Chapter 7 Describing Functions Analysis

Chengju Liu

Department of Control Science & Engineering

School of Electronic & Information Engineering

Tongji University

E-mail: liuchengju@tongji.edu.cn

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Outline of Chapter 7

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Concept of Frequency-Response Characteristic

$$r(t)=A\sin\omega t$$

$$\longrightarrow \boxed{\frac{1}{Ts+1}} \longrightarrow$$

$$c_s(t)=\frac{A}{\sqrt{1+\omega^2T^2}}\sin(\omega t\text{-}\arctan\omega T)$$

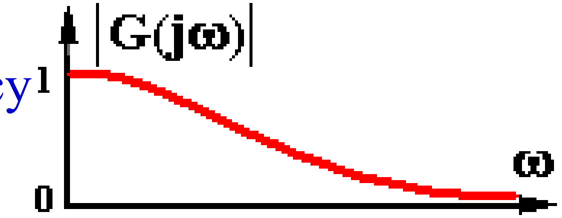
Definition of Frequency-Response Characteristic

$G(j\omega)$ Definition 1 $G(j\omega)=|G(j\omega)|\angle G(j\omega)$

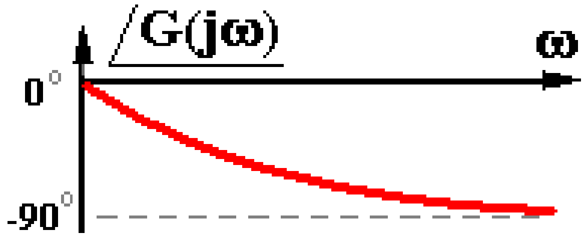
$$|G(j\omega)|=\frac{|c_s(t)|}{|r(t)|}=\frac{1}{\sqrt{1+\omega^2T^2}}$$

$$\angle G(j\omega)=\angle c_s(t)-\angle r(t)=-\arctan\omega T$$

Magnitude-Frequency Characteristic



Phase-Frequency Characteristic



$G(j\omega)$ Definition 2 $G(j\omega)=G(s)|_{s=j\omega}$

$$\frac{1}{\sqrt{1+\omega^2T^2}}\angle-\arctan\omega T=\left|\frac{1}{1+j\omega T}\right|\angle\frac{1}{1+j\omega T}=\frac{1}{1+j\omega T}=\left.\frac{1}{Ts+1}\right|_{s=j\omega}$$

Introduction

- **Motivations**

  Frequency response methods based on transfer functions are the main tool for linear system analyses and syntheses. Can we find something akin to “transfer functions” for nonlinear systems?

- **Observations**

  In response to a sinusoidal signal, most nonlinearities will produce a periodic (not necessarily sinusoidal) signal with frequencies being the harmonics of the input frequency.

Consider a nonlinear element $y = f(u)$ shown below.

$$u \longrightarrow \boxed{f(u)} \longrightarrow y$$

Let $u(t) = X \sin(\omega t)$, $y(t)$ will be periodic with a fundamental frequency ω.
Then, $y(t)$ can be described by the following Fourier series:

$$\begin{aligned} y(t) &= \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega t) + B_n \sin(n\omega t) \right] \\ &= \frac{A_0}{2} + \sum_{n=1}^{\infty} Y_n \sin(n\omega t + \varphi_n) \end{aligned}$$

where $\omega = \frac{2\pi}{T}$

$A_n = \frac{1}{\pi} \int_0^{2\pi} y(t) \cos(n\omega t)\, \mathrm{d}(\omega t), \quad B_n = \frac{1}{\pi} \int_0^{2\pi} y(t) \sin(n\omega t)\, \mathrm{d}(\omega t)$

$Y_n = \sqrt{A_n^2 + B_n^2}, \quad \varphi_n = \arctan \frac{A_n}{B_n}$

$$y(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega t) + B_n \sin(n\omega t)\right] = \frac{A_0}{2} + \sum_{n=1}^{\infty} Y_n \sin(n\omega t + \varphi_n)$$

$$\Downarrow$$

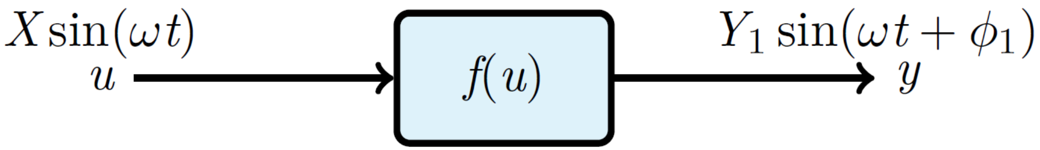
$$y(t) \approx A_1 \cos(\omega t) + B_1 \sin(\omega t) = Y_1 \sin(\omega t + \varphi_1)$$

- The nonlinear element can be described by the first fundamental component of the above series.
- It seems as if the function f were a linear system with a gain of Y_1 and phase of φ_1.

What is a describing function?

Definition

The **describing function** is the complex ratio of the amplitude of the fundamental component of the output of the nonlinear element to the sinusoidal input signal.



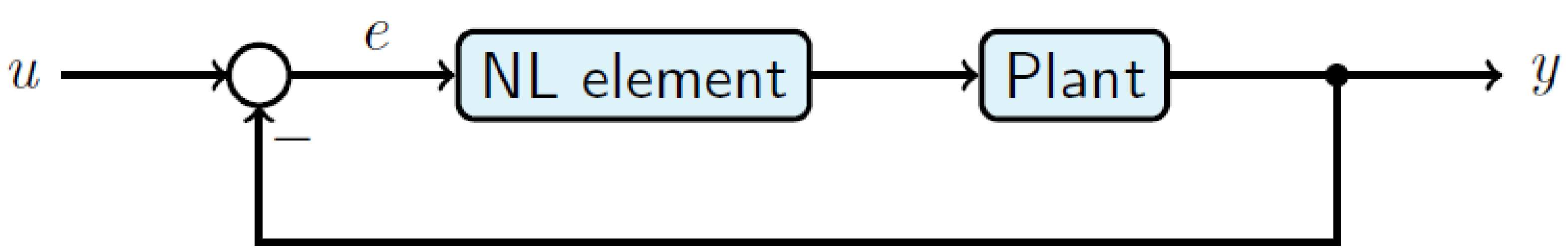
$$
\begin{aligned}
N(X,\omega) &= \frac{B_1 + jA_1}{X} \\
&= \frac{Y_1(X,\omega)}{X} e^{j\varphi_1}
\end{aligned}
$$

Y_1 : Amplitude of the fundamental harmonic component of the output.

φ_1 : Phase shift of the fundamental harmonic component of the output.

Outline of Chapter 7

Assumptions



- The nonlinearity is time invariant, there is a single nonlinear element in the system(非线性系统的结构图可简化成一个非线性环节N和一个线性部分$G(s)$串联的闭环结构).

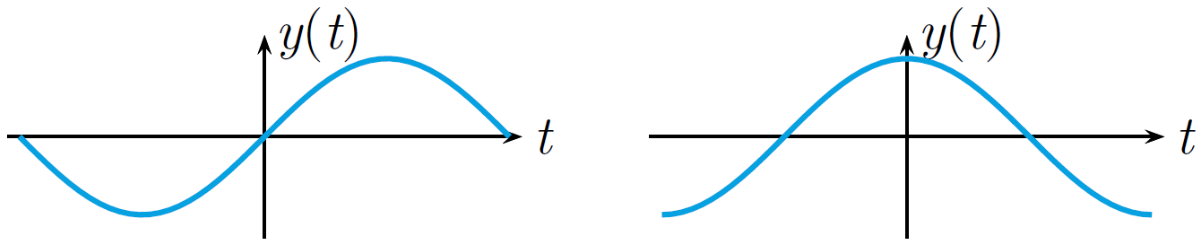
- Nonlinearity is symmetric about the origin (N(A)奇对称), this assumption makes the static term in the Fourier expansion of the output zero.

- Approximate the output by the first harmonic alone and neglect the(基波占优），the linear element has low-pass properties(系统的线性部分具有良好的低通滤波特性).

How to calculate describing functions?

- Computation of the describing function is generally straightforward, but tedious.
- It can be computed either analytically or numerically, and may be determined by experiments.
- Some tips for computing $y(t) \approx A_1 \cos(\omega t) + B_1 \sin(\omega t)$.

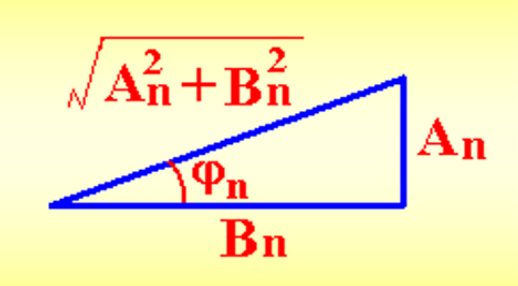
  - Odd function $y(t) = -y(-t)$ and even function $y(t) = y(-t)$



  - The describing functions for odd and memoryless nonlinearities (saturation, relay, deadzone) are frequency independent ($A_1 = 0$).

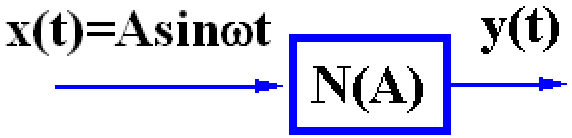
The definition of describing functions?

The Fourier series expansion for periodic function $y(t)$



$$y(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right)$$

$$= A_0 + \sum_{n=1}^{\infty} Y_n \sin(n\omega t + \phi_n)$$



$$\begin{cases} A_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cos n\omega t \; d(\omega t) \\ B_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\sin n\omega t \; d(\omega t) \end{cases} \qquad \begin{cases} Y_n = \sqrt{A_n^2 + B_n^2} \\ \phi_n = \arctan \dfrac{A_n}{B_n} \end{cases}$$

According to the assumptions, we get

$$y(t) = y_1(t) = Y_1 \sin(\omega t + \phi_1)$$

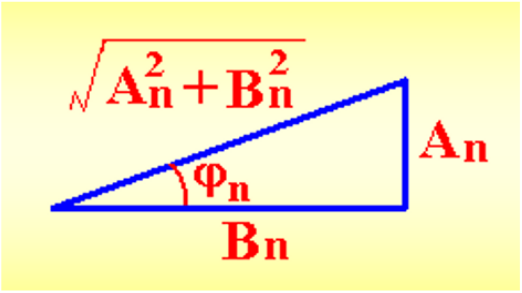
Definition of describing functions N(A)

$$\boxed{N(A) = \frac{Y_1}{A}\angle\phi_1 = \frac{\sqrt{A_1^2 + B_1^2}}{A}\angle\left(\arctan\frac{A_1}{B_1}\right)}$$

■ Determining the describing function of the two-position relay without hysteresis

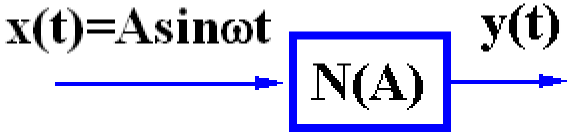
$$y_N(t) = \begin{cases} M & x(t) > 0 \\ -M & x(t) < 0 \end{cases}$$

■ The Fourier series expansion for periodic function $y(t)$

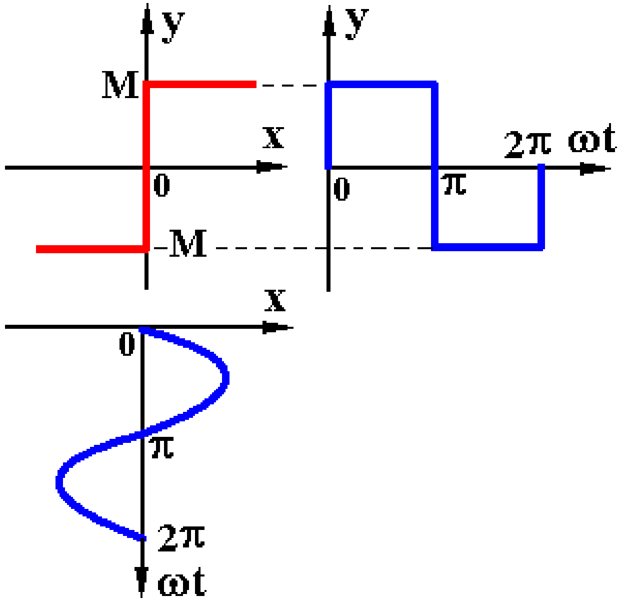


$$y(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right)$$

$$= A_0 + \sum_{n=1}^{\infty} Y_n \sin(n\omega t + \phi_n)$$



$$\begin{cases} A_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cos n\omega t \; d(\omega t) \\ B_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\sin n\omega t \; d(\omega t) \end{cases} \qquad \begin{cases} Y_n = \sqrt{A_n^2 + B_n^2} \\ \phi_n = \arctan \dfrac{A_n}{B_n} \end{cases}$$



$$y_1(t) = A_1 \cos\omega t + B_1 \sin\omega t = Y_1 \sin(\omega t + \phi_1)$$

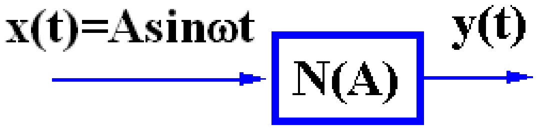
Fourier series

$$y(t) = \frac{4M}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots\right]$$

$$y_1(t) = \frac{4M}{\pi}\sin\omega t$$

Definition of describing functions

Input: $x(t) = A\sin\omega t$



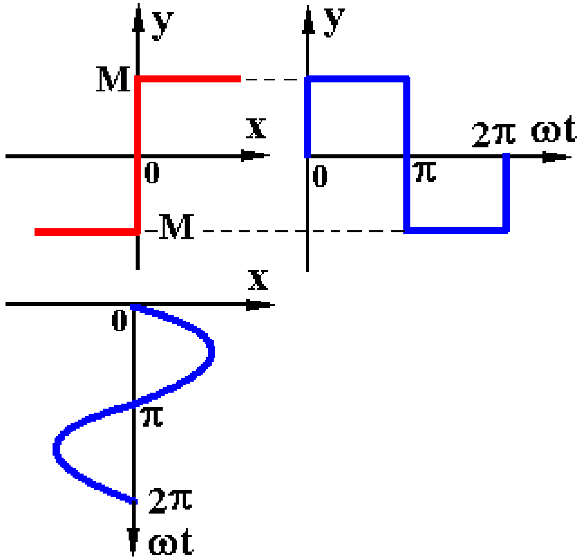
The fundamental wave of the output:

$$y_1(t) = Y_1 \sin(\omega t + \phi_1)$$

$$y_1(t) = \frac{4M}{\pi}\sin\omega t$$

Definition of describing functions N(A)

$$\boxed{N(A) = \frac{Y_1}{A}\angle\phi_1 = \frac{\sqrt{A_1^2 + B_1^2}}{A}\angle\left(\arctan\frac{A_1}{B_1}\right)}$$



The describing functions of ideal relays

$$N(A) = \frac{4M}{\pi A}\angle 0^\circ$$

How to evaluate describing functions by the analytical technique?

Example:
Determining the describing function of the following nonlinear element

$$y_N(t) = x(t)^3$$

$$y(t) = A^3 \sin^3 \omega t$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} A^3 \sin^4 \omega t \cdot d\omega t = \frac{4A^3}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{4}(1-\cos 2\omega t)^2 \cdot d\omega t$$

$$= \frac{A^3}{\pi}\int_0^{\frac{\pi}{2}} (1 - 2\cos 2\omega t + \cos^2 2\omega t)\cdot d\omega t = \frac{A^3}{\pi}\left[\frac{\pi}{2}\right] - \frac{A^3}{\pi}\left[\sin 2\omega t\right]_0^{\frac{\pi}{2}}$$

$$+\frac{A^3}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos 4\omega t + 1}{2}\cdot d\omega t$$

$$= \frac{A^3}{2} - 0 + \frac{A^3}{2\pi}\int_0^{\frac{\pi}{2}} \cos 4\omega t \cdot d\omega t + \frac{A^3}{2\pi}\int_0^{\frac{\pi}{2}} d\omega t = \frac{3A^3}{4}$$

$$N(A) = \frac{B_1}{A} + j\frac{A_1}{A} = \frac{3A^2}{4}$$

How to evaluate describing functions by the analytical technique?

