



# LOADING BALANCE PROBLEM

Solve by Linear Program

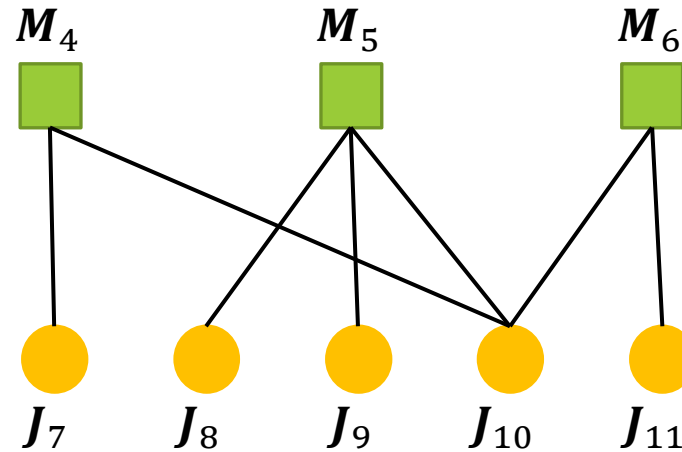
Zhiyuan Wang 12032878

Supervisor: Ke Tang

# CONTENT

1. Generate tree Structures and discuss how to choice tree and child machine node
2. Generate a graph with no cycle from a graph with cycles
3. LP-based Algorithm has best result

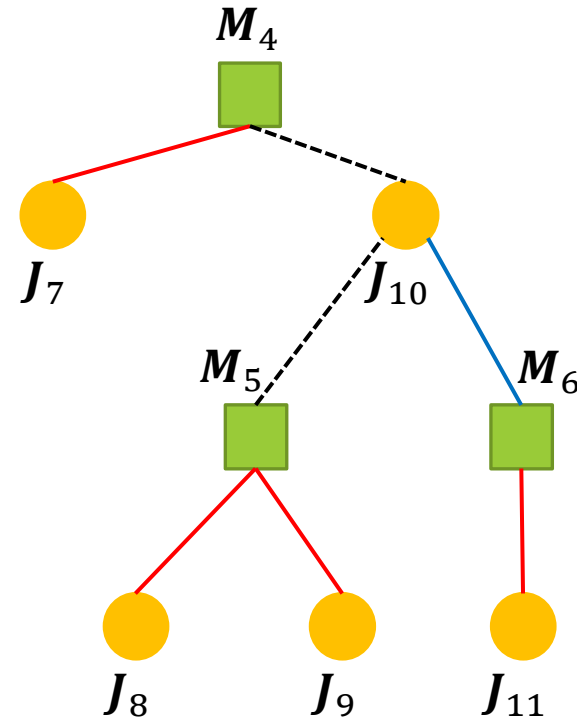
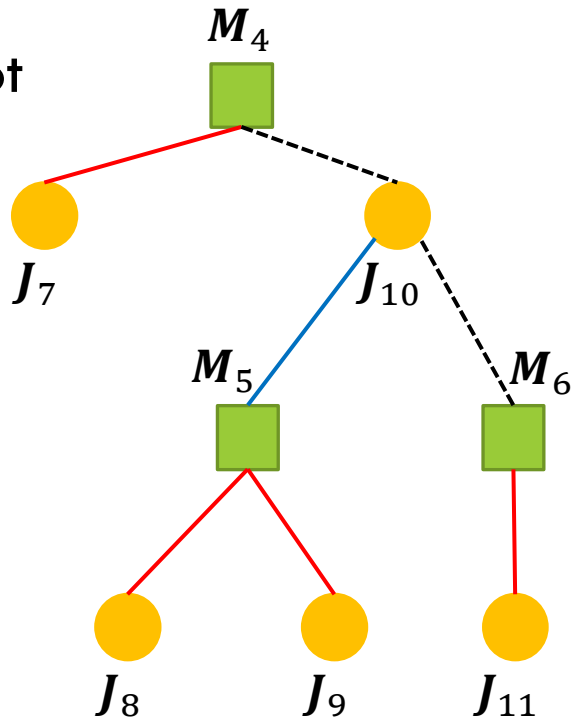
# GENERATE TREE STRUCTURES AND DISCUSS HOW TO CHOICE TREE AND CHILD MACHINE NODE



