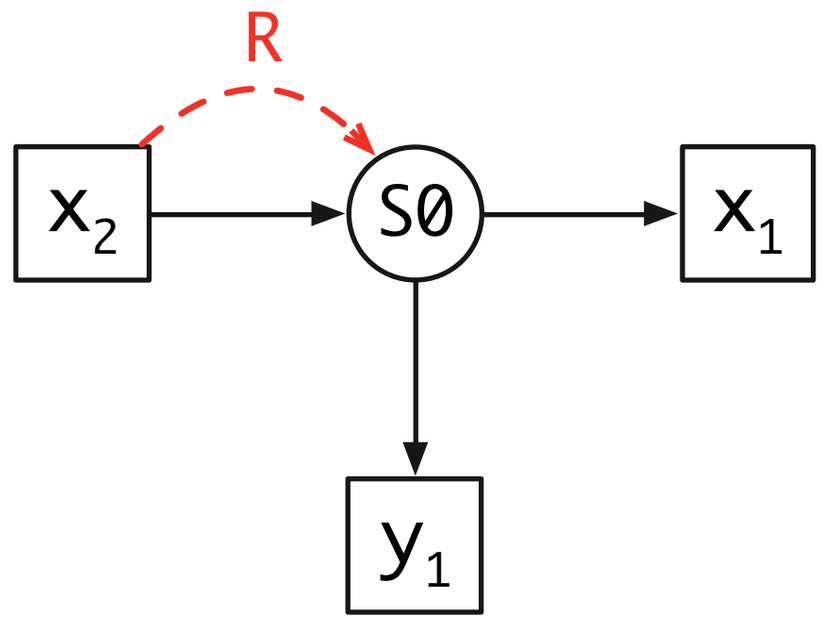


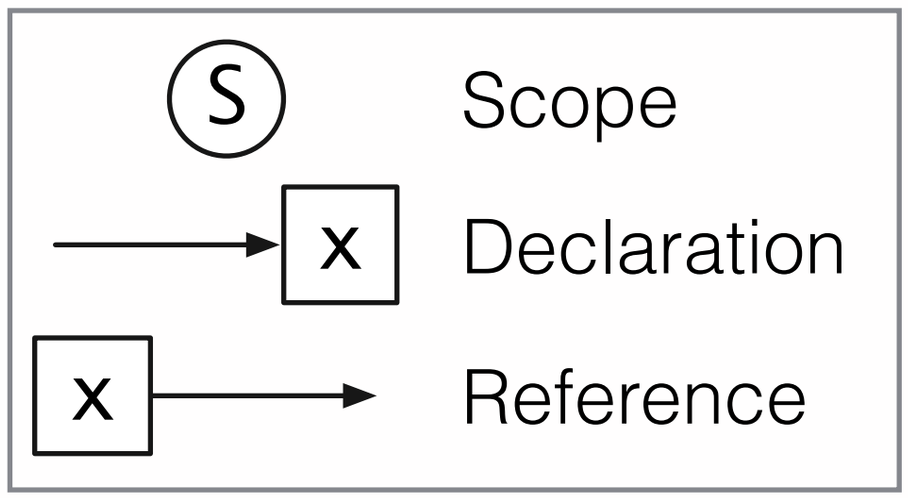


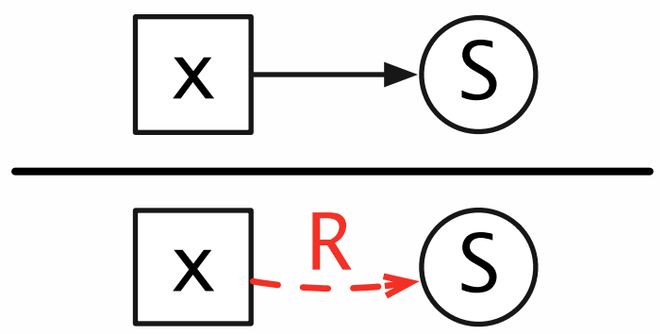
Reference Step

Simple Scopes

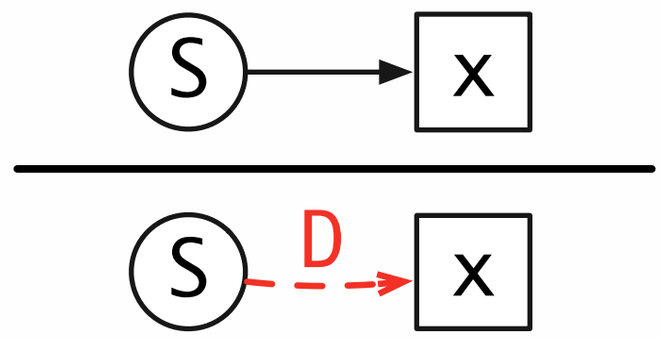






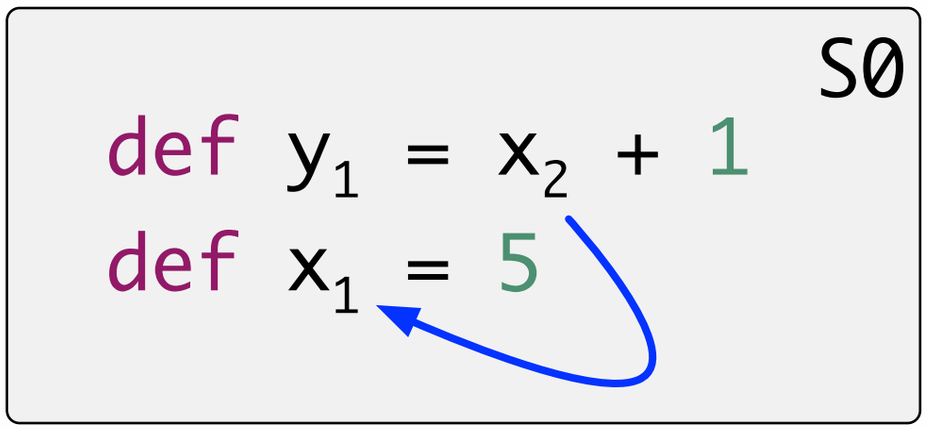


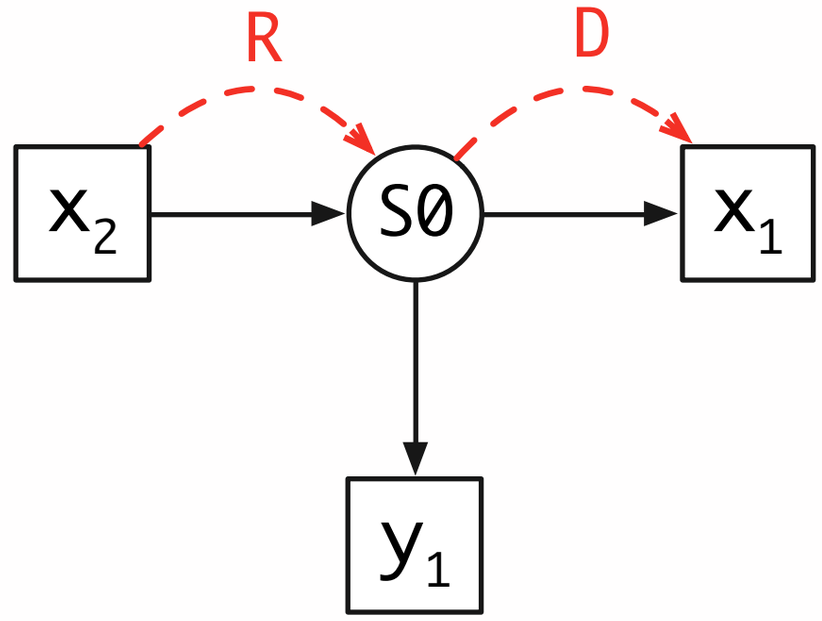
Reference Step

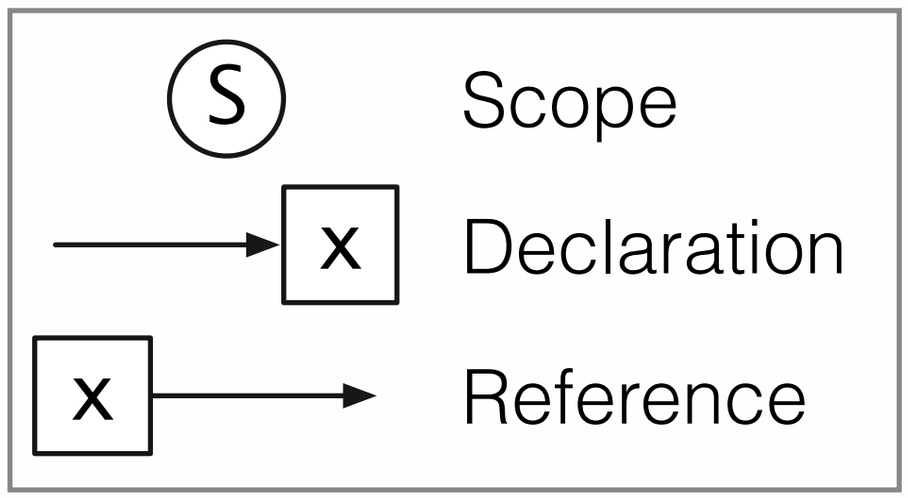


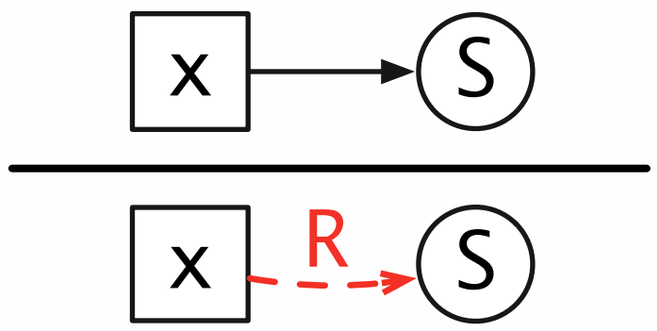
Declaration Step

Simple Scopes

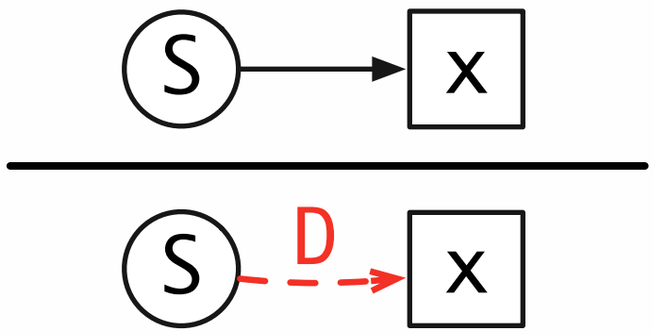








Reference Step



Declaration Step

Ambiguous Resolutions

S0

```
def x₁ = 5

def x₂ = 3

def z₁ = x₃ + 1
```

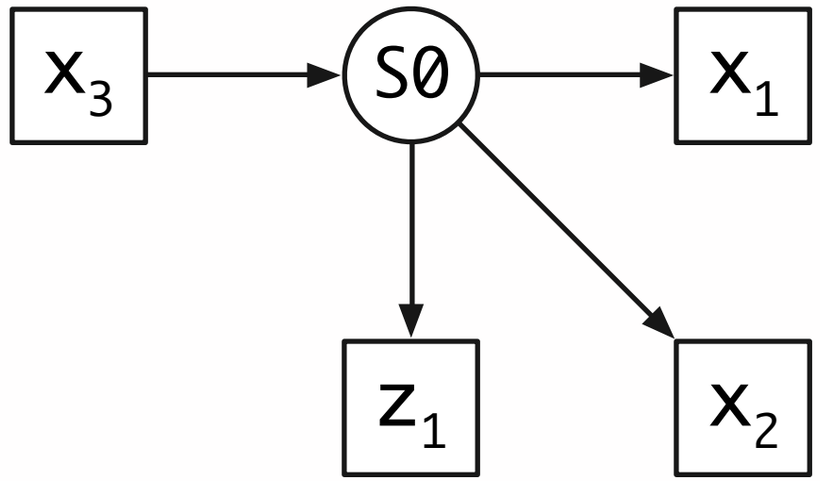
Ambiguous Resolutions

S0

```
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```



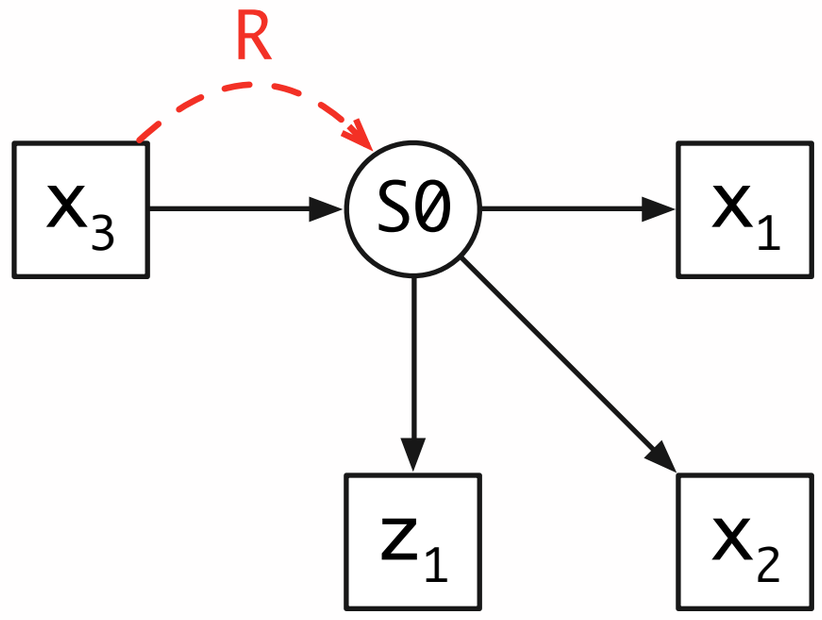
Ambiguous Resolutions

S0

```
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```



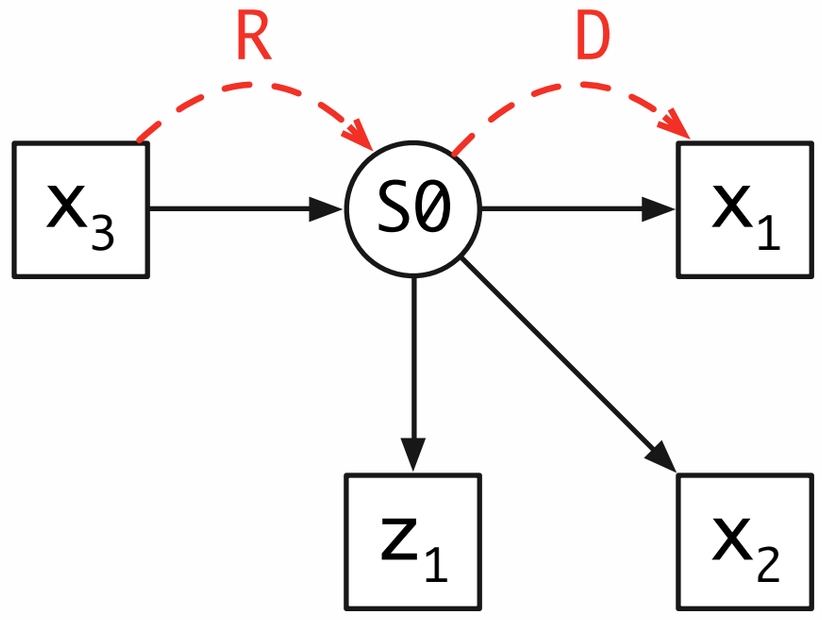
Ambiguous Resolutions

S0

```
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```



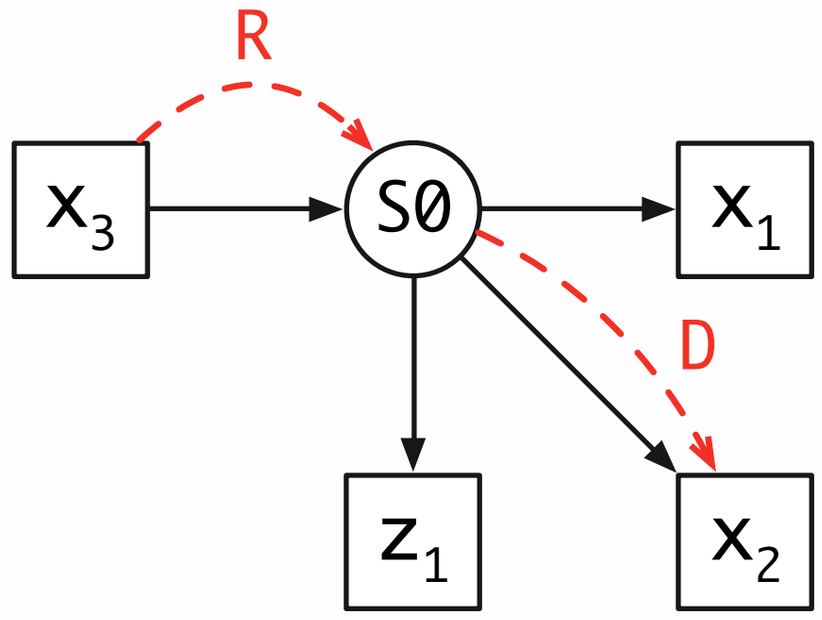
Ambiguous Resolutions

S0

```
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```



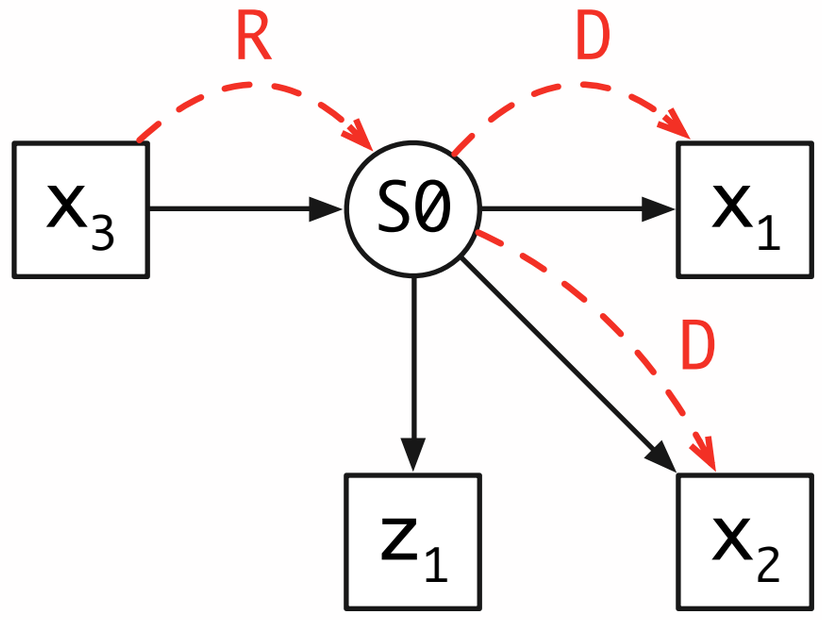
Ambiguous Resolutions

S0

```
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```

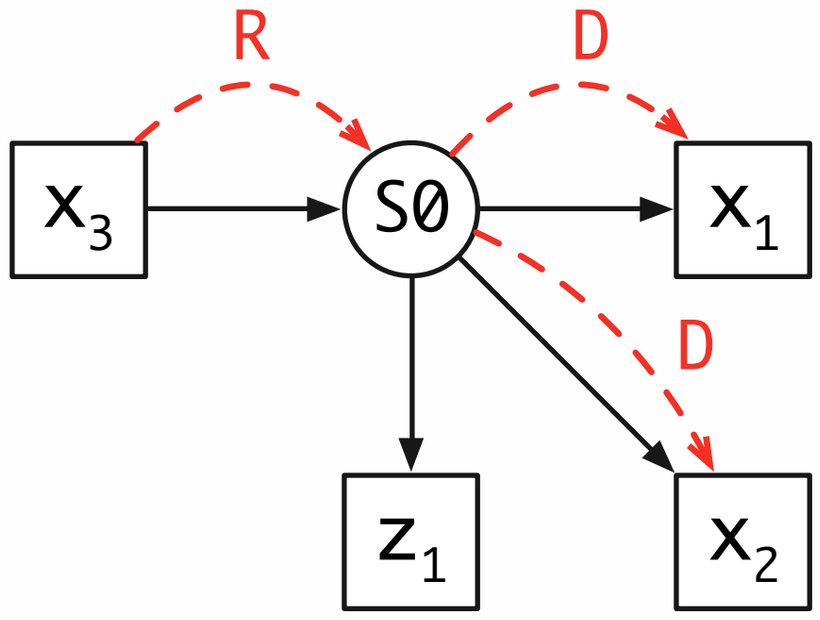


Ambiguous Resolutions

```
                                    S0
def x1 = 5

def x2 = 3

def z1 = x3 + 1
```




```
match t with
  | A x | B x => …
```

Lexical Scoping

```
def x1 = z2 5

def z1 =
  fun y1 {
    x2 + y2
  }
```

Lexical Scoping

```
                                S0
def x1 = z2 5


def z1 =
  fun y1 {
    x2 + y2
  }
```

Lexical Scoping

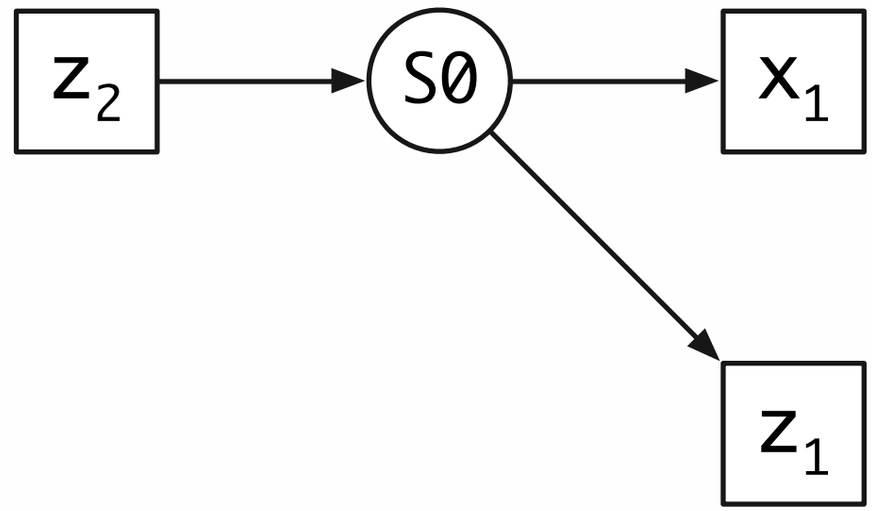
```
                                S0
def x1 = z2 5

def z1 =
   fun y1 {                     S1
     x2 + y2
   }
```

Lexical Scoping

```
                                    S0
def x1 = z2 5

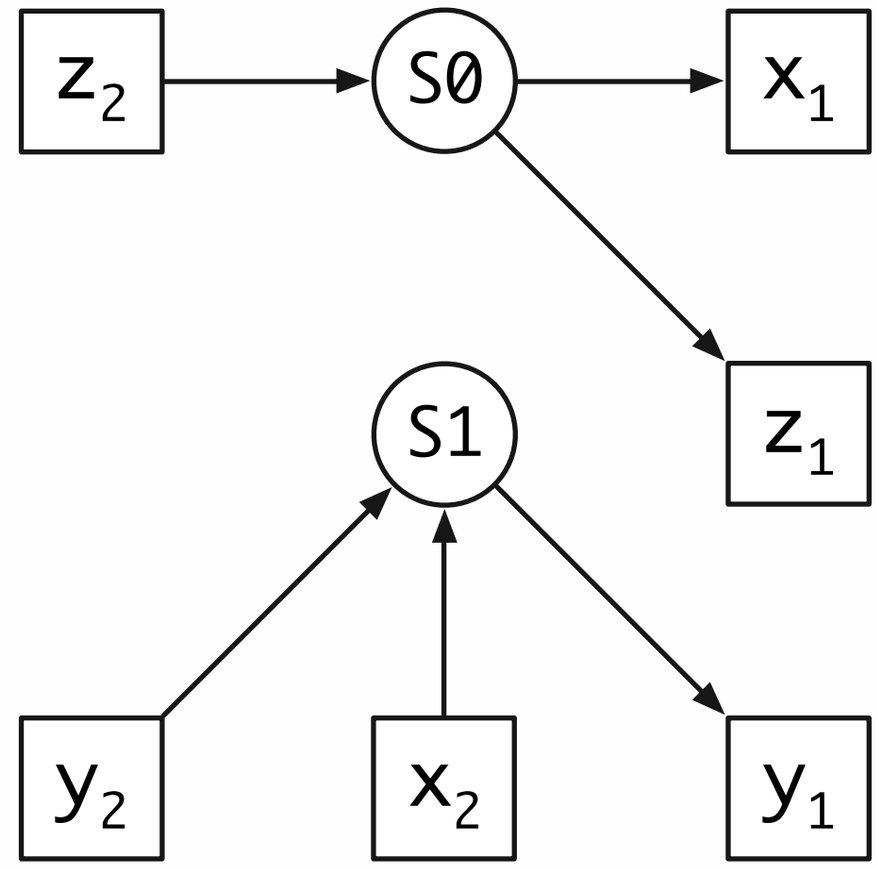
def z1 =
    fun y1 {                        S1
      x2 + y2
    }
```



Lexical Scoping

```
                                  S0
def x1 = z2 5

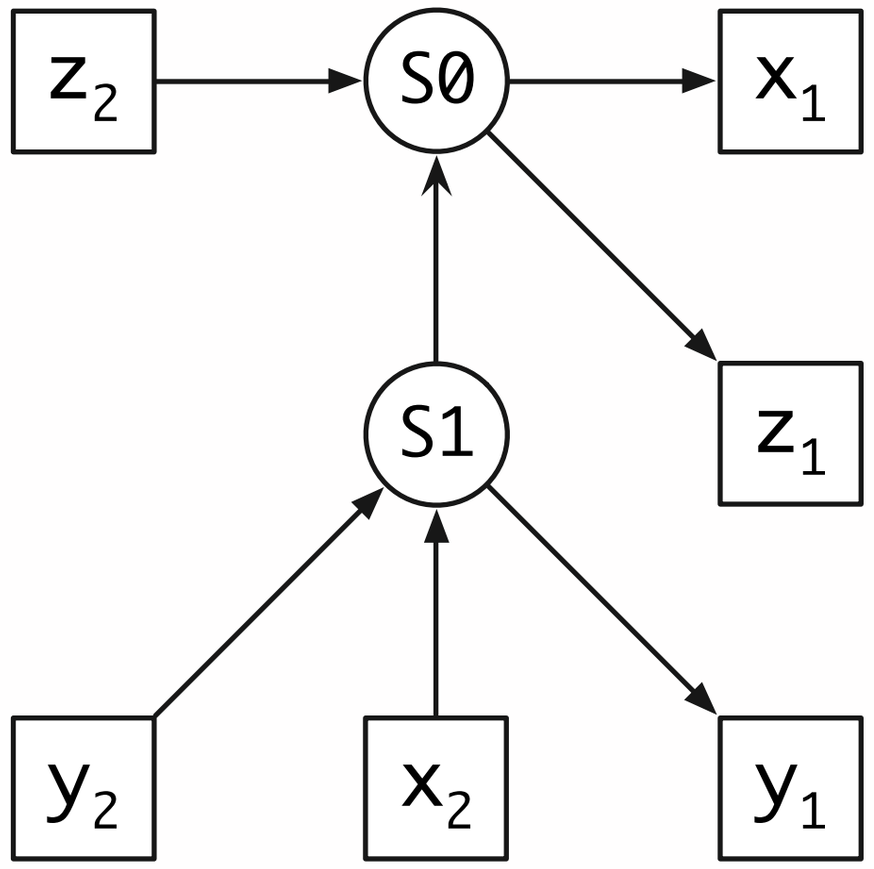
def z1 =
    fun y1 {                      S1
      x2 + y2
    }
```

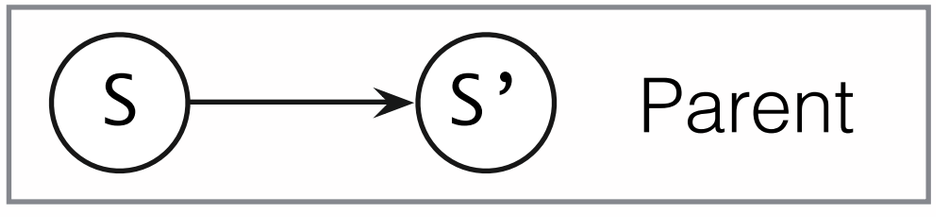


Lexical Scoping

```
                                  S0
def x1 = z2 5

def z1 =
    fun y1 {                      S1
      x2 + y2
    }
```

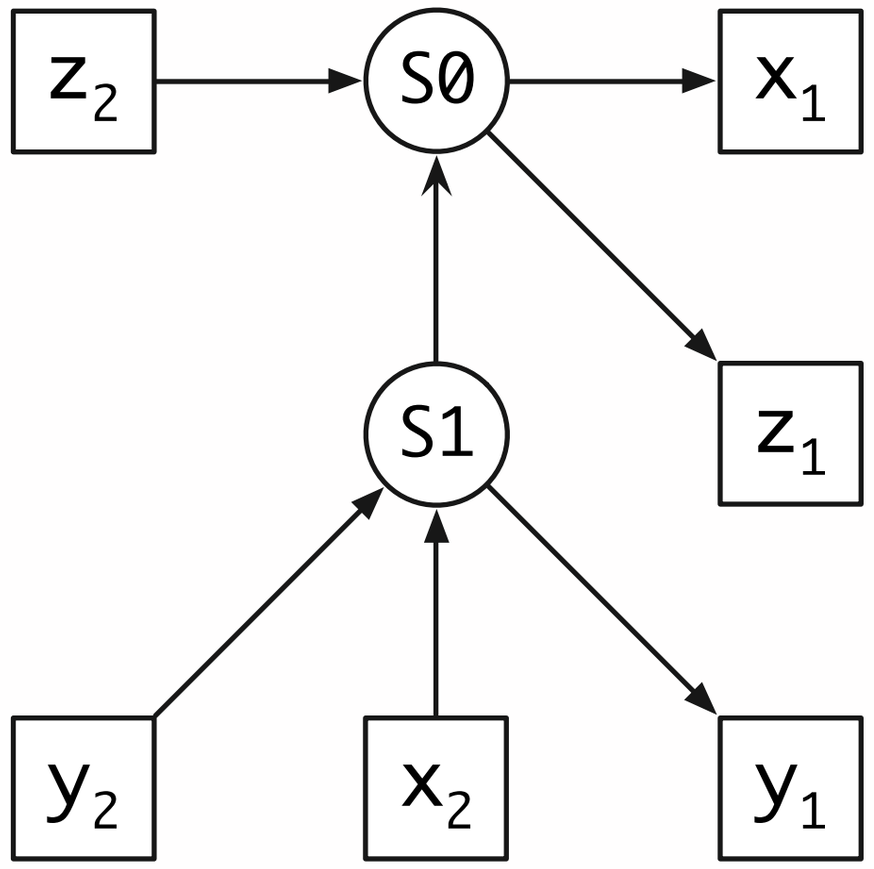


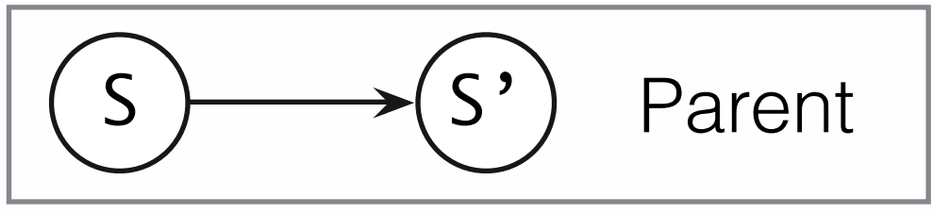


Lexical Scoping

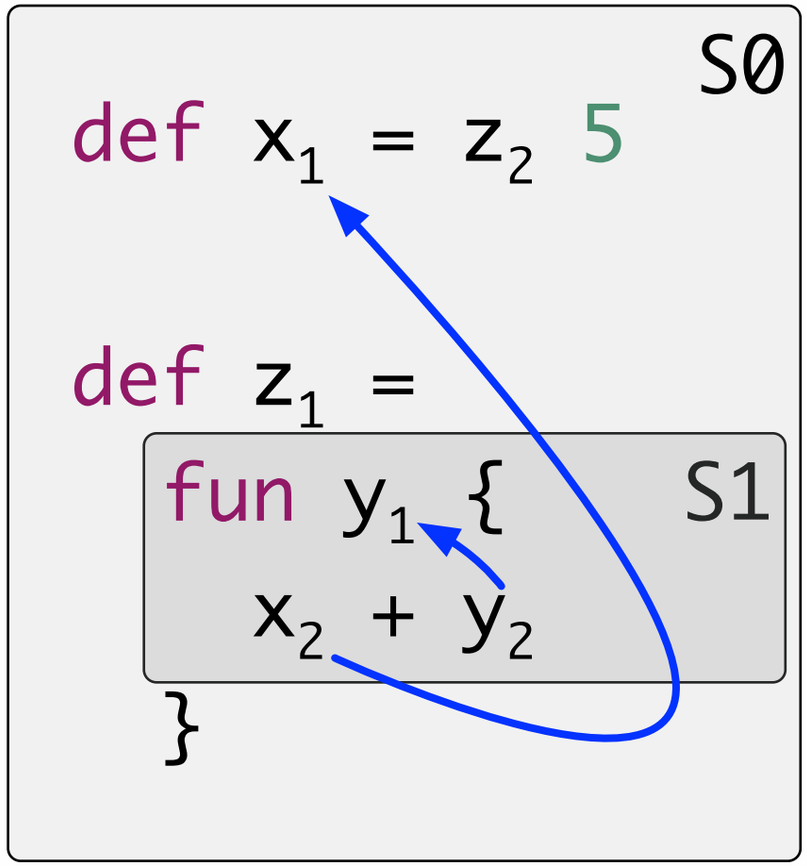
```
                                  S0
def x1 = z2 5

