Unique Paper Code : 3122611101

Name of the Paper : Single and Multivariable Calculus

Name of the Course : **B.Tech (IT & Mathematical Innovations)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Five** questions.
3. **All** questions carry equal marks.

1. (a) Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = \frac{1}{x^2}$, and the x – axis. 1 (6)

P.T.O.

(b) For what values of x does the following power series converge? Find series and radius of convergence for f′ and f″. (6)

(0) (0)

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}.$$ (−1, 1]

(c) The position P(x,y) of a particle moving in the xy-plane is given by the equations and parameter interval

$$x = 2t - 5,\ y = 4t - 7,\ -\infty < t < \infty.$$

Identify the path traced by the particle and describe the motion. (6)

2. (a) Consider the function f defined as follows

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0). \end{cases}$$

Find $f_{yx}(0,0)$ and $f_{xy}(0,0)$. (6)

(b) A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it.

How fast is the distance s(t) between the bicycle and balloon increasing 3 sec later? (6)

(c) Determine a rational function that meets the given conditions. The zero of the function is $x = -1$, the y – intercept is (0,2), the equations of the asymptotes are $x = -2$, $x = 3$, and $y = 0$. There is a removable discontinuity (hole) at $x = 4$. (6)

3. (a) A manufacturer of decorative end tables produces two models, basic and large. Its weekly profit function is modeled by

$$P(x, y) = -x^2 - 2y^2 - xy + 140x + 210y - 4300,$$

where x is the number of basic models sold each week and y is the number of large models sold each week. The warehouse can hold at most 90 tables. Assume that x and y must be nonnegative. How many of each model of end table should be produced to maximize the weekly profit, and what will the maximum profit be? 3400 (6)

(b) Change the order of integration in $\int_0^1 \int_y^{2-y} xydxdy$ and hence evaluate it. (6)

$\frac{8}{3}$

(c) Determine whether the following sequence is increasing, decreasing or not monotonic. Is the sequence bounded? (6)

(NO)

$$a_n = n + \frac{1}{n}$$

4. (a) A projectile is fired with an initial speed of 500 m/sec at an angle of elevation of 45°. (6)

(i) When and how far away will the projectile strike? 25k, 505 km

(ii) How high overhead will the projectile be when it is 5 km downrange? 75..

(iii) What is the greatest height reached by the projectile? 12500m

(b) Find the value of a (constant) that makes the following function differentiable for all x-values. (6)

$$f(x) = \begin{cases} ax, & \text{if } x < 0 \\ x^2 - 3x, & \text{if } x \geq 0 \end{cases}$$

(c) Find the lateral (side) surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \le x \le 4$, about the x–axis. (6)

5. (a) Graph the function $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ \frac{x}{2}, & x > 2 \end{cases}$. Then answer the following questions. (8)

(i) Find $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$, and $f(2)$.

(ii) Does $\lim_{x\to 2} f(x)$ exist? If so, what is it? If not, why not?

(iii) Find $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$.

(iv) Does $\lim_{x\to -1} f(x)$ exist? If so, what is it? If not, why not?

(b) A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph). It reaches a height of $s = 160t - 16t^2$ ft after t sec. (10)

(i) How high does the rock go?

(ii) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down? 96

(iii) What is the acceleration of the rock at any time t during its flight (after the blast)? -32

(iv) When does the rock hit the ground again? 10

6. (a) The number of professional services employees fluctuated during the period 2000–2009 as modeled by

$$E(t) = -28.31t^3 + 381.86t^2 - 1162.07t + 16905.87,$$

where t is the number of years since 2000 (t = 0 corresponds to 2000) and E is thousands of employees. Find the relative extrema of this function and sketch the graph. Interpret the meaning of the relative extrema. (10)

(b) The average temperature of certain city can be approximated by the function

$$T(x) = 43.5 - 18.4x + 8.57x^2 - 0.996x^3 + 0.0338x^4,$$

Where T represents the temperature, in degrees Fahrenheit, x = 1 represents the middle of January, x = 2 represents the middle of February, and so on. Use the second-derivative test to estimate the points of inflection for the function T(x). What is the significance of these points? (8)

7. (a) A clever college student develops an engine that is believed to meet all state standards for emission control. The new engine's rate of emission is given by

$$E(t) = 2t^2,$$

where E(t) is the emissions, in billions of pollution particulates per year, at time t, in years. The emission rate of a conventional engine is given by

$$C(t) = 9 + t^2.$$

(i) At what point in time will the emission rates be the same?

(ii) What reduction in emissions results from using the student's engine? (6)

(b) Find the first three nonzero terms of the Maclaurin series for the function $f(x) = \cos x - \frac{2}{1-x}$ and the values of x for which the series converges absolutely. (8)

$-1-2x-\frac{5x^2}{2}$

(c) Find $\frac{\partial w}{\partial x}$ at the point $(x,y,z) = (2,-1,1)$ if $w = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 1$, and x and y are the independent variables. (4)

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