MATH 243 Worksheet 2: Lines, Planes, Vector Functions

0: Problems from lecture slides:
a. Write a parametric equation for the line through $(2,-1,3)$ and $(1,4,-3)$
b. Find the equation of the plane containing $(1,-2,0),(3,1,4),(0,-1,2)$
c. Determine if $-x+2z=10$ and $(5,2-t,10+4t)$ are perpendicular, parallel, or neither
1: Compute the projections of
a. The vector $\langle 2,4,-1\rangle$ onto the vector $\langle 3,-3,1\rangle$
b. The vector $\langle 3,-3,1\rangle$ onto the vector $\langle 2,4,-1\rangle$.
2: Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS. The points are given by $P(3,0,1)$, $Q(-1,2,5)$, $R(5,1,-1)$, and $S(0,4,2)$.
3: Determine whether or not the following four points lie on the same plane (i.e., are co-planar): $A(0,-3,0)$, $B(0,3,0)$, $C(1,-3,4)$, and $D(-1,3,4)$.
4: What is the angle between the vectors $x\vec{i}-\vec{j}+\vec{k}$ and $x\vec{i}+2\vec{j}+3\vec{k}$? Justify your answer.

A. 0 degrees

B. less than 90 degrees.

C. greater than 90 degrees

D. It can be any of the above depending on the value of x.

5: Find the parametric equations for the line of intersection of the planes $2x+3y+5z=7$ and $x-y+2z=3$.
6: Find an equation for the plane through the points $A=(0,1,2)$, $B=(1,2,3)$, and $C=(2,3,5)$.
7: Find an equation for the plane that passes through the points $(0,-2,5)$ and $(-1,3,1)$ and is perpendicular to the plane $2z=5x+4y$.
8: Evaluate the limits

$$\lim_{t\to 1}\mathbf{r}(t)=\lim_{t\to 1}\left\langle \frac{t^2-1}{t^2-3t+2},\frac{t-1}{\sqrt{t+3}-2},\frac{\sin(t-1)}{t-1}\right\rangle$$

$$\lim_{t\to\infty}\mathbf{r}(t)=\lim_{t\to\infty}\left\langle \frac{1+t^2}{1-t^2},\arctan(t),\frac{1-e^{-2t}}{t}\right\rangle$$

9: Find a vector function that represents the curve of intersection of the surfaces $x^2+y^2=1$ and $z=y+2$.
10: Evaluate the derivative of the vector function $\mathbf{r}(t)=\left\langle e^{t^2+2t},\ln(\cos(t)),t\arctan(t)\right\rangle$.
11: Evaluate the integral

$$\int_0^1\left(\frac{1}{t+1}\mathbf{i}+\frac{1}{t^2+1}\mathbf{j}+\frac{t}{t^2+1}\mathbf{k}\right)dt$$

12: Given the vector functions $\mathbf{u}(t)=\langle t+1,2\sin(t),-3\ln(t)\rangle$ and $\mathbf{v}(t)=\langle 0,-t^3,e^{3t}\rangle$, find the derivatives of the following functions:
a. $\mathbf{u}-2\mathbf{v}$,
b. $\mathbf{u}\times\mathbf{v}$,
c. $\mathbf{u}\cdot\mathbf{v}$,
d. $\mathbf{u}(t^2+2)$