Example:

Determining the describing function of the following nonlinear element

$$y(x) = \frac{1}{2}x + \frac{1}{4}x^3$$

$y(x)$为x的奇函数，因此：$A_0 = 0$；当输入$x = A\sin\omega t$时，

$$y(t) = \frac{A}{2}\sin\omega t + \frac{A^3}{4}\sin^3\omega t$$

为t的奇函数，因此：$A_1 = 0$；因为$y(t)$具有半周期对称，有：

$$B_1 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} y(t)\sin\omega t d(\omega t) = \frac{4}{\pi}\left(\int_0^{\frac{\pi}{2}} \frac{A}{2}\sin^2\omega t d(\omega t) + \int_0^{\frac{\pi}{2}} \frac{A^3}{4}\sin^4\omega t d(\omega t)\right)$$

可得：

$$B_1 = \frac{A}{2} + \frac{3}{16}A^3$$

则该非线性元件的描述函数为：

$$N(A) = \frac{B_1}{A} = \frac{1}{2} + \frac{3}{16}A^2$$

Outline of Chapter 7

7.1 Introduction

7.2 Calculation of describing functions

7.3 Typical describing functions

7.4 Stability analysis by describing functions

7.5 Self-oscillation & its stability

7.6 Further discussions

7.7 Simulations with MATLAB

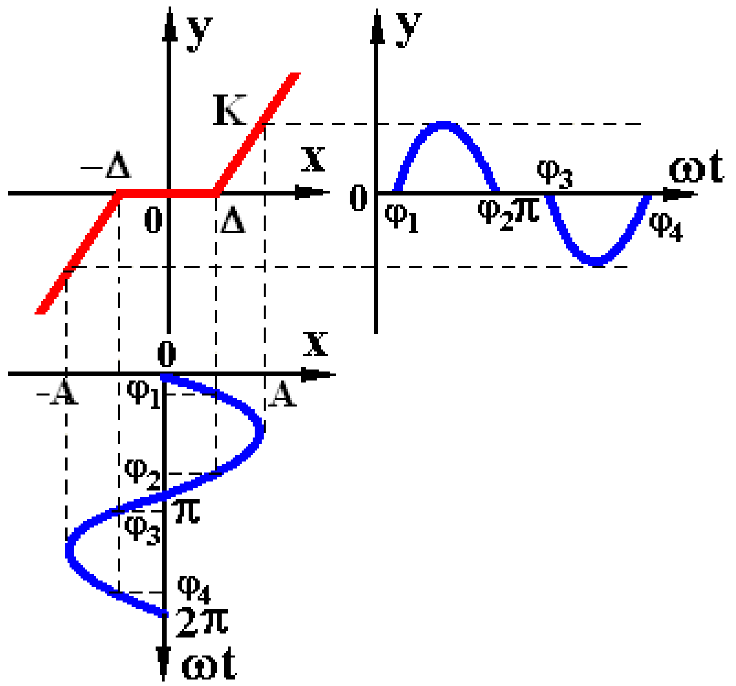
Typical describing functions: Dead zone

$$\begin{cases} A_0 = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} y(t)\cdot d\omega t \\ A_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot \cos n\omega t\cdot d\omega t \\ B_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot \sin n\omega t\cdot d\omega t \end{cases}$$

According to the odd symmetry characteristic of the dead zone nonlinearities, we have

$$\begin{cases} A_0 = 0 \\ A_1 = 0 \end{cases}$$

$$\begin{aligned} B_1 &= \frac{1}{\pi}\int_0^{2\pi} y(t)\cdot \sin\omega t\cdot d\omega t = \frac{2}{\pi}\int_0^{\pi} y(t)\cdot \sin\omega t\cdot d\omega t \\ &= \frac{2}{\pi}K\int_{\varphi_1}^{\varphi_2}\left(A\sin\omega t-\Delta\right)\cdot\sin\omega t\cdot d\omega t \\ &= \frac{4}{\pi}\int_{\varphi_1}^{\pi/2} K(A\sin\omega t-\Delta)\cdot\sin\omega t\cdot d\omega t \\ &= \frac{4KA}{\pi}\int_{\varphi_1}^{\pi/2}\sin^2\omega t\cdot d\omega t-\frac{4K\Delta}{\pi}\int_{\varphi_1}^{\pi/2}\sin\omega t\cdot d\omega t \\ &= \frac{4KA}{\pi}\int_{\varphi_1}^{\pi/2}\frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t-\frac{4K\Delta}{\pi}\int_{\varphi_1}^{\pi/2}\sin\omega t\cdot d\omega t \\ &= \frac{4KA}{\pi}\left[\frac{\omega t}{2}-\frac{1}{4}\sin 2\omega t+\frac{\Delta}{A}\cos\omega t\right]_{\varphi_1}^{\pi/2} \end{aligned}$$



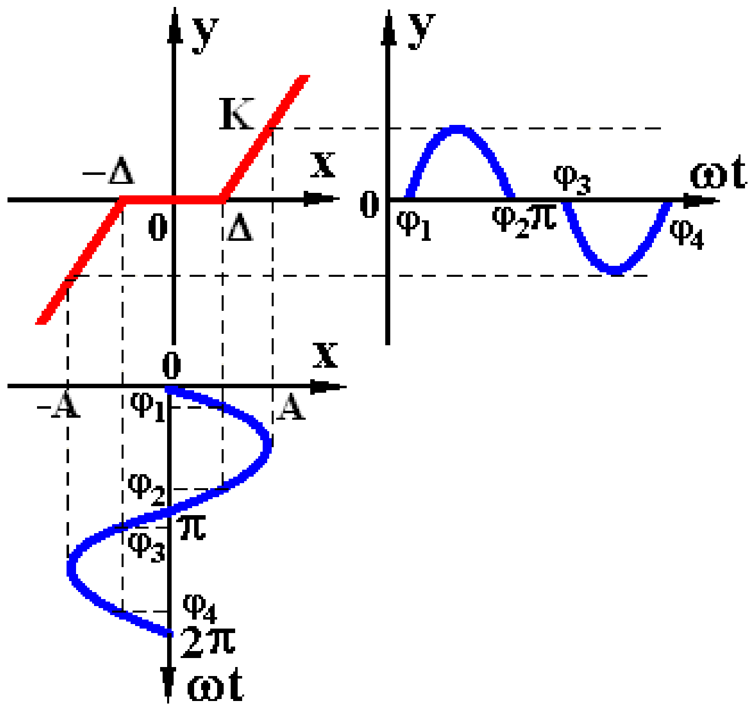
Fourier series

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\cdot\sin\omega t\cdot d\omega t = \frac{2}{\pi}\int_0^{\pi} y(t)\cdot\sin\omega t\cdot d\omega t$$

$$= \frac{4KA}{\pi}\left[\frac{\omega t}{2} - \frac{1}{4}\sin 2\omega t + \frac{\Delta}{A}\cos\omega t\right]_{\varphi_1}^{\pi/2}$$

$$\begin{cases} A_0 = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} y(t)\cdot d\omega t \\ A_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot\cos n\omega t\cdot d\omega t \\ B_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot\sin n\omega t\cdot d\omega t \end{cases}$$

$$A\sin\varphi_1 = \Delta \quad\Rightarrow\quad \sin\varphi_1 = \frac{\Delta}{A} \quad\Rightarrow\quad \varphi_1 = \arcsin\frac{\Delta}{A}$$



$$B_1 = \frac{4KA}{\pi}\left[\frac{\pi}{4} - \frac{1}{2}arcsin\frac{\Delta}{A} + \frac{1}{2}\frac{\Delta}{A}cos\left(arcsin\frac{\Delta}{A}\right) - \frac{\Delta}{A}cos\left(arcsin\frac{\Delta}{A}\right)\right]$$

$$= \frac{4KA}{\pi}\left[\frac{\pi}{4} - \frac{1}{2}arcsin\frac{\Delta}{A} - \frac{\Delta}{2A}\sqrt{1-\left(\frac{\Delta}{A}\right)^2}\right] \qquad A \geq \Delta$$

The describing function of the dead zone nonlinearities:

$$N(A) = \frac{B_1}{A} = \frac{2K}{\pi}\left[\frac{\pi}{2} - arcsin\frac{\Delta}{A} - \frac{\Delta}{A}\sqrt{1-\left(\frac{\Delta}{A}\right)^2}\right] \qquad A \geq \Delta$$

Typical describing functions: Saturation

$$\begin{cases} A_0 = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} y(t)\cdot d\omega t \\ A_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot \cos n\omega t\cdot d\omega t \\ B_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} y(t)\cdot \sin n\omega t\cdot d\omega t \end{cases}$$

$$y(t) = \begin{cases} KA\sin\omega t & 0 \le \omega t \le \phi_1 \\ Ka & \phi_1 \le \omega t \le \pi/2 \end{cases}$$

According to the odd symmetry characteristic of the dead zone nonlinearities, we have

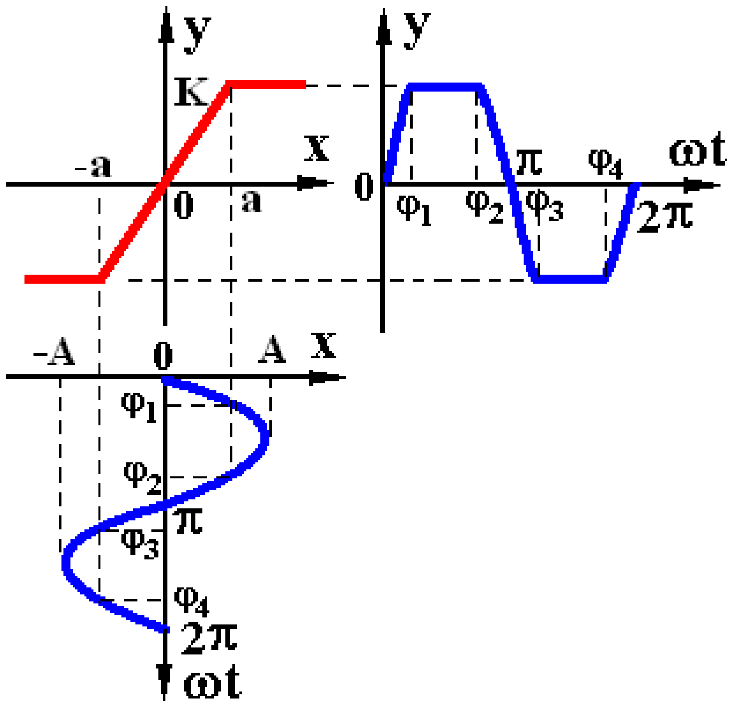
$$\begin{cases} A_0 = 0 \\ A_1 = 0 \end{cases} \qquad A\sin\phi_1 = a, \qquad \phi_1 = \arcsin\frac{a}{A}$$

$$B_1 = \frac{4}{\pi}\left[\int_0^{\phi_1} KA\sin^2\omega t\cdot d\omega t + \int_{\phi_1}^{\pi/2} Ka\sin\omega t\cdot d\omega t\right]$$

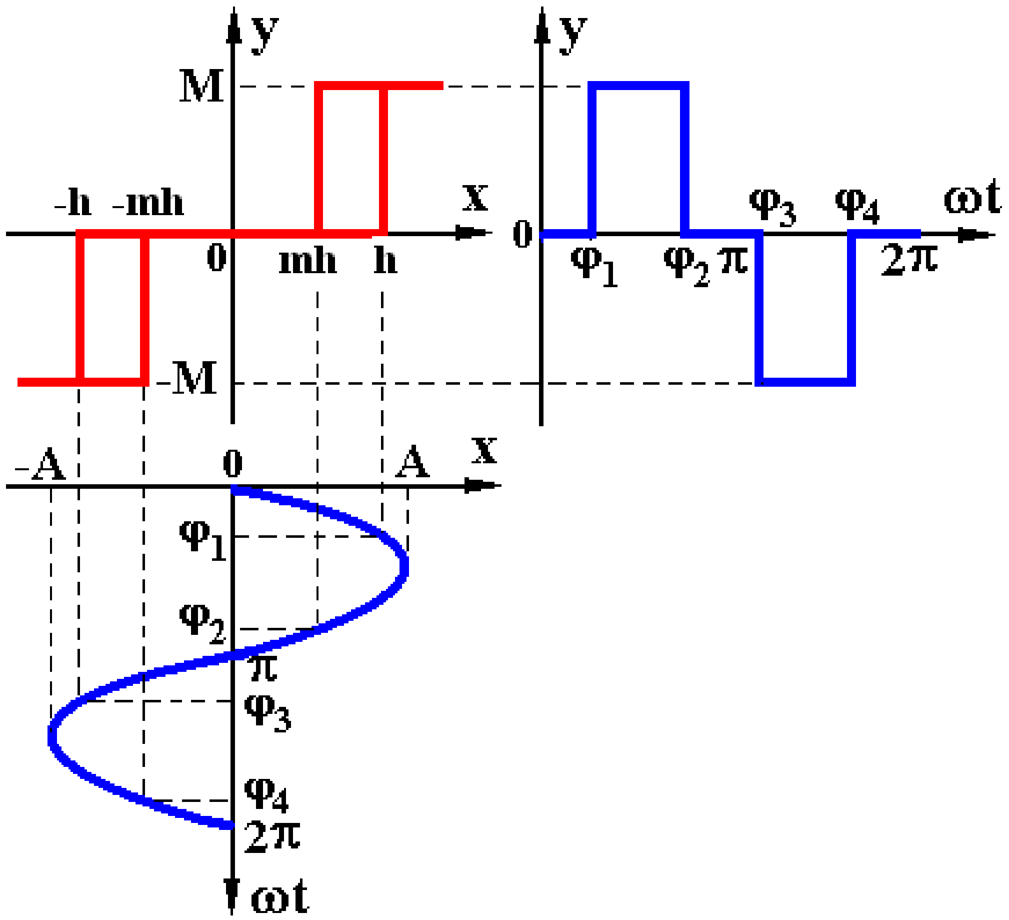
$$= \frac{4KA}{\pi}\left[\int_0^{\phi_1}\frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t + \frac{a}{A}\int_{\phi_1}^{\pi/2}\sin\omega t\cdot d\omega t\right]$$

$$= \frac{4KA}{\pi}\left\{\left[\frac{1}{2}\omega t - \frac{1}{4}\sin 2\omega t\right]_0^{\phi_1} + \left[\frac{a}{A}(-\cos\omega t)\right]_{\phi_1}^{\pi/2}\right\}$$

$$= \frac{4KA}{\pi}\left[\frac{1}{2}\arcsin\frac{a}{A} + \frac{a}{2A}\sqrt{1-(\frac{a}{A})^2}\right] \qquad A \ge a$$



Relay with dead zone and hysteresis



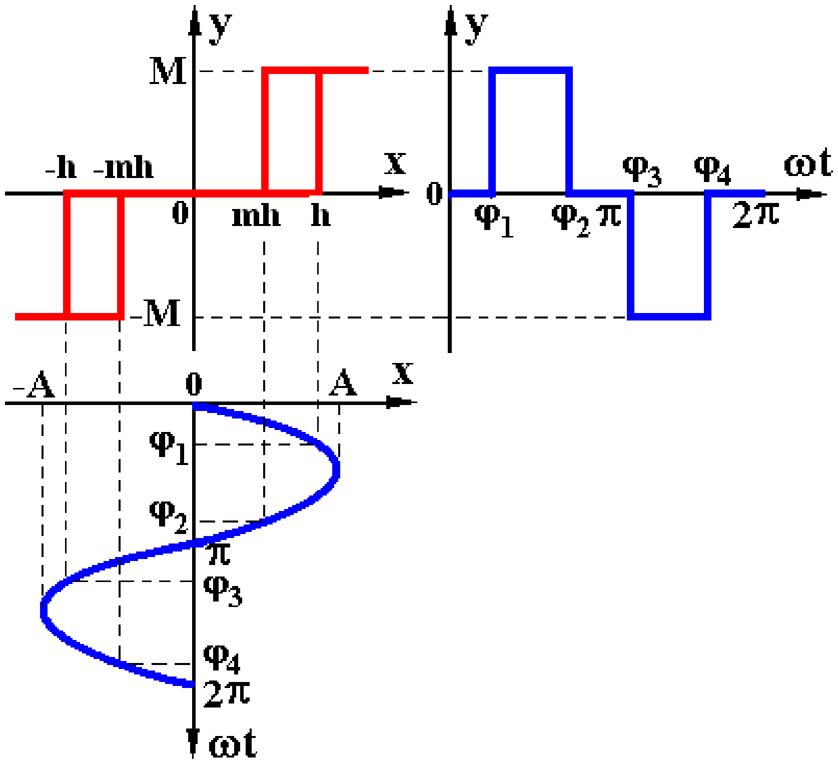
Describing functions for relays

$$\phi_1:\ A\sin\phi_1 = h \qquad \sin\phi_1 = h/A \qquad \cos\phi_1 = \sqrt{1-(h/A)^2}$$

$$\phi_2:\ A\sin\phi_2 = mh \qquad \sin\phi_2 = mh/A \qquad \cos\phi_2 = -\sqrt{1-(mh/A)^2}$$

$$\phi_3:\ A\sin\phi_3 = -h \qquad \sin\phi_3 = -h/A \qquad \cos\phi_3 = -\sqrt{1-(h/A)^2}$$

$$\phi_4:\ A\sin\phi_4 = -mh \qquad \sin\phi_4 = -mh/A \qquad \cos\phi_4 = \sqrt{1-(mh/A)^2}$$

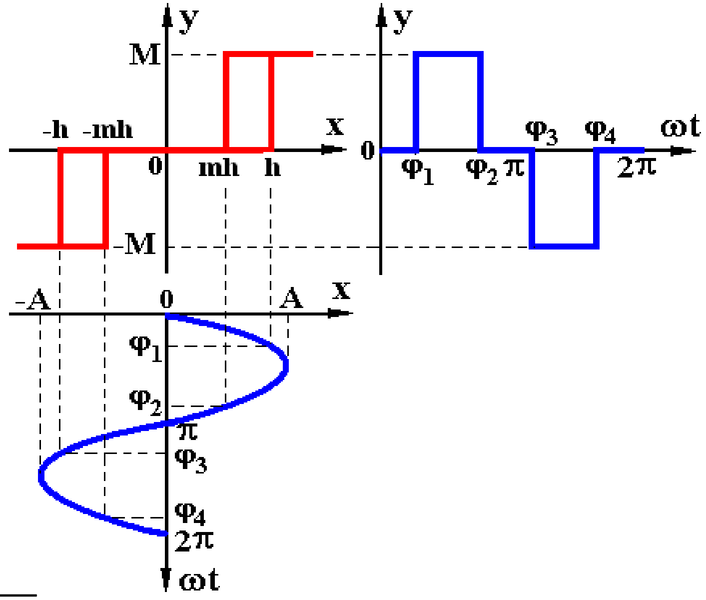


$$A_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\cos\omega t\, d(\omega t) = \frac{2}{\pi}\int_{\phi_1}^{\phi_2} M\cos\omega t\, d(\omega t) = \frac{2Mh}{\pi A}(m-1)$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\sin\omega t\, d(\omega t) = \frac{2}{\pi}\int_{\phi_1}^{\phi_2} M\sin\omega t\, d(\omega t) = \frac{2M}{\pi}\left[\sqrt{1-(\frac{mh}{A})^2}+\sqrt{1-(\frac{h}{A})^2}\right]$$

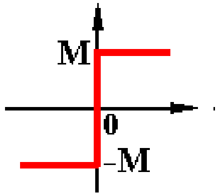
$$N(A) = \frac{B_1}{A} + j\frac{A_1}{A} = \frac{2M}{\pi A}\left[\sqrt{1-\left(\frac{mh}{A}\right)^2}+\sqrt{1-\left(\frac{h}{A}\right)^2}\right] + j\frac{2Mh}{\pi A^2}(m-1) \qquad (A \ge h)$$

Describing functions for relays



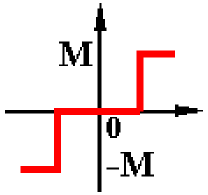
$$N(A)=\frac{2M}{\pi A}\left[\sqrt{1-\left(\frac{mh}{A}\right)^2}+\sqrt{1-\left(\frac{h}{A}\right)^2}\right]+j\frac{2Mh}{\pi A^2}(m-1) \qquad (A\ge h)$$

