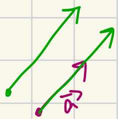
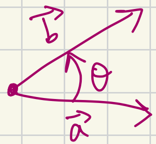
Recall: • $a = \langle a_1, a_2, a_3 \rangle$, $b = \langle b_1, b_2, b_3 \rangle \Rightarrow a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$.



• $a \cdot b = \|a\| \cdot \|b\| \cdot \cos\theta \Rightarrow$ a and b are orthogonal $\iff a \cdot b = 0$

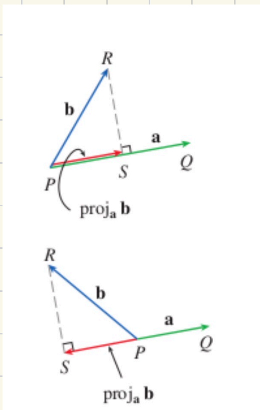


$\Rightarrow$ a and b are parallel $\iff$ a is a scalar multiple of b.

$\Updownarrow$

$\theta = 0$ or $\pi \iff a \cdot b = \pm \|a\| \cdot \|b\|$

•



Vector projection: $\text{proj}_a b = \underbrace{\text{comp}_a b}_{\text{scalar projection of } b \text{ on } a} \cdot$ unit vector along $\vec{a}$

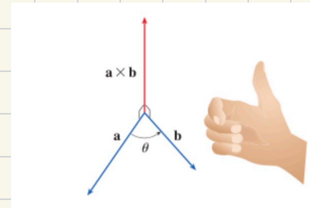
$$\boxed{\text{comp}_a b = \|b\| \cdot \cos\theta = \frac{a \cdot b}{\|a\|}}$$ ← signed length of the projection

$$\boxed{\Rightarrow \text{proj}_a b = \underbrace{\left(\frac{a \cdot b}{\|a\|} \right)}_{\text{signed length}} \cdot \left(\frac{\vec{a}}{\|a\|} \right)}$$ → unit vector along $\vec{a}$

• 2x2 determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

· Cross product: If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$ then the cross product of a and b is the vector

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



$$\boxed{\Rightarrow \quad a \times b = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - j \begin{vmatrix} a_1 & b_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}$$

Example: $a = \langle 1, 3, 4 \rangle$, $b = \langle 2, 7, -5 \rangle$

alternative notation $\hat{i}$

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 7 & -5 \end{vmatrix} i - j \begin{vmatrix} 1 & 4 \\ 2 & -5 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} k$$

$$= (3 \cdot (-5) - 4 \cdot 7)\, i - j\, (1 \cdot (-5) - 4 \cdot 2) + (1 \cdot 7 - 3 \cdot 2)\, k$$

$$= (-15 - 28)\, i - j\, (-5 - 8) + (7 - 6)\, k$$

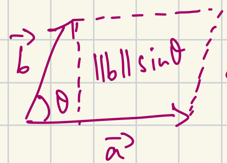
$$= -43 i + 13 j + k$$

DIY, also in discussion

Exercise: Show that $a \times b$ is orthogonal to a and b.

Theorem: $\|a \times b\| = \|a\| \cdot \|b\| \cdot \sin\theta$ where $0 \le \theta \le \pi$

Corollary: a and b are parallel $\Leftrightarrow$ $a \times b = 0$

Corollary: 

$\longleftarrow$ has area $A = \|a \times b\|$