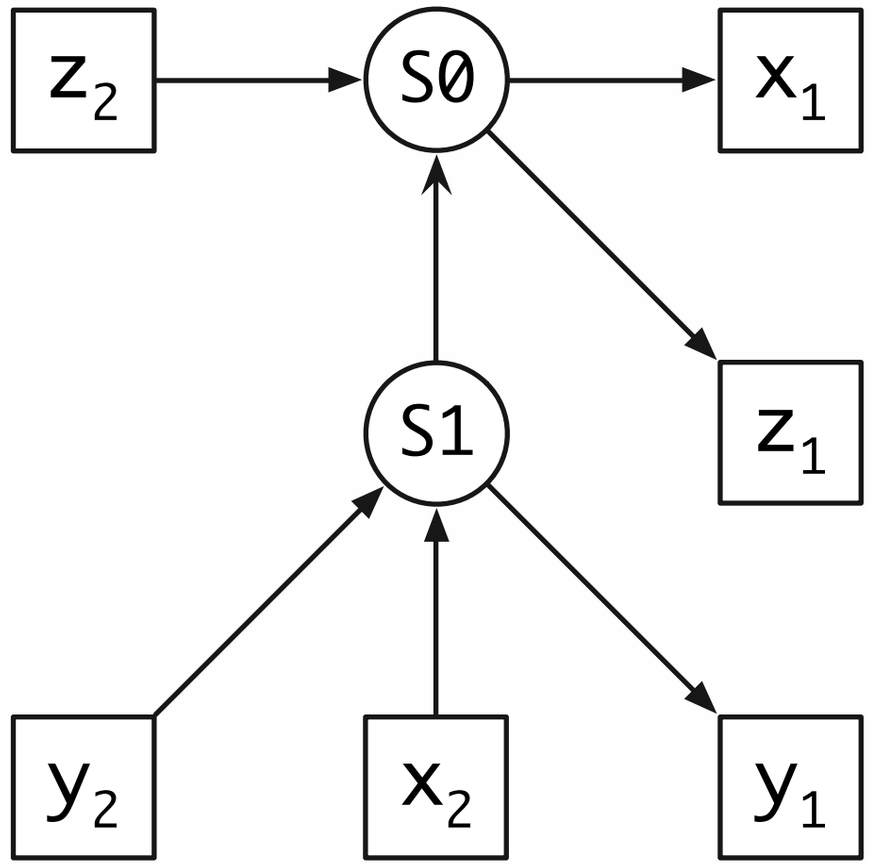
def z1 =
    fun y1 {                      S1
      x2 + y2
    }
```

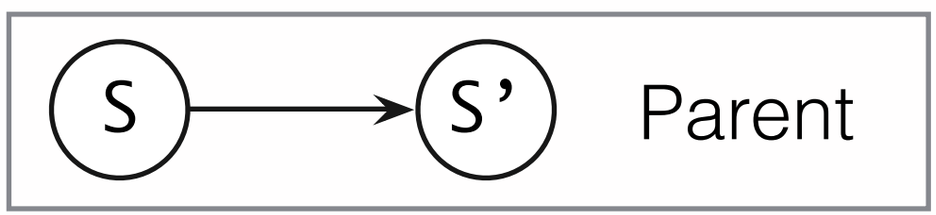




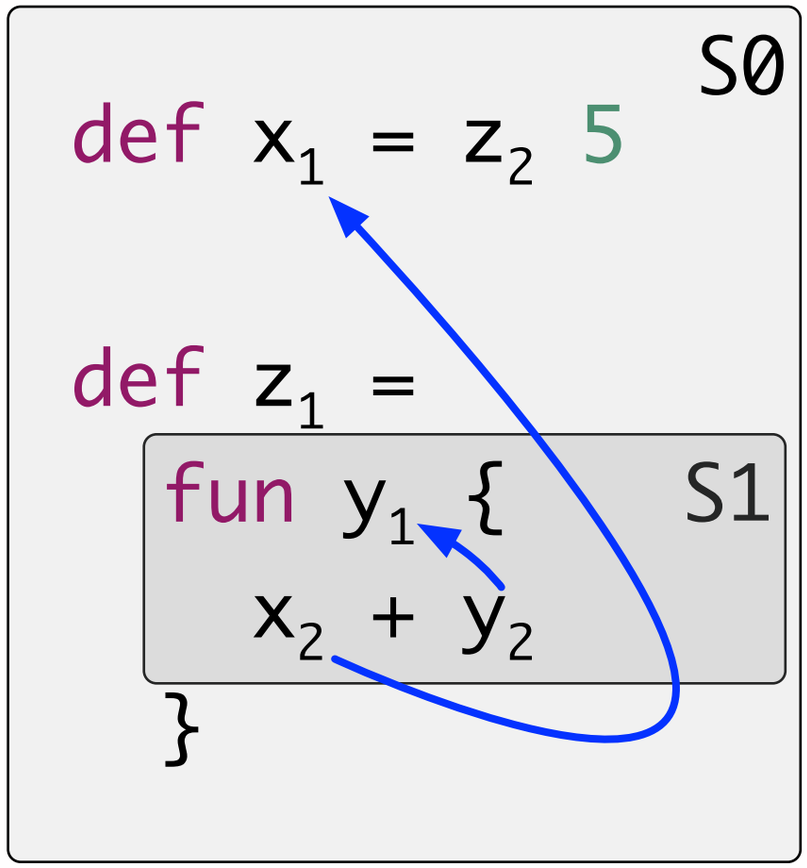
Lexical Scoping

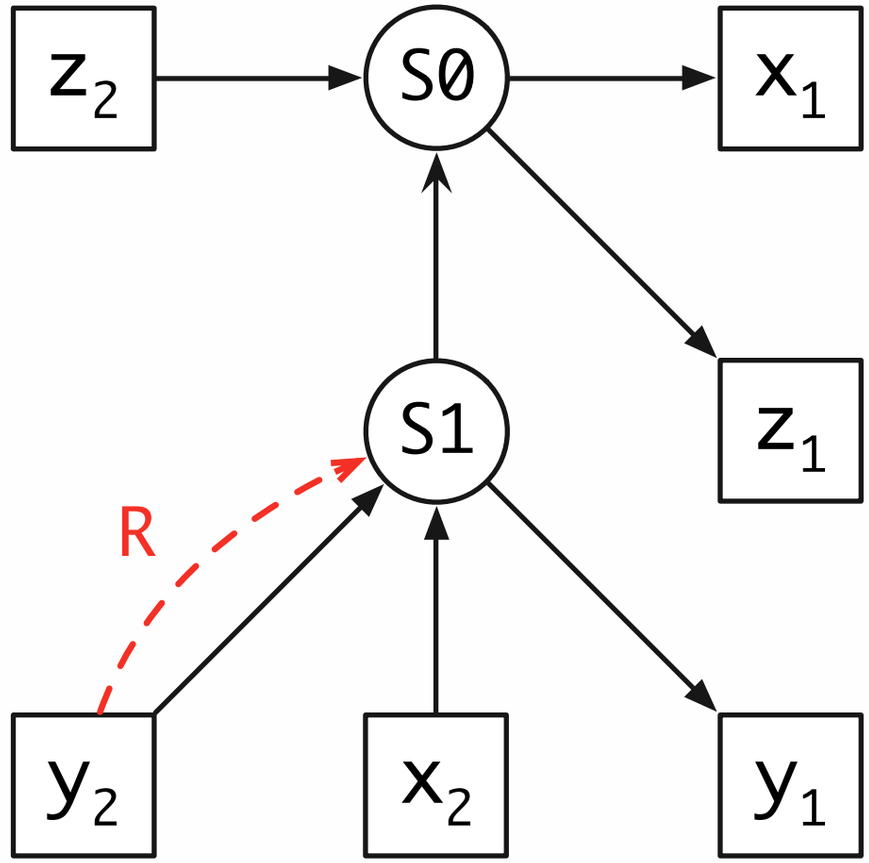


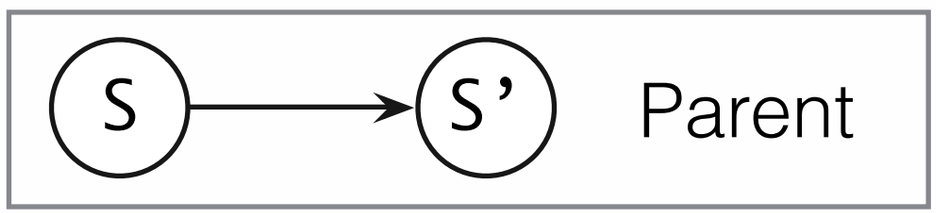




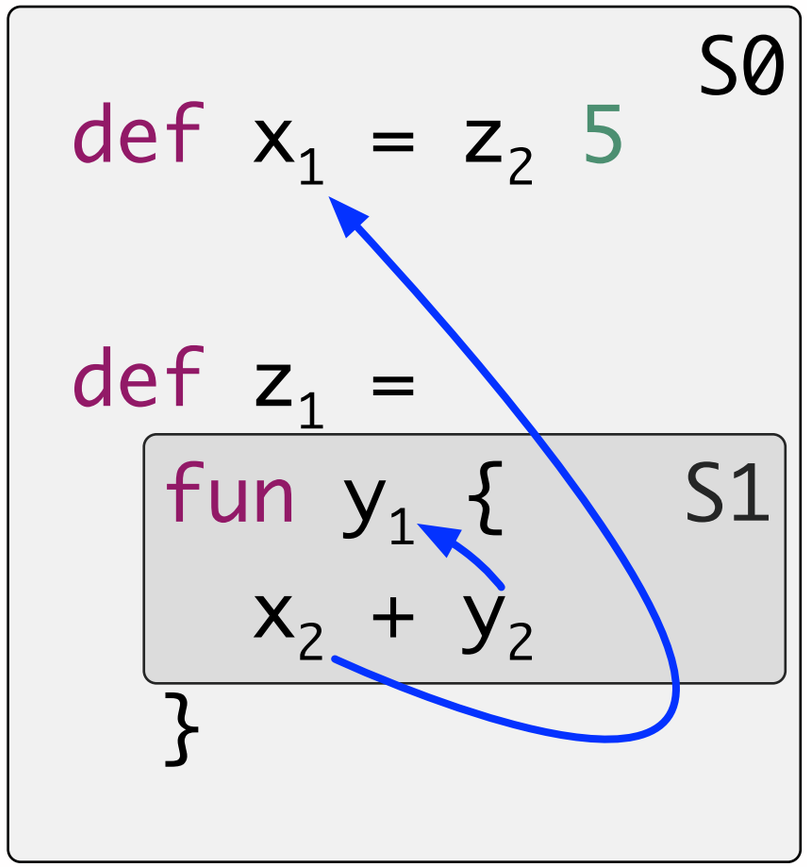
Lexical Scoping

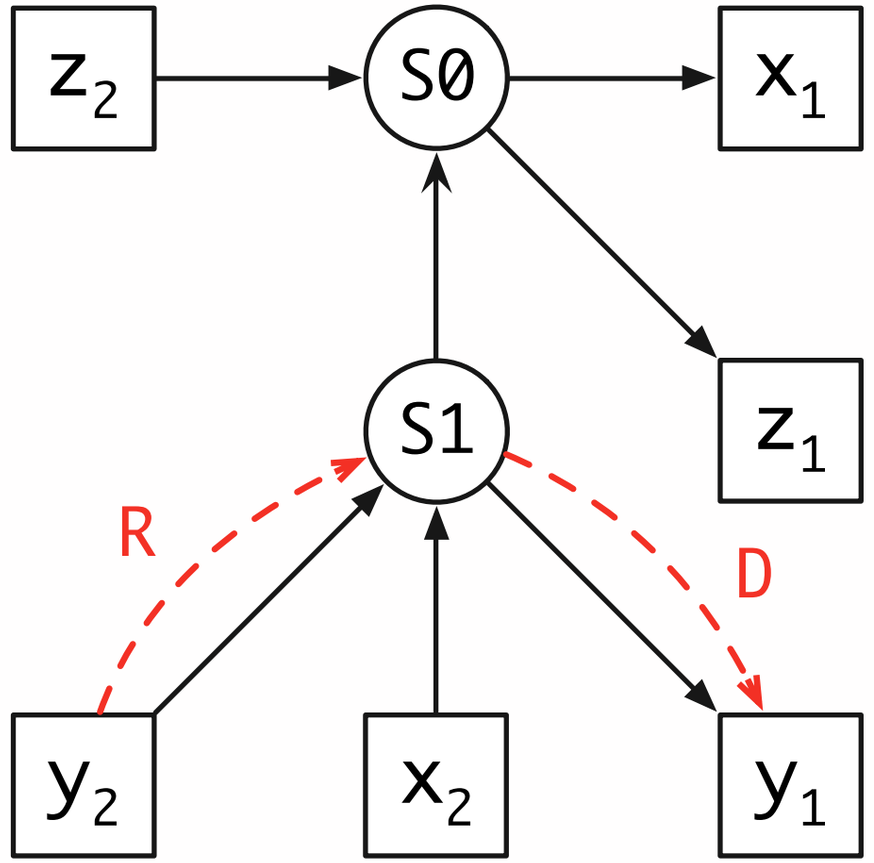


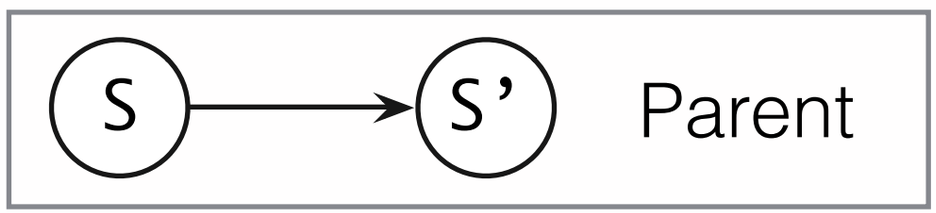




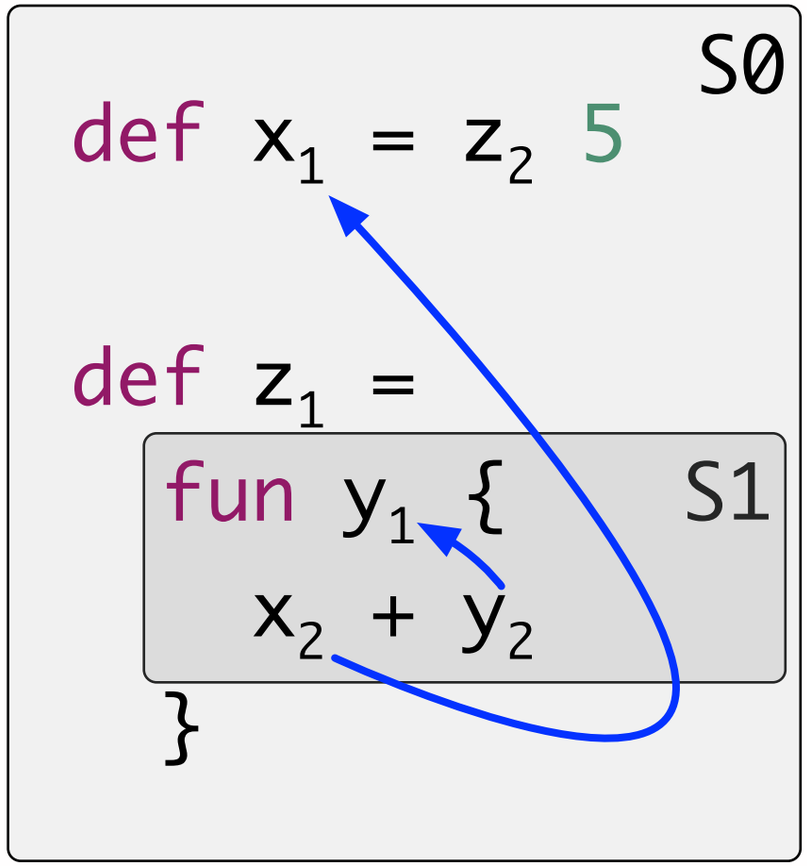
Lexical Scoping

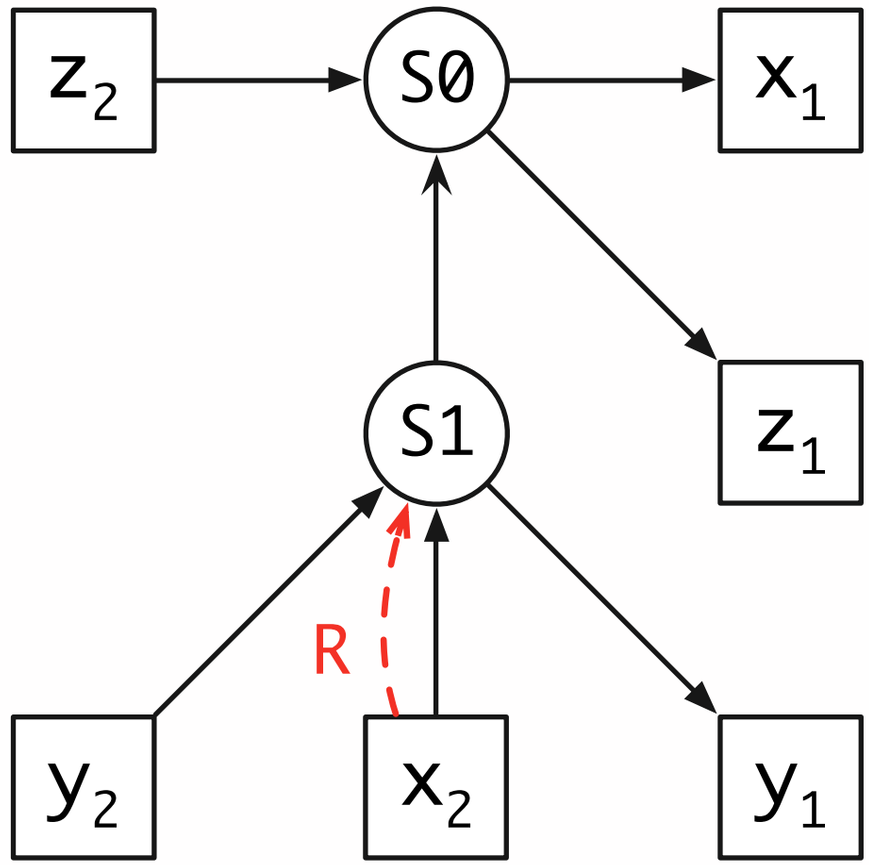


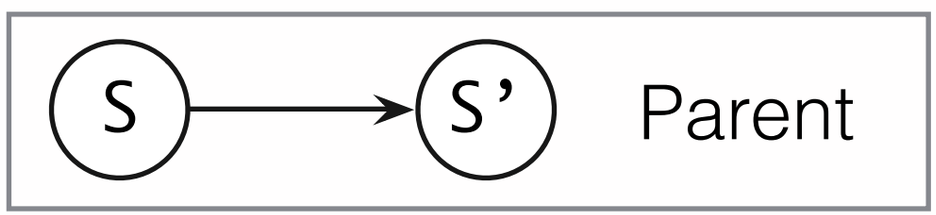




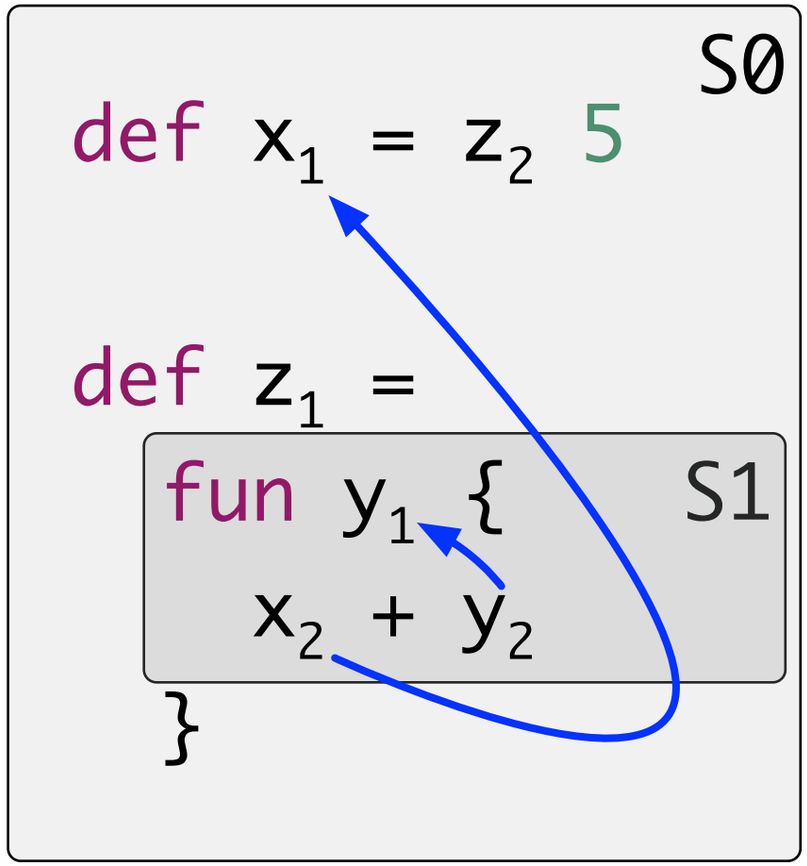
Lexical Scoping

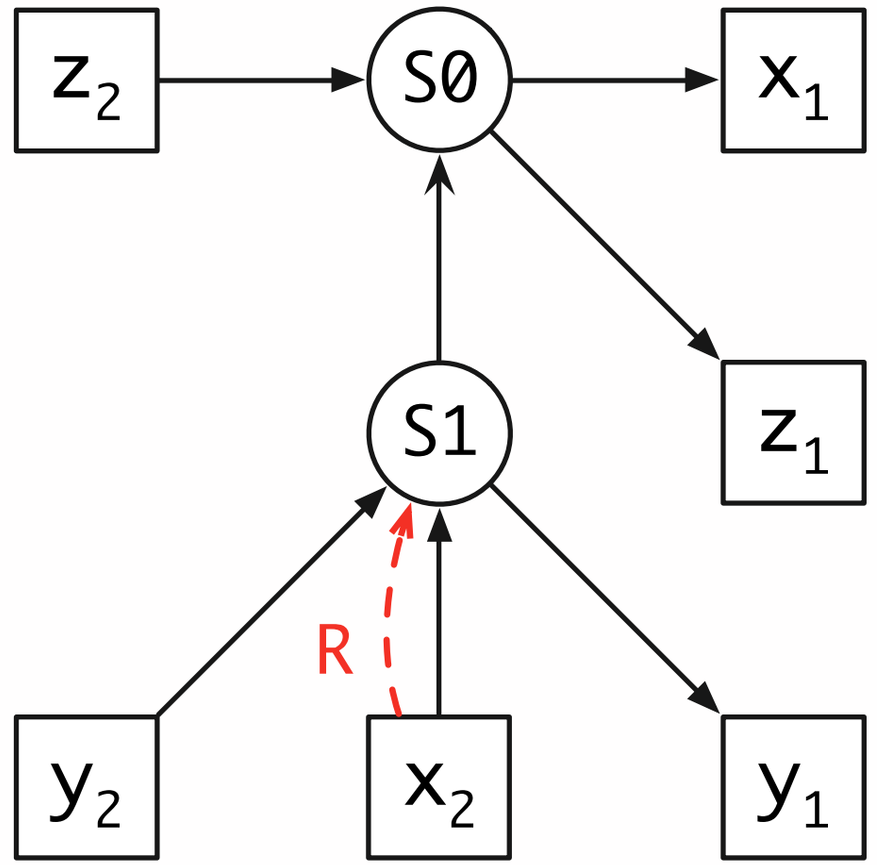


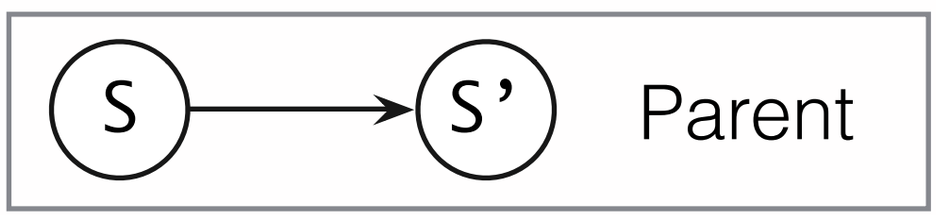


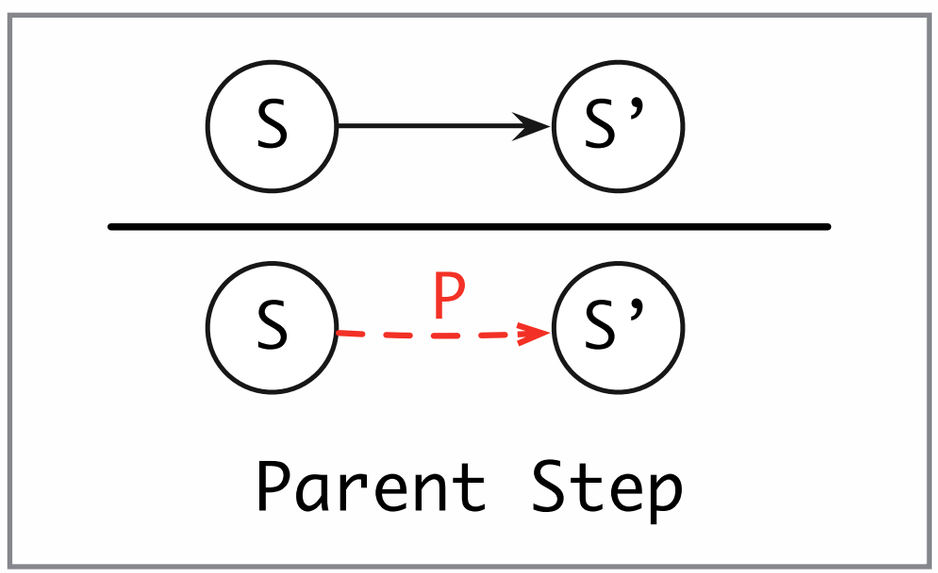


Lexical Scoping

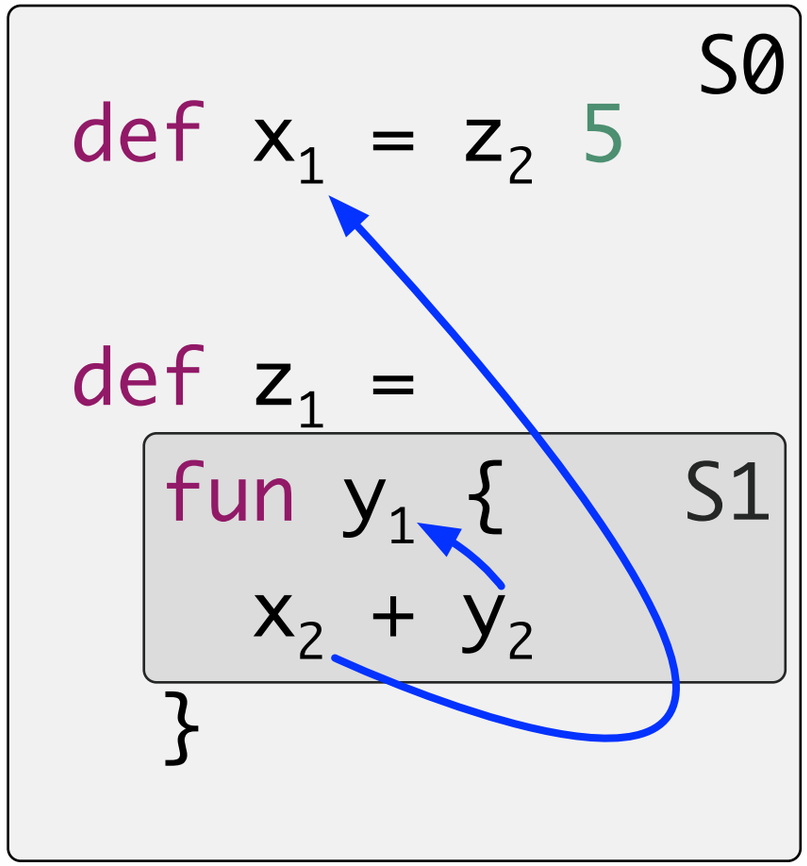


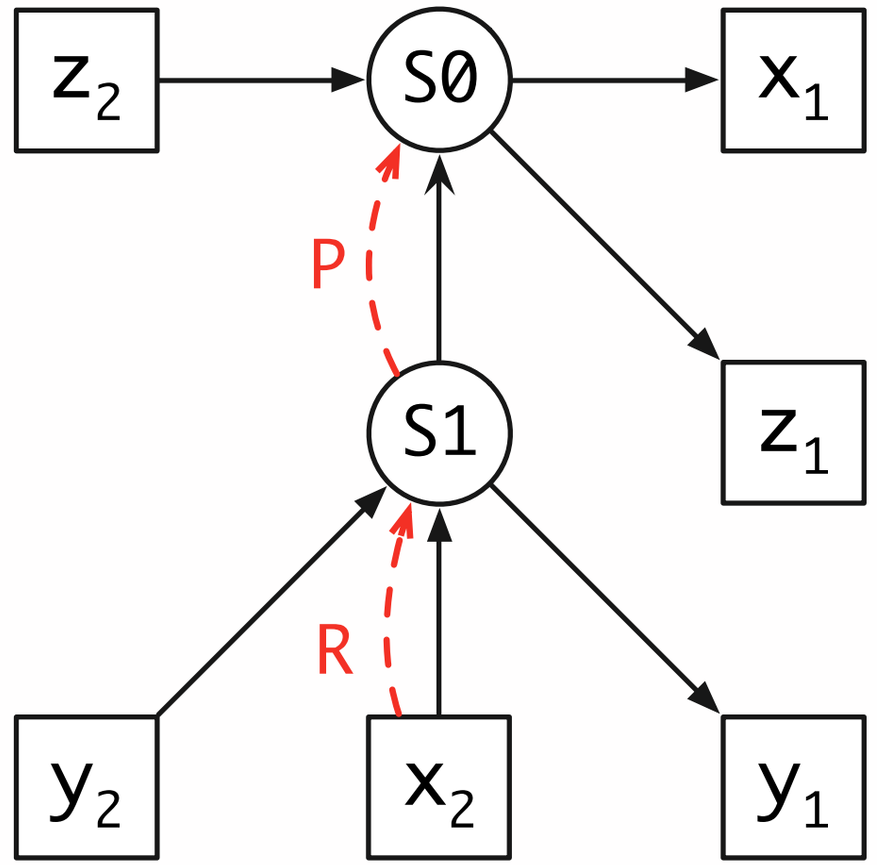


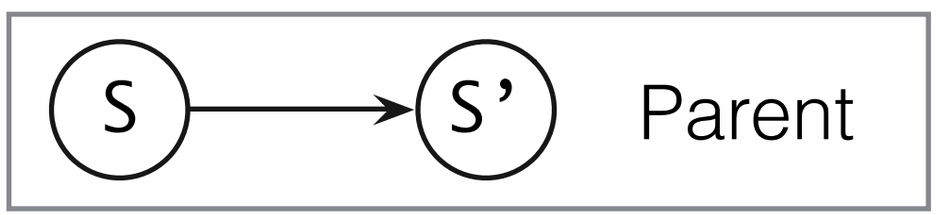


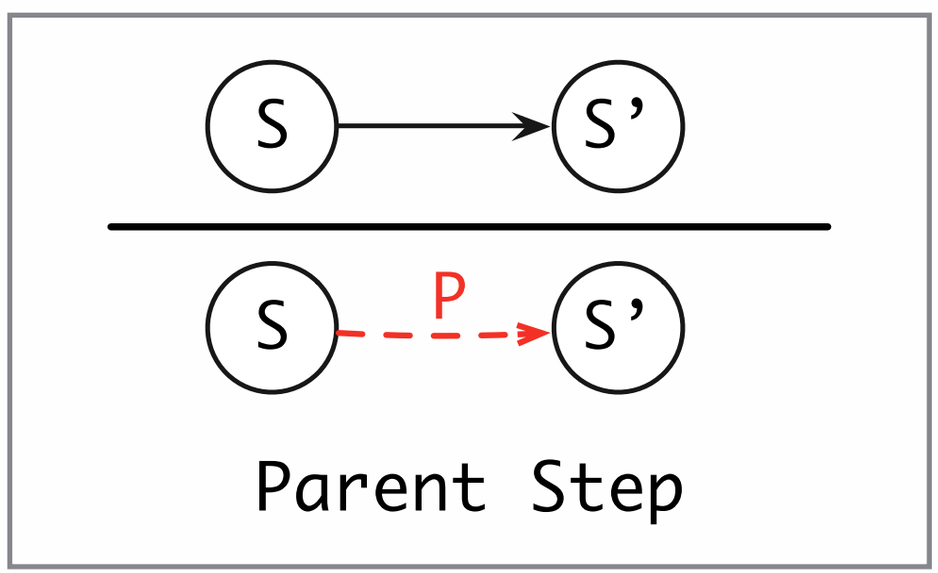


Lexical Scoping

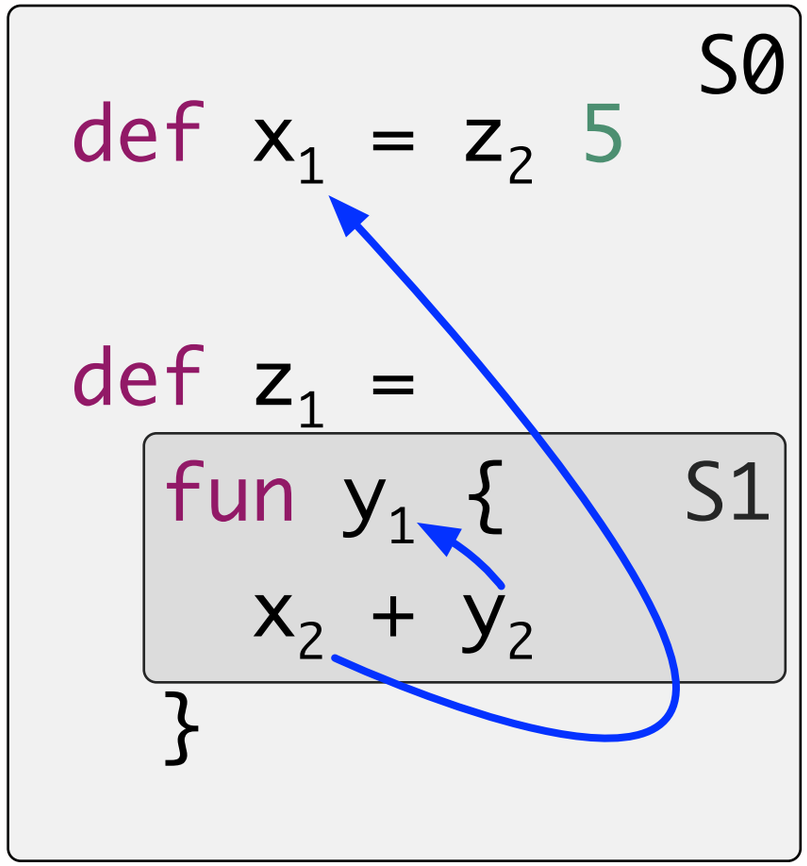


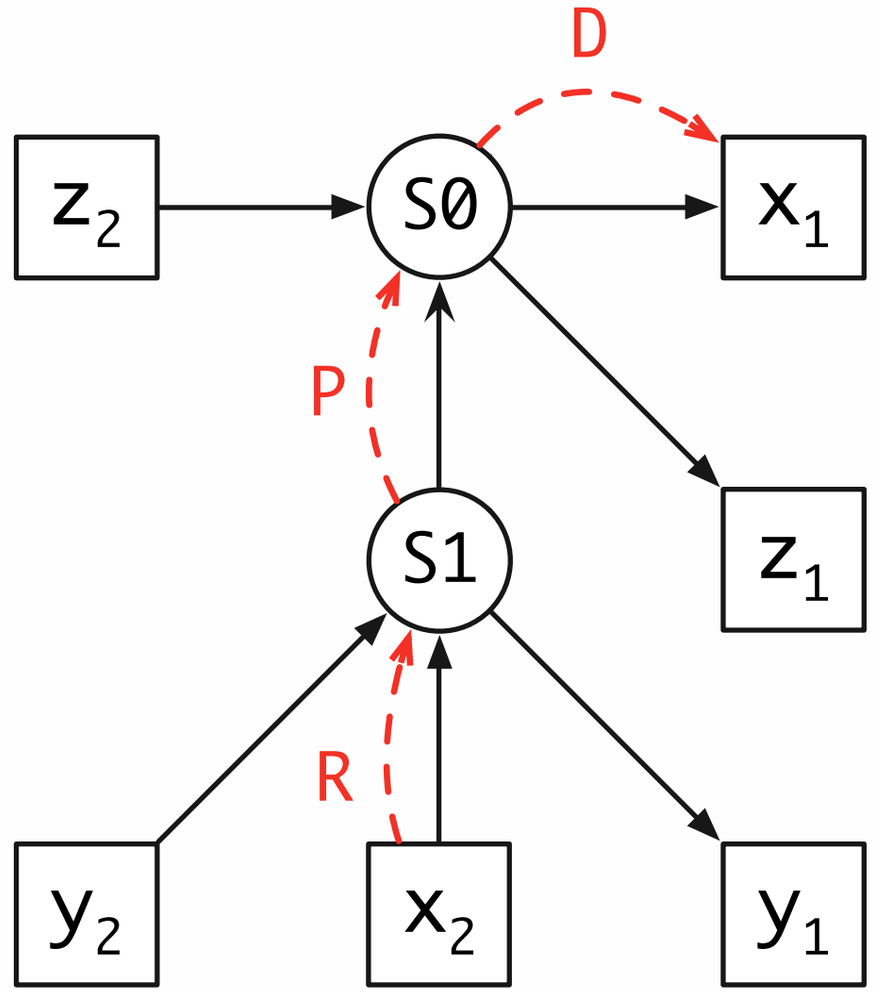


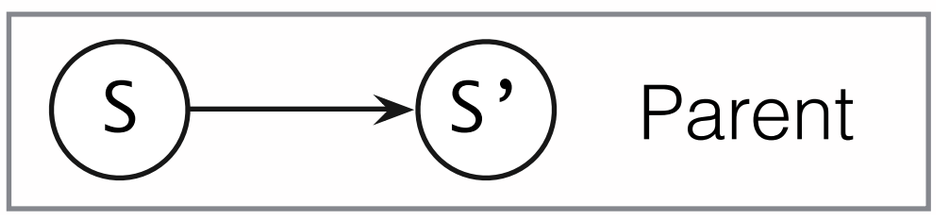


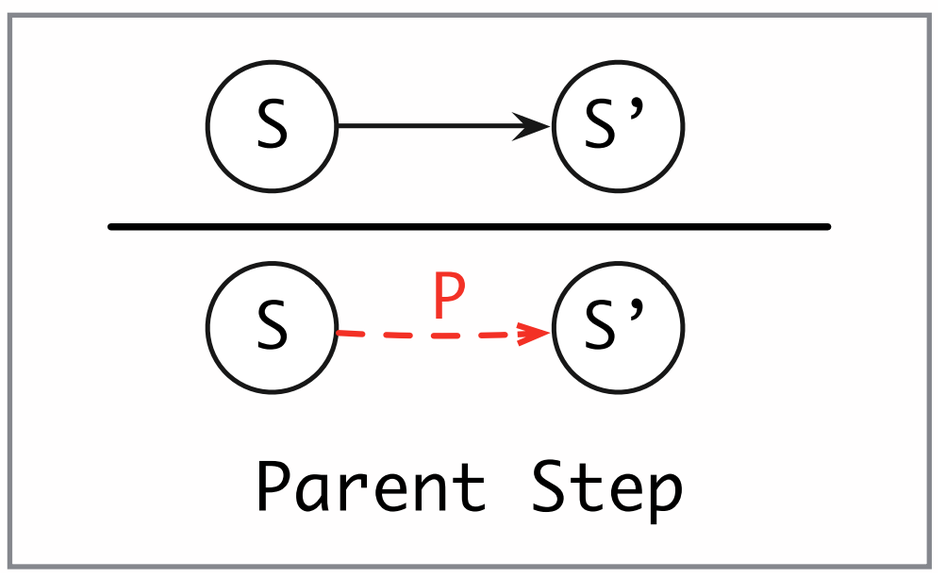


Lexical Scoping

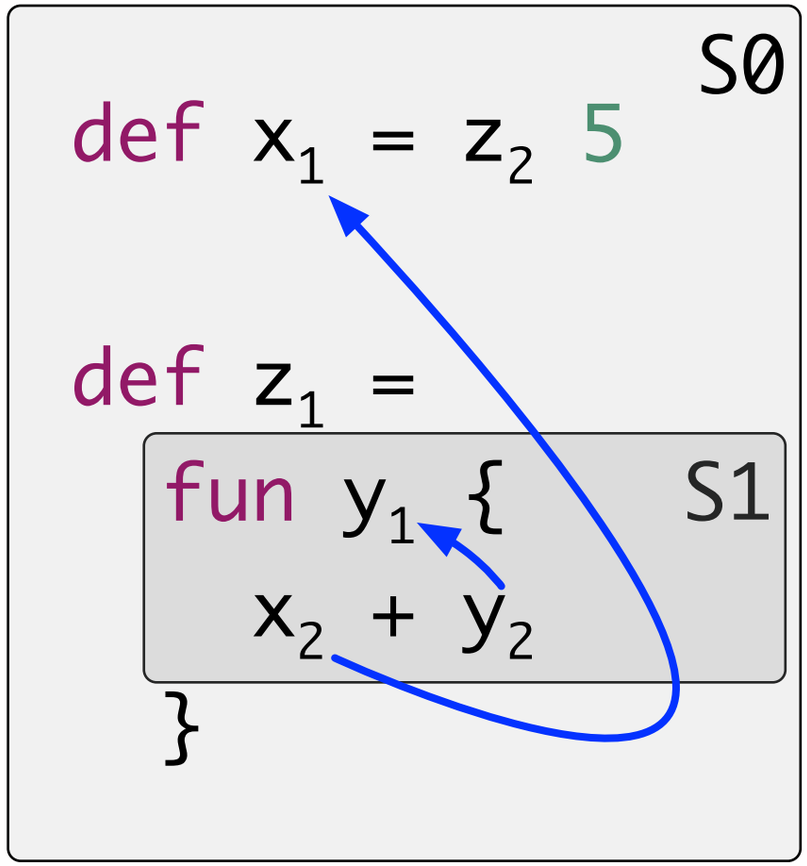


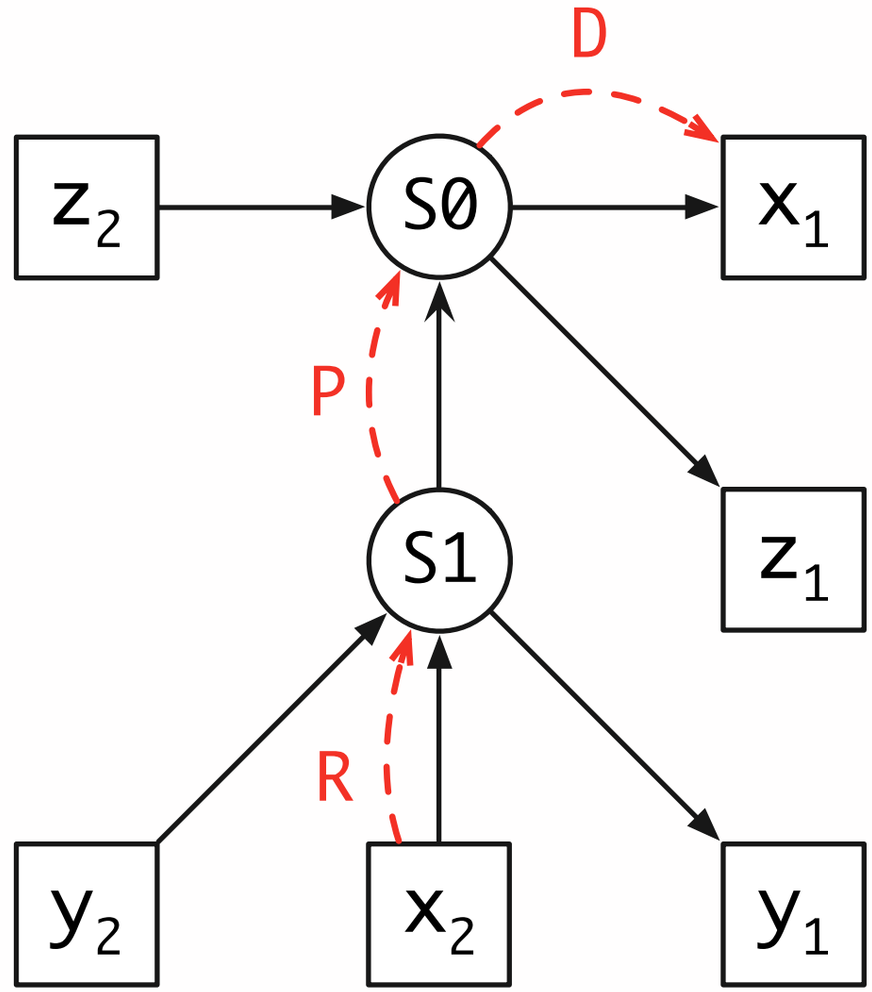


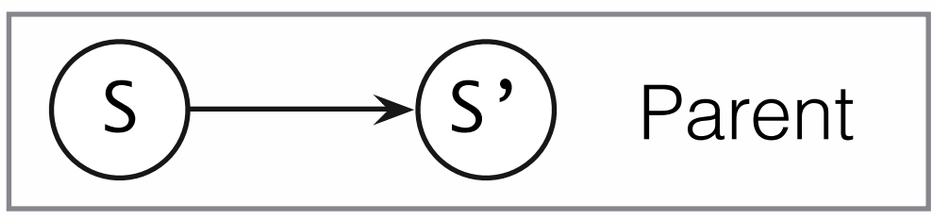




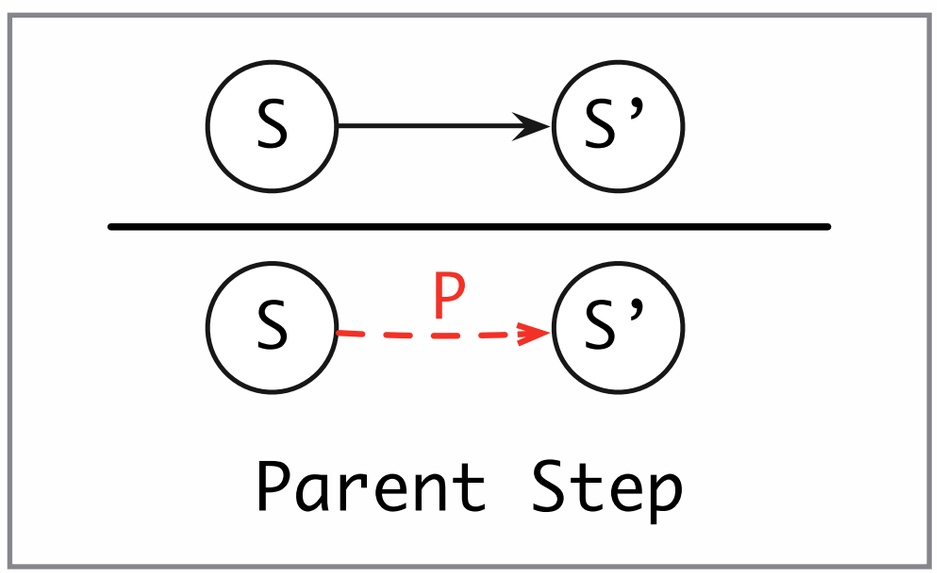
Lexical Scoping







Well formed path: R.P*.D



Shadowing

```
def x3 = z2 5 7

def z1 =
  fun x1 {
    fun y1 {
      x2 + y2
    }
  }
```

Shadowing

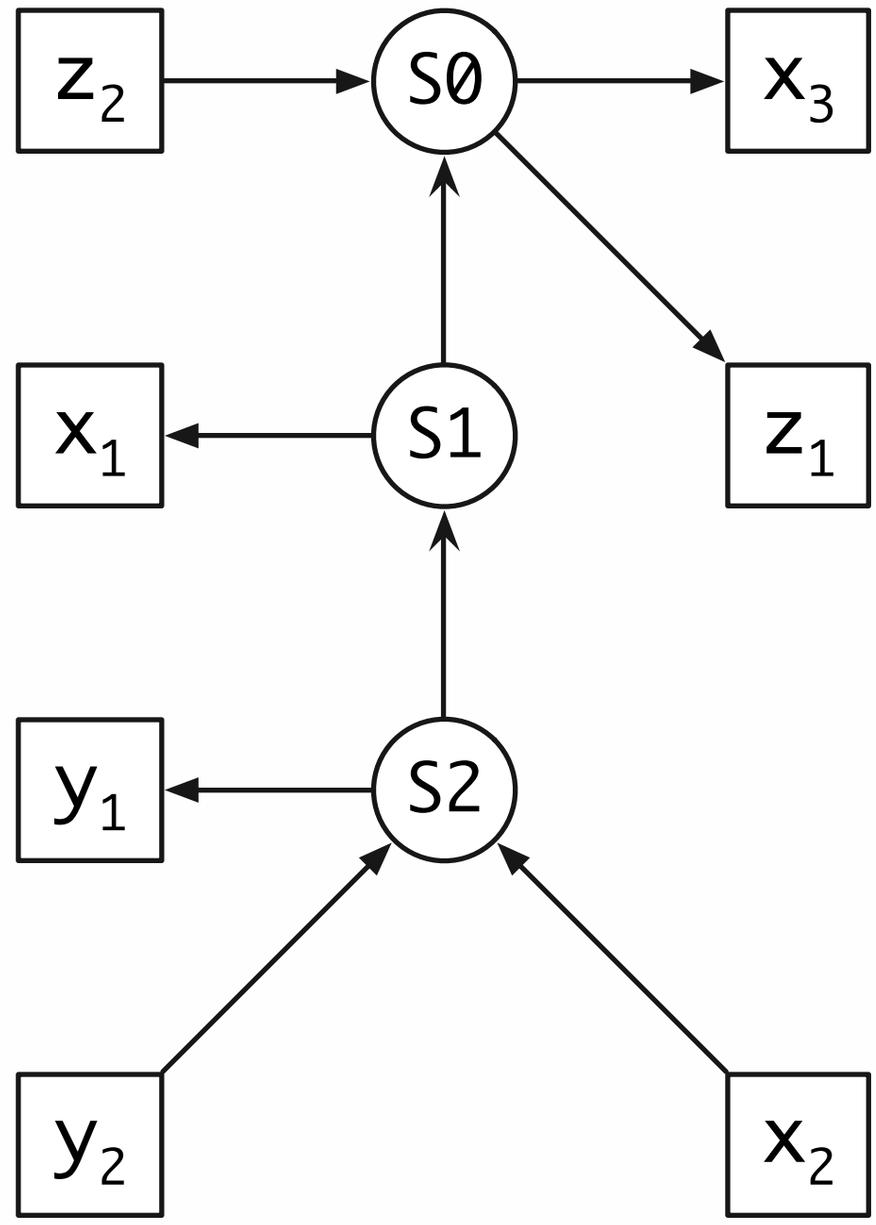
```
def x3 = z2 5 7                    S0


def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```

Shadowing

```
def x3 = z2 5 7                    S0

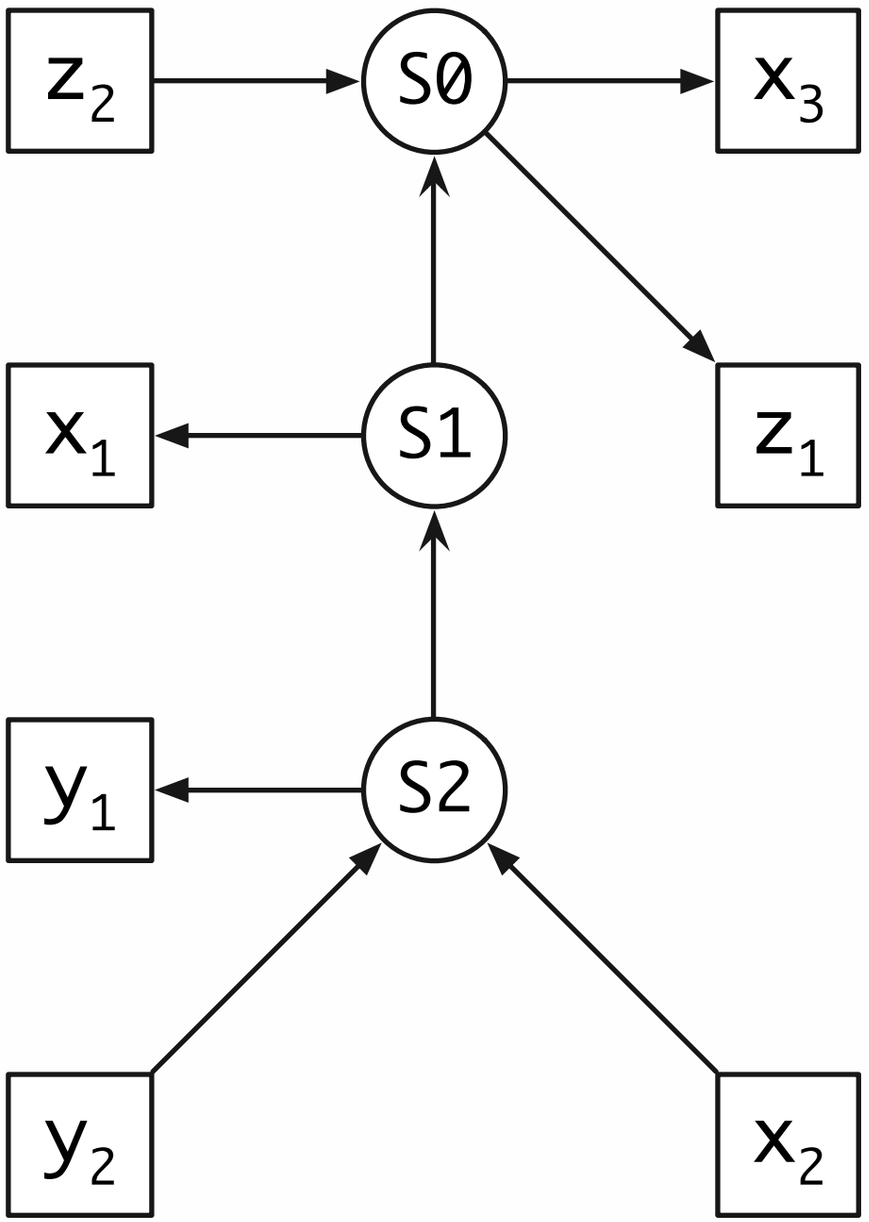
def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```



Shadowing