The source graph

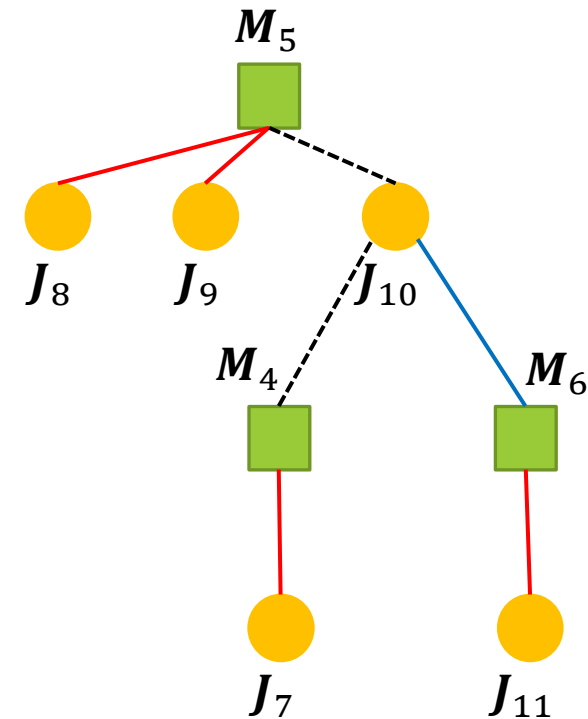
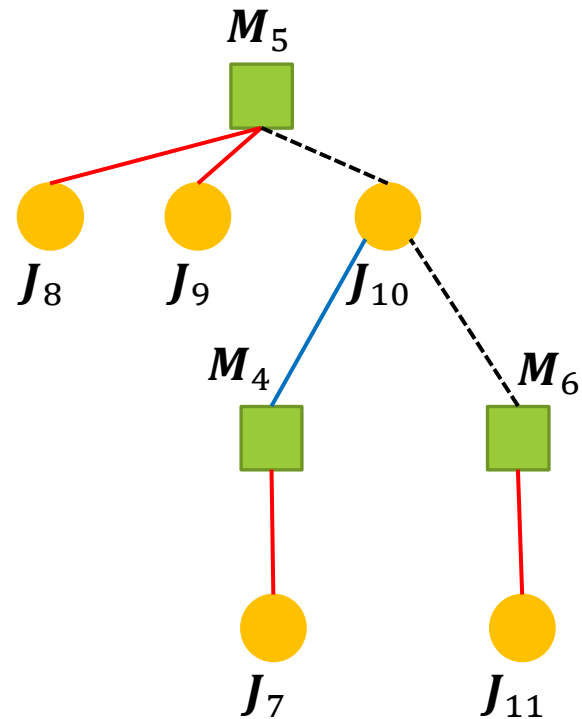
# GENERATE TREE STRUCTURES AND DISCUSS HOW TO CHOICE TREE AND CHILD MACHINE NODE

$M_4$  is the root



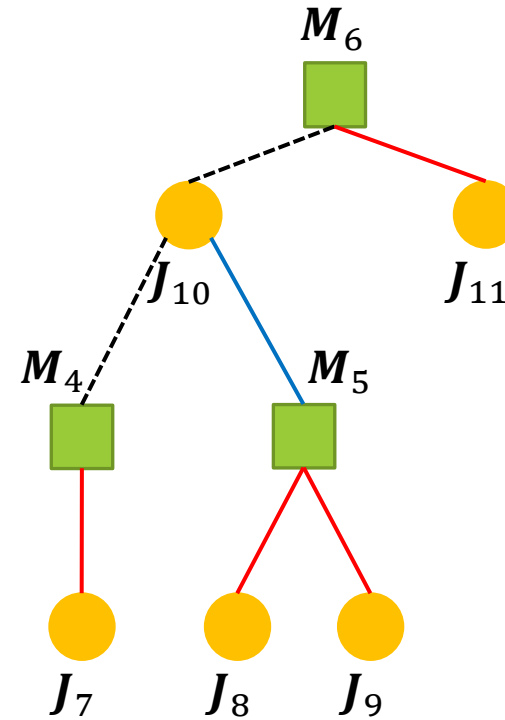
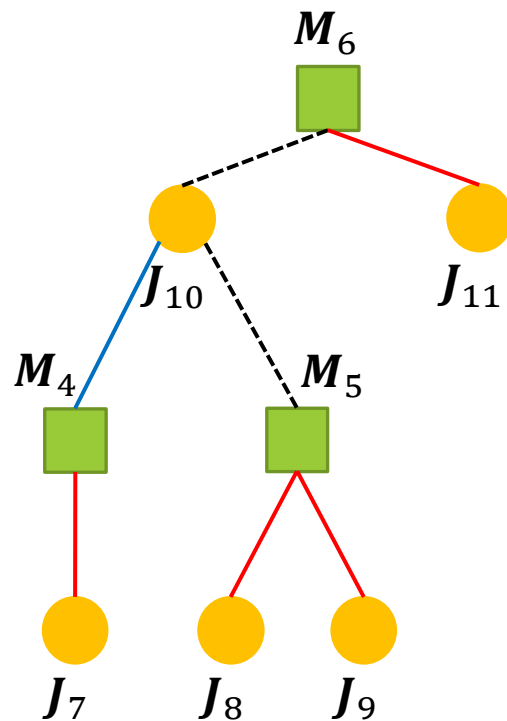
# GENERATE TREE STRUCTURES AND DISCUSS HOW TO CHOICE TREE AND CHILD MACHINE NODE

