Textbook Sections: 16.6 and 16.7
Topics: Parametric Surfaces and Their Areas, Surface Integrals
Instructions: Try each of the following problems, show the detail of your work.
Clearly mark your choices in multiple choice items. Justify your answers.
Cellphones, graphing calculators, computers and any other electronic devices are not to be used during the solving of these problems. Discussions and questions are strongly encouraged.

Parametric Surfaces and Their Areas:

1. Find parametric representations for the following surfaces:
   (a) The part of the hyperboloid $4x^2 - 4y^2 - z^2 = 4$ that lies in front of the yz-plane.
   (b) The part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$.
2. Find an equation of the tangent plane to the surface S given by the parametric equations
   $x = u^2 + 1,\ y = v^3 + 1,\ z = u + v$ at the point $P(5, 2, 3)$.
3. Find the area of the surface S that is the part of the paraboloid $y = x^2 + z^2$ that lies within the cylinder $x^2 + z^2 = 16$.

Surface Integrals

4. Evaluate the following surface integrals.
   (a) $\displaystyle\iint_S x^2yz\,dS$, where S is the part of the plane $z = 1+2x+3y$ that lies above the rectangle $[0, 3]\times[0, 2]$.
   (b) $\displaystyle\iint_S y^2z^2\,dS$,where S is the part of the cone $y = \sqrt{x^2 + z^2}$ given by $0 \le y \le 5$.
5. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and the oriented surface S. In other words, find the flux of $\mathbf{F}$ across S, for the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + 2z\mathbf{k}$, and S is the hemisphere $x^2 + y^2 + z^2 = 4, z \ge 0$, oriented downward.
6. Evaluate the surface integral $\displaystyle\iint_S z\,dS$, where S is the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

Suggested Textbook Problems

Section 16.6	1-5, 13-26, 33-36, 39-49, 62
Section 16.7	5-27

SOME USEFUL DEFINITIONS, THEOREMS AND NOTATION:

Definition 6 If a smooth parametric surface **S** is given by the equation

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$$

and S is covered just once as (u,v) ranges throughout the parameter domain D, then the surface area of S is

$$A(S) = \iint_D ||\mathbf{r}_u \times \mathbf{r}_v|| dA$$

where

$$\mathbf{r}_u = \frac{\partial x}{\partial u}\mathbf{i} + \frac{\partial y}{\partial u}\mathbf{j} + \frac{\partial z}{\partial u}\mathbf{k} \qquad \mathbf{r}_v = \frac{\partial x}{\partial v}\mathbf{i} + \frac{\partial y}{\partial v}\mathbf{j} + \frac{\partial z}{\partial v}\mathbf{k}$$

Formula 9

For the special case of a surface S with equation $z = f(x,y)$, where (x,y) lies in D and f has continuous partial derivatives, we take x and y as parameters. The surface area formula in Definition 6 becomes

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

Formula 2

The surface integral of f over the surface S is

$$\iint_S f(x,y,z)\ dS = \iint_D f(\mathbf{r}(u,v))||\mathbf{r}_u \times \mathbf{r}_v||\ dA$$

Formula 4

Any surface S with equation $z = g(x,y)$ can be regarded as a parametric surface with parametric equations $x = x, y = y, z = g(x,y)$, then

$$\iint_S f(x,y,z)\ dS = \iint_D f(x,y,g(x,y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\ dA$$

Definition 8: If **F** is a continuous vector field defined on an oriented surface S with unit normal vector **n**, then the surface integral of F over S is called the flux of **F** across S, and is given by

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\ dS$$

Formula 9: If S is given by a vector function $\mathbf{r}(u,v)$, then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\ dA$$

Formula 10: If S is given by the graph of the function $z = g(x,y)$, and S has an upward orientation, then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D (-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R)\ dA$$