$h=0$ Ideal relays



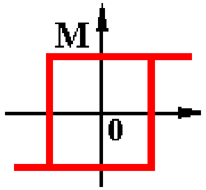
$$N(A)=\frac{4M}{\pi A}$$

$m=1$ Relays with dead zone



$$N(A)=\frac{4M}{\pi A}\sqrt{1-\left(\frac{h}{A}\right)^2}$$

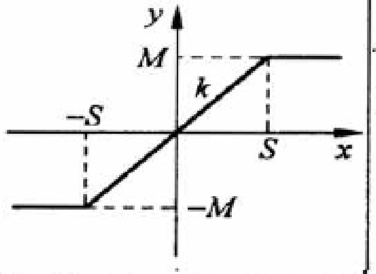
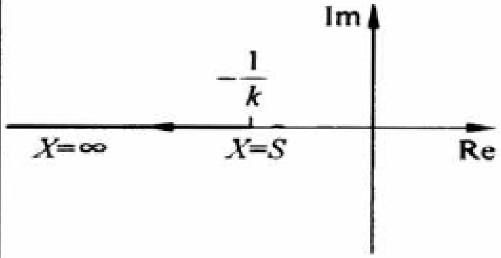
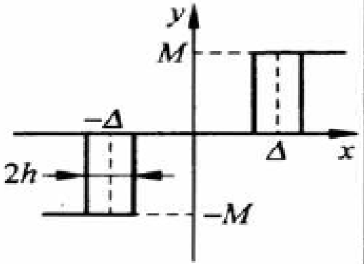
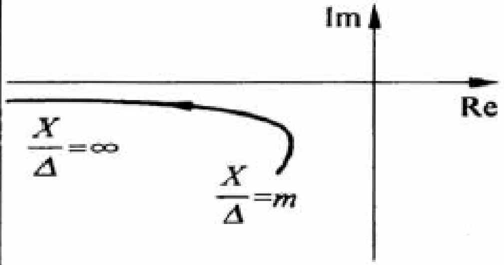
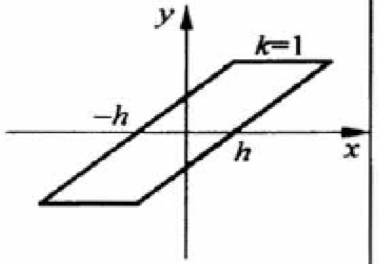
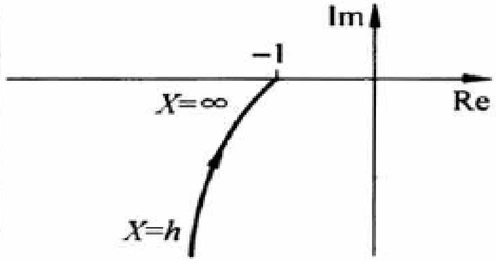
$m=-1$ Relays with hysteresis loop



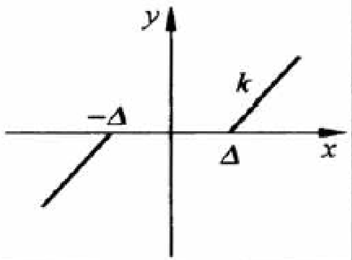
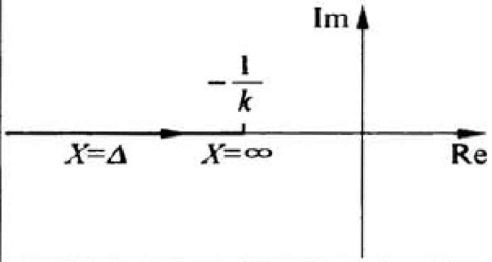
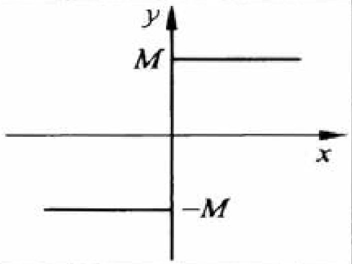
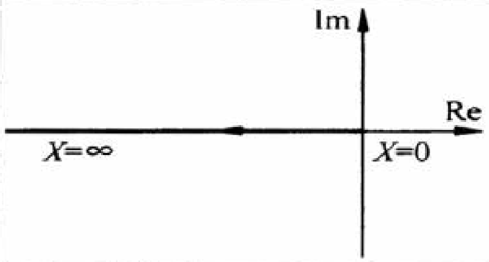
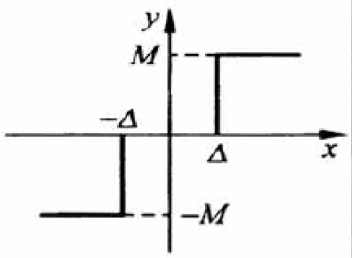
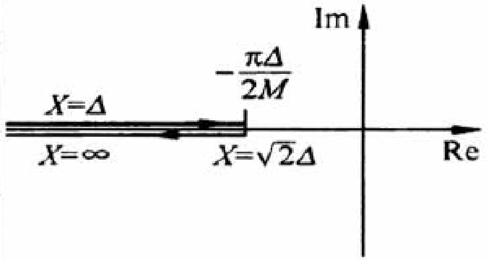
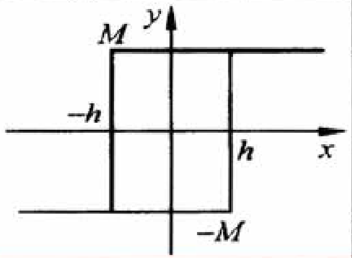
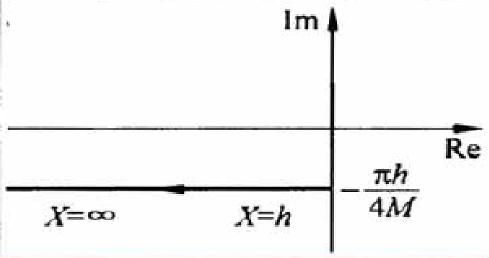
$$N(A)=\frac{4M}{\pi A}\sqrt{1-\left(\frac{h}{A}\right)^2}-j\frac{4Mh}{\pi A^2}$$

- Generally, the describing function N(A) is a function of A, and does not depend on *w*.
- If the nonlinear function is monotropic/non-monotropic, the N(A) is a real/complex function. 非线性环节为单/非单值函数时,N(A)是实/复数,虚部为/不为0.

Summary of typical describing functions

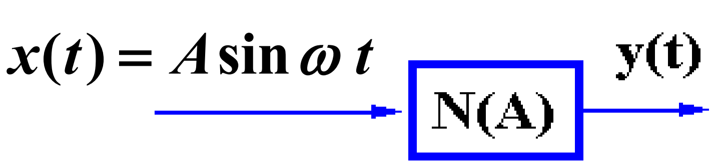
$y=f(x)$	$N(X)$	$-1/N$
	$N(X)=\frac{2k}{\pi}\left[\arcsin\frac{S}{X}+\frac{S}{X}\sqrt{1-\left(\frac{S}{X}\right)^2}\right]$ $(X\geqslant S)$	
	$N(X)=\sqrt{\left(\frac{a_1}{X}\right)^2+\left(\frac{b_1}{X}\right)^2}\,e^{j\arctan\frac{a_1}{b_1}}\ (X\geqslant\Delta+h)$ 其中 $\frac{a_1}{X}=-\frac{4\alpha\beta}{\pi}\left(\frac{\Delta}{X}\right)^2$ $\frac{b_1}{X}=\frac{2\beta}{\pi}\frac{\Delta}{X}\left[\sqrt{1-\left(\frac{\Delta}{X}\right)^2(1-\alpha)^2}\right.$ $\left.+\sqrt{1-\left(\frac{\Delta}{X}\right)^2(1+\alpha)^2}\right]$; $\alpha=\frac{h}{\Delta},\beta=\frac{M}{\Delta}$	 $m=1+h/\Delta$
	$N(X)=\sqrt{\left(\frac{a_1}{X}\right)^2+\left(\frac{b_1}{X}\right)^2}\,e^{j\arctan\frac{a_1}{b_1}}\ (X\geqslant h)$ 其中 $\frac{a_1}{X}=-\frac{4}{\pi}\left[\frac{h}{X}-\left(\frac{h}{X}\right)^2\right]$ $\frac{b_1}{X}=\frac{2}{\pi}\left[\frac{\pi}{4}+\frac{1}{2}\arcsin\left(1-\frac{2h}{X}\right)\right.$ $\left.+\left(1-\frac{2h}{X}\right)\sqrt{\frac{h}{X}-\left(\frac{h}{X}\right)^2}\right]$	

Summary of typical describing functions (con.)

$y=f(x)$	$N(X)$	$-1/N$
	$N(X)=$ $k-\frac{2k}{\pi}\left[\arcsin\frac{\Delta}{X}+\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\right]$ $(X\geqslant\Delta)$	
	$N(X)=\frac{4M}{\pi X}$	
	$N(X)=\frac{4M}{\pi X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$ $(X\geqslant\Delta)$	
	$N(X)=\frac{4M}{\pi X}e^{-j\arcsin\frac{h}{X}}$ $(X\geqslant h)$	

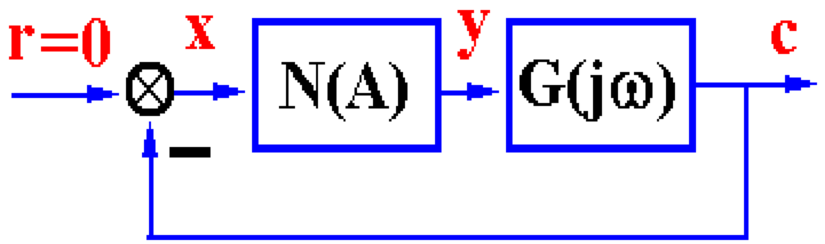
Outline of Chapter 7

1. 描述函数的概念、定义 $N(A)=\dfrac{Y_1}{A}\angle\varphi_1$



2. 描述函数分析方法

(1) 基本假设
$\begin{cases} \text{① 结构上：N(A), G(j}\omega\text{) 串联} \\ \text{② N(A)奇对称, } y_1\text{(t)幅值占优} \\ \text{③ G(j}\omega\text{)低通滤波特性好} \end{cases}$



(2) 稳定性分析

$G(j\omega)$ $\begin{cases}\text{不包围}\\ \text{包围}\\ \text{相交于}\end{cases}$ $\dfrac{-1}{N(A)}$ 则系统 $\begin{cases}\text{稳定}\\ \text{不稳定}\\ \text{可能自振}\end{cases}$

(3) 自振分析

$\dfrac{-1}{N(A)}$ $A\uparrow$ $\begin{cases}\text{穿入}\\ \text{穿出}\\ \text{相切于}\end{cases}$ $G(j\omega)$ 的点 $\begin{cases}\text{不是自振点}\\ \text{是自振点}\\ \text{对应半稳定的周期运动}\end{cases}$

定性分析

$\dfrac{-1}{N(A)}$, $G(j\omega)$

定量计算

$N(A)G(j\omega)=-1$

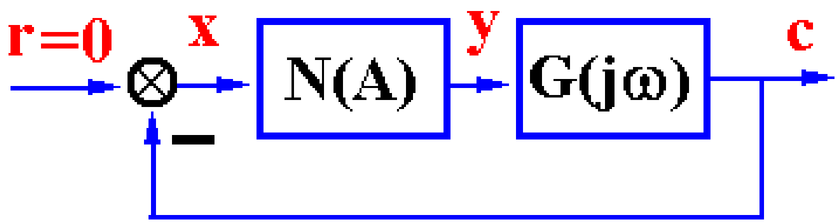
Basis for describing function analysis

- Stipulations

① N(A) and G(jω) are in cascade

② The input and the output of N(A) are symmetrical about the origin (odd symmetry characteristic)

③ G(jω) is a low-pass filter.



Characteristic equation

$$\Delta = 1 + N(A) \cdot G(j\omega) = 0$$

$$N(A) \cdot G(j\omega) = -1$$

$$G(j\omega) = \frac{-1}{N(A)}$$

系统稳定的充要条件 — 全部闭环极点均具有负的实部

代数稳定判据 — Ruoth判据

- 由闭环特征多项式系数（不解根）判定系统稳定性
- 不能用于研究如何调整系统结构来改善系统稳定性的问题

频域稳定判据 —
- Nyquist 判据
- 对数稳定判据

- 由开环频率特性直接判定闭环系统的稳定性
- 可以研究包含延迟环节的系统的稳定性问题
- 可研究如何调整系统结构参数改善系统稳定性及性能问题

Stability analysis

(1) Nyquist criterion (A revision)

$$Z=P+N$$

where

z — No. of CL poles in the right-half-plane (RHP)

p— No. of unstable OL poles

N — No. of counterclockwise encirclements of point $(-1, j0)$ by $G(j\omega)$

Z：系统在右半S平面极点个数（待求）；

P：右半S平面开环极点个数（已知）

N：开环幅相曲线$G(jw)$(不包含其镜像）包围G平面(-1,j0)点的圈数（逆时针为正）

显然，只有当Z=P+N=0时，闭环系统才是稳定的。

Stability analysis

(1) Nyquist criterion (A revision)

$$Z=P+N$$

where

z — No. of CL poles in the right-half-plane (RHP)

p— No. of unstable OL poles

N — No. of counterclockwise encirclements of point $(-1, j0)$ by $G(j\omega)$

- CL stability requires: 连续系统最小相位系统是所有极点和零点都位于s左半平面的系统

$G(j\omega)$is minimum phase system: $\mathrm{P}=0$,

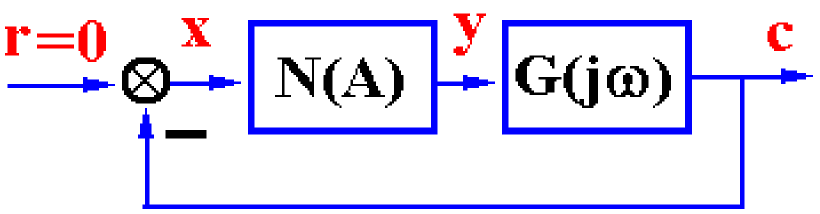
If the system is stable : $\mathrm{Z}=0$,

So, the CL stability requires N=0, that is the curve of -1/N(A) is not encircled by the curve of $G(j\omega)$

a. Calculate N(A);

b. Drawing the curve of frequency response $G(j\omega)$ and the negative reciprocal describing functions $-1/N(A)$;

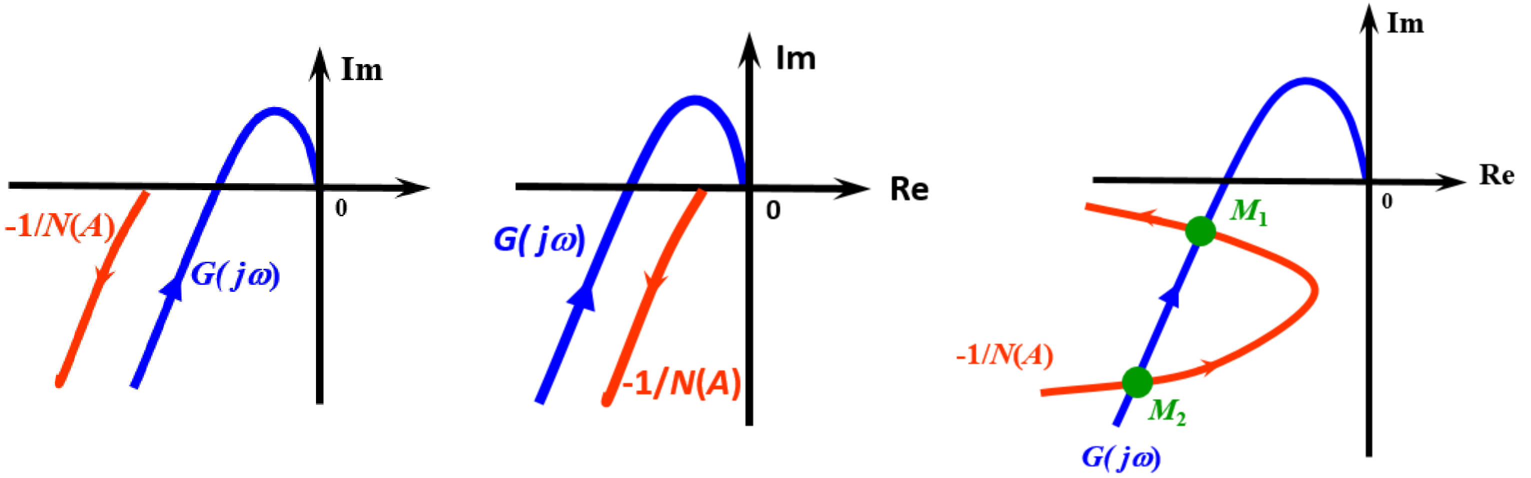
c. Check the position relationship of curve of $-1/N(A)$ and the curve of $G(j\omega)$.



$$\Delta = 1 + N(A)\cdot G(j\omega) = 0$$

$$N(A)\cdot G(j\omega) = -1$$

$$G(j\omega) = \frac{-1}{N(A)}$$



$G(j\omega)$ { not encircling / Encircling / crossing with } $\dfrac{-1}{N(A)}$

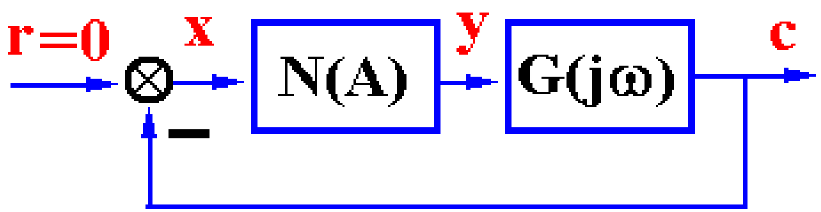
not encircling — stable

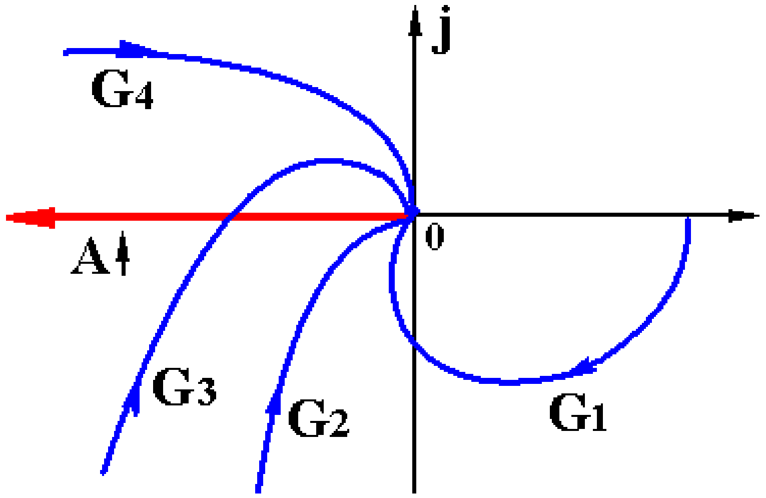
Encircling — unstable

crossing with — may be SSO(self sustained oscillation)

