Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination	**: End Semester Examination – Nov/Dec 2021**
Name of the Course	**: B.Tech (Information Technology and Mathematical Innovations)**
Name of the Paper	**: Complexity and Symmetry in Mathematics: Complex Analysis and Algebra**
Paper Code	**: 32861501**
Semester	**: V**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Instructions:

This question paper contains six questions, out of which any four are to be attempted. Each question carries equal marks.

---

1. Consider the function

$$f(z) = \begin{cases} z^5/|z|^4 & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

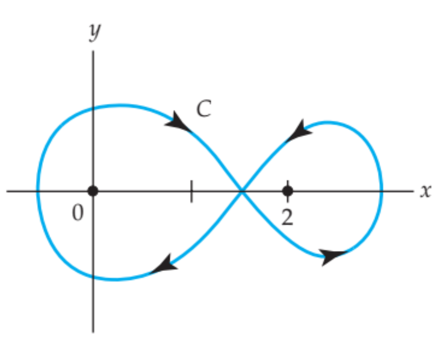
Show that this function is not analytic at $z = 0$ although the Cauchy Riemann's equations are satisfied at origin. Also show that function $f(z) = e^y \sin x + ie^y \cos x$ is nowhere differentiable.

2. Find the analytic function $f(z) = u + iv$, for which following conditions are satisfied

(a) $u + v = x^3 + 3x^2y - 3xy^2 - y^2 + 4x + 5$ and

(b) $f(0) = 2 + 3i$.

3. Evaluate $\displaystyle\int_C \frac{3z+1}{z(z-2)^2}dz$, where C is the contour given in figure below



and also evaluate $\oint_C (z^3 + z^2 + Re(z))dz$, where C is the triangle with vertices $z = 0$, $z = 1 + 2i$ and $z = 1$.

4. Use Cauchy's residue theorem to evaluate the integral

$$\oint_C \frac{z+1}{z^2(z-2i)}dz$$

along the contours (a) $|z| = 1$, (b) $|z - 2i| = 1$, (c) $|z - 2i| = 4$.

5. Check whether set $G = \{(a, b) \mid a, b \in Z\}$ under * operation, defined by

$$(a, b) * (c, d) = (ac + bd, ad + bc),$$

forms a group or not.

6. Let G be a group in which

$$(ab)^3 = a^3b^3$$
$$(ab)^5 = a^5b^5, \text{ for all } a, b \in G$$

Show that G is an abelian group.