$M_5$  is the root

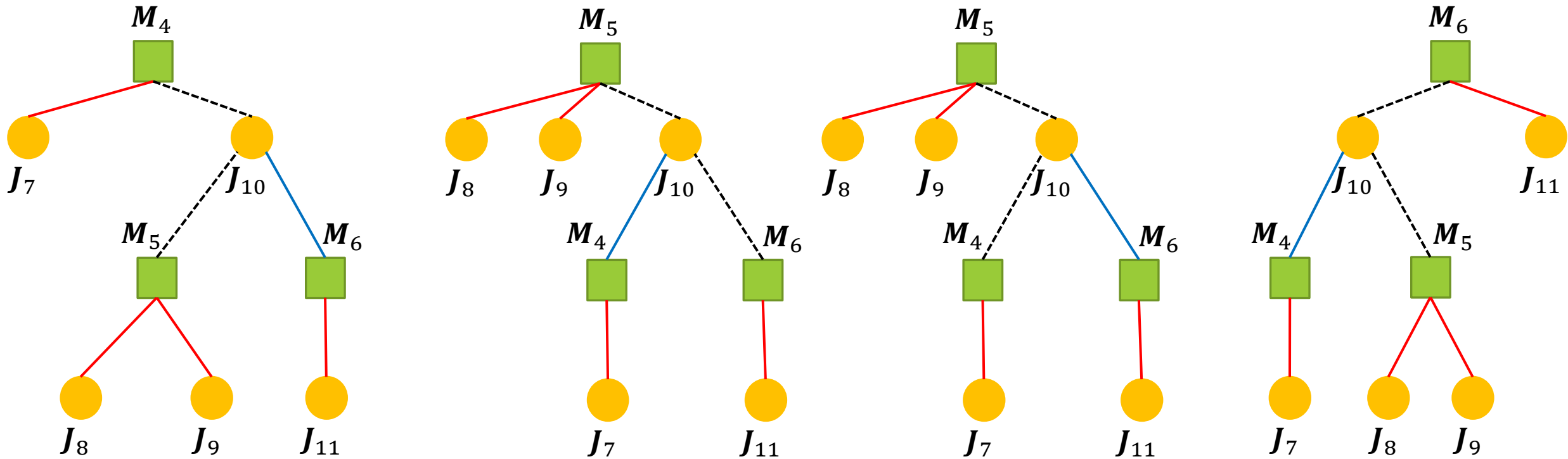


# GENERATE TREE STRUCTURES AND DISCUSS HOW TO CHOICE TREE AND CHILD MACHINE NODE

$M_6$  is the root

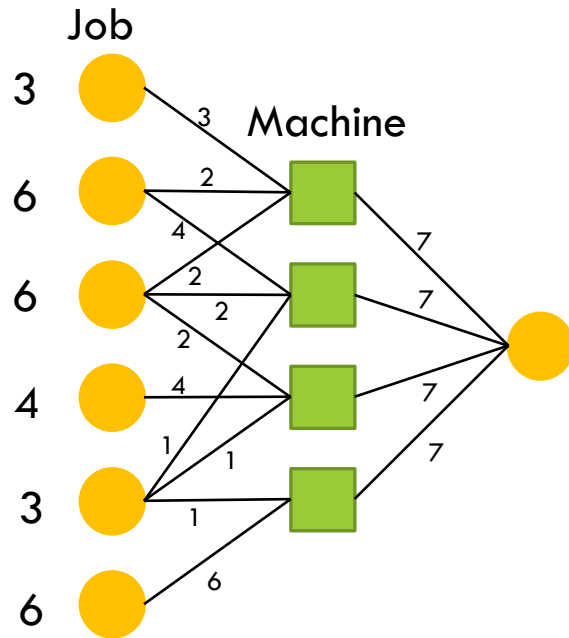


# GENERATE TREE STRUCTURES AND DISCUSS HOW TO CHOICE TREE AND CHILD MACHINE NODE

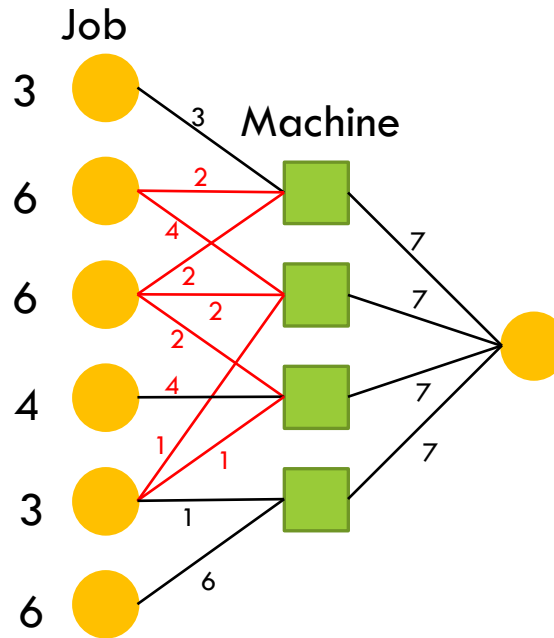


These are better case, the max number of jobs need to process in one machine is 2.

