

Focus effects in number sentences revisited

Abstract

There are very easy arguments for numbers: arguments that derive the existence of numbers in a few, simple steps from uncontroversial premises like the premise that I have ten fingers or that I have two legs. In recent literature some authors have refuted such arguments on linguistic grounds. They argue that easy arguments rely on a mistaken linguistic analysis of number sentences like ‘The number of my fingers is ten’ or ‘The number of my legs is two’: while such sentences are traditionally considered as identity sentences in which the numerals ‘ten’ and ‘two’ appear in singular term position, they are rather specificational sentences in which the numerals appear in determiner position. However, in a recent paper Jefferson Barlew (2017) has disputed this line of argument: he argues that *in easy arguments contexts* the pertinent number sentences do function as identity sentences even though they function as specificational sentences *in their standard use*. Hence, Barlew concludes, the rebuttal of easy arguments fails. The aim of the present paper is to defend the linguistic objection to easy arguments against Barlew’s criticism.

1 Introduction

When philosophers discuss whether numbers exist, they usually assume that they discuss a hard question that does not have an easy answer. However, surprisingly, there seem to be very easy arguments for the existence of numbers. Just look! I have ten fingers. But if I have ten fingers, then the number of my fingers is ten. Hence, there is a number! Or look at my legs! I have two legs. But if I have two legs, then the number of my legs is two. Hence, there is a number! In such arguments the existence of numbers is derived from completely uncontroversial premises, like the premise that I have ten fingers or that I have two legs. That makes the arguments very puzzling: How can it be that philosophers have discussed for thousands of years whether numbers exist if the existence of numbers can be derived from completely uncontroversial premises?

In recent literature some authors have argued that easy arguments fail to establish the existence of numbers on linguistic grounds. They argue that easy arguments rely on a mistaken linguistic analysis of number sentences like ‘The number of my fingers is ten’ or ‘The number of my legs is two’: while such sentences are traditionally considered as identity sentences in which the numerals ‘ten’ and ‘two’ appear in singular term position, they are rather specificational sentences in which the numerals

appear in determiner position.¹ However, in a recent paper Jefferson Barlew (2017) has disputed this line of argument: he argues that *in easy arguments contexts* the pertinent number sentences do function as identity sentences even though they function as specificational sentences *in their standard use*. Hence, Barlew concludes, the rebuttal of easy arguments fails. The aim of the present paper is to defend the linguistic objection to easy arguments against Barlew’s criticism.

The structure of the paper is as follows. The first section sketches the linguistic objection against easy arguments. The second section presents Barlew’s (2017) argument to the effect that number sentences function as identity sentences rather than as specificational sentences in easy argument contexts, in contrast to what opponents of easy arguments have claimed. In the final section I will argue that Barlew’s argument fails and, thus, that it is warranted to object to easy arguments on linguistic grounds.

2 A rebuttal of easy arguments

Paradigmatic easy arguments start from a fairly uncontroversial assumption that does not say anything about numbers. For instance, it is commonly assumed that Mars has two moons and, thus, that sentence (1) is true:

(1) Mars has two moons.

If sentence (1) is true, then sentence (2) is true as well:

(2) The number of moons of Mars is two.

But, so the argument goes, sentence (2) is true only if numbers exist. Hence, numbers exist!

Apart from the assumption that sentence (1) is true, the argument relies on the following two assumptions:

(P1) If sentence (1) is true, then sentence (2) is true.

(P2) The truth of sentence (2) requires the existence of numbers.

(P2) is based on a certain linguistic analysis of the pertinent number sentence that was most famously proposed by Gottlob Frege. In his *Foundations of Arithmetics* Frege writes:

[T]he proposition ‘Jupiter has four moons’ can be converted into ‘the number of moons of Jupiter is four’. Here the word ‘is’ should not be taken as a mere copula, as in the proposition ‘the sky is blue’ [...] Here ‘is’ has the sense of ‘is identical with’ or ‘is the same as’. (Frege 1980: §57)

Frege, thus, assumes the following:

Identity (ID)

Sentences of the form ‘The number of *F*s is *n*’, where ‘*n*’ is a placeholder for a numeral, are identity sentences in which ‘*n*’ functions as a singular term.

¹ The first elaborated criticism of the traditional analysis of the pertinent number sentences is due to Hofweber (2005). But, in contrast to Felka (2014, 2016) and Moltmann (2013), Hofweber does not defend a specificational analysis of those sentences.

