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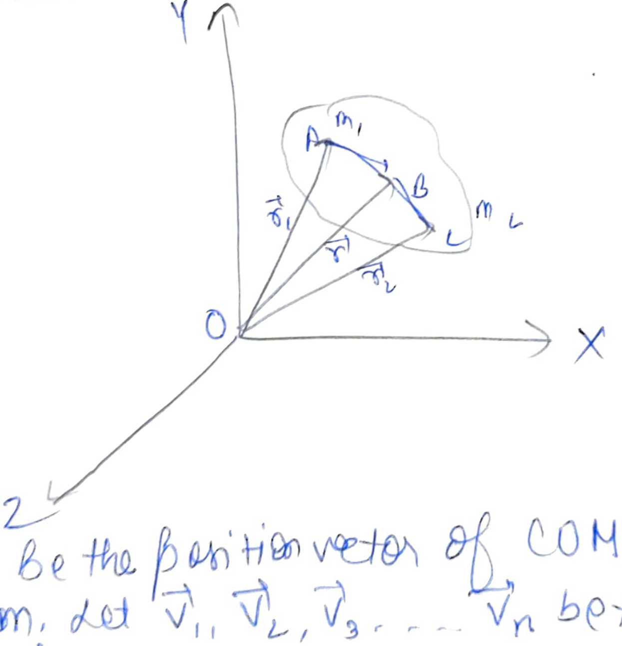
Solution 3

Le O be the origin of a rectangular co-ordinate system X Y Z
Consider a system of particles of masses $m_1, m_2, m_3, \ldots, m_n$, whose position in the co-ordinate system are given respectively by the position vectors $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_n$.



Let $\vec{r}$ be the position vector of COM of this system. Let $\vec{V}_1, \vec{V}_2, \vec{V}_3, \ldots \vec{V}_n$ be the instantaneous velocities of the particles. Suppose $F_1, F_2, F_3 \ldots \ldots F_n$ be the external force acting on the particles. Total force on the i^{th} particle can be written as

$$F_i = \sum_{i=1}^{n} \vec{F}_i.$$

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Now A/C to Newton 2nd Law

$$\frac{d}{dt}(m_i\vec{v}_i) = \vec{F}_i + \vec{f}_i \quad \text{--- (1)}$$

where, f_i is the external force on i^{th} particle.

Adding similar eqn for all the n particles.
we have,

$$\sum_{i=1}^{n} \frac{d}{dt}(m_i\vec{v}_i) = \cancel{\sum_{i=1}^{n} \vec{F}_i}^{\,0} + \sum_{i=1}^{n} \vec{f}_i \quad \text{--- (2)}$$

$\because$ the internal forces on all particle canceles out each other.,

$$\Rightarrow \sum_{i=1}^{n} \frac{d}{dt}(m_i\vec{v}_i) = \sum_{i=1}^{n} \vec{f}_i \quad \text{--- (3)}$$

$$\vec{v}_i = \frac{d\vec{r}_i}{dt} \Rightarrow \sum_{i=1}^{n} \frac{d^2(m_i\vec{r}_i)}{dt^2} = \vec{f}$$

$$\Rightarrow \frac{d^2}{dt^2} \sum_{i=1}^{n}(m_i\vec{r}_i) = \vec{f} \quad \text{--- (4)}$$

Mutiplyig & diviy L.H.S of (4) by $M = \sum_{i=1}^{n} m_i$ (i.e., total mass)

$$M \frac{d^2}{dt^2} \sum_{i=1}^{n} \frac{m_i\vec{r}_i}{M} = \vec{f} \Rightarrow \sum_{i=1}^{n} \frac{m_i\vec{r}_i}{M} = \vec{r}.$$

$$\left(\because M\frac{d^2\vec{r}}{dt^2} = \vec{f}\right)$$

→ from the above eqn.

$$\vec{r} = \sum_{i=1}^{n} \frac{m_i r_i}{M} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n r_n}{M}.$$

(where, $M = m_1 + m_2 + m_3 + \cdots + m_n$)

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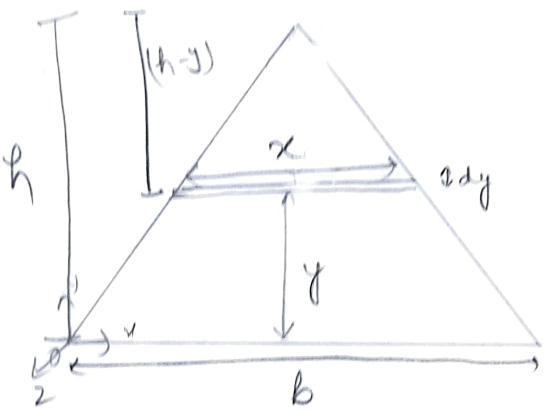
Now, let us consider a body of continous mass.
Now, every point mass is located at position $\vec{r}_i$

$$\vec{r}_{COM} = \frac{dm\,\vec{r}_1 + dm\,r_2 + \dots}{M}$$

$$\Rightarrow \boxed{\vec{r}_{COM} = \frac{\int \vec{r}\,dm}{\int dm}}$$

$\Rightarrow$



Let the vertex 0 the sheet be at origin, by observation we can say that z cordinate of the COM of sheet is at $t/2$.

Now, let the mass per unit area

$$\rho = \frac{M}{a} = \frac{M}{bh/2} = \frac{2M}{bh}$$

$\Rightarrow$ consider an elemental strip at distance y from base.

$$y_{COM} = \frac{\int y\,dm}{M}$$

also using similarity of triangles

$$\frac{h-y}{x} = \frac{h}{b} \Rightarrow x = \frac{b}{h}(h-y).$$

$dm \Rightarrow$ (area of elemental strip) $\times$ (area density)

$$dm = dy \cdot x \cdot \rho = \underline{\rho x\,dy}.$$

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$$y_{COM} = \frac{\int_0^h y\,\rho\,x\,dy}{M} = \frac{\int_0^h y\rho\left(\frac{b}{h}(h-y)\right)dy}{M}$$

$$= \frac{\int_0^h \left(\rho b y - \frac{\rho b y^2}{h}\right)dy}{M}$$

$$= \frac{\rho b\left[\frac{y^2}{2} - \frac{y^3}{2h}\right]_0^h}{M}$$

$$= \frac{\rho b h^2}{6M} = \frac{\cancel{b}h^{\cancel{2}}}{3\cancel{6}\cancel{M}} \times \frac{\cancel{2M}}{\cancel{bh}}$$

$$\boxed{y_{COM} = \frac{h}{3}}$$

$$x_{COM} = \frac{b}{h}\left(h - \frac{h}{3}\right)$$

$$= \frac{b}{\cancel{h}} \times \frac{2\cancel{h}}{3} = \frac{2b}{3}.$$

$\therefore$ Co-ordinates of COM: $\left(\frac{2b}{3}, \frac{h}{3}, \frac{t}{2}\right)$.