Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : End Semester Examination – Dec 2021
Name of the Course : B.Tech (Information Technology and Mathematical Innovations)
Name of the Paper : Modeling Change in the world around us: Partial Differential Equation
Paper Code : 32863101
Semester : III
Duration : 3 Hours
Maximum Marks : 75
Instructions:
This question paper contains six questions (and two sections), out of which any four are to be attempted, two from each section. Each question carries equal marks.

SECTION I

Q1) Find the PDE whose solution is given by $f(z^2 - xy, x^2 + y^2) = 0$. Classify whether the PDE $(4zy - x)p + (2z^2 + x)q = 2y + z$ is linear, non-linear, semi linear or quasi-inear. Also find the solution of the PDE $(4zy - x)p + (2z^2 + x)q = 2y + z$.

Q2) Classify the following PDE as parabolic, hyperbolic or elliptic. Reduce it to its canonical form and hence find its solution

$$x\frac{\partial^2 u}{\partial x^2} + (x+y)\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0$$

Q3) A function $f(x)$ is periodic with period 2 and in the interval $-1 < x < 1$ is defined as

$$f(x) = \begin{cases} 1+x & -1 < x < 0 \\ 2 & 0 < x < 1 \end{cases}$$

Find the Fourier Series for the function. Also, find the value of the function at $x = 0$ and $x = 1$.

SECTION II

Q4) The voltage distribution in an electric transmission line is given by $v_t = v_{xx}$, $0 < x < 1$, $t > 0$. A voltage equal to zero is maintained at $x = 1$, while at the end $x = 0$, the voltage varies according to the law $v(0, t) = Ct$, $t > 0$, where C is a constant. Find $v(x, t)$ if the initial voltage distribution is zero.

Q5) Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = r^2 \sin\theta, \quad 0 < r < 1, \ 0 < \theta < 2\pi$ associated with the boundary condition $u(1, \theta) = \sin 2\theta$.

Q6) Consider a vibrating string of fixed length π of constant tension T and density ρ such that $c^2 = T/\rho$ is a constant. The initial displacement of the string is 0 whereas the initial velocity is $\sin 2x$. Model the problem with an appropriate BVP and find its solution.