If *Identity* is correct, then the numeral ‘two’ contained in sentence (2) functions as a singular term. Since sentences containing singular terms can be true only if the singular terms refer, (2) can be true only if numbers exist.

However, in recent literature some authors have rejected *Identity* (Felka 2014, 2016; Hofweber 2007, 2016; Moltmann 2013). Some of them have argued that sentence (2) is a so-called specificational sentence while specificational sentences are the elliptical remainders of question-answer pairs (Felka 2014, 2016; Moltmann 2013). According to this analysis, sentence (2) is analysed as follows:²

(2*) [~~What~~ the number of moons of Mars is] is [~~Mars has two moons~~.]

If this analysis is correct, then the numeral ‘two’ is the elliptical remainder of sentence (1). Since the numeral functions in sentence (1) as a determiner, it functions in sentence (2) as a determiner as well. Hence, it does not function as a singular term and, thus, does not bring it about that the truth of sentence (2) requires the existence of numbers, as proponents of easy arguments assume.³

3 Barlew’s defence of easy arguments

Barlew (2017) concedes that number sentences of the form ‘The number of *F*s is *n*’ function as specificational sentences *in their standard use*. However, he argues that *in easy argument contexts* the number sentences function as identity sentences and, thus, that easy arguments go through. In the following I will first explain a distinction between narrow and broad focus on which Barlew relies in his argument and then explain how he uses this distinction to establish his claim.

3.1 Narrow and broad focus

Intuitively, the focus of an utterance of a sentence is that part of information conveyed with the utterance that is most important in the utterance context.⁴ Take, for instance, the sentence

(3) Paul entered quietly.

When the question under discussion is ‘Who entered quietly?’, the focus is on the information provided by ‘Paul’. When the question under discussion is ‘How did Paul enter?’, the focus is on the information provided by ‘quietly’. An utterance of (3) clearly is an appropriate contribution to the discussion either way.⁵

² This is the question-answer analysis proposed in Felka (2014, 2016). Cp. Moltmann (2013) for a different variant.

³ One might argue that the definite description still induces that (2) is true only if numbers exist. However, it has been argued that it only induces a *pragmatic* presupposition and, thus, that ‘Mars has two moons’ can be a true answer to the question even if numbers do not exist (cp. Felka (2016) and Brogaard (2007) for further discussion).

⁴ Cp. Hofweber (2016) for a more detailed explanation as well as the pertinent references from the linguistic literature.

⁵ At least given appropriate intonational stress (cp., e.g., Hofweber (2007: 12ff.)).

While (3), thus, has a flexible information structure some sentences differ in this respect. Consider, for illustration:

(4) Quietly, Paul entered.

(5) It was quietly how Paul entered.

Both (4) and (5) have a *fixed* information structure: they mark the information provided by ‘quietly’ as the focus of the utterance due to their syntactic structure. Sentences that exhibit such a structural focus are called *focus constructions*.

A striking feature of focus constructions is that they give rise to a specific question-answer behaviour which allows us to check (i) whether some sentence is a focus construction and (ii) what expression exactly carries the information marked as the focus. In relation to (i), consider the following exchanges:

(6) Who entered quietly? # Quietly, Paul entered.

(7) How did Paul enter? Quietly, Paul entered.

The question-answer behaviour of ‘Quietly, Paul entered’ makes obvious that the sentence marks the information provided by ‘quietly’ as the focus. For since that information is marked as the focus and, thus, as particularly important, the sentence cannot felicitously be uttered to answer the first question that does not ask about that information. In contrast, it can felicitously be uttered to answer the second question. In relation to (ii), notice that the expression that carries the information marked as the focus constitutes an appropriate short answer to question (8):

(8) How did Paul enter?

(9) Quietly, Paul entered.

(10) Quietly.

Thus, we can check what short answers are appropriate in order to determine what expression exactly carries the information marked as the focus.

The example sentences considered so far are cases of narrow focus in which the focus is on a single constituent (‘quietly’). Barlew points out that there are also cases of broad focus in which the focus is on the complete utterance. For illustration, consider a context in which (11) is the question under discussion:

(11) What happened?

(3) Paul entered quietly.

(12) # Quietly.

In this utterance context the focus of an utterance of sentence (3) is not on a single constituent like ‘quietly’. Rather, it is on the complete utterance. Accordingly, no single constituent will be an appropriate short answer to the question under discussion; we have to utter the complete sentence to answer the question appropriately. This is a case of broad focus.

