Examination : End Semester Examination – Dec 2019
Name of the Course : B.Tech. (Information Technology & Mathematical Innovations)
Name of the Paper :Seeing the world through Calculus.First steps through symbolic mathematics
Paper Code : 32861101
Semester : I
Duration : 2 Hours
Maximum Marks : 40
Instruction: All questions are compulsory. There is internal choice for some questions.

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1. A wire of length L is to be cut into two pieces, one of which will be bent to form a circle and
the other to form a square. Determine how the wire should be cut to   (3.5 X 2 7)

a) Maximize the sum of the areas enclosed by the two pieces.
b) Minimize the sum of the areas enclosed by the two pieces.

OR

A manufacturer of food-storage containers makes a cylindrical can with a volume of 500
milliliters (mL: 1 mL = 1 cm^3). What dimensions (height and radius) will minimize the material
needed to produce each can, that is, minimize the surface area?   (7)
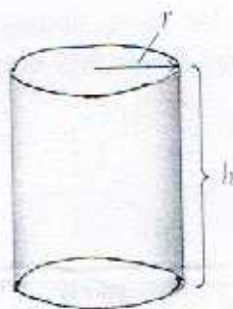
2. The cubical metal tank shown below has a parabolic gate held in place by bolts and designed
to withstand a fluid force of 160 lb without rupturing. The liquid you plan to store has a weight-
density of 50 lb/ft^3.   (3.5 X 2=7)

a) What is the fluid force on the gate when the liquid is 2 ft deep?

b) What is the maximum height to which the container can be filled without exceeding the gate's design limitation?

4 ft

4 ft

4 ft

Parabolic gate
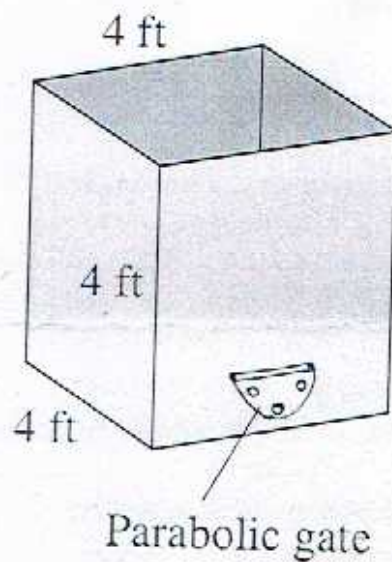
3. A bicycle wheel has radius a and a reflector is attached at a point P on a spoke of the bicycle wheel at a fixed distance d from the centre. Find parametric equations for the path traced by a point P as the wheel rolls along a straight line without slipping.

(7)

4. A golf ball is hit with an initial speed of 116 ft/sec at an angle of elevation of 45° from the tee to a green that is elevated 45 ft above the tee as shown in the diagram. Assuming that the pin, 369 ft downrange, does not get in the way, where will the ball land in relation to the pin?

(8)

Pin

Green

116 ft/sec

45°

45 ft

Tee

369 ft

NOT TO SCALE
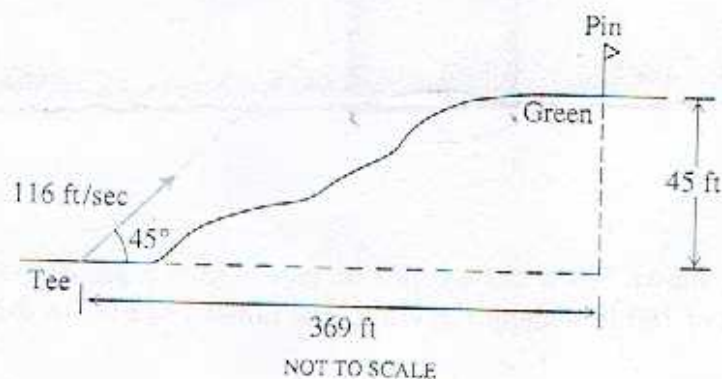
OR

Precisely 1.00 s after you kick it from ground level, a soccer ball passes just above the outstretched hands of a goaltender who is 5.00 m away from you, and lands on the goal line before bouncing into the net for the winning goal in a soccer match. The goaltender's fingertips are 2.50 m above the ground. Neglect air resistance, and use $g = 9.81$ m/s^2.

(a) At what angle did you launch the ball? (3)

(b) What was the ball's initial speed? (2)

(c) Assuming the ground is completely level, how long does the ball spend in the air? (3)

5. An object moves in the force field $\vec{F} = y^2\vec{i} + 2(x + 1)y\vec{j}$. How much work is performed as the object moves from the point $(2, 0)$ counterclockwise along the elliptical path $x^2 + 4y^2 = 4$ to $(0,1)$, then back to $(2, 0)$ along the line segment joining the two points, as shown in figure. (6)
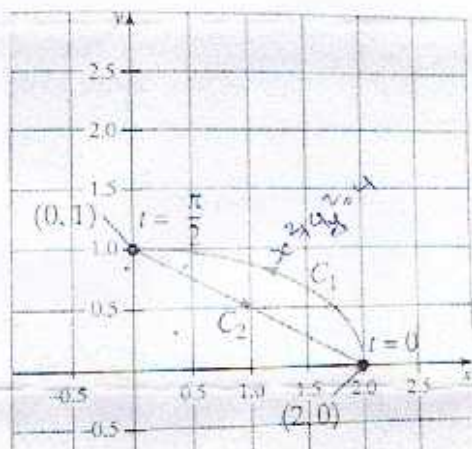
6. Show that Green's theorem is true for the line integral $\oint_C (-ydx + xdy)$, where C is the closed path as shown

closed path as shown (6)