Surface Integrals

Lecture for 7/8

Parameterizing Surfaces

1D curves needed 1 parameter, 2D surfaces will need 2 parameters

- Just as with curves, must include bounds on parameters

Surface Integral

Suppose S is parametrized by r(u, v)

- To calculate $\iint_S f \, dS$, we need bounds, convert dS, convert f
- If $(x_0, y_0, z_0) = r(u_0, v_0)$, then $f(x_0,y_0,z_0) = f(r(u_0, v_0))$
- Bounds will be the bounds on r, let's suppose $0 \leq u, v \leq 1$
- $dS = \|r_u \times r_v\| \, du \, dv$
- In the end, we get $\int_0^1 \int_0^1 f(r(u,v)) \|r_u \times r_v\| \, du \, dv$

Differential Conversion Derivation

Practice Problems

Evaluate $\iint_S f \, dS$ for the following functions and surfaces

- $f(x,y,z) = 6xy$, S is portion of $x+y+z = 1$ in 1st octant
- $f(x, y, z) = z$, S is upper half of sphere of radius 1
- $f(x,y,z) = y+z$, S is surface $x^2+y^2 \leq 3$ on the bottom, $z = 4-y$ on top, and the cylinder $x^2+y^2 = 3$ on the sides

Scratchwork