# Optimization Review

November 12, 2015 9:54 AM

**Opening:**

This document is prepared as a quick review of the optimization methods we've covered this term. It is based on extremely simple problems generated using POMA technique, so please don't treat it as a substitute for the practice questions Dr. Summerville prepared for us. Make sure you still do those! :)

- Bolun 2015/11/13

**Linear Optimization:**

Suppose you are an IAA student and there are two exams (let's give them generic names, like ML and OP)coming up on Wednesday. Since you take your studies very seriously, you've decided to dedicate the weekend to prepare for the two courses. The good news is you are a brilliant student and is very good at approximating how long you'll need to prepare, but the bad news is you haven't had much exposure to the two classes before.

You sat down and pondered what would happen if you were to write the exams without any prep. Since you love ML and always paid attention, you believe you should be able to get an 80 in the course. OP on the other hand is completely foreign to you and you decided the best you can do without studying is probably a pass (50).

Fortunately you are good at estimating what you need to do. Since you already know so much about ML, you decided each hour of study will probably only give you 2 additional points on the ML exam, while each hour spent studying OP will net you 5 points on the exam.

Looking at your schedule you have just about 16 hours to study over the weekend, so it'll be hard to get 100 on both exams. You decided to optimize the study process and see what's the maximum combined grade you could possibly achieve!

*Sets*:

$C \in Course = \{ML, OP\}$

*Inputs*:

$Base_C = \{80,50\}$

$GradePerHr_C = \{2,5\}$

$TotalTime = \{16\}$

$CourseMax = \{100\}$

*Decision Var*:

$Let\ Hour_C\ be\ the\ number\ of\ hours\ you\ want\ to\ study\ this\ weekend\ on\ course\ C$

*Formulation: Simple*

$$Max\ 80 + 2 \times Hour_{ML} + 50 + 5 \times Hour_{OP}$$
$$S.t.$$
$$80 + 2 \times Hour_{ML} \leq 100$$
$$50 + 5 \times Hour_{OP} \leq 100$$
$$Hour_{ML} + Hour_{OP} \leq 16$$
$$Hour_{ML} \geq 0$$
$$Hour_{OP} \geq 0$$

*Formulation: Summation*

$$Max\ Grade = \sum_{C \in Course} Base_C + Hour_C \times GradePerHr_C$$
$$S.t.$$
$$Base_C + Hour_C \times GradePerHr_C \leq CourseMax\ \forall C \in Course$$
$$\sum_{C \in Course} Hour_C \leq TotalTime$$
$$Hour_C \geq 0\ \forall C \in Course$$

**Integer Optimization:**

Suppose you are an IAA student and there are two exams (let's give them generic names, like ML and OP)coming up on Wednesday. Since you take your studies very seriously, you've decided to dedicate the weekend to prepare for the two courses. The good news is you are a brilliant student and is very good at approximating how long you'll need to prepare, but the bad news is you haven't had much exposure to the two classes before.

You sat down and pondered what would happen if you were to write the exams without any prep. Since you love ML and always paid attention, you believe you should be able to get an 80 in the course. OP on the other hand is completely foreign to you and you decided the best you can do without studying is probably a pass (50).

You know this weekend you can find just 3 blocks of time to yourself (SatMorn, SatAfter, SunAfter), and each block of time devoted to a ML course will net you 10 point increase in grade, while each block dedicated to OP will net you 20 point increase in grade.

*Sets*:
$C \in Course = \{ML, OP\}$
$B \in Block = \{SatMorn, SatAfter, SunAfter\}$

*Inputs*:
$Base_C = \{80,50\}$
$GradePerBlock_C = \{10, 20\}$
$MaxGrade = \{100\}$

*Decision Var*:
*Let* $StudyBlock_{C,B}$ *be the study decisions, specifically*:
$$StudyBlock_{C,B} = \begin{cases} 1 & if\ we\ decide\ to\ study\ course\ C\ for\ Block\ B \\ 0 & otherwise \end{cases}$$