Example: The negative reciprocal describing functions of ideal relays:

$$N(A)=\frac{4M}{\pi A} \quad \Rightarrow \quad \frac{-1}{N(A)}=-\frac{\pi A}{4M}$$





$$\Delta = 1 + N(A)\cdot G(j\omega) = 0$$

$$N(A)\cdot G(j\omega) = -1$$

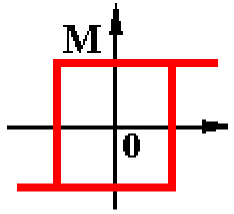
$$G(j\omega)=\frac{-1}{N(A)}$$

$G(j\omega)$ $\left\{ \begin{array}{l} \text{not encircling } (G1,\ G2) \\ \text{Encircling } (G4) \\ \text{crossing with} (G3) \end{array} \right.$ $\dfrac{-1}{N(A)}$

- not encircling ($G1$, $G2$) — stable
- Encircling ($G4$) — unstable
- crossing with($G3$) — may be SSO

$-1/N(A)$ 的绘制及其特点

例1 纯滞环继电特性的负倒描述函数



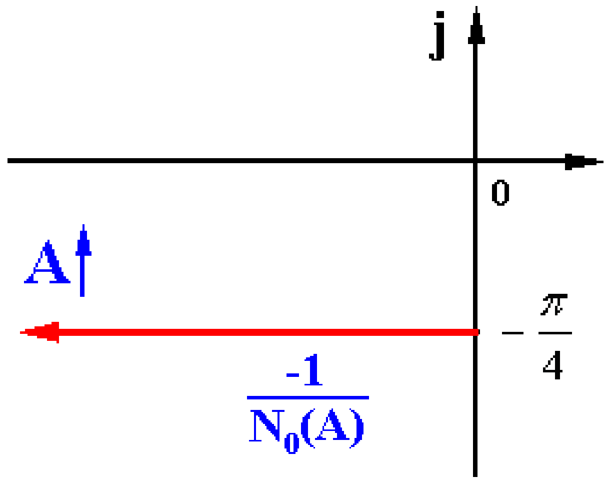
$(A > h)$

$$N(A)=\frac{4M}{\pi A}\sqrt{1-\left(\frac{h}{A}\right)^2}-j\frac{4Mh}{\pi A^2}=\frac{M}{h}\left[\frac{4h}{\pi A}\sqrt{1-\left(\frac{h}{A}\right)^2}-j\frac{4h^2}{\pi A^2}\right]$$

$$\frac{-1}{N_0(A)}=\frac{-1}{\frac{4h}{\pi A}\left[\sqrt{1-\left(\frac{h}{A}\right)^2}-j\frac{h}{A}\right]}$$

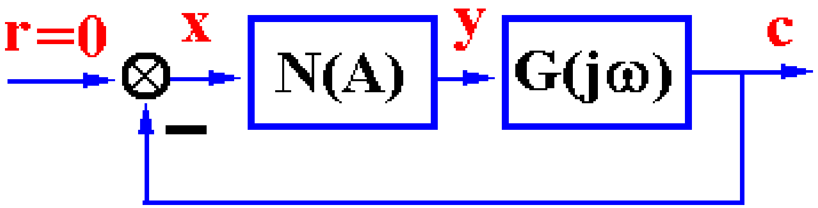
$$=\frac{-\pi A}{4h}\left(\sqrt{1-\left(\frac{h}{A}\right)^2}+j\frac{h}{A}\right)$$

$$=\frac{-\pi A}{4h}\sqrt{1-\left(\frac{h}{A}\right)^2}-j\frac{\pi}{4}=\frac{-\pi}{4h}\sqrt{A^2-h^2}-j\frac{\pi}{4}$$

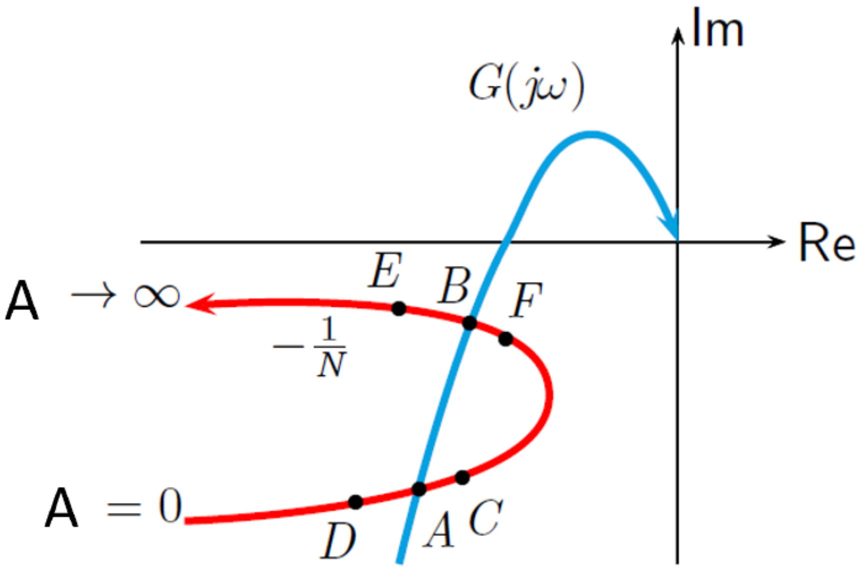


Outline of Chapter 7

Determination of self-oscillation

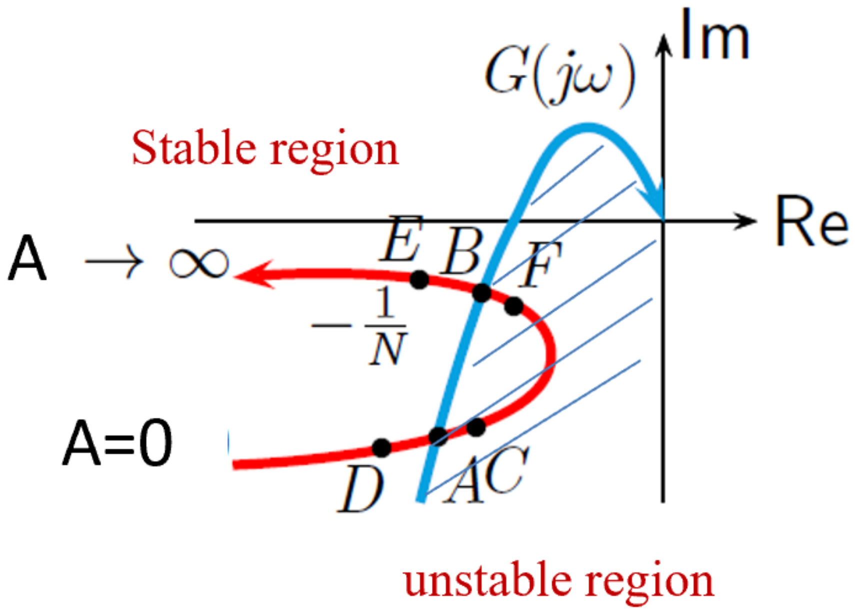


- $G\ (j\omega)$ locus crosses the $-\frac{1}{N}$ locus $\Longrightarrow$ (Self-oscillation)
- Crossing point determines the frequency and amplitude.



$A,\ B$: (self-oscillation)

$C,\ D,\ E,\ F$: Oscillations with their respective amplitudes



D: $G_p(j\omega)$ *does not encircle* $-1/N$

- Stable condition: A $\downarrow$
- System becomes more stable.

C, F: $G_p(j\omega)$ *encircles* $-1/N$

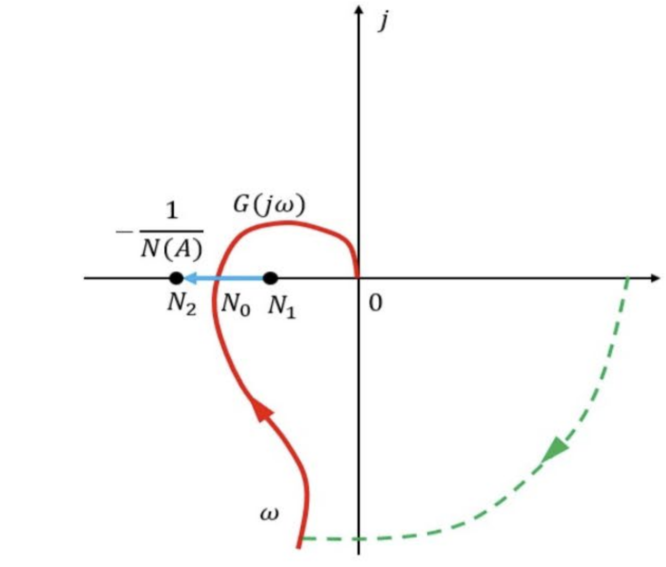
- Unstable condition: A$\uparrow$
- Point C, F will move to B.

E: $G_p(j\omega)$ *does not circle* $-1/N$

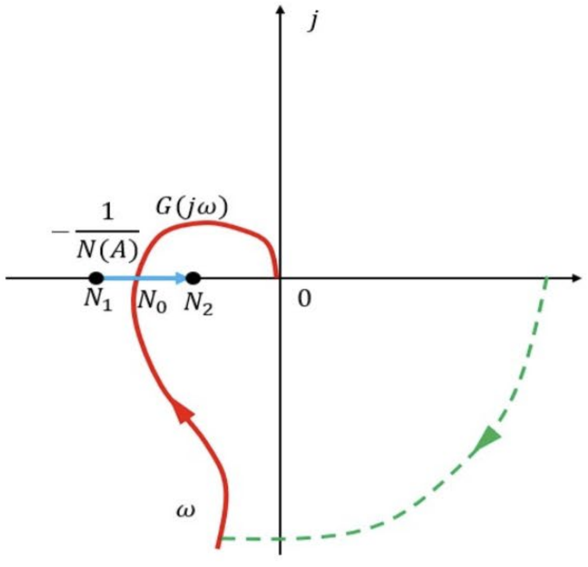
- Stable condition: A$\downarrow$
- Point E will move to B.

Conclusion:

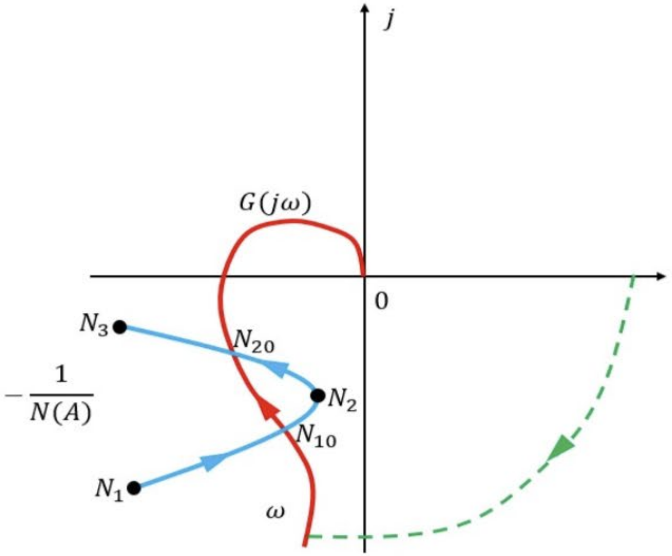
- Oscillation at point A: unstable oscillation (from unstable region to stable region)
- Oscillation at point B: stable self-oscillation(from unstable region to stable region)



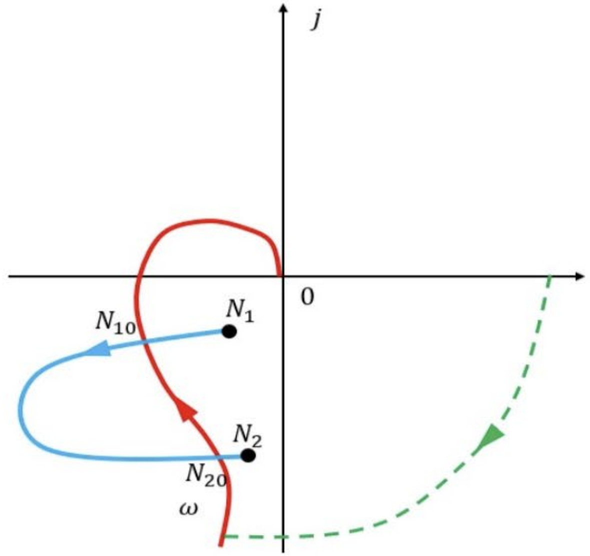
(a) $-1/N(A)$穿出$G(j\omega)$区域



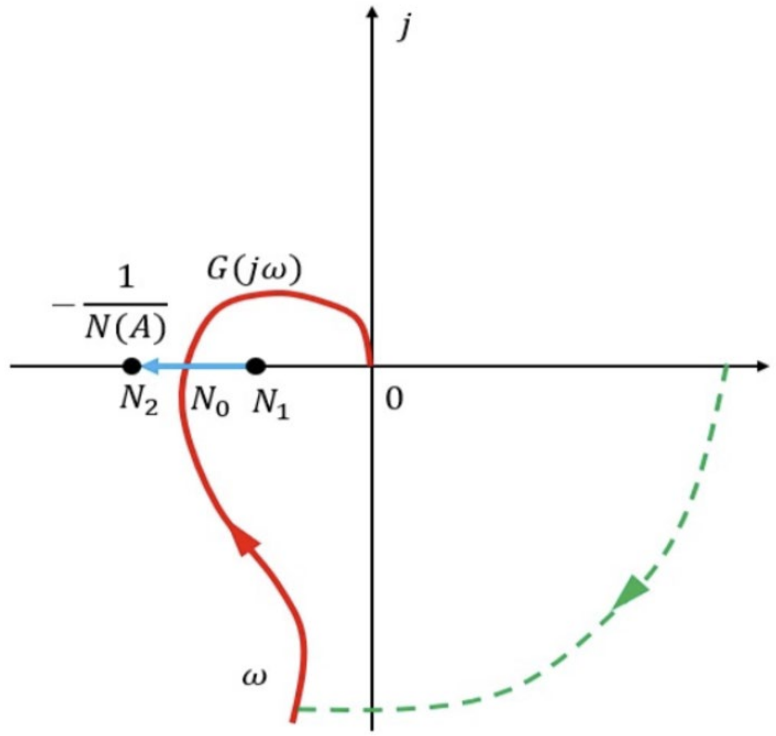
(b) $-1/N(A)$穿入$G(j\omega)$区域



(c) $-1/N(A)$先穿入再穿出$G(j\omega)$区域

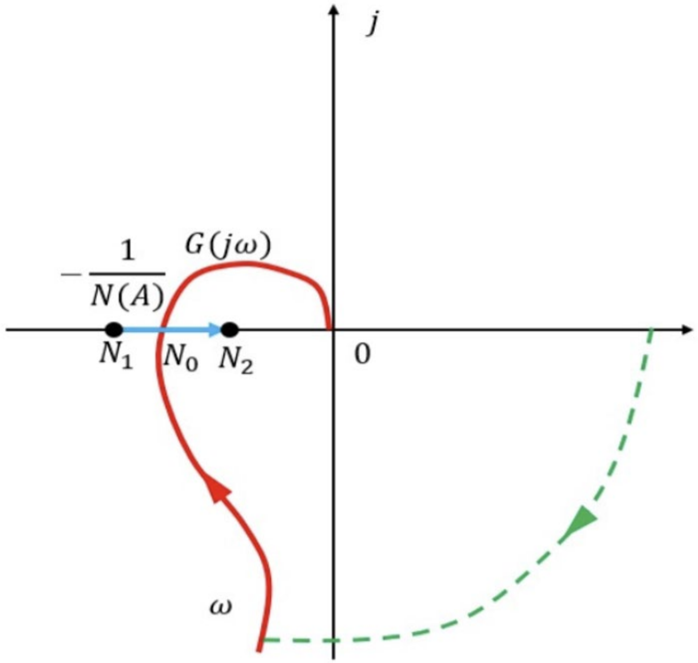


(d) $-1/N(A)$先穿出再穿入$G(j\omega)$区域



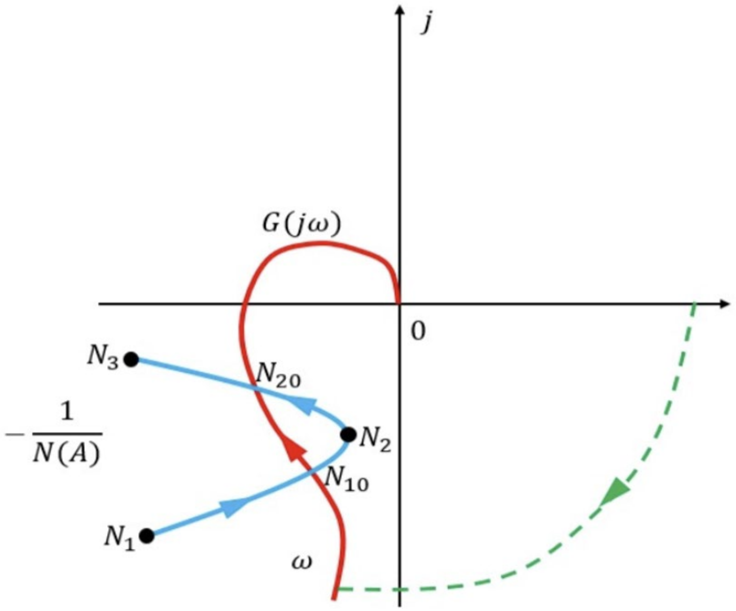
(a) $-1/N(A)$穿出$G(j\omega)$区域

- 图(a)：设系统周期运动的幅值为A_0，当外界扰动使非线性环节输入振幅减小为A_1时，由于Γ_G曲线包围$\left(-\frac{1}{N(A_1)},j0\right)$点，系统不稳定，振幅将增大，最终回到$N_0$点；当外界扰动使输入振幅增大为$A_2$，由于$\Gamma_G$曲线不包围$\left(-\frac{1}{N(A_2)},j0\right)$点，系统稳定，振幅将衰减，最终也回到$N_0$点；**即$N_0$点对应的周期运动是稳定的；**



