Simulated Annealing

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PART 1

Introduction

Simulated Annealing

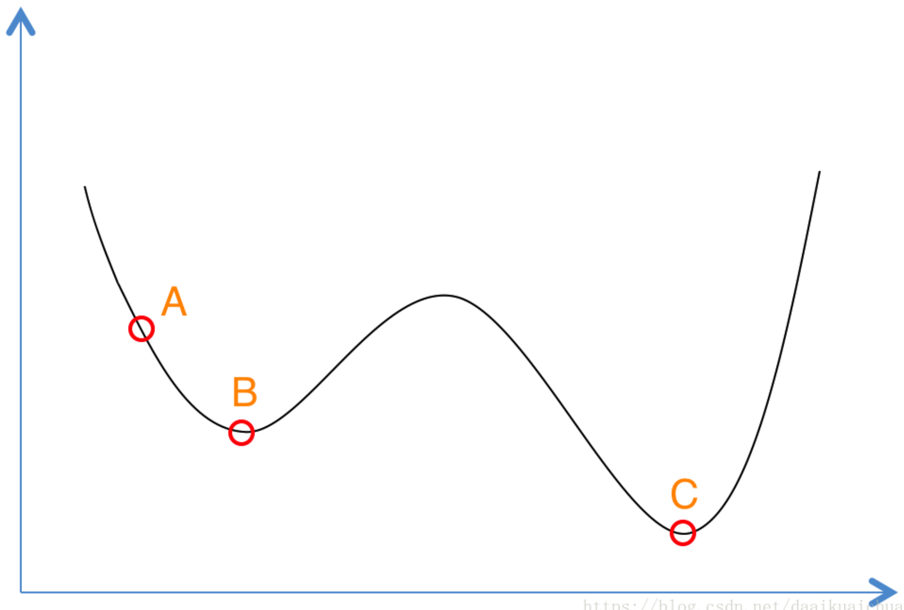
Simulated annealing(SA)：

Simulated annealing is a popular local search meta-heuristic used to address discrete and, to a lesser extent, continuous optimization problems. The key feature of simulated annealing is that it provides a means to escape local optima by allowing hill-climbing moves (i.e., moves which worsen the objective function value) in hopes of finding a global optimum.

Simulated Annealing

NP-hard problem

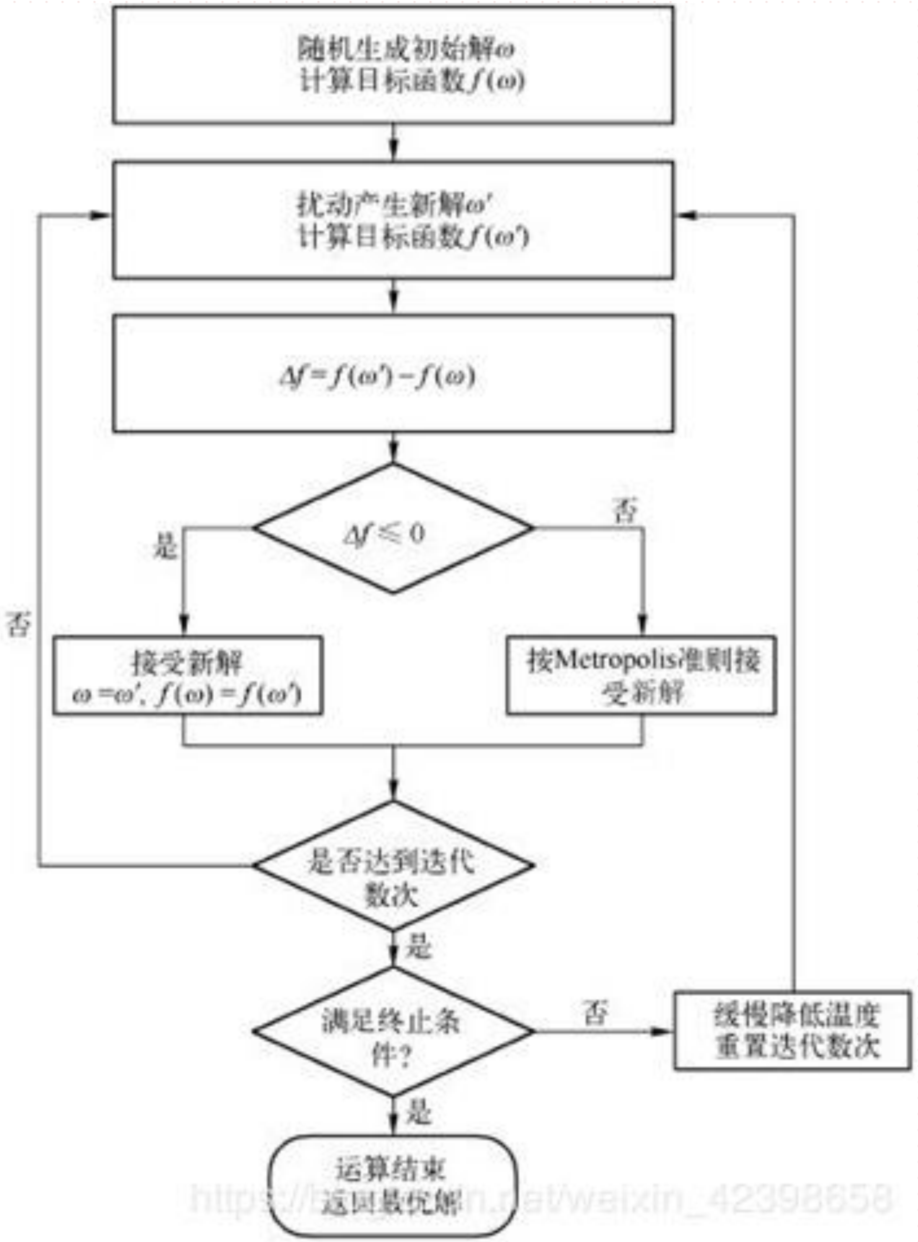
- Traveling Salesman Problem
- Graph Colouring Problem
- Flow shop and job shop scheduling
- Lot sizing



