Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination   : End Semester Examination – December 2022
Name of the Course   : B.Tech (IT and Math. Innovation)
Name of the Paper   : Complexity and Symmetry in Mathematics: Complex Analysis and Algebra
Paper Code   : 32861501
Semester   : V
Duration   : 2 Hours
Maximum Marks   : 50

Instructions:

(i) Attempt any <u>four</u> questions from Section A and <u>three</u> questions from Section B.

(ii) Each question in Section A is of 5 marks and in Section B of 10 marks.

(iii) Non-Scientific calculator is allowed.

SECTION A

Q1) Determine a so that the function $u = e^{\pi x} \cos ay$ is harmonic and find its harmonic conjugate.

Q2) Evaluate $(i + 1)^{i-1}$ and find its principal value.

Q3) Evaluate the integral $\oint_C \dfrac{2z-1}{z^2 - z} dz$, where C is the ellipse as given in the figure
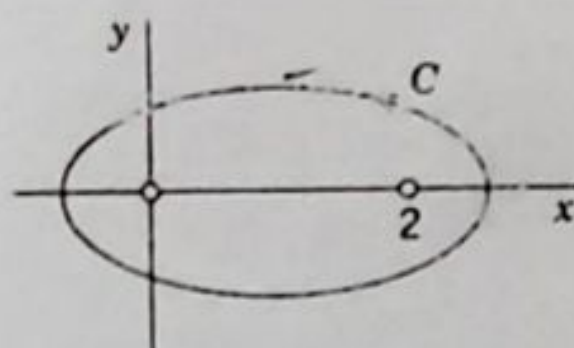
Q4) Find the Laurent's series of $f(z) = \dfrac{-2z+3}{z^2 - 3z + 2}$ at $z = 0$ in the region $1 \le |z| \le 2$.

Q5) Evaluate the integral $\oint_{|z|=1} \dfrac{30z^2 - 23z + 5}{(2z-1)^2 (3z-1)} dz$

Q6) Give the geometrical interpretation of the mapping of the circle $|z| = 1$ under the transformation $f(z) = z + (1/z)$.
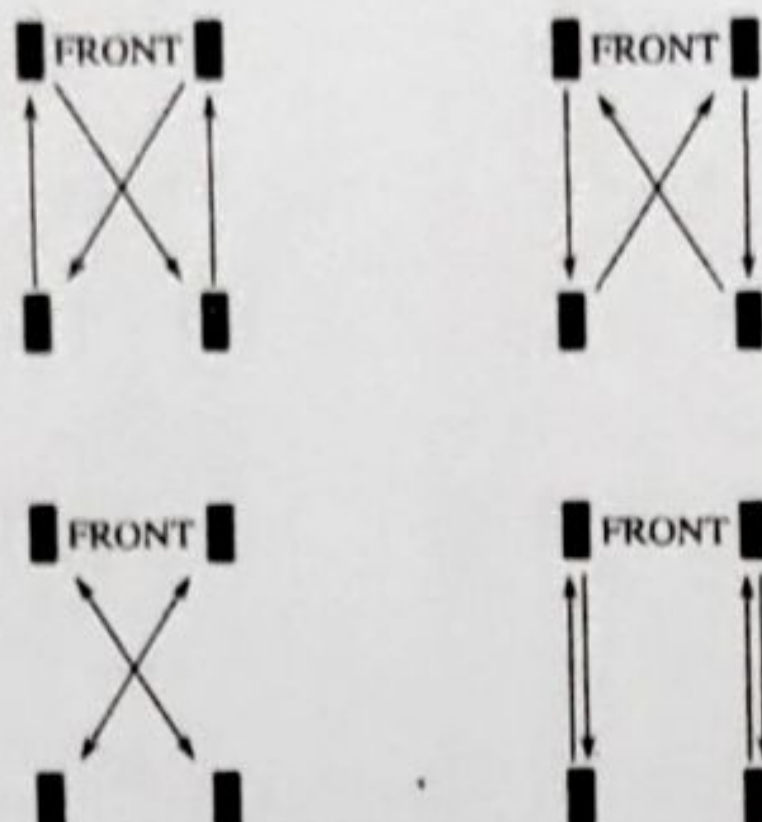
SECTION B

Q7) Let $G =$ Let $\{a \in Z_{20} |$ g.c.d $(a, 20) = 1\}$.

    (i)   Show that G is a group under multiplication modulo 20.

    (ii)   Find the order of each element and hence justify whether G is cyclic or not.

    (iii)   Find a subgroup of G of order 4 if it exists. Else justify the non-existence of such a subgroup.   (3 + 4 + 3 = 10 marks)

Q8) Given below are four tire rotation patterns that are generally recommended by tyre companies.
- (i) Explain these patterns as permutations in S_4.
- (ii) Find the smallest subgroup of S_4 that contains this pattern.
- (iii) Justify if this subgroup is abelian or not. (2 + 4 + 4 = 10 marks)

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Q9) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$.

- (i) Write α and β as product of transpositions.
- (ii) Write α and β as product of disjoint cycles.
- (iii) Evaluate $\alpha^3 \beta^{-2}$ (3 + 2 + 5 = 10 marks)

Q10) The integers 5 and 15 are among a collection of 12 integers that form a group under multiplication modulo 56.
- (i) List all the 12 elements of the group.
- (ii) Find the order of each element and hence justify if the group is cyclic. (5 + 5 = 10 marks)

Q11) Consider the following design.

- (i) Write the Cayley's table that represents the symmetric group of the design.
- (ii) What is the order of the group
- (iii) Find a subgroup of order 5 and give its geometrical interpretation. (6 + 1 + 3 = 10 marks)