MATH230 - Finite Mathematics with Applications

Last Name: **First Name:**   **DIS Sec #:** ____

1. (1+2=3 points) Give one example each of

   a) a *non-linear* equation in two variables. **Answer:** $x^2 + y = 2$

   b) a system of two linear equations representing *coinciding parallel* lines with slope $= -2$.
   Answer: $y = -2x + 1, \quad 3y = -6x + 3$

2. (3 points) Circle the option that represents all solutions *(if they exist)* of the linear system:
$$x - z = 2, \ y - 3z = 0$$
   a) DNE   b) $(2, 3, 1)$ c) $(2 - t, -3t, t)$ d) $(2 + t, 3t, t)$ ⟵

(Optional) show work for Question 2 for partial credits:

$$x = 2 + z, \quad y = -3z, \quad z = t$$

3. (2x5 = 10 points) True/False Questions. **Circle ONE option** each.

   (a) The inverse of a square matrix exists and is unique.   **True**   **False** ⟵

   (b) In a row-reduced matrix, all leading entries must be equal to 1.   **True** ⟵   **False**

   (c) If A and B are two independent events, then $p(B|A) = p(B)$.   **True**   **False** ⟵

   (d) If the feasible set of a system of linear inequalities is bounded, then the number of solutions of the system is finite.   **True**   **False** ⟵

   (e) A and B are independent if $p(A) = .2$, $p(B) = .3$, and $p(A \cup B) = .44$.   **True** ⟵   **False**

(Optional) show work for Item (e) for partial credits:

$p(A \cup B) = p(A) + p(B) - p(A \cap B) \qquad p(A)p(B) = .2(.3)$
$.44 = .2 + .3 - p(A \cap B) \Rightarrow p(A \cap B) = .5 - .44 \qquad = .06$
$= .06$

4. (2 points) Give an example of a *strict* linear inequality in three variables:

$$x - y + z < 0$$

5. (2 points) Given the matrix A, write a matrix B such that BA exists.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 5 & 4 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

2X3   3X1

1

6. (6 points) Solve for x, y:

$$\begin{bmatrix} -2 & -1 \\ 7 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ x \end{bmatrix}$$

Answers: $x = -5 \quad y = -1$

(Optional) show work for Question 6 for partial credits:

$$\Rightarrow \frac{1}{-6+7} \begin{bmatrix} 3 & 1 \\ -7 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ x \end{bmatrix} \Rightarrow \begin{bmatrix} 3+y \\ -7-2y \end{bmatrix} = \begin{bmatrix} 2 \\ x \end{bmatrix}$$

$$x = -7-2y \Leftarrow \quad 3+y = 2 \Leftarrow$$
$$= -7+2 = -5 \quad \Rightarrow y = -1$$

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7. In a survey to determine the opinions of Americans on health insurers, 200 baby boomers and 400 pre-boomers were asked this question: Do you believe that insurers are very responsible for high health costs? Of the baby boomers, 50 answered in the affirmative, whereas 200 of the pre-boomers answered in the affirmative.

(1 point) Introduce sets to describe various events related to this probability experiment.

Let,
  B = baby boomers
  P = pre-boomers
  A = respondents who answered in the affirmative to the survey question

(2 points) Draw a tree-diagram showing the probabilities.

Total no. surveyed
$= 200$
$+ 400$
$= 600$
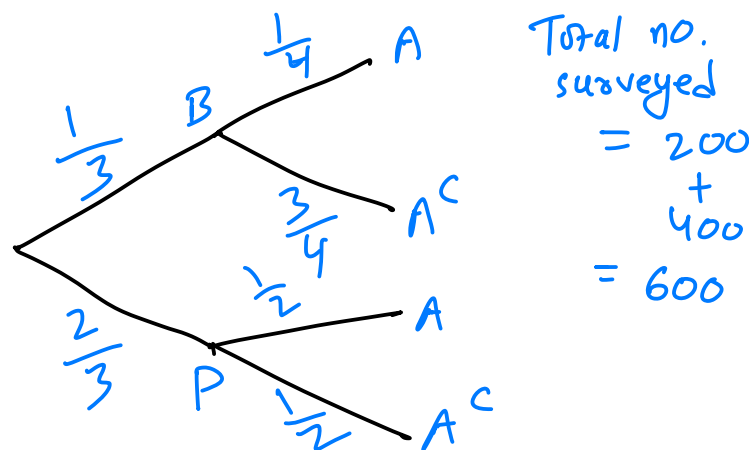
(3 points) If a respondent chosen at random from those surveyed answered the question in the affirmative, what is the probability that he or she is a baby boomer? **(Show all work)**