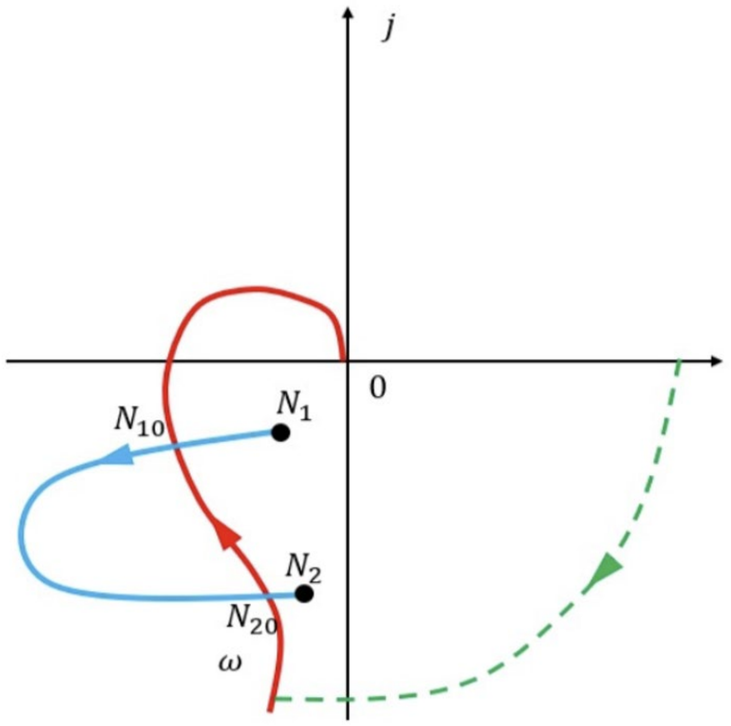
(b) $-1/N(A)$穿入$G(j\omega)$区域

- 图(b)：当外扰动使系统偏离周期运动至N_2点，即使其幅值由A_0增大为A_2时，系统不稳定，振幅将进一步增大，最终发散至无穷；当外扰动使系统偏离周期运动至N_1点，即使其幅值由A_0减小为A_1时，系统稳定，振幅将进一步减小，最终衰减为零；**即N_0点对应的周期运动是不稳定的；**



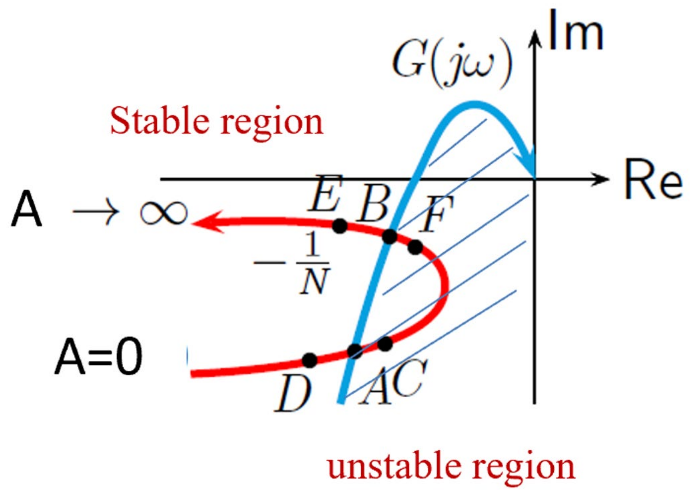
(c) −1/N(A)先穿入再穿出G(jω)区域

- 图(c)：Γ_G 曲线和 $\frac{-1}{N(A)}$ 曲线有两个交点 N_{10} 和 N_{20}，系统中存在两个周期运动，幅值分别为：A_{10} 和 A_{20}，在 N_{20} 点，外界小扰动使系统运动偏离该周期运动后，系统运动仍然能恢复该周期运动；在 N_{10} 点，只要有外界扰动使系统运动偏离该周期运动，则系统运动或收敛至零，或趋向于 N_{20} 点对应的周期运动；**N_{10} 点对应的周期运动是不稳定的，N_{20} 点对应的周期运动是稳定的；**



$(d)\ -1/N(A)$先穿出再穿入$G(j\omega)$区域

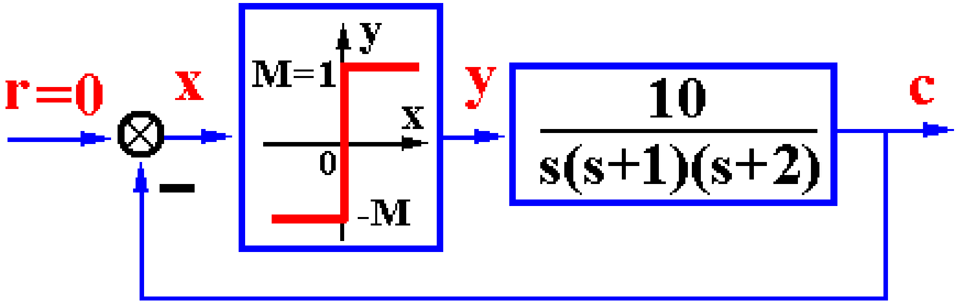
- 图(d)：**N_{10}点对应的周期运动是稳定的，N_{20}点对应的周期运动是不稳定的，外界小扰动或使系统运动发散至无穷，或趋向于幅值N_{10}点对应的周期运动；**



综上：在复平面上将Γ_G曲线包围的区域视为不稳定区域，Γ_G曲线不包围的区域视为稳定区域，则**周期运动稳定性判据：在Γ_G曲线和$-\dfrac{1}{N(A)}$曲线的交点处，若$-\dfrac{1}{N(A)}$曲线沿着振幅A增加的方向由不稳定区域进入稳定区域时，该交点对应的周期运动是稳定的；反之，若$-\dfrac{1}{N(A)}$曲线沿着振幅A增加的方向在交点处由稳定区域进入不稳定区域时，该交点对应的周期运动是不稳定的；**

Example 1: Determine the stability of the system (M=1), and obtain the parameters of SSO.

Solution: From the plot, there is SSO

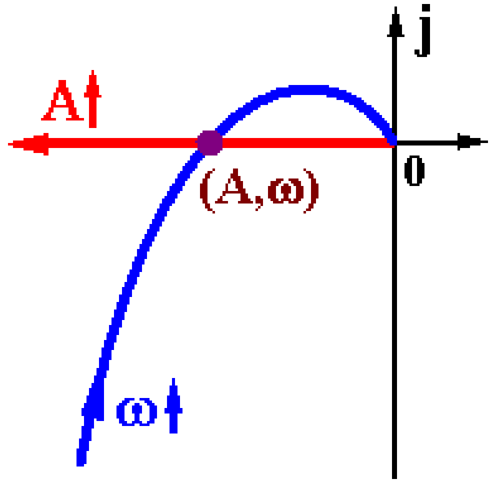


By the SSO condition:

$$N(A)\cdot G(j\omega) = -1$$

Thus: $$\frac{4}{\pi A}\cdot\frac{10}{j\omega(1+j\omega)(2+j\omega)} = -1$$

$$\frac{40}{\pi A} = -j\omega(1+j\omega)(2+j\omega) = 3\omega^2 - j\omega(2-\omega^2)$$

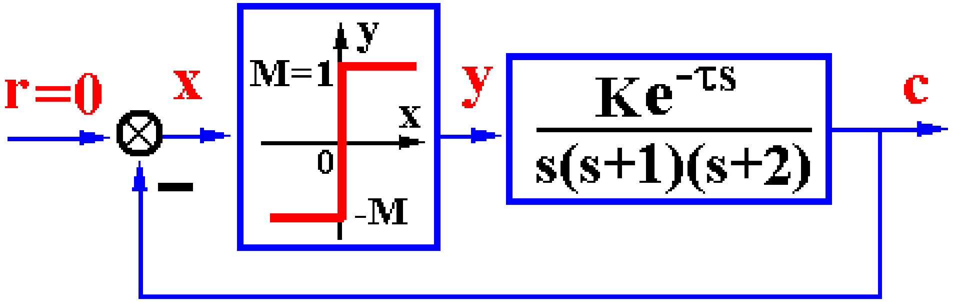


comparing the real/imaginary part

$$\begin{cases}\dfrac{40}{\pi A} = 3\omega^2 \\ \omega(2-\omega^2) = 0\end{cases} \qquad \begin{cases}\omega = \sqrt{2} \\ A = \dfrac{40}{6\pi} = 2.122\end{cases}$$

Example 2: Consider the system shown in the figure. Determine the value of K and t to obtain a periodic signal with $\omega = 1,\ A = 4$

Solution: Analysis: Obtain the required SSO by adjusting K and t.

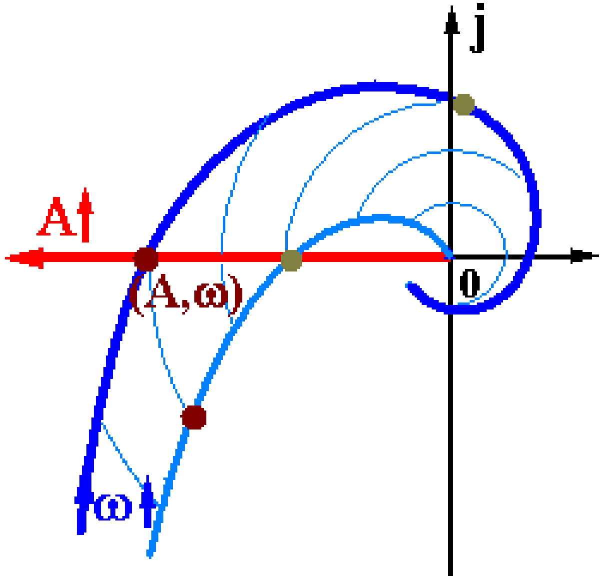


$$N(A)\cdot G(j\omega) = -1$$

$$\frac{4M}{\pi A}\cdot\frac{Ke^{-j\omega\tau}}{j\omega(1+j\omega)(2+j\omega)} = -1$$

$$\frac{4MKe^{-j\omega\tau}}{\pi A} = 3\omega^2 - j\omega\left(2-\omega^2\right)$$

$$= \omega\sqrt{4+5\omega^2+\omega^4}\ \angle\left(-\arctan\frac{2-\omega^2}{3\omega}\right)$$



By $\begin{cases} M = 1 \\ A = 4 \\ \omega = 1 \end{cases}$ and comparing the magnitude and phase $\begin{cases} K = \sqrt{10}\cdot\pi = 9.93 \\ \tau = \arctan\dfrac{1}{3} = 0.322 \end{cases}$

Example 3: Consider the nonlinear system shown in the figure, analyze if there is SSO. If there is, obtain the magnitude and frequency of the output signal $c(t)$.

Solution:

By SSO condition $\quad \boldsymbol{N(A)\cdot G(j\omega)=-1}$

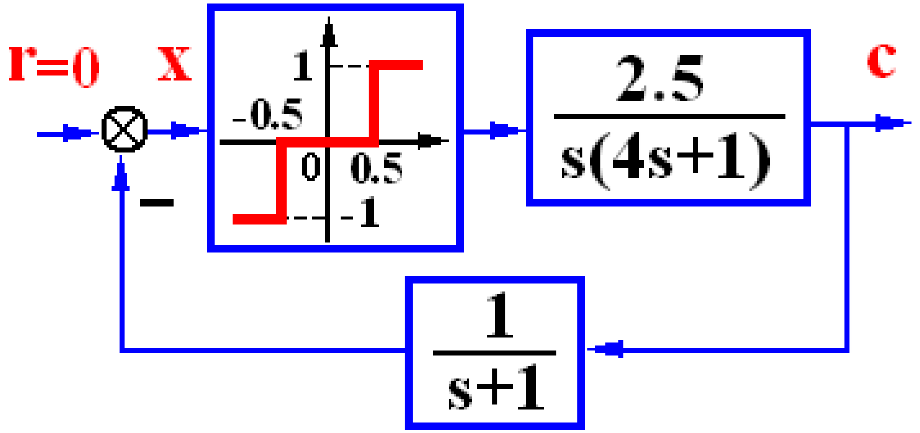
$$\frac{4M}{\pi A}\sqrt{1-\left(\frac{\hat{h}}{A}\right)^2}\cdot\frac{2.5}{j\omega(1+j\omega)(1+j4\omega)}=-1$$

$$\frac{10}{\pi A^2}\sqrt{A^2-0.5^2}=-j\omega\left(1-4\omega^2+j5\omega\right)$$

$$=5\omega^2-j\omega\left(1-4\omega^2\right)$$

Comparing the real/imaginary part

$$\begin{cases}\omega=0.5\\ \dfrac{10}{\pi A^2}\sqrt{A^2-0.5^2}=5\times0.5^2=1.25\end{cases}$$



注意：由推导自振荡产生的条件时可知，对于稳定的自振荡,计算所得到的振幅和频率是非线性环节的输入信号 $x(t)$=Asinωt 的振幅和频率，而不是系统的输出信号$c(t)$.

$$\begin{cases} \omega = 0.5 \\ \dfrac{10}{\pi A^2}\sqrt{A^2 - 0.5^2} = 5\times 0.5^2 = 1.25 \end{cases}$$

$$A^4 - 6.486A^2 + 1.621 = 0$$

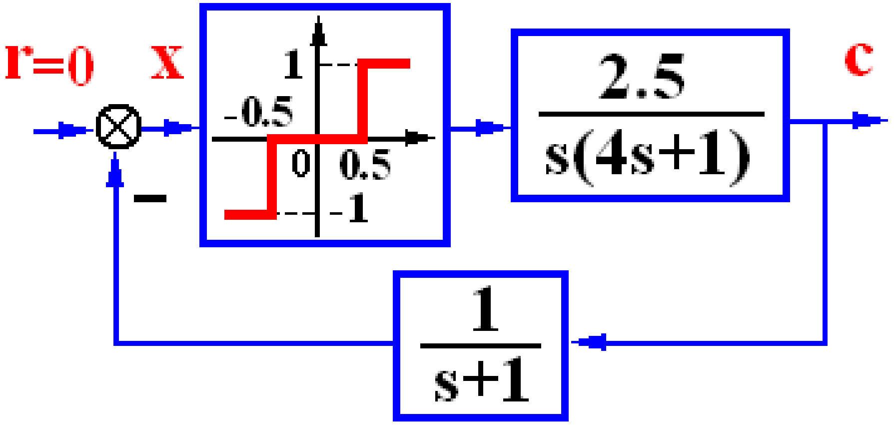
$$A^2 = \begin{cases} 0.2605 \\ 6.2241 \end{cases} \qquad \begin{cases} A_1 = 0.5104 \\ A_2 = 2.495 \end{cases}$$

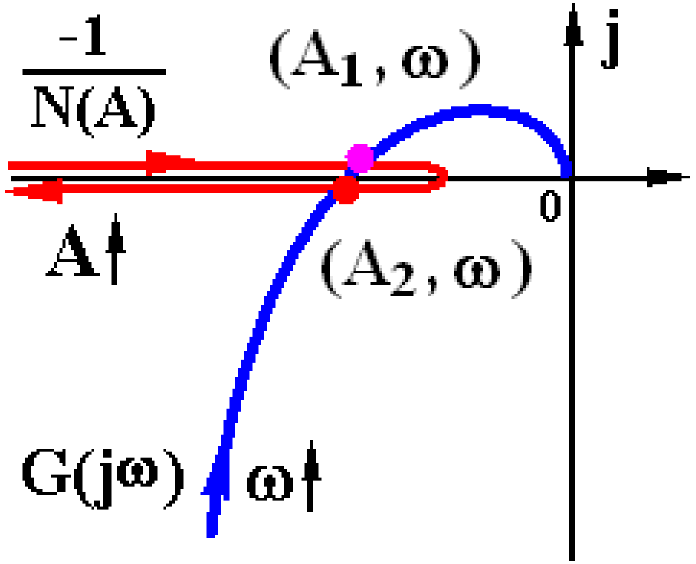
There is SSO with $\begin{cases} \omega = 0.5 \\ A = A_2 = 2.495 \end{cases}$

$$\frac{|x|}{|c|} = \frac{A}{A_c} = \frac{1}{\sqrt{\omega^2+1}} \overset{\omega=0.5}{=} \frac{1}{\sqrt{1.25}} = 0.894$$

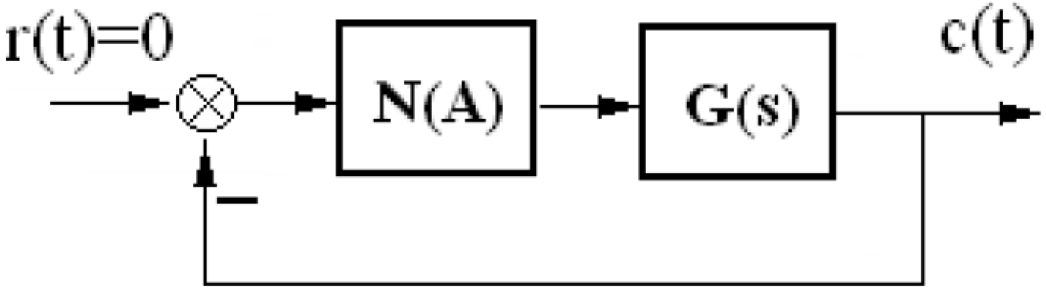
$$A_c = \frac{A}{0.894} = \frac{2.294}{0.894} = 2.79$$

$$\begin{cases} \omega = 0.5 \\ A_c = 2.79 \end{cases}$$

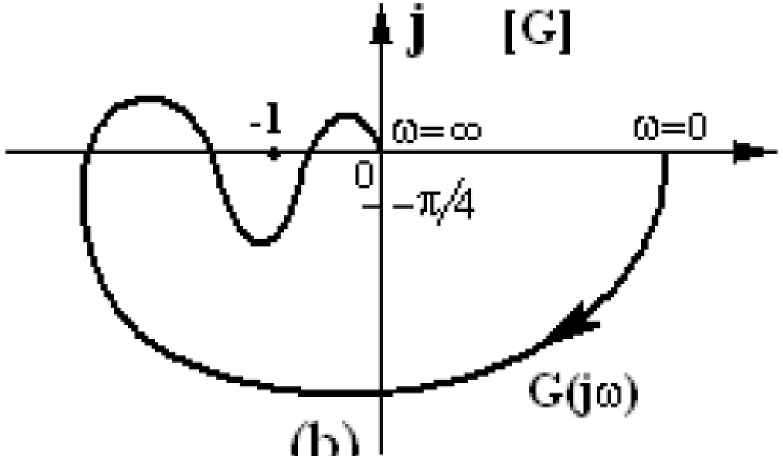




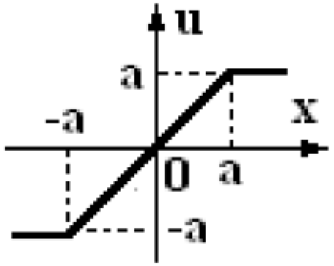
非线性控制系统的结构图如图 (a)所示，其中线性部分的幅相特性曲线如图(b)所示，非线性特性示于图 (c)~(g)。试应用描述函数法分析含图 (c)~(g)所示典型非线性特性的系统稳定性。



