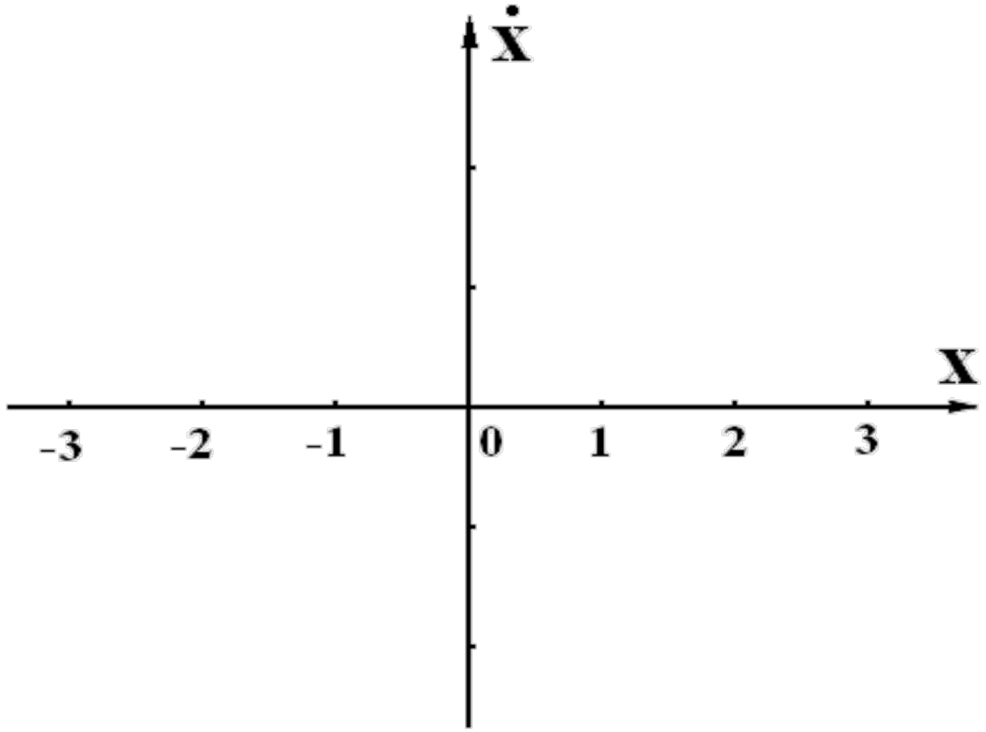
练习1：系统方程为 $\ddot{x}+x+\mathrm{sign}\dot{x}=0$，分析系统的自由响应。



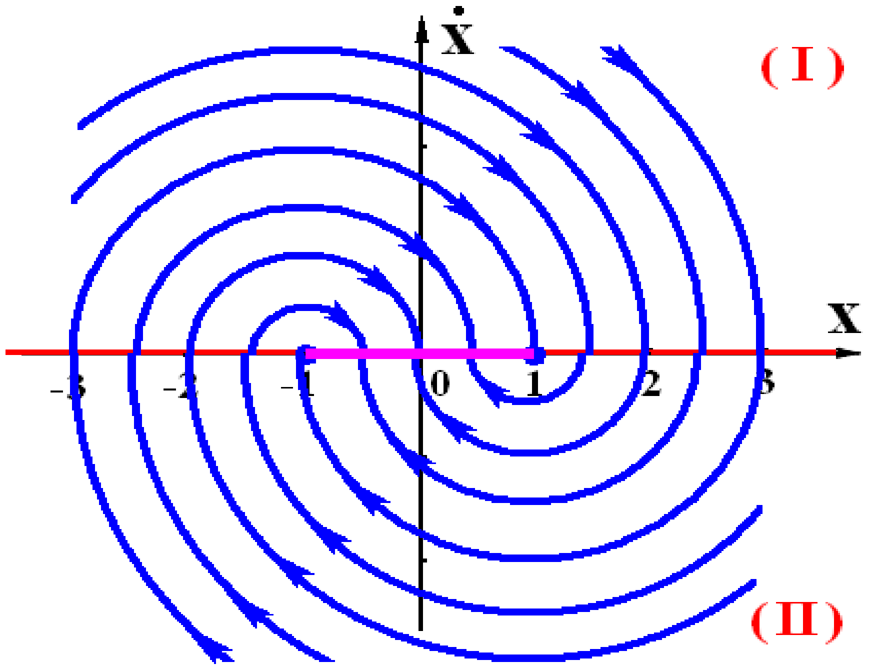
系统方程为 $\ddot{x}+x+\mathrm{sign}\dot{x}=0$，分析系统的自由响应。

