Partial Derivatives

Lecture for 6/16

Idea Behind Partial Derivatives

- Hold all but 1 variable constant to get 1 variable function
  - Example: $g(y) = f(2, y, 4)$
- We can now take usual derivative
- Chose $(x, z) = (2, 4)$, can keep doing this for more choices
- Can we get a formula in terms of our choice?

Definitions and Notations

- Define $f_x(x, y) = \lim_{h \to 0} [f(x+h, y) - f(x, y)]/h$
- Similarly, $f_y(x, y) = \lim_{h \to 0} [f(x, y+h) - f(x, y)]/h$
- If limits don't exist, partial derivatives don't exist
- Similarly if f has more variables

There are 3 notations:

- $(\partial/\partial x)\ f$, $\partial f/\partial x$, f_x

Properties of Partials

Same properties from Calculus 1 apply

- Sums: $(f+g)_x = f_x+g_x$
- Products: $(fg)_x = f_x g + fg_x$
- Quotient: $(f/g)_x = (f_x g - fg_x)/g^2$
- Chain: $(f \circ g)_x = (f' \circ g)g_x$ if $f : R \rightarrow R$

Variables besides the one differentiated act like constants:

- $(f(x, y)g(y))_x = f_x(x,y)g(y)$

Notions of Increasing and Decreasing

Consider the graph $z = f(x, y)$

- What happens to z if x fixed and y varies, or vice versa?
- Answer: consider f_x and f_y
- z is inc, dec. in x direction if $f_x \geq 0$, $f_x \leq 0$ respectively
- z is inc, dec in y direction if $f_y \geq 0$, $f_y \leq 0$

- Same principle applies to higher dimensions
- Will later see how to check inc/dec around the whole point

Back to Tangent Plane

We left off not knowing f_x, f_y for tangent plane video

- Rewatch any parts of the video not understood
- Try the practice problem with $z = x^2 + y^2$

Formula for plane tangent at $(x,y) = (x_0,y_0)$ from video:

- $f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0) = z-f(x_0,y_0)$

Practice Problems

Find all of the 1st order partial derivatives

- $x^y+y^x+e^{xy}$
- $x^4\sin(3y) - x/y + \cos(x/y)$
- $xyz/(x+y+z)$

Let c be a constant and $g(x) = f(x, c)$. Show that $f_x(x, c) = g'(x)$

Note: This vindicates our idea that taking the partial derivative produces the same value as plugging in constants and taking an ordinary derivative

Scratchwork

Scratchwork