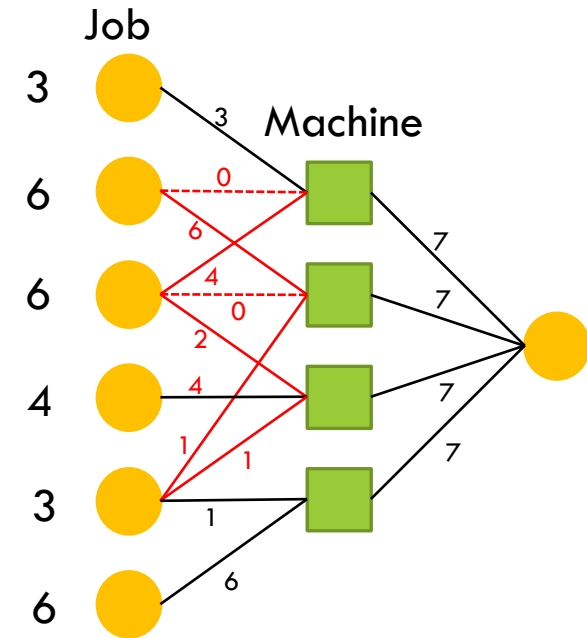
# GENERATE A GRAPH WITH NO CYCLE FROM A GRAPH WITH CYCLES



Origin Graph



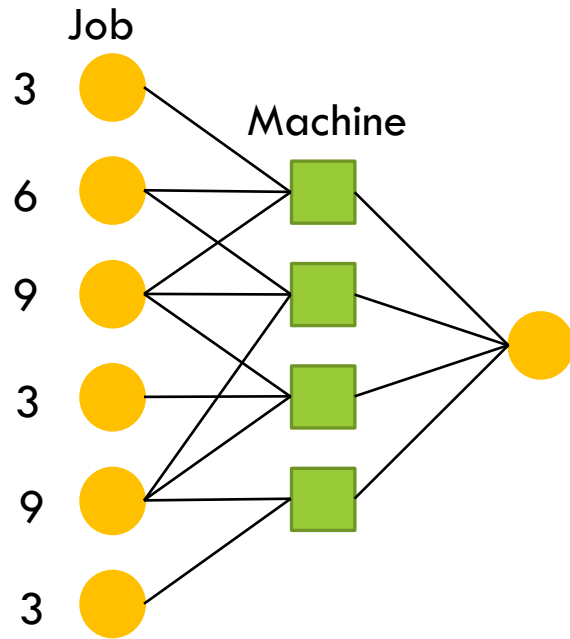
Find the Cycle



Remove the Cycle



# GENERATE A GRAPH WITH NO CYCLE FROM A GRAPH WITH CYCLES



Origin Graph

NMinimize[

数值最小化

{L,

$$t_{11} + t_{21} + t_{31} \leq L \ \&\& \ t_{22} + t_{32} + t_{52} \leq L$$

$$\&\& \ t_{33} + t_{43} + t_{53} \leq L \ \&\& \ t_{54} + t_{64} \leq L \ \&\&$$

