

# Cluster Innovation Centre, University of Delhi, Delhi-110007

## End Semester Examination – May 2023

Name of the Course : B.Tech (Information Technology and Mathematical Innovations)  
Semester : IV  
Paper Title : Does nature play dice: The amazing world of Probability & Statistics  
Paper Code : 32861401  
Maximum Marks : 40  
Instructions : All questions are compulsory. Each question carries 10 marks. Use of scientific calculator is allowed but not on mobile. Duration: 2 Hours

Q1) Attempt any two parts

- a. On a ten-question multiple-choice test there are five possible answers for each question, of which one is correct and four are incorrect. If a student guesses randomly and independently, find the probability of the student getting at least 8 correct answers? If each correct answer fetches +4 marks and each wrong answer or an unattempted question fetch -1 mark, what is the expected marks of a student giving the test? (5 marks)
- b. The number of surface flaws in plastic panels used in the interior of automobiles has 2 flaws per square foot of plastic panel. Assume an automobile interior contains 10 square feet of plastic panel, what is the probability that there are no surface flaws in an auto's interior? If 10 cars are sold to a rental company, what is the probability that at most one car has any surface flaws? (5 marks)
- c. CIC has 160 laptops. After the reopening of CIC post COVID, batteries of some laptops had drained out. The probability of the battery draining out is 0.4 and is independent of other laptops. The system administrator checks the laptops, one after another, to see if the battery has drained out. What is the probability that he has to test at least 5 laptops to find the first defective one? If he continues checking, what is the probability that that he would find the fourth defective laptop on checking the 20<sup>th</sup> laptop? (5 marks)

Q2) Attempt any two parts

- a. Two dice have the numbers marked as shown in the figure. Let  $X$  denotes the sum of the numbers shown on the two die. Write the probability mass function for  $X$ . Find the expected value of  $X$  and its standard deviation. (5 marks)

1		2
3	3	4
1		6
3	5	6
		8

- b. The life  $X$  (in years) of a refrigerator is given by the function

$$f(x) = kx(10-x)^4, 0 < x < 10$$

If  $f(x)$  is a pdf what is the value of  $k$ ? What is the probability that the refrigerator will last at least for 7 years? What is the expected life of the refrigerator? (5 marks)

- c. The moment generating function of a continuous random variable  $X$  is given by

$$m_x(t) = \frac{1}{(1-2t)^3}, \quad t < \frac{1}{2}$$

Find the mean and standard deviation of  $X$ .

(5 marks)

3  
216x8  
1296

Q3) Attempt any two parts

a. A poll is conducted to find out the popularity of Mahinder Singh Dhoni and Virat Kohli among the college going students. They were asked to rate the cricketers from 1 to 5; 5 being highly popular and 0 being not popular. The rankings given by the students is a random variable.  $X$  denotes the ranking for Mahinder Singh Dhoni and  $Y$  for Virat Kohli. The results of 50 students are shown in the following table.

X	5	4	3	2	1	0
Y						
4	11	4	2	1	1	1
3	8	3	2	1	1	
2	4	3	2	1		
1	3	1				
0	1					

Find the marginal distribution of  $X$  and  $Y$ . Find the average ranking for Virat Kohli given that the ranking of Mahinder Singh Dhoni is 4. (5 marks)

b. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 millimeters and a standard deviation of 0.2 millimeter.

- If the specifications require the thickness to be between 4.5 and 5.5 millimeters, what proportion of coverings do not meet specifications?
- The covering thickness of 95% of samples is below what value? (5 marks)

c. Consider the function  $f(x, y) = c(x + y)$  for  $0 \leq x \leq 3$  and  $x \leq y \leq x + 2$ . Determine the value of  $c$  such that the function  $f(x, y) = c(x + y)$  for  $0 \leq x \leq 3$  and  $x \leq y \leq x + 2$  satisfies the properties of a joint probability density function. Determine the marginal probability distribution of the random variable  $X$  and  $Y$  and state whether  $X$  and  $Y$  are independent or not. (5 marks)

Q4) Attempt any one part

a. Medical researchers have developed a new artificial heart constructed primarily of titanium and plastic. The heart will last and operate almost indefinitely once it is implanted in the patient's body, but the battery pack needs to be recharged about every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of these batteries is 4.05 hours. Assume that battery life is normally distributed with standard deviation of 0.2 hour. Is there evidence to support the claim that mean battery life exceeds 4 hours at 5% level of confidence? Compute the power of the test if the true mean battery life is 4.5 hours. What sample size would be required to detect a true mean battery life of 4.5 hours if we wanted the power of the test to be at least 0.9? (10 marks)

b. A feed dealer buys 20% protein feed from a feed manufacturer and resells the feed to local farmers. The feed dealer is interested in checking to make certain that the feed does not average less than 20% protein. He draws a sample of 10 bags and finds that the average protein content is 19.87%. Is there evidence to support the claim that the feed content is less than 20% at 5% level of confidence? What is the  $P$ -value for the test? What is the Type II error for the test if the true mean life is 42 hours? (10 marks)



$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

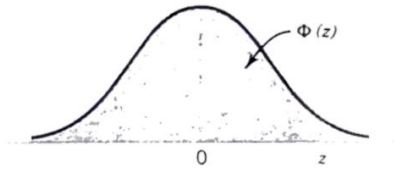


Table Cumulative Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967