```
def x3 = z2 5 7                    S0

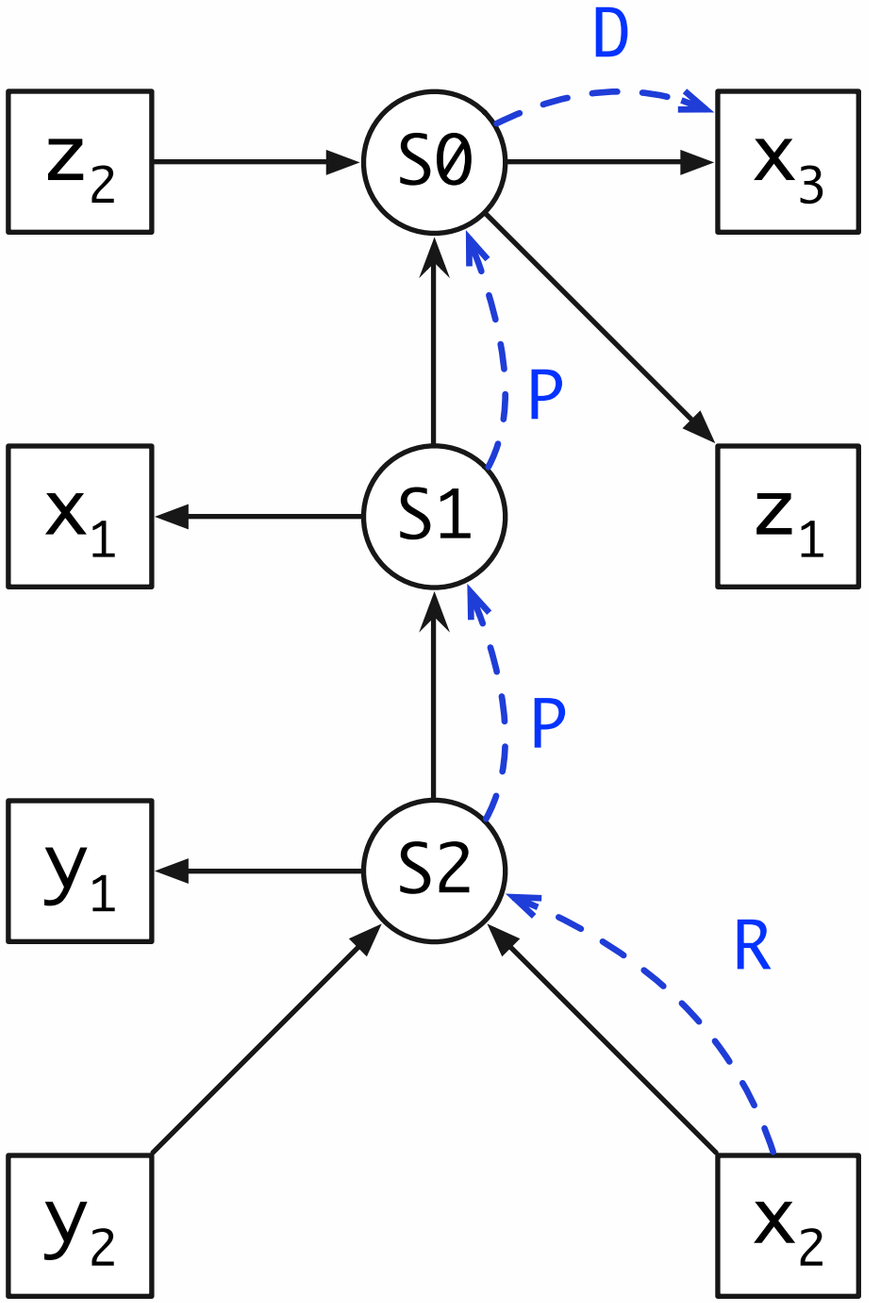
def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```



Shadowing

```
def x3 = z2 5 7                    S0

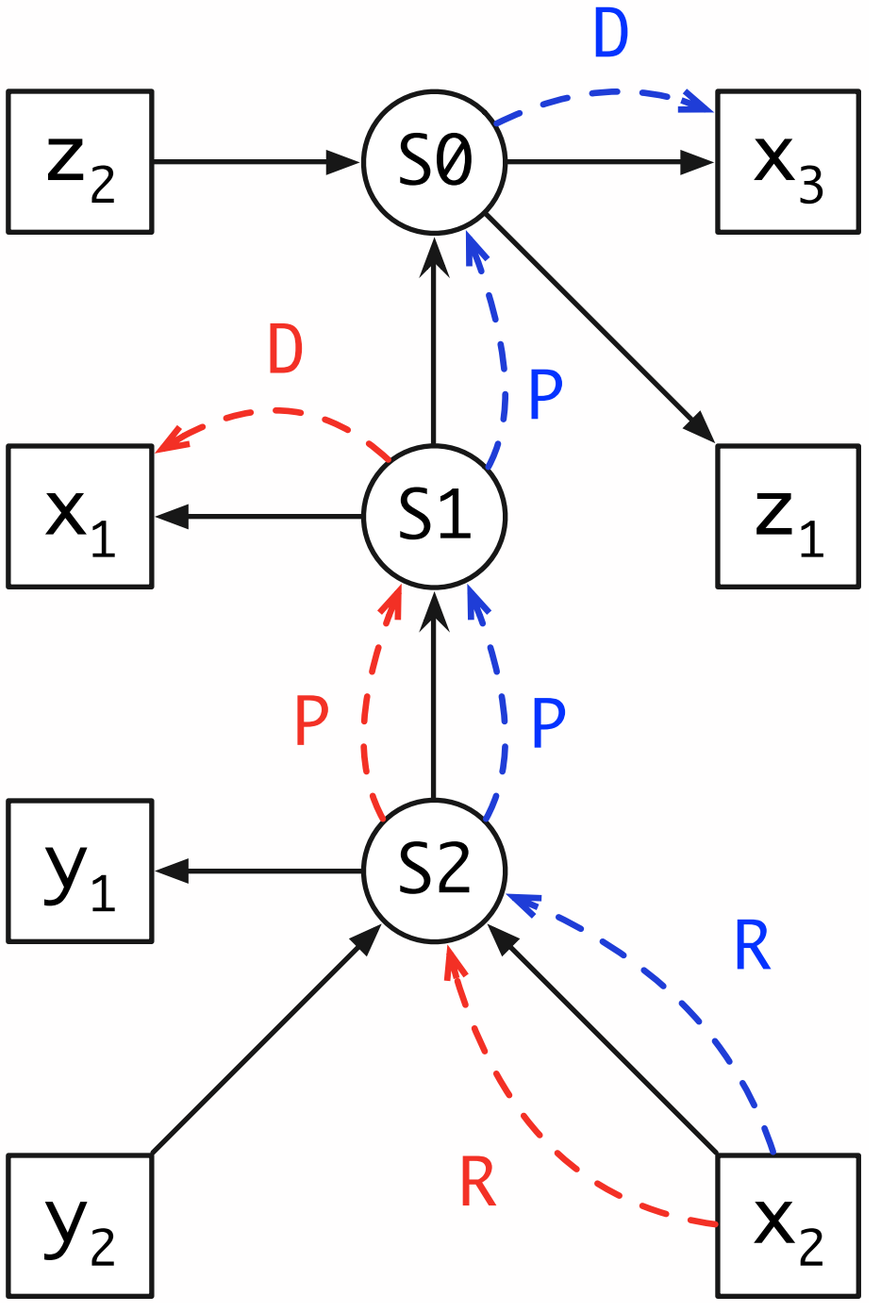
def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```



Shadowing

```
def x3 = z2 5 7                    S0

def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```

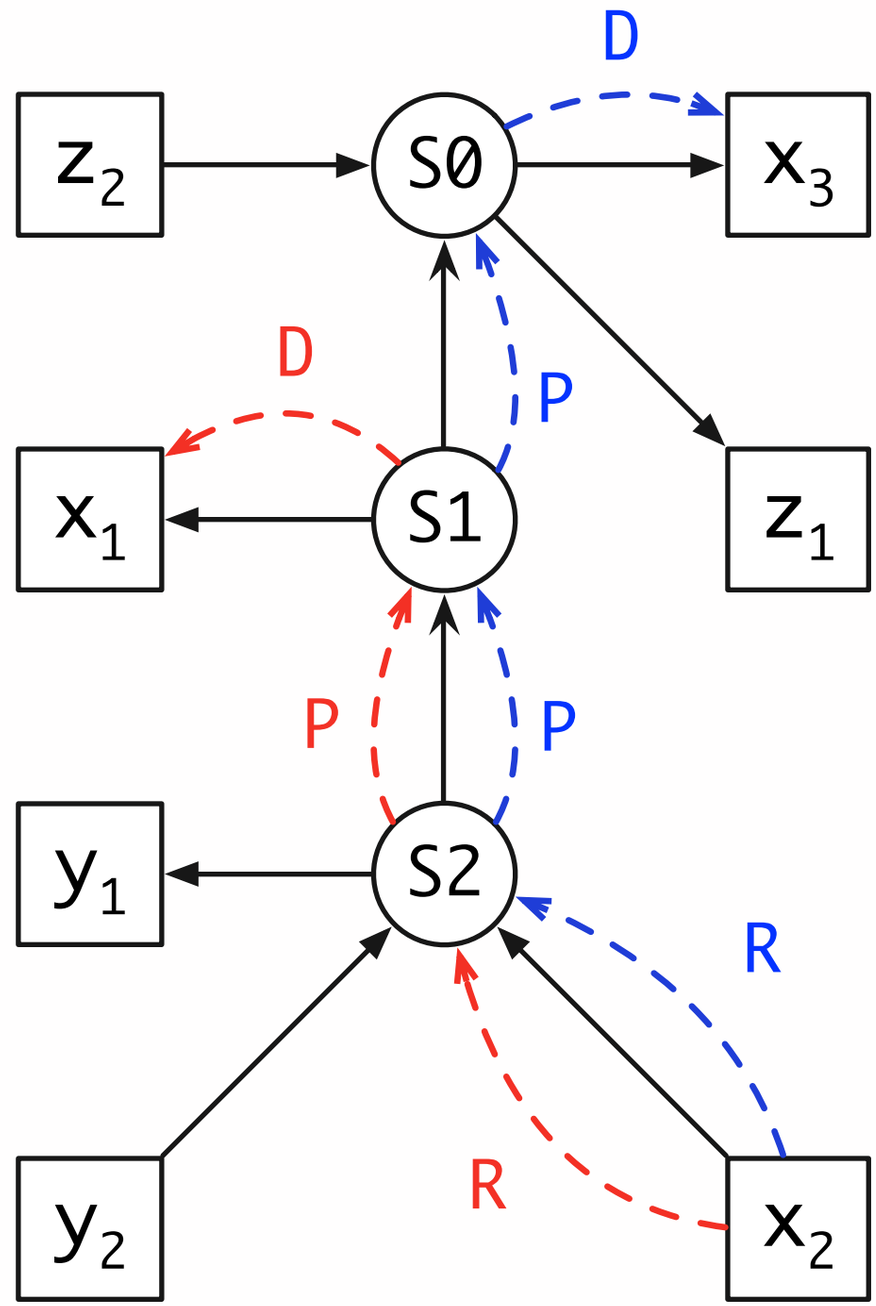


Shadowing

```
def x3 = z2 5 7                    S0

def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```

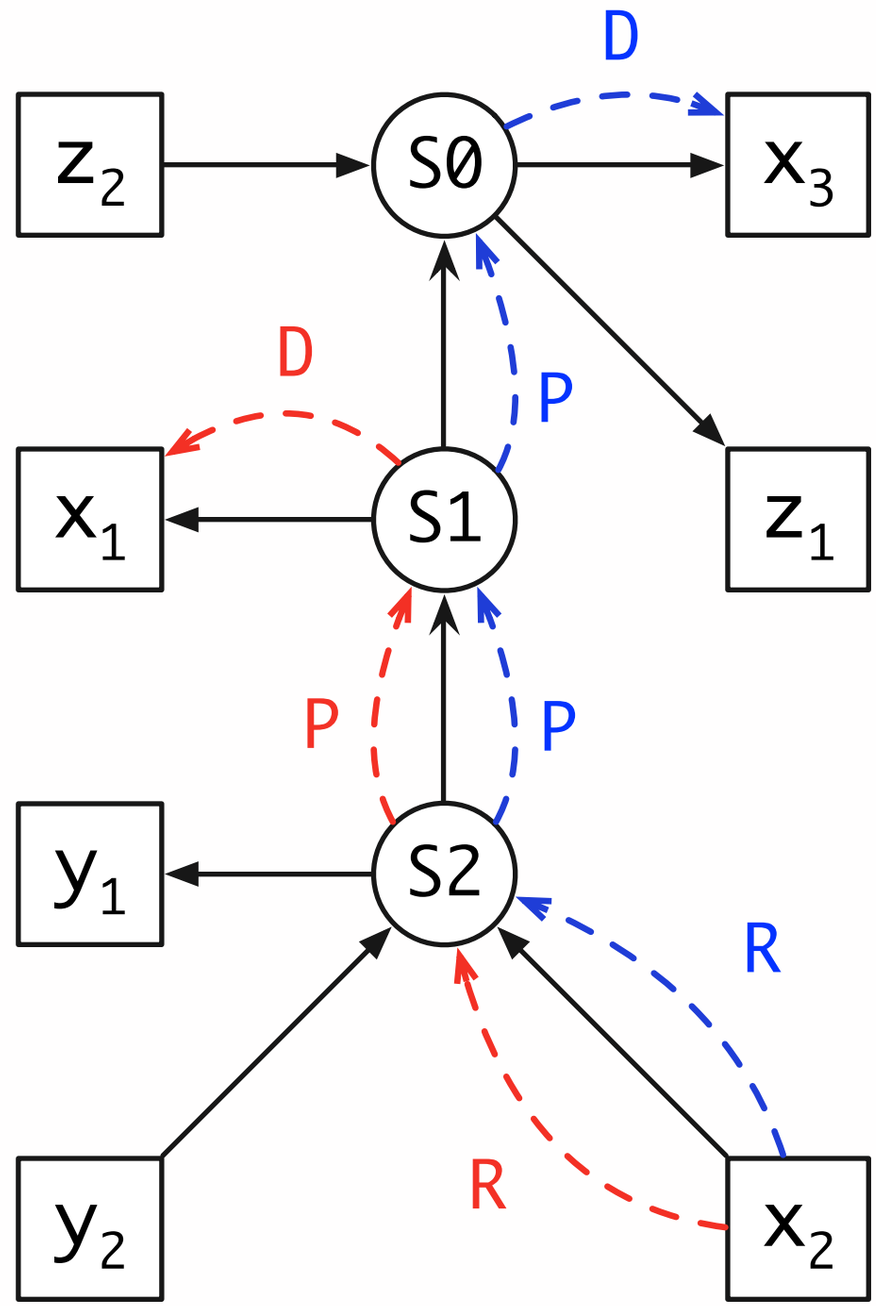
$$\frac{}{D < P.p}$$



Shadowing

```
def x3 = z2 5 7                    S0

def z1 =
  fun x1 {                         S1
    fun y1 {                       S2
      x2 + y2
    }
  }
```



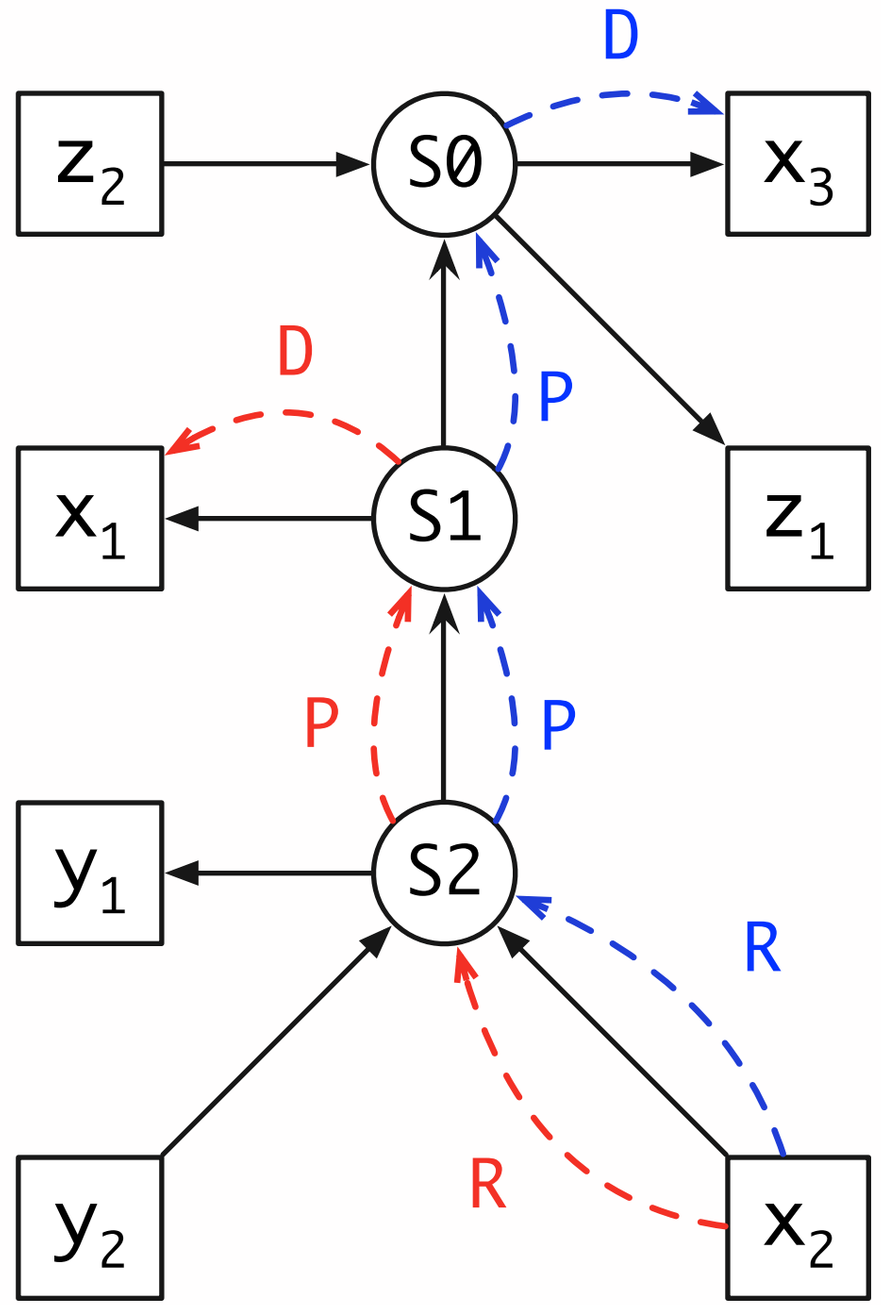
$$\frac{}{D < P.p}$$

$$\frac{p < p'}{s.p < s.p'}$$

Shadowing

```
def x3 = z2 5 7                                   S0

def z1 =
  fun x1 {                                        S1
    fun y1 {                                      S2
      x2 + y2
    }
  }
```



$$\frac{}{D < P.p}$$

$$\frac{p < p'}{s.p < s.p'}$$

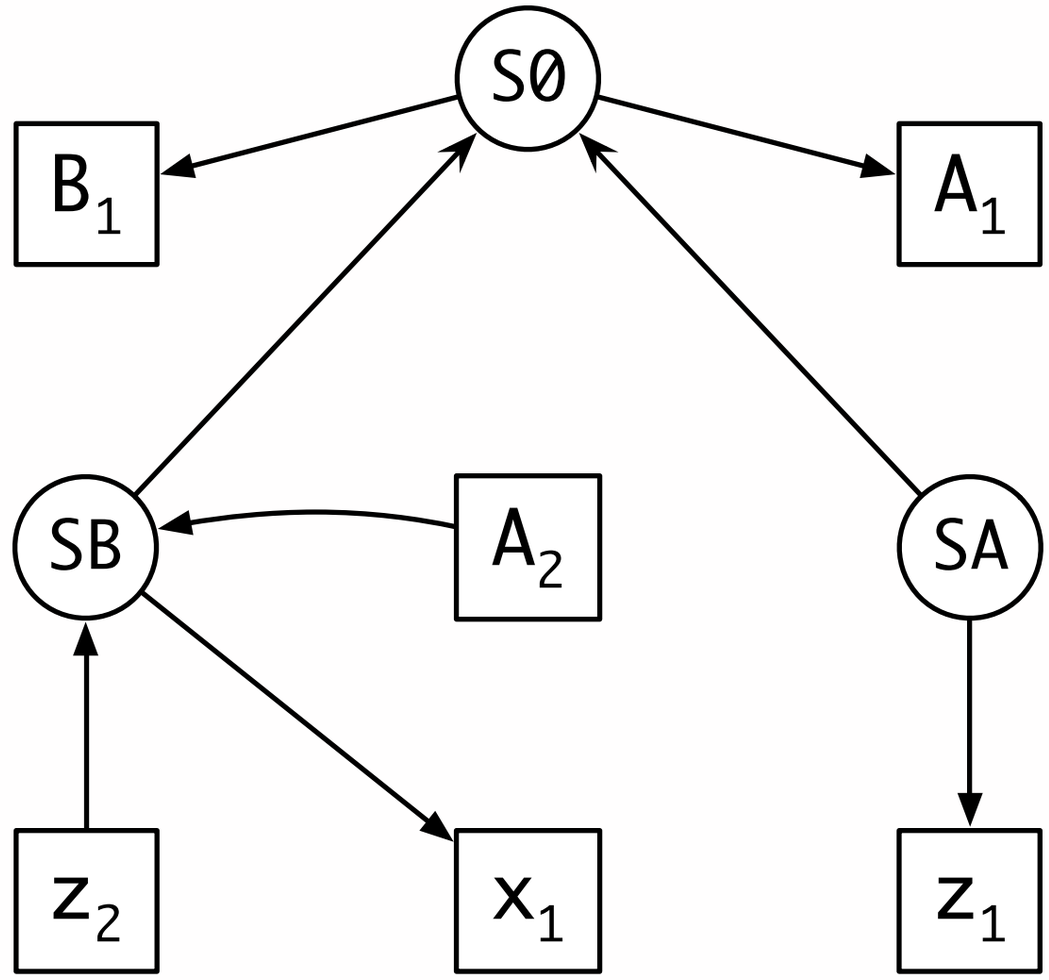
$$R.P.D < R.P.P.D$$

Imports

```
module A1 {                          S0
  def z1 = 5                         SA
}
module B1 {
  import A2                          SB
  def x1 = 1 + z2
}
```

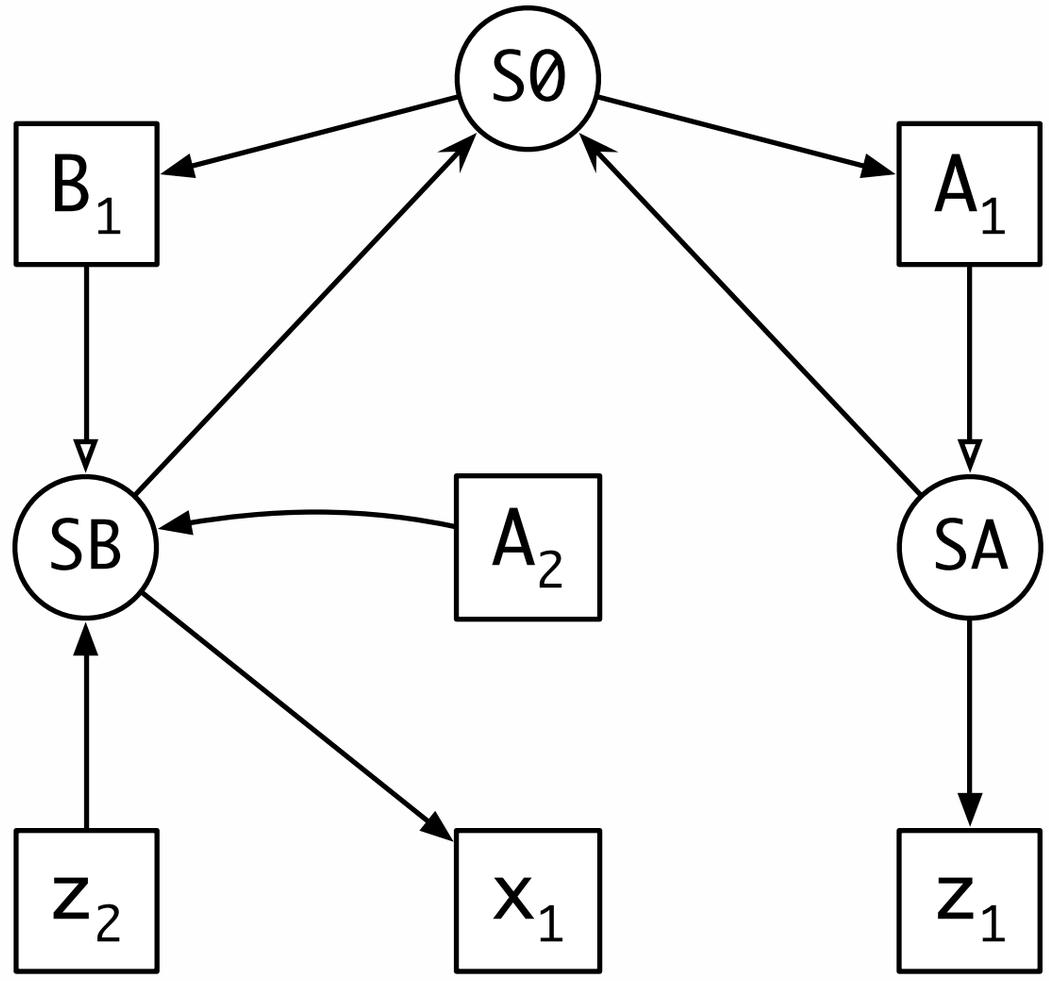
Imports

```
module A₁ {                    S0
  def z₁ = 5                   SA
}
module B₁ {
  import A₂                    SB
  def x₁ = 1 + z₂
}
```



Imports

```
module A1 {                    S0
  def z1 = 5                   SA
}
module B1 {
  import A2                    SB
  def x1 = 1 + z2
}
```



Imports

```
module A1 {                    S0
  def z1 = 5                   SA
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module B1 {
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  def x1 = 1 + z2
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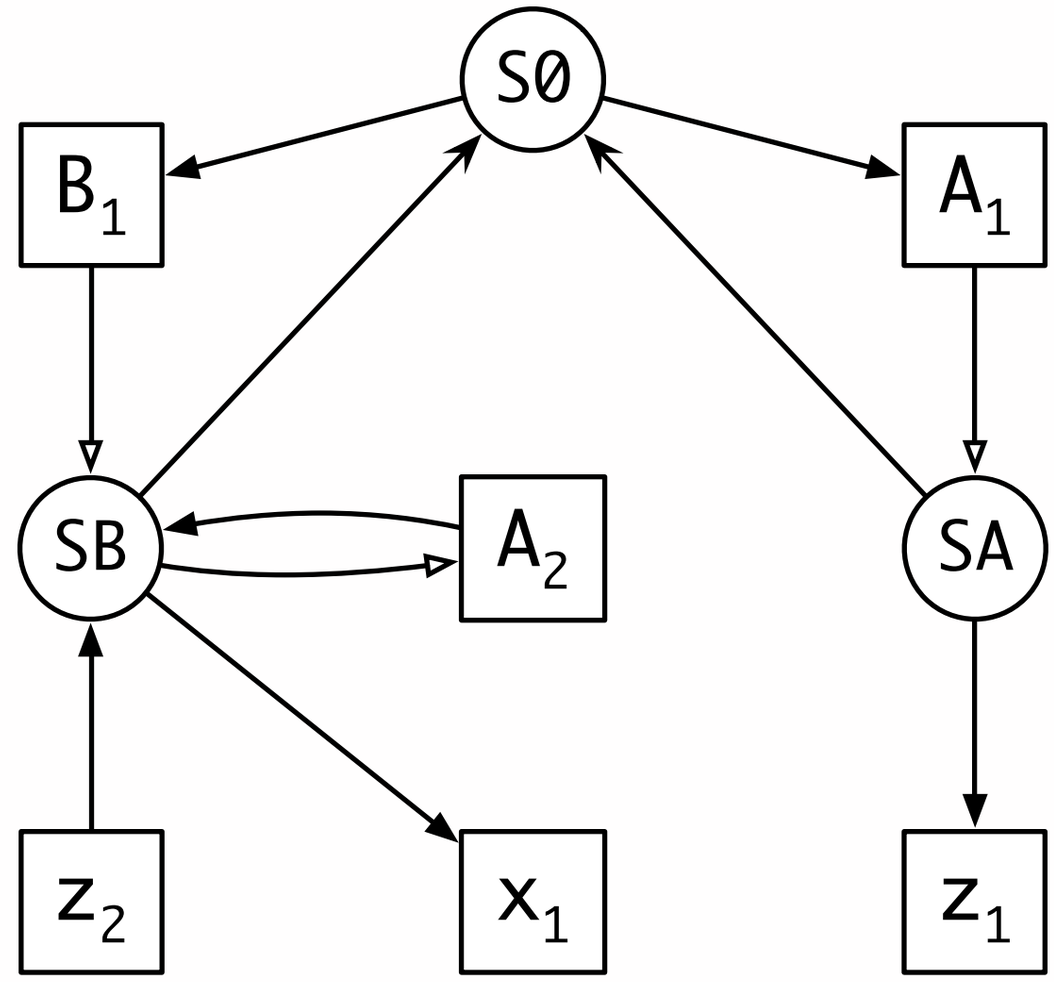

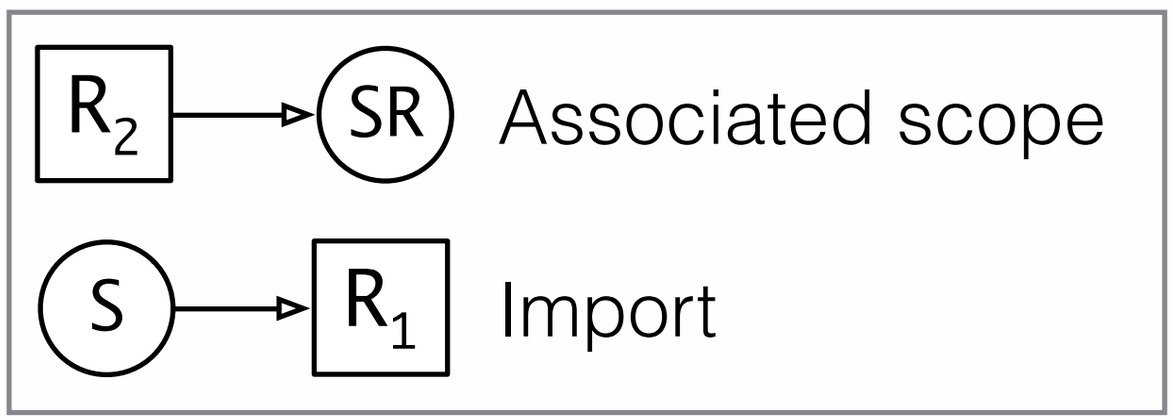

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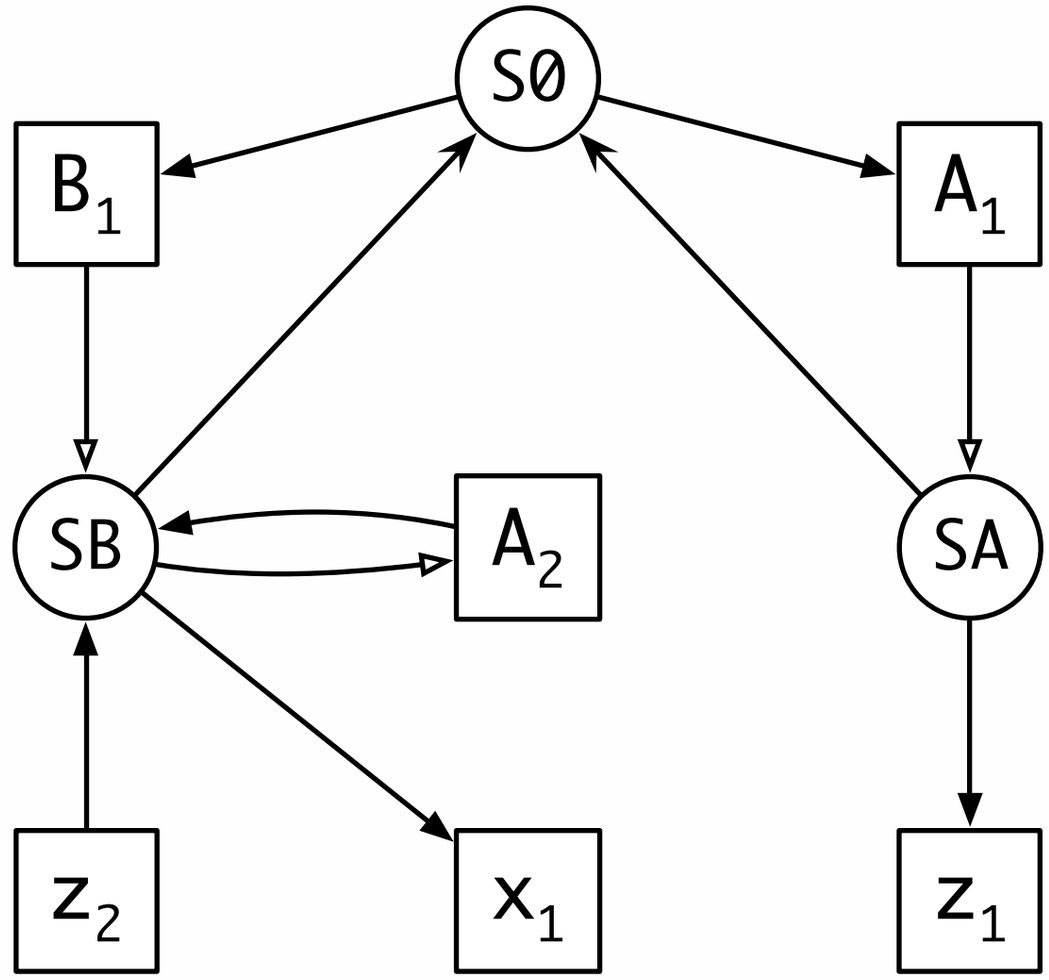

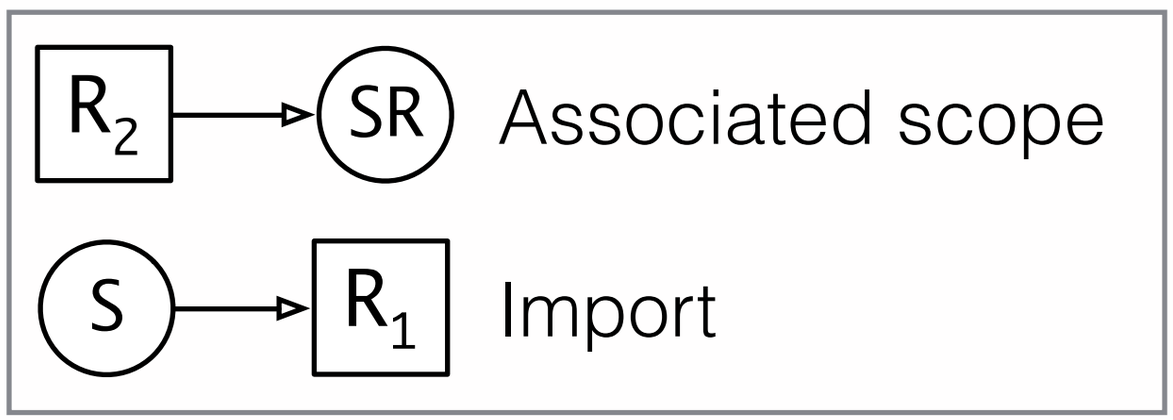

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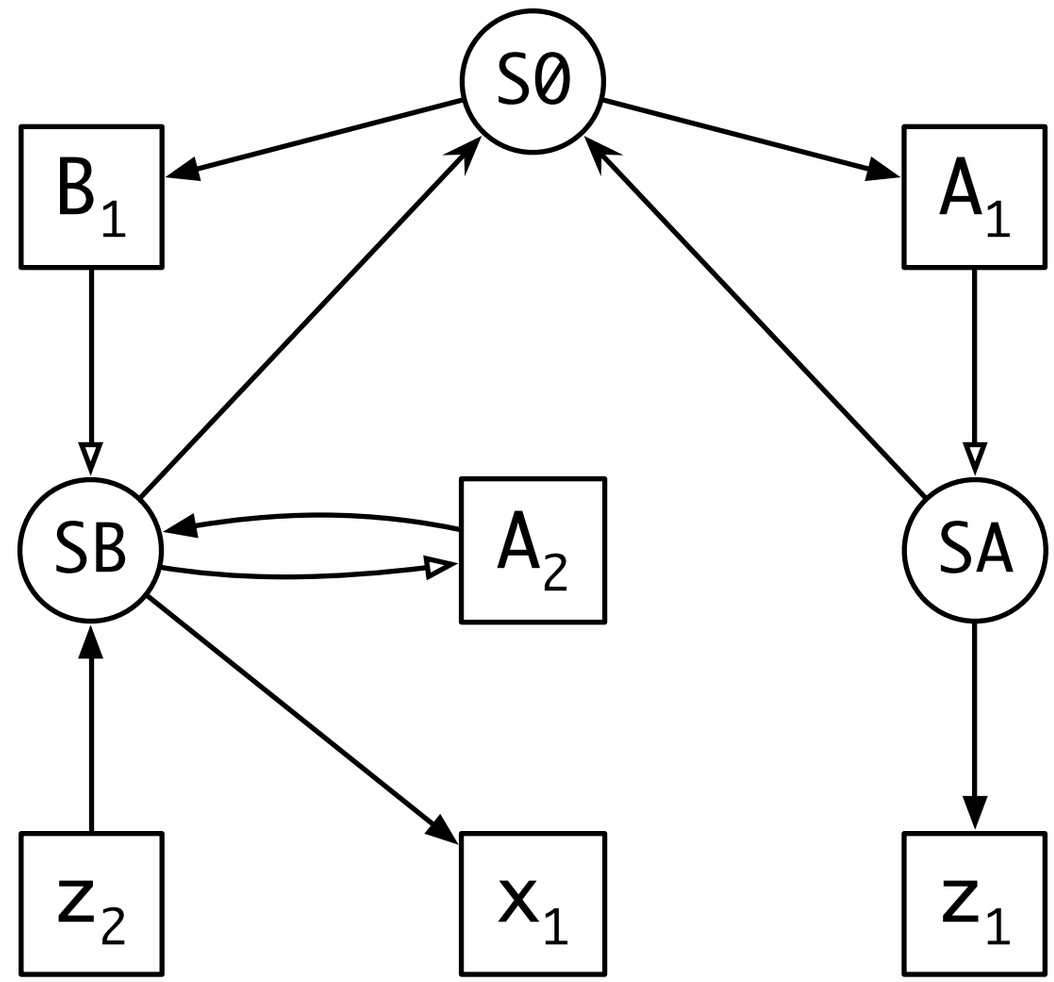