3.2 The number sentences in easy argument contexts

Both opponents of easy arguments and their critic Barlew assume that specificational sentences are copular sentences that are distinguished by exhibiting a structural focus on the post-copular expression.⁶ They also agree that at least in their standard use number sentences of the form ‘The number of *F*s is *n*’ exhibit a structural focus on the post-copular expression.⁷ The latter claim is based on the question-answer behaviour of the number sentences. Consider:

- (13) Who has ten fingers? # The number of Paula’s fingers is ten.
- (14) What is the number of Paula’s fingers? The number of Paula’s fingers is ten. // Ten.

An utterance of the number sentence (or simply the numeral ‘ten’) is an appropriate answer to a question that asks about the information provided by the numeral while it is not an appropriate answer to a question that does not ask about that information. Since this can be explained if the number sentence exhibits a structural focus on the post-copular expression, both opponents of easy arguments and Barlew assume that the sentence exhibits such a focus and, thus, functions as a specificational sentence in its standard use.

However, following Higgins (1973) and others, Barlew points out that many copular sentences allow for different uses and that it is thus ‘essential to determine which reading of [the number sentence] arises’ in easy argument contexts (Barlew 2017: 421). According to Barlew, easy argument contexts are not ‘contexts where the interlocutors are wondering about numbers of moons or planets’ since ‘a philosopher making the easy argument doesn’t actually care how many moons [Mars] has’ (Barlew 2017: 421). Rather, they are contexts in which philosophers discuss the entailments of ontologically innocent sentences like ‘Mars has two moons’.⁸ Thus, Barlew says, we have to determine how number sentences of the pertinent kind are used in contexts in which philosophers discuss the entailments of ontologically innocent sentences.

In order to do so, Barlew presents the following example of such a context:

- (C) Al and Betty are philosophers. Al is also an amateur astronomer with a decent telescope but not much background knowledge. After a night of star gazing Al tells Betty: ‘Guess what, Mars has two moons.’ Betty replies: ‘Hmm, I wonder what we can infer from this, or what other sentences we might say that are true in virtue of this.’

According to Barlew, this is an easy argument context since the question under discussion is (15):

⁶ Cp., e.g., Higgins (1973), Heycock (1994), Mikkelsen (2005) for this view. In the philosophical literature, a detailed defence can be found in Felka (2016).

⁷ This observation is due to Hofweber (2005) and is employed in Felka (2014, 2016) to defend the claim that the pertinent number sentences function as specificational sentences.

⁸ The distinction between ‘ontologically innocent’ and ‘ontologically loaded’ sentences is due to Hofweber (2007).

(15) What are the entailments of ‘Mars has two moons’?

However, Barlew observes, an utterance of the numeral ‘two’ is not an appropriate answer to the question under discussion, while an utterance of the complete sentence (2) is:

(2) The number of moons of Mars is two.

(16) # Two.

Thus, Barlew concludes, in the present context the focus is not on the numeral ‘two’ (or any other single constituent); rather, the focus is on the complete utterance. We thus have a case of broad focus, rather than a case of narrow focus on the numeral (or any other constituent of the sentence).

If Barlew’s consideration were correct, it would present a major difficulty for the objection to easy arguments presented above. As we have seen, the objection crucially relies on the claim that number sentences of the form ‘The number of *F*s is *n*’ are specificational sentences. But if in easy argument contexts the number sentences do not exhibit narrow focus on the post-copular term, they do not function as specificational sentences in such contexts. Rather, they function as identity sentences, just like proponents of easy arguments assume.

4 Rebuttal of Barlew’s defence

Barlew’s defence of easy arguments is successful only if he manages to establish (i) that the use of the sentence ‘The number of moons of Mars is two’ he considers is the one pertinent for easy arguments and (ii) that the sentence functions as an identity sentence in that use. In the following I will argue that Barlew fails on both counts.

4.1 What are the pertinent uses of the number sentences?

As presented above, Barlew concedes that the number sentence ‘The number of moons of Mars is two’ functions as a specificational sentence in its standard use. But, he argues, in the uses pertinent for easy arguments the sentence functions as an identity sentence and, thus, the arguments go through. According to Barlew, the pertinent uses are uses of the sentence in easy argument contexts, i.e. contexts in which metaphysicians are concerned with the entailments of ontologically innocent sentences rather than with astronomical facts concerning Mars and its moons. That is, they are uses in *philosophical* rather than in *ordinary* contexts.