解 $\begin{cases} \ddot{x}+x+1=0 & \dot{x}\geq 0 & \mathrm{I} \\ \ddot{x}+x-1=0 & \dot{x}<0 & \mathrm{II} \end{cases}$

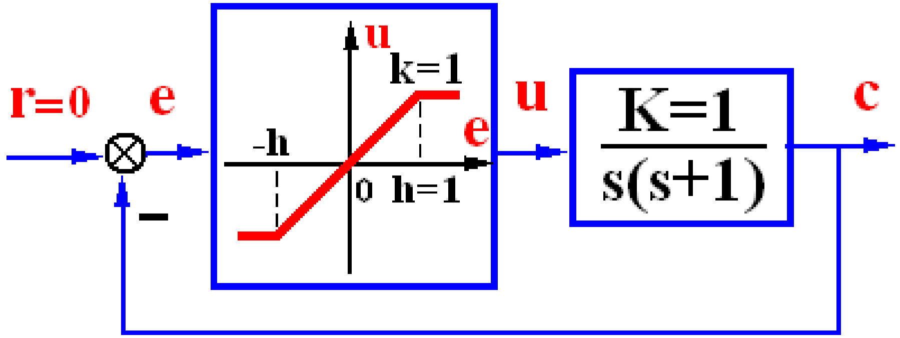
奇点 $\begin{cases} \mathrm{I} & x_{e1}=-1 \\ \mathrm{II} & x_{e2}=1 \end{cases}$

特征方程 $\begin{cases} \mathrm{I} & s^2+1=0 \\ \mathrm{II} & s^2+1=0 \end{cases}$

极点 $\begin{cases} s_{1,2}=\pm j1 & \text{中心点} \\ s_{1,2}=\pm j1 & \text{中心点} \end{cases}$



练习2：系统如右，在$(c \sim \dot{c})$平面上分析系统的自由响应运动。



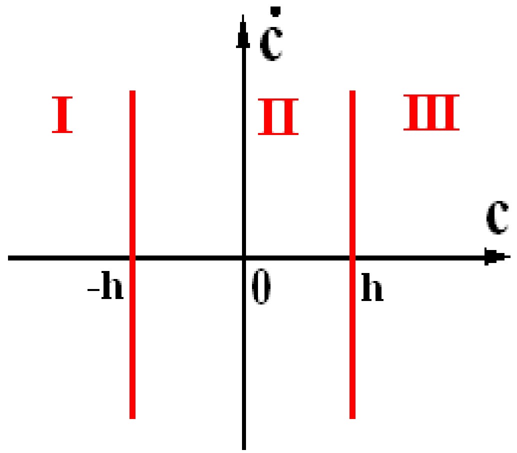
解　线性部分 $\dfrac{C(s)}{U(s)} = \dfrac{1}{s^2+s}$

$$\ddot{c} + \dot{c} = u$$

非线性部分 $u = \begin{cases} 1 & e > h \quad (\mathrm{I}) \\ e & |e| \le h \quad (\mathrm{II}) \\ -1 & e < -h \quad (\mathrm{III}) \end{cases}$

