Roll No.: 38

Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination	: End Semester Examination – December 2019
Name of the Course	: B. Tech. (I.T. & Mathematical Innovations)
Name of the Paper	: Modeling continuous changes through ordinary differential equations
Paper Code	: 32861102
Semester	: I
Duration	: 2 Hours
Maximum Marks	: 40

Instructions:

1. Write your Roll No. on top of the question paper immediately on receipt of this question paper.

2. Attempt any FIVE questions.

Q 1. Find the Frobenius series solution of [8]

$$2x^2y''+3xy'-(x^2+1)y=0.$$

Q 2. (a) Explain radius of convergence of a power series. Find the radius of convergence of the following series. [3]

$$\sum_{n=0}^{\infty}\frac{(-1)^n.2^n}{n!}x^n$$

(b) Consider a mass-spring-dashpot system with $m=\frac{1}{2}$, $k=17$, and $c=3$ (**Fig.1**). Let $x(t)$ denote the displacement of the mass m from its equilibrium position. If the mass is set in motion with $x(0)=x'(0)=0$ and with the imposed external force $f(t)=15\sin 2t$, find the resulting transient motion and steady periodic motion of the mass. [5]

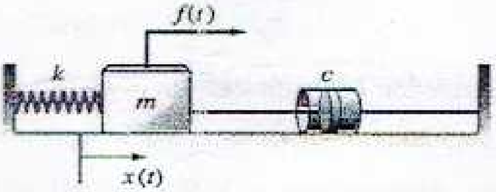

Fig. 1

Q. 3 (a) Transform the following second-order system into an equivalent system of first-order differential equations **[3]**

$$2y'' = -6y + 2x$$
$$x'' = 2y - 2x + 40\sin 3t.$$

(b) The following **Fig. 2** shows three brine tanks containing $V_1 = 30$, $V_2 = 15$, and $V_3 = 10$ gallons of brine, respectively. Fresh water flows into tank 1, while mixed brine flows from tank 1 into tank 2, from tank 2 into tank 3, and out of tank 3. Let $x_i(t)$ denote the amount (in pounds) of salt in tank i at time t for $i = 1, 2,$ and 3. If flow rate $r = 30$ (gal/min), and intial amounts of salt in the three brine tanks, in pounds, are $x_1(0) = 27$, $x_2(0) = x_3(0) = 0$. Find the amount of salt in each tank at time $t \geq 0$. **[5]**

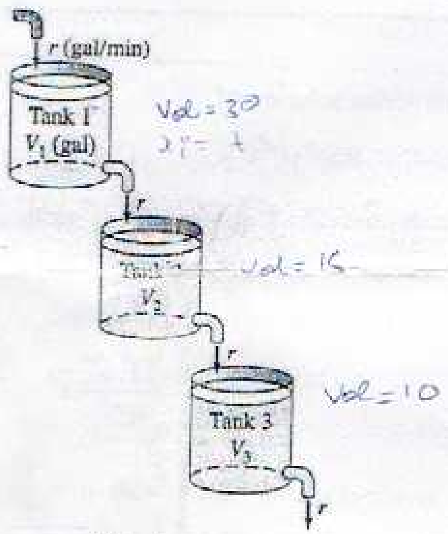

Fig. 2

Q. 4 (a) Solve the following initial value problem using method of undetermined coefficients. **[5]**

$$y'' - 3y' + 2y = 3e^{-x} - 10\cos 3x$$
$$y(0) = 1,\ y'(0) = 2.$$

(b) Consider an animal population $P(t)$ with constant death rate $\delta = 0.01$ (deaths per animal per month) and with birth rate β proportional to P. Suppose that $P(0) = 200$ and $P'(0) = 2$. **[3]**

(i) When is $P = 1000$?

(ii) When does doomsday occur?

Q. 5 **(a)** The roots of the characteristic equation of a certain differential equation are $0, 2, 3, -5, 0, 0, 0, 1, -5, 2\pm 3i$ and $2\pm 3i$. Write a general solution of this homogeneous differential equation. **[3]**

(b) The circuit in **Fig. 3** can be described by the equation **[5]**

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{R_1 C} \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

where i_L is the current passing through the inductor L and v_C is the voltage drop across the capacitor C. Suppose R_1 is 5 ohms, R_2 is 0.8 ohm, C is 0.1 farad, and L is 0.4 henry. Find formulas for i_L and v_C, if the initial current through the inductor is 3 amperes and the initial voltage across the capacitor is 3 volts.

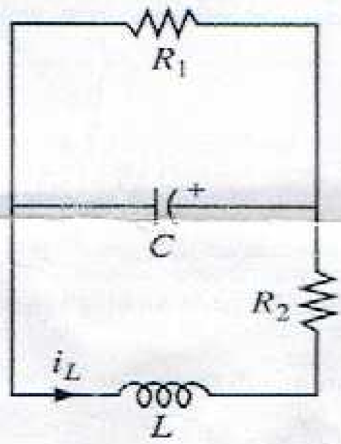

Fig. 3

Q. 6 **(a)** Determine the eigenvalues and associated eginfunctions for the endpoint problem

$$y'' + \lambda y = 0 \text{ with } y(0) = y'(1) = 0. \qquad [4]$$

(b) Consider the first-order differential equation **[4]**

$$xy' = 2y, \ \ y(a) = b. \qquad (1)$$

(i) Under what conditions can we be sure that a solution to (1) exists?

(ii) Under what conditions can we be sure that there is a unique solution to (1)? Justify your answers.

Q. 7 (a) Solve the following initial value problem using Laplace transformation method. **[3]**

$$y''+4y'+8y=e^{-t}$$
$$y(0)=2,\ y'(0)=-1.$$

(b) Consider two species (of animals, plants, or bacteria, for instance) with populations $x(t)$ and $y(t)$ at time t and which compete with each other for the food available in their common environment. We assume that competition has the effect of a rate of decline in each population that is proportional to their product xy. The populations $x(t)$ and $y(t)$ satisfy the following differential equations

$$\frac{dx}{dt}=60x-4x^2-3xy,$$
$$\frac{dy}{dt}=42y-2y^2-3xy.$$

Find all critical points of the above competition system, and investigate the type and stability of each. **[5]**

Q. 8 (a) A lunar lander is falling freely toward the surface of the moon at a speed of 450 meters per second (m/s). Its retrorockets, when fired, provide a constant deceleration of 2.5 meters per second per second (m/s^2) (the gravitational acceleration produced by the moon is assumed to be included in the given deceleration). At what height above the lunar surface should the retrorockets be activated to ensure a "soft touchdown" ($v=0$ at impact)? **[4]**

(b) Find a solution of the following differential equation using method of variation of parameters. **[4]**

$$y''+4y=\sin^2 x$$