



# A Tutorial on Learned Multi-dimensional Indexes

Abdullah-Al-Mamun  
Purdue University  
mamuna@purdue.edu

Hao Wu  
Purdue University  
wu1112@purdue.edu

Walid G. Aref  
Purdue University  
aref@purdue.edu

## ABSTRACT

Recently, Machine Learning (ML, for short) has been successfully applied to database indexing. Initial experimentation on *Learned Indexes* has demonstrated better search performance and lower space requirements than their traditional database counterparts. Numerous attempts have been explored to extend learned indexes to the multi-dimensional space. This makes learned indexes potentially suitable for spatial databases. The goal of this tutorial is to provide up-to-date coverage of learned indexes both in the single and multi-dimensional spaces. The tutorial covers over 25 learned indexes. The tutorial navigates through the space of learned indexes through a taxonomy that helps classify the covered learned indexes both in the single and multi-dimensional spaces.

## CCS CONCEPTS

• **Database Systems** → **Indexing**; • **Machine Learning** → *ML for Systems*.

## KEYWORDS

Learned Indexes, Spatial, Multi-dimensional

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## 1 INTRODUCTION

Due to recent successes in the field of Machine Learning (ML, for short), two trends of research have emerged in the systems community: *Systems for ML* and *ML for Systems*. *Systems for ML* aims at building large-scale systems for efficient ML workloads. In contrast, *ML for Systems* aims at using ML-based approaches to replace core components of systems for better performance and less space requirement. This tutorial falls under the broad category of *ML for System*. More specifically, this tutorial addresses the following question: Can one use ML techniques to guide data indexing? Can ML techniques replace and act in place of a multi-dimensional index?

Database Management Systems (DBMS) are designed to be general purpose. This general purpose nature of a modern DBMS does not consider the specifics of a particular application and data of the user [21]. In most DBMSs, for efficient data access, an index

structure, e.g., a B+-Tree is used. As a result, these index structures are highly optimized but are general-purpose data structures. In other words, they do not utilize knowledge of the underlying data distribution in the optimization process of an index. To illustrate, assume that we have 1 to 5M continuous integer keys. Now, in order to search a particular key, we can use the key itself (instead of a B+-Tree) as an offset. As a result, the logarithmic complexity of search operation can be reduced to  $O(1)$ .

By addressing this issue, the first work on “Learned Index” [22] has changed the perception of DBMS indexing. The key idea behind the above mentioned work is that “Indexes are models” of the data. Given a key, say  $k$ , an index simply predicts the position of  $k$  in the dataset. As a result, indexes can be learned. Surprisingly, learned indexes have demonstrated better search performance and lower space requirements.

Although the term “Learned Indexes” has been popular very recently, the idea of using a learning mechanism in data indexing is not completely new. An example of an earlier index that uses ML techniques is the handwritten trie that uses hidden Markov Models on a trie structure to index the learned models [1]. However, this earlier work focuses on *indexing the learned models* in contrast to the more recent trend of *learning the index*. In this tutorial, we will cover both trends: (1) Indexing the learned models, and (2) Learning the index or what is termed the learned indexes.

These initial works have been focused on read-only workloads. To deal with updates, a new class of updatable adaptive learned indexes has been proposed, e.g., [4]. It has been demonstrated that a careful space-time trade-off can lead to an updatable data structure.

In the area of Spatial Database Indexing, support for multi-dimensional data is required. The R-Tree and its variants, e.g., [2, 12], and the quadtree and its variants, e.g., [9, 36, 37], are widely studied and are used extensively in practice. Initial attempts have been made to replace the R-Tree with a learned counterpart, e.g., [21]. A series of followup works have followed to build learned multi-dimensional indexes.

