MATH 243: Worksheet 2 **Name and Discussion Section:** _____

Textbook Sections: 12.1, 12.2, 12.3
Topics: Regions in the 3 D space, the equation of the sphere, vectors and their properties, magnitude of a vector, operations with vectors, parallel vectors, the dot product, scalar and vector projections.
Instructions: Try each of the following problems, show the detail of your work. Clearly mark your choices in multiple choice items. Justify your answers. Cellphones, graphing calculators, computers and any other electronic devices are not to be used during the solving of these problems. Discussions and questions are strongly encouraged.

1. Consider the point $P(3,4,5)$.
   (a) What is the projection of the point on the xy-plane?
   (b) What is the projection of the point on the xz-plane?
   (c) Find the length of $\overline{OP}$, which is a line from the origin $O(0,0,0)$ and the point $P(3,4,5)$.
   (d) Find the position vector for the point $P(3,4,5)$ in **ijk**-form.
2. (a) Find an equation of the sphere that passes through the point $(1,8,5)$ and has center $(3,1,-3)$.
   (b) Using your answer from (a), find an equation to describe its intersection with the yz-plane. If the sphere does not intersect with the yz-plane, write DNE.
3. Find an equation of a sphere if one of its diameters has the endpoints at $(1,2,4)$ and $(4,3,10)$.
4. Given $\mathbf{a} = 9\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = \langle 7, 0, -9\rangle$, find the followings. Simplify your vectors completely.
   (a) $\mathbf{a} + \mathbf{b}$
   (b) $3\mathbf{a} - \mathbf{b}$
   (c) $||\mathbf{b}||$
   (d) $||\mathbf{b} - \mathbf{a}||$
5. Find the vector that has the opposite direction as $\langle 9, -6, -2\rangle$ but has length 5.
6. Determine if $\mathbf{a} = 8\mathbf{i} - 12\mathbf{j}$ and $\mathbf{b} = 6\mathbf{i} - 9\mathbf{j}$ are parallel vectors.

Definition of the Dot Product If $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, then the dot product of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by:

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Similarly, for two-dimensional vectors:

$$\langle a_1, a_2\rangle \cdot \langle b_1, b_2\rangle = a_1b_1 + a_2b_2$$

Theorem If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

7. Given two vectors, $\mathbf{a} = \langle 2, 5, 0 \rangle$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{k}$. Find $\mathbf{a} \cdot \mathbf{b}$.

8. Determine whether the given vectors are orthogonal, parallel, or neither.
   (a) $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 12\mathbf{j} - 2\mathbf{k}$
   (b) $\mathbf{a} = \langle 6, 5, -2 \rangle$ and $\mathbf{b} = \langle 5, 0, 9 \rangle$
   (c) $\mathbf{a} = \langle -18, 15 \rangle$ and $\mathbf{b} = \langle 12, -10 \rangle$

9. Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(4, 16)$.

10. Given the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in V_2, which of the following expressions are meaningful? Which are meaningless? Justify your answer.
    (a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$
    (b) $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c}$
    (c) $||\mathbf{a}|| \cdot ||\mathbf{c}||$

11. Given $\mathbf{a} = \langle 2, 0, -3 \rangle$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$. Find the following.
    (a) $\mathbf{a} \cdot \mathbf{b}$
    (b) the angle θ between $\mathbf{a}$ and $\mathbf{b}$.

Suggested Textbook Problems
Chapter 12.1: 1-42, 44-46
Chapter 12.2: 1-13, 15-19, 21, 23-32, 35, 37-41, 43-45, 47
Chapter 12.3: 1-13, 17, 19, 23, 25, 27, 28, 33, 40, 41, 43, 45, 47-52