Local Search and Constraint Programming

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PART 1

Introduction

Local Search and Constraint Programming

Real-world combinatorial optimization problems:

- They are usually large (real-world crew scheduling applications)
- They are not pure (involve a heterogeneous set of side constraints)

→ Exact approaches cannot be applied to solve real-world problems

Local Search and Constraint Programming

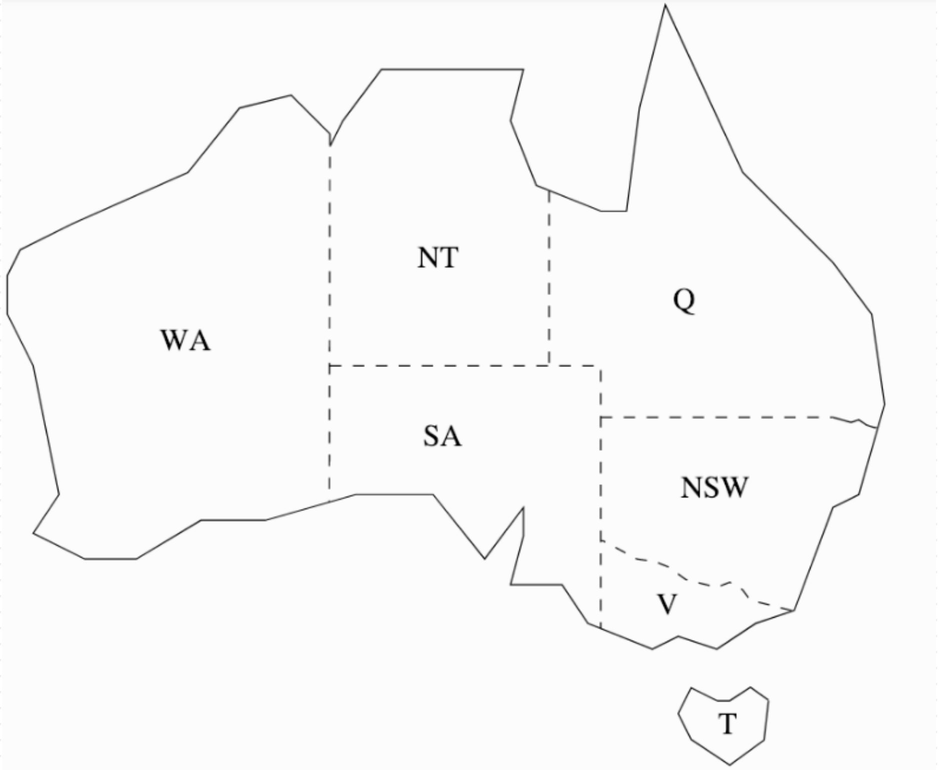
Three important issues of dealing with real-world problems：

- Huge problems require large neighborhoods whose exploration can be computationally expensive
- When dealing with problems involving many heterogeneous side constraints it is often preferable to consider them as hard constraints rather than transforming them into penalty functions.
- Real-world applications typically lead to frequent update/addition of constraints (recall again union contract regulations), thus the algorithmic approach requires flexibility.

Local Search and Constraint Programming

CP: Modeling and solving real-world combinatorial optimization problems

- Vehicle Routing Problem
- Graph Colouring Problem
- Job Shop Scheduling
- Airline Scheduling Problem



Local Search and Constraint Programming

- LS may use ideas from CP in order to make large neighborhoods more tractable.
- CP may use ideas from LS to explore a set of solutions close to the greedy path in a tree search and converge more quickly towards the optimum.

PART 2

A case of CP

Constraint Programming

CP includes:

- Constraint Satisfaction $\begin{cases} \mathit{Constraint\ Propagation} \\ \mathit{Domain\ Reduction} \\ \ldots \end{cases}$

- Constraint Solving

Variables

Vehicle Routing Problem:

- $i, j \in (1, \ldots, N)$ for locations (clients)
- $k \in (1, \ldots, M)$ for trucks (routes)
- $h \in (1, \ldots, 2M)$ for bins
- $l \in (1, \ldots, \mathrm{P})$ for types of goods

Constraints

Vehicle Routing Problem:

- Each route is associated to a truck, and starts and ends at the depot;
- Each client is visited exactly once, and within the time window;
- The bins’ capacity constraints are respected

CP model of Vehicle Routing Problem

$$\text{min} \quad totCost = \sum_{k=1}^{M} cost_k$$

on

$$\begin{aligned}
\forall k \in \{1,\ldots,M\} \quad & cost_k \geq 0, \\
& truck_k = UnaryResource(tt, c, cost_k) \\
\forall h \in \{1,\ldots,2M\} \quad & collects_h \in [1..P] \\
\forall i \in \{1,\ldots,N\} \quad & start_i \in [a_i..b_i], \\
& service_i = Activity(start_i, d_i, i), \\
& visitedBy_i \in [1..M], \\
& collectedIn_i \in [1..2M]
\end{aligned}$$

subject to

$$\forall i \in \{1,\ldots,N\} \quad service_i \text{ requires } truck[visitedBy_i] \tag{1}$$

$$\forall h \in \{1,\ldots,2M\} \quad \sum_{i \mid collectedIn_i = h} q_i \leq C \tag{2}$$

$$\forall i \in \{1,\ldots,N\} \quad collects[collectedIn_i] = type_i \tag{3}$$

$$\forall i \in \{1,\ldots,N\} \quad visitedBy_i = \left\lceil \frac{collectedIn_i}{2} \right\rceil \tag{4}$$

Propagation

$$\inf\left(start_j\right) + d_j + tt_{j\mathrm{i}} > \sup(start_i)$$

$$\wedge \inf(start_{\mathrm{i}}) + d_i + tt_{ij} > \sup\left(start_j\right)$$

$$\wedge\, value(visitedBy_i) = k$$

$$\Longrightarrow \quad visitedBy_j \neq k$$

Propagation

$$\forall i, j \in \{1, \ldots, N\} \qquad next_i = j \Rightarrow$$

$$succ_i = \{j\} \cup succ_j \tag{11}$$

$$\forall i, j \in \{1, \ldots, N\} \qquad j \in succ_i \Rightarrow$$

$$visitedBy_i = visitedBy_j \tag{12}$$

$$\forall k \in \{1, \ldots, M\}, \forall i \in \{1, \ldots, N\} \qquad first_k = i \Rightarrow$$

$$visitedBy_i = k \tag{13}$$

A Case of Vehicle Routing Problem

It is impractical to solve the model by complete global search

- Insertion Algorithms
- Greedy Insertion
- Discrepancy-based Search

PART

3

Characteristics of CP

Characteristics of CP

- Efficient
- Flexible
- It can reflect only part of the characteristics of the object
- Constraints can be established between different fields
- Constraint has no direction (X can restrict Y, Y can restrict X)
- Constraints are stackable
- Constraints are often not single, a variable can have multiple constraints