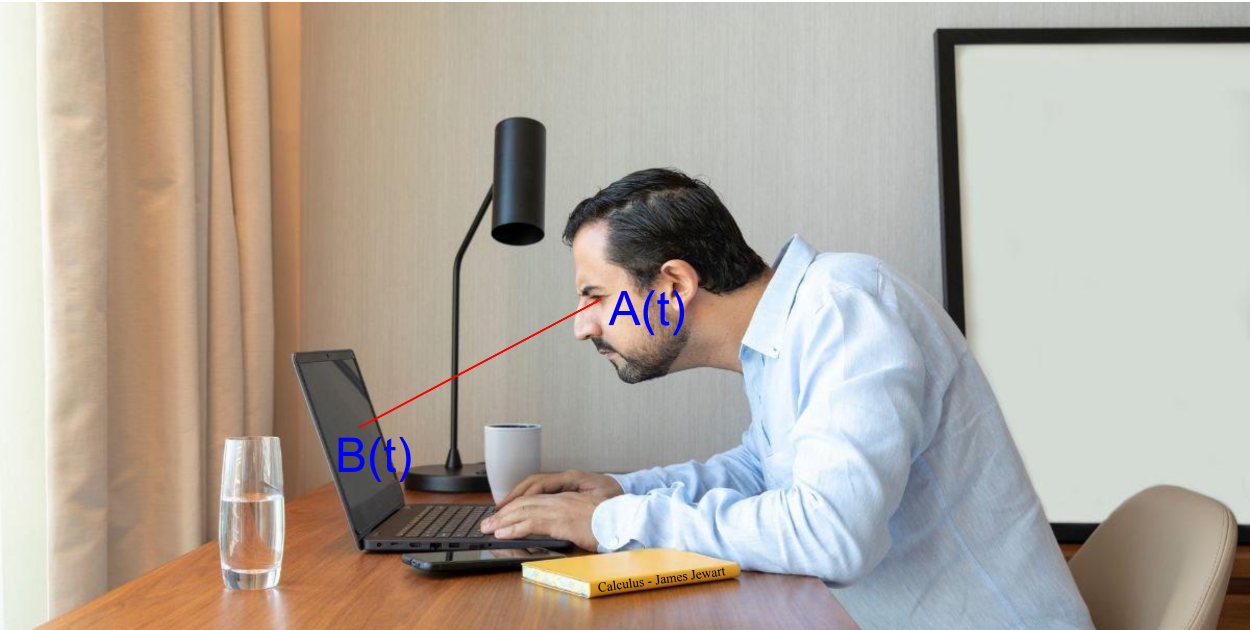
Vector Functions

Lecture for 6/11

Definition of Vector Functions

- Write r(t) = (f(t), g(t)) or (f(t), g(t), h(t))
- Same B-A trick to figure out r when given 2 vectors
- Can restrict domain



Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Limits, derivatives, integrals

- Limits are taken component-wise:
  - lim r(t) = (lim f(t), lim g(t), lim h(t))
- Vector function limit exists iff each component limit exists
- Derivatives and indefinite integrals also taken component-wise
- Constant of integration +C becomes vector +c = +(c_1,c_2,c_3)
- Definite integrals evaluated using antiderivatives as usual

Derivative Rules

- Let r, s be vectors, f scalar, c constant
- Basic properties still hold
  - Linearity: $(cr)' = cr'$, $(r+s)' = r'+s'$
- Product rules
  - $(fr)' = f'r + fr'$
  - $(r \cdot s)' = r' \cdot s + r \cdot s'$
  - $(r \times s)' = r' \times s + r \times s'$
- Chain rule: $[r(f(t))]' = f'(t)r'(f(t))$

Arc Length

- Can't reduce to components easily
- Call ds a tiny bit of the arc
- Line segment for ds is $r(t)$ to $r(t+dt)$
- Use this to get $ds = \|r'(t)\|\, dt$
- $L = \int ds = \int \sqrt{((f')^2+(g')^2+(h')^2)}\, dt$
- Now you have a basic integral

Practice problems

Understand your segments

- Find a vector equation for the line segment between (a, b, c) and (d, e, f)

Mixing vector products and derivatives

- Let $r(t) = (\cos(t), \sin(t), 0)$ and $s(t) = (\sin(t), -\cos(t), 1)$. Compute $(r \times s)'$, $(r \cdot s)'$ with and without the product rule

Scratch Work

Extra Problem

Arc length of helix

- Let $r(t) = (\cos(t), \sin(t), t)$, $0 \leq t \leq 2\pi$ represent one curl of a helix. Find the arc length of this curve