Tutorial Sheet II

Q1) Find the integral of the following complex functions

(a) $f(z) = \operatorname{Im} z^2$ along a triangle with vertices o, 1, i traversed in the counterclockwise direction.

(b) $f(z) = \dfrac{z^2+1}{z}$ along a unit circle with centre at origin in the clockwise direction.

(c) $f(z) = \dfrac{\tan\frac{z}{2}}{z^4-16}$ along the square with vertices ±1, ± i in the clockwise direction.

(d) $f(z) = \dfrac{2z^3+z^2+4}{z^4+4z^2}$ counterclockwise along a circle centred at 2 and of radius 4.

(e) $f(z) = \dfrac{z}{z^2+4z+3}$ counterclockwise along a circle centred at -1 and of radius 2.

(f) $f(z) = \dfrac{e^z}{ze^z-2iz}$ counterclockwise along a circle centred at origin and of radius 0.6.

Q2) Find the following integration around a unit circle centred at the origin

(a) $\displaystyle\oint \frac{z^6}{(2z-1)^6}dz$ (b) $\displaystyle\oint \frac{dz}{(z-2i)^2(z-i/2)^2}dz$

Q3) Find the Laurent series that converges for $0 < |z - z_0| < R$ and determine the precise region of convergence.

(a) $f(z) = \dfrac{e^z}{z^2-z^3}$, $z_0 = 0$

(b) $f(z) = \dfrac{1}{z^2(z-i)}$, $z_0 = i$

(c) $f(z) = \dfrac{\cos z}{(z-\pi)^2}$, $z_0 = \pi$

(d) $f(z) = \dfrac{e^z}{(z+1)^2}$, $z_0 = -1$

Q4) Evaluate the following integral when the curve is traversed in the anticlockwise direction

(a) $\displaystyle\oint_{C:|z-2-i|=3.2} \frac{z-23}{z^2-4z-5}dz$ (b) $\displaystyle\oint_{C:|z-1|=2} \frac{z+1}{z^4-2z^3}dz$

(c) $\displaystyle\oint_{C:|z-3|=2} \frac{dz}{z^3(z-2i)^2}$ (d) $\displaystyle\oint_{C:|z|=1} \frac{30z^2-23z+5}{(2z-1)^2(3z-1)^2}dz$