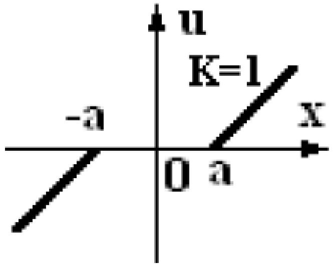
(a)



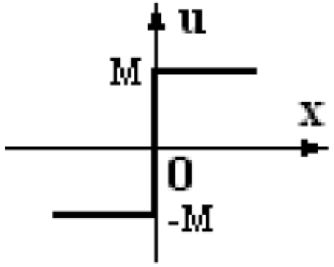
(b)



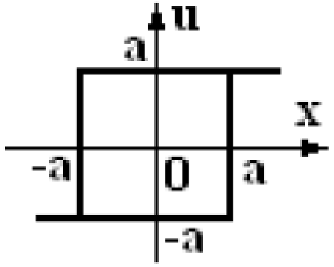
(c)



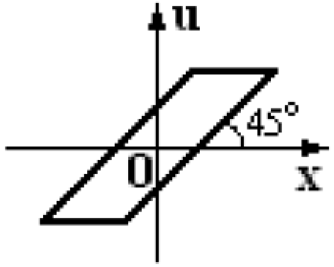
(d)



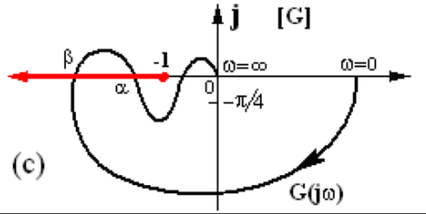
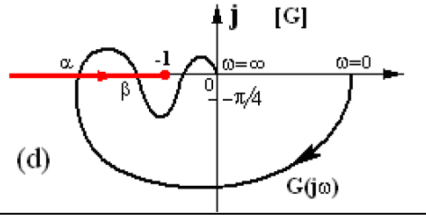
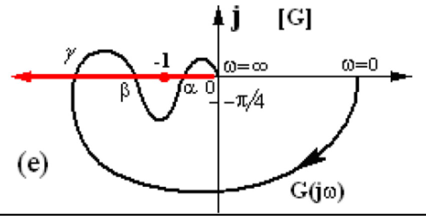
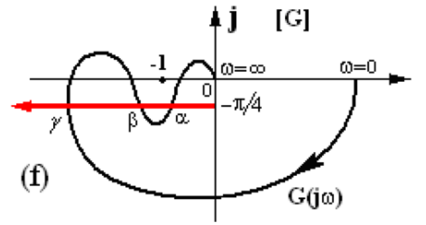
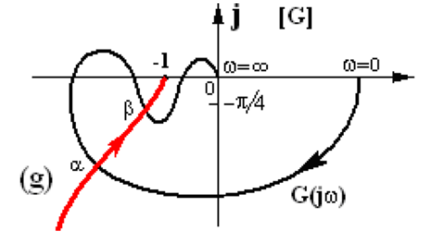
(e)



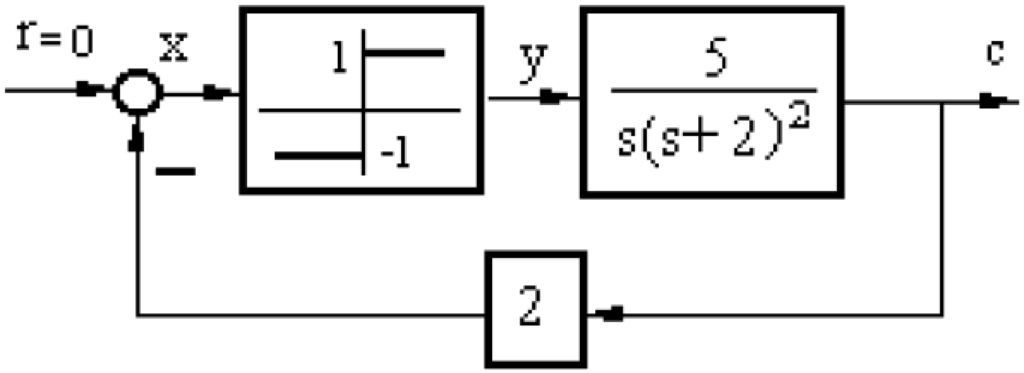
(f)



(g)

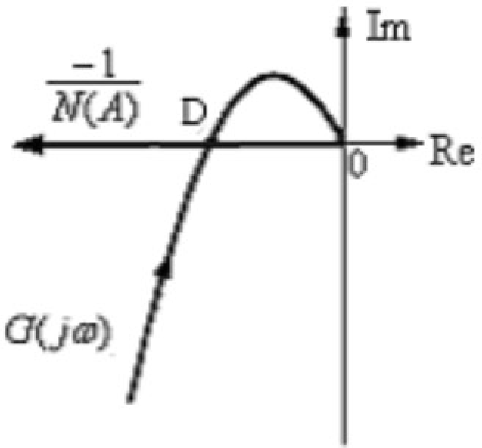
标号	自振点	非自振点	使系统稳定的 A 值范围	自振的 A 值范围	图 示
(c)	β	α	(a,α)	$(\alpha,\infty)\Rightarrow\beta$	
(d)	β	α	(a,α)	$(\alpha,\infty)\Rightarrow\beta$	
(e)	α、γ	β	/	$(0,\beta)\Rightarrow\alpha$ $(\beta,\infty)\Rightarrow\gamma$	
(f)	α、γ	β	/	$(0,\beta)\Rightarrow\alpha$ $(\beta,\infty)\Rightarrow\gamma$	
(g)	β	α	(a,α)	$(\alpha,\infty)\Rightarrow\beta$	

练习题：用描述函数法说明如下图所示系统必然存在自振，并确定输出信号c的自振振幅和频率，分别画出信号c, x, y的稳态波形。



解：

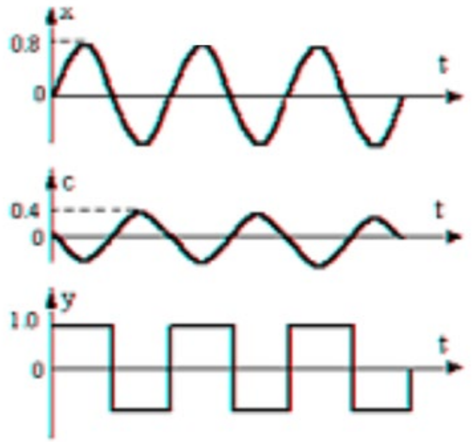
$$N(A)=\frac{4}{\pi A},\qquad \frac{-1}{N(A)}=\frac{-\pi A}{4}$$



绘出$-\mathbf{1}/\boldsymbol{N}(\boldsymbol{A})$和G($\boldsymbol{j\omega}$)的曲线如下图所示，可见**D**是自振点，系统一定会自振。由自振条件可得：

$$N(A)=\frac{-1}{G(j\omega)}\Rightarrow\frac{4}{\pi A}=\frac{-j\omega(j\omega+2)^2}{10}$$

$$=\frac{4\omega^2}{10}-\frac{j\omega(4-\omega^2)}{10}$$



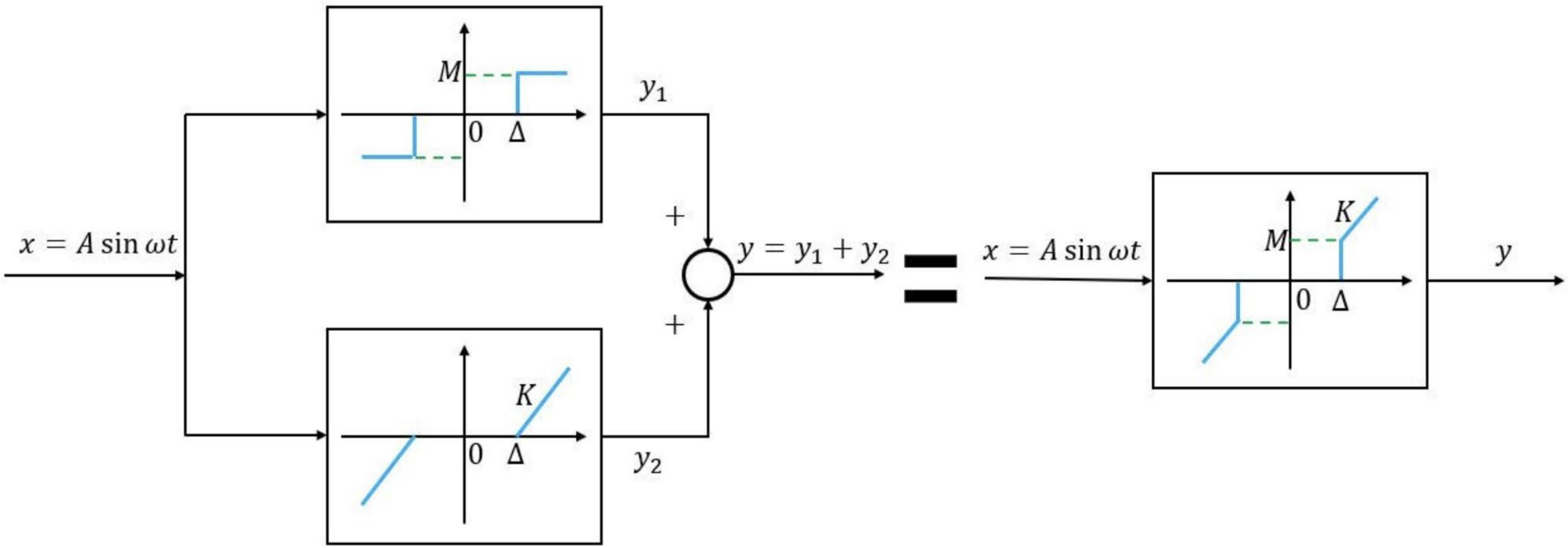
令虚部为零解出$\boldsymbol{\omega}=\mathbf{2}$，$\mathrm{A}_c=\boldsymbol{A}/\mathbf{2}=\mathbf{0.398}$

画出c, x, y 点的信号稳态波形，如图所示。

Outline of Chapter 7

Combination nonlinear characteristic (con.)

Nonlinear link having a dead zone, seeking the describing function N(A)



非线性特性的并联：若两个非线性特性输入相同，输出相加、减，则等效非线性为两个非线性特性的叠加，并联等效非线性特性的描述函数为各非线性描述函数的代数和。

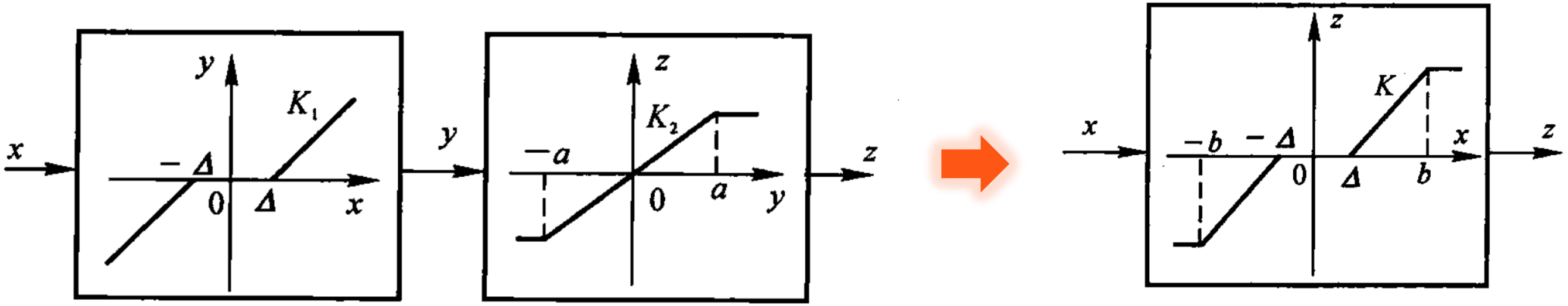
Combination nonlinear characteristic (con.)

Nonlinear characteristics of the dead zone can be decomposed into a dead zone relay characteristic and a typical dead parallel description function:

$$N(A) = \frac{4M}{\pi A}\sqrt{1-(\frac{\Delta}{A})^2} + \frac{2k}{\pi}\left(\frac{\pi}{2} - \arctan\frac{\Delta}{A} - \frac{\Delta}{A}\sqrt{1-(\frac{\Delta}{A})^2}\right)$$

$$= k - \frac{2k}{\pi}\sin^{-1}\frac{\Delta}{A} + \frac{4M-2k\Delta}{\pi A}\sqrt{1-(\frac{\Delta}{A})^2}, \text{where } A \geq \Delta$$

Combination nonlinear characteristic (con.)



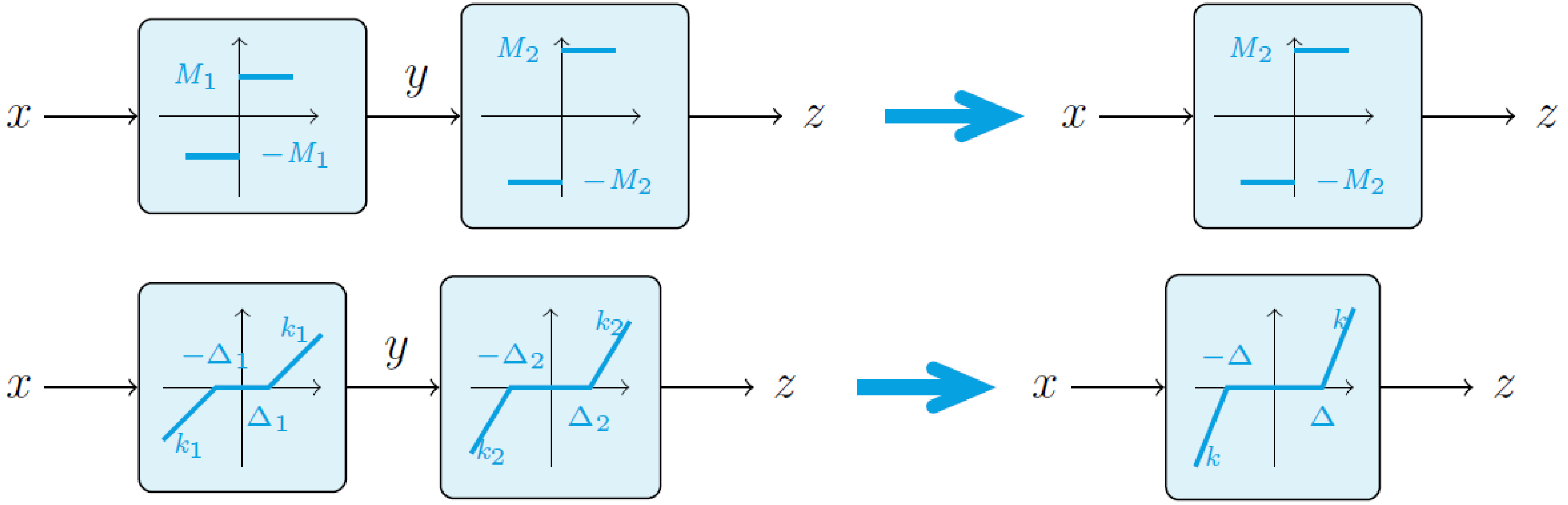
$$y=\begin{cases}K_1(x-\Delta) & (x>\Delta)\\ 0 & (|x|\leqslant\Delta)\\ K_1(x+\Delta) & (x<-\Delta)\end{cases}\qquad z=\begin{cases}K_2 a & (y>a)\\ K_2 y & (|y|\leqslant a)\\ K_2 a & (y<-a)\end{cases}\qquad y=a=K_1(x-\Delta)\Rightarrow x=\frac{a}{K_1}+\Delta$$

将上两式联立消去中间变量 y,可得

$$z=\begin{cases}Kb & (x>b)\\ K(x-\Delta) & (\Delta<x\leqslant b)\\ 0 & (|x|\leqslant\Delta)\\ K(x+\Delta) & (-b\leqslant x<-\Delta)\\ Kb & (x<-b)\end{cases}$$

$$N(A)=\frac{2K}{\pi}\left[\arcsin\frac{b}{A}-\arcsin\frac{\Delta}{A}+\frac{b}{A}\sqrt{1-\left(\frac{b}{A}\right)^2}-\frac{\Delta}{A}\sqrt{1-\left(\frac{\Delta}{A}\right)^2}\right]\qquad (A\geqslant b)$$

- For cascade nonlinearities, their describing function is usually NOT the multiplicity of individual describing functions.



$$k = k_1 k_2, \ \ \Delta = \Delta_1 + \frac{\Delta_2}{k_1}$$

1. Calculate the describing functions $N(X)$ of nonlinearities as shown in Fig. 1, and sketch the plots of $N(X)$ and $-1/N(X)$.

