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Advanced Programming

Algebraic Data Types

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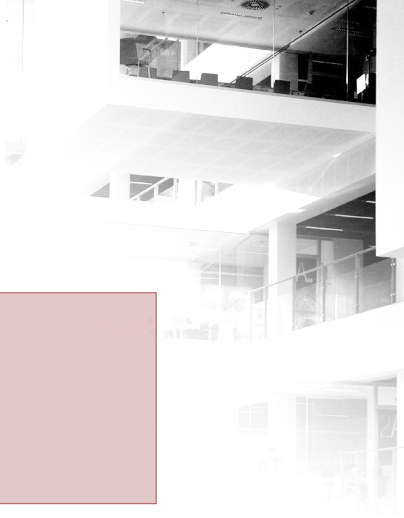
SOFTWARE QUALITY RESEARCH



- Algebraic Data Types
- Functional lists and trees
- Fold functions
- Style guidelines for functional programming in ADPRO

AGENDA

Algebraic Data Types (ADTs) as Enums and Classes

Enums less general, but used more often

Def. Algebraic Data Type

A type generated by one or more constructors, each taking zero or more arguments.

The sets of objects generated by each constructor are **summed** (unioned), each constructor can be seen as a representation of a Cartesian **product** (tuple) of its arguments; thus the name **algebraic**.

ADT as enum

```
1 enum List[+A]:
2   case Nil
3   case Cons(head: A, tail: List[A])
```

ADT as case class hierarchy

```
1 sealed trait List[+A]
2 case object Nil extends List[Nothing]
3 case class Cons[+A] (head: A, tail: List[A]) extends List[A]
```

sealed: extensible in the same file only

`Nothing`: **subtype of any type**

Algebraic Data Types (ADTs)

Def. Algebraic Data Type

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The sets of objects generated by each constructor are **summed** (unioned), each constructor can be seen as a representation of a Cartesian **product** (tuple) of its arguments; thus the name **algebraic**.

Example: lists

```
1 enum List[+A]:
2   case Nil
3   case Cons(head: A, tail: List[A])
```

operations on lists

```
1 object List:
2   def sum(ints: List[Int]): Int = ints match
3     case Nil => 0
4     case Cons(xxs) => x + sum(xs)
5   def apply[A](as: A*): List[A] =
6     if as.isEmpty then Nil
7     else Cons(as.head, apply(as.tail*))
```

companion object of `List[+A]`

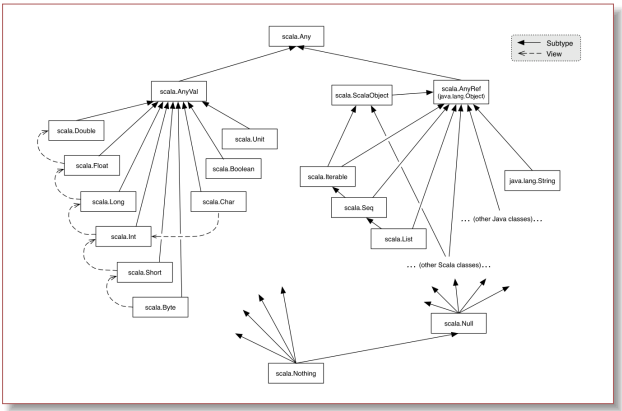
pattern matching against case constructors

overloading function application for the object

variadic function

Lists are covariant

All share the same tail!



For any type `A` we have that

`Nil <:List[Nothing] <:List[A]`

```
1 sealed trait List[+A]
2 case object Nil extends List[Nothing]
3 case class Cons[+A] (head: A, tail: List[A]) extends List[A]
```

Poll: How is your recursion?

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```
1 def f (a: List[Int]): Int = a match
2   case Nil => 0
3   case Cons(h, t) => h + f(t)
```

What is `f(List(42, -1, 1, -1, 1, -1))` ?

Function Values

- In functional programing **functions are values**
- Functions can be **passed to other functions**, composed, etc.
- Functions operating on function values are **higher order** (HOFs)

```
1  def map(a: List[Int])(f: Int => Int): List[Int] = a match
2    case Nil => Nil
3    case Cons(h, tail) => Cons(f(h), map(tail)(f))
```

A functional (pure) example

```
1  val mixed = List(-1, 2, -3, 4)
2  map(mixed)(abs)
```

```
1  map(mixed)((factorial) compose (abs))
```

An imperative (impure) example

```
1  val mixed = Array(-1, 2, -3, 4)
2  for i <- 0 until mixed.length do
3    mixed(i) = abs(mixed(i))
```

