DISJOINT PATHS PROBLEM

Greedy Algorithm

Zhiyuan Wang 12032878
Supervisor: Ke Tang

Southern University of Science and Technology

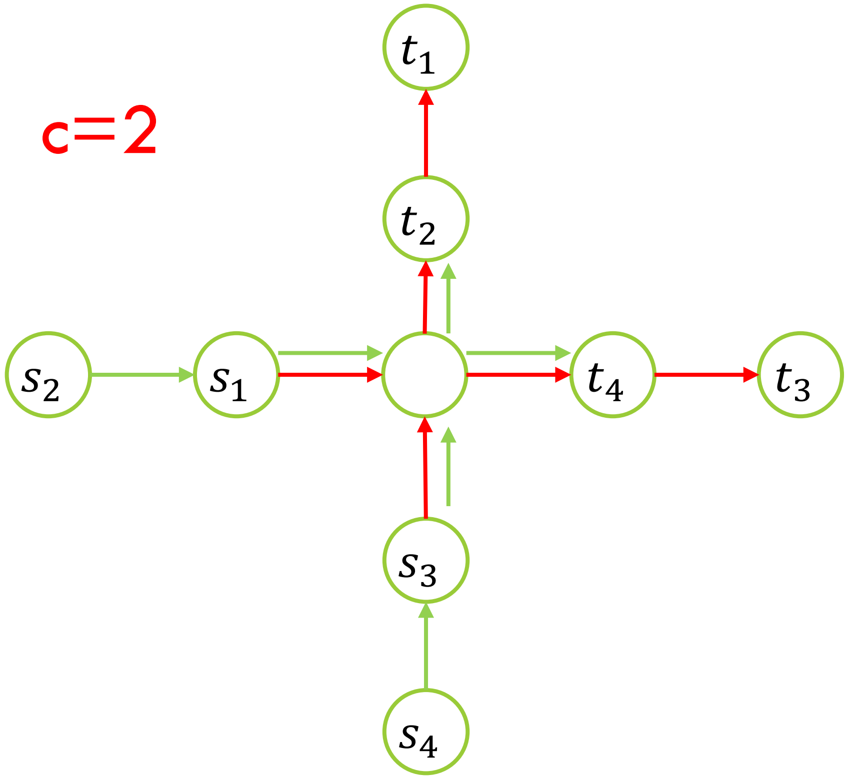
CONTENT

1. Introduction to the Disjoint Paths Problem.
2. The example about Disjoint Path Problem with c=2

DISJOINT PATHS PROBLEM

- Input:
  - A directed graph G, there are k pairs nodes$(s_1, t_1)\ldots\ (s_k, t_k)$. s_i is the source node of the path, t_1 is the target node of the path.
  - An integer capacity of each edge c.
- Constraint: In the graph, each node cannot be used by more than c paths.
- Objective: Maximization of the number of satisfied paths
- Output: Satisfied paths

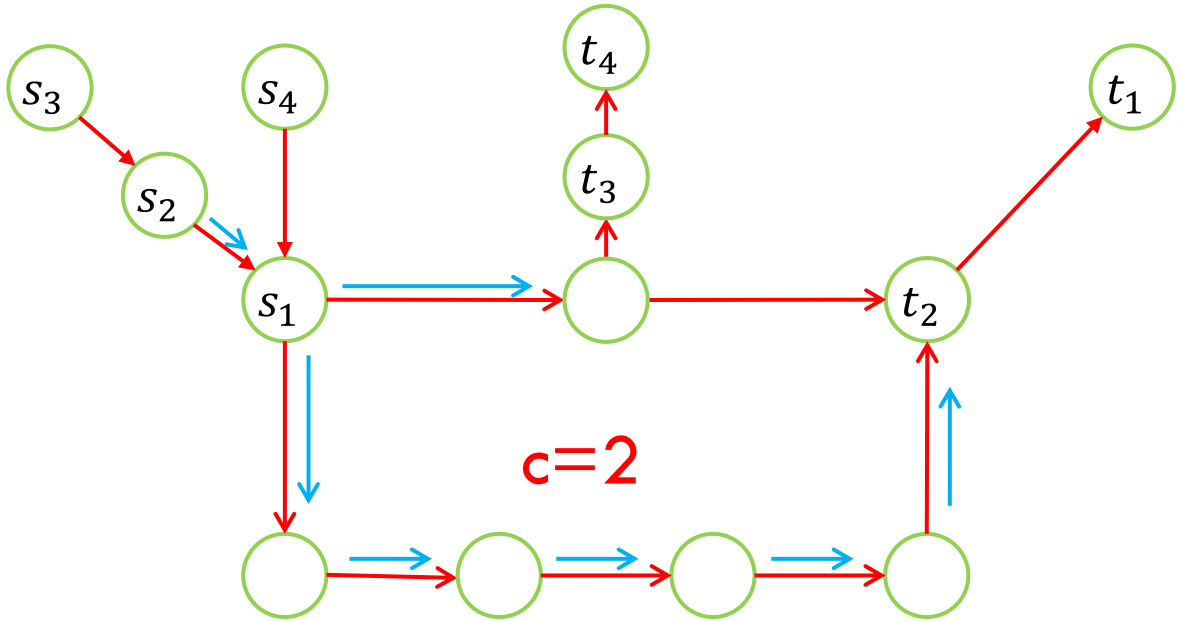
EXAMPLE1



- The optimal solution is always obtained by the greedy algorithm independent of the selection order of the shortest paths.
  - Whether how to choose the order of path when use greedy algorithm, all (s_1, t_1) , (s_2, t_2), (s_3, t_3) , and (s_4, t_4) will in the I.