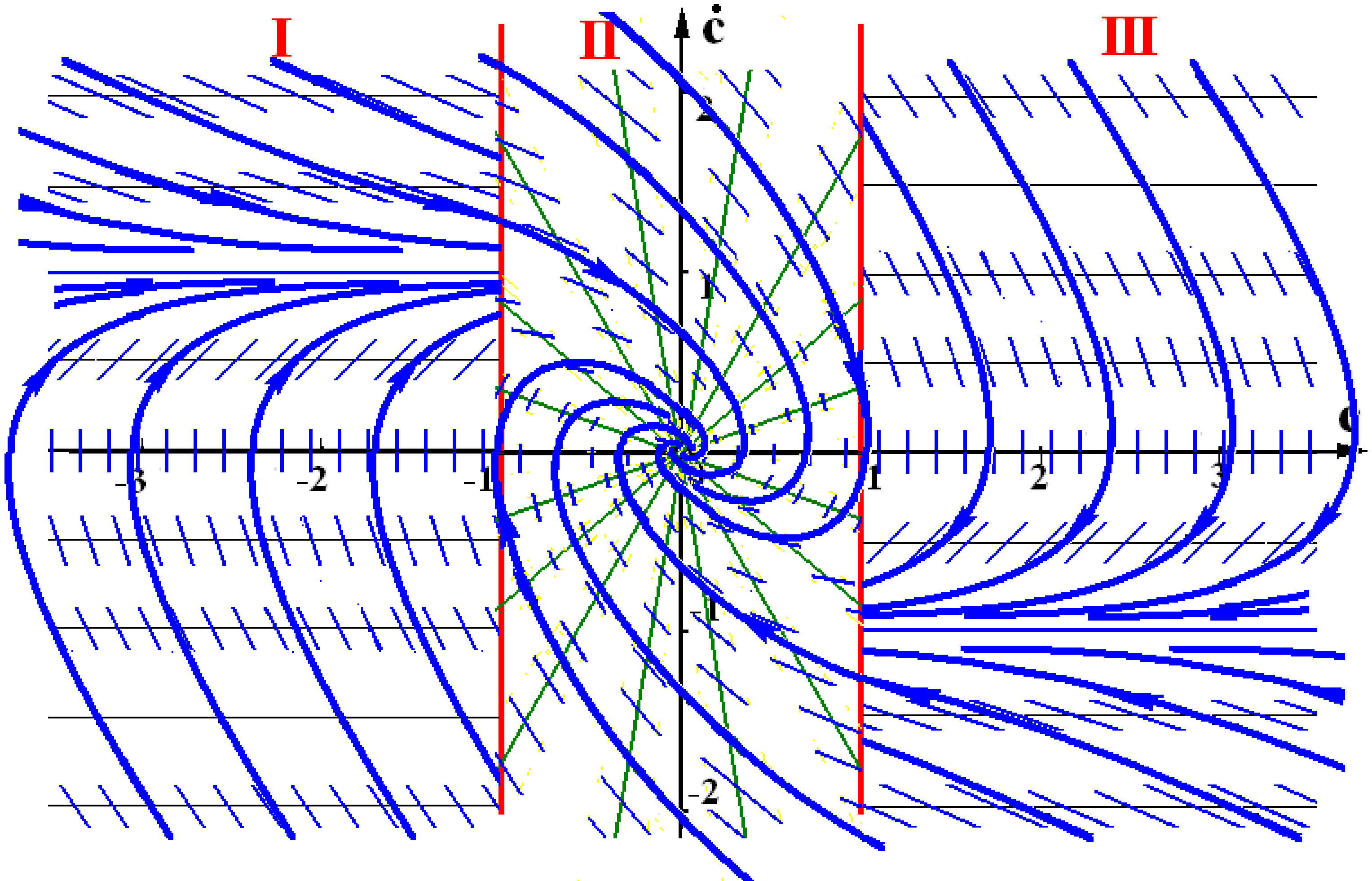
比较点　$e = r - c = -c$

整理　$\ddot{c} + \dot{c} = u = \begin{cases} 1 & c < -h \quad (\mathrm{I}) \\ -c & |c| \le h \quad (\mathrm{II}) \\ -1 & c > h \quad (\mathrm{III}) \end{cases}$