- Part 1: Learned Index Structures (20 minutes)
  - Introduction and Background (5 minutes)
  - Learned Single Dimensional Indexes (15 minutes)
- Part 2: Learned Multidimensional Indexes (25 minutes)
  - Motivation and Challenges (5 minutes)
  - State-of-the-art Learned Multidimensional Indexes (20 minutes)
- Part 3: Open Problems for Future Research (5 minutes)

**Figure 1: The outline of the tutorial (50 minutes).**

This tutorial will provide up-to-date coverage of learned multi-dimensional indexes. The target audience for the tutorial is students,

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academics, researchers and practitioners with basic knowledge on data structures and algorithms. We assume basic understanding of fundamental data indexing structures e.g., the B-tree, the R-Tree, the quadtree, space-filling curves, and the Bloom Filter. The tutorial is designed to be self-contained in providing all the necessary background on the concepts related to the "Learned" part of the Index Structures. The target outcomes of this tutorial are as follows:

- Understanding the limitations of traditional multi-dimensional indexes.
- Understanding the motivation behind developing learned multi-dimensional indexes.
- Familiarity and up-to-date coverage of the state-of-the-art learned multi-dimensional index structures.
- Highlighting the research challenges and new opportunities in the area of learned multi-dimensional indexes.

## 2 OUTLINE OF THE TUTORIAL

This tutorial consists of two main parts as illustrated in Figure-1. The first part will contain the overall problem setting and the learned indexes introduced for the one-dimensional case. The second part of the tutorial will cover the existing work on learned multi-dimensional indexes. We will navigate through the space of learned indexes using a simple taxonomy that we develop over the literature on the existing learned indexes. A sample snapshot of the taxonomy is given below.

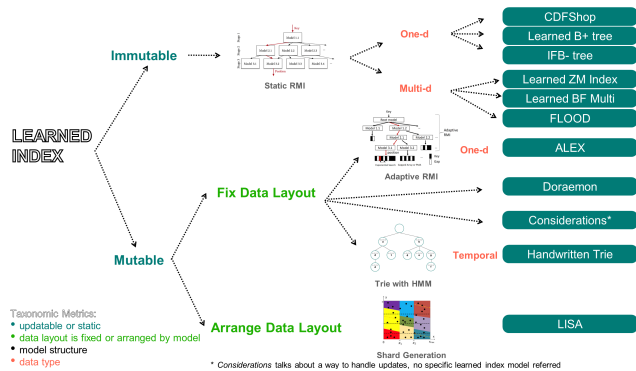


Figure 2: A sample taxonomy of Learned Index Structures

### 2.1 Part 1: Learned Index Structures

The chain of work in this particular area has started in 2018 with the paper titled "The Case for Learned Index Structures" [22]. In this paper, the key idea is that a one-dimensional index, e.g., the B+-Tree, can be treated as a learned model. For searching a key, a B+-Tree simply finds (predicts) the position of the key within a logical sorted array at the leaf level. If we follow this assumption, by learning the Cumulative Distribution Function (CDF) of the input data, the mapping function of an index can be learned. Due to the complexity of the CDF, a single ML model learned over the complete data cannot provide the desired accuracy [4]. To address this issue, a Recursive Model Index (RMI, for short) has been introduced. Several learned indexes utilize RMI, e.g., AIDEL [23], ASLM [25] and Hybrid-O [34]. A demo on tuning the learned indexes can be found in [29].

Handling dynamic data sets in the context of learned indexes is challenging. The reason is due to the following. Given a data set, it takes significant time to train an ML-based model to capture the CDF of the underlying data set. Given a new insert or update to the underlying dataset, this in a sense may change the CDF or at least perturb the distribution of the data and the learned model. Thus, upon multiple inserts, deletes, and updates, we need to retrain the model that, in term, reduces the utility of the learned index until it is retrained again. Updates and inserts in learned indexes have been addressed in several newly proposed indexes, e.g., [4, 13].

A series of followup works for addressing various aspects of learned indexes can be found in [7, 8, 17, 27, 30, 38, 40, 42, 43, 43, 44]. In [10], a data-aware index structure is proposed using interpolation. Hybrid approaches (with helper models) have been introduced [14, 15, 26]. Reinforcement Learning has been used for routing query and data in learned indexes [45]. Other related recent papers that we cover in the tutorial are: [19, 20, 28, 39] as well as a survey on learned data structures [6]. These as well as other learned indexes will be covered in Part 1 of this tutorial, and will serve as the foundation for Part 2 of the tutorial.

