第 4 章习题参考答案

1. The linearized (and normalized) magnetically suspended ball is described by

$$\dot{x}=\begin{bmatrix}0&1\\1&0\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}u$$

(1) Show that the system is unstable with $u\equiv 0$.

(2) Explain why it is possible to place the poles of the system at arbitrary locations (with the restriction of conjugate pairs) by linear state feedback.

(3) Find a state feedback which would place the poles of the closed-loop system at $-1\pm j$.

(4) Find a state feedback which would place the poles of the closed-loop system at $-1\pm j$ by using Matlab, and check whether the eigenvalues of $A-Bk$ are identical to the desired poles.

解

(1) 计算系统的极点， $\det(sI-A)=s^2-1$，由极点在左半平面，可知$u\equiv 0$时，系统不稳定。

(2) $Q_c=\begin{pmatrix}b & Ab\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}$， $rank(Q_c)=2$，系统可控，因此可以任意配置极点。

(3) $\Delta(s)=\det(Is-A)=\det\begin{pmatrix}s&-1\\-1&s\end{pmatrix}=s^2-1$

$$\Delta^*(s)=(s-1+j)(s-1-j)=s^2-1$$

$$p_{c1}=\begin{pmatrix}0&1\end{pmatrix}Q_c^{-1}=\begin{pmatrix}0&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}^{-1}=\begin{pmatrix}1&0\end{pmatrix}$$

$P_c^{-1}=\begin{pmatrix}p_{c1}\\p_{c1}A\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$， 得：$k=\begin{pmatrix}a_0^*-a_0 & a_1^*-a_1\end{pmatrix}P_c^{-1}=\begin{pmatrix}3&2\end{pmatrix}$

(4)
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>>A=[0 1;1 0];
B=[0;1];
Tc=ctrb(A,B);
rank(Tc);                %判断能控性
P=[−1+i   −1−i];         %输入期望极点
K=place(A, B, P);        %求反馈阵
Ac=A−B*K;
eig(Ac)                  %检验闭环特征值
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2. In the previous problem, the ball position x_1 can be measured using a photocell,

but the velocity x_2 is more difficult to obtain. Suppose, therefore, that the output is $y = x_1$.

(1) Design a full-order observer having eigenvalues at －5, －6, and use the observer feedback to produce closed-loop eigenvalues at $-1 \pm j$.

(2) Design a full-order observer having eigenvalues at －5, －6 by using Matlab.

解：

(1) $Q_o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$， $rank(Q_o) = 2$，系统可观。

取： $\begin{cases} \dot{z}(t) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} z(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} v(t) \\ \gamma(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} z(t) \end{cases}$

$\Delta(s) = \det(Is - A^T) = \det\begin{pmatrix} s & -1 \\ -1 & s \end{pmatrix} = s^2 - 1$

$\Delta^*(s) = (s+5)(s+6) = s^2 + 11s + 30$

$p_{\Sigma c1} = \begin{pmatrix} 0 & 1 \end{pmatrix} Q_{\Sigma c}^{-1} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \end{pmatrix}$ ， $P_{\Sigma c}^{-1} = \begin{pmatrix} p_{\Sigma c1} \\ p_{\Sigma c1} A^T \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ，

$k = \begin{pmatrix} a_0^* - a_0 & a_1^* - a_1 \end{pmatrix} P_{\Sigma c}^{-1} = \begin{pmatrix} 31 & 11 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 31 \end{pmatrix}$

因此，观测器增益矩阵 $f = k^T = \begin{pmatrix} 11 \\ 31 \end{pmatrix}$

全维观测器为：$\tilde{x}(t) = (A - fc)\tilde{x}(t) + bu(t) + fy(t)$

即：$\dot{\tilde{x}}(t) = \begin{pmatrix} -11 & 1 \\ -30 & 0 \end{pmatrix} \tilde{x}(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) + \begin{pmatrix} 11 \\ 31 \end{pmatrix} y(t)$

(2)带反馈的状态估计器系统的特征多项式为：$\Delta(s) = |sI - A + bk||sI - A + fc| = 0$

由分离特性得：

$|sI - A + bk| = \det\begin{pmatrix} s & -1 \\ k_1 - 1 & s + k_2 \end{pmatrix} = s^2 + k_2 s + k_1 - 1 = (s+1-j)(s+1+j)$

解得：$k = \begin{pmatrix} 3 & 2 \end{pmatrix}$

(3) >>A=[0 1;1 0];

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C=[1 0];
To=obsv(A,C);
rank(Tc);                    %判断能观性
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```
P=[−1+i   −1−i];                %输入期望极点
L=place(A', C', P)';            %求观测器反馈阵
Ao=A−L*C;
eig(Ao)                         %检验闭环特征值
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3. Consider the linear system with transfer function

$$G(s)=\frac{s+28}{(s+27)(s+29)}$$

Find a control law which places the poles at (﹣29, ﹣28), so that the zero is cancelled.

(1) Place the system in diagonal canonical form, and apply the feedback control law $u=r-k_m x$. Compute k_m for the desired closed-loop pole locations.

(2) Place the system in observable canonical form, and apply the feedback control law $u=r-k_o x$. Compute k_o for the desired closed-loop pole locations.

