MATH 243 Worksheet 8: Stokes' and Divergence Theorems

1: Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the following functions and surfaces
a. $\mathbf{F} = (3x, 2y, 1 - y^2)$, S is the portion of $z = 2 - 3y + x^2$ oriented downward and lying over triangle with vertices $(0,0), (2,0), (2,-4)$
b. $\mathbf{F} = (yz, x, 3y^2)$, S is the surface of solid bounded by $x^2 + y^2 = 4, z = x - 3, z = x + 2$ with negative orientation
c. $\mathbf{F} = \nabla \times G, G = (z^2 - 1, z + xy^3, 6)$, S is the portion of $x = 6 - 4y^2 - 4z^2$ in front of $x = -2$ with orientation in the negative x-direction
d. For extra computation practice, do part c without using Stokes' Theorem

2: Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the following functions and curves
a. $\mathbf{F} = (-yz, 4y + 1, xy)$, C is the circle of radius 3 centered at $(0,4,0)$, perpendicular to y-axis, and oriented CW when looking above $y > 4$
b. $\mathbf{F} = (3yx^2 + z^3, y^2, 4yx^2)$, C is the triangle with vertices $(0,0,3), (0,2,0), (4,0,0)$ oriented CCW when looking above C towards origin

3: Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for these with and without using divergence theorem
a. $\mathbf{F} = (\sin(\pi x), zy^3, z^2 + 4x)$, S is the surface of box with $-1 < x < 2, 0 < y < 1, 1 < z < 4$ oriented pointing out of the box
b. $\mathbf{F} = (2xz, 1 - 4xy^2, 2z - z^2)$, S is the surface of the solid bounded by $z = 6 - 2x^2 - 2y^2$ and $z = 0$, oriented pointing inside the solid

4: Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and the oriented surface S. In other words, find the flux of $\mathbf{F}$ across S, for the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + 2z\mathbf{k}$, and S is the hemisphere $x^2 + y^2 + z^2 = 4, z \geq 0$, oriented downward.

5: Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = \langle x^2, -y, z \rangle$ where E is the solid cylinder $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$.

6: Use the Divergence Theorem to calculate $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ for the following $\mathbf{F}$ and S:

A. $\mathbf{F}(x, y, z) = \langle xye^z, xy^2z^3, -ye^z \rangle$ and S is the surface of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$

B. $\mathbf{F}(x, y, z) = \langle xe^y, z - e^y, -xy \rangle$ and S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 4$

C.
$$\mathbf{F}(x, y, z) = \langle xy + 2xz, x^2 + y^2, xy - z^2 \rangle$$
and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = y - 2$ and $z = 0$

D. $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$ across the surface S of the solid region E which is the upper half of the ball of radius 1 given by the equations $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$