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B) \cdot P(A|B)}{P(B) \cdot P(A|B) + P(P) \cdot P(A|P)}$$

Therefore, the probability that a respondent who answered in the affirmative is a baby boomer is 1/5.

$$= \frac{\frac{1}{3} \cdot \frac{1}{4} \quad \times 3 \cdot 4 \cdot 2}{\frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{2} \quad \times 3 \cdot 4 \cdot 2}$$

$$= \frac{2}{2+8} = \frac{2}{10} = \frac{1}{5}$$

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8. (8 points) Find the inverse of matrix A using the **Gauss-Jordan** method. **Show all steps. No credit will be given for the final answer only without the work.**

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 6 \\ -4 & 0 & -9 \end{bmatrix}$$

Solution

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ 1 & 2 & 6 & 0 & 1 & 0 \\ -4 & 0 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 0 & 1 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 \\ -4 & 0 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 + 4R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 0 & 1 & 0 \\ 0 & -5 & -9 & 1 & -2 & 0 \\ 0 & 8 & 15 & 0 & 4 & 1 \end{array}\right]$$

$5R_1 + 2R_2$
$5R_3 + 8R_2$
$-R_2$
$$\left[\begin{array}{ccc|ccc} 5 & 0 & 12 & 2 & 1 & 0 \\ 0 & 5 & 9 & -1 & 2 & 0 \\ 0 & 0 & 3 & 8 & 4 & 5 \end{array}\right]$$

$R_1 - 4R_3$
$R_2 - 3R_3$
$$\left[\begin{array}{ccc|ccc} 5 & 0 & 0 & -30 & -15 & -20 \\ 0 & 5 & 0 & -25 & -10 & -15 \\ 0 & 0 & 3 & 8 & 4 & 5 \end{array}\right]$$

$\frac{1}{5}R_1$
$\frac{1}{5}R_2$
$\frac{1}{3}R_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & -3 & -4 \\ 0 & 1 & 0 & -5 & -2 & -3 \\ 0 & 0 & 1 & 8/3 & 4/3 & 5/3 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -6 & -3 & -4 \\ -5 & -2 & -3 \\ \frac{8}{3} & \frac{4}{3} & \frac{5}{3} \end{bmatrix}$$

or

$$\frac{1}{3}\begin{bmatrix} -18 & -9 & -12 \\ -15 & -6 & -9 \\ 8 & 4 & 5 \end{bmatrix}$$

9. **Consider the problem:** Joan and Dick spent 2 weeks (14 nights) touring four cities on the East Coast: Boston, New York, Philadelphia, and Washington. They paid $300, $200, $150, and $280 per night for lodging in each city, respectively, and their total hotel bill came to $3170. The number of days they spent in New York was the same as the total number of days they spent in Boston and Philadelphia, and the couple spent four times as many days in New York as they did in Boston. How many days did Joan and Dick stay in each city?

(a) (1 point) Introduce the variables in order to formulate the problem as a system of linear equations. Use complete sentence(s) in English language.

Let the no. of days spent by Joan and Dick in Boston, New York, Philadelphia, and Washington be x, y, z, and w respectively.

(b) (4 points) Formulate a system of linear equations for the problem. If necessary, rewrite the equations so that **they are in standard form**. (DO NOT solve)

$$x + y + z + w = 14$$
$$300x + 200y + 150w + 280z = 3170$$
$$y = x + z \implies x - y + z = 0$$
$$y = 4x \implies 4x - y = 0$$

Bonus Question (2 points): Prove that in any probability experiment, if two events are independent then so are the complements of the those events.

Suppose two events A and B are independent.

Then, $P(A \cap B) = P(A) P(B)$.

Now, $P(A^c) = 1 - P(A)$, $P(B^c) = 1 - P(B)$

$\therefore P(A^c)P(B^c) = (1 - P(A))(1 - P(B))$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - P(A \cup B) = P((A \cup B)^c)$

$\therefore$ De Morgan's Law $\implies = P(A^c \cap B^c)$

proves that $P(A^c \cap B^c) = P(A^c)P(B^c)$

i.e. A^c and B^c are independent