```
1  val mixed1 = Array(-1, 2, -3, 4)
2  for i <- 0 until mixed1.length do
3    mixed1(i) = factorial(abs(mixed1(i)))
```

Parametric Polymorphism

Monomorphic functions operate on fixed types:

A monomorphic map in Scala

```
def map(a: List[Int])(f: Int => Int): List[Int] = a match
  case Nil => Nil
  case Cons(h, tail) => Cons(f(h), map(tail)(f))
```

There is nothing specific here regarding Int.

A polymorphic map in Scala

```
def map[A, B](a: List[A])(f: A => B): List[B] = a match
  case Nil => Nil
  case Cons(h, tail) => Cons(f(h), map(tail)(f))
```

An example of use:

```
1 map[Int, String] (mixed) {
2   (_.toString) compose (factorial) compose (abs) }
```

- A **polymorphic** function operates on values of (m)any types
- A polymorphic **type constructor** defines a parameterized family of types
- Don't confuse with OO-polymorphism AKA "**dynamic dispatch**" (dependent on the inheritance hierarchy)

HOFs in the Standard Library

Methods of class `List[A]`, operate on `this` list, type `A` is bound in the class

`map[B](f: A =>B): List[B]`
Translate `this` list of `A`s into a list of `B`s using `f` to convert the values

`filter(p: A =>Boolean): List[A]`
A sublist of `this` containing elements satisfying predicate `p`

`flatMap[B](f: A =>List[B]): List[B]`
Apply `f` to elements of `this` and concatenate the produced lists

*type slightly simplified

`take(n: Int): List[A]`
A list of first `n` elements of `this`.

`takeWhile(p: A =>Boolean): List[A]`
A prefix of `this` containing elements satisfying `p`

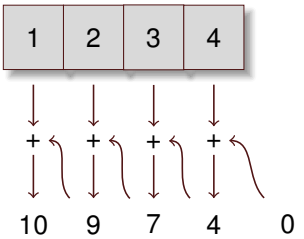
`forall(p: A =>Boolean): Boolean`
True iff `p` holds for all elements of `this`

`exists(p: A =>Boolean): Boolean`
True iff `p` holds for at least one element of `this`

More at `https://www.scala-lang.org/api/current/scala/collection/immutable/List.html`

Folds: Functional Loops

Sum of a list



What characterizes folds ?

- An **input list** `l = List(1,2,3,4)`
- An **initial value** `z = 0`
- A **binary operation** `f: (Int,Int) => Int = _ + _`
- An **iteration algorithm**

```
1  def foldRight[A,B](f: (A,B) => B)(z: B)(l: List[A]): B =
2    l match
3      case Cons(x, xs) => f(x, foldRight(f)(z)(xs))
4      case Nil => z
5
6  val l1      = List(1,2,3,4,5,6)
7  val sum     = foldRight[Int,Int](_+_)(0)(l1)
8  val product = foldRight[Int,Int](_*_)(1)(l1)
9  def map[A,B] (f: A => B) (l: List[A]): List[B] =
10   foldRight[A,List[B]]((x, z) => Cons(f(x), z))(Nil)(l)
```

Many HOFs are special cases of folding

Preferred Programming Style in ADPRO

Always choose the best possible style for an exercise and your abilities

Condemned (fail)	→ **Forgivable (medium grade*)**	→ **Enlightened (top grade)**
variables <		< values
assignments <		< value bindings
return statement <		< expression value
Any/Object type <		< parametric polymorphism
loops <	tail recursion* < simple recursion < folds*	< compose dedicated HOFs
	if conditions < pattern matching*	< use dedicated API
exceptions <		< Option or Either monad

* unless asked for explicitly, or really important for memory use.

Scala: Summary

- **Basics** (objects, modules, functions, expressions, values, variables, operator overloading, infix methods, interpolated strings.)
- **Pure functions** (referential transparency, side effects)
- **Loops and recursion** (tail recursion)
- **Functions as values** (higher-order functions)
- **Parametric polymorphism** (monomorphic functions, dynamic and static dispatch)
- **Standard HOFs** in Scala's library
- **Anonymous functions** (currying, partial function application)
- **Traits** (fat interfaces, multiple inheritance, mixins)
- **Algebraic Data Types** (pattern matching, case classes)
- **Folding**