Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : End Term Examination, May 2023
Name of the Course : B.Tech (Information Technology and Mathematical Innovations)
Name of the Paper : Linear Construction of actions: Engineering through linear programming and Game Theory
Paper Code : 32861601
Semester : VI
Duration : 2 Hours
Maximum Marks : 40

Attempt any 5 questions. Each question carries 8 marks. [8x5=40]

1. Bank One is in the process of devising a loan policy that involves a maximum of $12 million. The following table provides the pertinent data about available loans.

Type of loan	Interest rate	Bad-debt ratio
Personal	.140	.10
Car	.130	.07
Home	.120	.03
Farm	.125	.05
Commercial	.100	.02

Bad debts are unrecoverable and produce no interest revenue. Competition with other financial institutions dictates the allocation of at least 40% of the funds to farm and commercial loans. To assist the housing industry in the region, home loans must equal at least 50% of the personal, car, and home loans. The bank limits the overall ratio of bad debts on all loans to at most 4%. Assume that all loans are issued at approximately the same time, which allows to ignore differences in the time value of the funds allocated to the different loans. Formulate the Linear programming model for Bank One to maximize the net profit (Interest revenue is accrued on loans in good standing).

2. Perform one iteration of cutting plane method for the following linear problem. Also show graphically whether or not each of the following constraints can form a legitimate cut:

$$\text{Maximize } z = 7x_1 + 10x_2$$

subject to

$$-x_1 + 3x_2 \leq 6$$
$$7x_1 + x_2 \leq 35$$
$$x_1, x_2 \geq 0 \text{ and integers}$$

3. A Company needs to assign four jobs to four workers. The cost of performing a job is a function of the skills of the workers. Table (given below) summarizes the cost of the assignments. Worker 1 cannot do job 3, and worker 3 cannot do job 4

		Job			
		1	2	3	4
Worker	1	$50	$50	—	$20
	2	$70	$40	$20	$30
	3	$90	$30	$50	—
	4	$70	$20	$60	$70

If an additional (fifth) worker becomes available for performing the four jobs at the respective costs of $60, $45, $30, and $80. Determine the optimal job assignment to each of the five workers.

4. Three orchards supply crates of oranges to four retailers. The daily demand amounts at the four retailers are 150, 150, 400, and 100 crates, respectively. Supplies at the three orchards are dictated by available regular labor and are estimated at 150, 200, and 250 crates daily. The transportation costs per crate from the orchards to the retailers are given in Table

	Retailer			
	1	2	3	4
Orchard 1	$1	$2	$3	$2
Orchard 2	$2	$4	$1	$2
Orchard 3	$1	$3	$5	$3

Formulate the problem as a transportation model and perform 1 iteration of modified distribution method to find the optimum solution.

5. Two companies promote two competing products. Currently, each product controls 50% of the market. Because of recent improvements in the two products, each company plans to launch an advertising campaign. If neither company advertises, equal market shares will continue. If either company launches a stronger campaign, the other company is certain to lose its customers. Company A advertises in radio (A_1) and television (A_2). Company B, in addition to using radio (B_1), television (B_2), also mails brochures (B_3). The following matrix summarizes the percentage of the market captured or lost by company A

	B_1	B_2	B_3
A_1	3	4	1
A_2	-2	-3	3

What course of action should both the companies will follow to advertise their products.

6. HiTech produces two models of electronic gadgets that use resistors, capacitors, and chips. The following table summarizes the data of the situation:

Resource	Unit resource requirements		Maximum availability (units)
	Model 1 (units)	*Model 2 (units)*	
Resistor	2	3	1200
Capacitor	2	1	1000
Chips	0	4	800
Unit price ($)	3	4	

Let x_1 and x_2 be the amounts produced of Models 1 and 2, respectively. Following are the LP model and its associated optimal simplex tableau.

$$\text{Maximize } z = 3x_1 + 4x_2$$

subject to

$$2x_1 + 3x_2 \leq 1200 \quad \text{(Resistors)}$$
$$2x_1 + x_2 \leq 1000 \quad \text{(Capacitors)}$$
$$4x_2 \leq 800 \quad \text{(Chips)}$$
$$x_1, x_2 \geq 0$$

Basic	x_1	x_2	s_1	s_2	s_3	Solution
z	0	0	$\frac{5}{4}$	$\frac{1}{4}$	0	1750
x_1	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	450
s_3	0	0	-2	2	1	400
x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	100

Determine the status of each resource and in terms of the optimal revenue, determine the dual prices for the resistors, capacitors, and chips. Also if the available number of resistors is increased to 1300 units, find the new optimum solution.