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Title of the Paper :- Seeing the world through Calculus.

**Solution 3**

$$f(x,y) = \begin{cases} \dfrac{3xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$

at $x=0$;

$f(0,y) = 0 \quad ; y \neq 0.$

at $y=0$

$f(x,0) = 0 \quad ; x \neq 0.$

$\Rightarrow (0,0)$ is a Critical Point for $f(x,y)$

Let $\epsilon > 0$, then we have find $\delta > 0$ such that;

$$\left| \frac{3xy^2}{x^2+y^2} - 0 \right| < \epsilon \quad \text{when} \quad 0 < \sqrt{x^2+y^2} < \delta$$

$$\frac{3|x|y^2}{x^2+y^2} < \epsilon \quad \text{when} \quad 0 < \sqrt{x^2+y^2} < \delta.$$

As, $y^2 \leq x^2+y^2$ ; $x^2 \leq x^2+y^2$ | $|x| = \sqrt{x^2}$

OR $\dfrac{y^2}{x^2+y^2} \leq 1$ ; OR $\dfrac{x^2}{x^2+y^2} \leq 1$ |

$\Rightarrow \frac{3|x|y^2}{x^2+y^2} \leq \frac{3|x|}{} \Leftarrow 3\sqrt{x^2} \leq 3\sqrt{x^2+y^2}$

$\Rightarrow \left|\frac{3xy^2}{x^2+y^2}\right| \leq 3\sqrt{x^2+y^2}$

Let; if $\delta = \frac{\varepsilon}{3}$ & $0 < \sqrt{x^2+y^2} < \delta$

$\left|\frac{3xy^2}{x^2+y^2} - 0\right| \leq 3\sqrt{x^2+y^2} < 3\delta = \beta\left(\frac{\varepsilon}{3}\right) = \varepsilon$

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2} = 0$

A/Q. $f(0,0) = 0$

$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0.$

$\therefore f(x,y)$ is continous at $(0,0)$.

→ As there is no other CP of the func$^n$ $f(x,y)$, $\therefore$

$\therefore f(x,y)$ is continous everywhere.