Date :- 23/03/2022

Examination Roll No. :- 2131291501 7

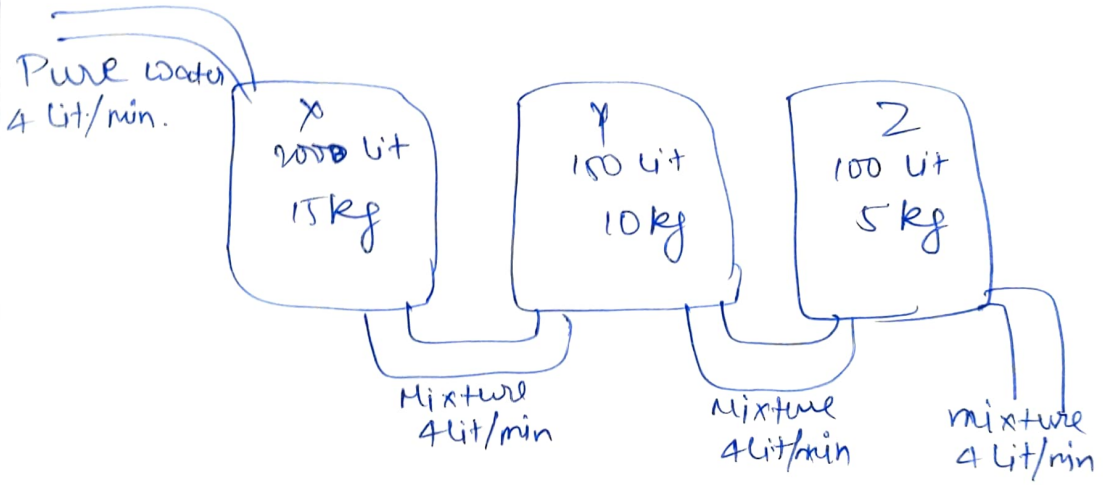
Name of Program :- B. Tech (Information Tech. and Mathematical Innovation).

Semester :- 1st Sem.

Unique Paper Code :- 32861102

Title of the Paper :- Modeling continous change through odinary Differential Equation.

Answer 3



$$\Delta x = r_{in} C_{in} \Delta t - r_{out} C_{out} \Delta t$$

$$\Rightarrow \Delta x = 0 - \frac{x}{200} \times 4 \times \Delta t$$

$$\frac{\Delta x}{\Delta t} = -\frac{x}{50} \Rightarrow \frac{dx}{dt} = -\frac{x}{50}$$

$$\Rightarrow \ln(x) = -\frac{1}{50} t + c$$

$$x = k e^{-t/50}$$

A/Q @ $t = 0$, $x = 15$

$$x = k e^0 \Rightarrow 15 = k$$

$$\Rightarrow \boxed{x = 15 e^{-t/50}}$$

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$$\Delta y = 4 \times \frac{x}{50} \times \Delta t - 4^2 \times \frac{y}{75} \times \Delta t$$

$$\frac{\Delta y}{\Delta t} = \frac{x}{50} - \frac{2y}{75}$$

$$\frac{dy}{dt} = \frac{x}{50} - \frac{2y}{75}$$

$$\frac{dy}{dt} + \frac{2y}{75} = \frac{3}{10} e^{-t/50}$$

$$\frac{dy}{dt} + \frac{2y}{75} = \frac{3e^{-t/50}}{10}$$

$$I.F = e^{\int \frac{2}{75} dt} = e^{\frac{2t}{75}}$$

$$y \cdot e^{2t/75} = \int \frac{3}{10} e^{-t/50} \cdot e^{2t/75} \, dt$$

$$y e^{2t/75} = \frac{3}{10} \int \left(e^{\frac{2t}{75} - \frac{t}{50}} \right) dt$$

$$y e^{2t/75} = \frac{3}{10} \int e^{+t/150} \, dt$$

$$y e^{2t/75} = \frac{3}{10} \times 150 \, e^{t/150} + C$$

$$y = 45 e^{\frac{t}{150} - \frac{2t}{75}} + C e^{-2t/75}$$

$$y = 45 e^{-t/50} + C e^{-2t/75}$$

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@ $t=0, y=10$

$10 = 45 + C$

$C = -35$

$\Rightarrow \boxed{y = 45e^{-t/50} - 35e^{-2t/75}}$

Now

$\Delta z = 4 \times \frac{y}{150} \Delta t - \frac{4 \times z}{100} \Delta t$

$\frac{\Delta z}{\Delta t} = \frac{2y}{75} - \frac{z}{25}$

$\frac{dz}{dt} + \frac{z}{25} = \frac{2}{75}\left(45e^{-t/50} - 35e^{-2t/75}\right)$

I.F $\Rightarrow e^{\int \frac{1}{25} dt} = e^{t/25}$

$z e^{t/25} = \int e^{t/25} \cdot \frac{2}{75}\left(45e^{-t/50} - 35e^{-2t/75}\right) dt$

$z e^{t/25} = \frac{2}{75}\int \left(45e^{\frac{-t}{50} + \frac{t}{25}} - 35e^{\frac{-2t}{75} + \frac{t}{25}}\right) dt$

$z e^{t/25} = \frac{2}{75}\int \left(45e^{t/50} - 35e^{t/75}\right) dt$

$z e^{t/50} = \frac{2}{75}\left[45 \times 50\, e^{t/50} - 35 \times 75\, e^{t/75}\right]$

$z e^{t/50} = \frac{2 \times 45 \times 50}{75}\, e^{t/50} - \frac{35 \times 75 \times 2}{75}\, e^{t/75}$

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$$ze^{t/25} = 60e^{t/50} - 70e^{t/75} + C$$

$$z = 60e^{-t/50} - 70e^{-2t/75} + ce^{-t/25}$$

(a) $t \to 0 \quad z = 5$

$$5 = 60 - 70 + C$$

$$\Rightarrow C = 15.$$

$$\boxed{z = 60e^{-t/50} - 70e^{-2t/75} + 15e^{-t/25}}$$

when $t \longrightarrow \infty$

$$x = 15e^{-t/5}$$

$x \to 0. \ (t \to \infty)$

$$y = 45e^{-t/50} - 35e^{-2t/75}$$

$y \to 0 \ (t \to \infty)$

$$z = 60e^{-t/50} - 70e^{-2t/75} + 15e^{-t/25}$$

$z \to 0 \ (t \to \infty)$.

$\Rightarrow$ At $t \to \infty$ the salts in all three tanks i.e., X, Y, Z will get finished.