### 2.2 Part 2: Learned Multi-dimensional Indexes

Naturally, researchers have explored how to extend the concept of learned indexes into the multi-dimensional space. Several works have explored projecting the multi-dimensional data into the one-dimensional space as a preprocessing step, and then a learned index is built over the one-dimensional projection of the data (e.g., as in [21]). Flood [31] is another learned in-memory read-optimized index that automatically adapts itself to a particular multi-dimensional data set and workload. In [16], an interpolation-friendly multi-dimensional index has been proposed. LISA [24] is a disk-based learned multi-dimensional index. In [41], the Z-order space filling curve has been incorporated with the staged learning model to build a multi-dimensional index. Other recent works are: [3, 5, 11, 32, 33]. The tutorial will cover these multi-dimensional learned indexes, and demonstrate how they work and the challenges they face. Finally, the tutorial will conclude by listing several open problems for future research.

## 3 RELATED TUTORIALS

One related tutorial titled "From Auto-tuning One Size Fits All to Self-designed and Learned Data-intensive Systems (Tutorial)" [18] has been offered in SIGMOD 2019. Another very closely related tutorial offered in ICDE 2020 is titled "Machine Learning Meets Big Spatial Data" [35]. These tutorials address how ML approaches can be used in place of the various systems components. While these tutorials are complementary and are related, they are not directly focused on the recent hot topic of learned indexes that is the topic of this tutorial.

## 4 PRIOR TUTORIALS

As it stands, this tutorial has not been offered by the authors in any other venue, and SIGSPATIAL 2020 will be the first venue where this tutorial will be offered.

One of the authors, Walid G. Aref, has offered several tutorials on different yet related subjects in the past. These are listed below:

- (1) Ahmed R. Mahmood and Walid G. Aref, “Query Processing Techniques for Big Spatial-Keyword Data”, International Conference on Management of Data (SIGMOD): 1777–1782, 2017.
- (2) Mohamed F. Mokbel and, Walid G. Aref, “Location-aware Query Processing and Optimization”. In the IEEE International Conference on Mobile Data Management (MDM), Mannheim, Germany May 2007.
- (3) Mohamed F. Mokbel and Walid G. Aref, “Location-aware Query Processing”, In the International Conference on Extending Database Technology (EDBT), Munich, Germany, March 2006.
- (4) Ihab F. Ilyas and Walid G. Aref, “Rank-aware Query Processing Tutorial”, In the IEEE International Conference on Data Engineering, Japan, April 2005.
- (5) Ihab F. Ilyas and Walid G. Aref. Rank-aware Query Processing Tutorial, the 9th International Conference on Extending Database Technology (EDBT), Heraklion - Crete, Greece, Mar. 2004.

## BIOGRAPHIES

**Abdullah-Al-Mamun** is a Ph.D. student at the Department of Computer Science (CS), Purdue University. His research interest is in the area of Database Systems (DB) + Machine Learning (ML): “ML for DB” and “DB for ML”. Particularly, he is interested in the area of Learned Multi-dimensional and Spatial Indexes. Previously, he completed his M.Sc. in CS from Memorial University of Newfoundland where he was a Fellow of the School of Graduate Studies.

**Hao Wu** is a senior undergraduate student at Purdue University with majors in Data Science, Statistics-Math, Aviation Management. He is interested in ML-oriented research as well as its application in data-driven multi-disciplinary projects.

**Walid G. Aref** is a professor of computer science at Purdue. His research interests are in extending the functionality of database systems in support of emerging applications, e.g., spatial, spatio-temporal, graph, biological, and sensor databases. He is also interested in query processing, indexing, data streaming, and geographic information systems (GIS). Walid’s research has been supported by the National Science Foundation, the National Institute of Health, Purdue Research Foundation, CERIAS, Panasonic, and Microsoft Corp. In 2001, he received the CAREER Award from the National Science Foundation and in 2004, he received a Purdue University Faculty Scholar award. Walid is a member of Purdue’s CERIAS. He is the Editor-in-Chief of the ACM Transactions of Spatial Algorithms and Systems (ACM TSAS), an editorial board member of the Journal of Spatial Information Science (JOSIS), and has served as an editor of the VLDB Journal and the ACM Transactions of Database Systems (ACM TODS). Walid has won several best paper awards including the 2016 VLDB ten-year best paper award. He is a Fellow of the IEEE, and a member of the ACM. Between 2011 and 2014, Walid has served as the chair of the ACM Special Interest Group on Spatial Information (SIGSPATIAL).

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