Date :- 21/03/2022

Examination Roll No. :- 21312915017

Name of Program :- B.Tech (Information Tech. and Mathematical Innovation).

Semester :- I^st Sem.

Unique Paper Code :- 32861101

Title of the Paper :- Seeing the World through Calculus.

Solution 1

We have, $P(x) = 4x^3 - 6x^2 - 24x + 5$

(i) Finding the critical points.

$$P'(x) = 12x^2 - 12x - 24$$
$$= 12(x^2 - x - 2).$$

for critical points.

$$P'(x) = 0$$
$$12(x^2 - x - 2) = 0$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$\boxed{x = 2, \; x = -1}$$

(ii) Checking for maxima, minima or point of inflection.

$$P''(x) = 12(2x - 1).$$

(a) For point of inflection.

$$P''(x) = 0$$
$$12(2x - 1) = 0.$$
$$2x - 1 = 0$$
$$\boxed{x = \frac{1}{2}}$$

Now checking for maxima and minima

@ $x=2$

$P''(2) = 12((2\cdot 2)-1) = 12\times 3 = 36 > 0$

@ $x=2$, the function attains a minima.

$P(2) = (4\times 8) - (6\times 4) - 24(2) + 5$

$= 32 - 24 - 48 + 5$

$= -35.$

also @ $x=-1$

$P''(-1) = 12(-2-1) = -36 < 0.$

$\therefore$ @ $x=-1$, the function attains a maxima.

$P(-1) = -4 - 6 + 24 + 5$

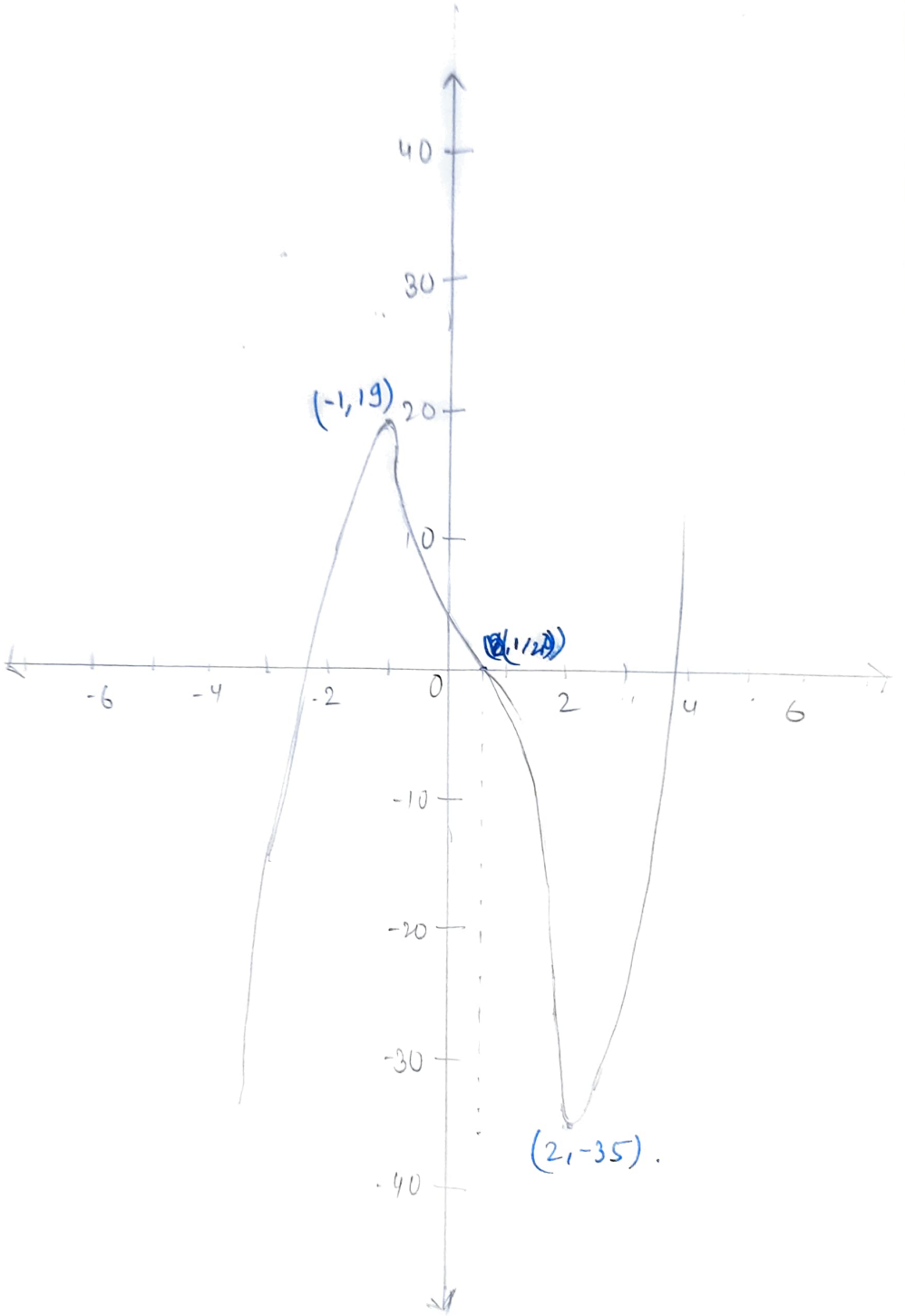
$= 19$

(iii) A/C to the C.P, the function can be further divided into several intervals :-

$(-\infty, -1), (-1, 1/2), (1/2, 2), (2, \infty)$.

Checking concavity & nature of function.

Interval	$P'(x)$	Nature of $P(x)$	$P''(x)$	Concavity
$(-\infty, -1)$	Let $x=-2$ $P'(-2) = 48 > 0$	Increasing	$P''(-2) = -60 < 0$	Concave down.
$(-1, 1/2)$	$x=0$ $P'(0) = -24 < 0$	Dec.	$P''(0) = -12 < 0$	Concave down
$(1/2, 2)$	$x=1$ $P'(1) = -24 < 0$	Dec.	$P''(1) = 12 > 0$	Concave up
$(2, \infty)$	$x=3$ $P'(3) = 48 > 0$	Inc.	$P''(3) = 60 > 0.$	Concave up


40
30
20
10
-10
-20
-30
-40
-6
-4
-2
0
2
4
6
(-1, 19)
(2, -35).