CHAPTER III

BOOLEAN ALGEBRA

BOOLEAN VALUES

INTRODUCTION

- Boolean algebra is a form of algebra that deals with single digit binary values and variables.
- Values and variables can indicate some of the following binary pairs of values:
  - ON / OFF
  - TRUE / FALSE
  - HIGH / LOW
  - CLOSED / OPEN
  - 1 / 0

BOOL. OPERATIONS

FUNDAMENTAL OPERATORS

- Three fundamental operators in Boolean algebra
  - **NOT**: unary operator that complements represented as $\overline{\mathbf{A}}$, $\mathbf{A}'$, or $\sim \mathbf{A}$
  - **AND**: binary operator which performs logical multiplication
    - i.e. **A** ANDed with **B** would be represented as $\mathbf{AB}$ or $\mathbf{A} \cdot \mathbf{B}$
  - **OR**: binary operator which performs logical addition
    - i.e. **A** ORed with **B** would be represented as $\mathbf{A} + \mathbf{B}$

NOT

A	$\overline{A}$
0	1
1	0

AND

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

BOOL. OPERATIONS

BINARY BOOLEAN OPERATORS

- Below is a table showing all possible Boolean functions $\mathbf{F_N}$ given the two-inputs **A** and **B**.

A	B	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
		0 Null	AB	Inhibition	A	Inhibition	B	$A \oplus B$	$A + B$	$\overline{A + B}$	$\overline{A \oplus B}$	$\overline{B}$	Implication	$\overline{A}$	Implication	$\overline{AB}$	1 Identity

BOOLEAN ALGEBRA

PRECEDENCE OF OPERATORS

- Boolean expressions must be evaluated with the following order of operator precedence
  - parentheses
  - NOT
  - AND
  - OR

Example:

$$F = (A(\overline{C + \bar{B}D}) + \overline{B\bar{C}})\bar{E}$$

$$F = \underbrace{\underbrace{\left(\underbrace{A\underbrace{\left(\underbrace{\overline{C + \underbrace{\underbrace{\bar{B}}D}}}\right)}} + \underbrace{\overline{\underbrace{B\underbrace{\bar{C}}}}}\right)}\underbrace{\bar{E}}}$$

BOOLEAN ALGEBRA

FUNCTION EVALUATION

- Example 1:

Evaluate the following expression when $\mathbf{A} = \mathbf{1}, \mathbf{B} = \mathbf{0}, \mathbf{C} = \mathbf{1}$

$$\mathbf{F} = \mathbf{C} + \overline{\mathbf{C}}\mathbf{B} + \mathbf{B}\overline{\mathbf{A}}$$

- Solution

$$\mathbf{F} = \mathbf{1} + \overline{\mathbf{1}} \cdot \mathbf{0} + \mathbf{0} \cdot \overline{\mathbf{1}} = \mathbf{1} + \mathbf{0} + \mathbf{0} = \mathbf{1}$$

- Example 2:

Evaluate the following expression when $\mathbf{A} = \mathbf{0}, \mathbf{B} = \mathbf{0}, \mathbf{C} = \mathbf{1}, \mathbf{D} = \mathbf{1}$

$$\mathbf{F} = \mathbf{D}(\mathbf{B}\overline{\mathbf{C}}\mathbf{A} + \overline{(\mathbf{A}\overline{\mathbf{B}} + \mathbf{C})} + \mathbf{C})$$

- Solution

$$\mathbf{F} = \mathbf{1} \cdot (\mathbf{0} \cdot \overline{\mathbf{1}} \cdot \mathbf{0} + \overline{(\mathbf{0} \cdot \overline{\mathbf{0}} + \mathbf{1})} + \mathbf{1}) = \mathbf{1} \cdot (\mathbf{0} + \overline{\mathbf{1}} + \mathbf{1}) = \mathbf{1} \cdot \mathbf{1} = \mathbf{1}$$