$$t_{11} == 3 \ \&\& \ t_{21} + t_{22} == 6 \ \&\& \ t_{31} + t_{32} + t_{33} == 9 \ \&\&$$

$$t_{43} == 3 \ \&\& \ t_{52} + t_{53} + t_{54} == 9 \ \&\& \ t_{64} == 3 \ \&\&$$

$$t_{11} \geq 0 \ \&\& \ t_{21} \geq 0 \ \&\& \ t_{22} \geq 0 \ \&\& \ t_{31} \geq 0 \ \&\& \ t_{32} \geq 0 \ \&\&$$

$$t_{33} \geq 0 \ \&\& \ t_{43} \geq 0 \ \&\& \ t_{52} \geq 0 \ \&\& \ t_{53} \geq 0 \ \&\& \ t_{54} \geq 0 \ \&\& \ t_{64} \geq 0$$

},

{t<sub>11</sub>, t<sub>21</sub>, t<sub>22</sub>, t<sub>31</sub>, t<sub>32</sub>, t<sub>33</sub>, t<sub>43</sub>, t<sub>52</sub>, t<sub>53</sub>, t<sub>54</sub>, t<sub>64</sub>, L

}]

Out[40]= {8.25, {t<sub>11</sub> → 3., t<sub>21</sub> → 0., t<sub>22</sub> → 6., t<sub>31</sub> → 5.25, t<sub>32</sub> → 2.25, t<sub>33</sub> → 1.5,  
t<sub>43</sub> → 3., t<sub>52</sub> → 0., t<sub>53</sub> → 3.75, t<sub>54</sub> → 5.25, t<sub>64</sub> → 3., L → 8.25}}

Calculate Process

# GENERATE A GRAPH WITH NO CYCLE FROM A GRAPH WITH CYCLES

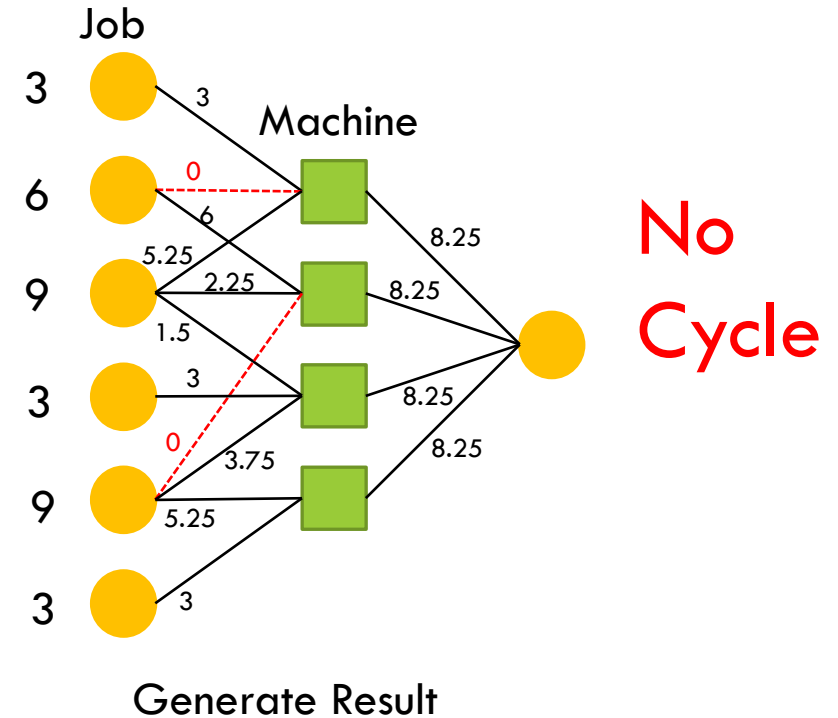
NMinimize[

数值最小化

```
{L,
  t11 + t21 + t31 ≤ L && t22 + t32 + t52 ≤ L
  && t33 + t43 + t53 ≤ L && t54 + t64 ≤ L &&
  t11 == 3 && t21 + t22 == 6 && t31 + t32 + t33 == 9 &&
  t43 == 3 && t52 + t53 + t54 == 9 && t64 == 3 &&
  t11 ≥ 0 && t21 ≥ 0 && t22 ≥ 0 && t31 ≥ 0 && t32 ≥ 0 &&
  t33 ≥ 0 && t43 ≥ 0 && t52 ≥ 0 && t53 ≥ 0 && t54 ≥ 0 && t64 ≥ 0
},
{t11, t21, t22, t31, t32, t33, t43, t52, t53, t54, t64, L
}]
```