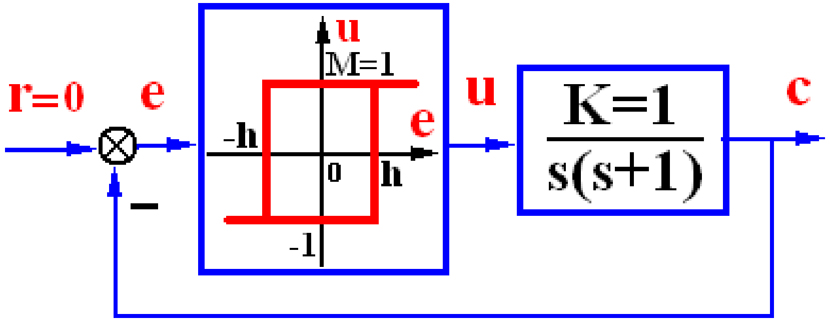


$$
\begin{cases}
(\mathrm{I}) & \ddot c = \dot c \cdot \dfrac{d\dot c}{dc} = \alpha \dot c = 1 - \dot c \\
(\mathrm{II}) & \ddot c = \dot c \cdot \dfrac{d\dot c}{dc} = \alpha \dot c = -(c + \dot c) \\
(\mathrm{III}) & \ddot c = \dot c \cdot \dfrac{d\dot c}{dc} = \alpha \dot c = -1 - \dot c
\end{cases}
\quad
\begin{cases}
c < -h \\
|c| < h \\
c > h
\end{cases}
\quad
\begin{cases}
\dot c = \dfrac{1}{1+\alpha} \\
\dot c = \dfrac{-c}{1+\alpha} \\
\dot c = \dfrac{-1}{1+\alpha}
\end{cases}
$$

$$
\ddot c + \dot c = u = \begin{cases} 1 & c < -h & (\mathrm{I}) \\ -c & |c| \le h & (\mathrm{II}) \\ -1 & c > h & (\mathrm{III}) \end{cases}
$$



练习3： 系统如右，在 $(c \sim \dot{c})$ 平面上分析系统的自由响应运动。



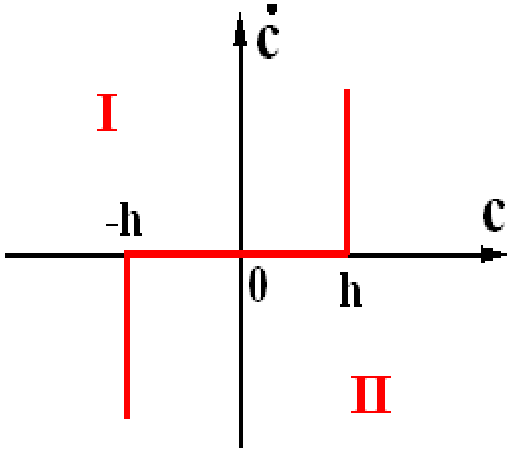
解 线性部分 $\dfrac{C(s)}{U(s)} = \dfrac{1}{s^2+s}$

$$\ddot{c} + \dot{c} = u$$

非线性部分 $u = \begin{cases} 1 & \begin{cases} e > h \\ e > -h,\ \dot{e} < 0 \end{cases} \\ -1 & \begin{cases} e < -h \\ e < h,\ \dot{e} > 0 \end{cases} \end{cases}$

比较点 $e = r - c$

整理 $\ddot{c} + \dot{c} = u = \begin{cases} 1 & \begin{cases} c < -h \\ c < h,\ \dot{c} > 0 \end{cases} \\ -1 & \begin{cases} c > h \\ c > -h,\ \dot{c} < 0 \end{cases} \end{cases}$

