Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : **End Semester Examination – December 2023**
Name of the Course : **B.Tech (IT and Math. Innovation)**
Name of the Paper : **Complexity and Symmetry in Mathematics: Complex Analysis and Algebra**
Paper Code : **32861501**
Semester : **V**
Duration : **2 Hours**
Maximum Marks : **50**
Instructions:
(i) Attempt any <u>four</u> questions from Section A and <u>three</u> questions from Section B.
(ii) Non-Scientific calculator is allowed.

<u>SECTION A</u>

$\cancel{\frac{\delta y}{\delta a}}$

Q1) Determine a and b so that the function $u = ax^3 + bxy^2$ is harmonic and find its harmonic conjugate. (5 marks)

$\frac{\delta u}{\delta x} = \frac{\delta v}{\delta y}$

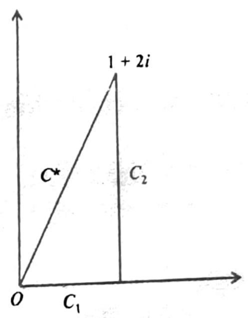
Q2) Evaluate $(i - 1)^{i+1}$ and find its principal value. (5 marks)

$\frac{\delta u}{\delta y} = -\frac{\delta v}{\delta x}$

Q3) Let $f(z) = Re\ (z)$. Integrate the function from the origin to the point $1 + 2i$ (a) along C^* (b) along C_1 followed by C_2. Are these two integrals equal? What can you deduce about the analyticity of $f(z)$ from these results? (5 marks)

$\delta^2 u \quad \frac{\delta^2 u}{\delta x^2} = \frac{\delta^2 v}{\delta y \delta x}$

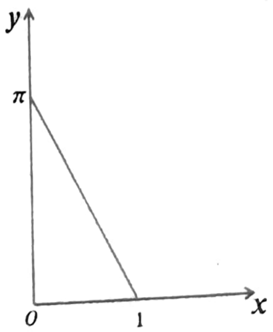
$\frac{\delta^2 u}{\delta y^2} = \frac{-\delta^2 v}{\delta y \delta x}$



Q4) Find the Laurent's series of $f(z) = \dfrac{1}{z^2(1-z)}$ at $z = 0$ in the region $0 \le |z| < \infty$. (5 marks)

Q5) Evaluate the integral $\displaystyle\oint_{|z|=1} \frac{30z^2 - 23z + 5}{(2z-1)^2(3z-1)}\,dz$ (5 marks)

Q6) Find the mapping of the right triangle given in the figure under the function $f(z) = Exp(z)$. (5 marks)



SECTION B

Q7) Let $G = \{6, 12, 18, 24, 30, 36\}$.

(i) Show that G is a group under multiplication modulo 42. Find the identity of the group and the inverse of each element.

(ii) Find the order of each element and hence justify whether G is cyclic or not.

(iii) Find a subgroup of G of order 3 if it exists. Else justify the non-existence of such a subgroup.

(5 + 3 + 2 = 10 marks)

Q8) Let G be a group under matrix multiplication generated by the complex matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}.$$

(i) Find the order of the group.

(ii) Find the order of the element A and B

(iii) Is the group G cyclic?

(iv) Find at least two non-trivial subgroups of G of different order.

(3 + 2 + 2 + 2 = 10 marks)

Q9) Let $\alpha, \beta \in S_7$ where $\alpha^4 = (1\ 2\ 5\ 6\ 4\ 3\ 7)$ and $\beta = (1\ 3\ 5\ 4\ 6)$,

(i) Find α and α^{-1}.

(ii) Find α^6.

(iii) Evaluate $\alpha^4 \beta^4$. Justify whether $\alpha^4 \beta^4 = (\alpha \beta)^4$

(3 + 1 + 6 = 10 marks)

Q10) The integers 5 and 15 are among a collection of 12 integers that form a group under multiplication modulo 56.

(i) List all the 12 elements of the group.

(ii) Find the order of each element and hence justify if the group is cyclic.

(5 + 5 = 10 marks)

Q11) Consider the following design.

(i) Write the Cayley's table that represents the symmetric group of the design.

(ii) What is the order of the group ?

(iii) Find a subgroup and give its geometrical interpretation.

(6 + 1 + 3 = 10 marks)