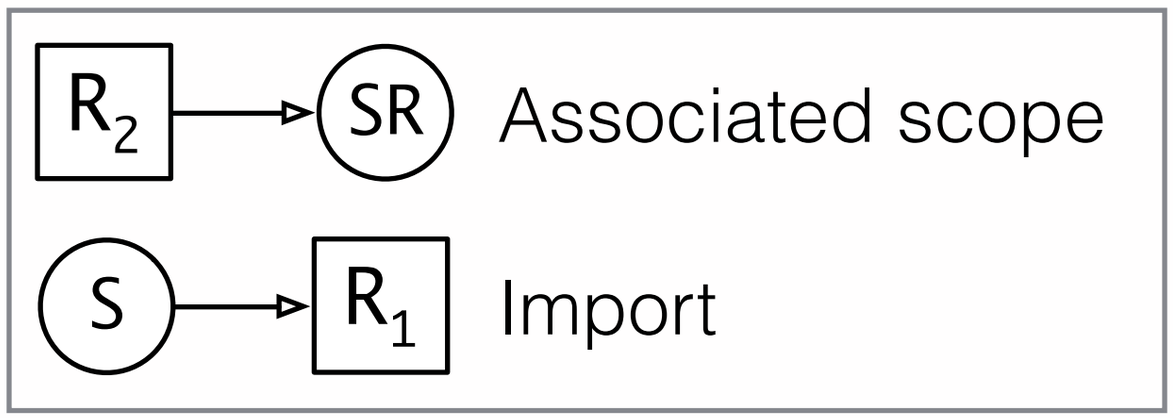

Imports

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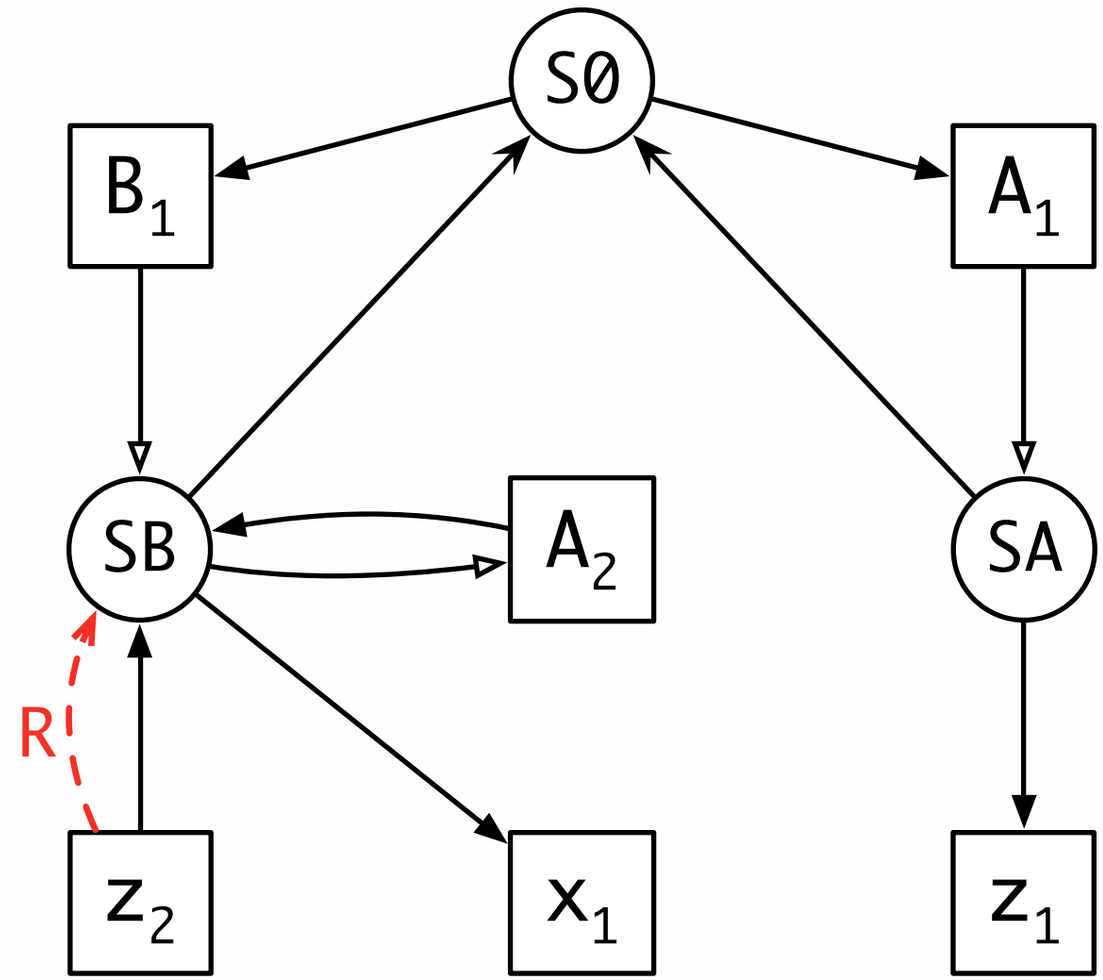

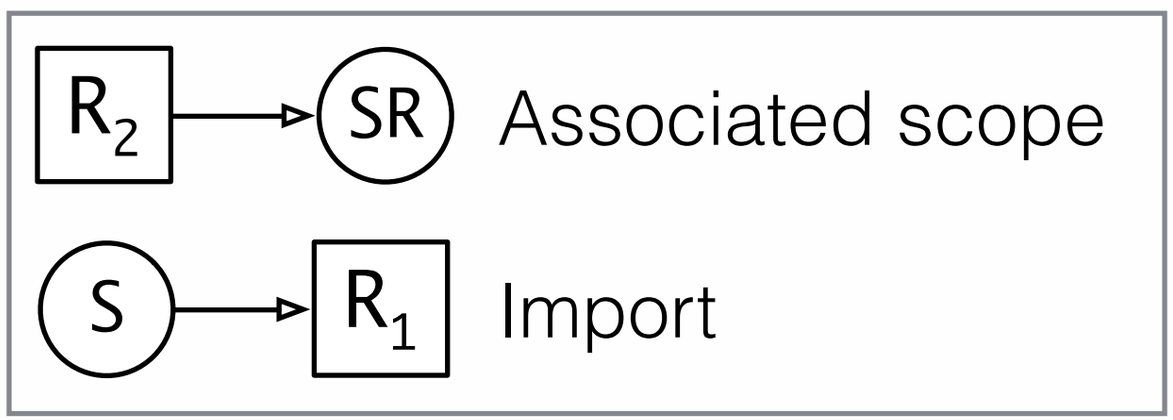

Imports

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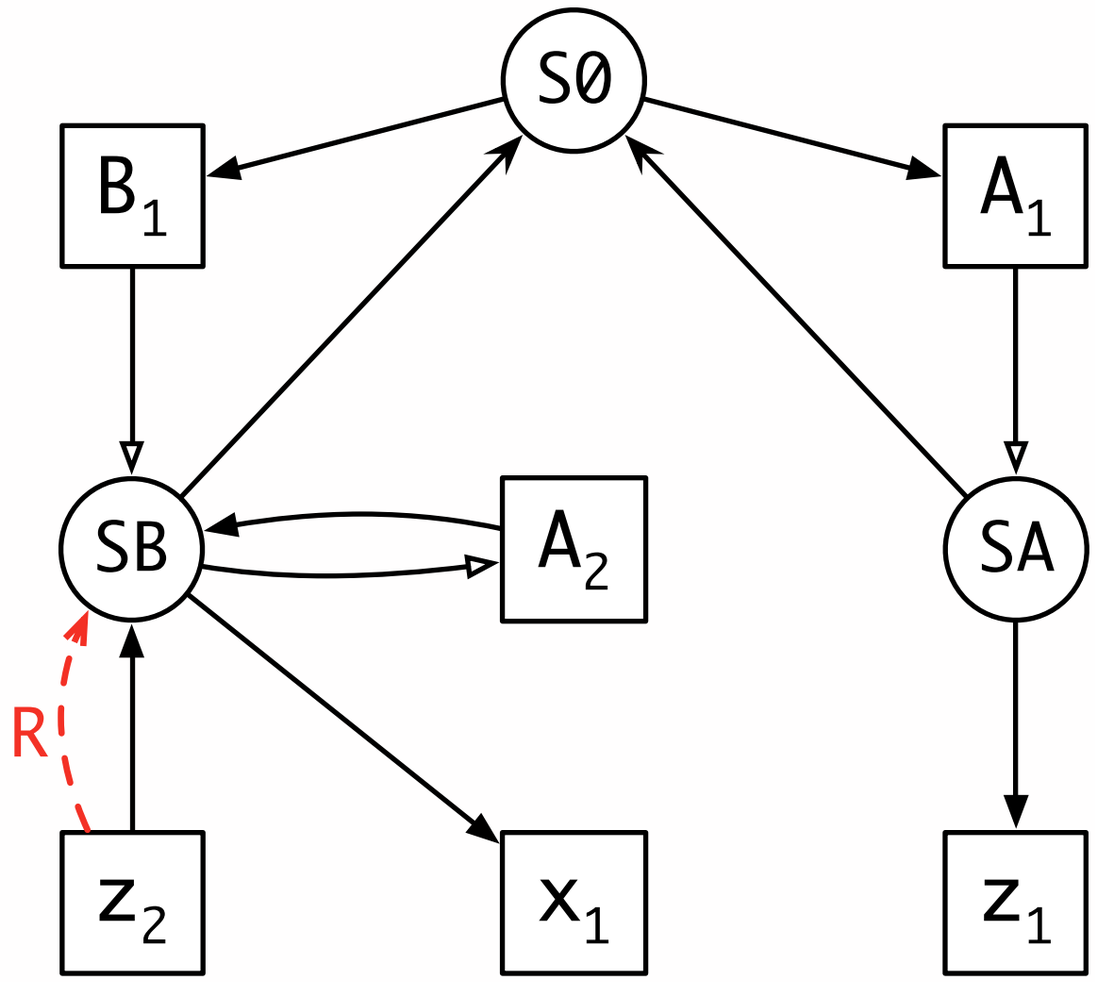

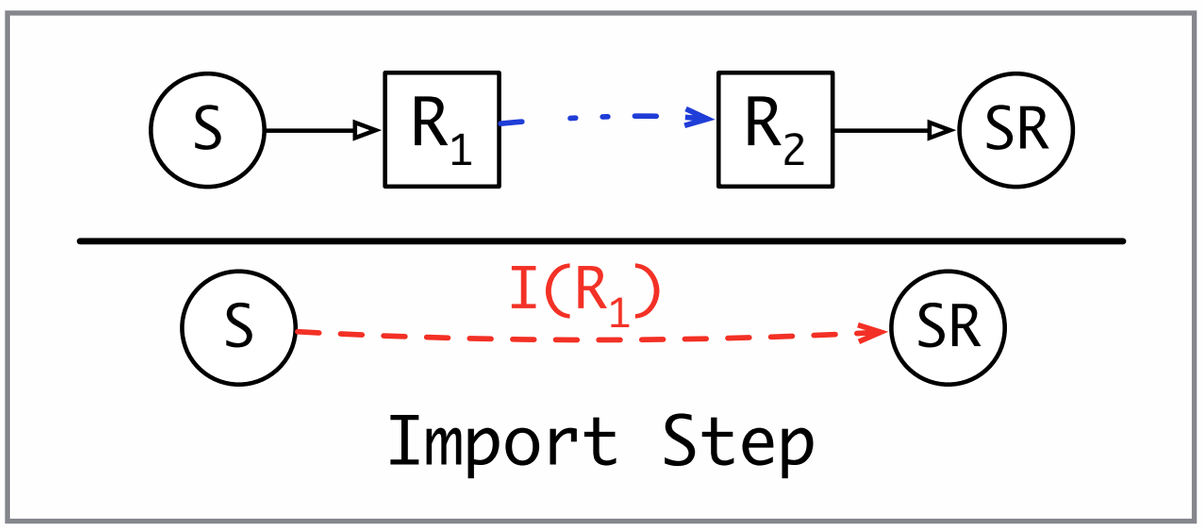

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}
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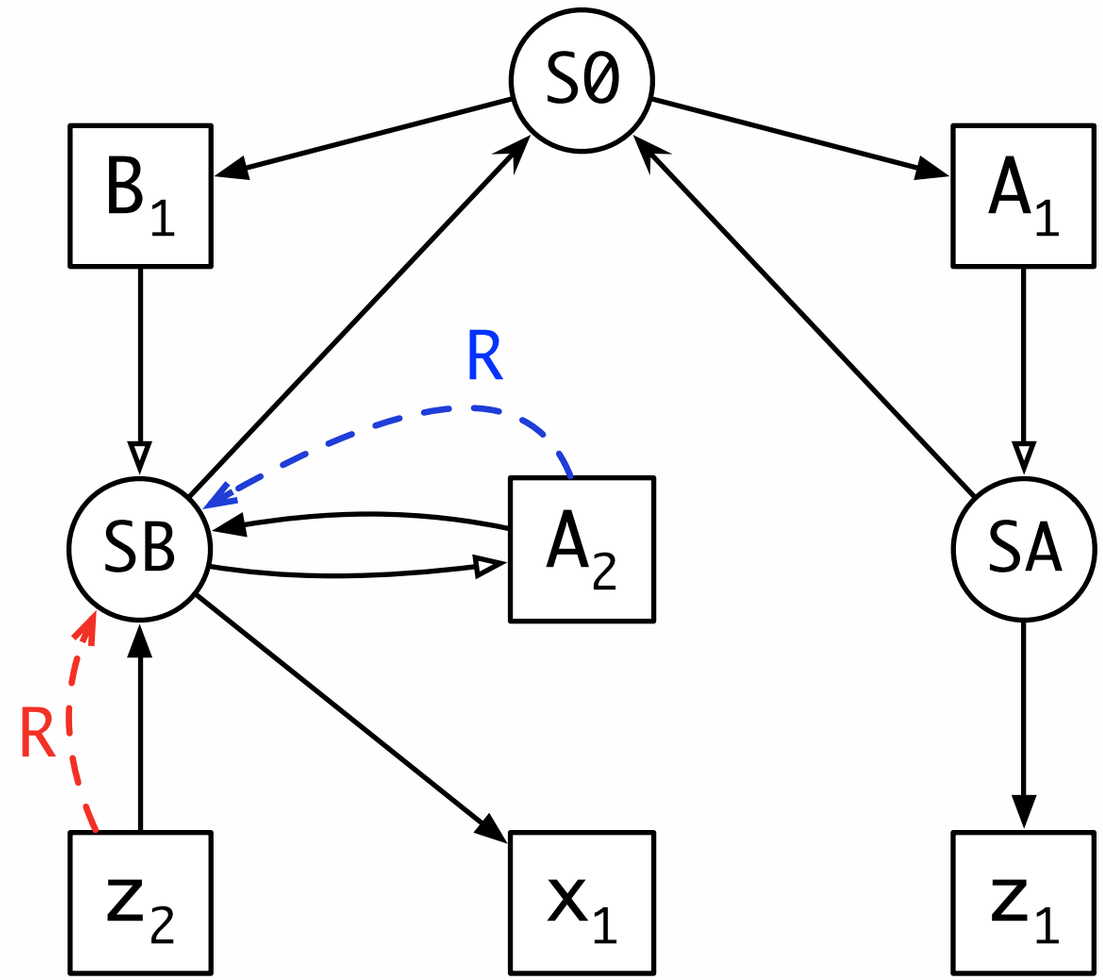

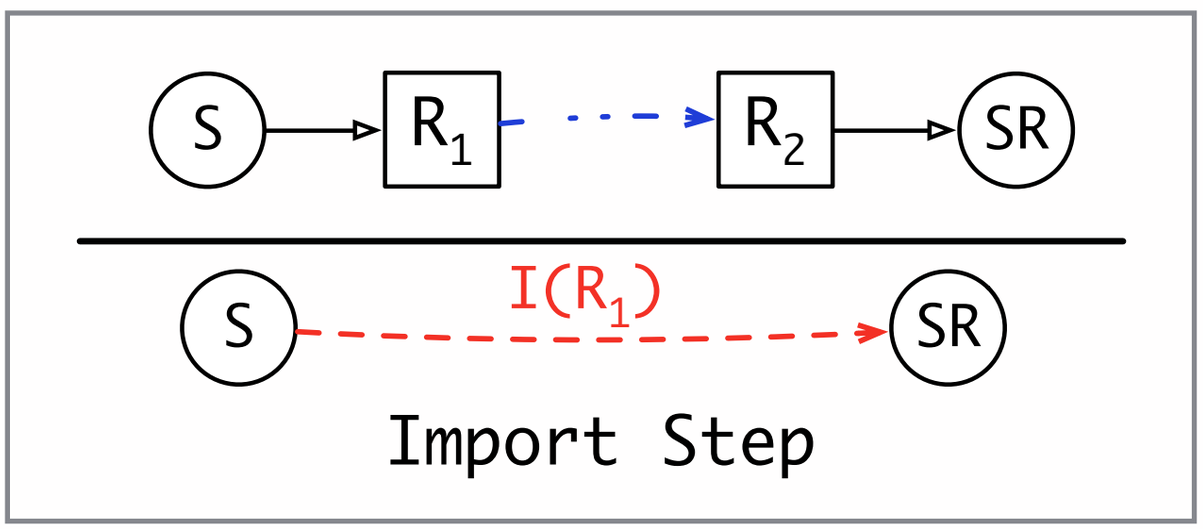

Imports

```
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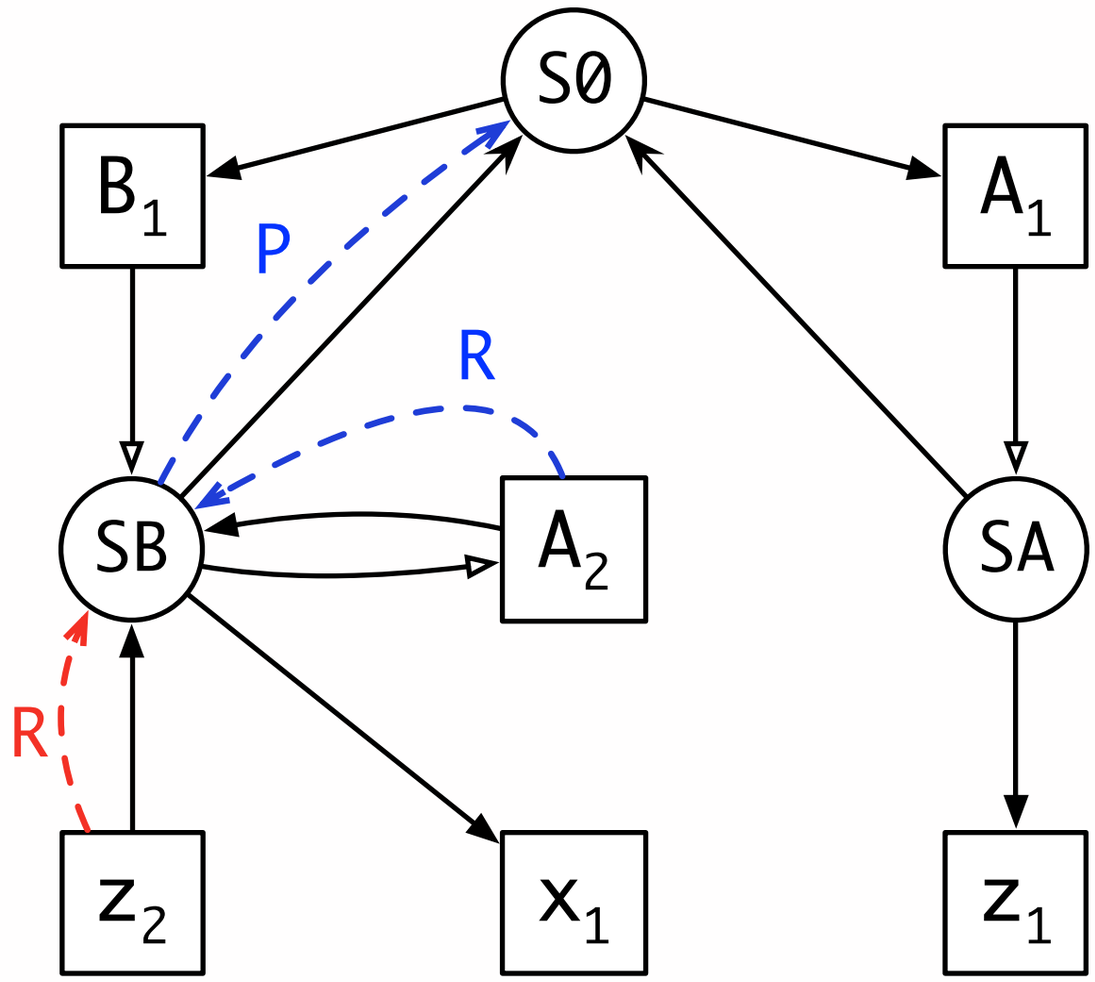
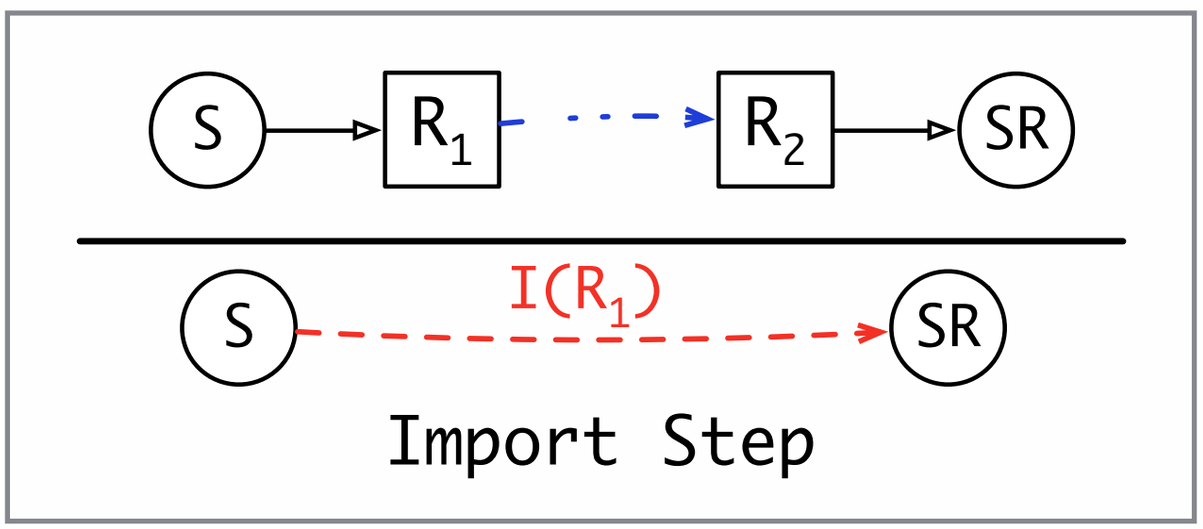
```





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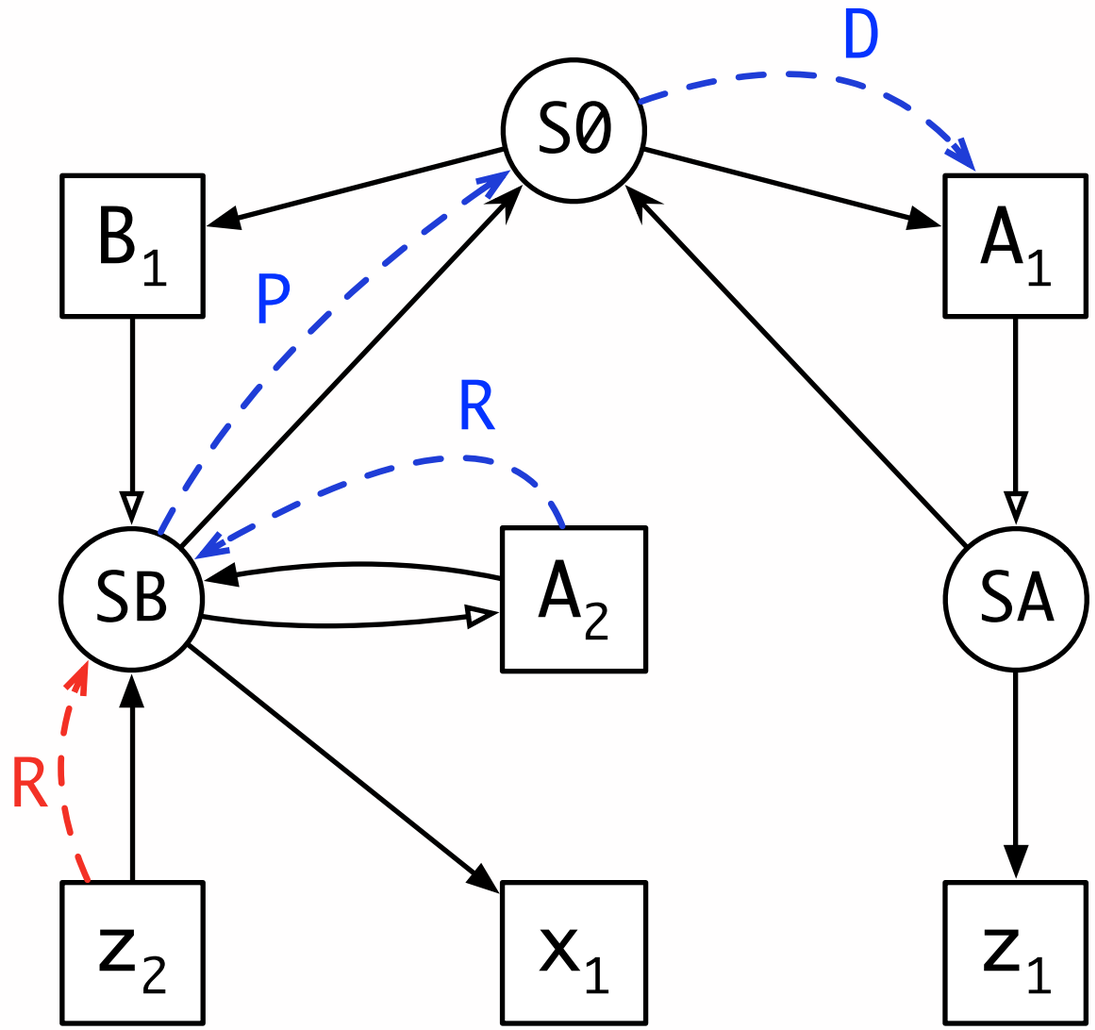

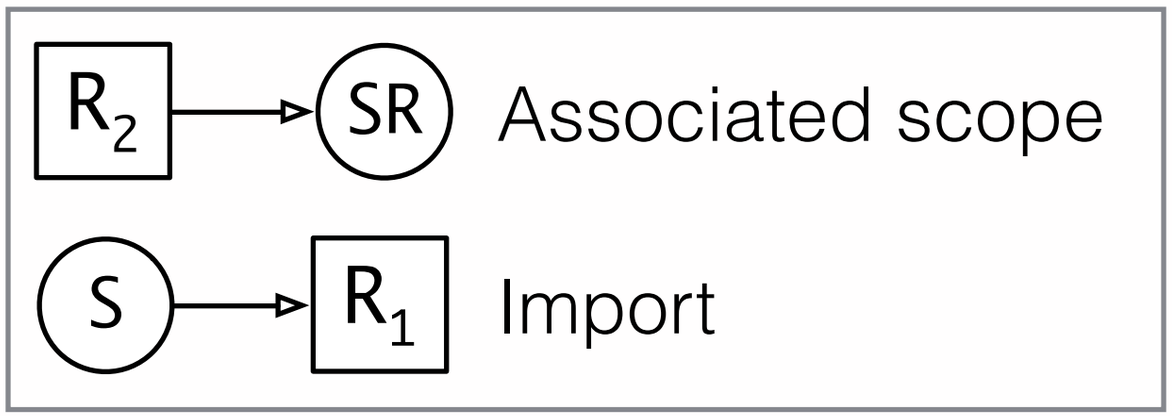

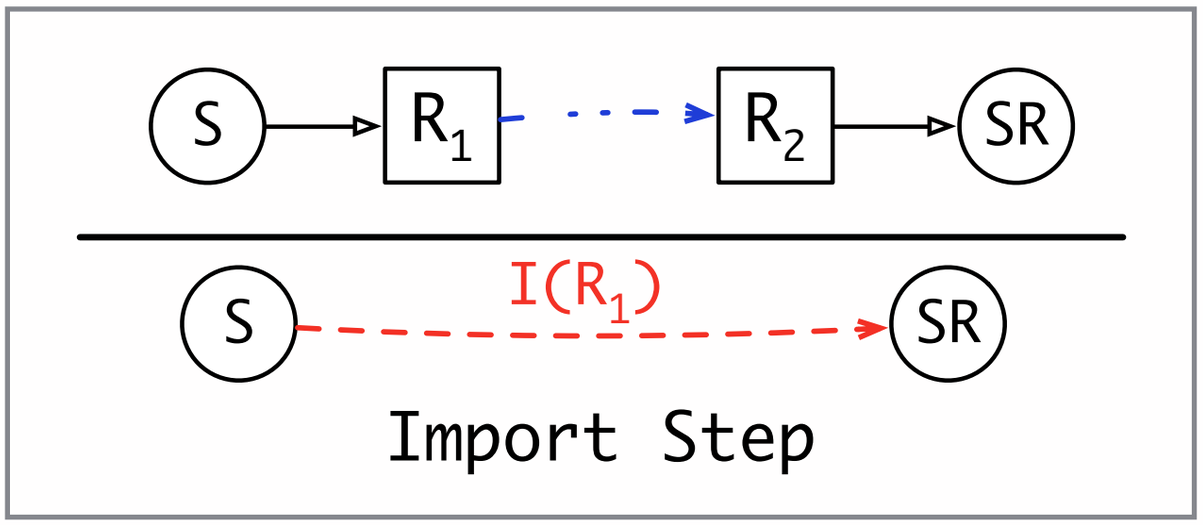

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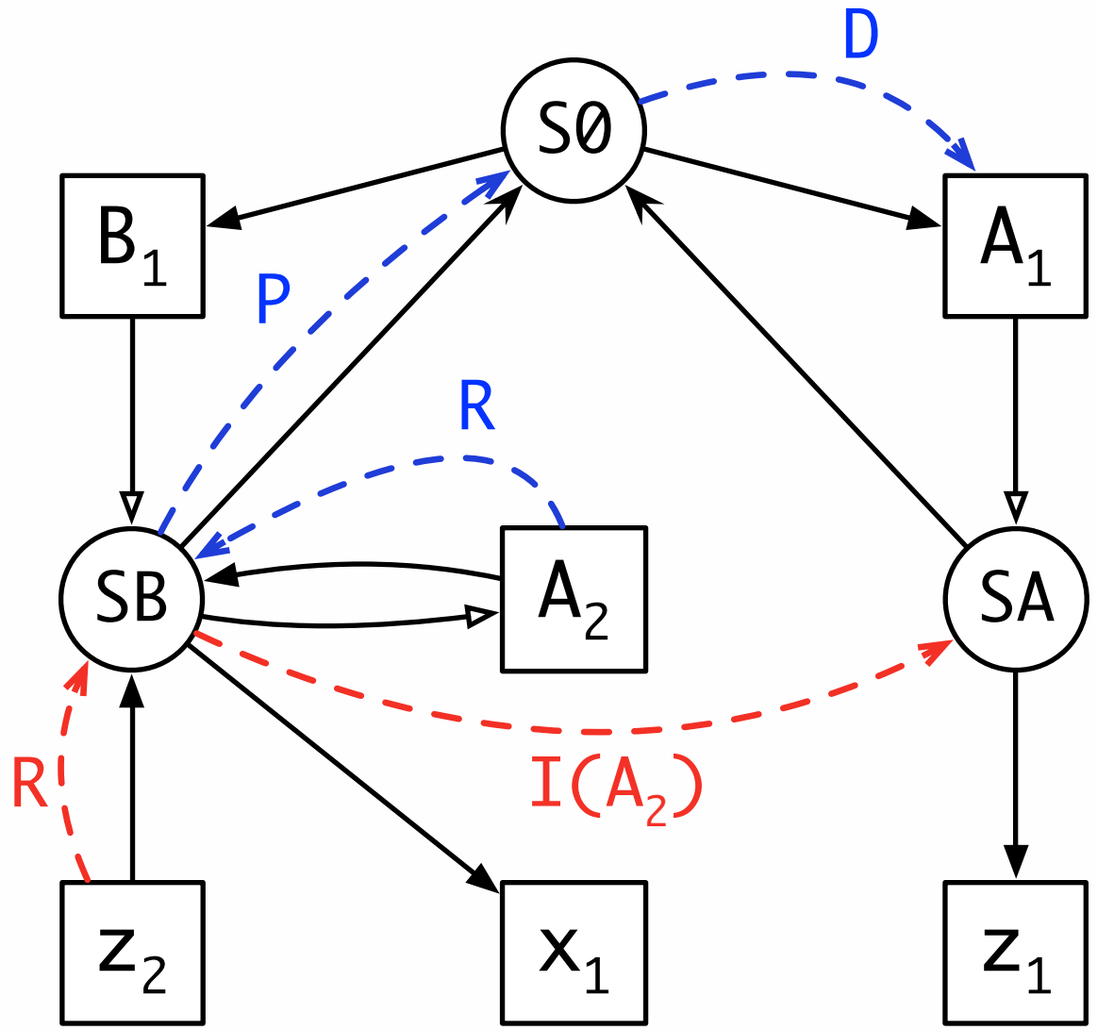

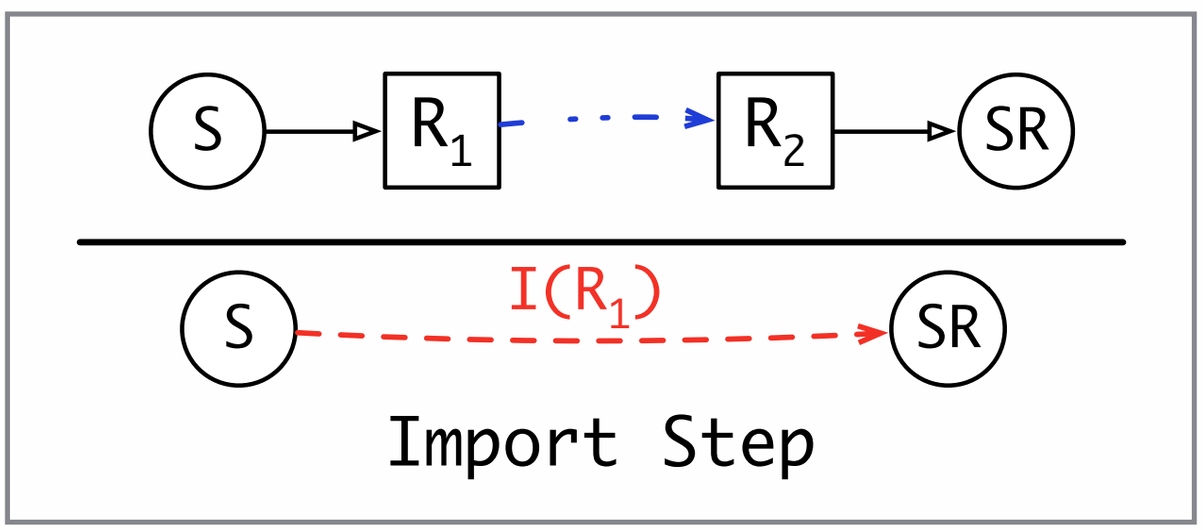

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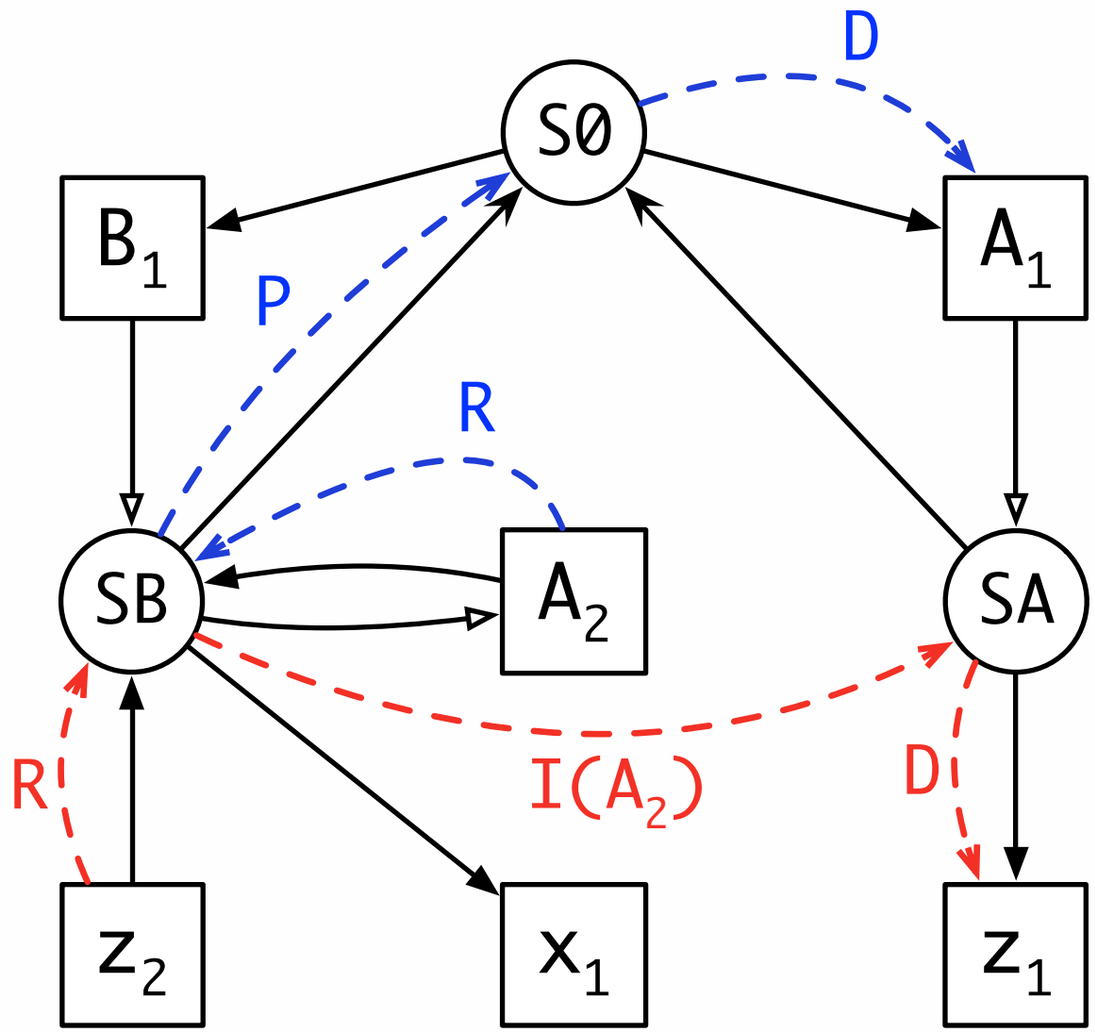
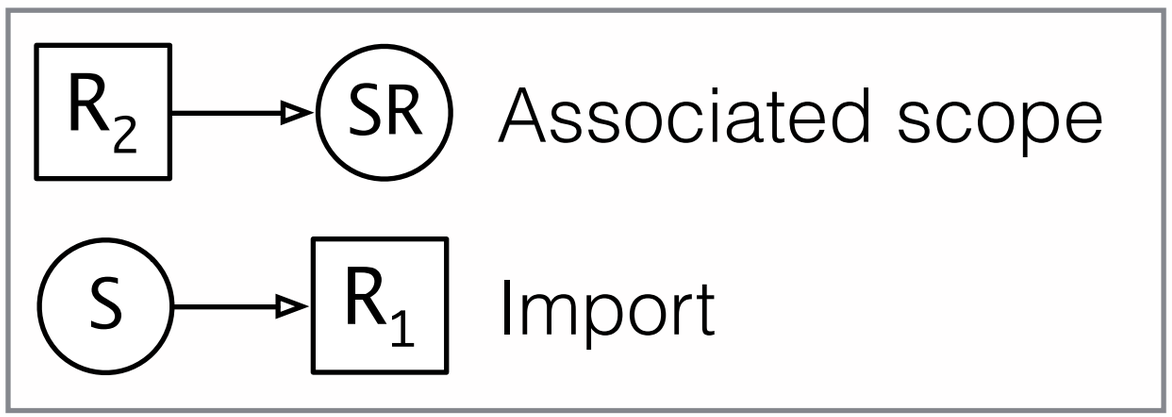
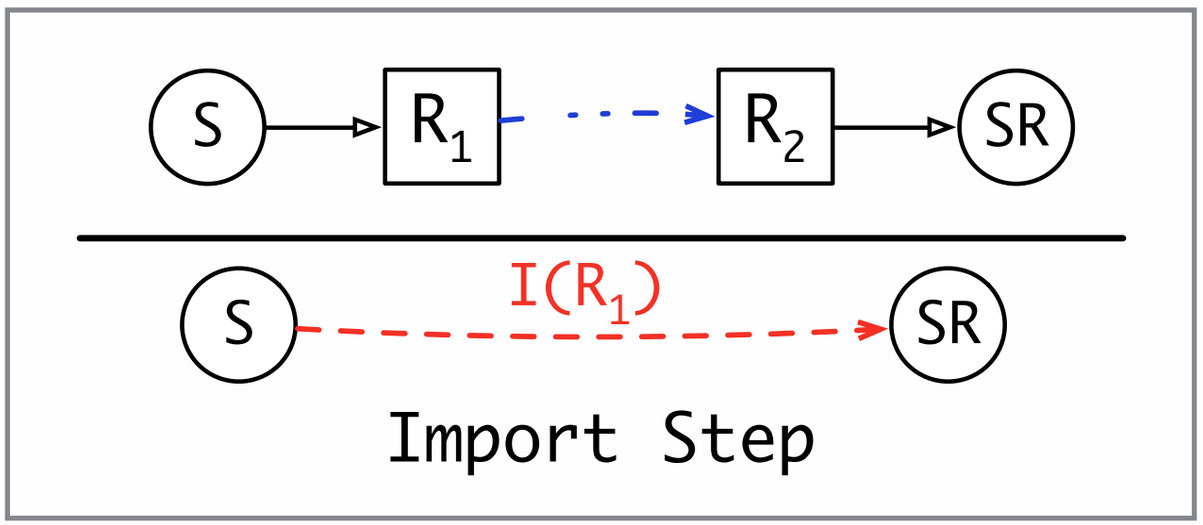
}
module B1 {
  import A2                    SB
  def x1 = 1 + z2
}
```







Imports shadow Parents

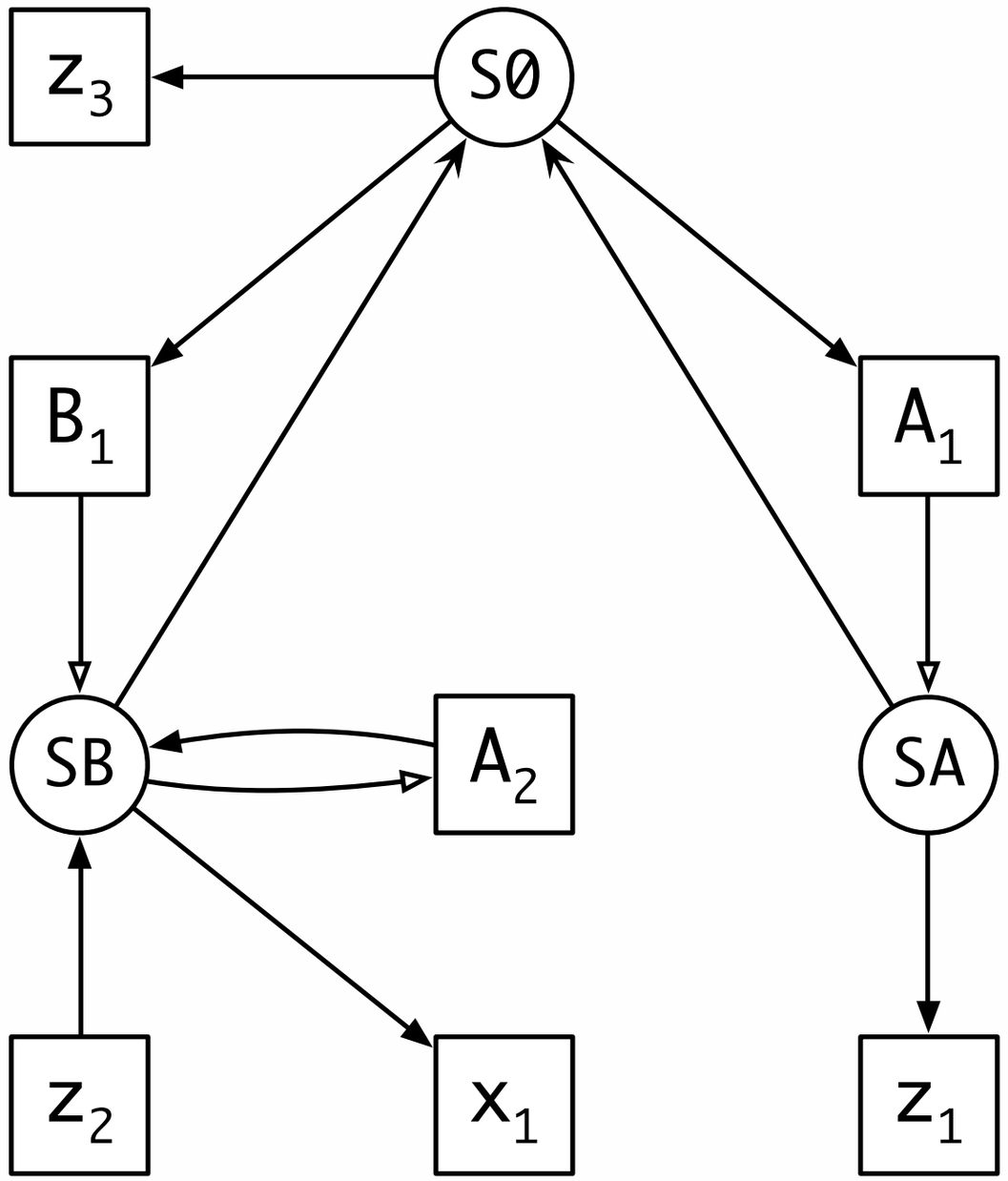
```
def z3 = 2                                  S0

module A1 {
    def z1 = 5                              SA
}
module B1 {
    import A2                               SB
    def x1 = 1 + z2
}
```

Imports shadow Parents

