Date :- 23/03/2022
Examination Roll No. :-21312915017
Name of Program :- B.Tech. (Information Tech.
and Mathematical Innovation).
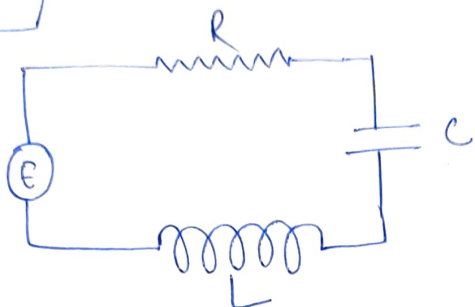Semester :- 1st Sem.
Unique Paper Code :- 32861102
Title of the Paper :- Modeling continous changes
through odinary differential equation.

Solution 4

A/Q,

$E(t) = 100 \sin(60t)$

$Q(0) = 0$; $I(0) = Q'(0) = 0$

$R = 2\,\Omega$; $L = 0.1\,H$; $C = \dfrac{1}{260}\,F$

we know that,

$$L\theta''(t) + R\theta'(t) + \frac{Q}{C} = E(t)$$

$\Rightarrow$ $0.1\,\theta''(t) + 2\theta'(t) + Q \times 260 = 100 \sin 60(t)$

$\underline{or}$ $\theta'' + 20\theta' + 26000 = 1000 \sin(60t)$ ——①

For Laplace Transformation of ①,

$\mathcal{L}(\theta') = s\,\mathcal{L}(\theta) - Q(0)$

$\qquad = s\,\mathcal{L}(\theta)$ ——ⓐ

$\mathcal{L}(\theta'') = s^2\,\mathcal{L}(\theta) - sQ(0) - \theta'(0)$

$\qquad = s^2\,\mathcal{L}(\theta)$ ——ⓑ

$\mathcal{L}\left(\sin(60t)\right) = \dfrac{60}{s^2 + (60)^2}$ ——ⓒ

Substituting ⓐ,ⓑ & ⓒ in ①

$$s^2 \mathcal{L}(\theta) + 20 s \mathcal{L}(\theta) + 2600 \mathcal{L}(\theta) = \frac{1000 \times 60}{s^2 + (60)^2}$$

$$\mathcal{L}(\theta) \left(s^2 - 20 s + 2600 \right) = \frac{60,000}{s^2 + (60)^2}$$

$$\mathcal{L}(\theta) = \frac{60,000}{\left(s^2 + 3600 \right) \left(s^2 - 20 s + 2600 \right)}$$

$$\mathcal{L}(\theta) = \frac{60,000}{\left(s^2 + 3600 \right) \left((s+10)^2 + (50)^2 \right)}$$

Let,

$$\frac{60,000}{\left(s^2 + 3600 \right) \left((s+10)^2 + (50)^2 \right)} = \frac{As + B}{s^2 + 3600} + \frac{Cs + D}{(s+10)^2 + (50)^2}$$

$$\Rightarrow 60,000 = (As + B)\left((s+10)^2 + (50)^2 \right) + (Cs + D)\left(s^2 + 3600 \right)$$

$$\Rightarrow s^3 (A + C) + s^2 (B + 20A + D) + s(2600 A + 20 B + 3600 C) + 2600 B + 3600 D = 60,000$$

$$A + C = 0 \qquad\qquad \text{——ⓐ}$$
$$B + 20A + D = 0 \qquad\qquad \text{——ⓑ}$$
$$2600 A + 20 B + 3600 C = 0 \qquad \text{——ⓒ}$$
$$2600 B + 3600 D = 60,000 \qquad \text{——ⓓ}$$

On solving ⓐ, ⓑ, ⓒ & ⓓ,

$$A = \frac{-30}{60} \; ; \; B = \frac{-1500}{61} \; ; \; C = \frac{30}{61} \; ; \; D = \frac{2100}{61}$$

$$\mathcal{L}(Q) = \frac{-30}{61}\left[\frac{s+50}{s^2+3600}\right] + \frac{30}{61}\left[\frac{s+70}{(s+10)^2+(50)^2}\right]$$

$$= -\frac{30}{61}\left(\frac{s}{s^2+3600}\right) + \left(\frac{-30\times50}{60\times61}\right)\cdot\left(\frac{60}{s^2+(60)^2}\right) +$$

$$\frac{30}{61}\left(\frac{(s+10)}{(s+10)^2+(50)^2}\right) + \frac{30\times60}{61\times50}\left(\frac{50}{(s+10)^2+(50)^2}\right)$$

$$\boxed{\begin{array}{l} Q(t) = \dfrac{-30}{61}\cos(60t) - \dfrac{25}{61}\sin(60t) \\[2ex] \qquad + \dfrac{30}{61}e^{-10t}\cos(50t) + \dfrac{36}{61}e^{-10t}\sin(50t) \end{array}}$$