

Bryton typo fixed  
Added exam strat doc, Disc. WS 1,  
WS for next disc. up later today  
removed to site. mid 2 is not cumul.,  
but final is cumulative

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Corrections

## Prerequisite quiz

Submitted quiz, got feedback

P16 typo:  $1/(k+1)$ , not  $1/k$

P18 upload test: "test by scratchwork"

You can still do/re-do quiz and  
get feedback if you show work

## Other

recordings in media gallery, rec.  
of 1st lecture is up

lectures will be up about a few  
hours after end

video on how to upload is up

Review of 6/10 pre-lecture

Distance formula:

$$d((x,y), (a,b)) = \sqrt{(x-a)^2 + (y-b)^2}$$

$$d((x,y,z), (a,b,c)) = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$

$$(x-a)^2 + (y-b)^2 = r^2 \text{ is a}$$

circle with radius  $r$ , center  
 $(a,b)$  in 2D

$$(z-c)^2 + (x-a)^2 + (y-b)^2 = r^2 \text{ is a}$$

Sphere with radius  $r$ , center  
 $(a,b,c)$  in 3D

2D: axes are  $x=0$ ,  $y=0$ , axes  
are lines

3D: coordinate planes are  $x=0$ ,  
 $y=0$ ,  $z=0$

Same equation now  $x \neq 0$  rep.  
2 plane instead of a line

$\|v\| = \text{length of } v = \text{length}$   
of the line segment from  
start to end of vector =  
 $\sqrt{a^2 + b^2 + c^2}$  if  $v = \langle a, b, c \rangle$

Unit vector  $v$  is where  $\|v\| = 1$   
any  $v \neq 0$  can be normalized;

$$w = \frac{v}{\|v\|} \text{ will be a unit vector}$$
$$= \frac{1}{\|v\|} v$$

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$$(a_1, a_2, a_3) \pm (b_1, b_2, b_3) =$$

$$(a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3)$$

$$c(a_1, a_2, a_3) = (ca_1, ca_2, ca_3)$$

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$$(3, -2, 2) * (2, 2, 1) = 1 - 6 + 2 = -3$$

$$\begin{array}{c|c}
 \begin{array}{cc}
 i & j \\
 \hline
 3 & -2 \\
 3 & 2
 \end{array} & 
 \begin{array}{cc}
 i & j \\
 \hline
 3 & -2 \\
 3 & 2
 \end{array}
 \end{array}
 + \begin{array}{l}
 -2 \cdot 1 = -2 \\
 2 \cdot 2 = 4 \\
 -2 - 4 = -6
 \end{array}$$

$$j: + 2 \cdot 3 - 3 \cdot 1 = 6 - 3 = 3$$

$$K: + 3 \cdot 2 - (-2) \cdot 3 = 6 + 6 = 12$$

can't cancel:  $a \cdot b = a \cdot c \nRightarrow b = c$

can't cancel:  $a \times b = a \times c \nRightarrow b = c$

can't undo:  $\frac{a \cdot b}{b} \neq a$  for

multiple reasons: dividing by vector  
 is not defined, and even if it was,  
 $a \cdot b$  is a scalar &  $b$  is vector

$\frac{a \times b}{b}$  doesn't make any sense

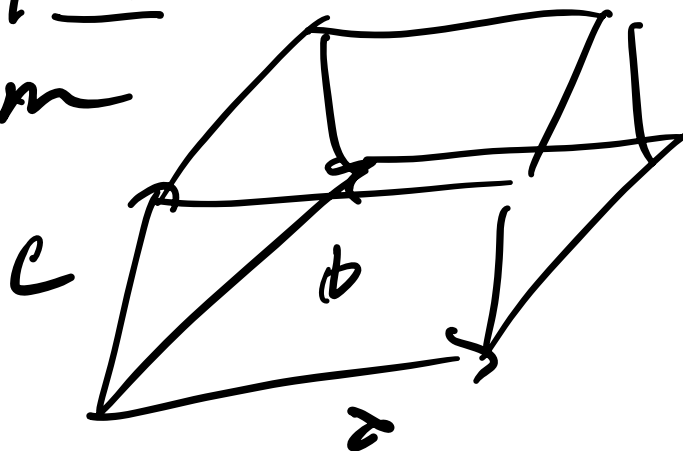
Cross not comm:  $a \times b \neq b \times a$