```
def z₃ = 2                                  S0

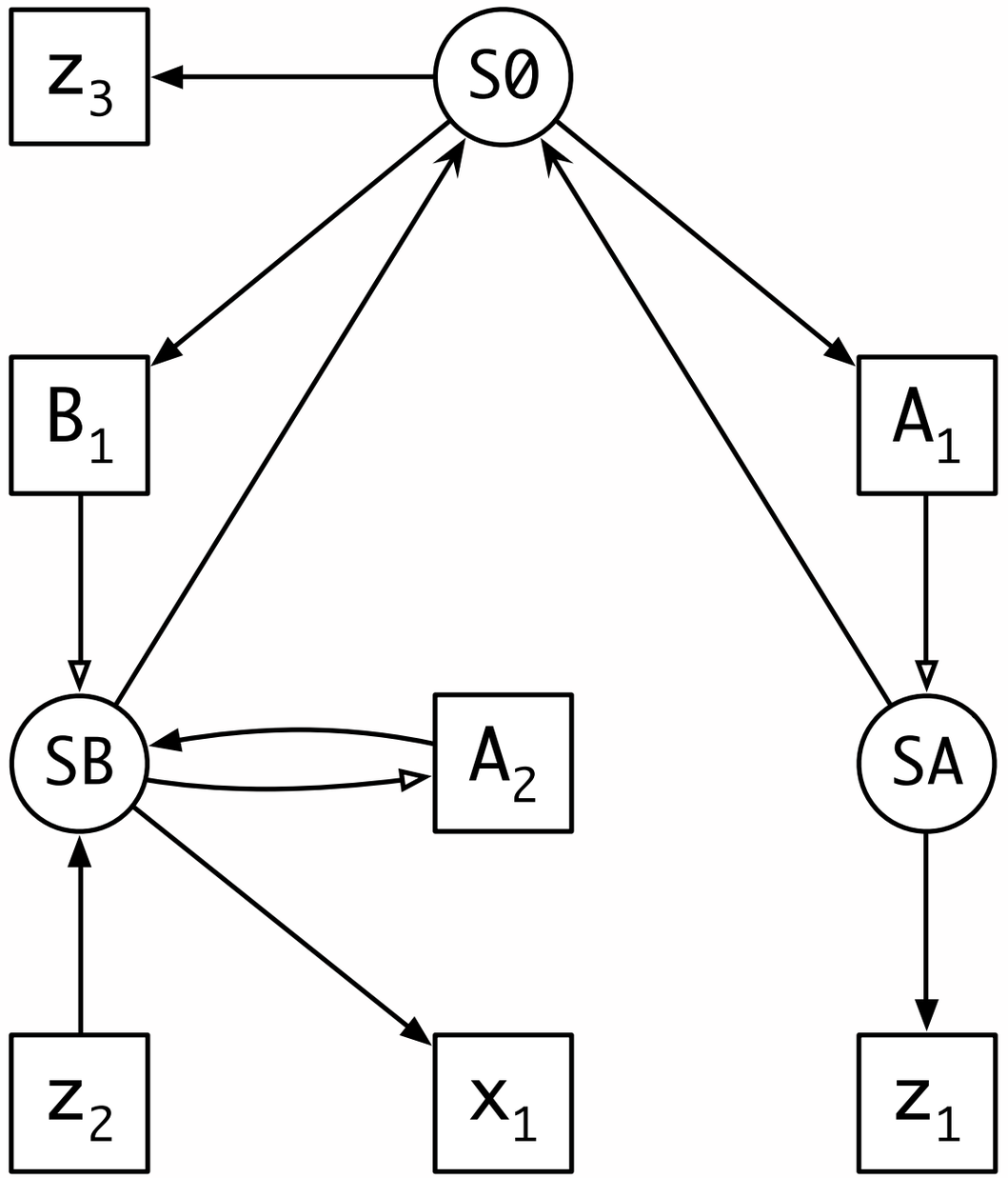
module A₁ {
    def z₁ = 5                              SA
}
module B₁ {
    import A₂                               SB
    def x₁ = 1 + z₂
}
```



Imports shadow Parents

```
def z₃ = 2                                  S0

module A₁ {
    def z₁ = 5                              SA
}
module B₁ {
    import A₂                               SB
    def x₁ = 1 + z₂
}
```



Imports shadow Parents

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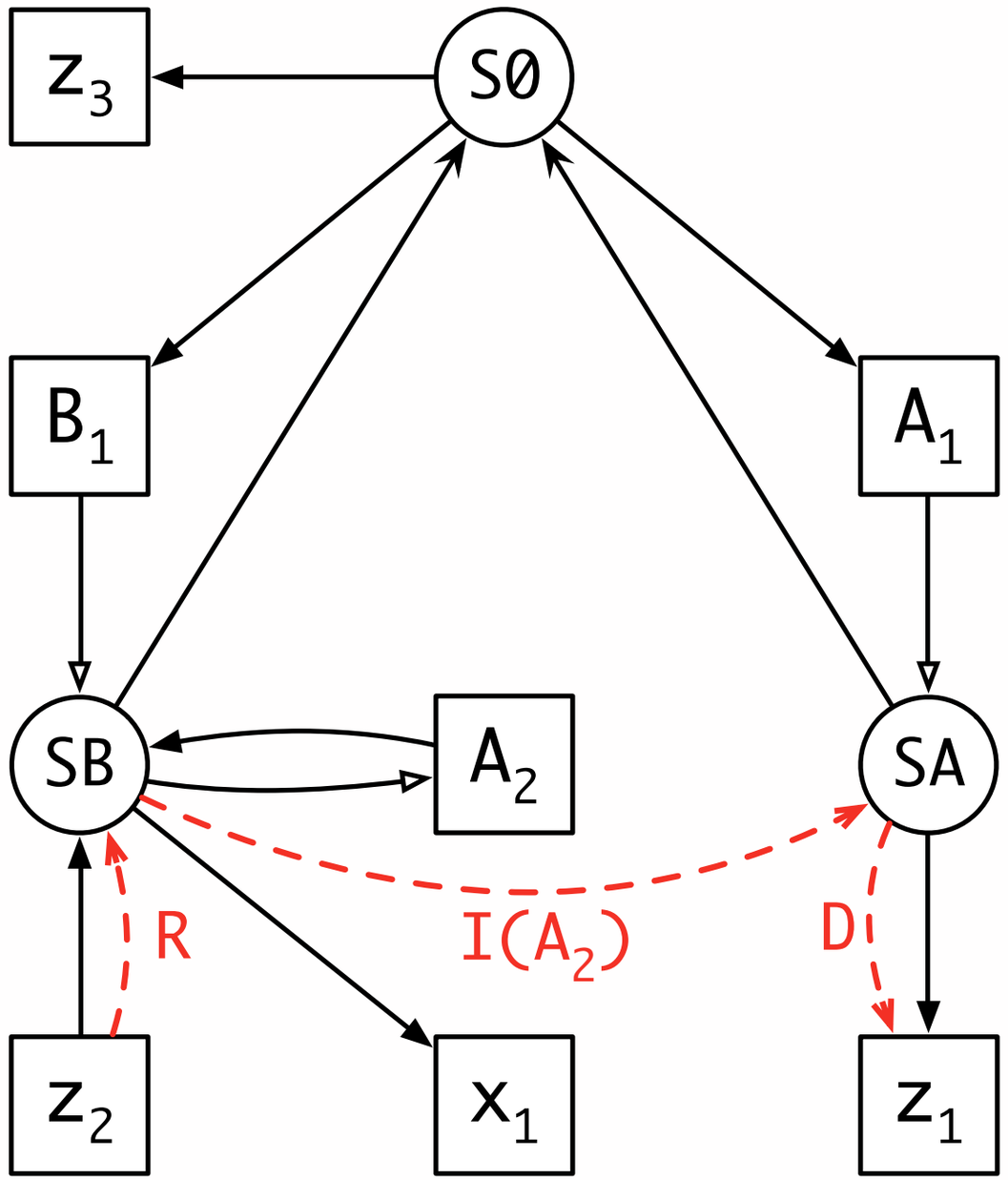
module A1 {
    def z1 = 5                              SA
}
module B1 {
    import A2                               SB
    def x1 = 1 + z2
}
```



Imports shadow Parents

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def z₃ = 2                                  S0

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}
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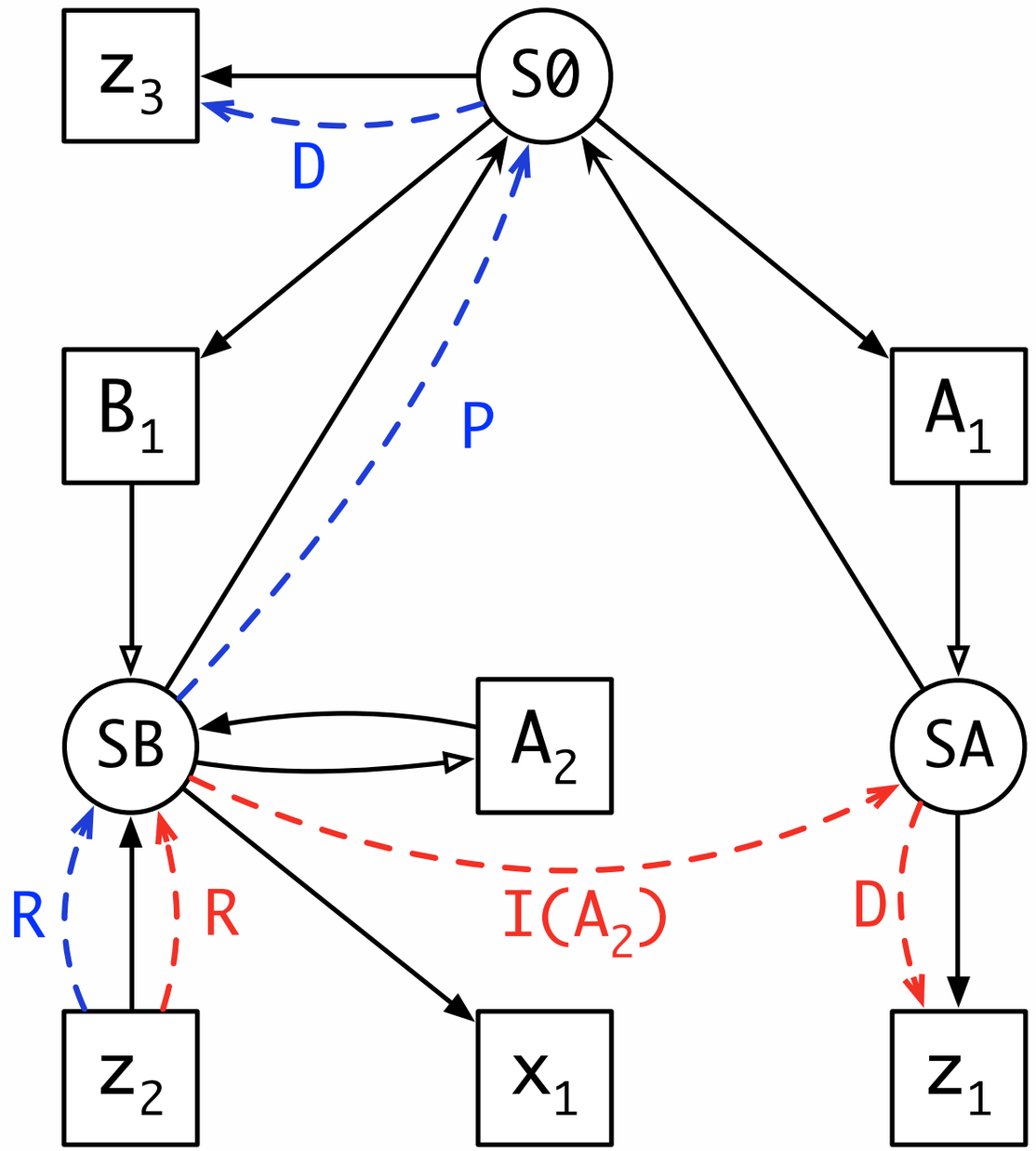
}
```



Imports shadow Parents

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module A1 {
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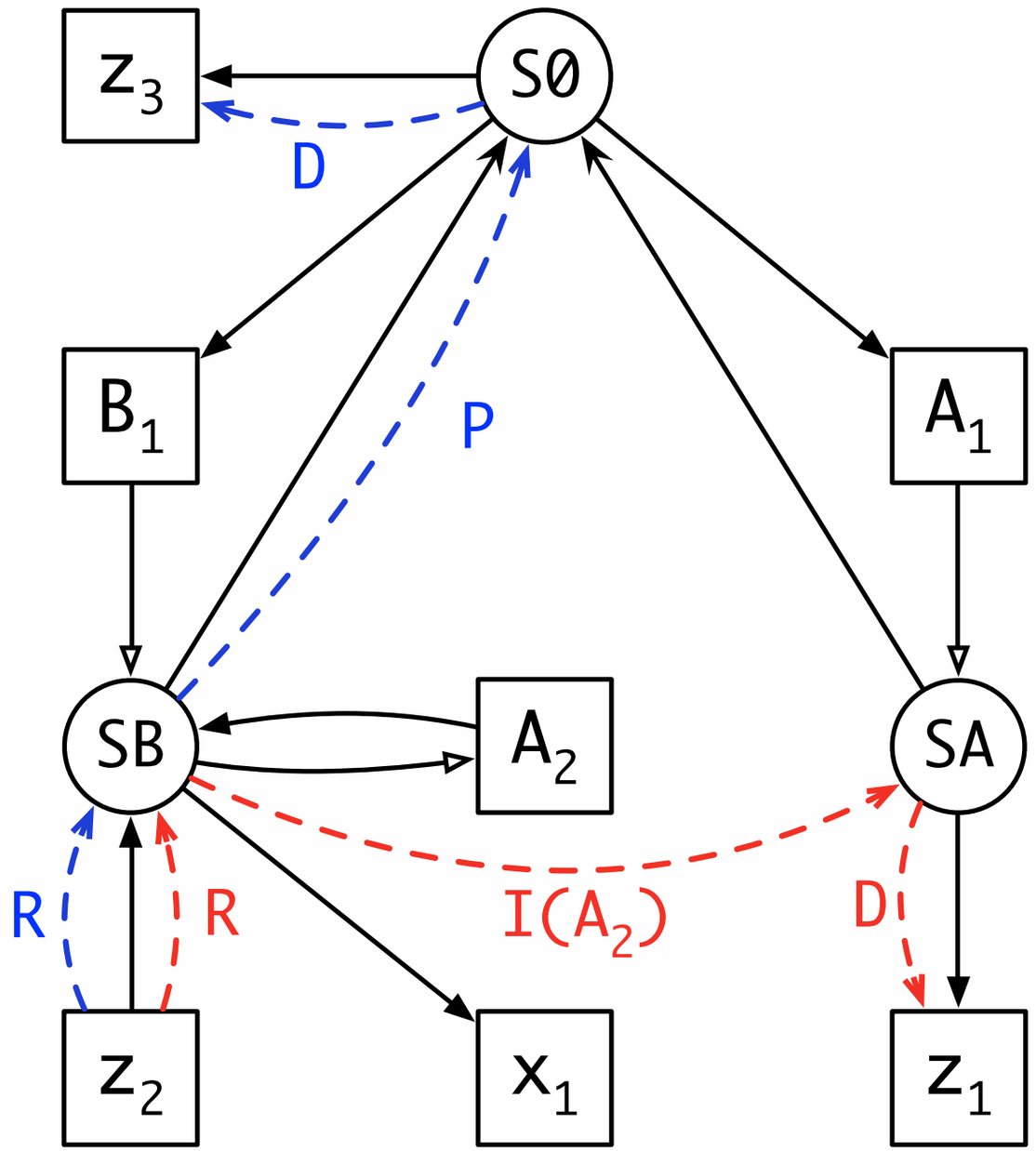

I(_).p' < P.p

Imports shadow Parents

```
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module A1 {
    def z1 = 5                              SA
}
module B1 {
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    def x1 = 1 + z2
}
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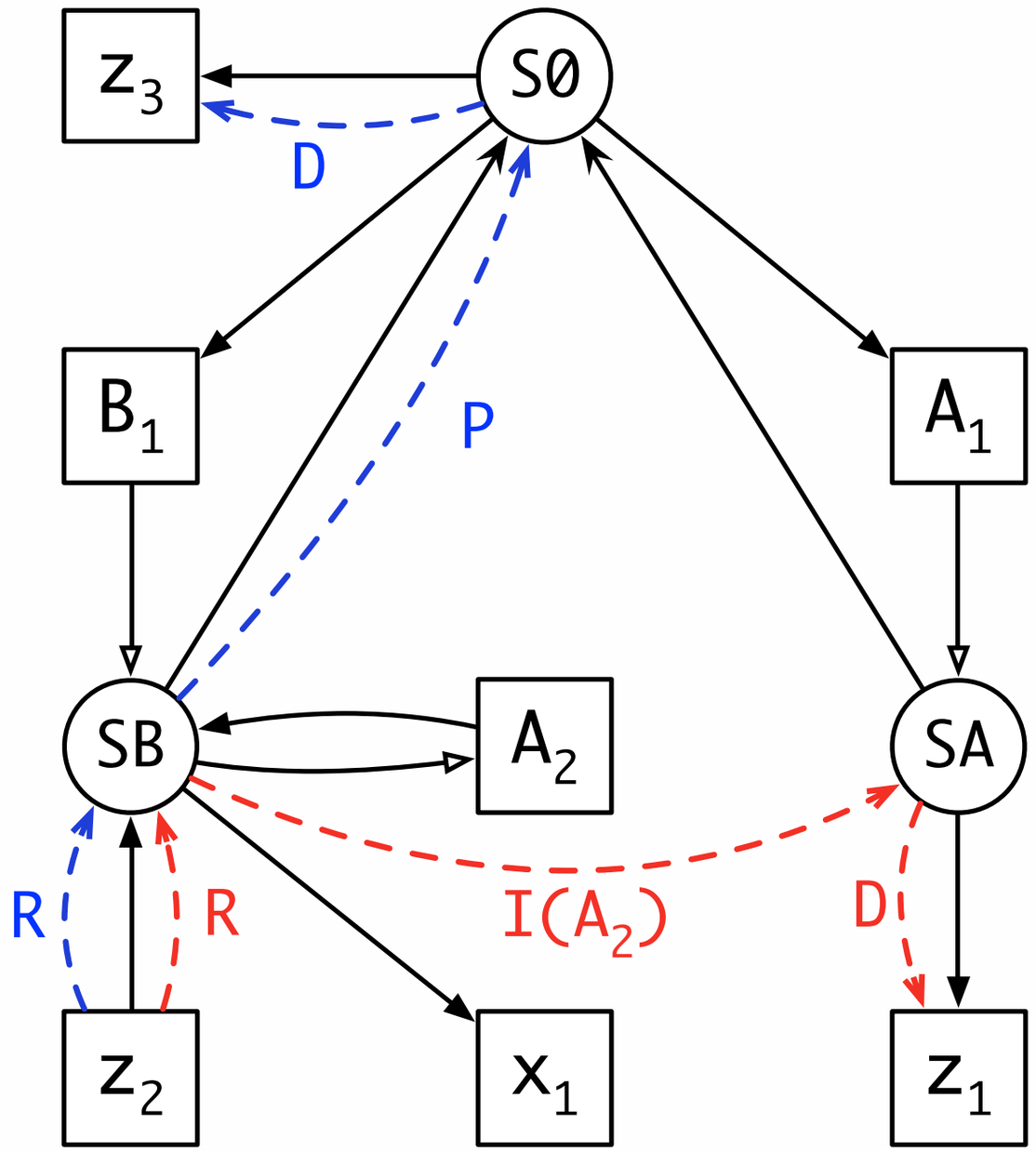
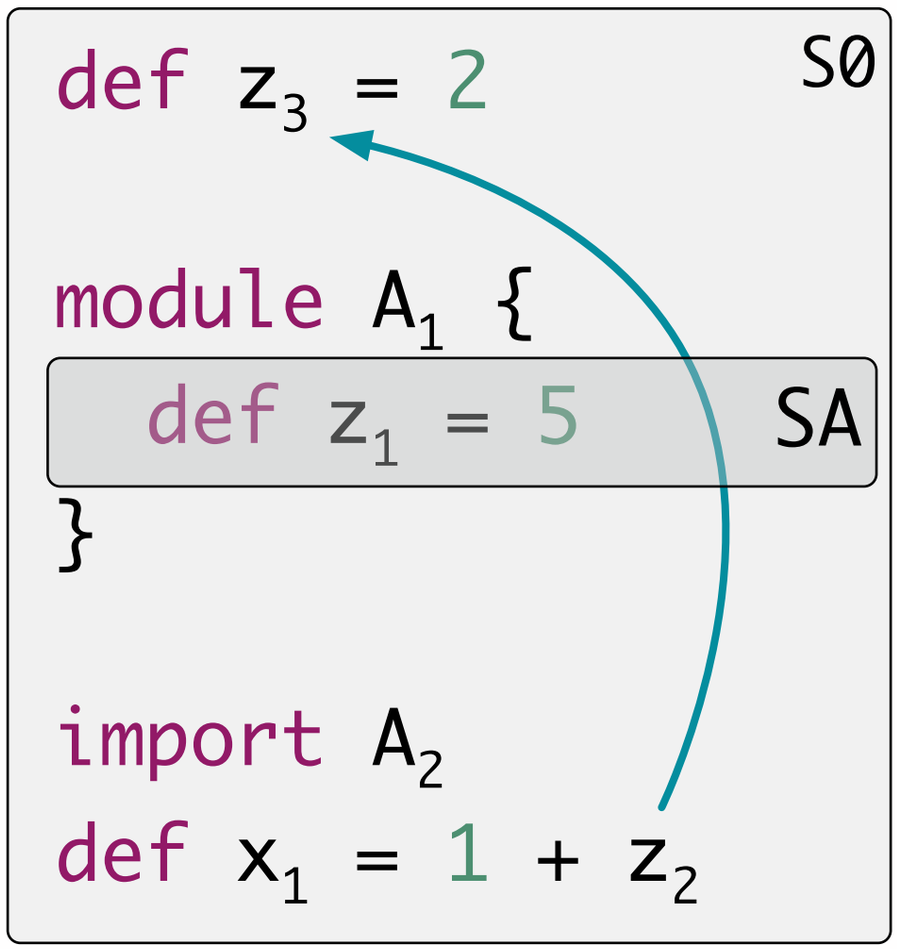


$$\frac{}{I(\_).p' < P.p} \quad \Rightarrow \quad R.I(A_2).D < R.P.D$$

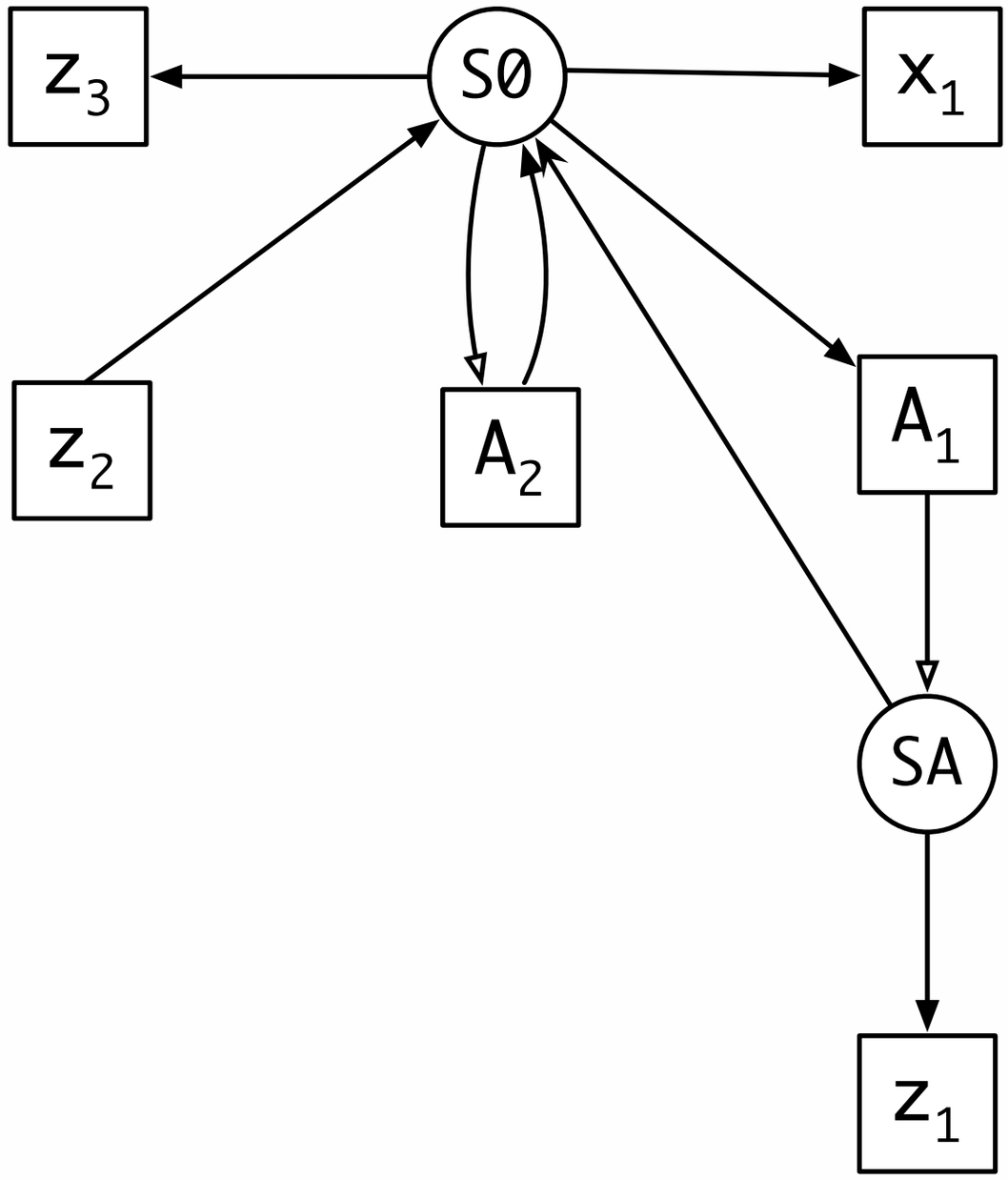
Imports vs. Includes



Imports vs. Includes

```
                                    S0
def z3 = 2
module A1 {
  def z1 = 5                        SA
}

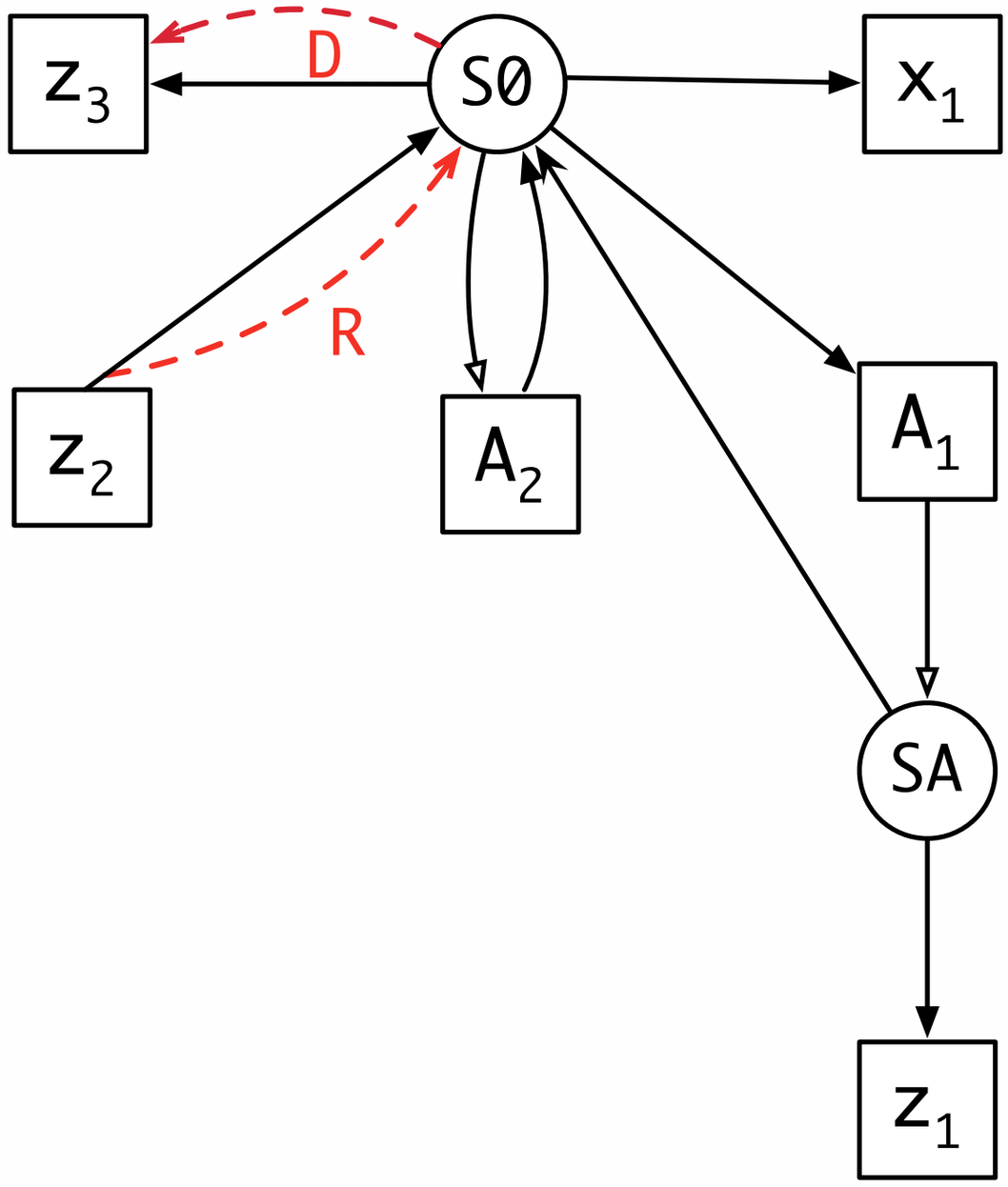
import A2
def x1 = 1 + z2
```



Imports vs. Includes

```
                                  S0
def z3 = 2
module A1 {
  def z1 = 5                      SA
}

import A2
def x1 = 1 + z2
```



Imports vs. Includes

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def z3 = 2                          S0

module A1 {
  def z1 = 5                        SA
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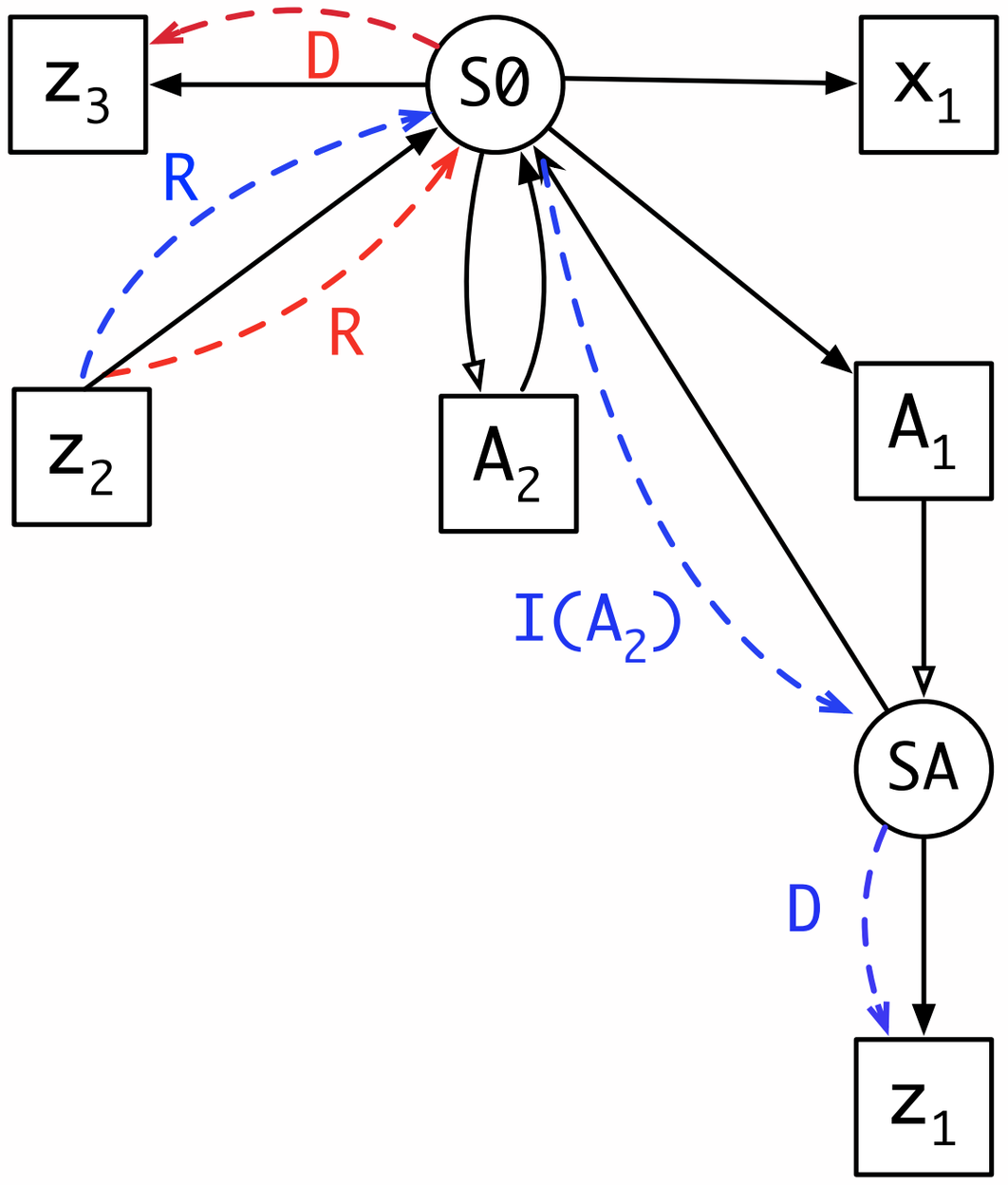
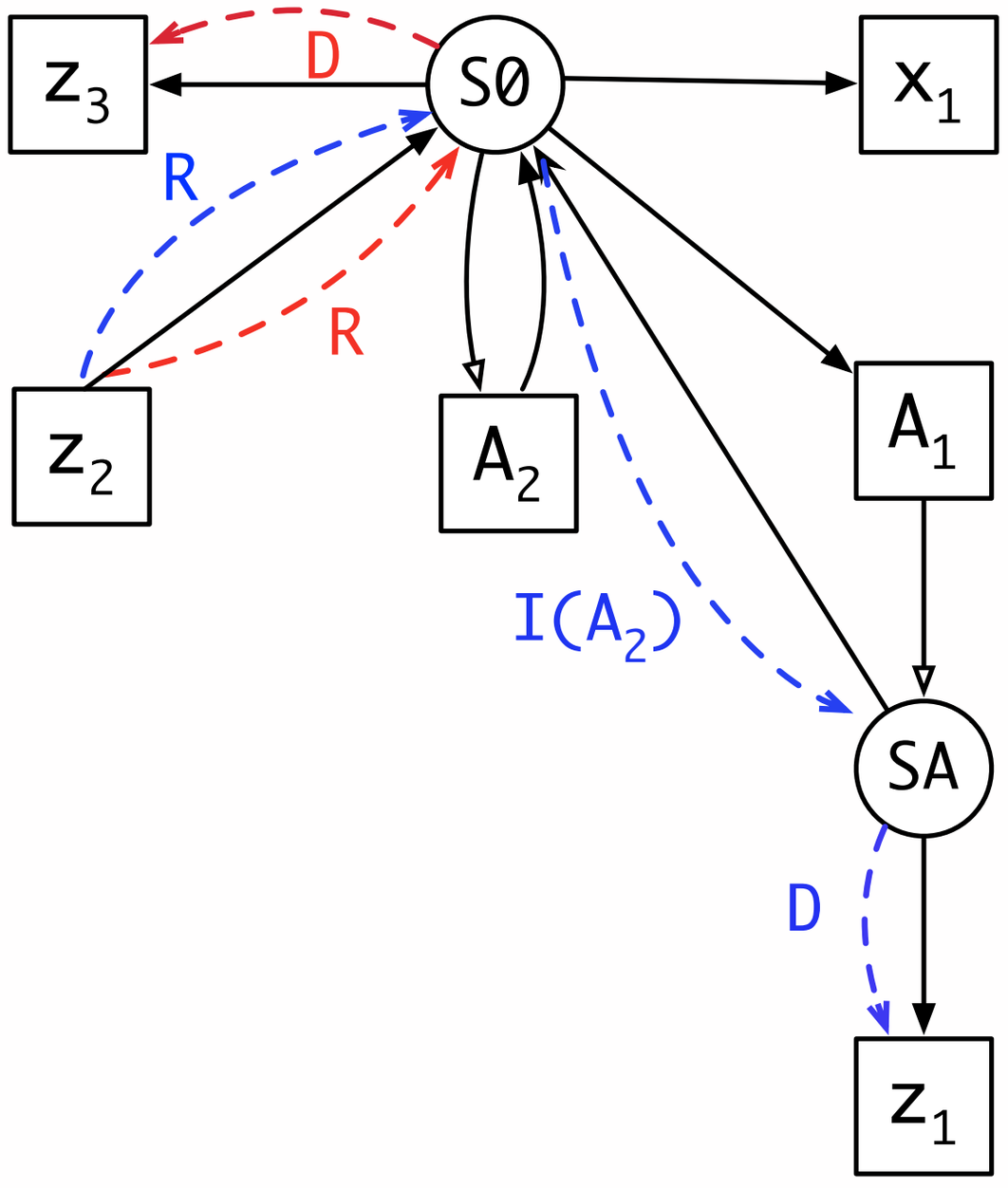
import A2
def x1 = 1 + z2
```



Imports vs. Includes

```
def z3 = 2                          S0
module A1 {
  def z1 = 5                        SA
}

import A2
def x1 = 1 + z2
```

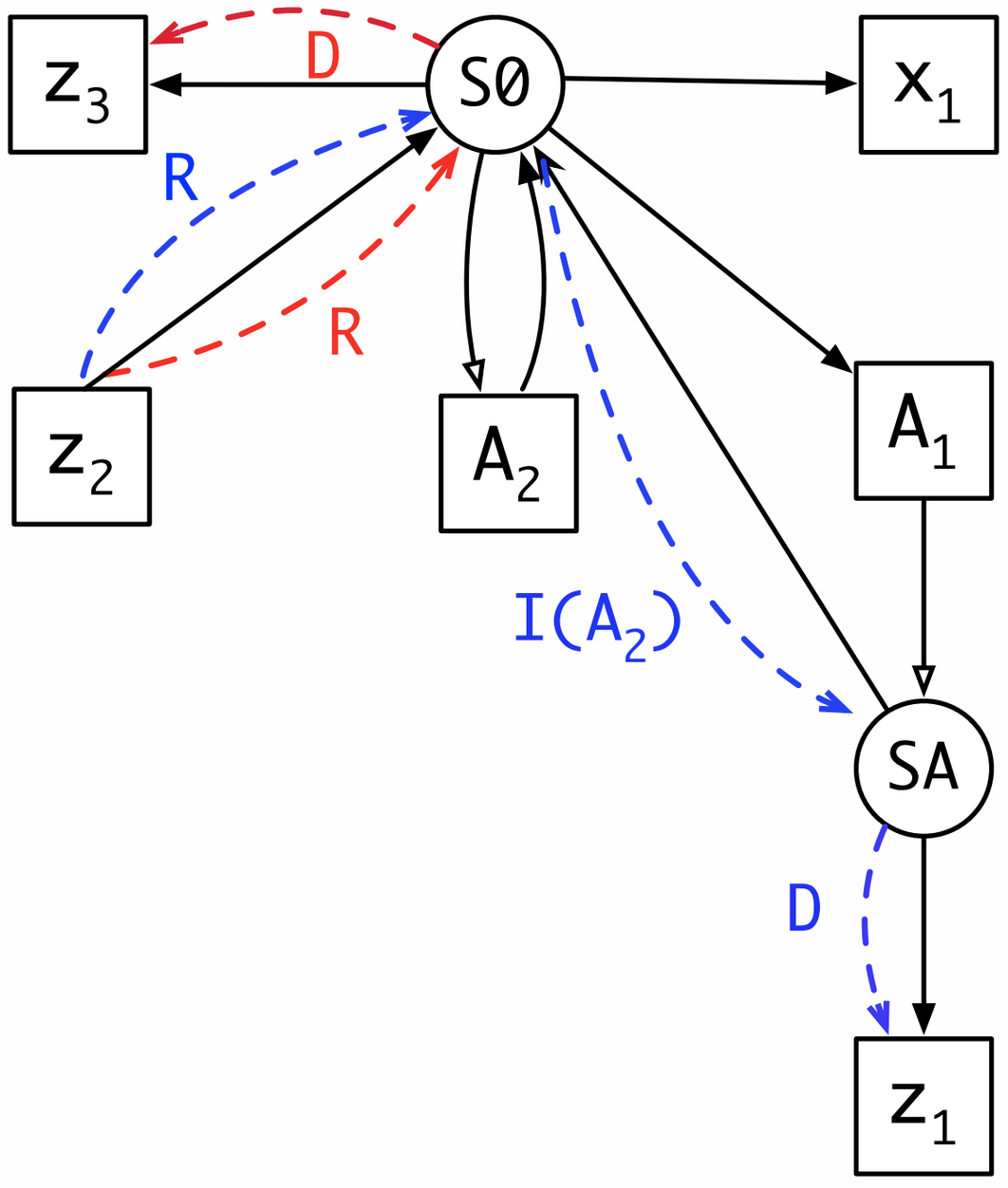


D < I(_).p’

Imports vs. Includes

