Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : **End Semester Examination – Dec 2022**
Name of the Course : **B.Tech (Information Technology and Mathematical Innovations)**
Name of the Paper : **Modelling change in the world around us: Partial Differential Equations**
Paper Code : **32863101**
Semester : **III**
Duration : **2 Hours**
Maximum Marks : **50**

SECTION-A

This section contains five questions, attempt any four from this section. Each carries equal marks. $(4 \times 5 = 20)$

1. Find the integral surface of the linear PDE
$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0, \quad z=1$.

2. Reduce the following equation to a canonical form and hence solve it
$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$

3. Find the region in the xy-plane in which the following equation is hyperbolic:
(i) $[(x-y)^2-1]u_{xx} + 2u_{xy} + [(x-y)^2-1]u_{yy} = 0$ **(2.5)**
(ii) Find the families of characteristics of the PDE $(1-x)^2u_{xx} - u_{yy} = 0$ in the elliptic and hyperbolic cases. **(2.5)**

4. Find the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using Charpit's method.

5. Show that the solution of the equation $yu_x - xu_y = 0$ containing the curve $x^2 + y^2 = a^2$, $u = y$, does not exist.

SECTION-B

This section contains total four questions, attempt any three from this section. Each carries equal marks. $(10 \times 3 = 30)$

6. Find the steady state temperature distribution in a semi-circular plate of radius 'a'. insulated on both the faces with its curved boundary kept at constant temperature U_0 and bounding diameter kept at zero temperature.

7. Find the Fourier series of following triangular wave function given by;

$$f(x) = |x| = \begin{cases} -x & -\pi \le x \le 0, \\ x & 0 \le x \le \pi \end{cases}$$

Then, deduce the following numerical series;

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots.$$

8. Find the D Alembert's solution for following cauchy problem of an infinite string with given initial conditions;

$$u_{tt} = c^2 u_{xx}, \quad x \in R, \quad t > 0$$
$$u(x,0) = sinx, \quad u_t(x,0) = cosx$$

9. Consider a finite plucked string of length 'l' fixed at both the end, Suppose the string is raised to a height 'h' at 'x = a' and then released. The string will oscillate freely. The problem is governed by following equation;

$$u_{tt} = c^2 u_{xx}$$

Find the displacement of the string at any position 'x' and time 't'. State all the boundary and initial conditions clearly.