MATH 243: Discussion Worksheet 3

Topics: 12.5 Equations of Lines and Planes; 13.1 Vector Functions and Space Curves
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1. Find the following equations of the line through the point $P(2,2,4)$ that is parallel to the vector $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 7\mathbf{k}$. For parts (a) and (b), use the parameter t.
   (a) Vector equation
   (b) Parametric equations
   (c) Symmetric equations

2. (a) Find symmetric equations of the line through the point $P(3,4,0)$ that is perpendicular to both vectors $2\mathbf{i} + 2\mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.
   (b) Find the point where the line from part (a) intersects the yz-plane.

3. Determine whether the lines L_1 and L_2 are parallel, intersecting, or skew.
   (a) $L_1 : \mathbf{r}(t) = \langle -1+3t,\ 2+4t,\ 3-2t \rangle$, $\qquad L_2 : \dfrac{x-1}{2} = \dfrac{y}{-3} = \dfrac{z+1}{-3}$
   (b) $L_1 : x = 2t,\ y = -3+t,\ z = 5-t$, $\qquad L_2 : x = 3-3s,\ y = 2 - \dfrac{3}{2}s,\ z = \dfrac{3}{2}s$

4. Find an equation of the plane through the points $A(0,1,2)$, $B(1,2,3)$, and $C(2,3,5)$.

5. Find an equation of the plane through the points $(0,-2,5)$ and $(-1,3,1)$ that is perpendicular to the plane $2z = 5x + 4y$.

6. Find parametric equations for the line of intersection of the planes
$$2x + 3y + 5z = 7 \quad \text{and} \quad x - y + 2z = 3.$$

7. Find an equation of the plane through $P(7,-2,-4)$ that is parallel to the plane $z = 4x - 5y$.

8. Find an equation of the plane that passes through the point $P(10,-1,5)$ and contains the line with symmetric equations $\dfrac{x}{4} = y + 6 = \dfrac{z}{5}$.

9. Determine whether the line L given by $\mathbf{r}(t) = \langle -2t, 2+7t, -1-4t \rangle$ intersects the plane given by $4x + 9y - 2z + 8 = 0$.

10. Find the domain of the vector function
$$\mathbf{r}(t) = \left\langle \ln(t+1), \frac{t}{4-t^2}, \sqrt{4-t} \right\rangle.$$

11. Evaluate the limit $\lim\limits_{t\to 1} \mathbf{r}(t)$, where
$$\mathbf{r}(t) = \left\langle \frac{t^2-1}{t^2-3t+2}, \frac{t-1}{\sqrt{t+3}-2}, \frac{\sin(t-1)}{t-1} \right\rangle.$$

SOME USEFUL DEFINITIONS, THEOREMS, AND NOTATION

Equations of a line. The line through the point (x_0, y_0, z_0) and parallel to the vector $\langle a, b, c\rangle$ has **vector equation**

$$\mathbf{r}(t) = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle.$$

The **parametric equations** for this line are

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct.$$

We can eliminate t to obtain the **symmetric equations**

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

provided a, b, and c are all nonzero.

Equations of a plane. The plane through (x_0, y_0, z_0) with normal vector $\langle a, b, c\rangle$ is given by

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

By collecting terms, we can rewrite this equation in the form

$$ax + by + cz = d$$

where $d = ax_0 + by_0 + cz_0$ is a constant. Two planes are parallel if their normal vectors are parallel, and they are perpendicular if their normal vectors are orthogonal.

Vector functions. A **vector-valued function** (or **vector function**) is a function whose domain is a set of real numbers and whose range is a set of vectors. In three dimensions, a vector-valued function is written as

$$\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}.$$

where f, g, and h are real-valued **component functions**. The domain of $\mathbf{r}(t)$ is the set of all values t for which $f(t)$, $g(t)$, and $h(t)$ are all defined. Limits of vector functions are defined component-wise: For a real number a,

$$\lim_{t\to a} \mathbf{r}(t) = \left\langle \lim_{t\to a} f(t),\ \lim_{t\to a} g(t),\ \lim_{t\to a} h(t) \right\rangle,$$

provided the three component limits exist.

Suggested Textbook Problems
Section 12.5: 1-72
Section 12.6: 1-49
Section 13.1: 1-58