Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : **End Semester Examination – March 2022**
Name of the Course : **B.Tech (Information Technology & Mathematical Innovation)**
Name of the Paper : **Seeing the world through Calculus. First steps through symbolic mathematics**
Paper Code : **32861101**
Semester : **I**
Duration : **3 Hours**
Maximum Marks : **75**
Instructions:
This question paper contains six questions, out of which any four are to be attempted. Each question carries equal marks.

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Q1) Sketch the graph of the following function.

$$P(x) = \ 4x^3 - 6x^2 - 24x + 5$$

State all the necessary concepts you have learnt such as point of inflection, relative or absolute maxima/minima, concavity, increasing or decreasing, etc.

Q2) Discuss the convergence or divergence of the sequence $< a_n >$ defined by

$a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$. Find the limit if it exists.

Q3) The function $f(x, y)$ defined as follows

$$f(x,y) = \begin{cases} \dfrac{3xy^2}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$$

Is $f(x, y)$ continuous at (0, 0) ? Is $f(x, y)$ continuous everywhere?

Q4) The revenue function in dollars is $r(x) = 500 + 660x$ and the cost function in dollars is $c(x) = \frac{-x^2}{2} + 1000x$ from producing *x* couches. Find the total and average profit functions. Graph the average profit function. Find the asymptotes for the graph of average profit.

Q5) Evaluate the integral $\int_0^b \int_{x/b}^{\sqrt{x/b}} (x^3 + y^3) dxdy$ by changing the order of integration.

Q6) Let x be the amount spent on advertising, in millions of dollars, and y be the price charged per item of the product, in dollars. The profit, in millions of dollars, of a one-product company is given by $P(x, y) = \ 60 - 15y^2 - \frac{x^2 y}{10} + 80y + 2xy$. Find the maximum value of P and the values of x and y at which it is attained.