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$\boxed{\text{Solution 2}}$

Given $a_1 = \sqrt{2}$ —— (I)

$a_{n+1} = \sqrt{2a_n}$. —— (II)

For convergence or divergence,
Squaring eq (II) both sides

$$(a_{n+1})^2 = 2a_n.$$

$$a_n = \frac{(a_{n+1})^2}{2} \quad \text{—— (III)}$$

Let.
$$\lim_{n\to\infty} a_n = \alpha.$$

then,
$$\lim_{n\to\infty} a_{n+1} = \alpha.$$

Hence from eq (III), Applying limits.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(a_{n+1})^2}{2}$$

$$\cancel{\alpha} = \frac{\alpha^{\cancel{2}}}{2} \Rightarrow \boxed{\alpha = 2}.$$

$$\Rightarrow \lim_{n\to\infty} a_n = 2.$$

$\Rightarrow a_n \leq 2$; for $n \in N$.

Hence, the sequence $\langle a_n \rangle$ is bounded above $\lim_{n\to\infty} a_n = 2$.

Now, for $n=1$

$a_2 - a_1 = \sqrt{2a_1} - a_1 = a_1(\sqrt{2}-1) = \sqrt{2}(\sqrt{2}-1) > 0$

$\therefore$ Suppose, $a_n - a_{n-1} > 0$.

Now, for $n=n$.

$a_{n+1} - a_n = \sqrt{2a_n} - \sqrt{2a_{n-1}} = \sqrt{2}(a_n - a_{n-1})$

≥ 0.

$\therefore$ The sequence $\langle a_n \rangle$ is increasing.

Also the sequence $\langle a_n \rangle$ is convergent.