SUSTech Southern University of Science and Technology

EXAMPLE2

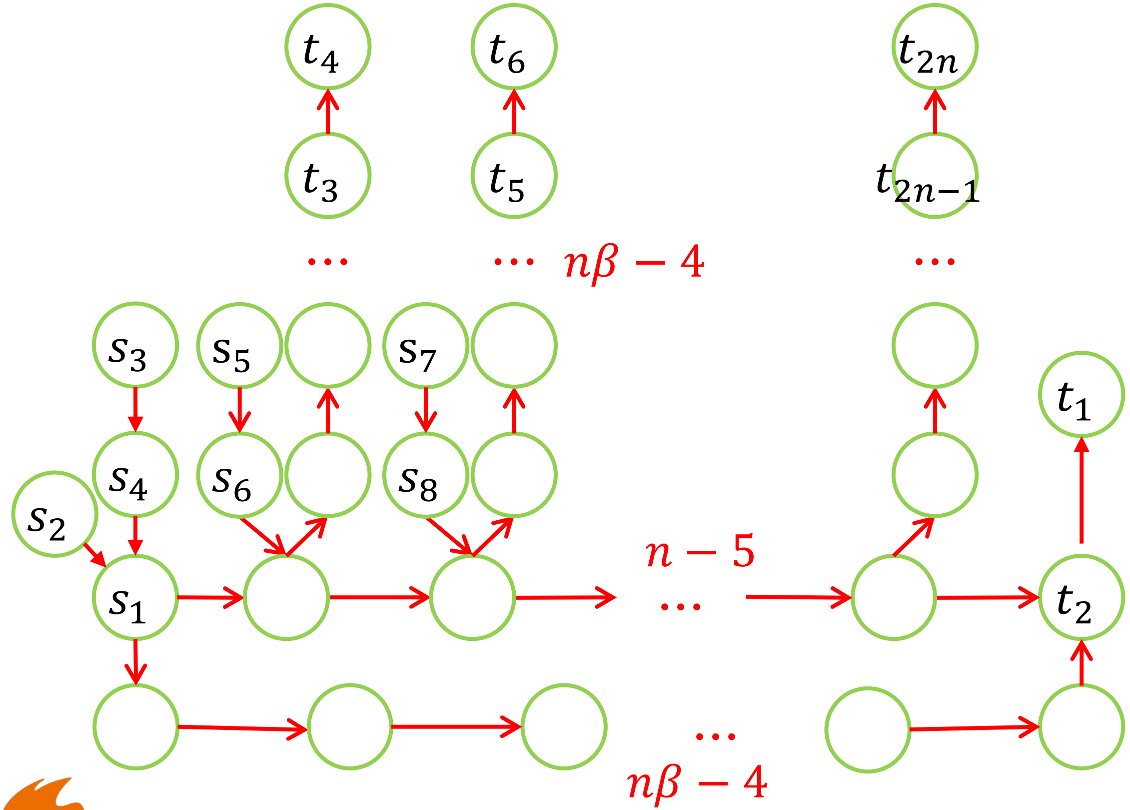


- The example where $|I|$ is close to $|I^*|/(4m^{\frac{1}{3}}+1)$ always holds depending on the selection order of the shortest paths
  - As shown as the left figure, if use greedy algorithm, there will be 2 paths: (s_1, t_1) , (s_2, t_2),
  - For the optimal solution, there will be 4 paths: (s_1, t_1) , (s_2, t_2), (s_3, t_3), (s_4, t_4), and (s_5, t_5).
  - There are 13 edges.
- $|I|$:$|I^*|/(4m^{\frac{1}{3}}+1) \approx 5.2$

SUSTech Southern University of Science and Technology

EXAMPLE2

- From the last page, we can get an pattern and extend it.



$m = n(n\beta + 1) + n + 2$

$\beta = \sqrt[3]{m}, |I| = 2, |I^*| = 2n$

$\beta^3 = n^2\beta + 2n + 2$

$$\frac{\beta}{n} = \frac{n}{\beta} + \frac{2}{\beta^2} + \frac{2}{n\beta^2}$$

$$\lim_{n\to\infty} \beta = \sqrt[3]{m} \to \infty$$

$$\lim_{n\to\infty} \frac{\beta}{n} = \frac{n}{\beta} + \frac{2}{\beta^2} + \frac{2}{n\beta^2} = \frac{n}{\beta} + 0 + 0 = \frac{n}{\beta}$$

$$\lim_{n\to\infty} \frac{\beta}{n} = \frac{n}{\beta} = 1$$

So we can know, if n tend to ∞, $\frac{\beta}{n}$ equal to 1, and $\lim_{n\to\infty} |I| = \frac{|I^*|}{n} = \frac{|I^*|}{\beta} = \frac{|I^*|}{m^{\frac{1}{3}}+1}$

THANK YOU !

Q&A

Southern University of Science and Technology