*Formulation: Simple*
$$Max\ [80 + 10 \times StudyBlock_{ML,SatMorn} + 10 \times StudyBlock_{ML,SatAfter} + 10 \times StudyBlock_{ML,SunAfter}] + [50 + 20 \times StudyBlock_{OP,SatMorn} + 20 \times StudyBlock_{OP,SatAfter} + 20 \times StudyBlock_{OP_{SunAfter}}]$$
$S.t.$
$$[80 + 10 \times StudyBlock_{ML,SatMorn} + 10 \times StudyBlock_{ML,SatAfter} + 10 \times StudyBlock_{ML,SunAfter}] \leq 100$$
$$[50 + 20 \times StudyBlock_{OP,SatMorn} + 20 \times StudyBlock_{OP,SatAfter} + 20 \times StudyBlock_{OP,SunAfter}] \leq 100$$
$$StudyBlock_{ML,SatMorn} + StudyBlock_{OP,SatMorn} \leq 1$$
$$StudyBlock_{ML,SatAfter} + StudyBlock_{OP,SatAfter} \leq 1$$
$$StudyBlock_{ML,SunAfter} + StudyBlock_{OP,SunAfter} \leq 1$$

*Formulation: Summation*
$$Max\ Grade = \sum_{C \in Course} Base_C + (\sum_{B \in Block} StudyBlock_{C,B} \times GradePerBlock_C)$$
$S.t.$
$$Base_C + \sum_{B \in Block} StudyBlock_{C,B} \times GradePerBlock_C \leq MaxGrade\ \forall C \in Course$$
$$\sum_{C \in Course} StudyBlock_{C,B} \leq 1 \quad \forall B \in Block$$

**Goal Programming:**

Suppose you are an IAA student and there are two exams (let's give them generic names, like ML and OP)coming up on Wednesday. Since you take your studies very seriously, you've decided to dedicate the weekend to prepare for the two courses. The good news is you are a brilliant student and is very good at approximating how long you'll need to prepare, but the bad news is you haven't had much exposure to the two classes before.

You sat down and pondered what would happen if you were to write the exams without any prep. Since you love ML and always paid attention, you believe you should be able to get an 80 in the course. OP on the other hand is completely foreign to you and you decided the best you can do without studying is probably a pass (50).

Fortunately you are good at estimating what you need to do. Since you already know so much about ML, you decided each hour of study will probably only give you 2 additional points on the ML exam, while each our spent studying OP will net you 5 points on the exam.

Looking at your schedule you have just about 16 hours to study over the weekend, so it'll be hard to get 100 on both exams. You decided to optimize the study process and see what's the maximum combined grade you could possibly achieve!

You estimated that you just need a grade of 85 in both to strike a happy balance between studying all weekend and having some time to yourself.

*Sets:*
$C \in Course = \{ML, OP\}$

*Inputs:*
$Base_C = \{80,50\}$
$GradePerHr_C = \{2,5\}$
$TotalTime = \{16\}$
$CourseMax = \{100\}$
$Target = \{85\}$

*Decision Var:*
$Let\ Hour_C\ be\ the\ number\ of\ hours\ you\ want\ to\ study\ this\ weekend\ on\ course\ C$
$Let\ Penalty_C\ be\ the\ penalty\ for\ over\ or\ under\ studying\ i.e.difference\ between\ actual\ and\ target\ grade\ for\ course$

*Formulation: Simple*
$Min\ Penalty_{ML} + Penalty_{OP}$
$S.t.$
$Penalty_{ML} \geq 80 + 2 \times Hour_{ML} - 85$
$Penalty_{ML} \geq 85 - (80 + 2 \times Hour_{ML})$
$Penalty_{OP} \geq 50 + 5 \times Hour_{OP} - 85$
$Penalty_{OP} \geq 85 - (50 + 5 \times Hour_{OP})$
$80 + 2 \times Hour_{ML} \leq 100$
$50 + 5 \times Hour_{OP} \leq 100$
$Hour_{ML} + Hour_{OP} \leq 16$
$Hour_{ML} \geq 0$
$Hour_{OP} \geq 0$