However, Barlew’s assumption that easy arguments target uses of the number sentence in philosophical contexts is mistaken. There certainly are contexts in which metaphysicians discuss entailments of ontologically innocent sentences rather than astronomical facts concerning Mars and its moons. And in these contexts metaphysicians are concerned with uses of number sentences. But this does *not* imply that the uses of number sentences they discuss are uses in philosophical contexts: surely, in a given

context C_1 , one can discuss features of sentences (including their apparent entailments) *as they are used in a different context C_2* . And this is exactly what is going on in easy argument contexts: in such contexts, metaphysicians discuss features of number sentences as they are used by ordinary speakers in non-philosophical contexts. Proponents of easy arguments take every opportunity to emphasise this. Here is a representative quotation from Thomasson:

[...] the relevant conditions of existence are determined by the application [...] conditions for the terms speakers use [...] the truths [...] uncovered by metaphysicians are just ways of making explicit the ontological implications of the rules we master in learning to use expressions. (Thomasson 2009: 450)

As Thomasson emphasises in this quotation, in easy argument contexts metaphysicians take expressions in their standard uses by ordinary speakers and investigate their existence entailments in that very use.

Indeed, it is not a coincidence that proponents of easy arguments focus on standard uses of number sentences. They do so since the first premise of easy arguments—to the effect that the sentence ‘The number of moons of Mars is two’ is true if the sentence ‘Mars has two moons’ is true—is based on the observation that *ordinary speakers* take the two sentences to be truth-conditionally equivalent. Thus, in order to justify their first premise proponents of easy arguments need to consider the number sentence in its standard use since that is the use that underlies the pertinent speakers’ intuitions. But if they do so, then they need to take the number sentence in its standard use in the second premise as well; otherwise, their argument would suffer from a simple fallacy of ambiguity.

Barlew might want to try the following defence strategy:

Proponents of easy arguments like Thomasson are simply mistaken, but their mistake is easily fixed. Rather than appealing to what ordinary speakers think about standard uses, they *should* appeal to their own intuitions about easy argument uses of number sentences. Since even in those uses the number sentences seem to be entailed by their ontologically innocent counterparts, the modified easy argument strategy has a justification for its first premise. Since philosophical uses of number sentences are identity rather than specificational uses, their second premise is also true.

The next subsection shows that this defence strategy fails as well, since Barlew is unable to establish that number sentences are not used as specificational sentences even in easy argument contexts.

4.2 A case of broad focus?

Let us, thus, consider the question of whether Barlew has established that there are special uses of number sentences of the pertinent kind in which they function as identity sentences. Recall that in order to establish this result, he tries to present a context in which there is no focus on the post-copular term of sentence (2). If he had presented such a context,

this would be strong evidence for the claim that sentence (2) does not function as a specificational sentence in that context since specificational sentences are distinguished by exhibiting a structural focus on the post-copular term. However, in this subsection I will argue that Barlew has not presented such a context.

Recall that in his example the question under discussion is supposed to be (15):

(15) What are the entailments of ‘Mars has two moons’?

To this question, Barlew claims, sentence (2) is an appropriate answer:

(2) The number of moons of Mars is two.

This could not be the case if the sentence were exhibiting a structural focus on the numeral ‘two’ since in that case an utterance of the sentence could only be an appropriate answer to a question that asks about the information provided by the numeral. Thus, Barlew says, the sentence does not exhibit such a focus and, hence, does not function as a specificational sentence in the context.

However, Barlew’s claim that sentence (2) is an appropriate answer to the question under discussion is mistaken. For the question under discussion requires *examples of sentences*. In particular, it requires examples of sentences that are entailed by the sentence ‘Mars has two moons’. But an utterance of sentence (2) does not give an example of such a sentence: an utterance of sentence (2) does not say anything about sentences or other linguistic expressions; it only says something about Mars and its moons. Therefore, it does not constitute an answer to the question under discussion. In contrast, an utterance of sentence (2_Q) does constitute an answer to the question under discussion:

(2_Q) ‘The number of moons of Mars is two.’

An utterance of sentence (2_Q) is the short version of the following complete answer to the question under discussion, which, indeed, is also an appropriate answer to (15):

(2_L) ‘Mars has two moons’ entails ‘The number of moons of Mars is two’.

But from the observation that (2_L) constitutes an appropriate answer to the question under discussion we cannot derive anything about the information structure of some other sentence. In particular, we cannot derive anything about the information structure of sentence (2), with which opponents of easy arguments are concerned.