BOOLEAN ALGEBRA

BASIC IDENTITIES

$X + 0 = X$	$X \cdot 1 = X$	Identity
$X + 1 = 1$	$X \cdot 0 = 0$	
$X + X = X$	$X \cdot X = X$	Idempotent Law
$X + X' = 1$	$X \cdot X' = 0$	Complement
$(X')' = X$		Involution Law
$X + Y = Y + X$	$XY = YX$	Commutativity
$X + (Y + Z) = (X + Y) + Z$	$X(YZ) = (XY)Z$	Associativity
$X(Y + Z) = XY + XZ$	$X + YZ = (X + Y)(X + Z)$	Distributivity
$X + XY = X$	$X(X + Y) = X$	Absorption Law
$X + X'Y = X + Y$	$X(X' + Y) = XY$	Simplification
$(X + Y)' = X'Y'$	$(XY)' = X' + Y'$	DeMorgan's Law
$XY + X'Z + YZ = XY + X'Z$	$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$	Consensus Theorem

BOOLEAN ALGEBRA

DUALITY PRINCIPLE

- **Duality principle:**
  - States that a Boolean equation remains valid if we take the dual of the expressions on both sides of the equals sign.
  - The dual can be found by interchanging the **AND** and **OR** operators along with also interchanging the **0**'s and **1**'s.
  - This is evident with the duals in the basic identities.
    - For instance: DeMorgan's Law can be expressed in two forms

$(\mathbf{X}+\mathbf{Y})' = \mathbf{X}'\mathbf{Y}'$ as well as $(\mathbf{X}\mathbf{Y})' = \mathbf{X}'+\mathbf{Y}'$

BOOLEAN ALGEBRA

FUNCTION MANIPULATION (1)

- Example: Simplify the following expression

$$\mathbf{F} = \mathbf{BC} + \mathbf{B\overline{C}} + \mathbf{BA}$$

- Simplification

$$\mathbf{F} = \mathbf{B}(\mathbf{C} + \overline{\mathbf{C}}) + \mathbf{BA}$$

$$\mathbf{F} = \mathbf{B} \cdot \mathbf{1} + \mathbf{BA}$$

$$\mathbf{F} = \mathbf{B}(\mathbf{1} + \mathbf{A})$$

$$\mathbf{F} = \mathbf{B}$$

BOOLEAN ALGEBRA

FUNCTION MANIPULATION (2)

- Example: Simplify the following expression

$$\mathbf{F} = \mathbf{A} + \bar{\mathbf{A}}\mathbf{B} + \bar{\mathbf{A}}\bar{\mathbf{B}}\mathbf{C} + \bar{\mathbf{A}}\bar{\mathbf{B}}\bar{\mathbf{C}}\mathbf{D} + \bar{\mathbf{A}}\bar{\mathbf{B}}\bar{\mathbf{C}}\bar{\mathbf{D}}\mathbf{E}$$

- Simplification

$$\mathbf{F} = \mathbf{A} + \bar{\mathbf{A}}(\mathbf{B} + \bar{\mathbf{B}}\mathbf{C} + \bar{\mathbf{B}}\bar{\mathbf{C}}\mathbf{D} + \bar{\mathbf{B}}\bar{\mathbf{C}}\bar{\mathbf{D}}\mathbf{E})$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \bar{\mathbf{B}}\mathbf{C} + \bar{\mathbf{B}}\bar{\mathbf{C}}\mathbf{D} + \bar{\mathbf{B}}\bar{\mathbf{C}}\bar{\mathbf{D}}\mathbf{E}$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \bar{\mathbf{B}}(\mathbf{C} + \bar{\mathbf{C}}\mathbf{D} + \bar{\mathbf{C}}\bar{\mathbf{D}}\mathbf{E})$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \bar{\mathbf{C}}\mathbf{D} + \bar{\mathbf{C}}\bar{\mathbf{D}}\mathbf{E}$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \bar{\mathbf{C}}(\mathbf{D} + \bar{\mathbf{D}}\mathbf{E})$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \bar{\mathbf{D}}\mathbf{E}$$

$$\mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E}$$

BOOLEAN ALGEBRA

FUNCTION MANIPULATION (3)