Out[40]= {8.25, {t11 → 3., t21 → 0., t22 → 6., t31 → 5.25, t32 → 2.25, t33 → 1.5,  
t43 → 3., t52 → 0., t53 → 3.75, t54 → 5.25, t64 → 3., L → 8.25}}

Calculate Process



# GENERATE A GRAPH WITH NO CYCLE FROM A GRAPH WITH CYCLES

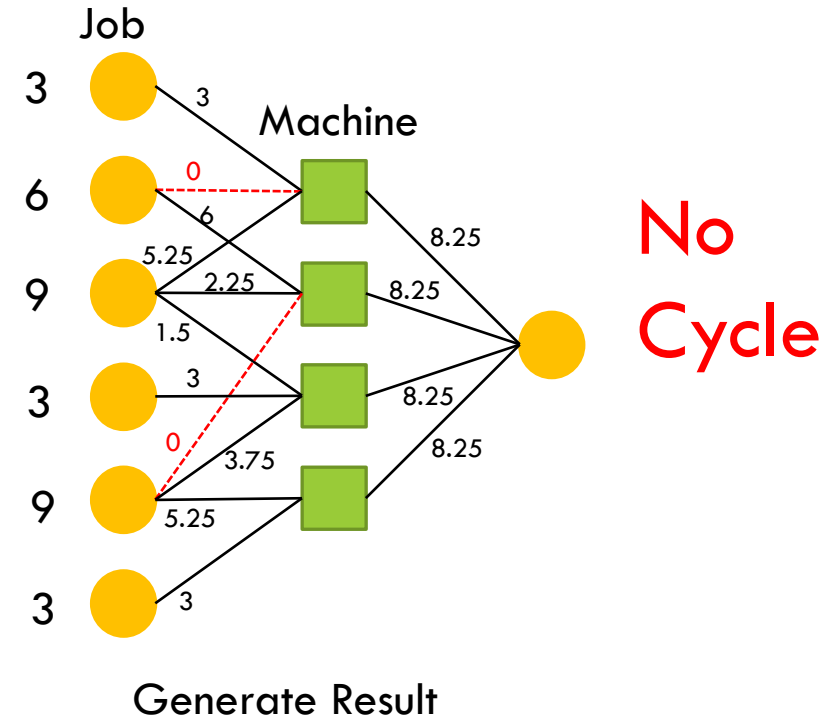
NMinimize[

数值最小化

```
{L,
  t11 + t21 + t31 ≤ L && t22 + t32 + t52 ≤ L
  && t33 + t43 + t53 ≤ L && t54 + t64 ≤ L &&
  t11 == 3 && t21 + t22 == 6 && t31 + t32 + t33 == 9 &&
  t43 == 3 && t52 + t53 + t54 == 9 && t64 == 3 &&
  t11 ≥ 0 && t21 ≥ 0 && t22 ≥ 0 && t31 ≥ 0 && t32 ≥ 0 &&
  t33 ≥ 0 && t43 ≥ 0 && t52 ≥ 0 && t53 ≥ 0 && t54 ≥ 0 && t64 ≥ 0
},
{t11, t21, t22, t31, t32, t33, t43, t52, t53, t54, t64, L
}]
```

Out[40]= {8.25, {t11 → 3., t21 → 0., t22 → 6., t31 → 5.25, t32 → 2.25, t33 → 1.5,  
t43 → 3., t52 → 0., t53 → 3.75, t54 → 5.25, t64 → 3., L → 8.25}}