To drive my point home, consider the following argument that is analogous to the one that Barlew presents. In the previous section we considered the sentence ‘Quietly, Paul entered’ as an example of a focus construction that marks the information provided by ‘quietly’ as the focus. One might now try to establish that in some contexts the sentence does not mark the information provided by ‘quietly’ as the focus. Take, for instance, a context in which the question under discussion is (16):

(16) What is an example of a focus construction?

To this question, one might argue, an utterance of sentence (4) is an appropriate answer while an utterance of (17) is not:

(4) Quietly, Paul entered.

(17) # Quietly.

Thus, so the argument would go, the sentence ‘Quietly, Paul entered’ does not mark the information provided by ‘quietly’ as the focus in the present context since in that case an utterance of the sentence could only be an appropriate answer to a question that asks about that information. But, again, the argument fails since it relies on the mistaken assumption that an utterance of sentence (17) is an appropriate answer to the question under discussion while in fact only an utterance of sentence (18) or of its short version (19) is—and it is exactly *since* sentence (4) marks the information provided by ‘quietly’ as the focus:

(18) An example of a focus construction is ‘Quietly, Paul entered.’

(19) ‘Quietly, Paul entered.’

For the very same reason Barlew’s argument fails. Therefore, it also fails to establish that sentence (2) functions as an identity sentence in the specified context.⁹

5 Conclusion

Barlew recently argued that number sentences like ‘The number of moons of Mars is two’ function as identity sentences in easy argument contexts. Thus, Barlew claims, a rebuttal of easy arguments on linguistic grounds remains unconvincing. The present paper defended the linguistic objection to easy arguments against Barlew’s criticism. In particular, it has been argued that (i) only standard uses of the number sentences are pertinent for easy arguments and (ii) his considerations do not even show that there are non-standard uses of the number sentences in which they function as identity sentences. Since Barlew’s defence of easy arguments thus fails, the linguistic objection against easy arguments stands. Arguing from ‘Mars has two moons’ to ‘The number of moons of Mars is two’ is no quick and easy way to establish the existence of numbers, since such an argument has to rely on a mistaken linguistic analysis of the pertinent number sentence.

References

- Barlew, J. 2017: ‘Focus on numbers’. *Linguistics and Philosophy* 40, pp. 401–426.
- Brogaard, B. 2007: ‘Number Words and Ontological Commitment’. *Philosophical Quarterly* 57, pp. 1–20.

⁹ Barlew might try to modify the question under discussion such that it does not ask for examples of sentences anymore. For instance, the question could also be: ‘What does follow from the fact that Mars has two moons?’ But an utterance of sentence (2) is not a correct answer to that question either; only ‘(From the fact that Mars has two moons it follows) that the number of moons of Mars is two’ is.

- Felka, K. 2014: ‘Number Words and Reference to Numbers’. *Philosophical Studies* 168, pp. 261–283.
- 2016: *Talking About Numbers. Easy Arguments for Mathematical Realism*. Frankfurt am Main: Klostermann.
- Frege, G. 1980: *The Foundations of Arithmetic: A Logico-Mathematical Enquiry Into the Concept of Number*. Illinois: Northwestern University Press.
- Heycock, C. 1994: ‘The Internal Structure of Small Clauses: New Evidence from Inversion’. In Beckmann, J. (ed.): *Proceedings of the North Eastern Linguistic Society (=NELS) 25*, University of Pennsylvania, pp. 223–238.
- Higgins, F. R. 1973: *The Pseudo-Cleft Construction in English*. Ph.D. thesis, Cambridge MA: Massachusetts Institute of Technology.
- Hofweber, T. 2005: ‘Number Determiners, Numbers, and Arithmetic’. *Philosophical Review* 114, pp. 179–225.
- 2007: ‘Innocent Statements and Their Metaphysically Loaded Counterparts’. *Philosophers’ Imprint* 7, pp. 1–33.
- 2016: *Ontology and the Ambitions of Metaphysics*. Oxford: Oxford University Press.
- Mikkelsen, L. 2005: *Copular Clauses: Specification, Predication and Equation*. Amsterdam: John Benjamins.
- Moltmann, F. 2013: ‘Reference to Numbers in Natural Language’. *Philosophical Studies* 162, pp. 499–536.
- Thomasson, A. L. 2009: ‘Answerable and Unanswerable Questions’. In Manley, D., D. J. Chalmers and R. Wasserman (eds.): *Metametaphysics: New Essays on the Foundations of Ontology*, Oxford University Press.