

Cylindrical and Spherical Integrals

Lecture for 6/30

Converting Integrals

Cylindrical: $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$

- Bounds: $0 \leq \theta < 2\pi$

Spherical: $(x, y, z) = \rho(\cos(\theta) \sin(\varphi), \sin(\theta)\sin(\varphi), \cos(\varphi))$

- Bounds: $0 \leq \varphi \leq \pi, 0 \leq \theta < 2\pi$

We can convert functions, but what about $dA = dx \, dy \, dz$?

- Cylindrical: $dA = r \, dr \, d\theta \, dz$
- Spherical: $dA = \rho^2 \sin(\varphi) \, d\rho \, d\varphi \, d\theta$

Differential Derivation

Practice Problems

Evaluate $\iiint_E f \, dV$ for the following functions and regions:

- $f = z$, E is region inside $y^2 + z^2 = 1$ and between $x + y + z = 2$, $x = 0$
- $f = x^2 + y^2$, E is portion of $x^2 + y^2 + z^2 = 4$ with $y \geq 0$
- $f = x^2$, E is region inside $x^2 + y^2 + z^2 = 36$ and $z = -(3x^2 + 3y^2)^{1/2}$

Scratchwork