PART 2

Details of SA

Flowchart of SA



Metropolis

Metropolis：

$$P = \begin{cases} 1 & f(\omega') \le f(\omega) \\ e^{-\frac{f(\omega') - f(\omega)}{t_k}} & f(\omega') > f(\omega) \end{cases}$$

Details of SA

Select an initial solution $\omega \in \Omega$

Select the temperature change counter $k=0$

Select a temperature cooling schedule, t_k

Select an initial temperature $T = t_0 \geq 0$

Select a repetition schedule, $\mathbf{M}_k$ that defines the number of iterations executed at each temperature, t_k

Repeat

Set repetition counter $m = 0$

    Repeat

    Generate a solution $\omega' \in \mathrm{N}(\omega)$

    Calculate $\Delta_{\omega,\omega'} = f(\omega') - f(\omega)$

    If $\Delta_{\omega,\omega'} \leq 0$, then $\omega \leftarrow \omega'$

    If $\Delta_{\omega,\omega'} > 0$, then $\omega \leftarrow \omega'$ with probability $\exp(-\Delta_{\omega,\omega'}/t_k)$

    $m \leftarrow m + 1$

    Until $m = \mathrm{M}_k$

$k \leftarrow k + 1$

Until stopping criterion is met

Steps of SA

Effect of Parameters

- Initial temperature t_0： If t_0 is too low at the beginning of the traditional simulated annealing phase, the algorithm can get trapped in an inferior solution, while if the initial temperature is too high, the algorithm can waste too many iterations (hence computing time) by accepting too many hill-climbing moves.
- Iterations executed at each temperature: Global search performance of simulated annealing algorithm is strongly correlated with iterations executed at each temperature. Sufficient search at the same temperature is quite necessary, but it also takes time to compute.
- Temperature cooling schedule: The setting of temperature cooling schedule is very important, the most common way is :

$$t_{k+1} = \alpha \times t_k \qquad \alpha \in (0,1)$$

Stopping Condition

- Maximum CPU time
- Maximum number of iterations
- Current temperature is lower than the threshold temperature

PART 3

Advantages and disadvantages

Advantages and disadvantages

Advantages:

- Iterative search is efficient and can be parallelized.
- There is a certain probability to accept a worse solution than the current solution, so it may jump out of the local optimal solution.
- The algorithm is robust.
- It is a global optimization algorithm that can converge theoretically.

Disadvantages:

- Convergence speed will be slow if cooling is too slow, the global optimal solution probably not be found if cooling is too fast.
- The algorithm performance is related to the initial parameter value.