- Example: Show that the following equality holds

$$\overline{A(\bar{B}\bar{C} + BC)} = \bar{A} + (B + C)(\bar{B} + \bar{C})$$

- Simplification

$$\overline{A(\bar{B}\bar{C} + BC)} = \bar{A} + \overline{(\bar{B}\bar{C} + BC)}$$

$$= \bar{A} + (\overline{\bar{B}\bar{C}})(\overline{BC})$$

$$= \bar{A} + (B + C)(\bar{B} + \bar{C})$$

STANDARD FORMS

SOP AND POS

- Boolean expressions can be manipulated into many forms.
- Some standardized forms are required for Boolean expressions to simplify communication of the expressions.
  - **Sum-of-products** (**SOP**)
    - Example:

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}) = \mathbf{AB} + \overline{\mathbf{B}}\mathbf{C}\overline{\mathbf{D}} + \mathbf{AD}$$

  - **Products-of-sums** (**POS**)
    - Example:

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}) = (\mathbf{A} + \mathbf{B})(\overline{\mathbf{B}} + \mathbf{C} + \overline{\mathbf{D}})(\mathbf{A} + \mathbf{D})$$

STANDARD FORMS

MINTERMS

- The following table gives the minterms for a **three-input** system

			m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
A	**B**	**C**	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$	$\overline{A}B\overline{C}$	$\overline{A}BC$	$A\overline{B}\,\overline{C}$	$A\overline{B}C$	$AB\overline{C}$	ABC
0	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**
0	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**
0	**1**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**
1	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**
1	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**
1	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**

STANDARD FORMS

SUM OF MINTERMS

- **Sum-of-minterms** standard form expresses the Boolean or switching expression in the form of a **sum of products** using **minterms**.
  - For instance, the following Boolean expression using minterms

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \overline{\mathbf{A}}\,\overline{\mathbf{B}}\,\overline{\mathbf{C}} + \overline{\mathbf{A}}\,\overline{\mathbf{B}}\mathbf{C} + \mathbf{A}\overline{\mathbf{B}}\,\overline{\mathbf{C}} + \mathbf{A}\overline{\mathbf{B}}\mathbf{C}$$

could instead be expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = m_0 + m_1 + m_4 + m_5$$

or more compactly

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 5) = \textit{one-set}(0, 1, 4, 5)$$

STANDARD FORMS

MAXTERMS

- The following table gives the maxterms for a **three-input** system

A	B	C	M_0 $A+B+C$	M_1 $A+B+\overline{C}$	M_2 $A+\overline{B}+C$	M_3 $A+\overline{B}+\overline{C}$	M_4 $\overline{A}+B+C$	M_5 $\overline{A}+B+\overline{C}$	M_6 $\overline{A}+\overline{B}+C$	M_7 $\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

STANDARD FORMS

PRODUCT OF MAXTERMS

- **Product-of-maxterms** standard form expresses the Boolean or switching expression in the form of **product of sums** using **maxterms**.
  - For instance, the following Boolean expression using maxterms

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = (\mathbf{A} + \mathbf{B} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \mathbf{C})(\overline{\mathbf{A}} + \overline{\mathbf{B}} + \overline{\mathbf{C}})$$

could instead be expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = M_1 \cdot M_4 \cdot M_7$$

or more compactly as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(1, 4, 7) = \textit{zero-set}(1, 4, 7)$$

STANDARD FORMS

MINTERM AND MAXTERM EXP.

