RBEND delta p analytic calculation

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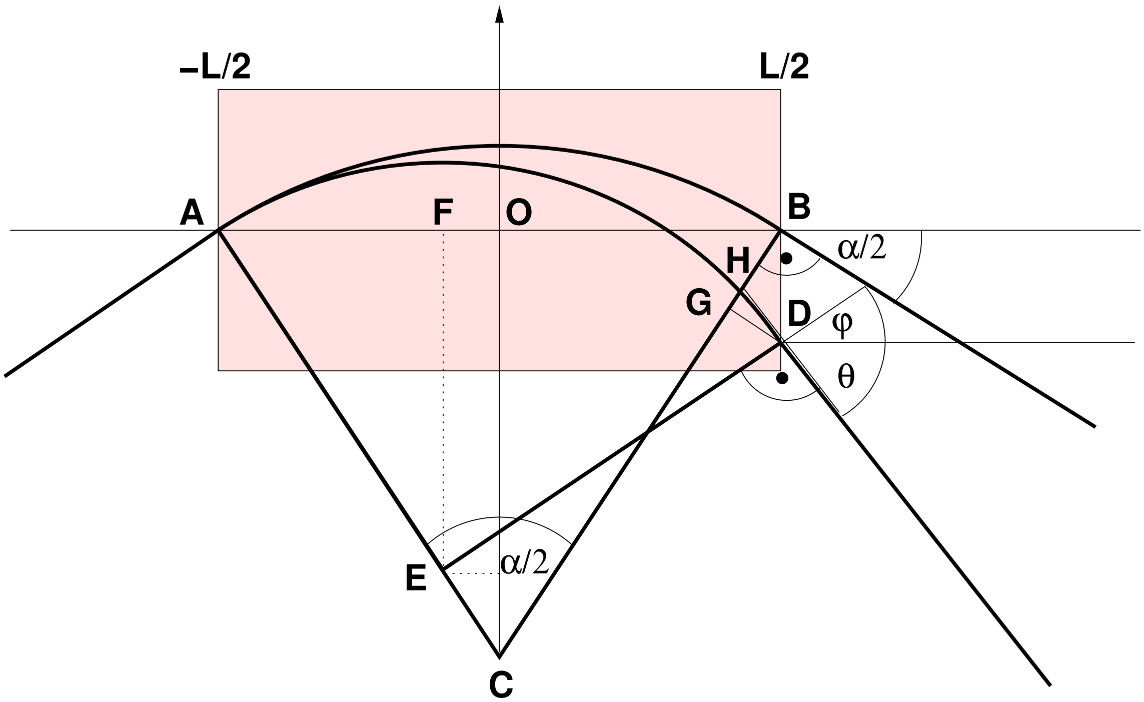

Figure 1: Rectangular bend of length L and bending angle α. On momentum reference particle (x=0 px=0) enters the magnet at A and leaves at B. Particle with delta p enters also at A and exits at D.

1 The goal is to find coordinates of point D and angle θ

$R = |AC| = \frac{L/2}{sin(\alpha/2)}$ - nominal particle radius
$r = |AE| = R(1+\delta)$ - off momentum particle radius. Center of this circle is $E(x_r, y_r)$ and is on the line passing through A and C.

First, I find coordinates of point $E(x_r, y_r)$. For this I consider the triangle AFE:

$$y_r = -|FE| = -(r \cdot cos(\alpha/2)) \tag{1}$$

$$x_r = -|FO| = -(|AO| - |AF|) = -((L/2) - r \cdot sin(\alpha/2)) \tag{2}$$

Point $D(x, y)$ is on crsossing of the trajectory (circle centered at E and radius r) $(x - x_r)^2 + (y - y_r)^2 = r^2$ and exit edge of the bend $x = L/2$. Solving this set of equations gives

$$y^2 - 2 \cdot y_r \cdot y + ({x_r}^2 + {y_r}^2 - r^2 + L^2/4 - L \cdot x_r) = 0 \tag{3}$$

Solutions are (the first one is the correct one):

$$\begin{aligned} y &= (-b + \sqrt{\Delta})/(2 \cdot a) \\ y2 &= (-b - \sqrt{\Delta})/(2 \cdot a) \end{aligned} \tag{4}$$

where a, b, Δ are respective coefficients of eq.3.

The exit position in the bend exit refernce frame is $|BH| = |GB| - |GH|$. First, considering triangle GDB I find

$$|GB| = y \cdot cos(\alpha/2) \tag{5}$$

$$|GD| = y \cdot sin(\alpha/2) \tag{6}$$

Finally, considering triangle GHD I find $|GH|$

$$|GH|/|GD| = tan(\theta - \alpha/2) => |GH| = |GD| \cdot tan(\theta - \alpha/2) \tag{7}$$

$$|GH| = y \cdot sin(\alpha/2) \cdot tan(\theta - \alpha/2) \tag{8}$$

Applying these equations to a 2 m long 45 degrees bend with deltap=-0.4 I find that PTC with nst=20 is correct within $10^{-6}m$. Increasing number of steps to 100 yields precision of $10^{-10}m$