DW13 3a: Find f with $F = \nabla f$, $F = \langle (1+xy)e^{xy}, x^2 e^{xy} \rangle$

$F = \nabla f = \langle f_x, f_y \rangle$, so $f = \int f_y \, dy = \int F_2 \, dy$

$= \int x^2 e^{xy} \, dy = x e^{xy} + g(x)$.

Now we solve for g. Note $(1+xy)e^{xy} = f_x =$

$(1+xy)e^{xy} + g'(x) \Rightarrow g'(x) = 0 \Rightarrow g = c$.

Now note the problem says find a function f, not find all functions f. So we can set $c = 0$ and $f(x,y) = x e^{xy}$ works.

b: From part a, find $\int F \cdot dr$, $r(t) = \langle \cos t, 2\sin t \rangle$, $0 \le t \le \frac{\pi}{2}$

By FTOLC since $F = \nabla f$, $\int F \cdot dr = \int \nabla f \cdot dr$

$= f(r(\text{end})) - f(r(\text{start})) = f(0,2) - f(1,0) =$

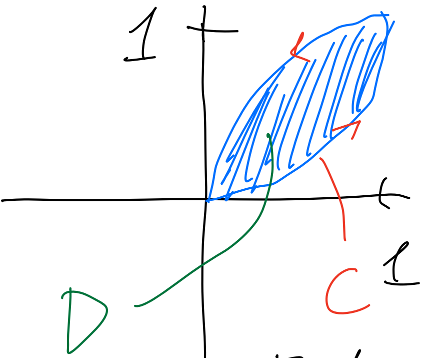
$0e^0 - 1e^0 = 0 - 1 = -1$

end: $t = \frac{\pi}{2} \Rightarrow r(\frac{\pi}{2}) = \langle 0, 2 \rangle$

start: $t = 0 \Rightarrow r(0) = \langle 1, 0 \rangle$

Recall: $f = x e^{xy}$

Q5: Find $\int_C (y + e^{x^{5/3}}) \, dx + (2x + \cos(y^2)) \, dy$ with Green where C is positively oriented curve formed by the boundary of the region enclosed by $y=x^2$ & $x=y^2$.



Green's Theorem converts line integrals into double integrals over the whole region, so 1st graph the region.

Let D be the enclosed region so that $C=\partial D$. Bounds for D are $0\le x\le 1$, and for y, y is bounded below by $y=x^2$ & above by $x=y^2 \Rightarrow y=\sqrt{x}$. So $x^2\le y\le \sqrt{x}$.

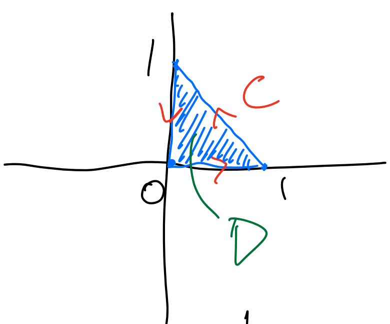
Let $P = y+e^{x^{5/3}}$, $Q = 2x+\cos(y^2)$, then

$$\text{int} = \int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA = \iint_D (2-1)\,dA$$

$$= \iint_D dA = \int_0^1\int_{x^2}^{\sqrt{x}} dy\,dx = \int_0^1 (\sqrt{x}-x^2)\,dx =$$

$$\left(\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{2}{3}-\frac{1}{3} = \frac{1}{3}$$

Q6: Use Green to find work $F=\langle x(x+y), xy^2\rangle$ does on particle moving $(0,0)\to(1,0)\to(0,1)\to(0,0)$ in straight lines.



Let C be the curve, D be the region, then $W=\int_C F\cdot dr$. Let $P=x(x+y)$, $Q=xy^2$. Note that $r=(x,y) \Rightarrow dr=(dx,dy) \Rightarrow F\cdot dr = P\,dx+Q\,dy$, so

$W = \int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA =$

$\iint_D (y^2 - x)\,dA$. Now we need to find bounds for D. Note $0 \le x \le 1$ and $0 \le y \le 1-x$ because the diagonal line in the graph is the line $y = 1-x$

$$W = \int_0^1 \int_0^{1-x} (y^2 - x)\,dy\,dx = \int_0^1 \left[\frac{(1-x)^3}{3} - x(1-x)\right] dx$$

$$= \int_0^1 \left(-\frac{(x-1)^3}{3} - x + x^2\right) dx = -\frac{(x-1)^4}{12} - \frac{x^2}{2} + \frac{x^3}{3}\Big|_0^1$$

$$= -\frac{1}{2} + \frac{1}{3} - \left(-\frac{1}{12}\right) = -\frac{1}{6} + \frac{1}{12} = -\frac{1}{12}.$$

$$\int (1+xy)e^{xy} = \int e^{xy}\,dx + \int xye^{xy}\,dx$$

$$= \frac{1}{y}e^{xy} + y\underbrace{\int xe^{xy}\,dx}_{\text{IBP, take more time but possible}}$$

Note: The functions P, Q chosen for problems in math 243 about Green's Theorem are rigged so that $Q_x - P_y$ is simple so that $\iint_D (Q_x - P_y)\, dA$ is easy to integrate. Often $Q_x - P_y$ is just a constant. If you get a complicated double integral that seems impossible to evaluate, check that you didn't make a mistake by swapping P & Q, or swapping x & y partial derivatives.