Date :- 23/03/2022
Examination Roll No. :- 21312915017
Name of Program :- B.Tech (Information Tech. and Mathematical Innovation)
Semester :- Ist Sem.
Unique Paper Code :- 32861102
Title of the Paper :- Modeling continous change through odinary differential equation.

Solution-1

we have,

$$\frac{dx}{dt} = kx - \lambda x^2 \quad \text{---} (1) \quad ; \; k > 0 \;\&\; \lambda > 0.$$

$$k = \frac{1}{400} \qquad \lambda = 10^{-8}$$

Let $h \rightarrow$ emigration factor (no. of people leaving the village)

A/Q, $h = 100$

eq (1) $- h$.

$$\Rightarrow \frac{dx}{dt} = kx - \lambda x^2 - 100 \quad \text{---} (2)$$

$$\frac{dx}{dt} = \frac{x}{400} - 10^{-8}x^2 - 100 \quad \text{---} (3)$$

for critical points;

$$\frac{dx}{dt} = 0$$

$$\frac{x}{400} - 10^{-8}x^2 - 100 = 0$$

$$x - 400 \times 10^{-8}x^2 - 100 \times 400 = 0$$

$$\Rightarrow 4 \times 10^{-6}x^2 - x + 40000 = 0.$$

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$$x = \frac{-1 \pm \sqrt{1 - 4(4\times10^{-6})(4\times10^{4})}}{2\times4\times10^{-6}}$$

$$x = \frac{-1 \pm \sqrt{1 - 64\times10^{-2}}}{8\times10^{-2}}$$

$\Rightarrow x_1 = 5\times10^4$

$x_2 = 2\times10^5$

→ Ploting the phase portraits

Let $x = 3\times10^5$

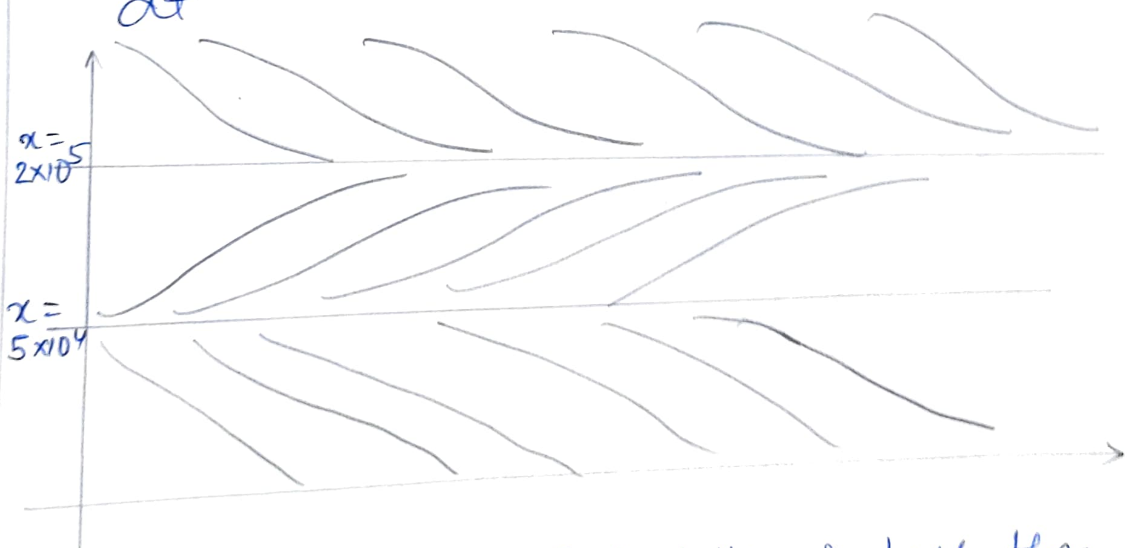
$\Rightarrow \frac{dx}{dt} = -250 < 0 \Rightarrow$ slope decreases.

$x = 10^4$

$\frac{dx}{dt} = -76 < 0 \Rightarrow$ slope decreases.

$x = 6\times10^4$

$\frac{dx}{dt} = 14 > 0 \Rightarrow$ slope increases.



→ If initial value of Population is less than 5×10^4, the village will get empty.

→ At ~~2x~~ 10^5 → the population in village is stable

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→ $5\times10^4 < x < 2\times10^5$
the population of village unstable.

→ Now ~~the~~ to find when the village will get empty or it's population stable time,

$$\frac{dx}{dt} = \frac{x}{400} - 10^{-8} x^2 - 100 \quad (\text{eq } 3).$$

$$\frac{dx}{dt} = (-10^{-8}) x^2 + \frac{x}{400} - 100$$

$$\Rightarrow \int \frac{dx}{\left[(-10^{-8})x^2 + \frac{x}{100} - 100\right]} = \int dt$$

$$\Rightarrow x = \frac{50000\left(4e^{3t/2000} - e^{150000 C_1}\right)}{e^{3t/2000} - e^{150000 C_1}}$$

→ In the year 1990
$t = 0$.
$\Rightarrow x(0) = 20,000$

$$\Rightarrow 2\cancel{0,000} = \frac{5\cancel{0,000}\left(4 - e^{150000 C_1}\right)}{1 - e^{150000 C_1}}$$

$$2 - 2e^{150000 C_1} = 20 - 5e^{150000 C_1}$$

$$\cancel{3} e^{150000 C_1} = \cancel{18}\ 6$$

$$\boxed{e^{150000 C_1} = 6}$$

$$\Rightarrow C_1 = \frac{\ln(6)}{150,000}$$

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$$\Rightarrow \boxed{x(t) = \frac{50000\left(4e^{3t/2000} - 6\right)}{\left(e^{3t/2000} - 6\right)}}$$

where $x(t) \rightarrow$ Poppulation of the village
$t \rightarrow$ time in years