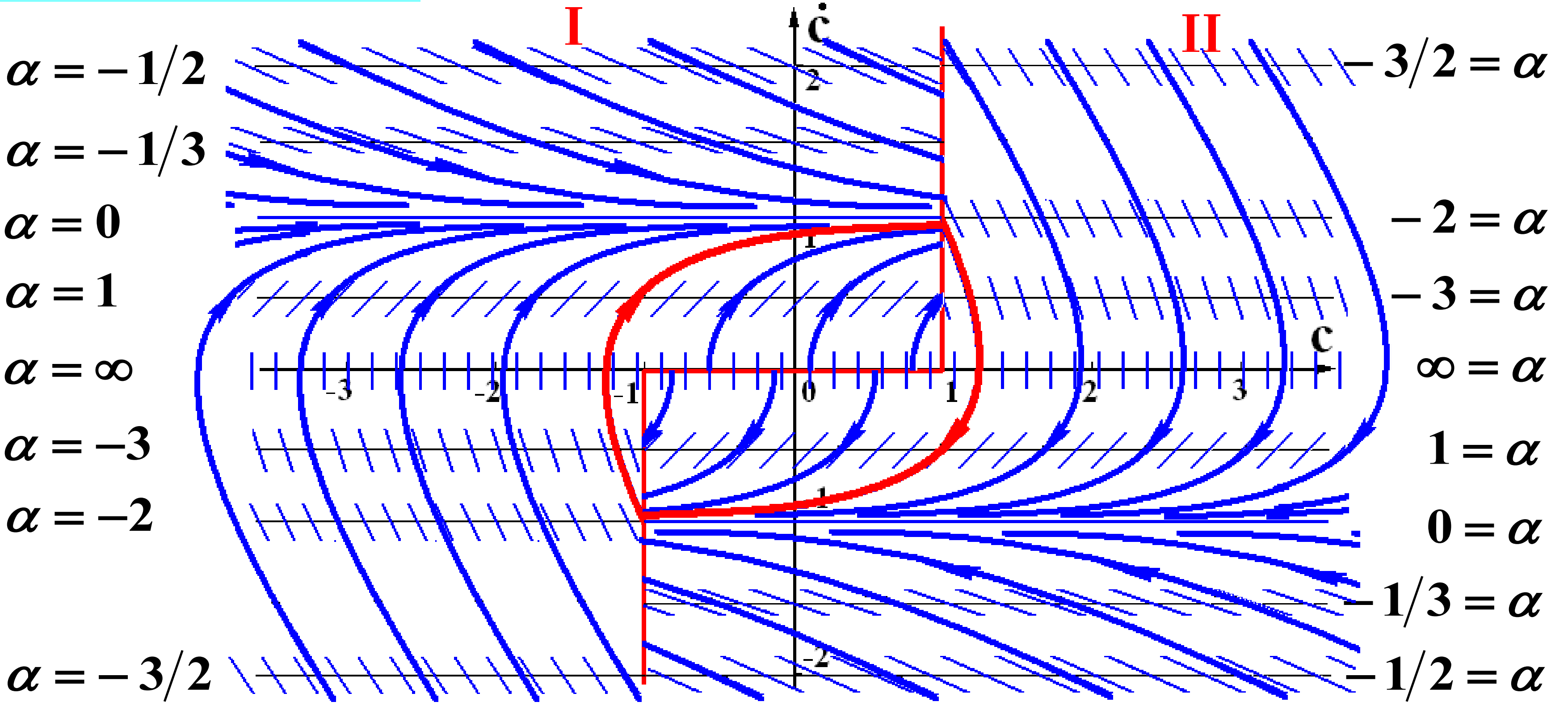


α		$-1/2$	$-1/3$	0	1	∞	-3	-2	$-3/2$
I	$1/(1+\alpha)$	2	$3/2$	1	$1/2$	0	$-1/2$	-1	-2
II	$-1/(1+\alpha)$	-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	2

