

Cluster Innovation Centre

Examination : End Semester Examination – May 2024

Name of the Course: B.Tech (Information Technology & Mathematical Innovations)

Name of the Paper: Linear Construction of Actions: Engineering through Linear Programming and Game Theory

Paper Code : ~~911610~~ 32861601

Semester : VI

Duration: 2 hours

Maximum Marks (MM): 40

Instructions: This question paper contains total seven question, out of which attempt any five questions, each question carries equal 8 marks.

1. Three electric power plants with capacities of 25, 40 and 30 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 30, 35, and 25 million kWh. The price per million kWh at the three cities is given in Table below;

		City		
		1	2	3
Plant	1	\$600	\$700	\$400
	2	\$320	\$300	\$350
	3	\$500	\$480	\$450

During the month of August, there is a 20% increase in demand at each of the three cities, which can be met by purchasing electricity from another network at a premium rate of \$1000 per million kWh. The network is not linked to city 3, however. The utility company wishes to determine the most economical plan for the distribution and purchase of additional energy.

- Formulate the problem as a transportation problem. (2)
- Determine an optimal distribution plan for the utility company. (3)
- Determine the cost of the additional power purchased by each of three cities. (3)

2. Suppose a company can manufacture three types of candy bars. Each candy bar consists of sugar and chocolate. Below is the composition and profit for each type of candy bar. 50 oz of sugar and 100 oz of chocolate are available.

Candy Bar	Sugar (ounce)	Chocolate (ounce)	Profit (cent)
A	1	2	3
B	1	3	7
C	1	1	5

The LP for maximising the profit is as follows.

$$\text{Maximize } Z = 3x_1 + 7x_2 + 5x_3$$

such that

$$x_1 + x_2 + x_3 \leq 50$$

$$2x_1 + 3x_2 + x_3 \leq 100$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

Suppose the optimal simplex tableau is

Basic Variable	Row	Z	x_1	x_2	x_3	s_1	s_2	RHS	Ratios
Z	(0)	1	3	0	0	4	1	300	
x_3	(1)	0	0.5	0	1	1.5	-0.5	25	
x_2	(2)	0	0.5	1	0	-0.5	0.5	25	

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- For what value of candy bar A profit does the current set of basic variables remain optimal? If the profit for candy bar A were 7 cents, what would be the new optimal solution? (3)
→ 7, → RHS = 350, 71 = 50 units
 - For what amount of available sugar would the current basic variables remain optimal? (2)
→ 60 oz
 - Suppose a type A candy bar used only 0.5 oz of sugar and 0.5 oz of chocolate. Should the company now make type A candy bars? *they should, but stick with original and same chocolate* (3)
3. TOYCO assembles three types of toys: trains, trucks and cars using three operations. Available assembly times for the three operations are 430, 460 and 420 minutes per day, respectively, and the revenue per toy train, truck and car are \$ 3, \$2, and \$ 5 respectively. The assembly times per train at the three operations are 1, 3, and 1 minutes. The corresponding times per truck and per car are (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used). The optimal primal solution calls for producing no toy trains, 100 toy trucks, and 230 toy cars. Suppose that TOYCO is studying the possibility of introducing a fourth toy: fire trucks. The assembly does not make use of operation 1. Its unit assembly times on operations 2 and 3 are 1 and 3 minutes respectively. The revenue per unit is \$4. Would you advise TOYCO to introduce the new product? (8)
4. An intelligent student Rudra, who usually makes good grades, provided that he can review the syllabus the night before the test. For tomorrow's test, Rudra has a small problem: His cousin brothers are having an all-night pool party in which he would like to participate. Rudra has three options:
- A_1 = Party all night
 - A_2 = Divide the night equally between studying and partying
 - A_3 = Study all night

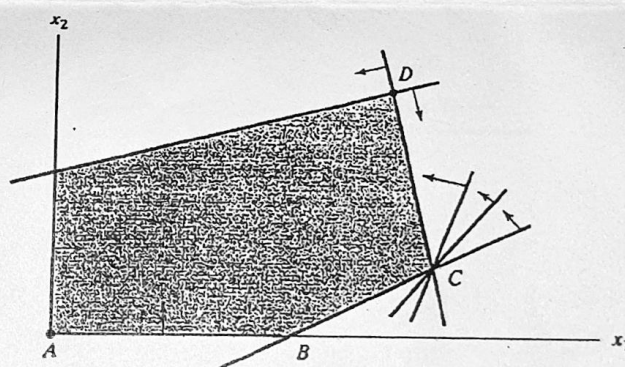
Tomorrow's exam can be very easy (B_1), easy (B_2), moderate (B_3), or tough (B_4), depending on the professor's mood. Rudra anticipates the following scores:

	B_1	B_2	B_3	B_4
A_1	9	6	2	8
A_2	8	9	4	5
A_3	7	5	2	5

Find a course of action for Rudra.

(8)

5. Consider the graphical solution space given in the figure. Suppose that the simplex iterations starts at A and that the optimum solution occurs at D. Further, assume that the objective function is defined such that at A, x_1 enters the solution first.
 - a. Identify (on the graph) the corner points that define the simplex method path to the optimum point. (4)
 - b. Determine the maximum possible number of simplex iterations needed to reach the optimum solution, assuming no cycling. (4)



6. A business executive must make the four round trips listed in the table between the head office in Dallas and a branch office in Atlanta.

The price of a round trip ticket from Dallas is \$400. A discount of 25% is granted if the dates of arrival and departure of a ticket span a weekend (Saturday and Sunday). If the stay in Atlanta lasts more that 21 days, the discount is increased to 30%. A one-way ticket between Dallas and Atlanta (either direction) costs \$250. How should the executive purchase the tickets? (8)

Departure date from Dallas	Return date to Dallas
Monday, June 3	Friday, June 7
Monday, June 10	Wednesday, June 12
Monday, June 17	Friday, June 21
Tuesday, June 25	Friday, June 28

7. Consider the solution space given in the figure, where it is desired to find the optimum extreme point that uses the dual simplex method to minimise $z = 2x_1 + x_2$. The optimum solution occurs at point $F = (0.5, 1.5)$ on the graph.

a. Can the dual simplex iterations start at point A? Explain. (2)

b. If the starting basic (infeasible but better than optimum) solution is given by point G, would it be possible for the iterations of the dual simplex method to follow the path $G \rightarrow E \rightarrow F$? (3)

c. If the starting basic (infeasible) solution starts at point L, identify a possible path of the dual simplex method that leads to the optimum feasible point at point F. (3)