解：(1)

$G(s)=\dfrac{c_1}{s+27}+\dfrac{c_2}{s+29}$， 求得 $c_1=\dfrac{1}{2}$， $c_2=\dfrac{1}{2}$

化成对角阵的状态方程为：$\begin{cases}\dot{x}=\begin{pmatrix}-27 & 0\\ 0 & -29\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}1\\ 1\end{pmatrix}u\\ y=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{cases}$

$Q_c=\begin{pmatrix}b & Ab\end{pmatrix}=\begin{pmatrix}1 & -27\\ 1 & -29\end{pmatrix}$， $rank(Q_c)=2$，系统可控，因此可以任意配置极点。

$$\Delta(s)=\det(Is-A)=s^2+56s+783$$

$$\Delta^*(s)=(s+28)(s+29)=s^2+57s+812$$

$$p_{c1}=\begin{pmatrix}0 & 1\end{pmatrix}Q_c^{-1}=\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}29/2 & -27/2\\ 1/2 & -1/2\end{pmatrix}^{-1}=\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\end{pmatrix}$$

$$P_c^{-1}=\begin{pmatrix}p_{c1}\\ p_{c1}A\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{27}{2} & \frac{29}{2}\end{pmatrix}$$

得：$k=\begin{pmatrix}a_0^*-a_0 & a_1^*-a_1\end{pmatrix}P_c^{-1}=\begin{pmatrix}29 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{27}{2} & \frac{29}{2}\end{pmatrix}=\begin{pmatrix}1 & 0\end{pmatrix}$

(2) 化成可观标准型的状态方程为：$\begin{cases} \dot{x} = \begin{pmatrix} 0 & -783 \\ 1 & -56 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 28 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases}$

$rank(Q_c) = 2$，系统可控，因此可以任意配置极点。

$$p_{c1} = \begin{pmatrix} 0 & 1 \end{pmatrix} Q_c^{-1} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 28 & -783 \\ 1 & -28 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -28 \end{pmatrix}$$

$$P_c^{-1} = \begin{pmatrix} p_{c1} \\ p_{c1}A \end{pmatrix} = \begin{pmatrix} 1 & -28 \\ -28 & 785 \end{pmatrix}$$

得：$k = \begin{pmatrix} a_0^* - a_0 & a_1^* - a_1 \end{pmatrix} P_c^{-1} = \begin{pmatrix} 29 & 1 \end{pmatrix} \begin{pmatrix} 1 & -28 \\ -28 & 785 \end{pmatrix} = \begin{pmatrix} 1 & -27 \end{pmatrix}$

4. Consider the SISO, LTI system

$$\dot{x}(t) = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

(1) Find the feedback gain matrix with the desired closed-loop poles －3,－4;

(2) Design an observer for the system such that the observer eigenvalues are －8,－8.

(3) Draw the block diagram of the whole system.

解：(1)

$Q_c = \begin{pmatrix} b & Ab \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}$，$rank(Q_c) = 2$，系统可控，因此可以任意配置极点。

$$\Delta(s) = \det(Is - A) = \det\begin{pmatrix} s & -2 \\ 0 & s-3 \end{pmatrix} = s^2 - 3s$$

$$\Delta^*(s) = (s+3)(s+4) = s^2 + 7s + 12$$

$$p_{c1} = \begin{pmatrix} 0 & 1 \end{pmatrix} Q_c^{-1} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix}$$

$P_c^{-1} = \begin{pmatrix} p_{c1} \\ p_{c1}A \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$，得：$k = \begin{pmatrix} a_0^* - a_0 & a_1^* - a_1 \end{pmatrix} P_c^{-1} = \begin{pmatrix} 6 & 10 \end{pmatrix}$

(2)

$Q_o = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$，$rank(Q_o) = 2$，系统可观。

取：$\begin{cases} \dot{z}(t)=\begin{pmatrix}0 & 0\\ 2 & 3\end{pmatrix}z(t)+\begin{pmatrix}1\\ 0\end{pmatrix}v(t) \\ \gamma(t)=\begin{pmatrix}0 & 1\end{pmatrix}z(t) \end{cases}$

$$\Delta(s)=\det(Is-A^T)=\det\begin{pmatrix}s & 0\\ -2 & s-3\end{pmatrix}=s^2-3s$$

$$\Delta^*(s)=(s+8)(s+8)=s^2+16s+64$$

$$p_{\Sigma c1}=\begin{pmatrix}0 & 1\end{pmatrix}Q_{\Sigma c}^{-1}=\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix}^{-1}=\begin{pmatrix}0 & \frac{1}{2}\end{pmatrix}\text{，}\qquad P_{\Sigma c}^{-1}=\begin{pmatrix}p_{\Sigma c1}\\ p_{\Sigma c1}A^T\end{pmatrix}=\begin{pmatrix}0 & \frac{1}{2}\\ 1 & \frac{3}{2}\end{pmatrix}\text{，}$$

$$k=\begin{pmatrix}a_0^*-a_0 & a_1^*-a_1\end{pmatrix}P_{\Sigma c}^{-1}=\begin{pmatrix}64 & 19\end{pmatrix}\begin{pmatrix}0 & \frac{1}{2}\\ 1 & \frac{3}{2}\end{pmatrix}=\begin{pmatrix}19 & 60.5\end{pmatrix}$$

因此，$f=k^T=\begin{pmatrix}19\\ 60.5\end{pmatrix}$

全维观测器为：$\tilde{x}(t)=(A-fc)\tilde{x}(t)+bu(t)+fy(t)$

即：$\dot{\tilde{x}}(t)=\begin{pmatrix}-19 & 2\\ -60.5 & 3\end{pmatrix}\tilde{x}(t)+\begin{pmatrix}0\\ 1\end{pmatrix}u(t)+\begin{pmatrix}19\\ 60.5\end{pmatrix}y(t)$

(4) 整个系统的框图为：