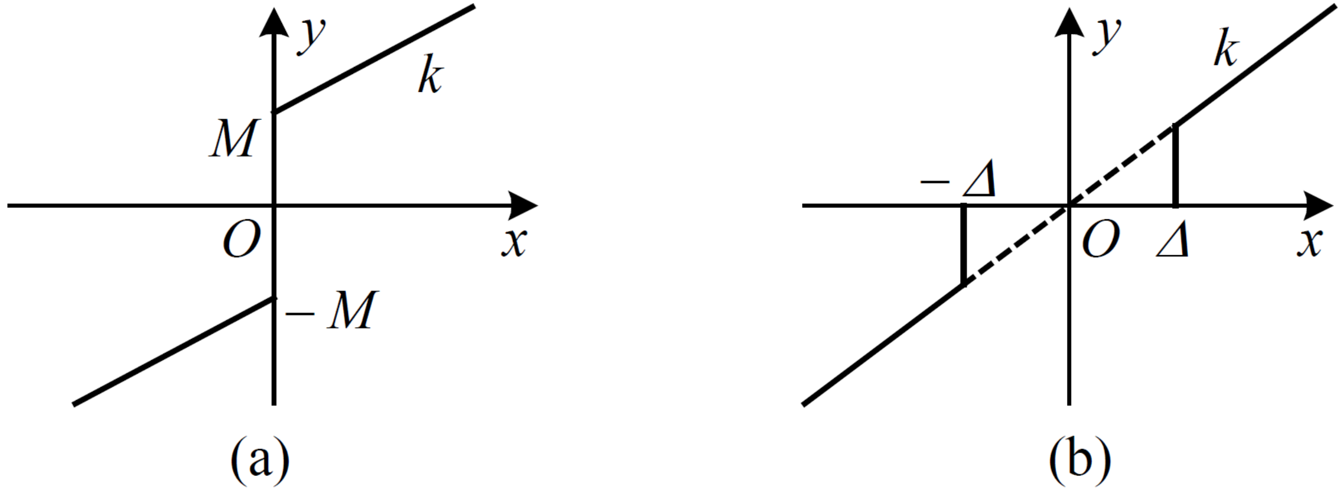
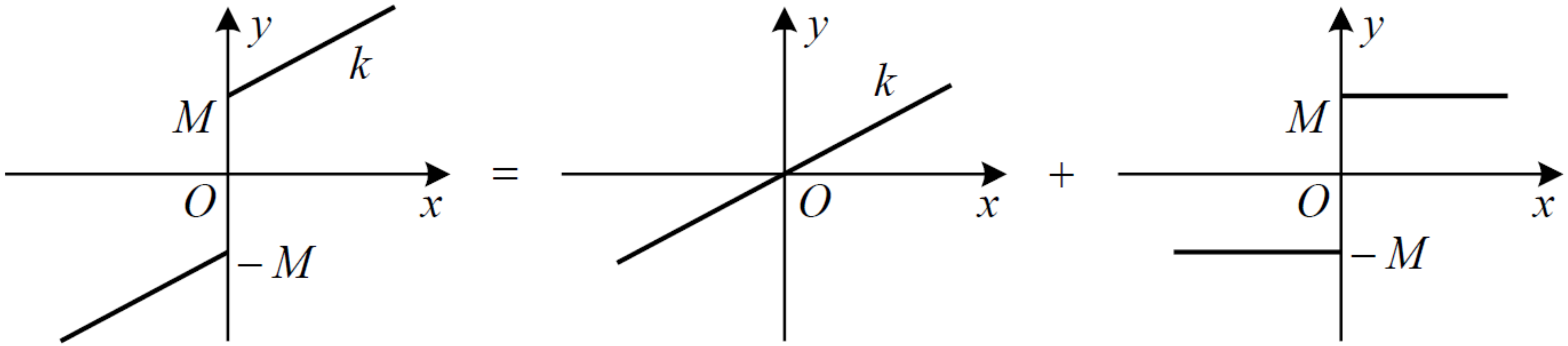


Fig. 1 Nonlinearity in Problem 1

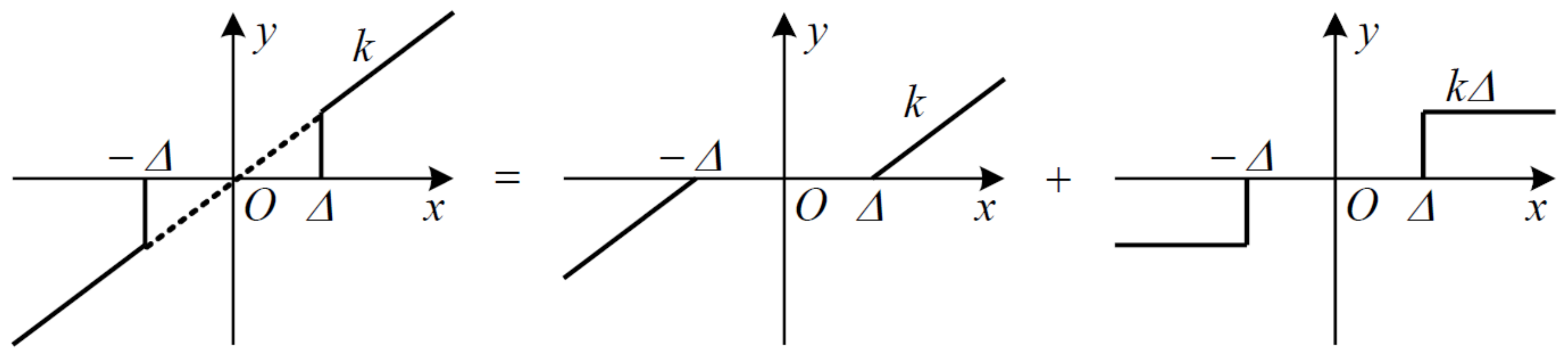
解：

(a)



$$N(x)=N_1+N_2=k+\frac{4M}{\pi X}$$

（b）

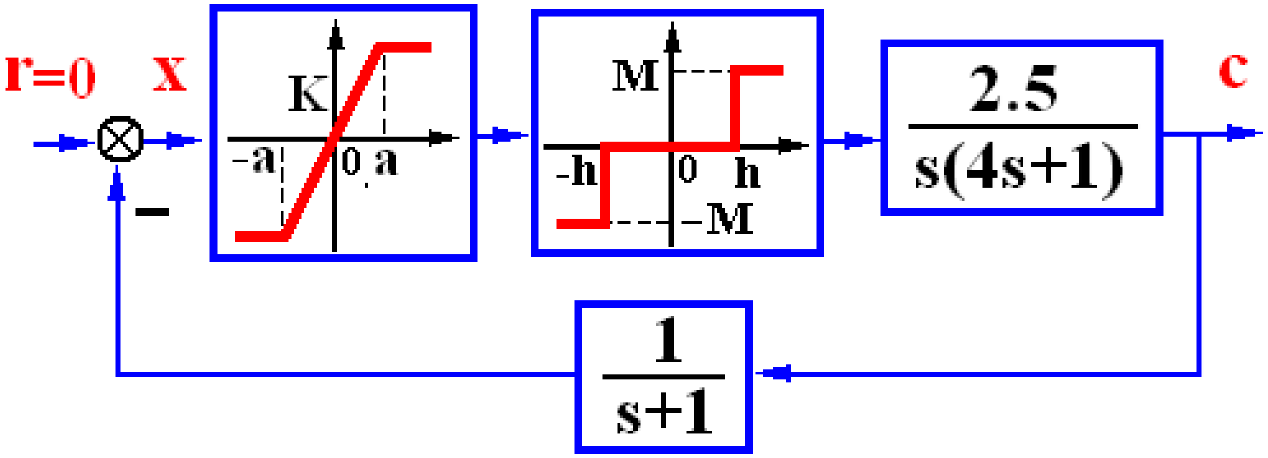


$$N(x)=N_1+N_2=k-\frac{2k}{\pi}\left[\arcsin\frac{\Delta}{X}+\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\right]+\frac{4M}{\pi X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

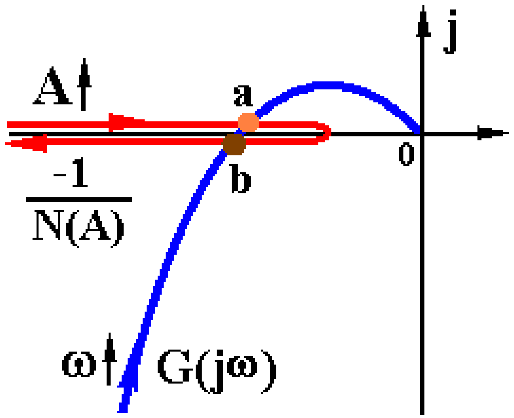
$$=k-\frac{2k}{\pi}\left[\arcsin\frac{\Delta}{X}+\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\right]+\frac{4k\Delta}{\pi X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

$$=k-\frac{2k}{\pi}\arcsin\frac{\Delta}{X}-\frac{2k}{\pi}\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}+\frac{4k\Delta}{\pi X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

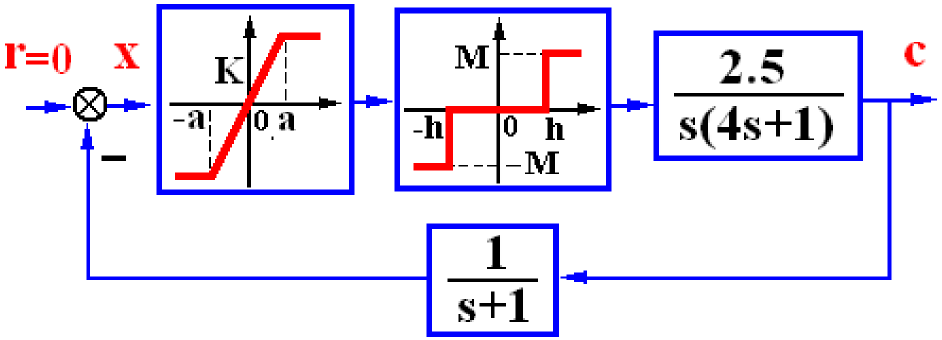
作业：非线性系统如图所示，$a=M=h=1,\ K=2,$ 分析系统是否存在自振；若存在自振，确定输出端信号$c(t)$的振幅和频率。



补充：非线性系统如图所示，$a=M=h=1,\ K=2,$ 分析系统是否存在自振；若存在自振，确定输出端信号$c(t)$的振幅和频率。



解：将两非线性环节等效合并，结构图化为

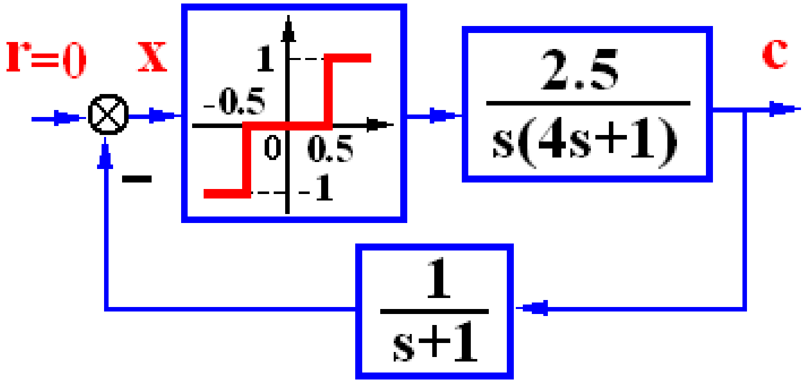


依自振条件 $N(A)\cdot G(j\omega)=-1$

$$\frac{4M}{\pi A}\sqrt{1-\left(\frac{\hat{h}}{A}\right)^2}\cdot\frac{2.5}{j\omega(1+j\omega)(1+j4\omega)}=-1$$

$$\frac{10}{\pi A^2}\sqrt{A^2-0.5^2}=-j\omega(1-4\omega^2+j5\omega)$$

$$=5\omega^2-j\omega(1-4\omega^2)$$



比较虚实部 $\begin{cases}\omega=0.5\\ \dfrac{10}{\pi A^2}\sqrt{A^2-0.5^2}=5\times0.5^2=1.25\end{cases}$

$$\begin{cases}\omega = 0.5\\ \dfrac{10}{\pi A^2}\sqrt{A^2-0.5^2} = 5\times 0.5^2 = 1.25\end{cases}$$

$$A^4 - 6.486A^2 + 1.621 = 0$$

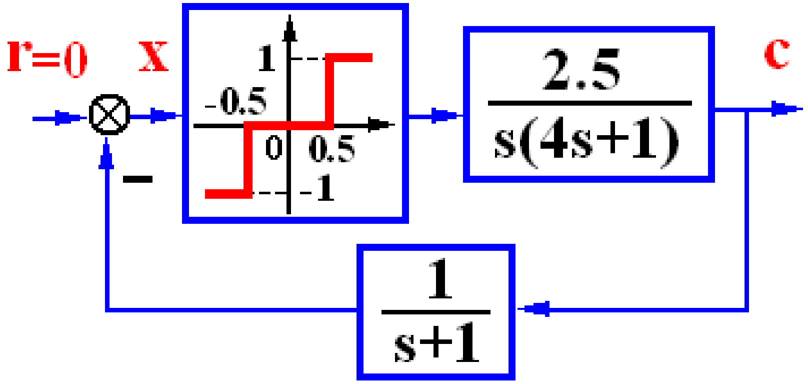
$$A^2 = \begin{cases}0.2605\\ 6.2241\end{cases} \qquad \begin{cases}A_1 = 0.5104\\ A_2 = 2.495\end{cases}$$

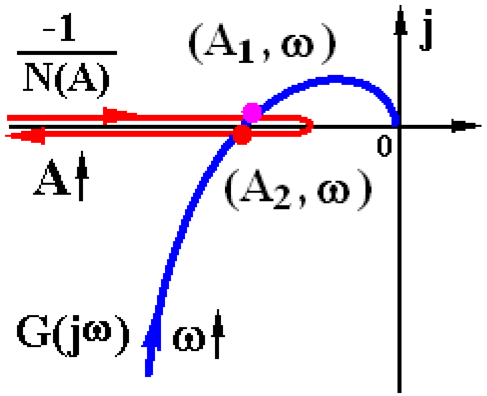
分析可知：系统存在自振 $\begin{cases}\omega = 0.5\\ A = A_2 = 2.495\end{cases}$

$$\frac{|x|}{|c|} = \frac{A}{A_c} = \frac{1}{\sqrt{\omega^2+1}} \overset{\omega=0.5}{=} \frac{1}{\sqrt{1.25}} = 0.894$$

$$\mathrm{A_c} = \frac{\mathrm{A}}{0.894} = \frac{2.495}{0.894} = 2.79$$

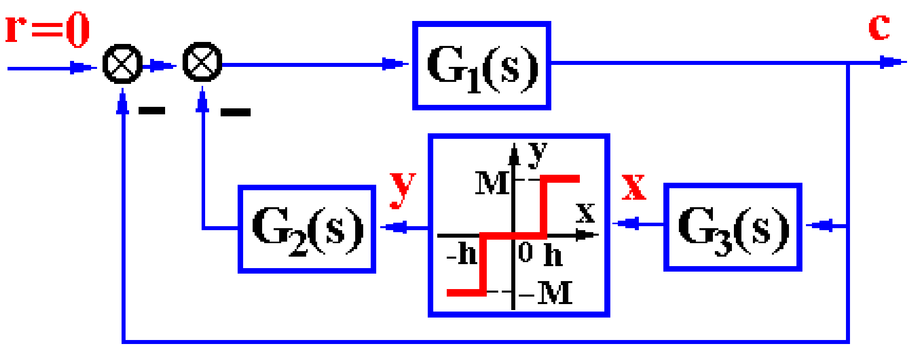
$$\begin{cases}\omega = 0.5\\ A_c = 2.79\end{cases}$$





补充：非线性系统结构图如右图所示，

已知：
$$\begin{cases} G_1(s)=\dfrac{1}{s(s+1)}, \quad G_2(s)=\dfrac{K}{s} \\ N(A)=\dfrac{4M}{\pi A}\sqrt{1-\left(\dfrac{h}{A}\right)^2} \quad (A\geq h) \end{cases}$$

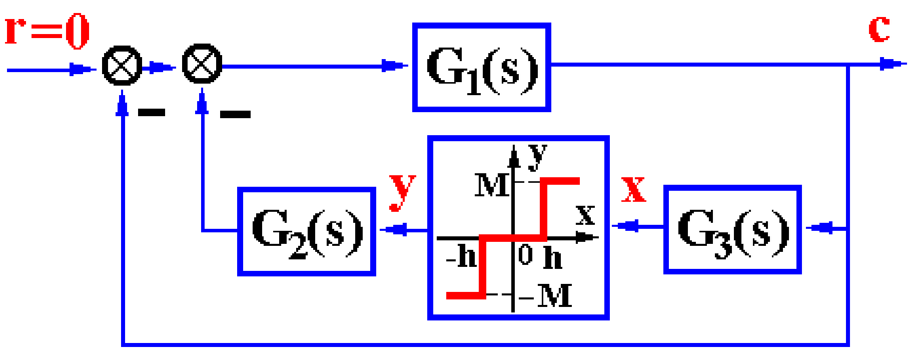


（1） $G_3(s)=1$ 时，系统是否自振？

确定使系统自振的K值范围；求K=2时的自振参数。

（2） $G_3(s)=s$ 时，分析系统的稳定性。

解：先将系统结构图化为典型结构

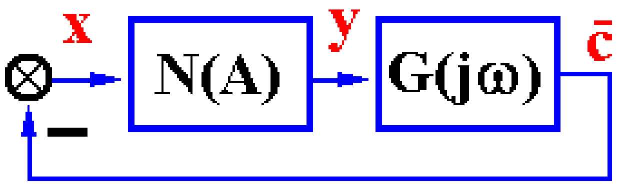


解法I　等效变换法　$G(s)=\dfrac{G_1}{1+G_1}G_2G_3$

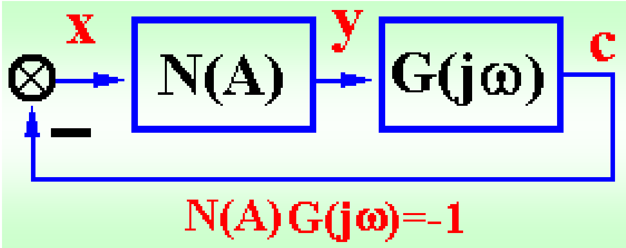
解法II　特征方程法　$\Phi(s)=\dfrac{G_1}{1+G_1G_2G_3N+G_1}$

$D(s)=1+G_1G_2G_3N+G_1=0$　$NG_1G_2G_3=-(1+G_1)$　$N\dfrac{G_1G_2G_3}{1+G_1}=-1$

$$G(s)=\frac{G_1G_2G_3}{1+G_1}$$



N(A) G(jω)=-1



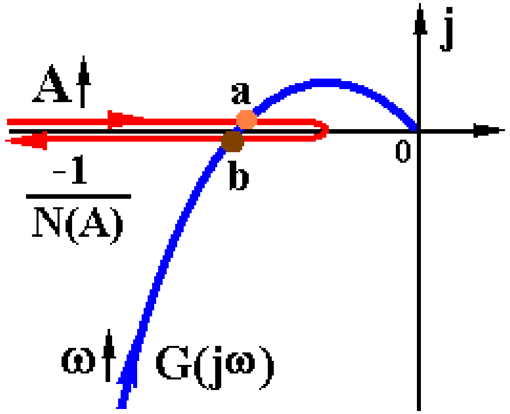
解(1) $G_3(s)=1$时

$$G(s)=\frac{G_1G_2G_3}{1+G_1}=\frac{\frac{1}{s(s+1)}\cdot\frac{K}{s}\cdot 1}{1+1/s(s+1)}=\frac{K}{s[s^2+s+1]}$$

$$\begin{cases} A\to h: & \dfrac{-1}{N(A)}\to-\infty \\ A\to\infty: & \dfrac{-1}{N(A)}\to-\infty \end{cases}$$

$$\frac{-1}{N(A)}=\frac{-\pi A}{4M\sqrt{1-(h/A)^2}}=\frac{-\pi A^2}{4M\sqrt{A^2-h^2}}$$

