Date :- 23/03/2022 ①

Examination Roll No. :- 21312915017

Name of Program :- B. Tech. (Information Tech. and Mathematical Innovation).

Semester :- Ist Sem.

Unique Paper Code :- 32861102

Title of the Paper :- Modeling continous change through odinary differential equation.

$\boxed{\text{Solution - 5}}$

$x(t) \to$ Population of deers

$y(t) \to$ Population of rabbits.

We have,

$$\frac{dx}{dt} = 5x - x^2 - xy$$

$$\frac{dy}{dt} = -2y + xy$$

On comparing with std. eqn's

$$\frac{dx}{dt} = a_1 x - b_1 x^2 - c_1 xy \quad \& \quad \frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy$$

We have,

$a_1 = 5$ ; $b_1 = 1$ ; $c_1 = 1$

$a_2 = -2$ ; $b_2 = 0$ ; $c_2 = -1$

$c_1 \cdot c_2 = -1$

$b_1 \cdot b_2 = 0$

$\Rightarrow b_1 \cdot b_2 > c_1 \cdot c_2$

$\Rightarrow$ This is case of Peaceful co-existince of 2 species.

Now, Finding the Critical Points,

$5x - x^2 - xy = 0$ & $-2y + xy = 0$

$x(5 - x - y) = 0$ $\quad$ $y(x - 2) = 0$

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- $x = 0$ , $y = 0$. $\Rightarrow (0,0)$
- $5 - x - y = 0$ & $(x-2) = 0$

$x = 2.$

$5 - 2 - y = 0$

$y = 3$ $\Rightarrow (2,3)$

- $5 - x - y = 0$ & $y = 0$

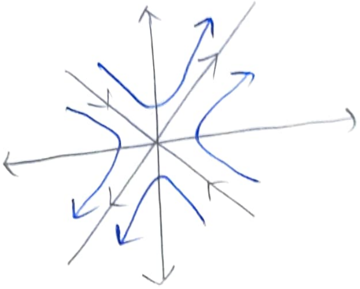
$x = 5$ $\Rightarrow (5,0)$.

Critical Points are,

$(0,0)$, $(2,3)$, $(5,0)$.

$\rightarrow$ $J(x,y) = \begin{bmatrix} 5-2x-y & -x \\ -y & -2+x \end{bmatrix}$

(i) $J(0,0) = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$



$\lambda_1 = 5$ $\lambda_2 = -2.$

$\Rightarrow (0,0)$ is a sedal Point.

(ii) $J(2,3) = \begin{bmatrix} 5-4-3 & -2 \\ -3 & -2+2 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 \\ -3 & 0 \end{bmatrix}$

$(-2-\lambda)(0-\lambda) - (-3)(-2) = 0$

$\lambda^2 + 2\lambda - 6 = 0$

$\lambda = \dfrac{-2 \pm \sqrt{4 - 4\cdot(-6)\cdot 1}}{2}$

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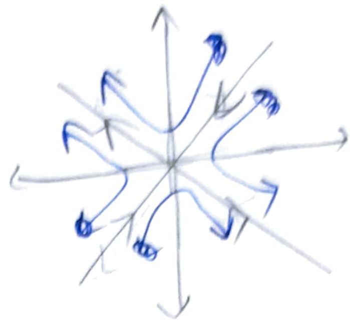
$$\Rightarrow \lambda = \frac{-2 \pm \sqrt{4+24}}{2}$$

$$\lambda = \frac{\cancel{-2} \pm \cancel{2}\sqrt{7}}{\cancel{2}}$$

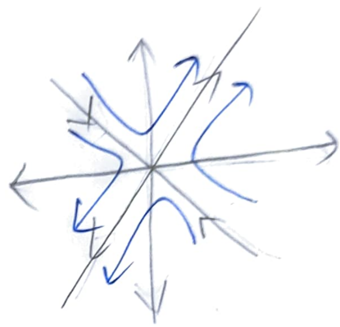


$$\lambda_1 = -1+\sqrt{7} > 0$$

$$\lambda_2 = -1-\sqrt{7} < 0$$

$\Rightarrow$ (2, 3) is a Sedal Point.

(iii) $J(5,0) = \begin{bmatrix} -5 & -5 \\ 0 & +3 \end{bmatrix}$



$\lambda_1 = -5, \ \lambda_2 = +3$

(5, 0) is a sedal Point.