- Given an arbitrary Boolean function, such as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \mathbf{AB} + \overline{\mathbf{B}}(\overline{\mathbf{A}} + \overline{\mathbf{C}})$$

how do we form the canonical form for:

- **sum-of-minterms**
  - Expand the Boolean function into a sum of products. Then take each term with a missing variable $\mathbf{X}$ and **AND** it with $\mathbf{X} + \overline{\mathbf{X}}$.
- **product-of-maxterms**
  - Expand the Boolean function into a product of sums. Then take each factor with a missing variable $\mathbf{X}$ and **OR** it with $\mathbf{X}\overline{\mathbf{X}}$.

STANDARD FORMS

FORMING SUM OF MINTERMS

- Example

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \mathbf{AB} + \overline{\mathbf{B}}(\overline{\mathbf{A}} + \overline{\mathbf{C}}) = \mathbf{AB} + \overline{\mathbf{A}}\overline{\mathbf{B}} + \overline{\mathbf{B}}\overline{\mathbf{C}}$$

$$= \mathbf{AB}(\mathbf{C} + \overline{\mathbf{C}}) + \overline{\mathbf{A}}\overline{\mathbf{B}}(\mathbf{C} + \overline{\mathbf{C}}) + (\mathbf{A} + \overline{\mathbf{A}})\overline{\mathbf{B}}\overline{\mathbf{C}}$$

$$= \overline{\mathbf{A}}\overline{\mathbf{B}}\overline{\mathbf{C}} + \overline{\mathbf{A}}\overline{\mathbf{B}}\mathbf{C} + \mathbf{A}\overline{\mathbf{B}}\overline{\mathbf{C}} + \mathbf{AB}\overline{\mathbf{C}} + \mathbf{ABC}$$

$$= \sum m(0, 1, 4, 6, 7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as 1s in Truth Table

STANDARD FORMS

FORMING PROD OF MAXTERMS

- Example

$$\mathbf{F}(\mathbf{A},\mathbf{B},\mathbf{C}) = \mathbf{AB} + \overline{\mathbf{B}}(\overline{\mathbf{A}} + \overline{\mathbf{C}}) = \mathbf{AB} + \overline{\mathbf{A}}\overline{\mathbf{B}} + \overline{\mathbf{B}}\overline{\mathbf{C}}$$

$$= (\mathbf{A} + \overline{\mathbf{B}})(\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}}) \qquad \text{(using distributivity)}$$

$$= (\mathbf{A} + \overline{\mathbf{B}} + \mathbf{C}\overline{\mathbf{C}})(\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}})$$

$$= (\mathbf{A} + \overline{\mathbf{B}} + \mathbf{C})(\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}})$$

$$= \prod M(2, 3, 5)$$

A	B	C	F	
0	0	0	1	
0	0	1	1	
0	1	0	0	← 2
0	1	1	0	← 3
1	0	0	1	
1	0	1	0	← 5
1	1	0	1	
1	1	1	1	

Maxterms listed as 0s in Truth Table

STANDARD FORMS

CONVERTING MIN AND MAX

- Converting between sum-of-minterms and product-of-maxterms
  - The two are complementary, as seen by the truth tables.
  - To convert interchange the $\sum$ and $\prod$, then use missing terms.
    - Example: The example from the previous slides

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 6, 7)$$

is re-expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(2, 3, 5)$$

where the numbers 2, 3, and 5 were missing from the minterm representation.

SIMPLIFICATION

KARNAUGH MAPS

- STANDARD FORMS
  - -SUM OF MINTERMS
  - -PRODUCT OF MAXTERMS
  - -CONVERTING MIN & MAX

- Often it is desired to simplify a Boolean function. A quick graphical approach is to use Karnaugh maps.

2-variable Karnaugh map

A \ B	0	1
0	0	0
1	0	1

$$F = AB$$

3-variable Karnaugh map

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	1	1

$$F = AB + C$$

4-variable Karnaugh map

AB \ CD	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	1	1	1	1
10	0	1	0	0

$$F = AB + \overline{C}D$$

SIMPLIFICATION

KARNAUGH MAP ORDERING

- Notice that the ordering of cells in the map are such that moving from one cell to an adjacent cell only changes one variable.

