Discussion Quiz 5 Solutions

1: Parametrize all solutions for the system {w+x+y+z = 16, -2w+y+z = 8}. Then given the additional equation w+x = 2, eliminate one of the parameters and reparametrize.

Answers may vary depending how you choose the parameters. Here's one way:

Subtract the 2nd equation from 1st to get 3w+x = 8, which becomes x = 8-3w. Rearrange 2nd equation to y = 8+2w-z. Now notice that x and y are both in terms of w and z. Thus, we can let w = s and z = t be our parameters, making the parametrization (w,x,y,z) = (s, 8-3s, 8+2s-t, t).

The additional equation w+x = 2 becomes 8-2s = 2, so s = 3 and we have eliminated one of the parameters. Plugging this in, we get (w, x, y, z) = (3, -1, 14-t, t).

2: Find matrices A, B such AB, BA are defined and AB = 0 ≠ BA, or show why this task is impossible.

One example is A = [0 1] and B = [1 0]^T. Note that A is 1x2 and B is 2x1, so AB and BA are defined. Now let's check that AB = 0 and BA ≠ 0:

AB = [1*0 + 0*1] = [0], a 1x1 matrix.

The 2nd column of BA is 1 * [1 0]^T = [1 0]^T ≠ 0, so BA ≠ 0.

There are many others examples too, examples are practically unlimited.

3: Mark the following true or false

(a): If XY and YX are defined for matrices X and Y, then XY and YX are square.

True. Suppose X, Y have dimensions a x b, c x d respectively. Since XY is defined, we get b = c by matching the inner dimensions. Since YX is defined, we get d = a. The dimensions of XY, YX are a x d, c x b respectively. Using the 2 equations we found, these become a x a, b x b, which are square.

(b): If ABA is invertible, then A and B are both invertible

True. Let C = $(ABA)^{-1}$ so that I = (ABA)C = A(BAC). This means A^{-1} exists and A^{-1} = BAC. Then B(ACA) = (BAC)A = I, so B^{-1} exists and B^{-1} = ACA.

(c): Suppose B is invertible and A, B are known matrices. For the matrix equation AX = AB, the only solution for X is X = B.

False. Suppose B = I, A = 0. Then A, B are fixed and B is invertible, so the hypotheses are satisfied. Let's see if the conclusion is satisfied. The equation AX=AB reduces to 0*X = 0*I, which reduces to 0 = 0, which is true regardless of what X is.

Since every X works, there are solutions besides X = B (for example, X = 0) and the statement "the only solution for X is X = B" is false.