Coordinates and Vectors

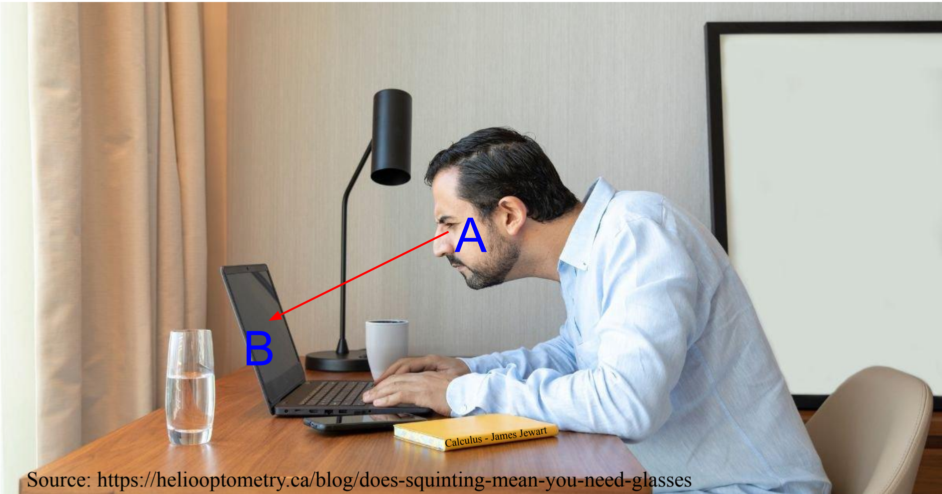
Pre-lecture video for 6/10

Coordinates

- In 3D, we use x, y, and z
- Points are of the form (x, y, z)
- 2D Formulas easily extend to 3D
  - Distance formula
  - Equation of sphere
  - Coordinate planes
- Warning: $y = mx+b$ is a plane, $x^2+y^2 = 1$ is a cylinder

Basics of Vectors

- Vectors have direction and magnitude
- A vector is defined by its start and end
- Represent as (a, b, c) or $\langle a, b, c \rangle$ or ai + bj + ck
- Start A, end B means B-A
- Denote magnitude of **v** by $||\mathbf{v}||$



Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Vector Arithmetic

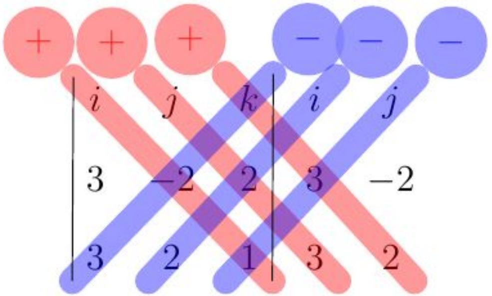
- Addition and subtraction are coordinate-wise
- Multiplication by scalar is what you expect
- Rules you expect to hold do hold
  - For example, $c(\mathbf{v}+\mathbf{u}) = c(\mathbf{u}+\mathbf{v}) = c\mathbf{u}+c\mathbf{v}$
- Can we do multiplication?
  - Defining $(a,b,c) * (d,e,f) = (ad, be, cf)$ won't pay off

Introducing Dots and Crosses

- Define dot product by $(x_1,y_1,z_1) \cdot (x_2,y_2,z_2) = x_1x_2+y_1y_2+z_1z_2$
- Define cross product as $(y_1z_2-y_2z_1,\ z_1x_2-z_2x_1, x_1y_2-x_2y_1)$
- Ways to remember cross
  - Cyclicity x->y->z->x
  - Rule of Sarrus

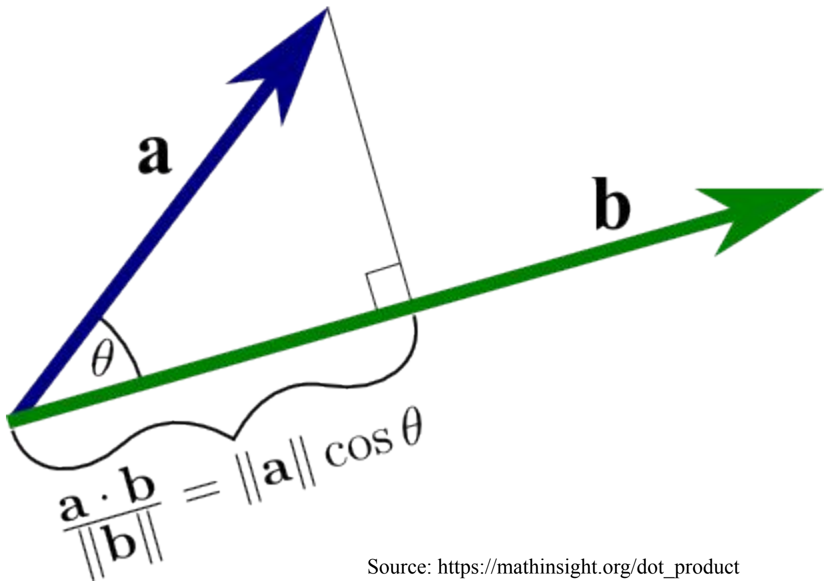
$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 3 & -2 & 2 & 3 & -2 \\ 3 & 2 & 1 & 3 & 2 \end{array}\right.$$

Source: https://tex.stackexchange.com/q/59567/



Dot Product Properties

- For non-zero **u** and **v**, **u** · **v** = 0 if and only if **u**, **v** perpendicular
- Why? Axes perpendicular and "·" is invariant under rotation
- Let k**b** be projection of **a** onto **b**
- Solve for k via dot product property
- Obtain formula for angle:
    - $\cos(\theta) = (\mathbf{a} \cdot \mathbf{b})/(\|\mathbf{a}\|\ \|\mathbf{b}\|)$



Source: https://mathinsight.org/dot_product

More Properties

- The cross product $\mathbf{u} \times \mathbf{v}$ is perpendicular to $\mathbf{u}$ and $\mathbf{v}$
- $\cdot$ and x are distributive in traditional ways
  - $(a\mathbf{u}+b\mathbf{v})\cdot\mathbf{w} = a(\mathbf{u}\cdot\mathbf{w})+b(\mathbf{v}\cdot\mathbf{w})$
- $\cdot$ is commutative, but x is anti-commutative
  - $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$, $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$
- $\mathbf{v} \times \mathbf{v} = \mathbf{0}$, so parallel vectors have cross-product zero
- $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta)$ where θ is angle between $\mathbf{a}$ and $\mathbf{b}$

Be Careful

- You can't cancel out · or x
  - Vector division doesn't exist
- You can't undo · or x
  - Products destroy information
- Cross is not commutative
- Cross is not associative

Review problems

Verify the perpendicularity property of the cross product by using the perpendicularity property of the dot product

Write down some phone number abc-def-ghij you know. Let $\mathbf{u} = \langle b, c, d \rangle$, $\mathbf{v} = \langle e, f, g \rangle$, $\mathbf{w} = \langle h, i, j \rangle$. Find $\|\mathbf{u}\|$, $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \times (a\mathbf{v} \times \mathbf{w})$, and $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$. You may check your work using Wolfram-Alpha

Find an example to show cancelling · or x is impossible. Find an example to show x is not associative.