Calculate Process

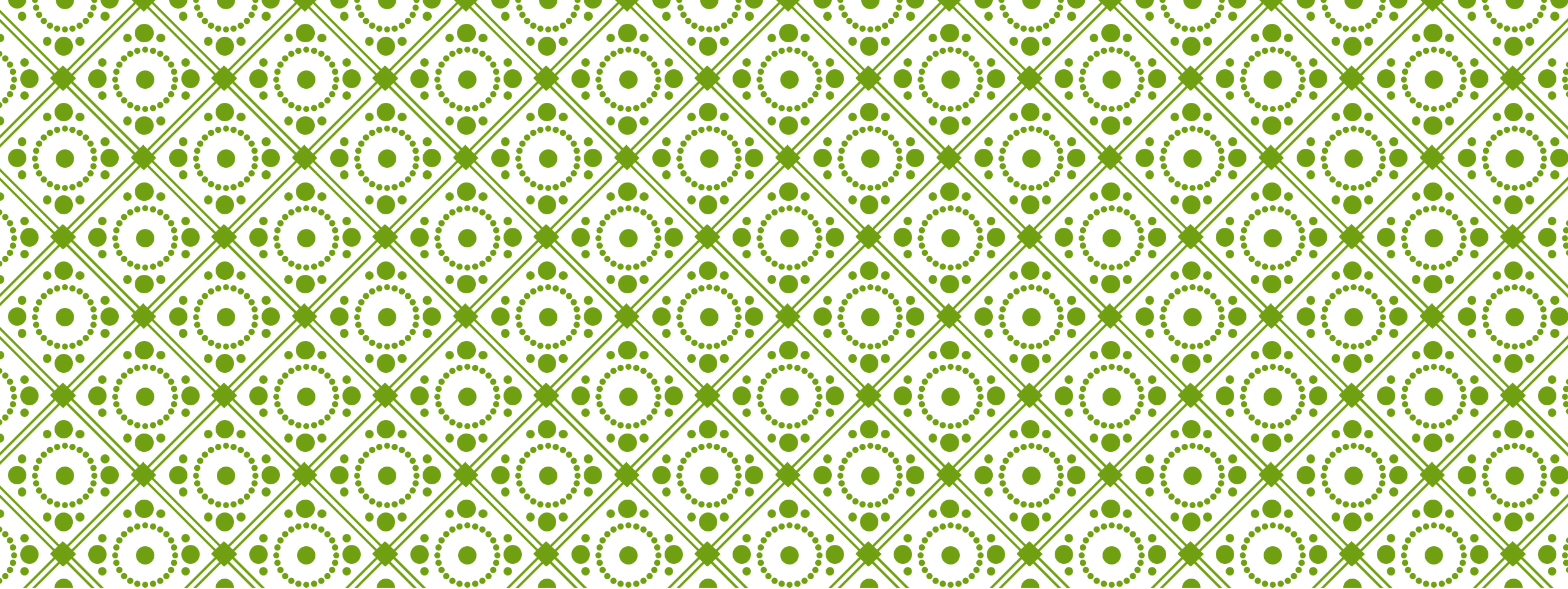


# GENERATE A GRAPH WITH NO CYCLE FROM A GRAPH WITH CYCLES

- I can't find an example that the graph drawn by my LP software has a cycle.
- I try to construct many examples that can get the optimal value with a cycle, but my LP program prefers to set one edge of the cycle to 0.

```
In[92]:= FindMinimum[  
  [求极小值和其坐标  
  {L,  
    t11 ≥ 0 && t12 ≥ 0 && t13 ≥ 0 && t14 ≥ 0 && t21 ≥ 0 && t22 ≥ 0 &&  
    t23 ≥ 0 && t24 ≥ 0 && t31 ≥ 0 && t32 ≥ 0 && t33 ≥ 0 && t34 ≥ 0 &&  
    t11 + t12 + t13 + t14 == 72 && t21 + t22 + t23 + t24 == 32 &&  
    t31 + t32 + t33 + t34 == 36 &&  
    t11 + t21 + t31 ≤ L && t12 + t22 + t32 ≤ L &&  
    t13 + t23 + t33 ≤ L && t14 + t24 + t34 ≤ L  
  },  
  {t11, t12, t13, t14, t21, t22, t23, t24, t31, t32, t33, t34, L},  
  Method → "InteriorPoint"]  
  [方法  
Out[92]= {35., {t11 → 0., t12 → 2., t13 → 35., t14 → 35., t21 → 0., t22 → 32.,  
  t23 → 0., t24 → 0., t31 → 35., t32 → 1., t33 → 0., t34 → 0., L → 35.}}
```





# THANK YOU !

Q&A