2-variable Karnaugh map

A \ B	0	1	
0	0	1	$\bar{A}$
1	2	3	A
	$\bar{B}$	B	

3-variable Karnaugh map

	$\bar{C}$	C	C	$\bar{C}$	
A \ BC	**00**	**01**	**11**	**10**	
0	0	1	3	2	$\bar{A}$
1	4	5	7	6	A
	$\bar{B}$	$\bar{B}$	B	B	

4-variable Karnaugh map

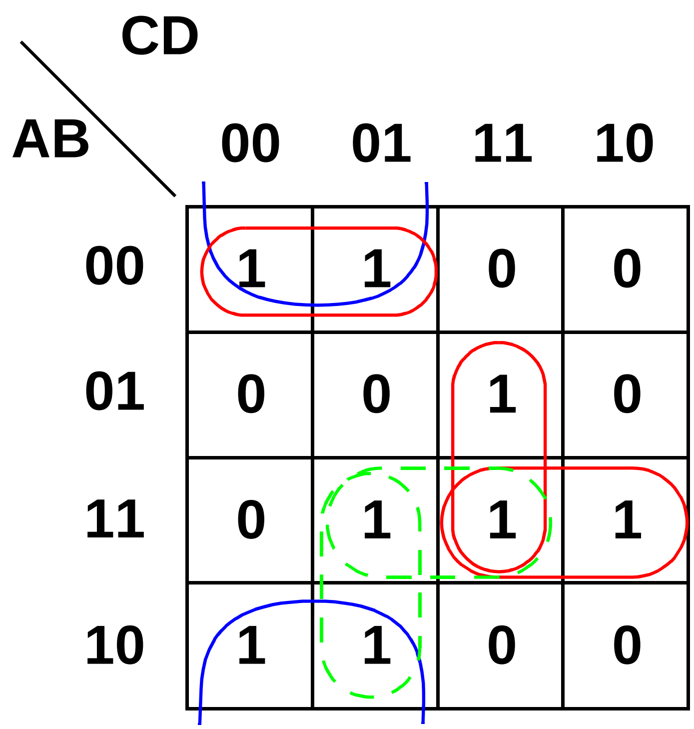
		$\bar{D}$	D	D	$\bar{D}$	
	AB \ CD	**00**	**01**	**11**	**10**	
$\bar{A}$	**00**	0	1	3	2	$\bar{B}$
$\bar{A}$	**01**	4	5	7	6	B
A	**11**	12	13	15	14	B
A	**10**	8	9	11	10	$\bar{B}$
		$\bar{C}$	$\bar{C}$	C	C	

- This ordering allows for grouping of minterms/maxterms for simplification.

SIMPLIFICATION

IMPLICANTS

- **Implicant**
  - Bubble covering only **1**s (size of bubble must be a power of 2).
- **Prime implicant**
  - Bubble that is expanded as big as possible (but increases in size by powers of 2).
- **Essential prime implicant**
  - Bubble that contains a **1** covered only by itself and no other prime implicant bubble.
- **Non-essential prime implicant**
  - A **1** that can be bubbled by more then one prime implicant bubble.



SIMPLIFICATION

PROCEDURE FOR SOP

- **Procedure for finding the SOP from a Karnaugh map**
  - Step 1: Form the 2-, 3-, or 4-variable Karnaugh map as appropriate for the Boolean function.
  - Step 2: Identify all essential prime implicants for **1**s in the Karnaugh map
  - Step 3: Identify non-essential prime implicants for **1**s in the Karnaugh map.
  - Step 4: For each essential and one selected non-essential prime implicant from each set, determine the corresponding product term.
  - Step 5: Form a sum-of-products with all product terms from previous step.

