1: You can check your calculation with https://matrix.reshish.com/multiplication.php

2: (a) False, the 0 matrix has the property $A*0 = 0*A = 0$

(b) True. AA is defined and has dimensions n x n since A has dimensions n x n. Then (AA)A and A(AA) are both defined, so AAA is well-defined and AAA = (AA)A = A(AA) by associativity.

(c) False. If A has dimensions m x n, then A^T has dimensions n x m by definition of the transpose. We have a (m x n) x (n x m) multiplication for AA^T. Since the inner numbers match, AA^T is defined with dimensions m x m. This is true for any A, so "exists … not defined" is false.

3: Suppose B has dimensions m x n. For ABA = (AB)A = A(BA) to be defined, we need AB and BA to be defined. For AB to be defined we need m = 5 and for BA to be defined we need n = 3. Hence B is a 5 x 3 matrix. BB is not defined due to mismatch in (5x3)x(5x3), so ABB = A(BB) is not defined, so ABBA = (ABB)A is not defined.

4: Almost any random pair of matrices will work. For example, let A, B be both 2x2 matrices where every entry is 0 except the upper left entry of A is 1 and the upper right entry of B is 1.

5: False. Let C = 0 and A, B be the same as in the example for problem 4.

6: Let $x_{ij} = [X]_{ij}$ be the (i, j)th entry of X and note that X=Y if and only if $x_{ij} = y_{ij}$ for all (i,j). Now we compute the (i,j)th entry of both sides using the summation definition of matrix multiplication. $[(AB)^T]_{ij} = [AB]_{ji} = \Sigma_k\, a_{jk}b_{ki} = \Sigma_k\, b_{kj}a_{jk} = \Sigma_k\, [B^T]_{ik}[A^T]_{kj} = [B^TA^T]_{ij}$

7: Let A have dimensions m x n with $m \neq n$. If B is both a left and a right inverse, then AB and BA are both defined, so B has dimensions n x m. But then AB is an m x m matrix and BA is an n x n matrix, so AB = BA is impossible, so "AB = BA = I" is impossible.

8: If B, C are both inverses of A, then B = BI = B(AC) = BAC = (BA)C = IC = C.

9, 10: Enter what you picked and check with https://matrix.reshish.com/inverse.php

11: Let $X = C^{-1} B^{-1} A^{-1}$. Inverses are unique, so we only need to check (ABC)X = I = X(ABC). By problem 13, it suffices to check the 1st inequality. Use associativity to get $(ABC)X = AB(C C^{-1}) B^{-1} A^{-1} = AB\, I\, B^{-1} A^{-1} = ABB^{-1} A^{-1} = AA^{-1} = I$.

12-14: Email me for a solution.