Tutorial Sheet III

Q1) Show that the set $\{5, 15, 25, 35\}$is a group under multiplication modulo 40. What is the identity element of this group? What is the inverse of each element of the group? Find the order of each element of this group.

Q2) The integers 5 and 15 are among a collection of 12 integers that form a group under multiplication modulo 56. List all 12.

Q3) Suppose the table below is a group table. Fill in the blank entries.

	e	a	b	c	d
e	e				
a		b			e
b		c	d	e	
c		d		a	b
d					

Q4) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in R, a \neq 0 \right\}$ Show that G is a group under usual matrix multiplication. What is the identity of the group? Find the inverse of an element of G.

Q5) Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in Z \right\}$ be a group under matrix addition. Let $H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in Z; a+b+c+d=0 \right\}$. Show that H is a subgroup of G.

Q6) Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| ad - bc = \pm 1;\ a,b,c,d \in R \right\}$. Show that G is a group under matrix multiplication. What is the order of the group? Can you relate it to the symmetries of a square?

Q7) Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| ad - bc = 1;\ a,b,c,d = \pm 1, \pm i \right\}$ Show that G is a group under matrix multiplication. What is the order of the group? Try to find at least one non-trivial subgroup f G. (Hint: Consider a square with the centre at the origin)

Q8) Let $G = \left\{ \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \middle| \theta \in R \right\}$. Is G a group under matrix multiplication? Does the commutative property hold? Can you give a geometrical interpretation about this set? Geometrically visualize the inverse of an element of this set.

Q9) Find the product of the following permutations. Determine if they are odd or even permutations:

(a) (1, 3, 4, 5) (2, 5, 4) (b) (1, 2) (1, 3, 4) (1, 2, 5)

Q10) Express the following permutation as a product of disjoint cycles

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 8 & 7 & 1 & 5 & 2 & 4 \end{bmatrix}$$

Q11) Let the arrangement on the left of the figure be the identity. The arrangement on the right is the cyclic permutation (1, 2, 3, 4, 5, 6). What permutation will bring back the arrangement on the right to the original position? What is the inverse of the permutation (1, 2, 3, 4, 5, 6)? Using a similar logic find the inverse of the permutation (1, 3, 5, 4, 6, 2).