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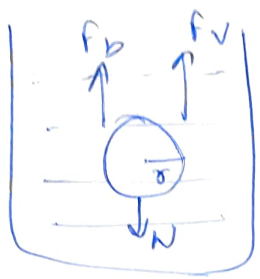
Title of the Paper :- Physics at Work I : Deconstructing Machines.

Solution 1



Here, Radius of metallic sphere $= r$

Density of spere $= \rho$

Density of fluid $= a$

Cofficient of viscocity $= \eta$

$F_b \rightarrow$ buyont force

$F_v \rightarrow$ viscous force

$W \rightarrow$ weight.

Let V_t be the terminal velocity.

we know,

$$F_b = \frac{4}{3} \pi r^3 g a$$

↳ weight of liq. displaced.

$$F_v = 6\pi r \eta V_t$$

↳ Stokes Law.

$$W = \frac{4}{3} \pi r^3 \rho g$$

↳ weight of metallic sphere.

As, the spehere reaches it's terminal velocity the system will attain an equilibrium state.

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Mathematically,

$$W = F_b + F_v$$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta V_t$$

$$\cancel{6}^{3}\pi r \eta V_t = \frac{\cancel{4}^{2}}{\cancel{3}}\pi r^3 g(\rho-\sigma)$$

$$V_t \cdot \eta = \frac{2}{9} r^2 g[\rho-\sigma]$$

or $$\boxed{V_t = \frac{2r^2 g[\rho-\sigma]}{9\eta}} \quad \text{— (1)}$$

→ Measuring cofficient of viscocity experimentally.

$$\eta = \frac{2r^2 g[\rho-\sigma]}{V_t} \quad \text{— (2)}$$

We can perform an experiment in which we will drop the ball of known radius (r) & density (ρ) in a liquid of density (σ) and measure the time taken by ball to cover a distance h inside the liquid. Using h and observed time t we can calculate it's V_t.

i.e., $\boxed{V_t = \frac{h}{t}}$

Now, we have values of all unknown quantitie of eq (2)

Putting the value in eq (2) we can get η.

By doing this exp. several times, ~~and~~ we can calculate an averge value of η which will be more presize.

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→ The formula we derived for η is after the assumption that the width & length of liquid column is ∞.
But in real practice we perform the exp. in finite length and width liquid column
So, we need a correction in our formulae, to get more presize value of η.
This correction is called ~~Lad~~ Ladenburg correction and it is :-

$$V_t = V_t'\left(1 + \frac{2.4r}{R}\right)\left(1 + \frac{3.3r}{H}\right) \quad \text{---} \quad (3)$$

V_t' → observed terminal velocity
r → radius of sphere
R → Radius of cylinder (inner)
H → Height of cylinder / liquid.

the first terms indicates ~~of~~ the finite radius and the other one is for finite depth.

→ Let consider a falling raindrop and taking it's characteristic time equals τ
we can express it's motion as.

$$\frac{dv}{dt} = -\frac{v}{\tau} + g \quad \text{---} \quad (4)$$

Solve the D.E. (4), we get.

$$\boxed{V_t = \tau g}$$

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Replacing $v \to u$, $u = v + \alpha$
↳ α is a const.

eq ④ will become

$$\frac{du}{dt} = -\frac{1}{\tau} u + \frac{\alpha}{\tau} g$$

taking ~~a~~ $\alpha = -\tau g$

$$\frac{du}{dt} = -\frac{u}{\tau}$$

on solving ;

$$\boxed{u = Ae^{-t/\tau}} \quad \text{or} \quad v = u + \tau g$$

$$\boxed{v = Ae^{-t/\tau} + \tau g.}$$

we know, $t = 0 \longrightarrow v = 0$

$$\boxed{A = -\tau g}$$

Hence,

$$v = \tau g\left(1 - e^{-t/\tau}\right)$$

& $\underline{v_t = \tau g}$

$$\Rightarrow \boxed{v = v_t\left(1 - e^{-t/\tau}\right)}$$

Now, if $t \gg \tau$, $e^{-t/\tau} \to 0$

$\underline{v \to v_t.}$