*Formulation: Summation*
$$Min\ Penalty = \sum_{C \in Course} Penalty_C$$
$S.t.$
$Penalty_C \geq Base_C + GradePerHr_C \times Hour_C - Target$
$Penalty_C \geq Target\ - (Base_C + GradePerHr_C \times Hour_C)$
$Base_C + Hour_C \times GradePerHr_C \leq CourseMax\ \forall C \in Course$
$$\sum_{C \in Course} Hour_C \leq TotalTime$$
$Hour_C \geq 0\ \forall C \in Course$

**Multiple Criteria Programming:**
Suppose you are an IAA student and there are two exams (let's give them generic names, like ML and OP)coming up on Wednesday. Since you take your studies very seriously, you've decided to dedicate the weekend to prepare for the two courses. The good news is you are a brilliant student and is very good at approximating how long you'll need to prepare, but the bad news is you haven't had much exposure to the two classes before.

You sat down and pondered what would happen if you were to write the exams without any prep. Since you love ML and always paid attention, you believe you should be able to get an 80 in the course. OP on the other hand is completely foreign to you and you decided the best you can do without studying is probably a pass (50).

Fortunately you are good at estimating what you need to do. Since you already know so much about ML, you decided each hour of study will probably only give you 2 additional points on the ML exam, while each our spent studying OP will net you 5 points on the exam.

Looking at your schedule you have just about 16 hours to study over the weekend, so it'll be hard to get 100 on both exams. You decided to optimize the study process and see what's the maximum combined grade you could possibly achieve!

You decided you want to both maximize grade AND minimize study time. You estimated that you want a minimum of 80 in both courses to be able to sleep at night.

*Sets*:
$C \in Course = \{ML, OP\}$

*Inputs*:
$Base_C = \{80,50\}$
$GradePerHr_C = \{2,5\}$
$TotalTime = \{16\}$
$CourseMax = \{100\}$
$CourseMin = \{80\}$

*Decision Var*:
*Let* $Hour_C$ *be the number of hours you want to study this weekend on course* $C$

*Formulation: Simple*
$Max\ 80 + 2 \times Hour_{ML} + 50 + 5 \times Hour_{OP}$
$Min\ Hour_{Ml} + Hour_{OP}$
$S.t.$
$80 + 2 \times Hour_{ML} \leq 100$
$50 + 5 \times Hour_{OP} \leq 100$
$80 + 2 \times Hour_{ML} \geq 80$
$50 + 5 \times Hour_{OP} \geq 80$
$Hour_{ML} + Hour_{OP} \leq 16$
$Hour_{ML} \geq 0$
$Hour_{OP} \geq 0$

*Formulation: Summation*

$$Max\ Grade: \sum_{C \in Course} Base_C + Hour_C \times GradePerHr_C$$

$$Min\ Studytime: \sum_{C \in Course} Hour_C$$

$S.t.$
$Base_C + Hour_C \times GradePerHr_C \leq CourseMax\ \forall C \in Course$
$Base_C + Hour_C \times GradePerHr_C \geq CourseMin\ \forall C \in Course$

$$\sum_{C \in Course} Hour_C \leq TotalTime$$

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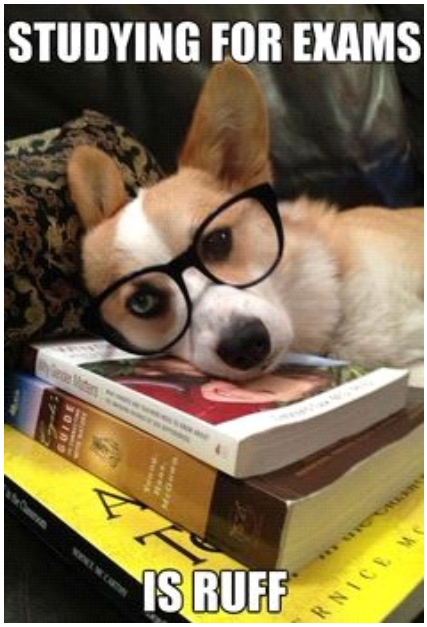
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