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$\boxed{\text{Solution 5}}$

$$U = \epsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right] \quad \text{--- (1)}$$

$$\frac{dU}{dr} = \epsilon\left[\left(-12\,\frac{r_0^{12}}{r^{13}}\right) - \left(2(-6)\,.\,\frac{r_0^{6}}{r^{7}}\right)\right] \quad \text{--- (11)}$$

$$= \epsilon\left[-12\left(\frac{r_0}{r}\right)^{12}\frac{1}{r} + 12\left(\frac{r_0}{r}\right)^{6}\frac{1}{r}\right]$$

$$\frac{dU}{dr} = 12\,\epsilon\left(\frac{r_0}{r}\right)^{6}\frac{1}{r}\left[-\left(\frac{r_0}{r}\right)^{6} + 1\right] \quad \text{--- } \cancel{(11)}$$

For extrima (i.e., minima)

$$\frac{dU}{dr} = 0$$

$$\Rightarrow \frac{12\epsilon}{r}\left(\frac{r_0}{r}\right)^{6}\left(-\left(\frac{r_0}{r}\right)^{6} + 1\right) = 0$$

$$\Rightarrow \left(1 - \left(\frac{r_0}{r}\right)^{6}\right) = 0$$

$$\Rightarrow \left(\frac{r_0}{r}\right)^{6} = 1 \quad \Rightarrow \boxed{r_0 = r}$$

Susbtituting $(r_0 = r)$ in eq (1)

$$U = \epsilon\left(\left(\frac{\cancel{r_0}}{r_0}\right)^{12} - 2\cancel{\left(\frac{r_0}{r_0}\right)^{6}}\right] = \epsilon(1-2)$$

$$\boxed{U_{min} = -\epsilon}$$

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On differentiating eq (1)

$$\frac{d^2U}{dr^2} = \epsilon\left[\frac{(-12)(-13)r_0^{12}}{r^{14}} + \frac{12(-7)r_0^6}{r^8}\right]$$

$$\frac{d^2U}{dr^2} = \epsilon\left[\frac{156\, r_0^{12}}{r^{14}} - \frac{84 r_0^6}{r^8}\right]$$

$$\frac{d^2U}{dr^2} = \frac{12\epsilon r_0^6}{r^8}\left[\frac{13 r_0^6}{r^6} - 7\right]$$

$$\left.\frac{d^2U}{dr^2}\right|_{r=r_0} = \frac{12\epsilon \cancel{r_0^6}}{r_0^{\cancel{8}2}}\left[\frac{13\cancel{r_0^6}}{\cancel{r_0^6}} - 7\right]$$

$$= \frac{12\epsilon \times 6}{r_0^2}$$

$$\boxed{K = \frac{72\epsilon}{r_0^2}}$$

→ frequency of ocillation.

we have

$$T = 2\pi\sqrt{\frac{K}{\mu}} \quad \text{where } \mu = \frac{m_1 m_2}{m_1+m_2} \text{ for } m_1 = m_2 = m.$$

$$\mu = \frac{m}{2}.$$

$$T = 2\pi\sqrt{\frac{72\epsilon}{r_0^2}\times\frac{2}{m}}$$

$$T = 2\pi\,\frac{12}{r_0}\sqrt{\frac{\epsilon}{m}}$$

$$\Rightarrow f = \frac{1}{T}$$

$$\Rightarrow \boxed{f = \frac{r_0}{24\pi}\sqrt{\frac{m}{\epsilon}}}$$