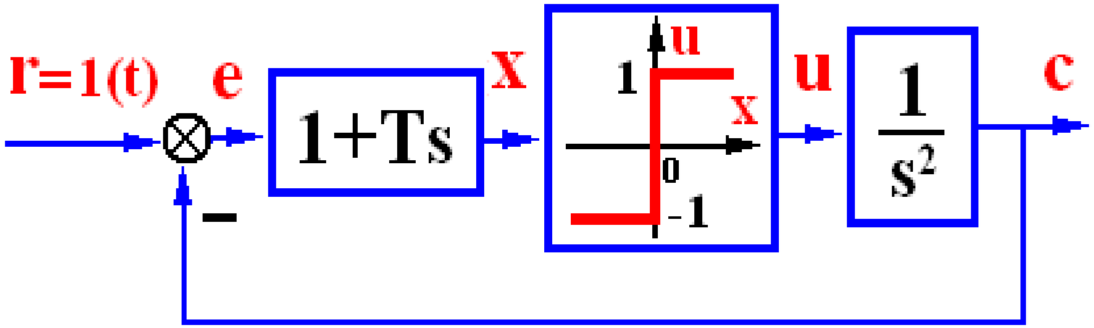
$$\ddot{c}+\dot{c}=\begin{cases}1 & \begin{cases}c<-h\\ c<h,\ \dot{c}>0\end{cases}\\ -1 & \begin{cases}c>h\\ c>-h,\ \dot{c}<0\end{cases}\end{cases}$$

$$\text{(I)}\quad \ddot{c}=\dot{c}\cdot\frac{d\dot{c}}{dc}\Rightarrow\ \alpha\dot{c}=1-\dot{c}\qquad \text{等倾斜线}\quad \dot{c}=\frac{1}{1+\alpha}$$

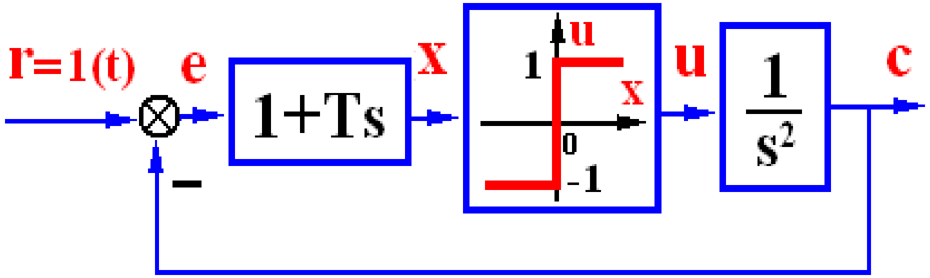
$$\text{(II)}\quad \ddot{c}=\dot{c}\cdot\frac{d\dot{c}}{dc}\Rightarrow\ \alpha\dot{c}=-1-\dot{c}\qquad \text{等倾斜线}\quad \dot{c}=\frac{-1}{1+\alpha}$$



练习4： 系统如右 $r(t)=1(t)$　$T=0,\ 0.5$　讨论系统运动。



解：线性部分 $\ddot{c}(t)=u(t)$



非线性部分 $u=\begin{cases}1 & e+T\dot{e}>0 \quad (\mathrm{I}) \\ -1 & e+T\dot{e}<0 \quad (\mathrm{II})\end{cases}$

比较点 $e=r-c=1-c$

整理 $\ddot{e}=-\ddot{c}=-u=\begin{cases}-1 & e+T\dot{e}>0 \quad (\mathrm{I}) \\ 1 & e+T\dot{e}<0 \quad (\mathrm{II})\end{cases}$

开关线方程 $\dot{e}=\dfrac{-1}{T}e$

在 I 区：$\ddot{e}=\dfrac{d\dot{e}}{de}\dfrac{de}{dt}=\dot{e}\dfrac{d\dot{e}}{de}=-1 \Rightarrow \dot{e}^2=-2e+C_{\mathrm{I}}$

