Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination	:	End Semester Examination – March 2021
Name of the Course	:	B.Tech. (Information Technology & Mathematical Innovation)
Name of the Paper	:	Modeling continuous changes through ordinary differential equations
Paper Code	:	32861102
Semester	:	I
Duration	:	3 Hours
Maximum Marks	:	75

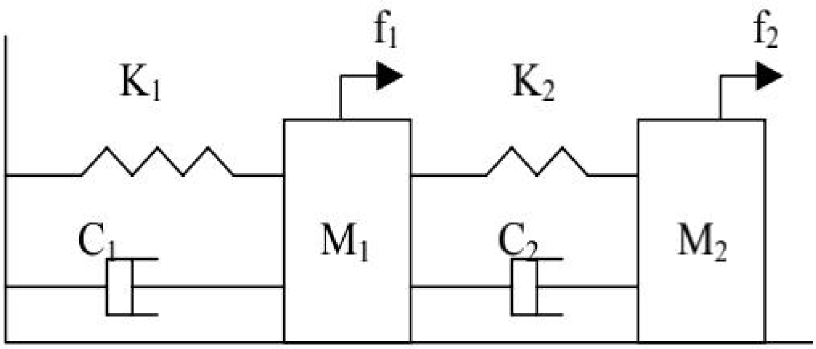
Instructions:
This question paper contains six questions, out of which any four are to be attempted. Each question carries equal marks.

Q1. Determine whether or not the second order differential equation

$$(x^3 - x^2)\frac{d^2y}{dx^2} + 2x^2\frac{dy}{dx} + 2xy = 0$$

has two linearly independent solutions. Justify your answers.

Q2. Consider the following system with $C_1 = C_2 = 0.001, K_1 = K_2 = 1, M_1 = 1$ *and* $M_2 = 2$.



Write the ordinary differential equation of the motion and find the general solution.

Q3. Find solution of the following non-homogeneous equation using method of undetermined coefficients.

$$\frac{d^3x}{dt^3} - 5\frac{d^2x}{dt^2} + 4x = 1 + t^2e^{2t} - t\,sin2t$$

Q4. Assume that Susceptible S(t), Infected I(t), and Recovered R(t) satisfy the following SIR epidemic model

$$\frac{dS}{dt} = 0.4N - 0.5SI - 0.4S$$
$$\frac{dI}{dt} = 0.5SI - 0.6I$$
$$\frac{dR}{dt} = 0.2I - 0.4R$$

Find all critical points and discuss the stability of the model. The total population is given by $N = S + I + R$.

Q5. Consider the two ordinary differential equations

$$\left(\frac{dp}{dt}\right)^2 = 9p$$
$$\frac{dp}{dt} = 3\sqrt{p}$$

Do they have the same solution set? Justify your answers. Determine the points (t_1, p_1) for which the equation $\frac{dp}{dt} = 3\sqrt{p}$ with initial condition $p(t_1) = p_1$ has a unique solution, no solution and infinitely many solutions.

Q6. Suppose that the trajectory $(x_1(t), x_2(t))$ of a particle moving in the plane satisfies the following system of differential equations

$$\frac{\mathrm{d}^2x_1}{\mathrm{dt}^2} - 2\frac{dx_2}{dt} - 3x_1 = 0$$
$$\frac{\mathrm{d}^2x_2}{\mathrm{dt}^2} - 2\frac{dx_1}{dt} + 3x_2 = 0$$

with $x_1(0) = 2, x_2(0) = 0, \left(\frac{dx_1}{dt}\right)_{\mathrm{t=0}} = \left(\frac{dx_2}{dt}\right)_{\mathrm{t=0}} = 0.$ Find the solution of the system.