SIMPLIFICATION

EXAMPLE FOR SOP (1)

- Simplify the following Boolean function

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 5) = \overline{\mathbf{A}}\,\overline{\mathbf{B}}\,\overline{\mathbf{C}} + \overline{\mathbf{A}}\,\overline{\mathbf{B}}\mathbf{C} + \mathbf{A}\overline{\mathbf{B}}\,\overline{\mathbf{C}} + \mathbf{A}\overline{\mathbf{B}}\mathbf{C}$$

- Solution:

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

zero-set(*2, 3, 6, 7*)
one-set(*0, 1, 4, 5*)

- The essential prime implicants are $\overline{\mathbf{B}}$.
- There are no non-essential prime implicants.
- The sum-of-products solution is $\mathbf{F} = \overline{\mathbf{B}}$.

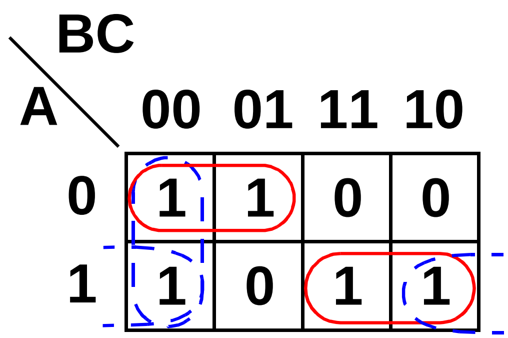
SIMPLIFICATION

EXAMPLE FOR SOP (2)

- Simplify the following Boolean function

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 6, 7) = \overline{\mathbf{A}}\overline{\mathbf{B}}\overline{\mathbf{C}} + \overline{\mathbf{A}}\overline{\mathbf{B}}\mathbf{C} + \mathbf{A}\overline{\mathbf{B}}\overline{\mathbf{C}} + \mathbf{A}\mathbf{B}\overline{\mathbf{C}} + \mathbf{A}\mathbf{B}\mathbf{C}$$

- Solution:



zero-set$(2, 3, 5)$

one-set$(0, 1, 4, 6, 7)$

- The essential prime implicants are $\overline{\mathbf{A}}\overline{\mathbf{B}}$ and $\mathbf{AB}$.
- The non-essential prime implicants are $\overline{\mathbf{B}}\overline{\mathbf{C}}$ or $\mathbf{A}\overline{\mathbf{C}}$.
- The sum-of-products solution is

$$\mathbf{F} = \mathbf{AB} + \overline{\mathbf{A}}\overline{\mathbf{B}} + \overline{\mathbf{B}}\overline{\mathbf{C}} \text{ or } \mathbf{F} = \mathbf{AB} + \overline{\mathbf{A}}\overline{\mathbf{B}} + \mathbf{A}\overline{\mathbf{C}}.$$

SIMPLIFICATION

PROCEDURE FOR POS

- **Procedure for finding the SOP from a Karnaugh map**
  - Step 1: Form the 2-, 3-, or 4-variable Karnaugh map as appropriate for the Boolean function.
  - Step 2: Identify all essential prime implicants for **0**s in the Karnaugh map
  - Step 3: Identify non-essential prime implicants for **0**s in the Karnaugh map.
  - Step 4: For each essential and one selected non-essential prime implicant from each set, determine the corresponding sum term.
  - Step 5: Form a product-of-sums with all sum terms from previous step.

SIMPLIFICATION

EXAMPLE FOR POS (1)

- Simplify the following Boolean function

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(2, 3, 5) = (\mathbf{A} + \overline{\mathbf{B}} + \mathbf{C})(\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}})$$

- Solution:

A \ BC	00	01	11	10
0	1	1	0	0
1	1	0	1	1

zero-set$(2, 3, 5)$

one-set$(0, 1, 4, 6, 7)$

- The essential prime implicants are $\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}}$ and $\mathbf{A} + \overline{\mathbf{B}}$.
- There are no non-essential prime implicants.
- The product-of-sums solution is $\mathbf{F} = (\mathbf{A} + \overline{\mathbf{B}})(\overline{\mathbf{A}} + \mathbf{B} + \overline{\mathbf{C}})$.