```
def z3 = 2                          S0
module A1 {
  def z1 = 5                        SA
}

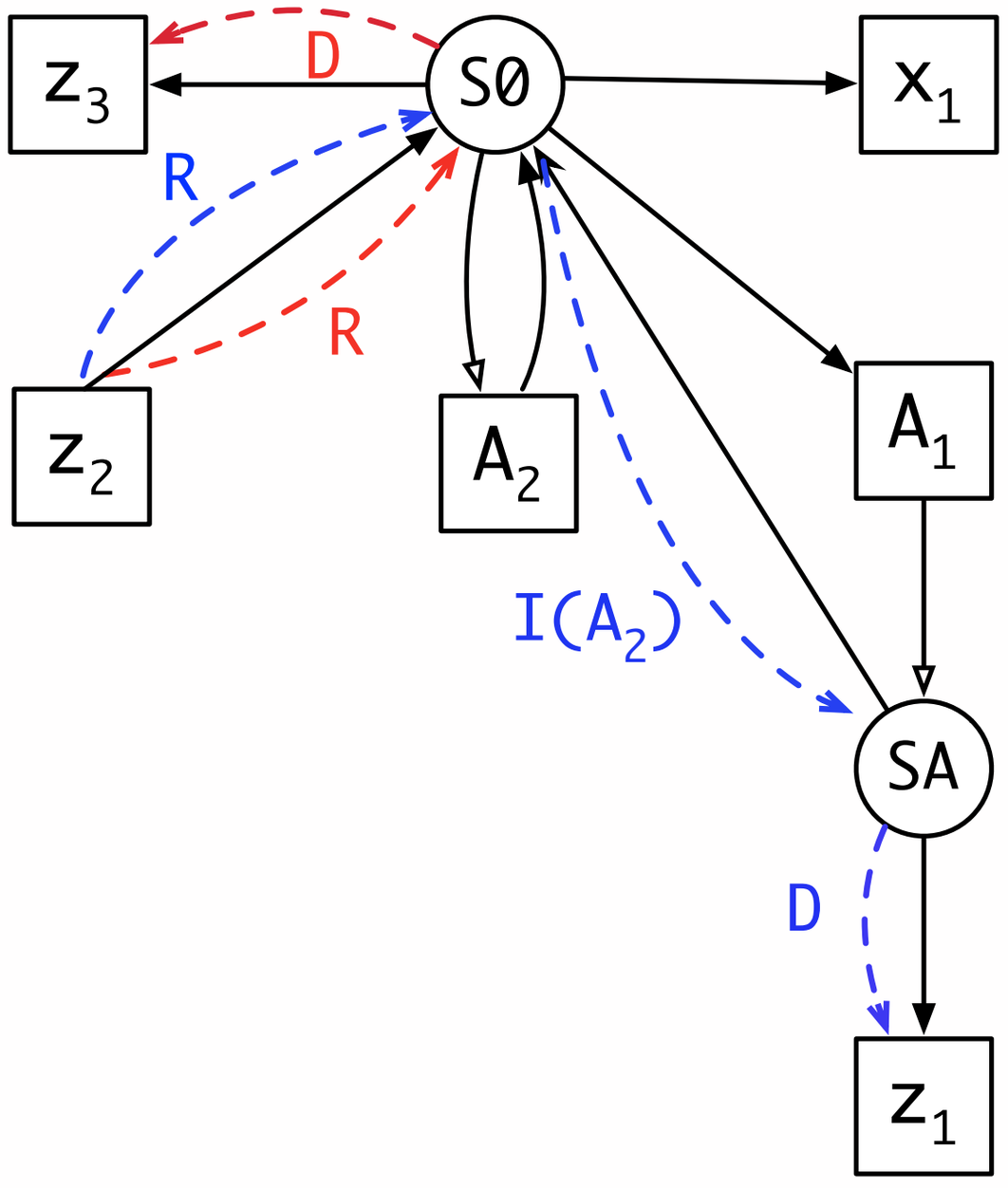
import A2
def x1 = 1 + z2
```



$$\frac{}{\texttt{D} < \texttt{I(\_).p'}} \quad \Rightarrow \quad \texttt{R.D} < \texttt{R.I}(A_2)\texttt{.D}$$

Imports vs. Includes

```
                                  S0
def z3 = 2
module A1 {
  def z1 = 5                      SA
}
include A2
def x1 = 1 + z2
```



$$\frac{}{D < I(\_).p'} \quad \Rightarrow \quad R.D < R.I(A_2).D$$

Imports vs. Includes

```
def z3 = 2                          S0
module A1 {
  def z1 = 5                        SA
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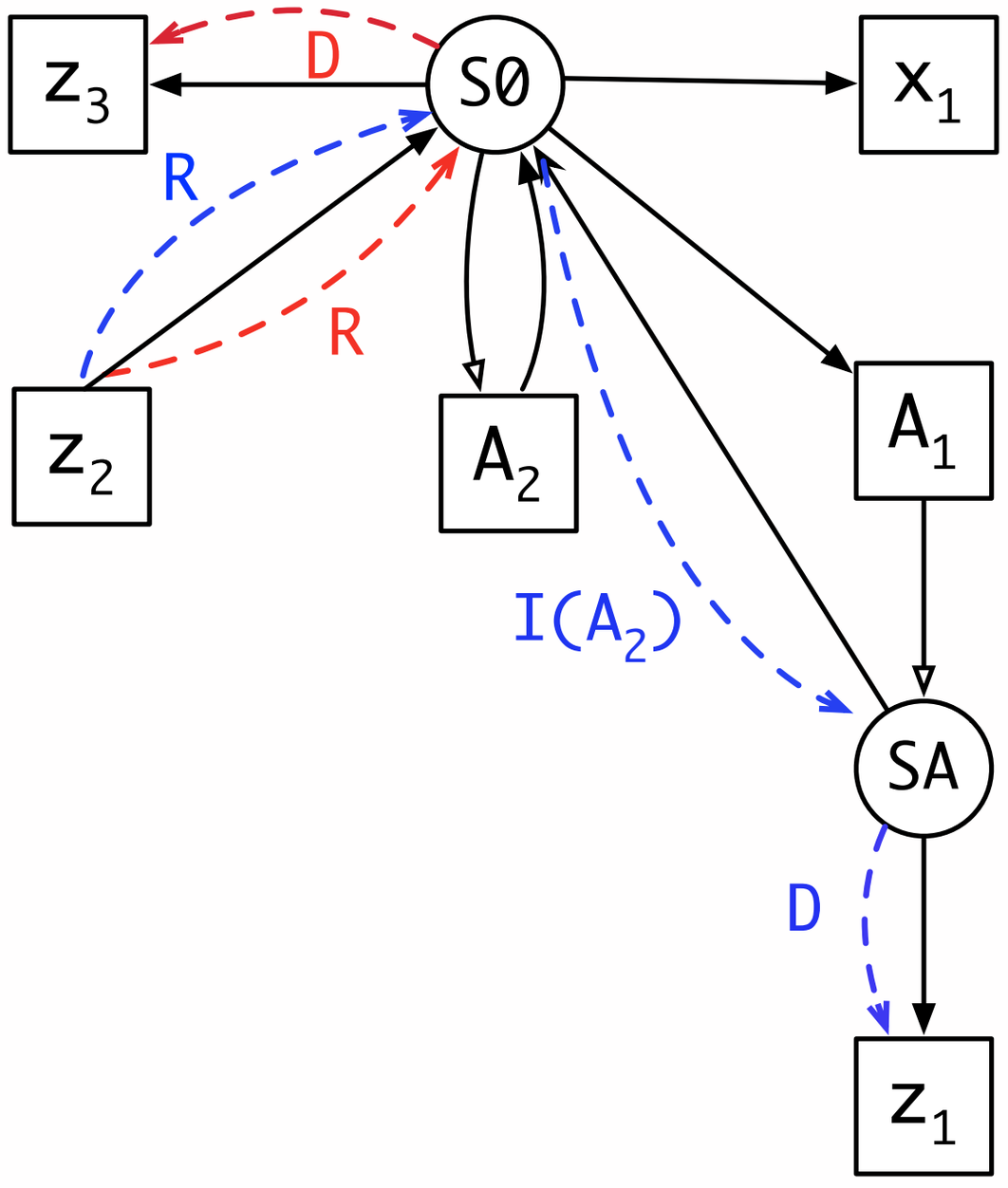
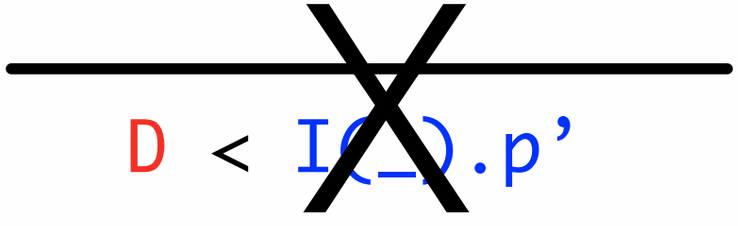
include A2
def x1 = 1 + z2
```





$\Rightarrow$ $R.D < R.I(A_2).D$

Imports vs. Includes

```
def z3 = 2                          S0
module A1 {
  def z1 = 5                        SA
}
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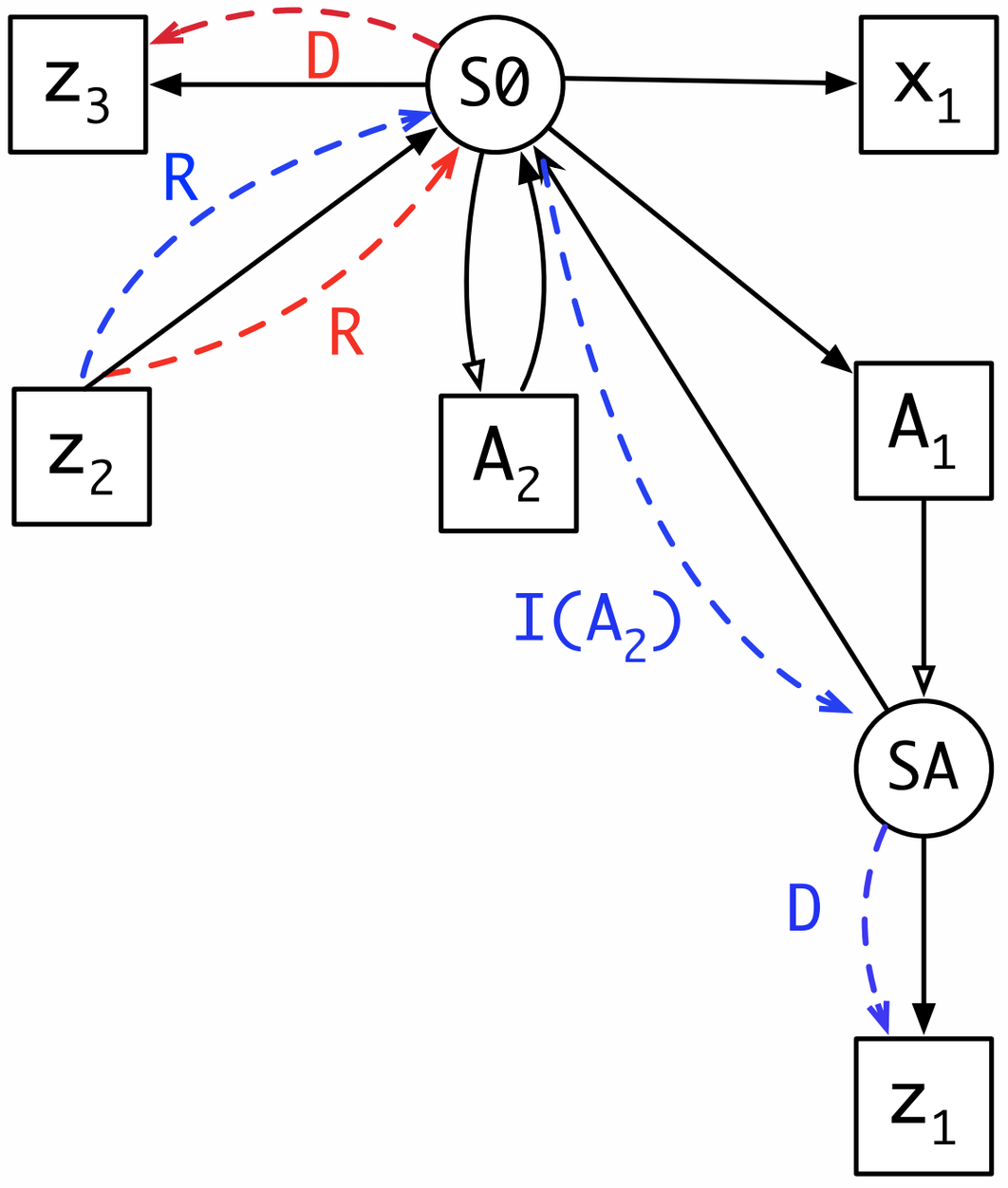
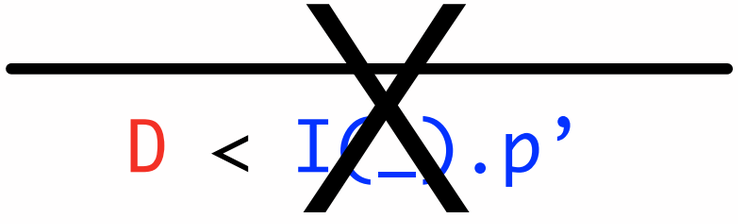
def x1 = 1 + z2
```





Qualified Names

S0

```
module N₁ {
    def s₁ = 5
}

module M₁ {
    def x₁ = 1 + N₂.s₂
}
```

Qualified Names

S0

```
module N1 {
    def s1 = 5
}

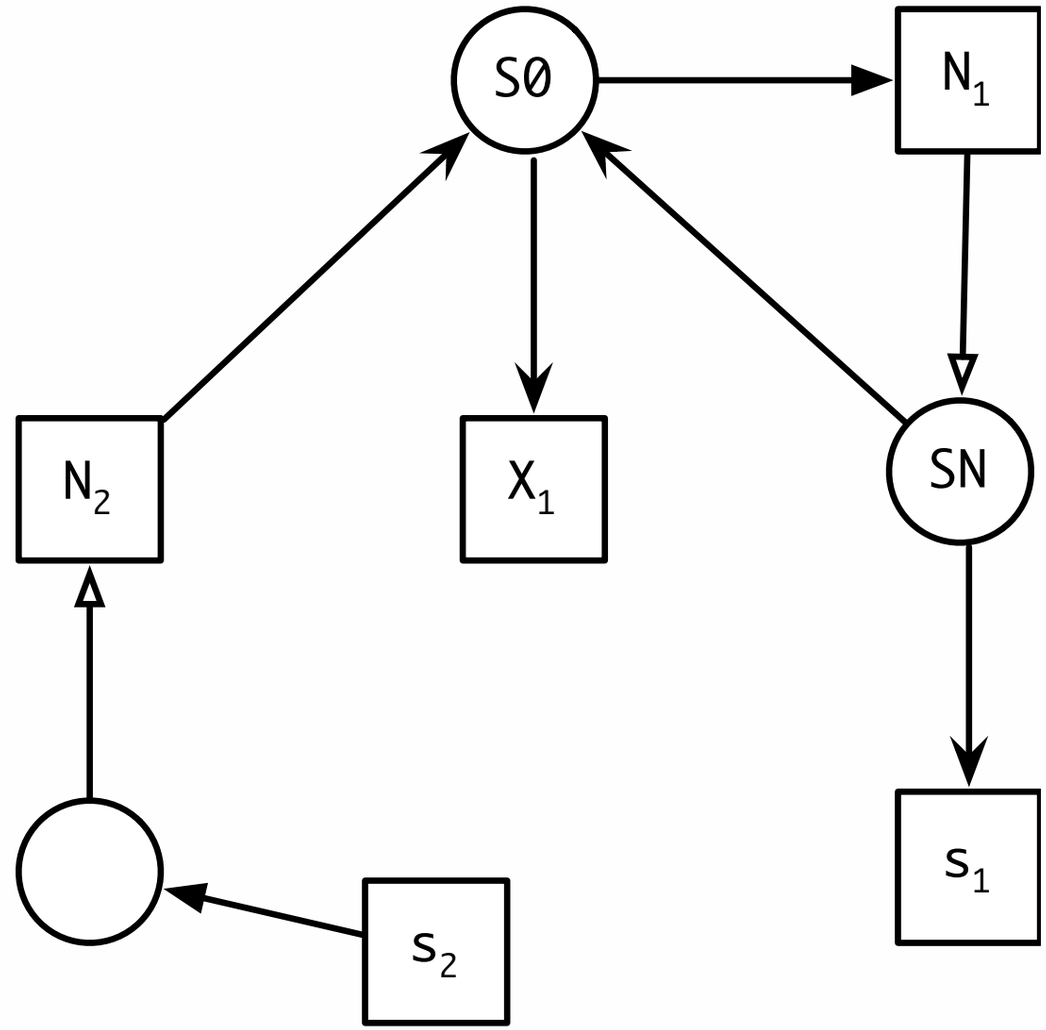
module M1 {
    def x1 = 1 + N2.s2
}
```

Qualified Names

S0

```
module N₁ {
    def s₁ = 5
}

module M₁ {
    def x₁ = 1 + N₂.s₂
}
```



Qualified Names

S0

```
module N1 {
    def s1 = 5
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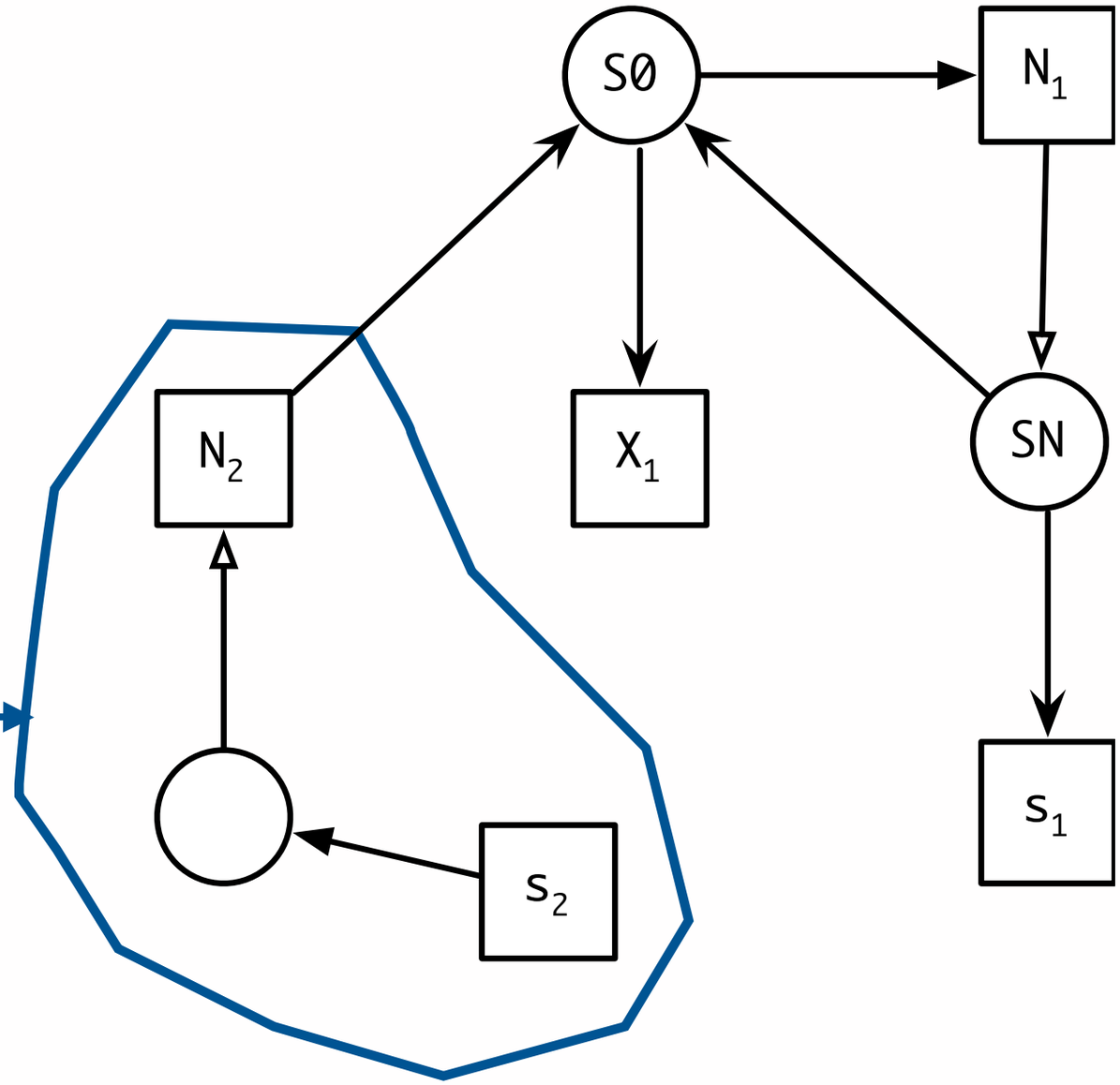
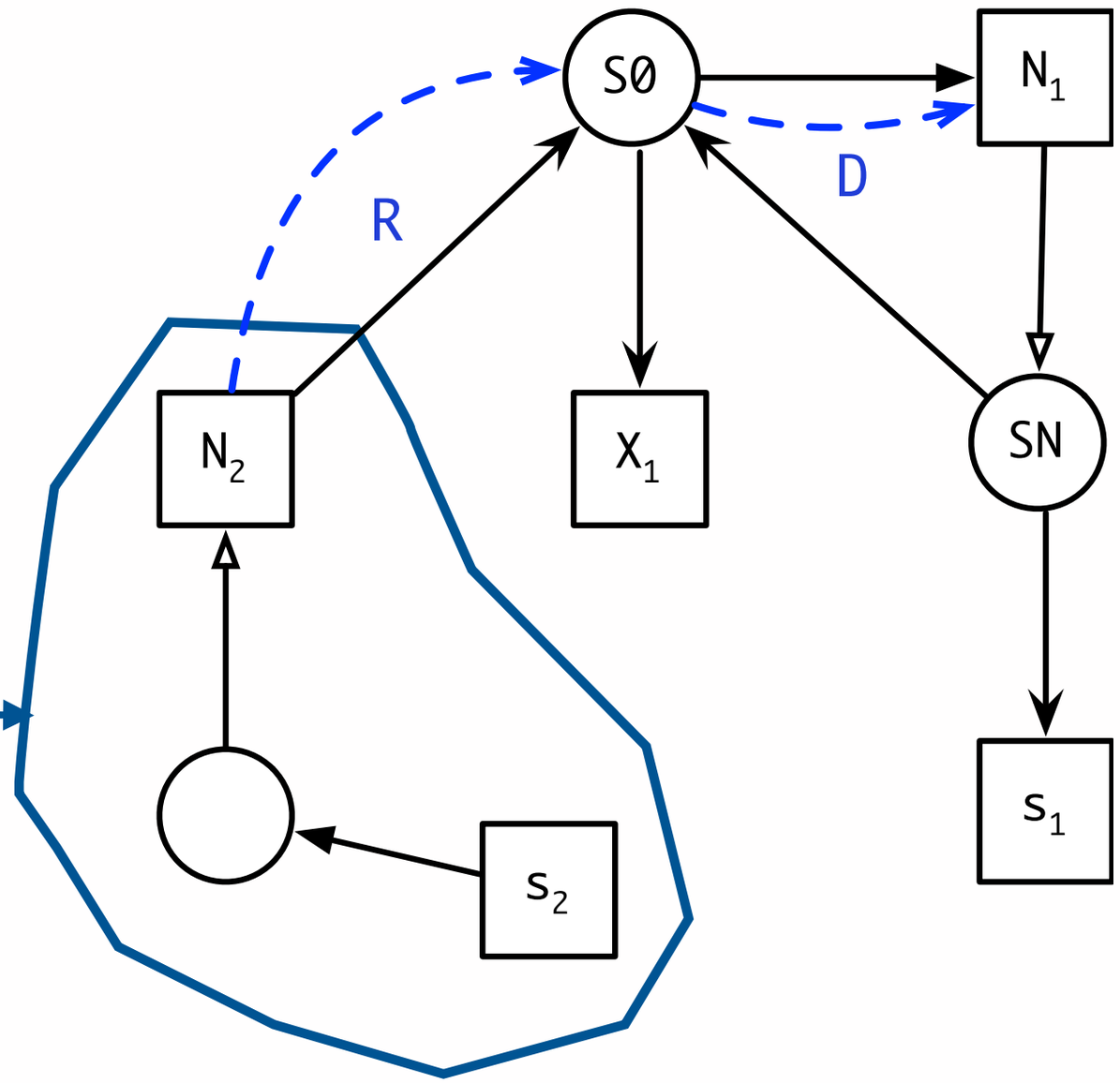
module M1 {
    def x1 = 1 + N2.s2
}
```



Qualified Names

```
module N1 {                    S0
    def s1 = 5
}

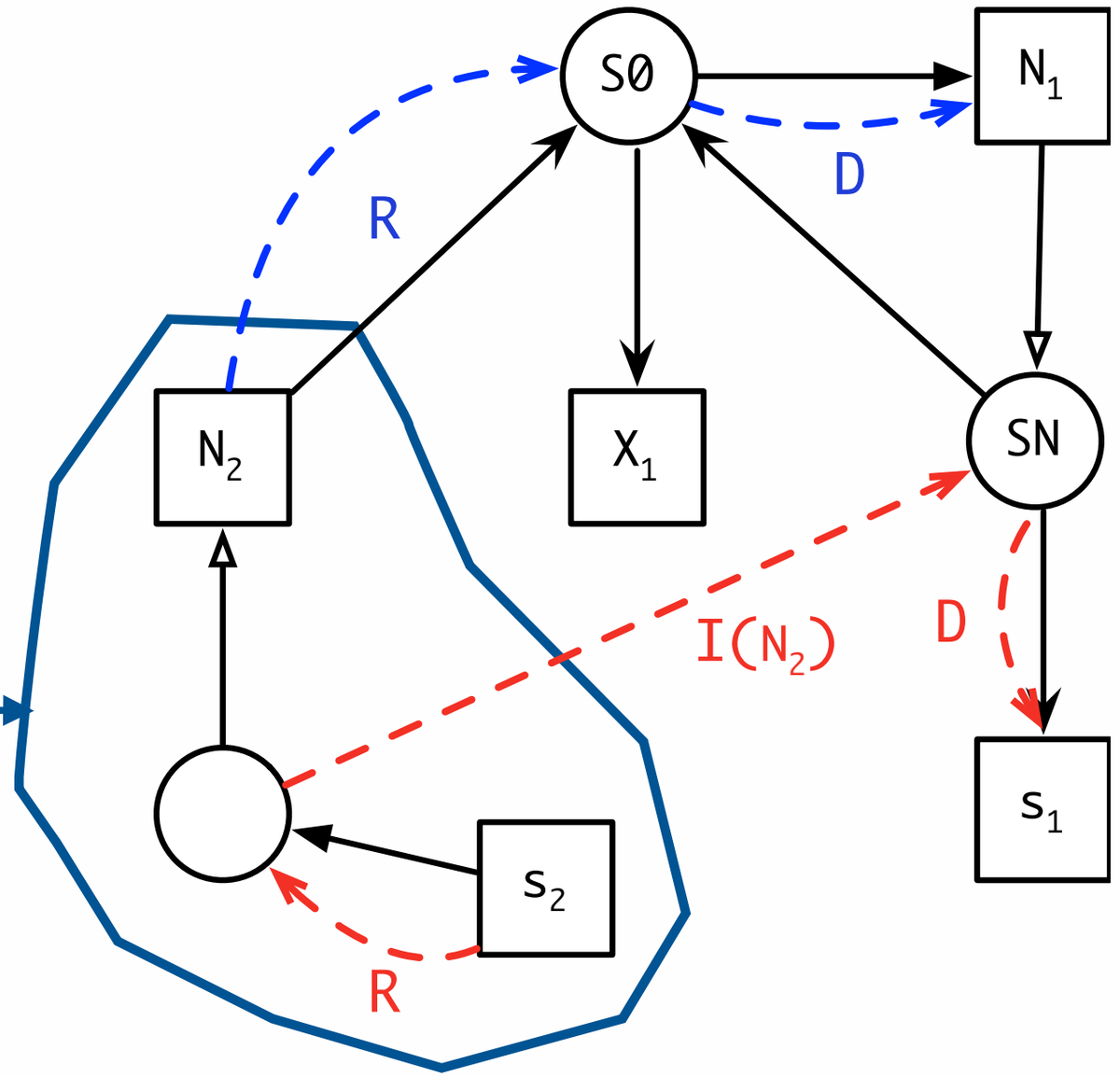
module M1 {
    def x1 = 1 + N2.s2
}
```



Qualified Names

```
module N1 {                    S0
    def s1 = 5
}

module M1 {
    def x1 = 1 + N2.s2
}
```



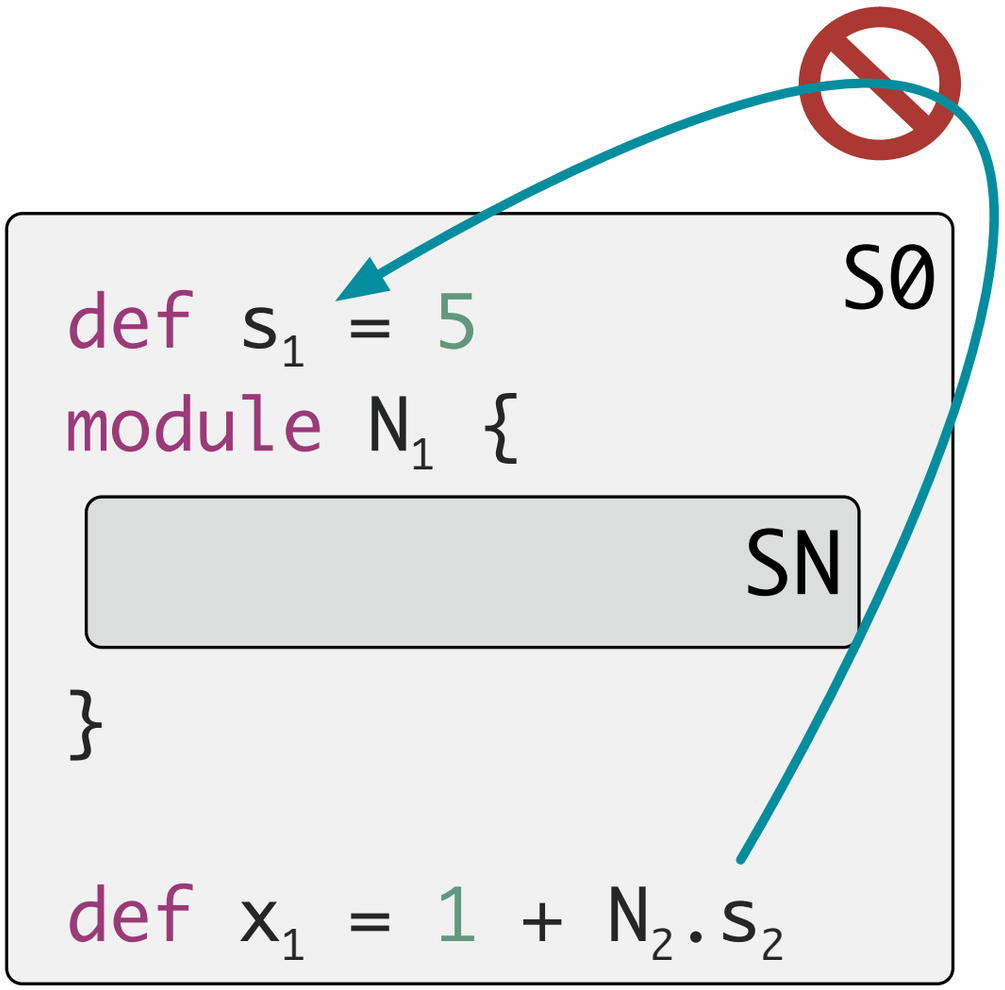
Import Parents

S0

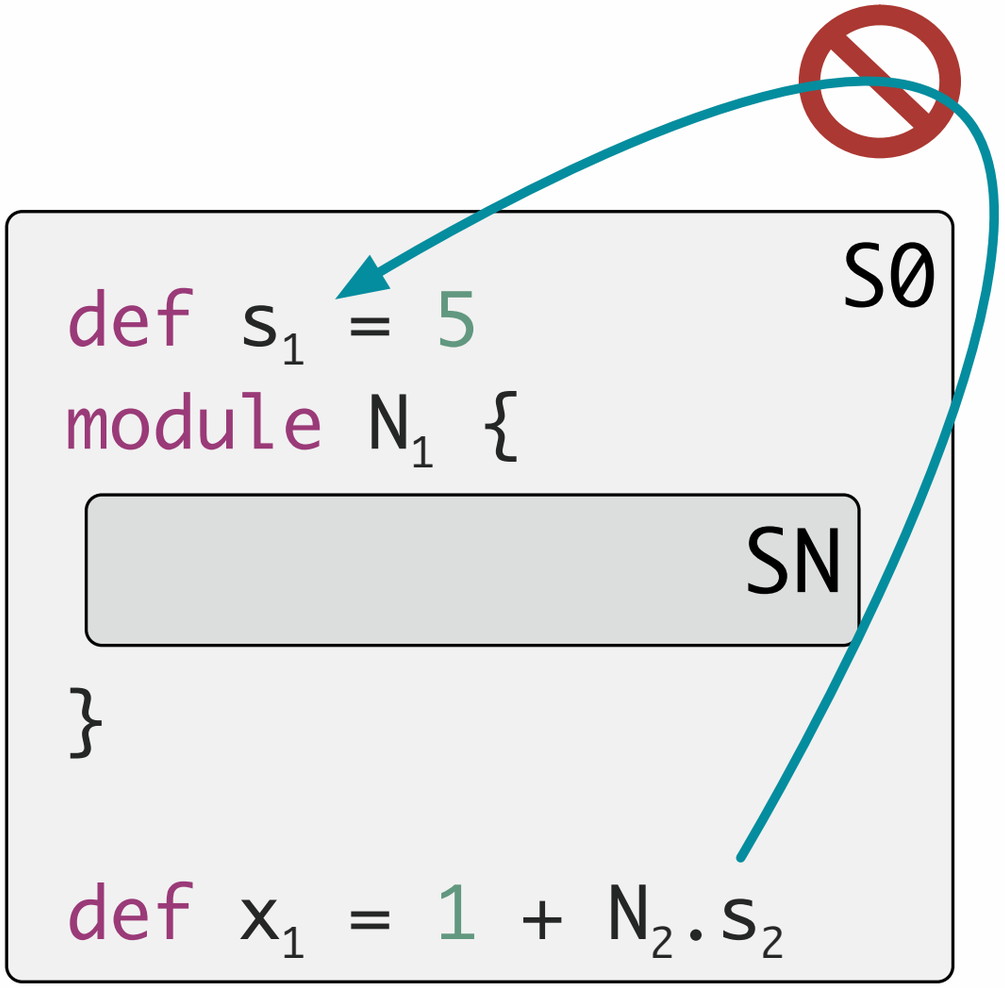
```
def s₁ = 5
module N₁ {
  SN
}

