MATH 243: Worksheet 8 **Discussion Section:** _____

This worksheet covers Chap14.8 Lagrange Multiplier, Chap15.1 Double Integrals over Rectangles.

The Lagrange Multipliers

1. In order to maximize and minimize a function

$$f(x, y) \text{ subject to the given constraint } g(x, y) = k,$$

we can use the method of Lagrange multiplier.

   (a) Using the method of Lagrange multipliers, set up a system of equations for finding the minimum and the maximum for the function $f(x, y) = 9xe^y$ subject to the given constraint $x^2 + y^2 = 2$. Do NOT evaluate.

   (b) Using (a), find the extreme values of the function $f(x, y) = 9xe^y$ subject to the given constraint $x^2 + y^2 = 2$.

2. In order to maximize and minimize a function

$$f(x, y, z) \text{ subject to the given constraint } g(x, y, z) = k,$$

we can use the method of Lagrange multiplier.

   (a) Using the method of Lagrange multipliers, set up a system of equations for finding the minimum and the maximum for the function $f(x, y, z) = 8x + 8y + 3z$ subject to the given constraint $4x^2 + 4y^2 + 3z^2 = 35$. Do NOT evaluate.

   (b) Using (a), find the extreme values of the function $f(x, y, z) = 8x + 8y + 3z$ subject to the given constraint $4x^2 + 4y^2 + 3z^2 = 35$.

3. Find the maximum and minimum values of $f(x, y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$.

4. Find the absolute minimum and absolute maximum of $f(x, y) = 2x^2 - y^2 + 6y$ on the disk of radius 4, $x^2 + y^2 \leq 16$.

Double Integrals over Rectangles

5. Calculate the iterated integral $\displaystyle\int_{-3}^{3}\int_{0}^{\frac{\pi}{2}} (y + y^2 \cos x)\, dx\, dy$

6. Evaluate the double integral by first identifying it as the volume of a solid.

$$\iint_R (y + xy^{-2})\, dA, \quad R = \{\, (x, y) |\ 0 \leq x \leq 2,\ 1 \leq y \leq 2 \,\}$$

7. Evaluate $\displaystyle\iint_R x \cos^2(y)\ dA$, where $R = [-2, 3] \times [0, \pi/2]$.

8. Evaluate $\displaystyle\iint_R \frac{1}{(2x + 3y)^2}\ dA$, where $R = [0, 1] \times [1, 2]$.

9. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$.

10. Find the average value of $f(x, y) = x^2 y$ over the rectangle R with the vertices $(-1, 0)$, $(-1, 5)$, $(1, 5)$, $(1, 0)$.

Suggested Textbook Problems
Chapter 14.8: 3, 9, 11-17, 19-21, 31, 34, 36, 39, 42, 43
Chapter 15.1: 1a, 3, 9-43, 47