1: Probability of $X = 0, 1, 2, 3$ is 1/64, 9/64, 27/64, 27/64 respectively. You can use this to draw a histogram with those probabilities as bar heights.

Mean is $1/64*0 + 9/64 * 1 + 27/64 * 2 + 27/64 * 3 = 2.25$. You can also calculate it as $E[X] = ¾ * 3 = 9/4$ using linearity of expectation. See solution to #3 for another example of linearity.

Median is half-way through and ½ = 32/64, which lands on $X = 2$, so median is 2. Mode is both 2 and 3 since they are tied for the highest probability.

2: $P(X = 0)$ is negative and you can't have a negative probability. The probabilities add up to 0.9 instead of adding up to 1.

3: For an event X, let I_X be a random variable defined as 1, 0 if X happens, doesn't happen respectively. Let A, B, C be the event the 1st, 2nd, 3rd battery picked respectively is defective. The number of defective batteries is $I_A+I_B+I_C$ since every defect contributes exactly 1 to this sum and every non-defect contributes 0.

We seek $E[I_A+I_B+I_C] = E[I_A] + E[I_B] + E[I_C] = P(A)+P(B)+P(C) = 3P(A)$, where the last equality is by similarity. But $P(A) = 2/10$ since 2 out of all 10 are defective and the 1st battery is picked randomly, so the answer is $3*2/10 = 0.6$.

4: Let $I_k = 1, 0$ if the kth card does, doesn't remain in the same order respectively. Just like problem 3, we seek $E[I_1]+E[I_2]+... = 52*P(X)$ where X is the probability the 1st card remains in the same order. The 1st card is equally likely to go to any of the 52 spots, so $P(X) = 1/52$ and we get 1 as the expected number.

5: The company pockets $260. If she lives, they do nothing. If she dies, which happens with probability 1-0.99 = 0.01, they lose $20000. So the expected gain is 260-20000*0.01 = $60.

6a: If the cycle has k bets, he loses 1, 2, 4, …, 2^{k-1} on the 1st, 2nd, …, (k-1)st bet respectively, then wins 2^k on the kth bet, so his profit is $2^k - (1+2+...+2^{k-1}) = 2^k - (2^k - 1) = \$1$

6b: On the kth bet, he bets 2^k and gets back $2 * 2^k$ if the ball lands on black, which happens with probability 18/37. So his expectation is $-2^k + 2 * 2^k * 18/37 = - 2^k/37 < 0$

6c: It seems Martin profits every completed cycle despite every bet in the cycle having negative expected value. The catch is he has to complete a cycle. With a finite amount of money, eventually he will get unlucky and lose so many bets in a row during a cycle that he does not have enough money to place the next bet.

7-8: Email me for a solution. If you don't want to do these bonus problems, don't worry.