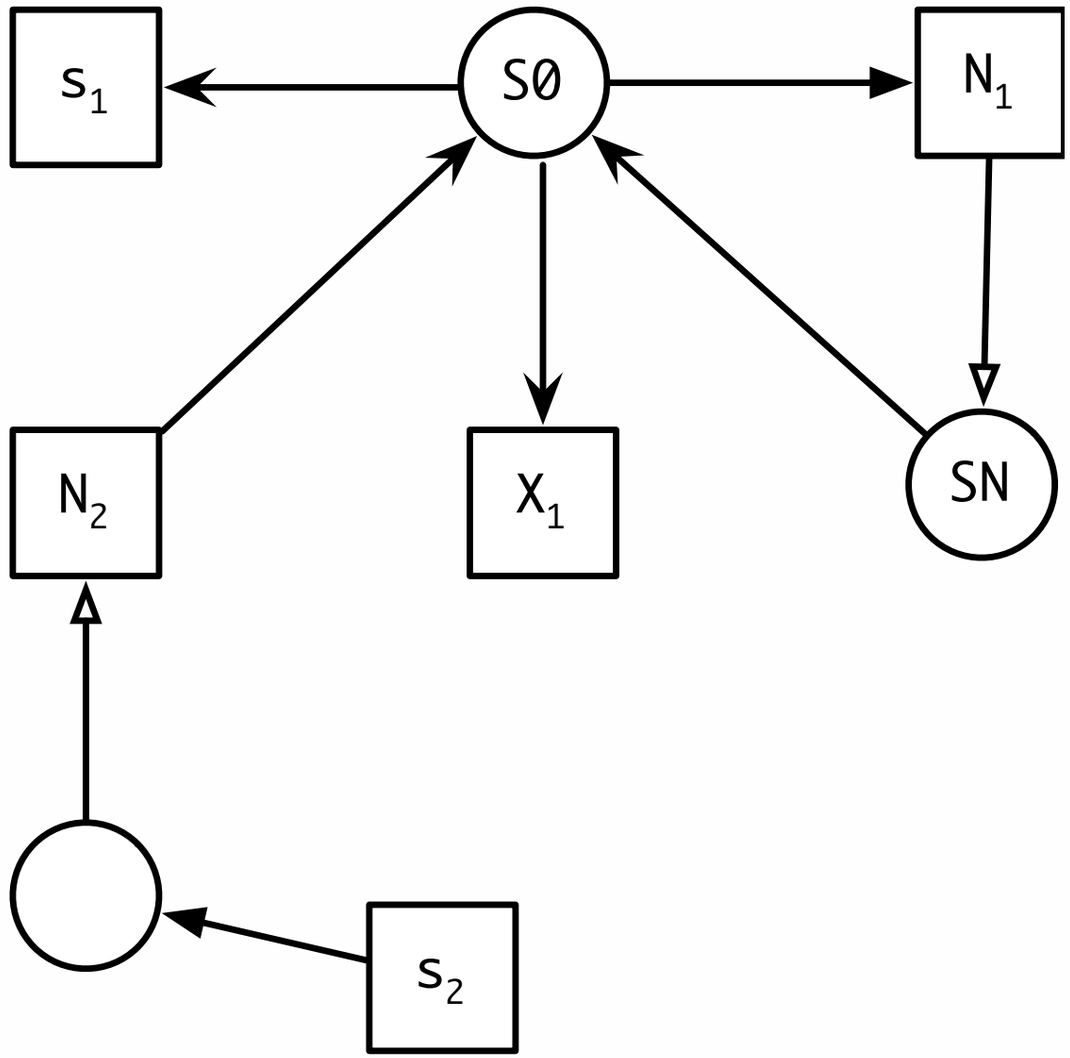
def x₁ = 1 + N₂.s₂
```

Import Parents

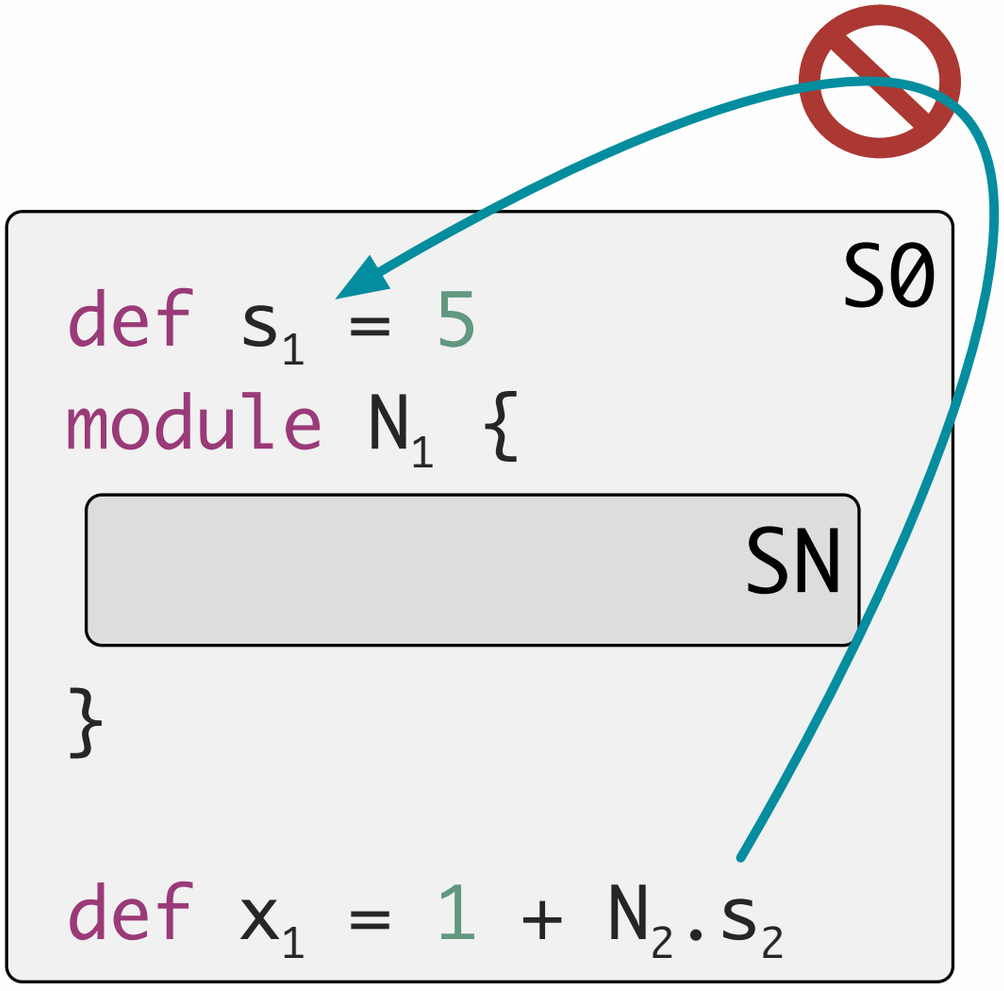


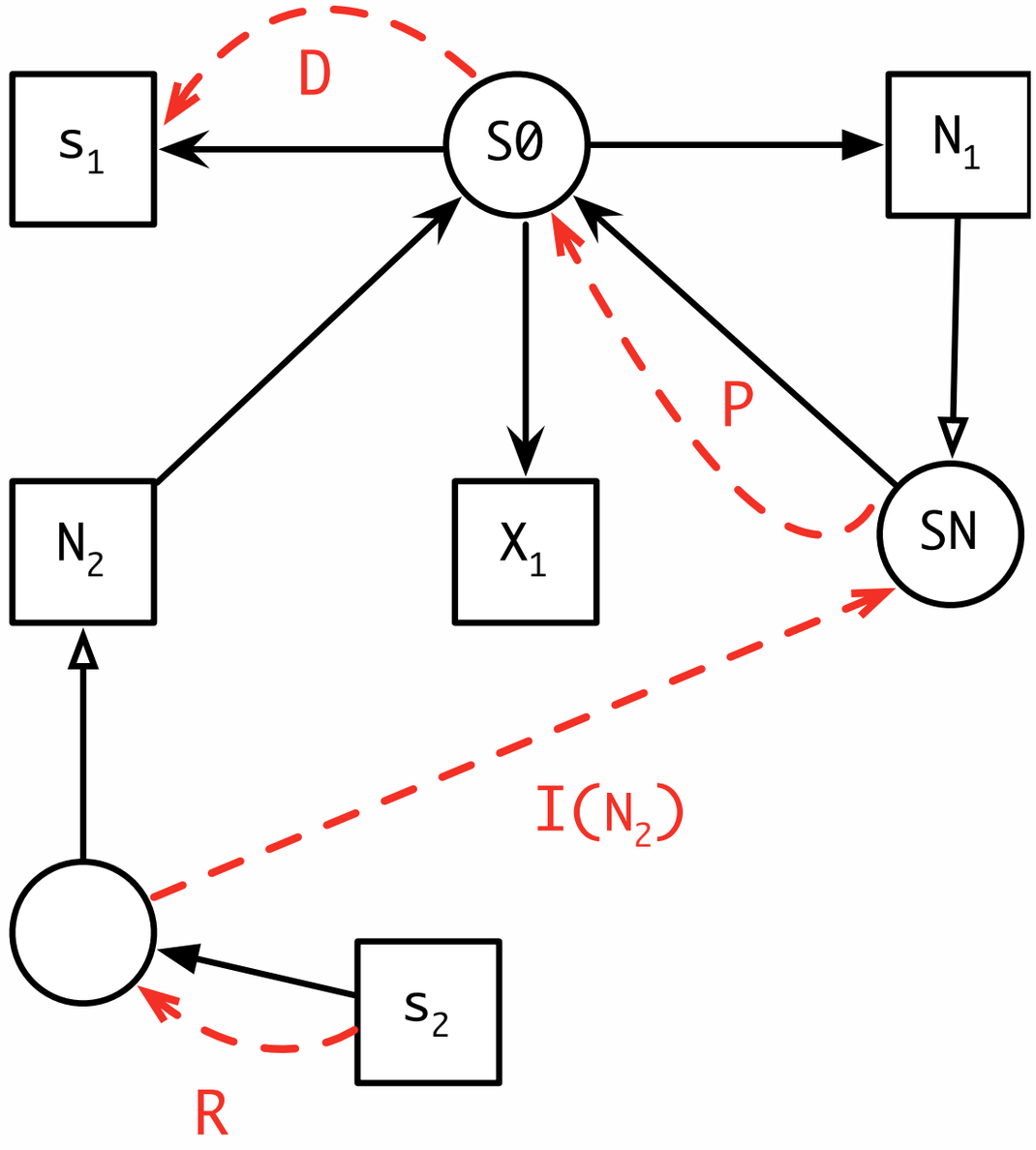
Import Parents



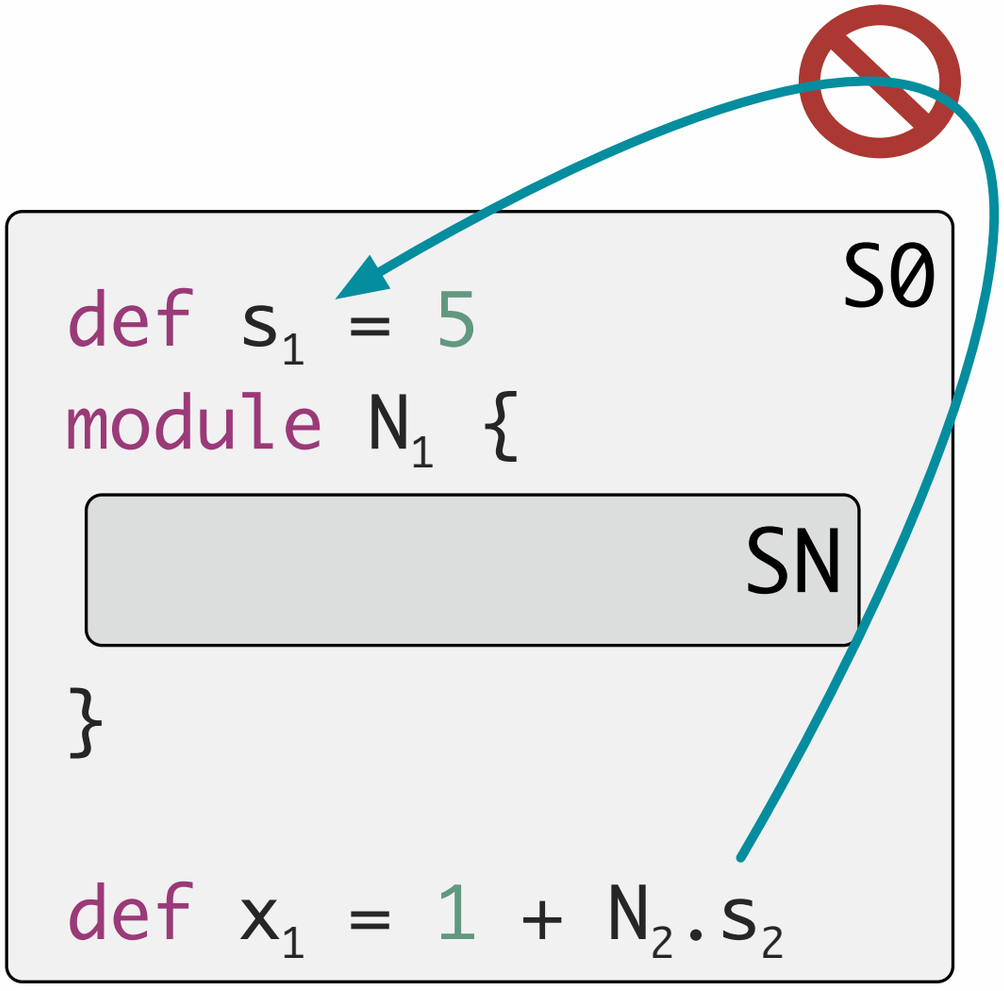


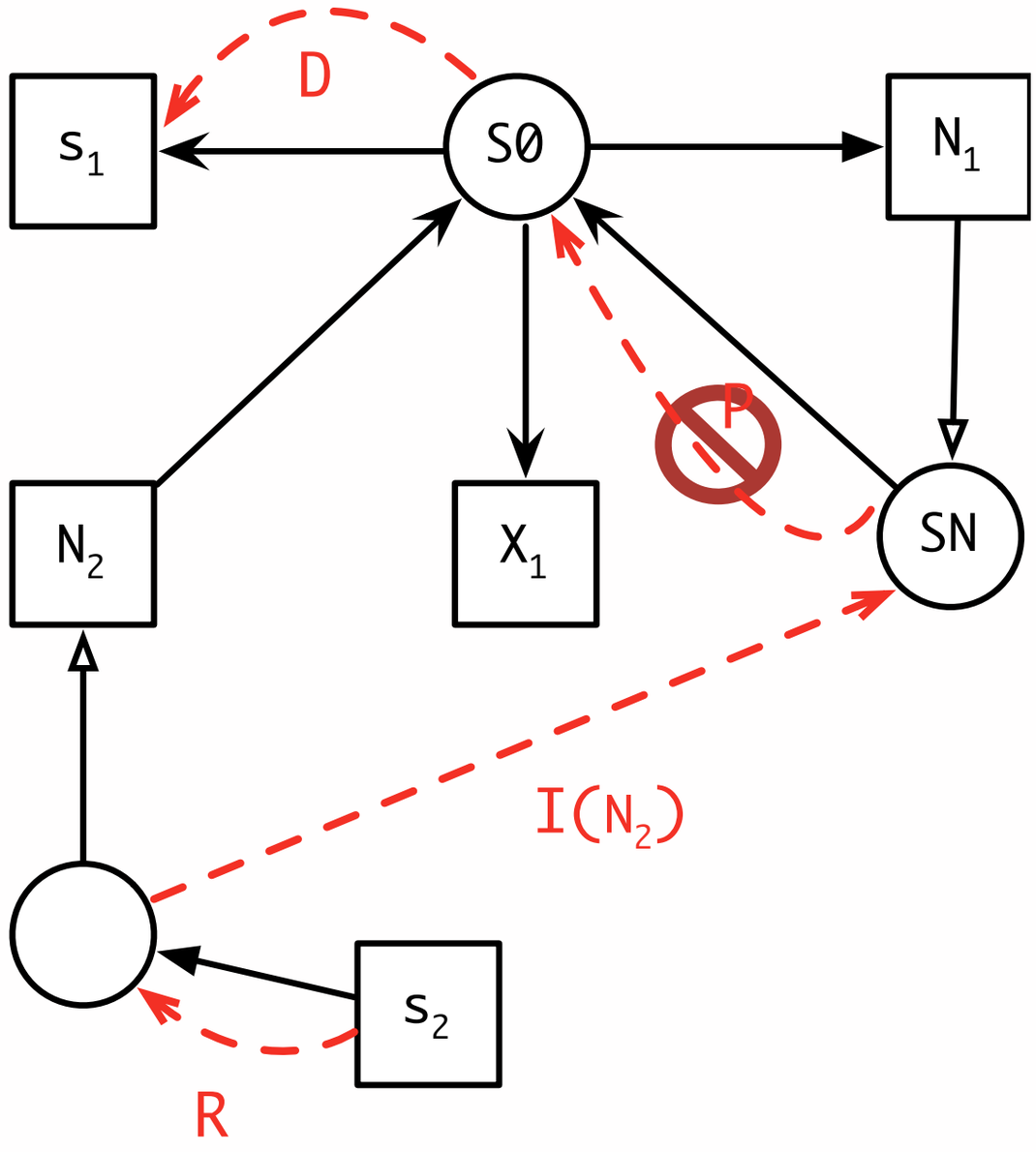
Import Parents





Import Parents





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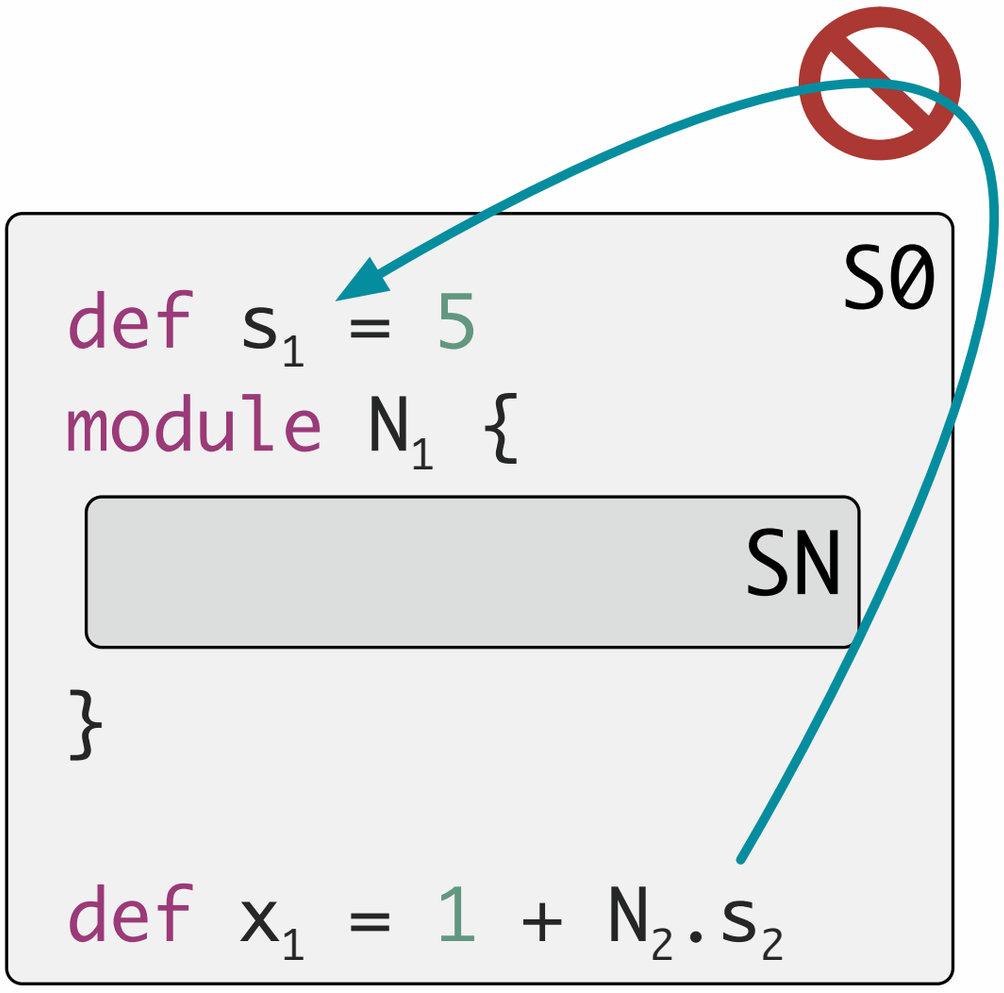
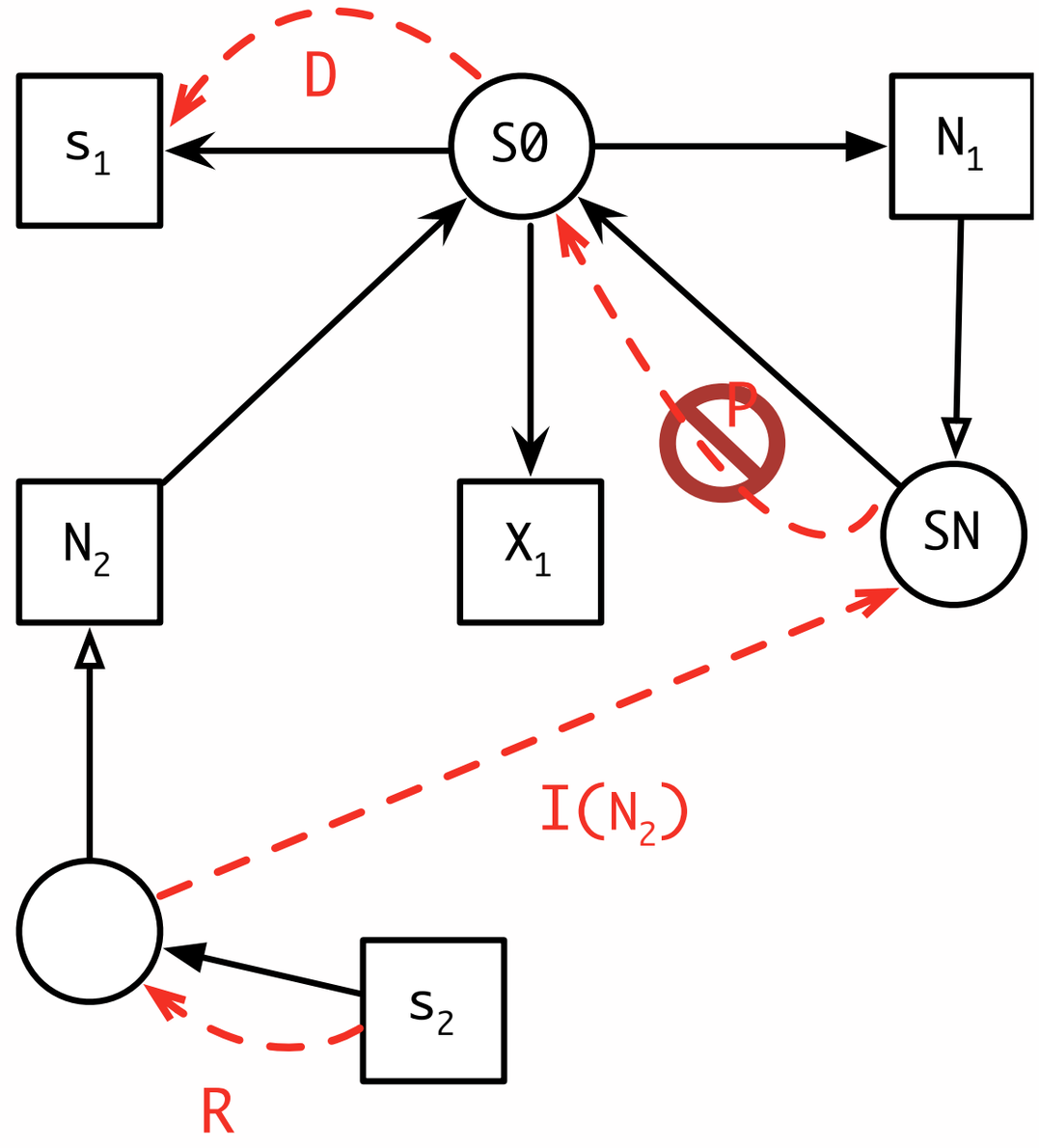


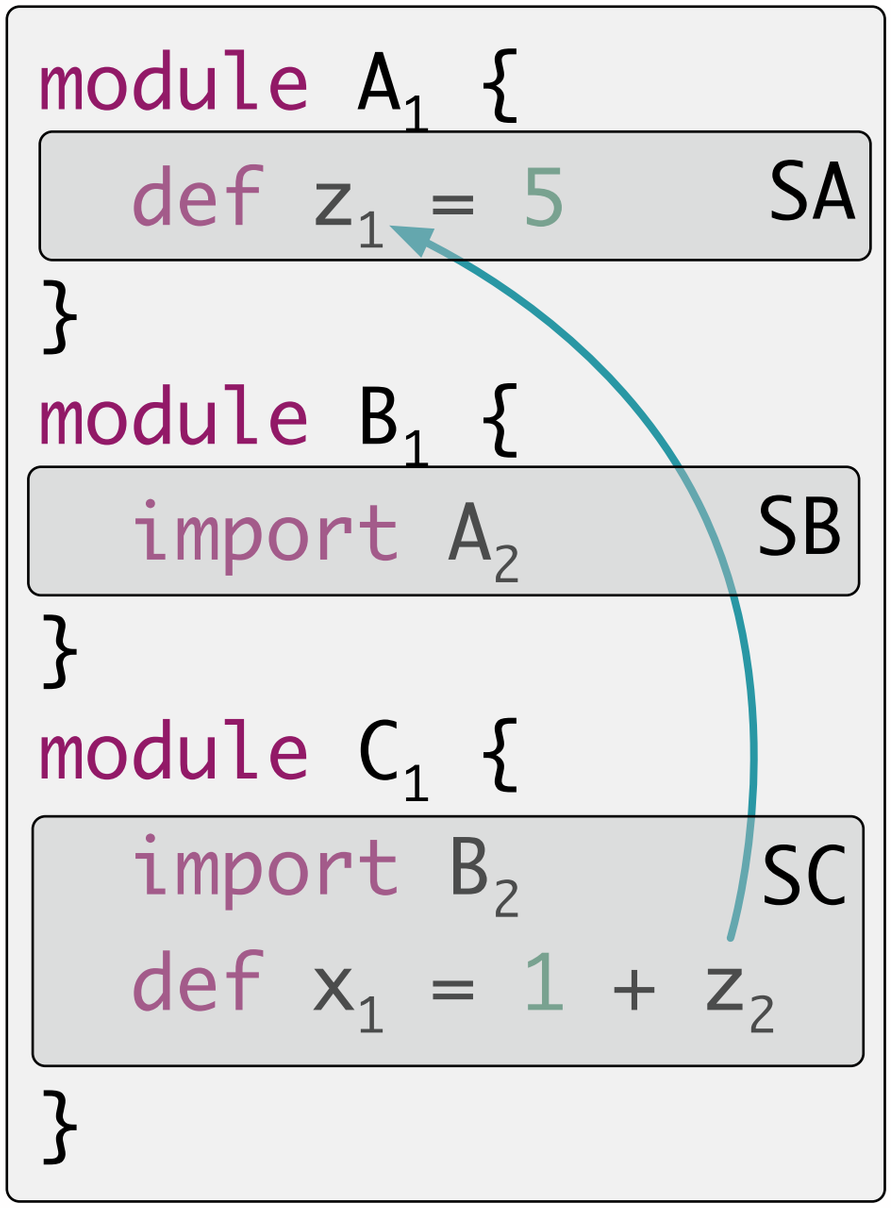


Well formed path: R.P*.I(_)*.D

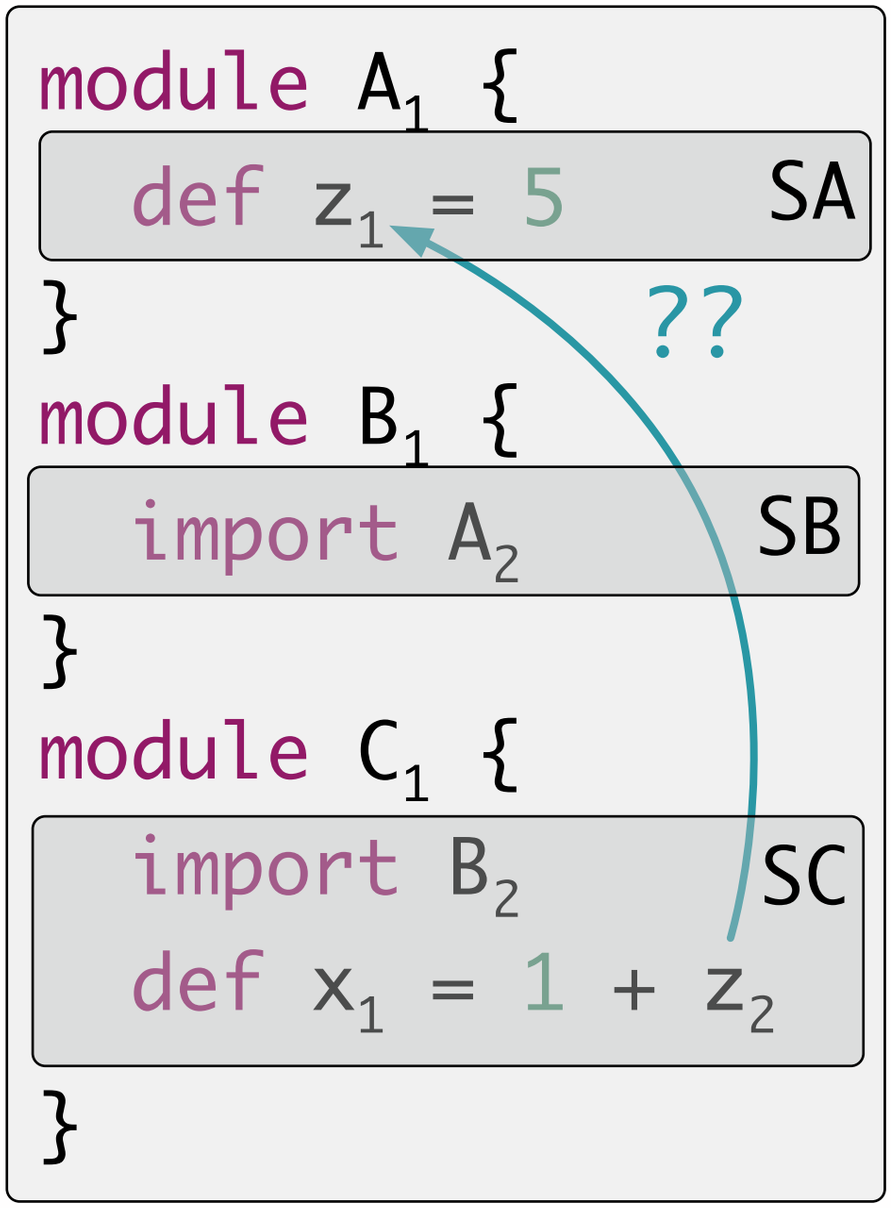
Transitive vs. Non-Transitive

```
module A1 {
  def z1 = 5                    SA
}
module B1 {
  import A2                     SB
}
module C1 {
  import B2                     SC
  def x1 = 1 + z2
}
```

Transitive vs. Non-Transitive



Transitive vs. Non-Transitive



Transitive vs. Non-Transitive

```
module A1 {
  def z1 = 5                    SA
}                          ??
module B1 {
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  def x1 = 1 + z2
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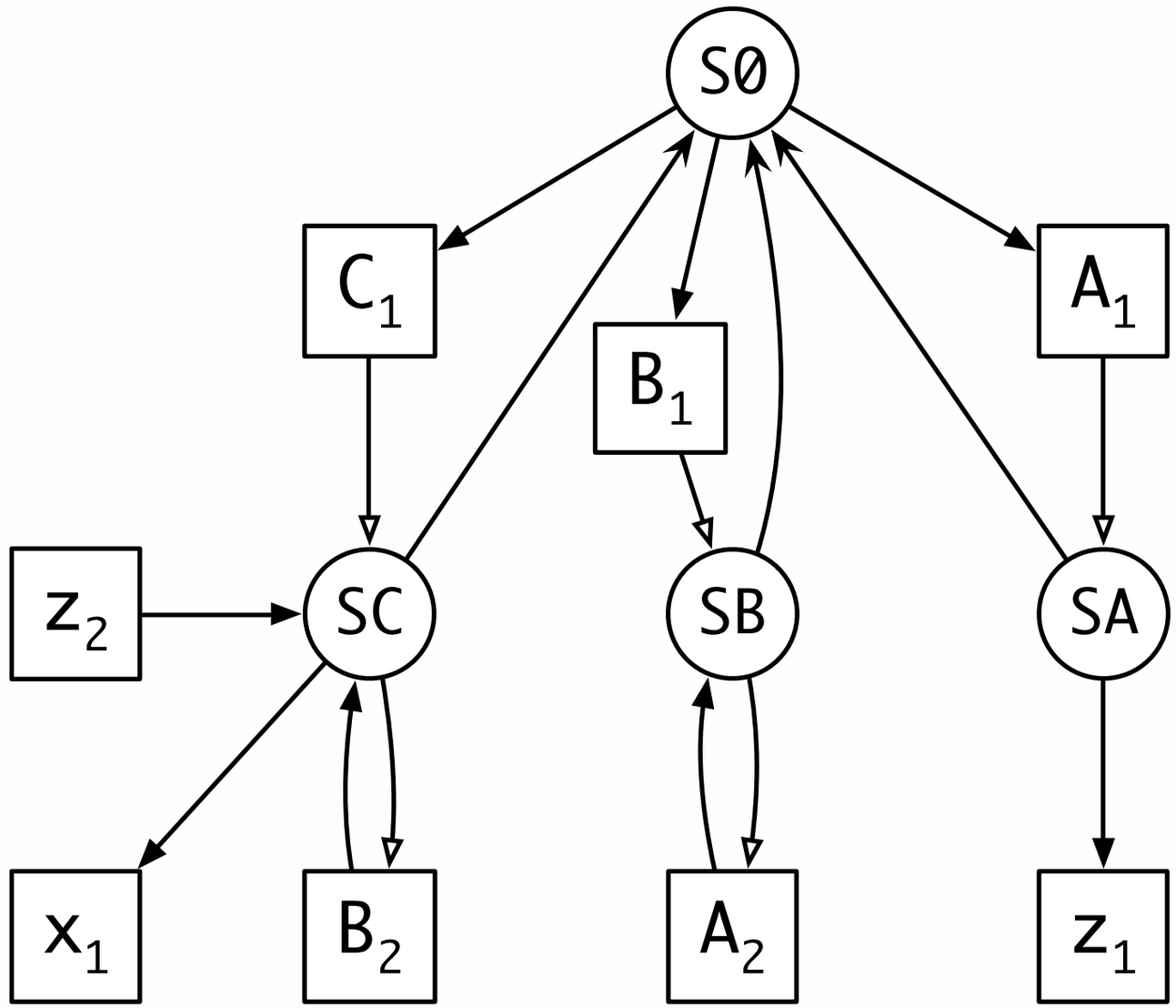
```



Transitive vs. Non-Transitive

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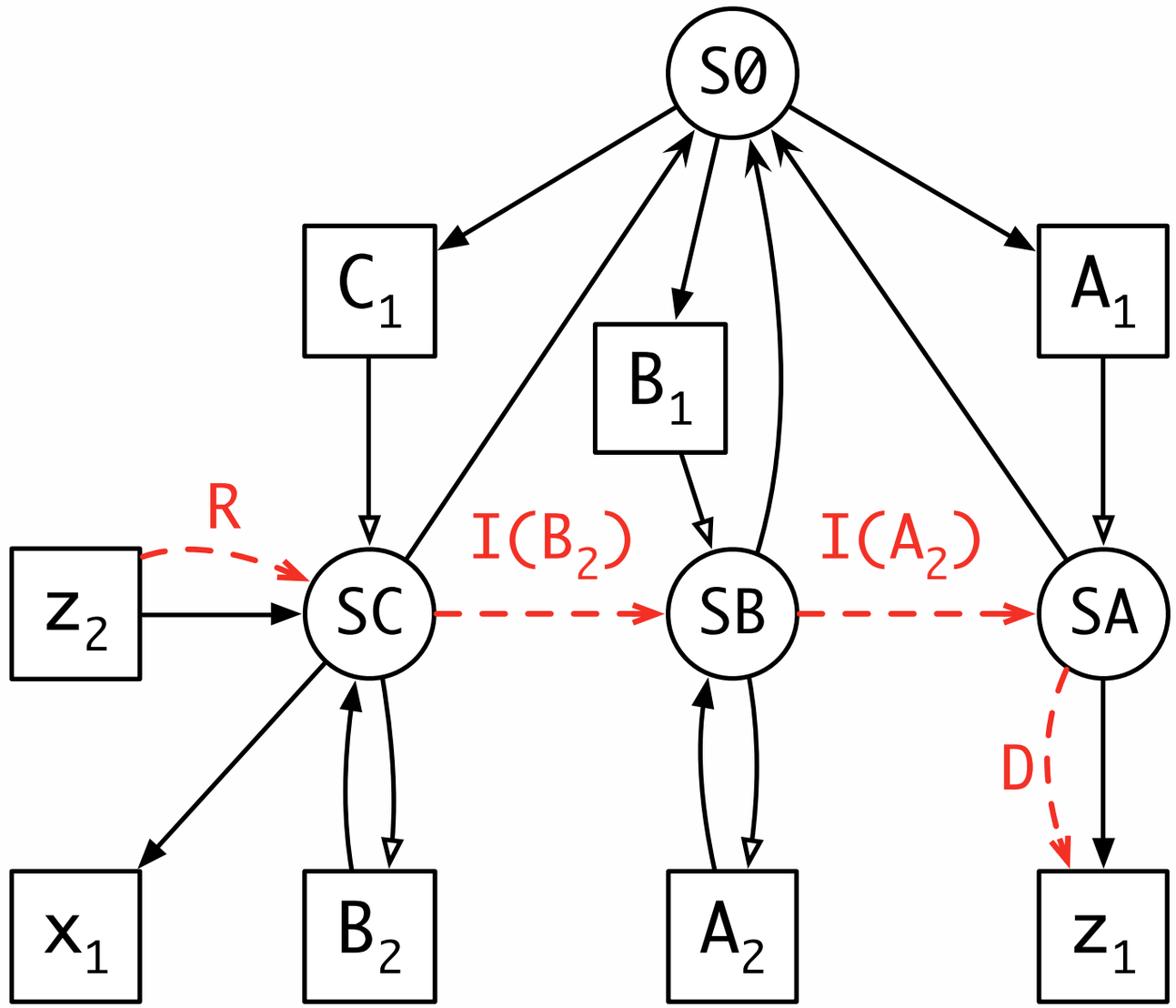

Transitive vs. Non-Transitive

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module A1 {
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module B1 {
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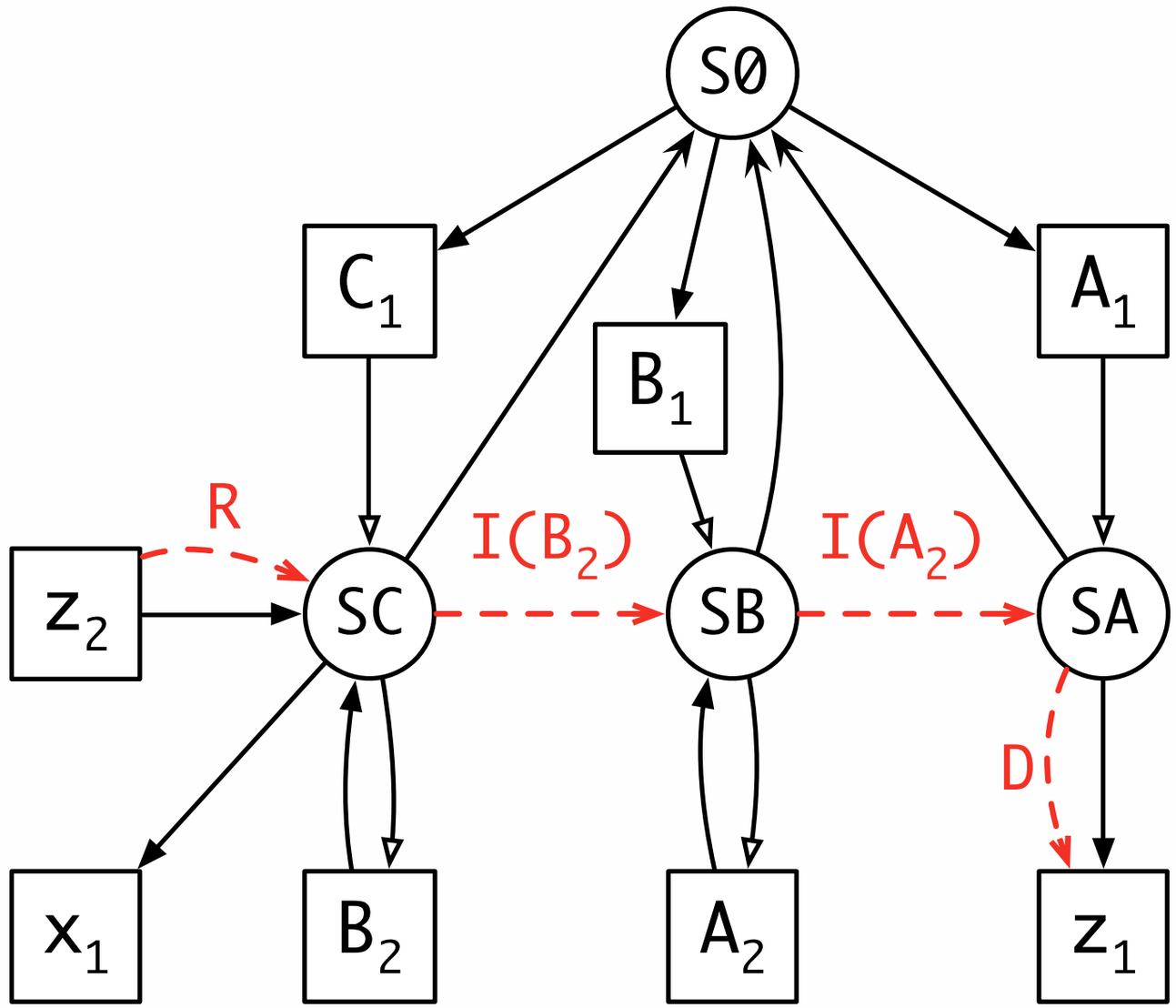
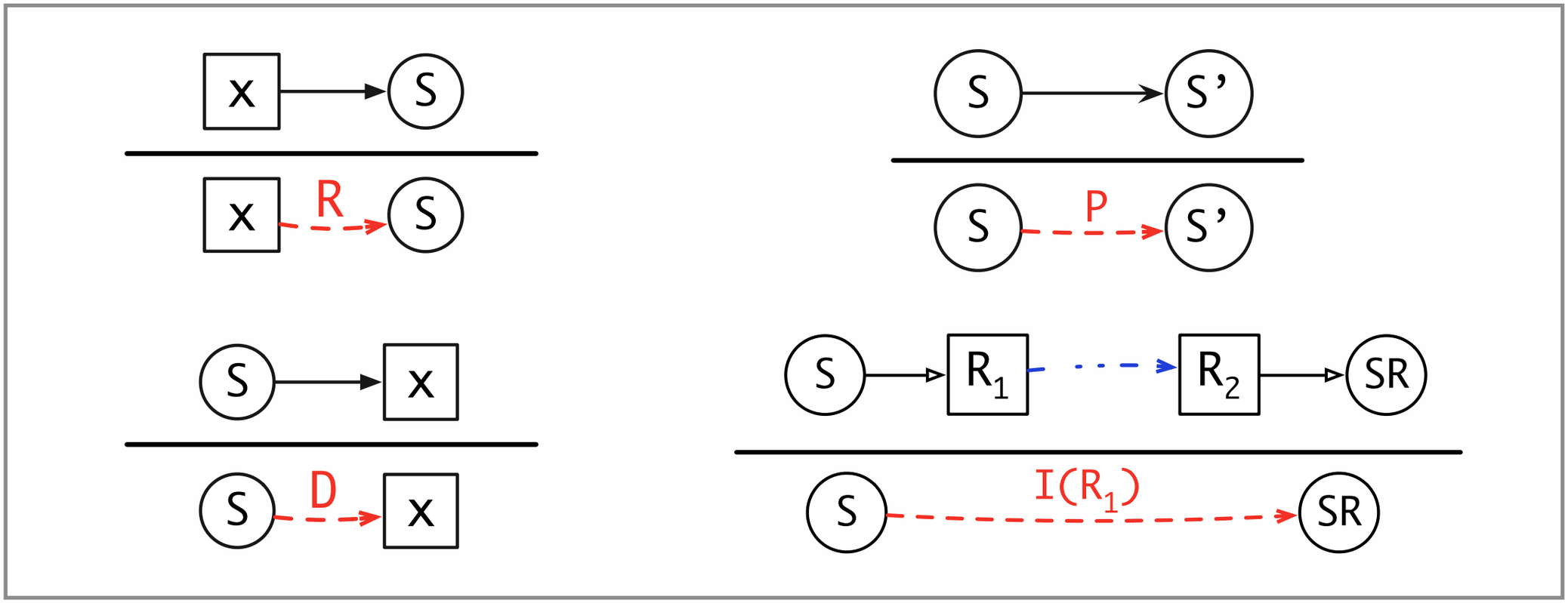
  import B2                     SC
  def x1 = 1 + z2
}
```



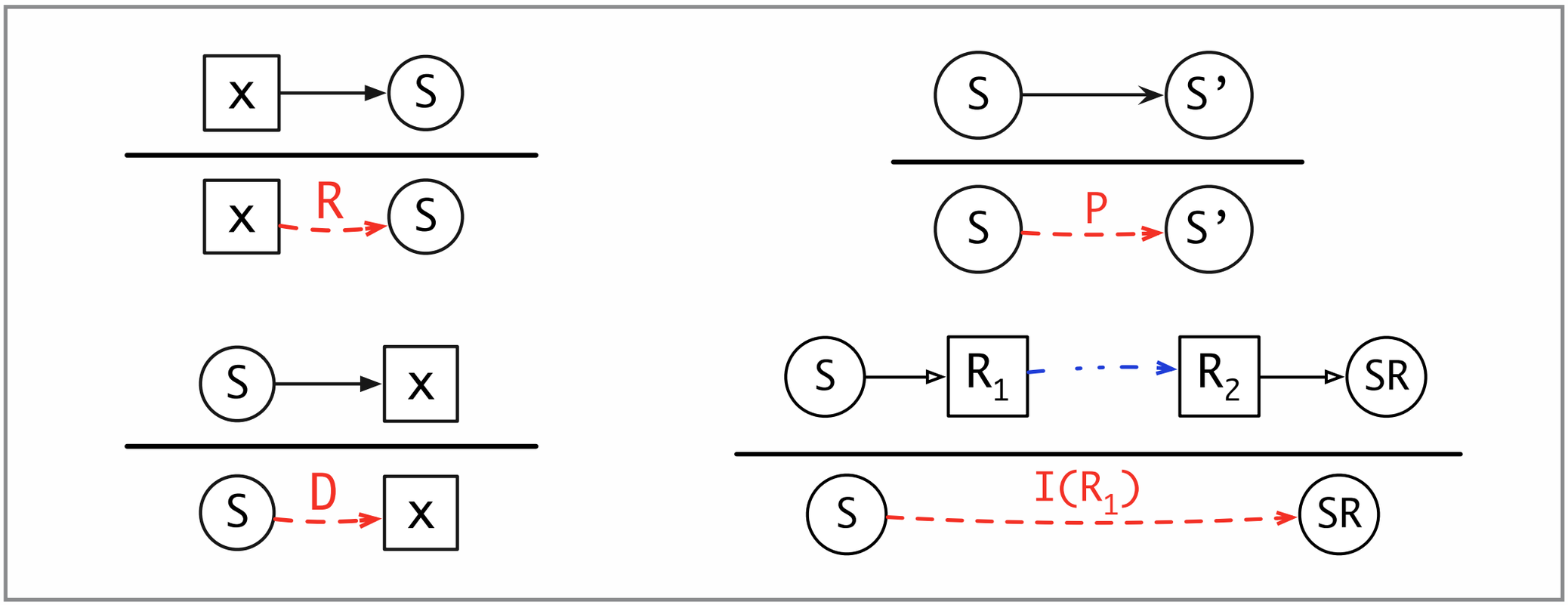
With transitive imports, a well formed path is `R.P*.I(_)*.D`

