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Title of the paper :- Seeing the world through Calculas.

Solution 6

$$P(x,y) = 60 - 15y^2 - \frac{x^2 y}{10} + 80y + 2xy.$$

$$\frac{\partial P}{\partial x} = 0 - 0 - \frac{2xy}{10} + 0 + 2y$$

$$\frac{\partial P}{\partial x} = 2y - \frac{xy}{5}$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{-y}{5}$$

Now

$$\frac{\partial P}{\partial y} = 0 - 30y - \frac{x^2}{10} + 80 + 2x$$

$$\frac{\partial P}{\partial y} = 80 - 30y - \frac{x^2}{10} + 2x$$

$$\frac{\partial^2 P}{\partial y^2} = -30$$

$$\frac{\partial^2 P}{\partial x \partial y} = 2 - \frac{x}{5}.$$

Now finding the stationary Point.

Put $\frac{\partial P}{\partial x} = 0$

$$2y - \frac{xy}{5} = 0$$

$$10y - xy = 0$$

$$y(10 - x) = 0$$

$y = 0$ OR $x = 10$

Now $\frac{\partial P}{\partial y} = 0$

$$80 - 30y - \frac{x^2}{10} + 2x = 0$$

At $x = 10$,

$$80 - 30y - 10 + 20 = 0$$

$$90 - 30y = 0 \Rightarrow \underline{y = 3}$$

At $y = 0$

$$80 - 0 - \frac{x^2}{10} + 2x = 0$$

$$x^2 - 20x - 800 = 0$$

$$(x - 40)(x + 20) = 0$$

$x = 40$ OR, $x = -20$.

So we have three stationary points.

$(10, 3)$; $(40, 0)$; $(-20, 0)$

Now, checking whether the pts are maxima minima or saddle point.

finding $\left(\frac{\partial^2 P}{\partial x^2}\right) \cdot \left(\frac{\partial^2 P}{\partial y^2}\right) - \left(\frac{\partial^2 P}{\partial x \partial y}\right)^2$

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for (10,3):

$\Delta = \frac{-3}{5} \times (-30) - \cancel{(2-2)^{2}}\ 0$

$= 18 > 0.$

(10,3) is not a saddle point.

for (40,0)

$\Delta = 0(-30) - \left(2 - \frac{\cancel{40}\ 8}{\cancel{5}}\right)^2$

$= 0 - (-6)^2 = -36 < 0$

$\Rightarrow$ (-20,0) is a saddle point.

for (-20,0)

$\Delta = 0(-20) - \left(2 + \frac{\cancel{(-20)}\ 4}{\cancel{5}}\right)$

$= 0 - (6)^2 = -36 < 0$

(-20,0) is a saddle point.

Checking weather (10,3) is maxima or minima.

$\frac{\partial^2 p}{\partial x^2} = \frac{-3}{5}$ & $\frac{\partial^2 p}{\partial y} = -30$ ~~(cond^n)~~.

$\Rightarrow \frac{\partial^2 p}{\partial x^2} < 0$ & $\frac{\partial^2 p}{\partial y} < 0$

$\Rightarrow$ (10,3) is a maxima.

P(x,y) is maximum at (10,3).

$P_{max} = 60 - (15 \times 9) - 30 + 240 + 60$

$= 360 - 135 - 30$

$= 195$ Ans