SIMPLIFICATION

EXAMPLE FOR POS (2)

- Simplify the following Boolean function

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(0, 1, 5, 7, 8, 9, 15)$$

- Solution:
  - The essential prime implicants are $\mathbf{B} + \mathbf{C}$ and $\overline{\mathbf{B}} + \overline{\mathbf{C}} + \overline{\mathbf{D}}$.
  - The non-essential prime implicants can be $\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{D}}$ or $\mathbf{A} + \mathbf{C} + \overline{\mathbf{D}}$.
  - The product-of-sums solution can be either

$$\mathbf{F} = (\mathbf{B} + \mathbf{C})(\overline{\mathbf{B}} + \overline{\mathbf{C}} + \overline{\mathbf{D}})(\mathbf{A} + \overline{\mathbf{B}} + \overline{\mathbf{D}})$$

or

$$\mathbf{F} = (\mathbf{B} + \mathbf{C})(\overline{\mathbf{B}} + \overline{\mathbf{C}} + \overline{\mathbf{D}})(\mathbf{A} + \mathbf{C} + \overline{\mathbf{D}})$$

zero-set(*0, 1, 5, 7, 8, 9, 15*)

one-set(*2, 3, 4, 6, 10, 11, 12, 13, 14*)

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	0	0	1
11	1	1	0	1
10	0	0	1	1

SIMPLIFICATION

DON’T-CARE CONDITION

- Switching expressions are sometimes given as **incomplete**, or with **don’t-care conditions**.
  - Having don’t-care conditions can simplify Boolean expressions and hence simplify the circuit implementation.
  - Along with the *zero-set*( ) and *one-set*( ), we will also have *dc*( ).
  - Don’t-cares conditions in Karnaugh maps
    - Don’t-cares will be expressed as an “**X**” or “**-**” in Karnaugh maps.
    - Don’t-cares can be bubbled along with the **1**s or **0**s depending on what is more convenient and help simplify the resulting expressions.

SIMPLIFICATION

DON'T-CARE EXAMPLE (1)

- Find the SOP simplification for the following Karnaugh map

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	0	0	1
11	1	X	0	X
10	0	0	1	X

zero-set(*0, 1, 5, 7, 8, 9, 15*)

one-set(*2, 3, 4, 6, 11, 12*)

dc(*10, 13, 14*)

Taken to be 0

Taken to be 1

- Solution:
  - The essential prime implicants are $\mathbf{B\bar{D}}$ and $\mathbf{\bar{B}C}$.
  - There are no non-essential prime implicants.
  - The sum-of-products solution is $\mathbf{F} = \mathbf{\bar{B}C} + \mathbf{B\bar{D}}$.

SIMPLIFICATION

DON'T-CARE EXAMPLE (2)

- Find the POS simplification for the following Karnaugh map

AB \ CD	00	01	11	10
00	**0**	**0**	**1**	**1**
01	**1**	**0**	**0**	**1**
11	**1**	**X**	**0**	**X**
10	**0**	**0**	**1**	**X**

Taken to be 0

Taken to be 1

zero-set(*0, 1, 5, 7, 8, 9, 15*)

one-set(*2, 3, 4, 6, 11, 12*)

dc(*10, 13, 14*)

- Solution:

  - The essential prime implicants are $\mathbf{B} + \mathbf{C}$ and $\overline{\mathbf{B}} + \overline{\mathbf{D}}$.
  - There are no non-essential prime implicants.
  - The product-of-sums solution is $\mathbf{F} = (\mathbf{B} + \mathbf{C})(\overline{\mathbf{B}} + \overline{\mathbf{D}})$.