With non-transitive imports, a well formed path is `R.P*.I(_)?.D`

A Calculus for Name Resolution



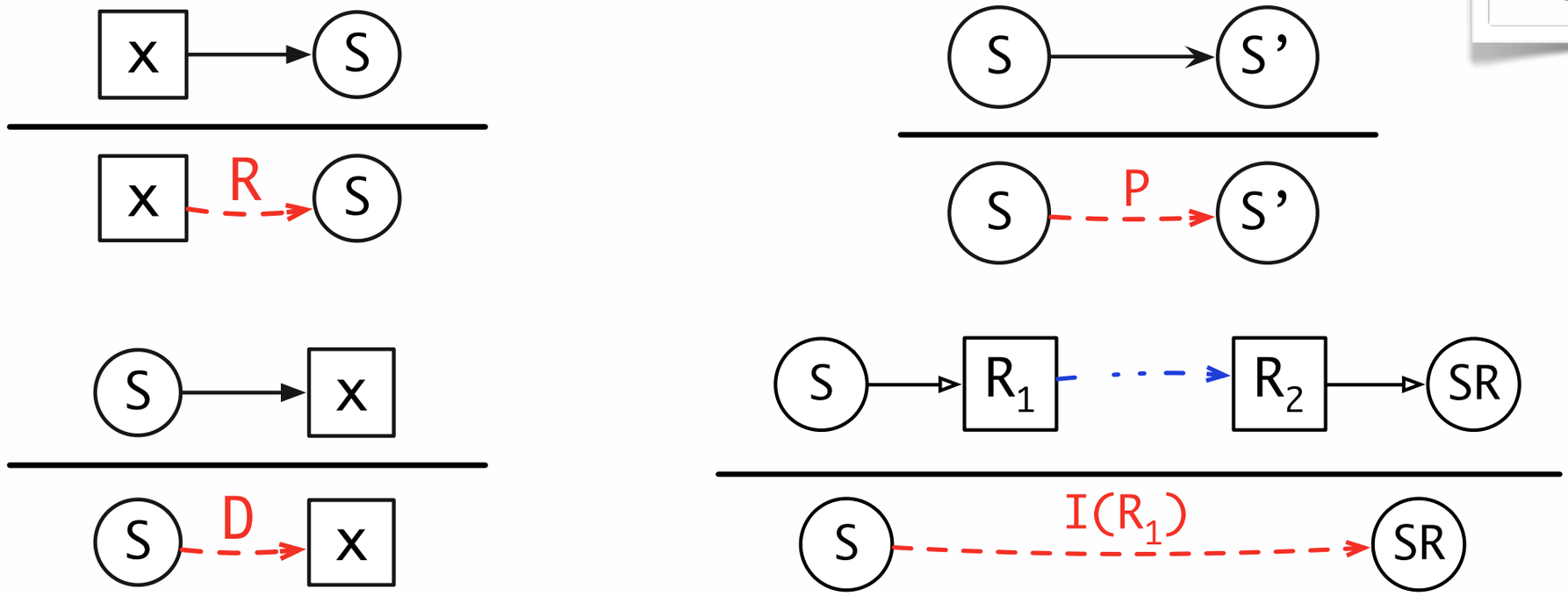
A Calculus for Name Resolution



Well formed path: `R.P*.I(_)*.D`

A Calculus for Name Resolution

Reachability



Well formed path: `R.P*.I(_)*.D`

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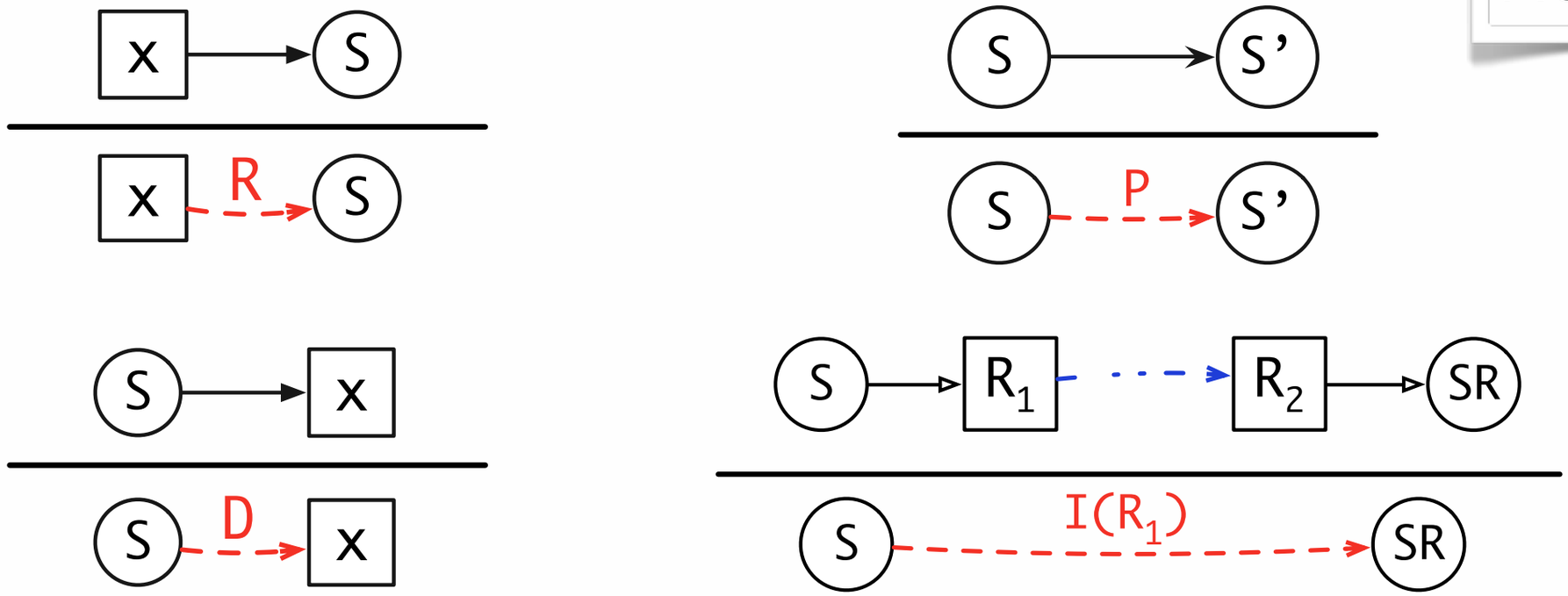

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$$\frac{}{D < P.p} \qquad \frac{}{I(\_).p' < P.p}$$

$$\frac{p < p'}{s.p < s.p'} \qquad \frac{}{D < I(\_).p'}$$

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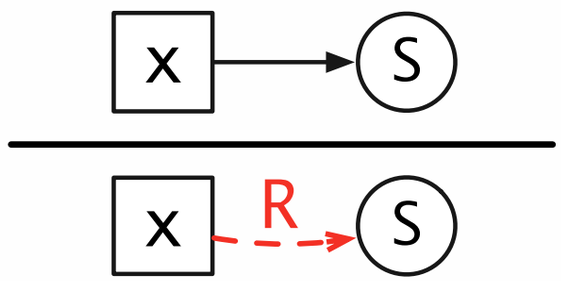

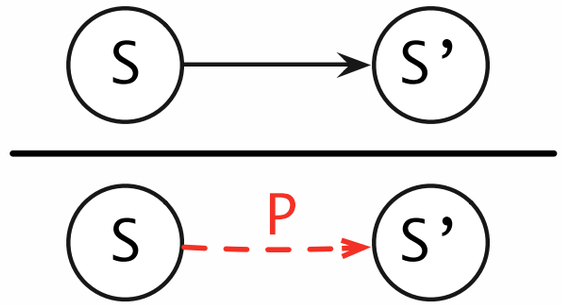

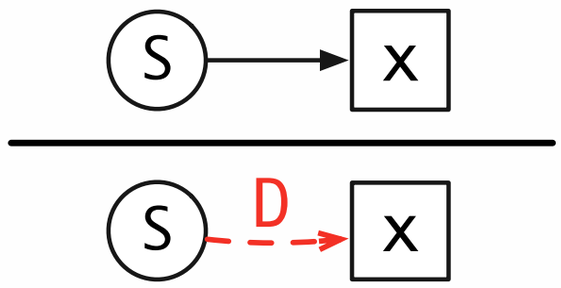

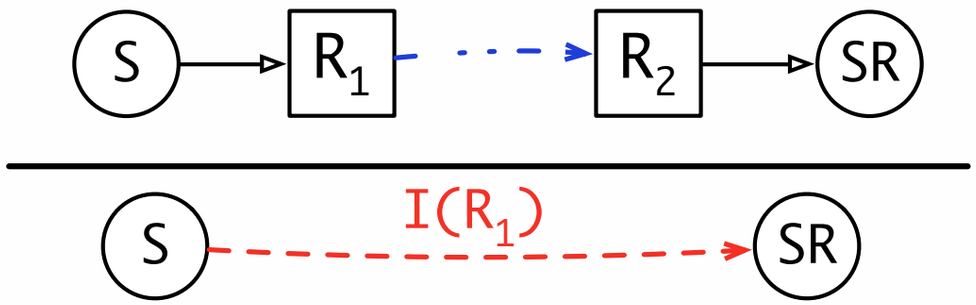

Well formed path: `R.P*.I(_)*.D`

Visibility

$$\frac{}{D < P.p} \qquad \frac{}{I(\_).p' < P.p}$$

$$\frac{p < p'}{s.p < s.p'} \qquad \frac{}{D < I(\_).p'}$$

Theory of Scope Graphs

Scope graphs, formally

$x_i^{\mathsf{D}}{:}S$ declaration with name x at position i with optional associated scope S

x_i^{R} reference with name x at position i

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$\mathcal{G}$: scope graph
$\mathcal{S}(\mathcal{G})$: scopes S in $\mathcal{G}$
$\mathcal{D}(S)$: declarations $x_i^{\mathsf{D}}{:}S'$ in S
$\mathcal{R}(S)$: references x_i^{R} in S
$\mathcal{I}(S)$: imports x_i^{R} in S
$\mathcal{P}(S)$: parent scope of S

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- $\mathcal{P}(S)$ is a partial function
- The parent relation is well-founded
- Each x_i^{R} and x_i^{D} appears in exactly one scope S

Resolution calculus, formally

Resolution paths

$$\frac{x_i^{\mathsf{R}} \in \mathcal{R}(S) \quad \{x_i^{\mathsf{R}}\} \cup \mathbb{I} \vdash p : S \longmapsto x_j^{\mathsf{D}}}{\mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}}}$$

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$$\frac{\mathbb{I} \vdash p : S \rightarrowtail x_i^{\mathsf{D}} \qquad \forall j, p' (\mathbb{I} \vdash p' : S \rightarrowtail x_j^{\mathsf{D}} \Rightarrow \neg(p' < p))}{\mathbb{I} \vdash p : S \longmapsto x_i^{\mathsf{D}}}$$

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Visibility paths

$$\frac{\mathbb{I} \vdash p : S \rightarrowtail x_i^{\mathsf{D}} \qquad \forall j, p'(\mathbb{I} \vdash p' : S \rightarrowtail x_j^{\mathsf{D}} \Rightarrow \neg(p' < p))}{\mathbb{I} \vdash p : S \longmapsto x_i^{\mathsf{D}}}$$

Reachability paths

$$\frac{\vdots}{\mathbb{I} \vdash p : S \rightarrowtail x_i^{\mathsf{D}}}$$

...may include import steps

$$\frac{y_i^{\mathsf{R}} \in \mathcal{I}(S_1) \setminus \mathbb{I} \quad \mathbb{I} \vdash p : y_i^{\mathsf{R}} \longmapsto y_j^{\mathsf{D}} : S_2}{\mathbb{I} \vdash \mathbf{I}(y_i^{\mathsf{R}}, y_j^{\mathsf{D}} : S_2) : S_1 \longrightarrow S_2}$$

Resolution calculus, formally

Resolution paths

$$\frac{x_i^{\mathsf{R}} \in \mathcal{R}(S) \quad \{x_i^{\mathsf{R}}\} \cup \mathbb{I} \vdash p : S \longmapsto x_j^{\mathsf{D}}}{\mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}}}$$

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Well-founded!

...may include import steps

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- But there is a terminating resolution algorithm
  - for path well-foundedness `RP*I*D`
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- Resolution calculus says how to validate paths but not how to find them
  - and reachability paths can have cycles
- But there is a terminating resolution algorithm
  - for path well-foundedness `RP*I*D`
  - and path ordering `D<I<P`
- Uses familiar notions of environments and shadowing

Resolution Algorithm v1

$$E_D(S) := \mathcal{D}(S)$$

$$E_P(S) := E_V(\mathcal{P}(S))$$

$$E_I(S) := \bigcup \left\{ E_L(S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \wedge y_j^{\mathsf{D}} {:} S_y \in Resolve(y_i^{\mathsf{R}}) \right\}$$

$$E_L(S) := E_D(S) \triangleleft E_I(S)$$

$$E_V(S) := E_L(S) \triangleleft E_P(S)$$

$$Resolve(x_i^{\mathsf{R}}) := \{x_j^{\mathsf{D}} \mid \exists S \ s.t. \ x_i^{\mathsf{R}} \in \mathcal{R}(S) \wedge x_j^{\mathsf{D}} \in E_V(S)\}$$

where $E_1 \triangleleft E_2 := E_1 \cup \{x_i^{\mathsf{D}} \in E_2 \mid \nexists \ x_{i'}^{\mathsf{D}} \in E_1\}$

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where $E_1 \triangleleft E_2 := E_1 \cup \{x_i^{\mathsf{D}} \in E_2 \mid \nexists \ x_{i'}^{\mathsf{D}} \in E_1\}$

- still need to incorporate “seen imports”

Resolution Algorithm v2

$$E_D[\mathbb{I}](S) := \mathcal{D}(S)$$

$$E_P[\mathbb{I}](S) := E_V[\mathbb{I}](\mathcal{P}(S))$$

$$E_I[\mathbb{I}](S) := \bigcup \left\{ E_L[\mathbb{I}](S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \setminus \mathbb{I} \wedge y_j^{\mathsf{D}} {:} S_y \in \mathit{Resolve}[\mathbb{I}](y_i^{\mathsf{R}}) \right\}$$

$$E_L[\mathbb{I}](S) := E_D[\mathbb{I}](S) \triangleleft E_I[\mathbb{I}](S)$$

$$E_V[\mathbb{I}](S) := E_L[\mathbb{I}](S) \triangleleft E_P[\mathbb{I}](S)$$

$$\mathit{Resolve}[\mathbb{I}](x_i^{\mathsf{R}}) := \{x_j^{\mathsf{D}} \mid \exists S \ s.t. \ x_i^{\mathsf{R}} \in \mathcal{R}(S) \wedge x_j^{\mathsf{D}} \in E_V[\{x_i^{\mathsf{R}}\} \cup \mathbb{I}](S)\}$$

where $E_1 \triangleleft E_2 := E_1 \cup \{x_i^{\mathsf{D}} \in E_2 \mid \nexists \ x_{i'}^{\mathsf{D}} \in E_1\}$

Resolution Algorithm v2

$$
\begin{aligned}
E_D[\mathbb{I}](S) &:= \mathcal{D}(S) \\
E_P[\mathbb{I}](S) &:= E_V[\mathbb{I}](\mathcal{P}(S)) \\
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E_L[\mathbb{I}](S) &:= E_D[\mathbb{I}](S) \triangleleft E_I[\mathbb{I}](S) \\
E_V[\mathbb{I}](S) &:= E_L[\mathbb{I}](S) \triangleleft E_P[\mathbb{I}](S)
\end{aligned}
$$

$$
\mathit{Resolve}[\mathbb{I}](x_i^{\mathsf{R}}) := \{ x_j^{\mathsf{D}} \mid \exists S \ s.t. \ x_i^{\mathsf{R}} \in \mathcal{R}(S) \wedge x_j^{\mathsf{D}} \in E_V[\{x_i^{\mathsf{R}}\} \cup \mathbb{I}](S) \}
$$

where $E_1 \triangleleft E_2 := E_1 \cup \{ x_i^{\mathsf{D}} \in E_2 \mid \nexists \ x_{i'}^{\mathsf{D}} \in E_1 \}$

- but still might not terminate due to cycles (e.g. consider a scope that imports itself)

Resolution Algorithm v3

$$E_D[\mathbb{I},\mathbb{S}](S) \quad := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ \mathcal{D}(S) \end{cases}$$

$$E_P[\mathbb{I},\mathbb{S}](S) \quad := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ E_V[\mathbb{I},\{S\} \cup \mathbb{S}](\mathcal{P}(S)) \end{cases}$$

$$E_I[\mathbb{I},\mathbb{S}](S) \quad := \begin{cases} \emptyset \text{ if } S \in \mathbb{S} \\ \bigcup \left\{ E_L[\mathbb{I},\{S\} \cup \mathbb{S}](S_y) \mid y_i^{\mathsf{R}} \in \mathcal{I}(S) \setminus \mathbb{I} \wedge y_j^{\mathsf{D}} {:} S_y \in \mathit{Resolve}[\mathbb{I}](y_i^{\mathsf{R}}) \right\} \end{cases}$$

$$E_L[\mathbb{I},\mathbb{S}](S) \quad := E_D[\mathbb{I},\mathbb{S}](S) \triangleleft E_I[\mathbb{I},\mathbb{S}](S)$$

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$$\mathit{Resolve}[\mathbb{I}](x_i^{\mathsf{R}}) := \{x_j^{\mathsf{D}} \mid \exists S \ s.t. \ x_i^{\mathsf{R}} \in \mathcal{R}(S) \wedge x_j^{\mathsf{D}} \in E_V[\{x_i^{\mathsf{R}}\} \cup \mathbb{I}, \emptyset](S)\}$$

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Lemma: visibility paths never have cycles

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Lemma: visibility paths never have cycles

Theorem: algorithm is sound and complete for calculus:

$$(x_j^{\mathsf{D}} \in Res[\mathbb{I}](x_i^{\mathsf{R}})) \iff (\exists p \ s.t. \ \mathbb{I} \vdash p : x_i^{\mathsf{R}} \longmapsto x_j^{\mathsf{D}})$$

Language-independent α-equivalence

Program similarity

`P` $\simeq$ `P'` if have same AST ignoring identifier names

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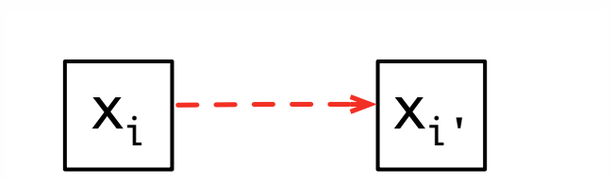
Position equivalence



$$\frac{}{i \overset{\texttt{P}}{\sim} i'}$$

$$\frac{i' \overset{\texttt{P}}{\sim} i}{i \overset{\texttt{P}}{\sim} i'}$$

$$\frac{i \overset{\texttt{P}}{\sim} i' \quad i' \overset{\texttt{P}}{\sim} i''}{i \overset{\texttt{P}}{\sim} i''}$$

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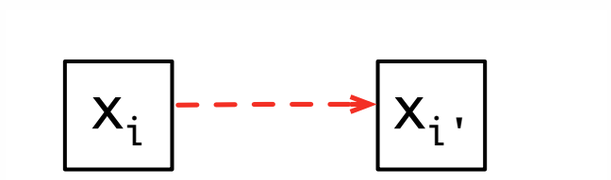

$$\frac{}{i \overset{\texttt{P}}{\simeq} i'} \qquad \frac{i' \overset{\texttt{P}}{\simeq} i}{i \overset{\texttt{P}}{\simeq} i'} \qquad \frac{i \overset{\texttt{P}}{\simeq} i' \quad i' \overset{\texttt{P}}{\simeq} i''}{i \overset{\texttt{P}}{\simeq} i''} \qquad \frac{}{i \overset{\texttt{P}}{\simeq} i}$$

Alpha equivalence

$$\texttt{P1} \overset{\alpha}{\approx} \texttt{P2} \triangleq \texttt{P1} \simeq \texttt{P2} \wedge \forall\ e\ e',\ e \overset{\texttt{P1}}{\simeq} e' \Leftrightarrow e \overset{\texttt{P2}}{\simeq} e'$$

(with some further details about free variables)

Preserving ambiguity

```
module A1 {
  def x2 := 1
}
module B3 {
  def x4 := 2
}
module C5 {
  import A6 B7;
  def y8 := x9
}
module D10 {
  import A11;
  def y12 := x13
}
module E14 {
  import B15;
  def y16 := x17
}
```

P1

```
module AA1 {
  def z2 := 1
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module BB3 {
  def z4 := 2
}
module C5 {
  import AA6 BB7;
  def s8 := z9
}
module D10 {
  import AA11;
  def u12 := z13
}
module E14 {
  import BB15;
  def v16 := z17
}
```

P2

```
module A1 {
  def z2 := 1
}
module B3 {
  def x4 := 2
}
module C5 {
  import A6 B7;
  def y8 := z9
}
module D10 {
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  def y12 := z13
}
module E14 {
  import B15;
  def y16 := x17
}
```

P3

$P1 \overset{\alpha}{\approx} P2$

$P2 \not\overset{\alpha}{\approx} P3$

Applying Scope Graphs

(ongoing work)

Validation

Validation

- We have modeled a large set of example binding patterns
  - definition before use
  - different let binding flavors
  - recursive modules
  - imports and includes
  - qualified names
  - class inheritance
  - partial classes
  - …

Validation

- We have modeled a large set of example binding patterns
  - definition before use
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  - recursive modules
  - imports and includes
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  - class inheritance
  - partial classes
  - …
- Next goal: fully model some real languages
  - Java
  - ML
  - …

Generating Scope Graphs from AST

$$
\begin{array}{rcll}
[\![ds]\!]^{prog} & := & P(S) := \bot \ \wedge \ [\![ds]\!]^{decl^*}_{S} & \textit{(new } S\textit{)} \\
[\![\texttt{module}\ X_i\ \{\, ds \,\}]\!]^{decl}_{s} & := & X_i^{\mathsf{D}}{:}S' \in \mathcal{D}(s) \ \wedge \ P(S') := s \ \wedge \ [\![ds]\!]^{decl^*}_{S'} & \textit{(new } S'\textit{)} \\
[\![\texttt{import}\ Xs\,\texttt{.}\,X_i]\!]^{decl}_{s} & := & X_i^{\mathsf{R}} \in \mathcal{I}(s) \ \wedge \ [\![Xs\,\texttt{.}\,X_i]\!]^{qid}_{s} & \\
[\![\texttt{def}\ b]\!]^{decl}_{s} & := & [\![b]\!]^{bind}_{s,s} & \\
[\![x_i = e]\!]^{bind}_{s_r,s_d} & := & x_i^{\mathsf{D}} \in \mathcal{D}(s_d) \ \wedge \ [\![e]\!]^{exp}_{s_r} & \\
[\![x_i]\!]^{qid}_{s} & := & x_i^{\mathsf{R}} \in \mathcal{R}(s) & \\
[\![\texttt{fun}\ (x_i\texttt{:}t)\ \{e\}]\!]^{exp}_{s} & := & P(S') := s \ \wedge \ x_i^{\mathsf{D}} \in \mathcal{D}(S') \ \wedge \ [\![e]\!]^{exp}_{S'} & \textit{(new } S'\textit{)} \\
[\![\texttt{letrec}\ bs\ \texttt{in}\ e]\!]^{exp}_{s} & := & P(S') := s \ \wedge \ [\![bs]\!]^{bind^*}_{S',S'} \ \wedge \ [\![e]\!]^{exp}_{S'} & \textit{(new } S'\textit{)} \\
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[\![Xs\,\texttt{.}\,x_i]\!]^{exp}_{s} & := & [\![Xs\,\texttt{.}\,x_i]\!]^{qid}_{s} & \\
[\![e_1\ e_2]\!]^{exp}_{s} & := & [\![e_1]\!]^{exp}_{s} \ \wedge \ [\![e_2]\!]^{exp}_{s} &
\end{array}
$$

generate smallest graph satisfying constraints

Binding gives Types

Static type-checking (or inference) is one obvious client for name resolution

In many cases, we can perform resolution **before** doing type analysis

```
def x : int = 6

def f = fun (y : int) { x + y }
```

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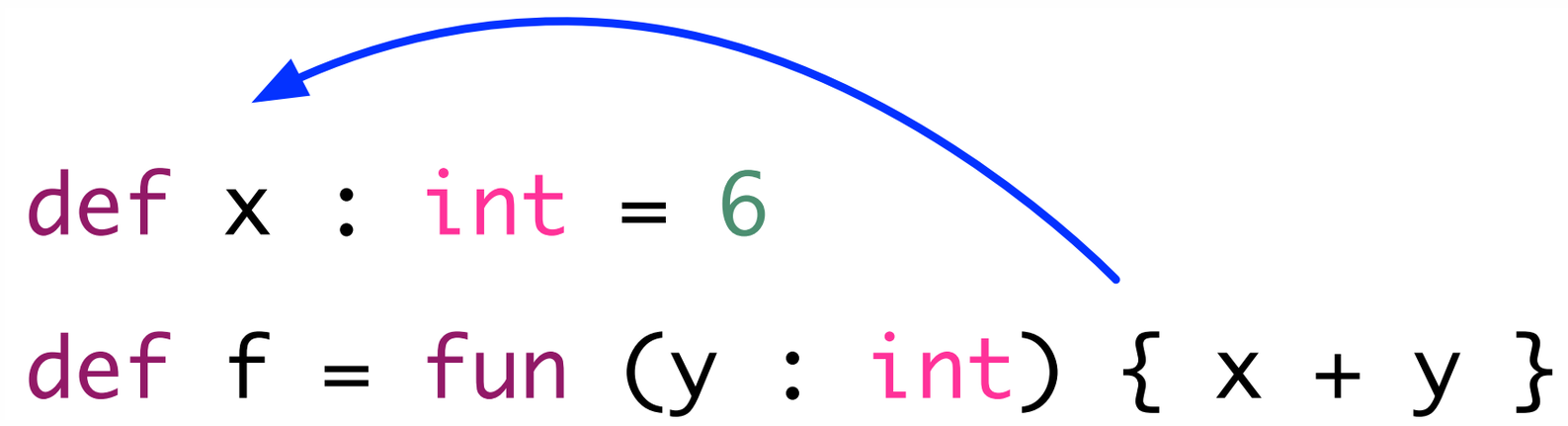

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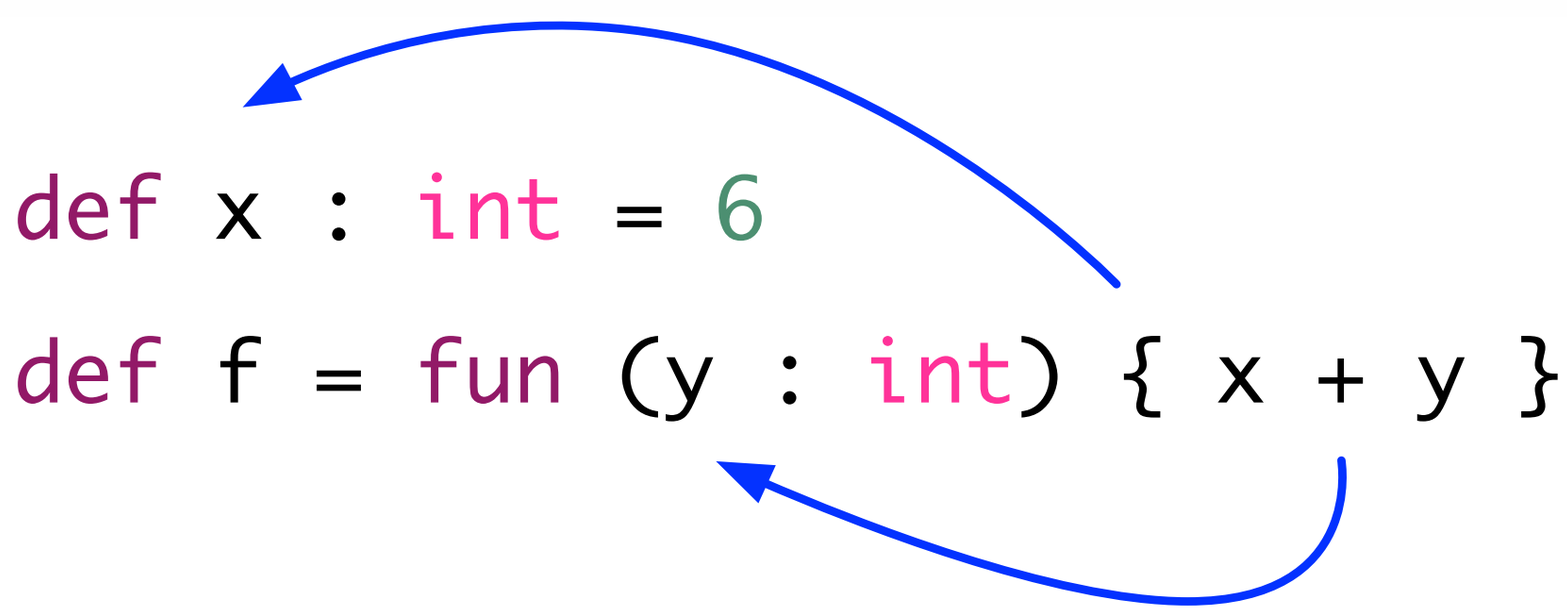

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But sometimes we need types **before** we can do name resolution

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record A₁ { x₁ : int }
record B₁ { a₁ : A₂ ; x₂ : bool}

def z₁ : B₂ = ...

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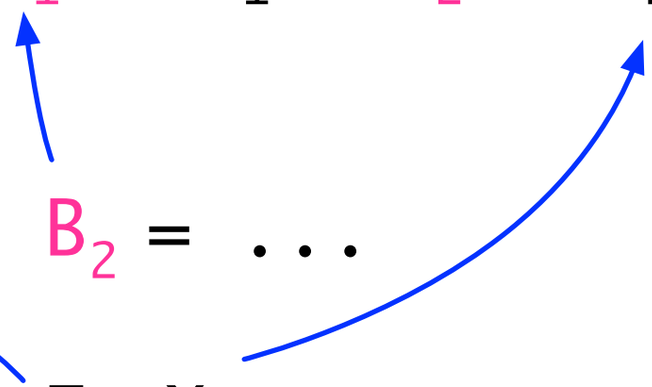

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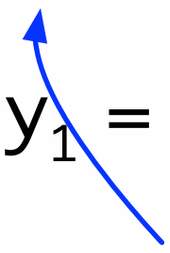

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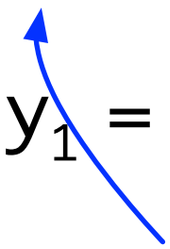

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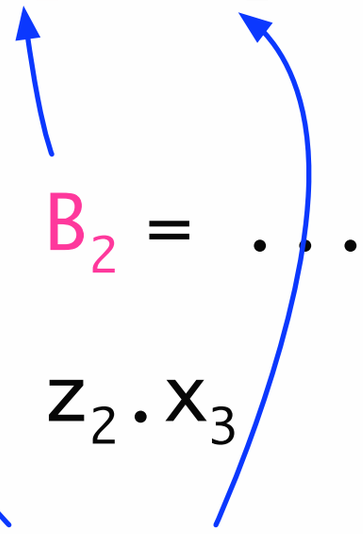

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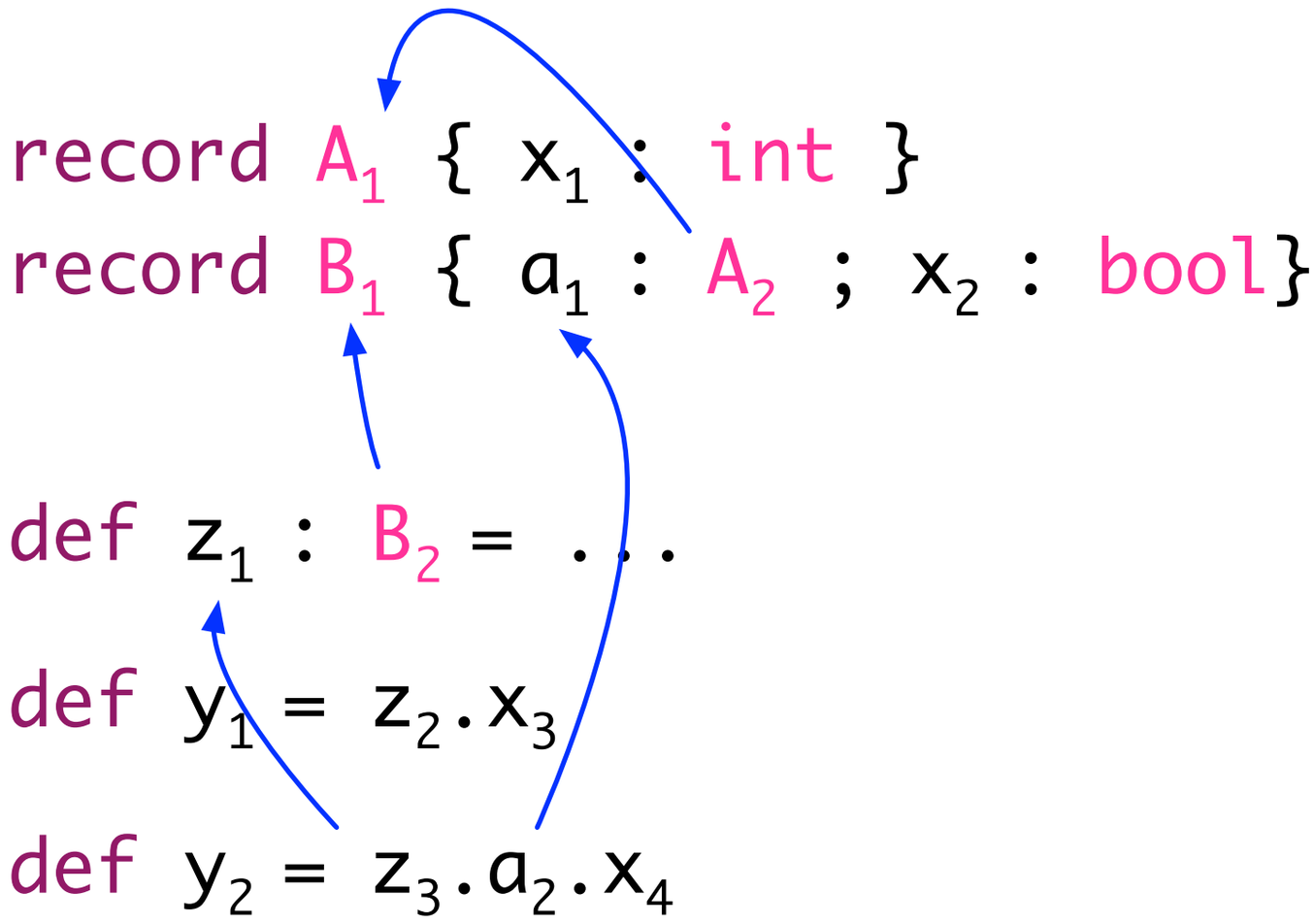

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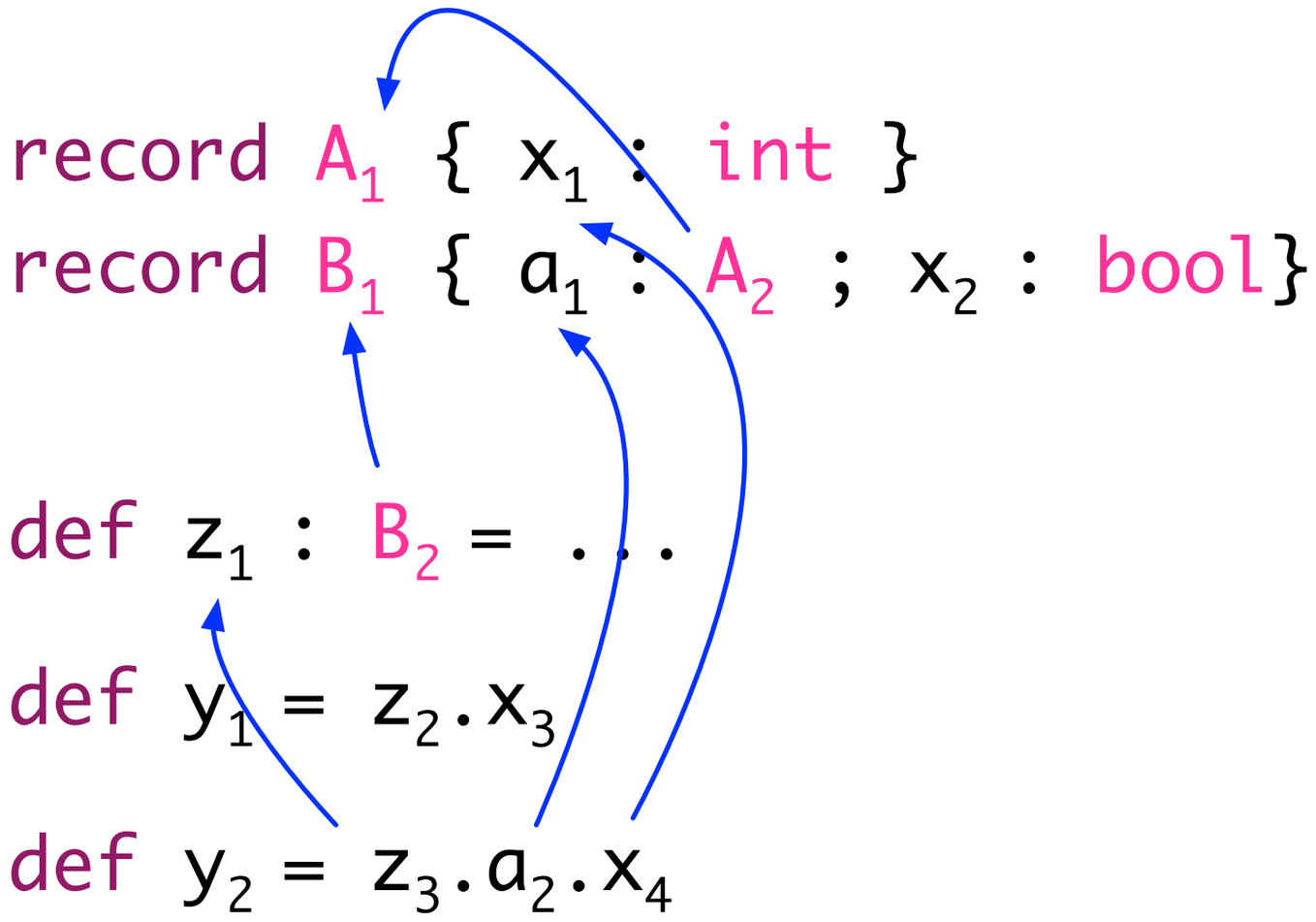

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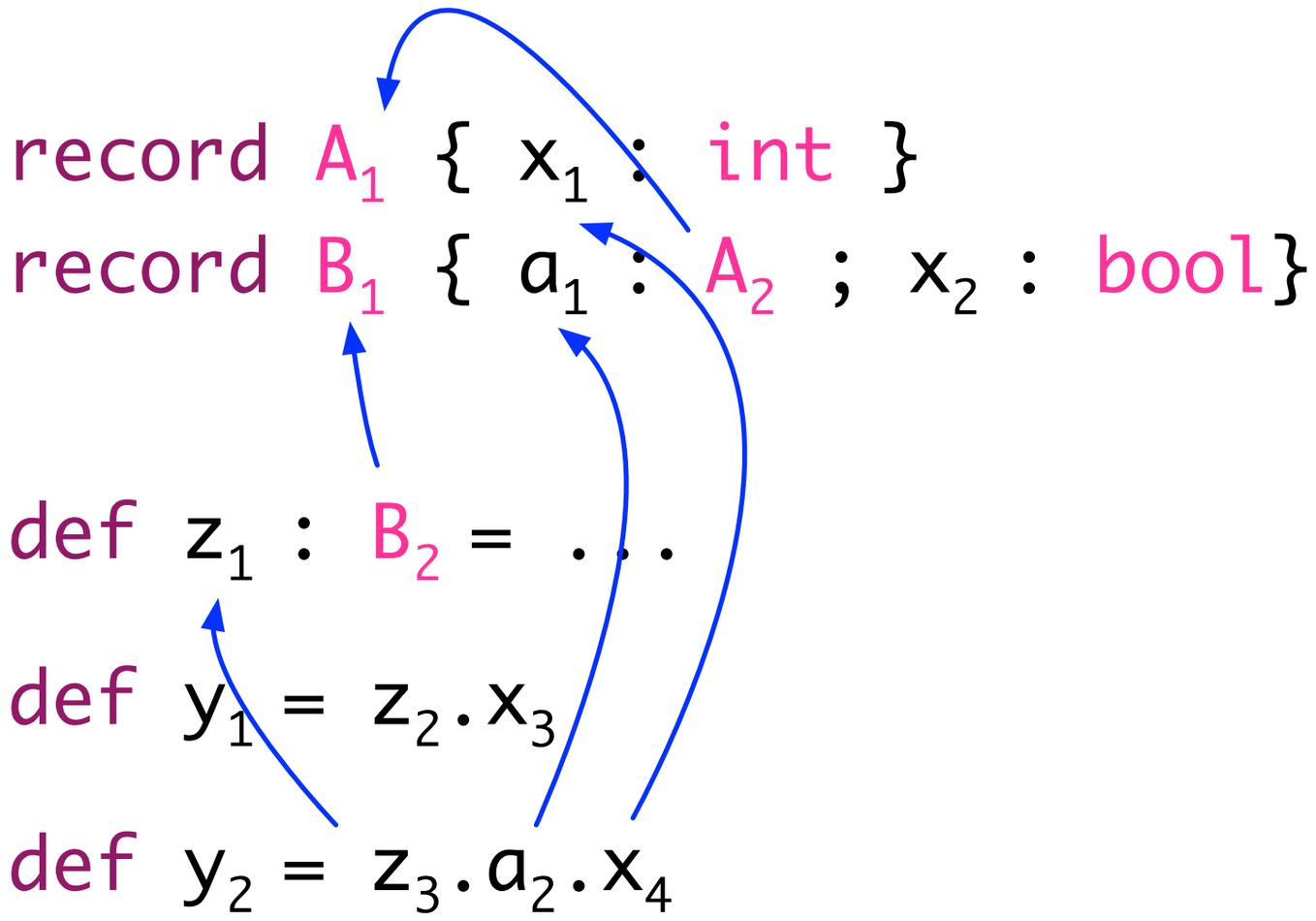

Our approach: interleave **partial** name resolution with type resolution (also using constraints)

Also in the works...

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Use Scope Graphs to Describe Name Resolution

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