Tangencies and Curvature

Pre-lecture for 6/12

Tangent Vector

- Tangent to r(t) is r’(t)
- Unit tangent is $T(t) = r'(t)/\|r'(t)\|$

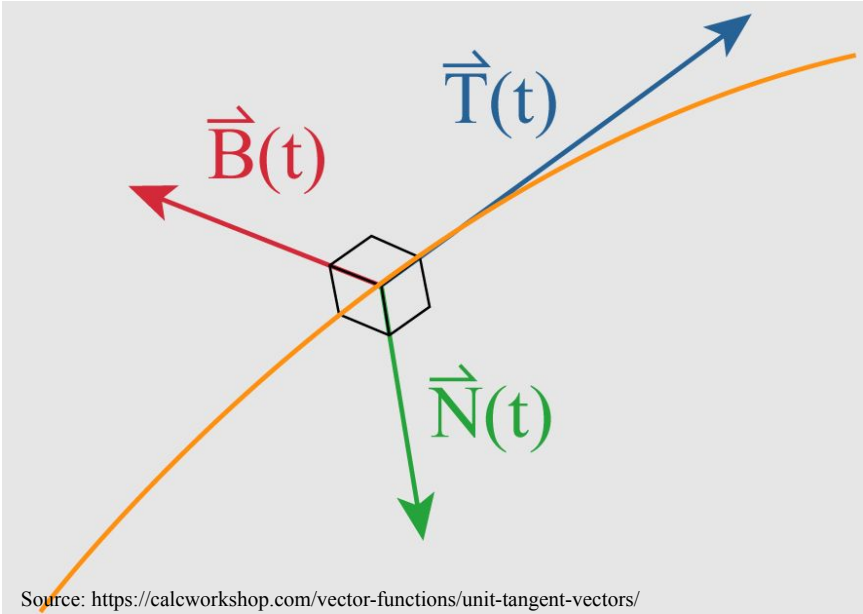
Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Normal Vector

- Define $N(t) = T'(t)/\|T'(t)\|$
- Fact: If u(t) is a unit vector, then u' and u are orthogonal
- Fact: N is orthogonal to T

Binormal Vector

- Define B(t) = T(t) x N(t)
- Now T, N, B are pairwise orthogonal



Source: https://calcworkshop.com/vector-functions/unit-tangent-vectors/

Curvature

- Measures how fast a curve is changing direction
- Defined by $\kappa = ||dT/ds||$ where s is arc length
- Where this comes from:

Reformulating κ for Calculations

To find κ, we need a convenient formula

- $\kappa = ||T'(t)||/||r'(t)||$
- $\kappa = ||r'(t) \times r''(t)||/||r'(t)||^3$

Scratch Work

Practice Problems

Let r(t) = (t, 3sin(t), 3cos(t)). Find the tangent, normal, and binormal vectors for r. Then determine the curvature of r.

Curvature of single-variable function

- Use one of the reformulations to show that the curvature of the graph of y = f(x) is $||f''(x)||/(1+f'(x)^2)^{3/2}$