同理在 II 区：$\Rightarrow \dot{e}^2=2e+C_{\mathrm{II}}$

$\left.\begin{matrix}\dot{e}^2=-2e+C_{\mathrm{I}} \\ \dot{e}^2=2e+C_{\mathrm{II}}\end{matrix}\right\}$ 抛物线方程

当 $T=\begin{cases}0 \\ 0.5\end{cases}$ 时，开关线为：$\begin{cases}e=0 \\ \dot{e}=-2e\end{cases}$

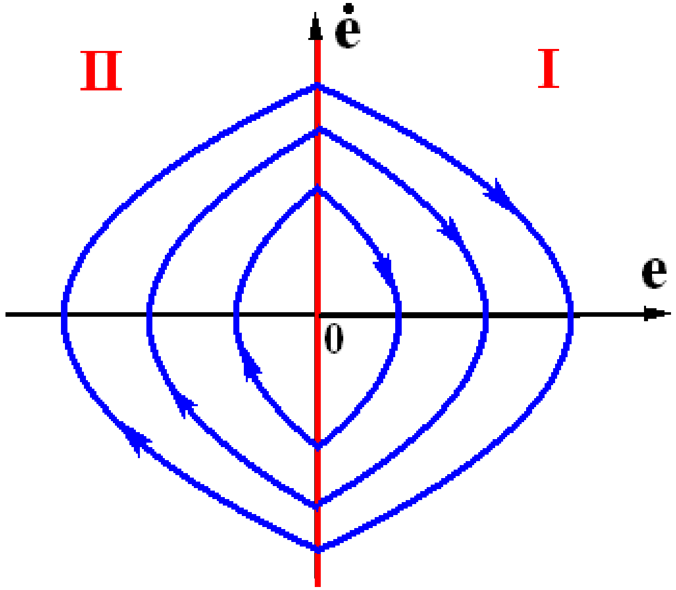
$$\ddot{e}=\begin{cases}-1 & e+T\dot{e}>0 \quad (\mathrm{I}) \\ 1 & e+T\dot{e}<0 \quad (\mathrm{II})\end{cases}$$

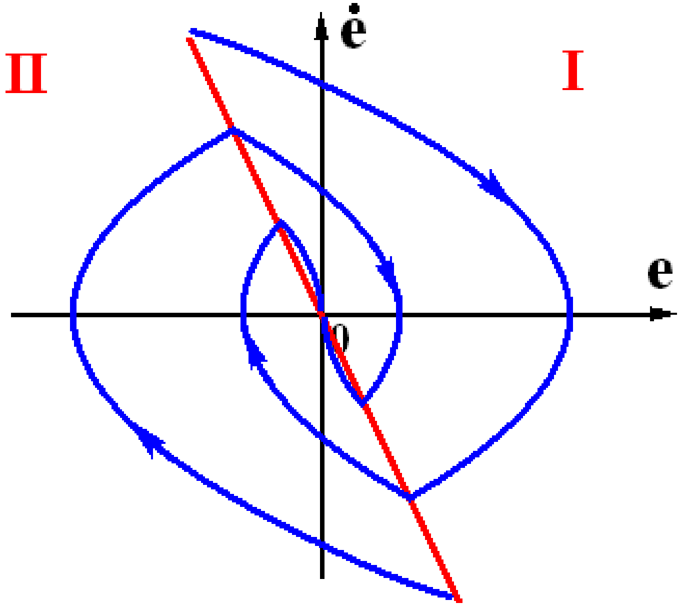
(I) $e+T\dot{e}>0$ (II) $e+T\dot{e}<0$

系统方程

$$\begin{cases}\ddot{e}=-1 \\ \dot{e}^2=-2e+C_{\mathrm{I}}\end{cases} \qquad \begin{cases}\ddot{e}=1 \\ \dot{e}^2=2e+C_{\mathrm{II}}\end{cases}$$

相轨迹图





开关线

$$\begin{cases}T=0 \\ e=0\end{cases} \qquad \begin{cases}T=0.5 \\ \dot{e}=-2e\end{cases}$$