University of Delaware
Department of Mathematical Sciences
MATH 243 Midterm Exam 2
Fall 2025

Monday 3rd November, 2025

Instructions:

- The time allowed for completing this exam is **90** minutes in total.
- Check your examination booklet before you start. There should be **8** questions on **9** pages.
- Turn off your cell phone and put it away. Headsets, and any other electronic devices are prohibited.
- No calculators.
- Answer the questions in the space provided. If you need more space for an answer, continue your answer on the back of the page and/or the margins of the test pages. No extra paper. *Do not separate the pages from the exam booklet.*
- For full credit, sufficient work must be shown to justify your answer.
- Partial credit will not be given if appropriate work is not shown.
- Write legibly and clearly; indicate your final answer to every problem. Cross out any work that you do not want graded. If you produce multiple solutions for a problem, indicate clearly which one you want graded.
- **Any form of academic misconduct will result in a failing grade.**

Question:	1	2	3	4	5	6	7	8	Total
Points:	9	18	8	10	24	10	10	11	100
Score:									

1. (9 points) Let $W = xy\sqrt{z}$,

$$\text{where } x = \ln(st), \quad y = \tan(s+2t) \quad \text{and} \quad z = s^2e^{2t}.$$

Find $\dfrac{\partial W}{\partial t}$. Your final answer **must be** in terms of only s and t. Do NOT simplify.

2. Consider the function

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5.$$

(a) (5 points) Find the critical point(s) of f that lie in the region $x^2 + y^2 < 16$.

(b) (8 points) Use the method of Lagrange multipliers to find the extreme values of the function subject to the constraint

$$g(x, y) = x^2 + y^2 = 16.$$

(c) (5 points) Using the results from part (a) and (b), determine the absolute maximum and absolute minimum of the function

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5 \text{ on the disk } D = \{(x, y) \mid x^2 + y^2 \leq 16\}.$$

3. (8 points) Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^2 + 2xy + y^2}{x^2 + y^2}$ does not exist. Justify your answer.

4. (10 points) Determine the **average value** of the function $f(x, y) = e^x + 2y$ over the rectangular region R,

$$R = \{(x, y) \,|\, 0 \leq x \leq 1,\ 0 \leq y \leq 3\}.$$

5. Let $f(x, y) = xe^y + x^2 \sin(3y)$.

   (a) (4 points) Find all of the first partial derivatives of f.

   (b) (4 points) Find the gradient of f at the point $(1, 0)$.

   (c) (4 points) Find the **rate of change** of f at the point $(1, 0)$ in the direction of the vector $3\mathbf{i} + 4\mathbf{j}$.

(d) (6 points) Find the **linearization** of $f(x, y) = xe^y + x^2 \sin(3y)$ at the point $(1, 0)$ and use it to **approximate** $f(0.9, 0.1)$.

(e) (6 points) Find **parametric equations of the normal line** to the surface

$$z = xe^y + x^2 \sin(3y)$$

at the point $(1, 0, 1)$.

6. (a) (5 points) Change the order of integration for the iterated integral

$$\int_0^{\ln(4)} \int_{e^x}^4 f(x,y)\,dydx.$$

(b) (5 points) Change the order of integration for the iterated integral

$$\int_0^2 \int_{y^3}^8 g(x,y)\,dxdy.$$

7. (10 points) Evaluate the double integral

$$\iint_D (2xy + 1)\, dA,$$

where the plane region D is bounded by the parabola $x = y^2$ and the vertical line $x = 1$.

8. (11 points) Use **polar coordinates** to **evaluate** the double integral

$$\iint_D (x^2 + y^2)^{\frac{3}{2}}\, dA,$$

where D is the plane region in the first quadrant bounded by the circle $x^2 + y^2 = 1$, the line $y = \dfrac{x}{\sqrt{3}}$ and the x-axis.