Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : **End Semester Examination – March 2022**
Name of the Course : **B.Tech (Information Technology and Mathematical Innovations)**
Name of the Paper : **Modeling continuous changes through ordinary differential equations**
Paper Code : **32861102**
Semester : **I**
Duration : **3 Hours**
Maximum Marks : **75**

Instructions:

This question paper contains six questions, out of which any four are to be attempted. Each question carries equal marks.

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1. The human population of a village satisfies the logistic law of the population

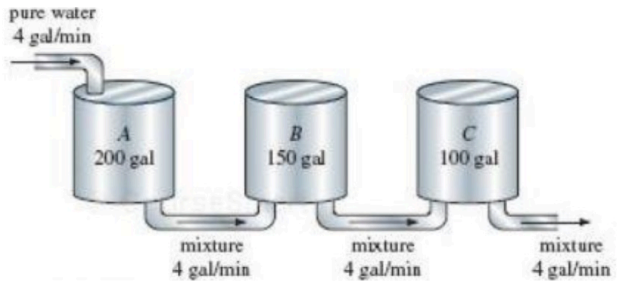
$$\frac{dx}{dt} = kx - \lambda x^2,\ k > 0,\ \lambda > 0$$

with $k = \frac{1}{400}$, $\lambda = (10)^{-8}$ and t measured in years, and the annual emigration from village is neglected. However, every year 100 people disillusioned with village life, and move from village to urban area. Develop a new population modal with the given k, λ so as to include the stated annual emigration. Assuming that the population of village in 1990 is 20,000, solve the resulting initial value problem and thus find the population of the village as a function of time.

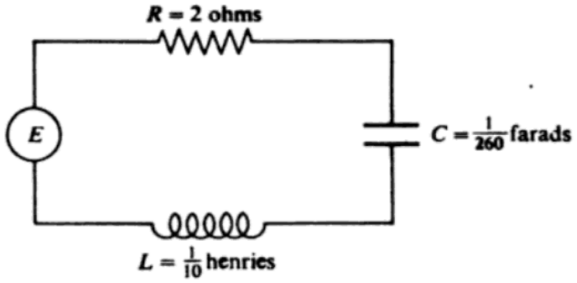
2. A 32 lb weight is attached both to the lower end of a vertically suspended spring that it stretched 5 meters by force 10 N and to a dashpot that provided negative 3 N of resistance for each foot per second of velocity. If the weight is pulled down 1 ft below its static equilibrium position and then released from rest at time t=0. Beginning at t=0 an external force $F(t) = 2t^2 + e^t + 2te^t + 4e^{3t}$ is applied to the system. Determine the resulting displacement for non homogenous system as a function of time.

3. Three tanks X, Y and Z are interconnected. Tank X initially contains 200 liters of brine in which there is dissolved 15 kg of salt, tank Y initially contain 150 liters of brine dissolved with 10 kg of salt ,and Z initially contains100 liters of brine dissolved with 5 kg of salt. Starting at time t=0, pure water flows into tank X at the rate of 4 liters/min. Brine flows in tank Y and Z at the rate of 4 liters/min. Brine solution pumped out of each tank at a rate of 4 liters/min. The mixture in the tank is kept uniform by stirring. Set up a set of differential equation that modals

amount of salt in each tank. Predict the limiting values of them at time $t \to \infty$ and solve the mathematical modal for given initial conditions. How much salt left as $t \to \infty$?



4. A circuit has in series an electromotive force $E(t) = 100 \sin 60t$, a resistor of 2 ohms, an inductor of 0.1H, and a capacitor of $\frac{1}{260}$ farads. As shown below



If the initial current and the initial charge on the capacitor are both zero. Use Laplace transform method to find the charge on the capacitor at any time t>0.

5. Suppose that two different species of animals occupy the same environment, not as a pre-predator, but rather as competitor for the same resources. A standard example is a population of deers and rabbits in the woodland. Let $x(t)$ denotes the population of deer and let $y(t)$ denoted the population of rabbits (time measured in years). The differential equations that modals the population are:

$$\frac{dx}{dt} = 5x - x^2 - xy$$
$$\frac{dy}{dt} = -2y + xy$$

Find the critical points and construct a phase plane portrait for the linearization at the indicated critical point.

6. Find the power series solution of the differential equation

$$2x^2\frac{d^2}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

in some interval $0 < x < R$.