

Examination : End Semester Examination – May 2022
Name of the Course : B.Tech (Information Technology and Mathematical Innovations)
Name of the Paper : Does Nature Play dice: The amazing world of probability and statistics
Paper Code : 32861401
Semester : IV
Duration : 2 Hours+ 30 minutes
Maximum Marks : 40

Instructions:

1. This question paper contains six questions, out of which any four are to be attempted. Each question carries equal marks.
2. Read the question paper carefully. Use of scientific calculator is allowed in the exam. Use provided Z-tables, t-tables and χ^2 -tables for calculations.

1. In group testing for a certain disease, a blood sample was taken from each of n individuals and part of each sample was placed in a common pool. The latter was then tested. If the result was negative, there was no more testing and all n individuals were declared negative with one test. If, however, the combined result was found positive, all individuals were tested, requiring $n+1$ tests. If $p = 0.05$ is the probability of a person's having the disease and $n = 5$, compute the expected number of tests needed, assuming independence. (10)

2. A bakery sells rolls in units of a dozen. The demand X (in 1000 units) for rolls has a gamma distribution with parameters $\alpha = 3$, $\theta = 0.5$, where θ is in units of days per 1000 units of rolls. It costs \$2 to make a unit that sells for \$5 on the first day when the rolls are fresh. Any leftover units are sold on the second day for \$1. How many units should be made to maximise the expected value of the profit? (10)

3. Let X equal the weight in grams of a "52-gram" snack pack of candies. Assume that the distribution of X is $N(\mu, 4)$. A random sample of $n = 10$ observations of X yielded the following data:

55.95	56.54	57.58	55.13	57.48
56.06	59.93	58.30	52.57	58.46

- a. Give a point estimate for μ . (5)
- b. Find the endpoints for a 95% confidence interval for μ . (2.5)
- c. On the basis of these very limited data, what is the probability that an individual snack pack selected at random is filled with less than 52 grams of candy? (2.5)

4. Vitamin B₆ is one of the vitamins in a multiple vitamin pill manufactured by a pharmaceutical company. The pills are produced with a mean of 50 mg of vitamin B₆ per pill. The company believes that there is a deterioration of 1 mg/month, so that after 3 months it expects that $\mu = 47$. A consumer group suspects that $\mu < 47$ after 3 months.

a. Define a critical region to test $H_0: \mu = 47$ against $H_1: \mu < 47$ at an $\alpha = 0.05$ significance level based on a random sample of size $n = 20$. (5)

b. If the 20 pills yielded a mean of $\bar{x} = 46.94$ with a standard deviation of $s = 0.15$, what is your conclusion? (2.5)

c. What is the approximate p-value of this test? (2.5)

5. In a large multinational company, the following is the joint PMF between an employee's gender (X) and whether the person holds an executive position Y (= 1 executive position; = 0 non-executive position):

P(X, Y)		Y	
		0	1
X	F	0.49	0.01
	M	0.41	0.09

Determine whether X and Y are independent and use your knowledge about conditional probability and independence to decide whether there is sexual bias in promotion to an executive position within the company. (10)

6. Let X_1, X_2, X_3, X_4 represent the random times in days needed to complete four steps of a project. These times are independent and have gamma distributions with common $\theta = 2$ and $\alpha_1 = 3, \alpha_2 = 2, \alpha_3 = 5, \alpha_4 = 3$, respectively. One step must be completed before the next can be started. Let Y equal the total time needed to complete the project.

a. Find an integral that represents $P(Y \leq 25)$. (5)

b. Using a normal distribution, approximate the answer to part (a). Is this approach justified? (5)