1. True or false

(1) Any linear-invariant system described by a state space model can be converted into a unique transfer function, and vice versa.

(2) When converting a state space model into a transfer function, only its controllable and observable subsystem matters.

(3) To model a physical system, choices of state variables are unique.

(4) For continuous linear time invariant systems, similar transformations can change the controllability and observability of the system.

(5) Given the transfer function of a linear system, we can always transform it into a state-space model in a diagonal or Jordan canonical form.

(6) For a linear time-invariant system, if the state-space model of the system is stable, the system is controllable.

2. Short Answer Questions

(1) Draw the block diagram of the linear time-invariant continuous control system represented in state space.

(2) After doing similarity transformations for LTI systems, which properties of the systems are unchanged?

(3) What are the controllability and observability, respectively?

(4) When a state observer is needed? What is the Separation Principle?

(5) What are Dual systems and what is the principle of duality?

(6) Give the controllable canonical form, observable canonical form, diagonal canonical form, Jordan canonical form.

3. Prove the following property of the state transition matrix

$$\phi(-t) = \phi^{-1}(t)$$

4. Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Determine its discretized systems through zero-order holder where the sampling period $T = 0.1$s.

5. One of the state-space model of this system can be simplified as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t) \end{cases}$$

(1) Determine the stability, controllability and observability of the system and give the reasons; (5 marks)

(2) Calculate the transfer function.

(3) Try to find the state feedback gain matrix $k(k_1, k_2)$ such that the closed-loop system poles are located at -2 and -3.

(4) Design an observer for the system such that the observer eigenvalues are –6, –6.

(5) Can we design the state feedback controller and the state observer separately and then combined together to construct the observer-state feedback closed-loop control system?