Coordinates and Vectors

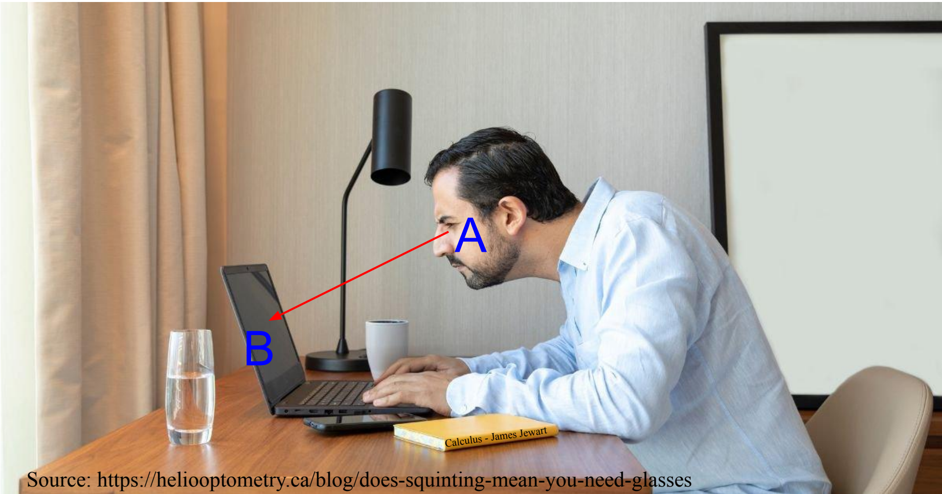
Pre-lecture video for 6/10

Coordinates

- In 3D, we use x, y, and z
- Points are of the form (x, y, z)
- 2D Formulas easily extend to 3D
  - Distance formula
  - Equation of sphere
  - Coordinate planes
- Warning: $y = mx+b$ is a plane, $x^2+y^2 = 1$ is a cylinder

Basics of Vectors

- Vectors have direction and magnitude
- A vector is defined by its start and end
- Represent as (a, b, c) or $\langle a, b, c\rangle$ or ai + bj + ck
- Start A, end B means B-A
- Denote magnitude of v by $||v||$



Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Vector Arithmetic

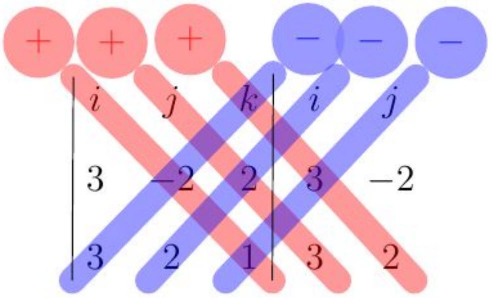
- Addition and subtraction are coordinate-wise
- Multiplication by scalar is what you expect
- Rules you expect to hold do hold
  - For example, $c(v+u) = c(u+v) = cu+cv$
- Can we do multiplication?
  - Defining $(a,b,c) * (d,e,f) = (ad, be, cf)$ won't pay off

Introducing Dots and Crosses

- Define dot product by $(x_1,y_1,z_1) \cdot (x_2,y_2,z_2) = x_1x_2+y_1y_2+z_1z_2$
- Define cross product as $(y_1z_2-y_2z_1,\ z_1x_2-z_2x_1, x_1y_2-x_2y_1)$
- Ways to remember cross
  - Cyclicity x->y->z->x
  - Rule of Sarrus

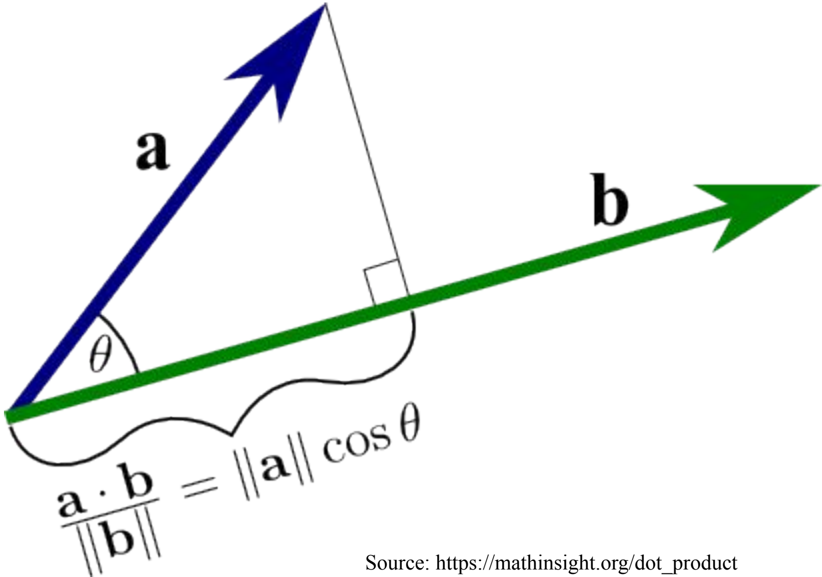
$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 3 & -2 & 2 & 3 & -2 \\ 3 & 2 & 1 & 3 & 2 \end{array}\right.$$

Source: https://tex.stackexchange.com/q/59567/



Dot Product Properties

- For non-zero u and v, $u \cdot v = 0$ if and only if u, v perpendicular
- Why? Axes perpendicular and "·" is invariant under rotation
- Let k*b be projection of a onto b
- Solve for k via dot product property
- Obtain formula for angle:
  - $\cos(\theta) = (a \cdot b)/(||a||\ ||b||)$



Source: https://mathinsight.org/dot_product

More Properties

- For non-zero u and v, **u** x **v** is perpendicular to u and v
- · and x are distributive in traditional ways
  - (au+bv)·w = a(u·w)+b(v·w)
- · is commutative, but x is anti-commutative
  - **u**·**v** = **v**·**u**, **u** x **v** = -(**v** x **u**)
- **v** x **v** = 0, so parallel vectors have cross-product zero
- ||**a** x **b**|| = ||**a**|| ||**b**|| sin(θ) where θ is angle between **a** and **b**

Be Careful

- You can’t cancel out · or x
  - Vector division doesn’t exist
- You can’t undo · or x
  - Products destroy information
- Cross is not commutative
- Cross is not associative

Review problems

Verify the perpendicularity property of the cross product by using the perpendicularity property of the dot product

Write down some phone number abc-def-ghij you know. Let u = (b, c, d), v = (e, f, g), w = (h, i, j). Find ||u||, u·v, u x (av x w), and u · (v x w). You may check your work using Wolfram-Alpha

Find an example to show cancelling · or x is impossible. Find an example to show x is not associative.