由自振条件 $N(A)\cdot G(j\omega)=-1$

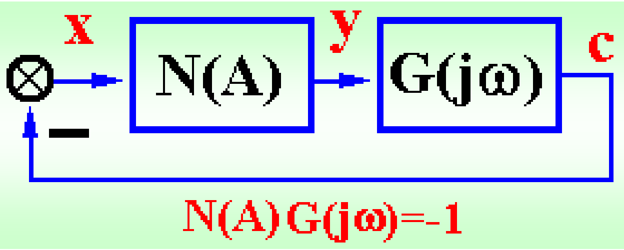


$$\frac{4M}{\pi A}\sqrt{1-(\frac{h}{A})^2}\ \frac{K}{j\omega(1-\omega^2+j\omega)}=-1$$

$$\frac{4MK}{\pi A}\sqrt{1-(\frac{h}{A})^2}=-j\omega(1-\omega^2+j\omega)=\omega^2-j\omega(1-\omega^2)$$

虚部 $\omega=1$

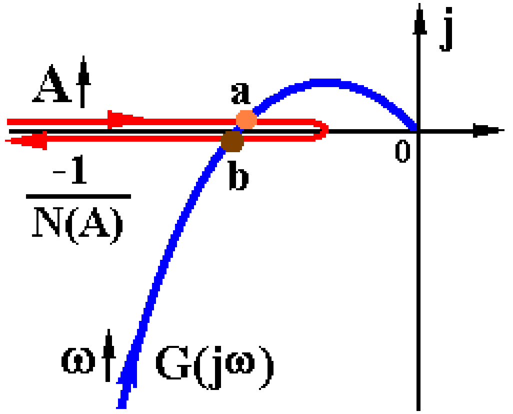
实部 $\dfrac{4K}{\pi A^2}\sqrt{A^2-1}=1\qquad\sqrt{A^2-1}=\dfrac{\pi A^2}{4K}$



$$\sqrt{A^2-1}=\frac{\pi A^2}{4K} \qquad (\frac{\pi}{4K})^2A^4-A^2+1=0 \qquad A^2=\frac{1\pm\sqrt{1-4(\pi/4K)^2}}{2(\pi/4K)^2}$$

有解条件： $K\geq\pi/2$

$$K=2 \qquad A^2=\frac{1\pm\sqrt{1-4(\pi/4K)^2}}{2(\pi/4K)^2}\overset{K=2}{=}\begin{cases}5.25\\1.1122\end{cases}$$

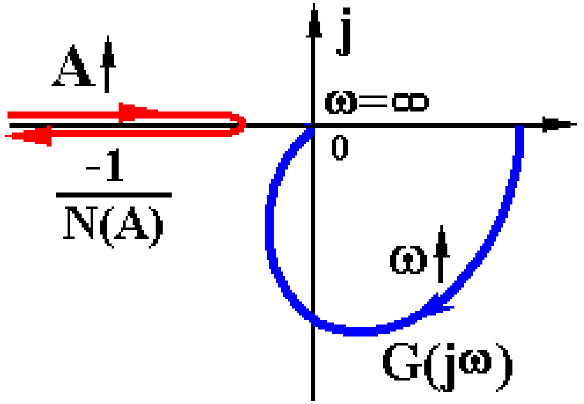


$$\begin{cases}A_1=2.29\\A_2=1.055\end{cases} \qquad \begin{cases}A=2.29\\\omega=1\end{cases}$$

(2) $G_3(s)=s$ 时

$$G(s)=\frac{G_1G_2G_3}{1+G_1}=\frac{\frac{1}{s(s+1)}\cdot\frac{K}{s}\cdot s}{1+\frac{1}{s(s+1)}}=\frac{K}{s^2+s+1}$$

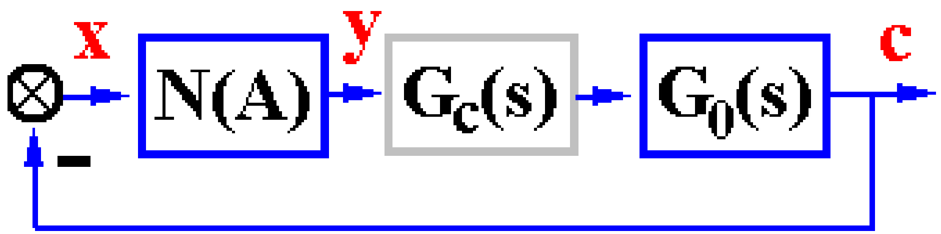
此时系统稳定

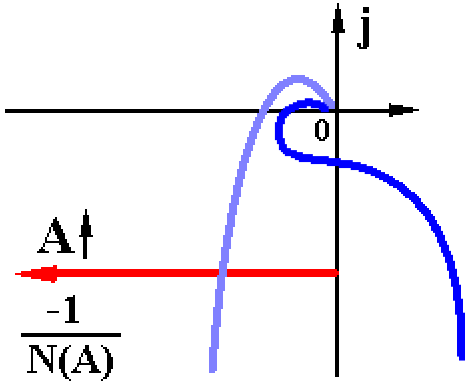


Outline of Chapter 7

改善非线性系统性能的措施

1. 调整线性部分的结构参数

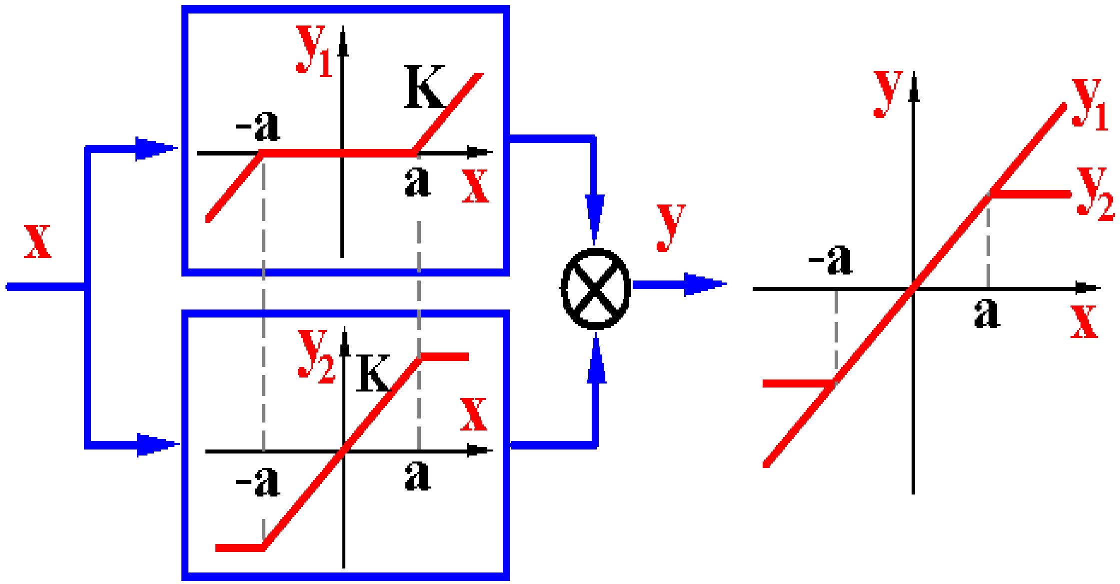




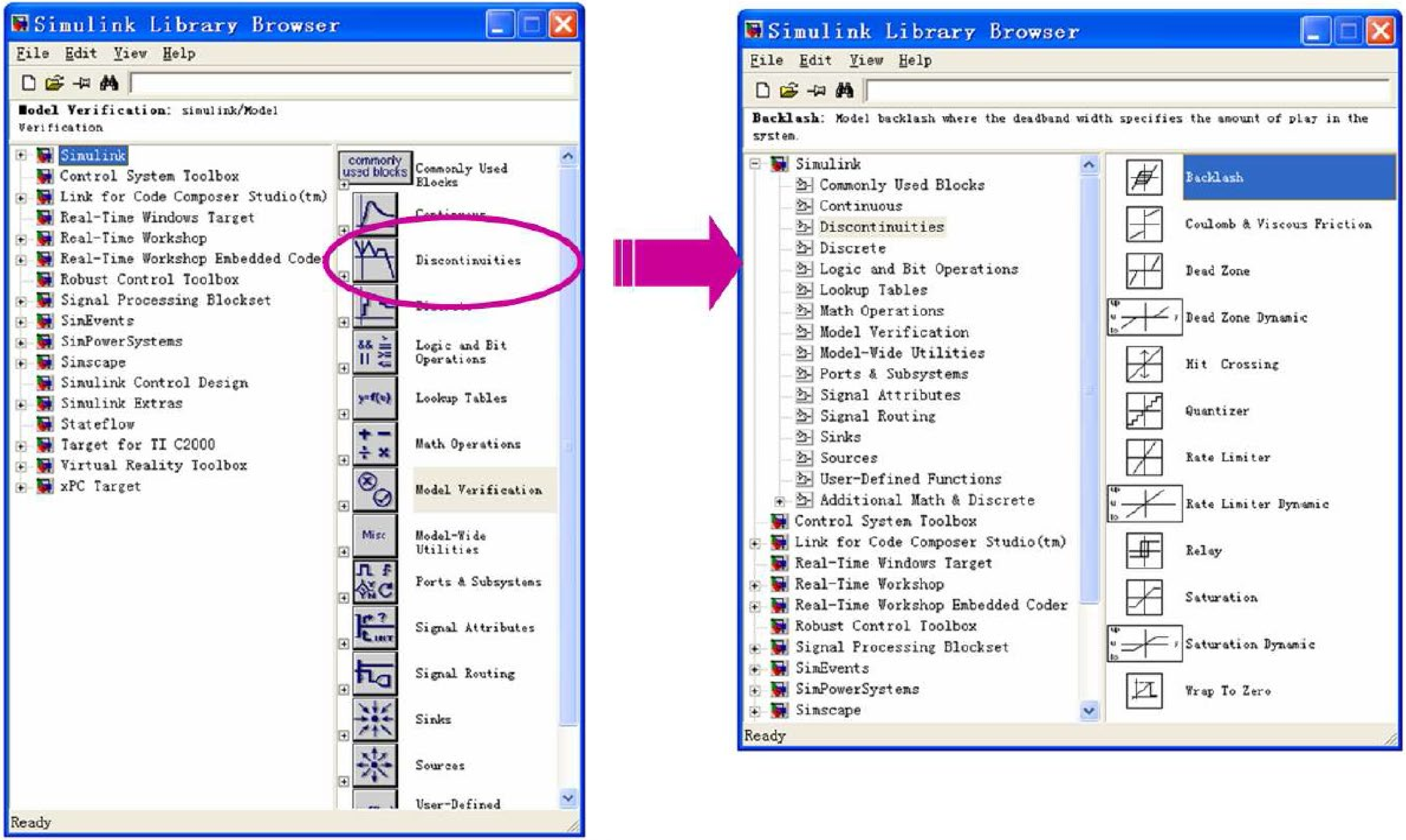
改善非线性系统性能的措施

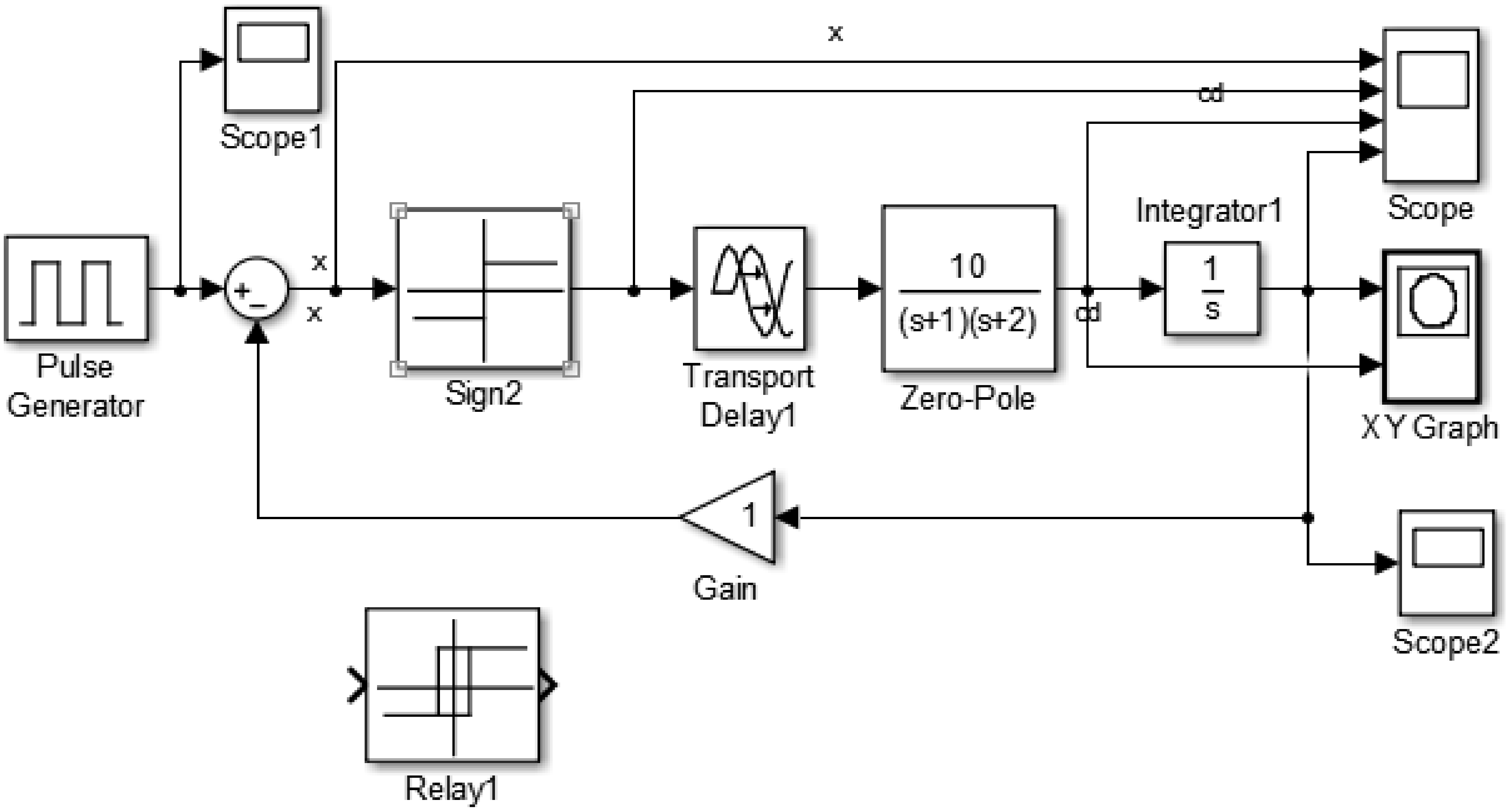
2. 改变非线性特性

饱和 + 死区



Outline of Chapter 7


Simulink Library Browser
File Edit View Help
Model Verification: simulink/Model Verification
Simulink
Control System Toolbox
Link for Code Composer Studio(tm)
Real-Time Windows Target
Real-Time Workshop
Real-Time Workshop Embedded Coder
Robust Control Toolbox
Signal Processing Blockset
SimEvents
SimPowerSystems
Simscape
Simulink Control Design
Simulink Extras
Stateflow
Target for TI C2000
Virtual Reality Toolbox
xPC Target
commonly used blocks
Commonly Used Blocks
Continuous
Discontinuities
Discrete
Logic and Bit Operations
Lookup Tables
Math Operations
Model Verification
Misc
Model-Wide Utilities
Ports & Subsystems
Signal Attributes
Signal Routing
Sinks
Sources
User-Defined
Ready
Simulink Library Browser
File Edit View Help
Backlash: Model backlash where the deadband width specifies the amount of play in the system.
Simulink
Commonly Used Blocks
Continuous
Discontinuities
Discrete
Logic and Bit Operations
Lookup Tables
Math Operations
Model Verification
Model-Wide Utilities
Ports & Subsystems
Signal Attributes
Signal Routing
Sinks
Sources
User-Defined Functions
Additional Math & Discrete
Control System Toolbox
Link for Code Composer Studio(tm)
Real-Time Windows Target
Real-Time Workshop
Real-Time Workshop Embedded Coder
Robust Control Toolbox
Signal Processing Blockset
SimEvents
SimPowerSystems
Simscape
Backlash
Coulomb & Viscous Friction
Dead Zone
Dead Zone Dynamic
Hit Crossing
Quantizer
Rate Limiter
Rate Limiter Dynamic
Relay
Saturation
Saturation Dynamic
Wrap To Zero
Ready



Scope1
x
cd
Pulse
Generator
x
x
Sign2
Transport
Delay1
10
(s+1)(s+2)
Zero-Pole
Integrator1
1
s
cd
Scope
XY Graph
1
Gain
Scope2
Relay1

Nonlinear blocks in Simulink

Backlash Model behavior of system with play

Coulomb and viscous friction Model discontinuity at zero, with linear gain elsewhere

Dead zone Provide region of zero output

Dead Zone dynamic Set inputs within bounds to zero

Hit crossing Detect crossing point

Quantizer Discretize input at specified interval

Rate limiter Limit rate of change of signal

Rate limiter Dynamic Limit rising and falling rates of signal

Relay Switch output between two constants

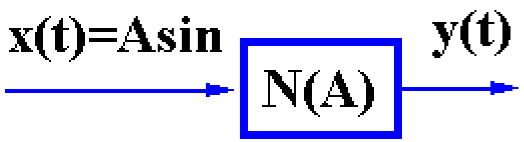
Saturation Limit range of signal

Saturation dynamic Bound range of input

Wrap to zero Set output to zero if input is above threshold

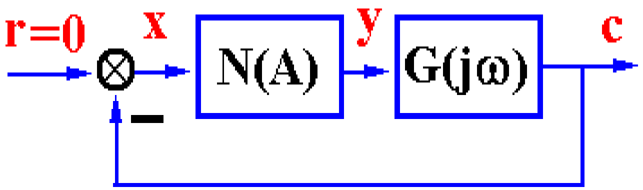
Summary

1. Describing function $N(A)=\frac{Y_1}{A}\angle\varphi_1$



2. System analysis by describing functions

(1) Stipulation
- ① N(A) and G(jω) are in cascade.
- ② The input and the output of N(A) are symmetry about the origin. y(x)=-y(-x)
- ③ G(jω) is a low-pass filter.



(2) Stability analysis $G(j\omega)$
- not circling $\frac{-1}{N(A)}$ stable
- circling $\frac{-1}{N(A)}$ unstable
- crossing with $\frac{-1}{N(A)}$ May be SSO

(3) SSO $\frac{-1}{N(A)}$ $\xrightarrow{A\uparrow}$
- Crossing in $G(j\omega)$ Non-SSO point
- Crossing out $G(j\omega)$ SSO Point
- Tangential to $G(j\omega)$ Neither stable nor unstable