not assoc.:  $a \times (b \times c) \neq (a \times b) \times c$

But:  $(a \cdot b) \cdot c$  doesn't make sense because  $a \cdot b$  is scalar

$((a \times b) \cdot c, a \cdot (b \times c))$  does make sense

interprets the volume that  $a, b, c$  form



Volume of this figure is  $|a \cdot (b \times c)|$

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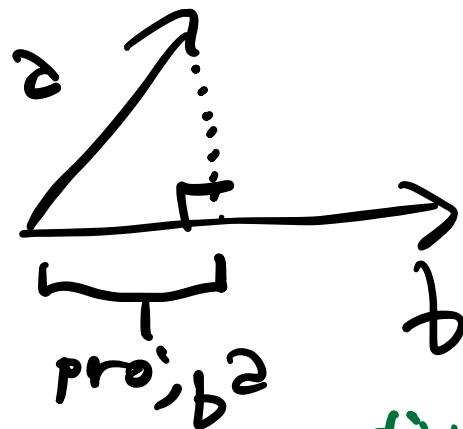
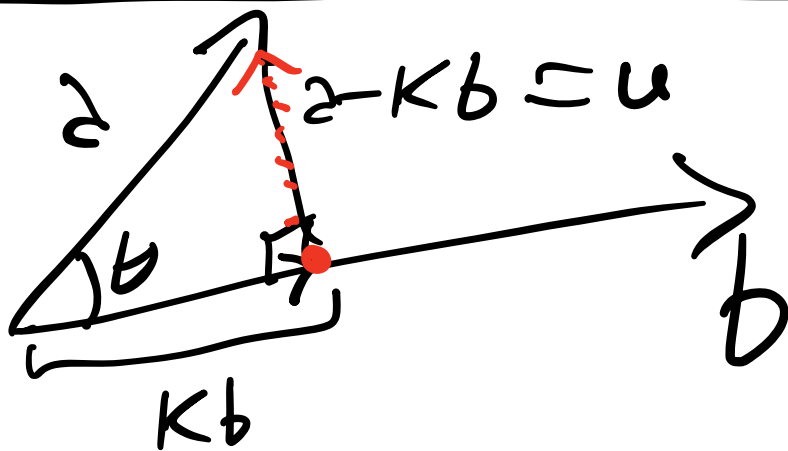
Verify that:  $a \times b$  is perp. to both  $a$  &  $b$

Find: vectors  $a, b, c$  such that

$$a \cdot b = a \cdot c \quad \text{but} \quad b \neq c$$

find  $a, b, c$  w/  $a \times b = a \times c, b \neq c$

find  $a, b, c$  w/  $(a \times b) \times c \neq a \times (b \times c)$



$\checkmark$   $2 - Kb$ ,  $Kb$  are perpendicular <sup>div by K</sup>  
 $0 = \underline{(2 - Kb) \cdot Kb} = \underline{K(a \cdot b) - K^2 b \cdot b}$

$$0 = \underline{a \cdot b - K(b \cdot b)} \Rightarrow \underline{a \cdot b = K(b \cdot b)}$$

$$\left[ K = \frac{a \cdot b}{b \cdot b} = \frac{a \cdot b}{\|b\|^2} \right]$$

Exercise: use the definition of dot product to confirm  $v \cdot v = \|v\|^2$  for any vector  $v$

Also know  $\|Kb\| = \|a\| \cos \theta$

from definition of  $\cos \theta$  as ratio between 2 lengths  $\xrightarrow{K}$

$$\cos \theta = \frac{\|Kb\|}{\|a\|} = \frac{a \cdot b}{\|b\|^2} \frac{\|b\|}{\|a\|} =$$

$$\frac{2 \cdot b}{\|b\| \|a\|}$$

Additional notes about  
the formula for  $\theta$ :

$\text{proj}_b a$  lies on same line as  $b$ ,  
it is parallel to  $b$ , so it is  
 $Kb$  for some  $K$

$u$  is the last vector making  
up the right triangle, then

$$Kb + u = a \Rightarrow u = a - Kb$$

Example of vector arithmetic:

$$a = b - c \quad \text{in usual math,} \\ a, b, c \in \mathbb{R}$$



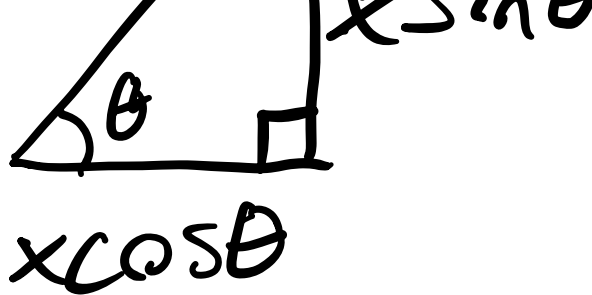
$$a + c = b - c + c = b$$

Same exact thing works for  
vectors

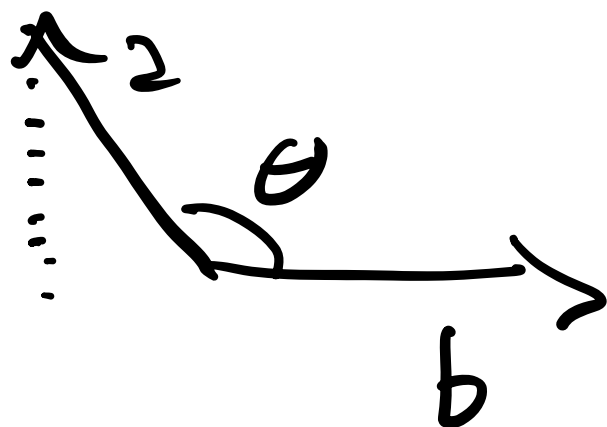
In general,

$$\|cv\| = |c| \|v\|$$





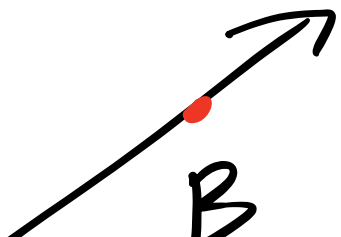
For example,  
if  $c = -1$  then  
$$\| -v \| = 1 - (-1) \| v \|$$
$$= \| v \|$$



it is possible  
for  $\theta > 90^\circ$ ,  
in which case  
 $\cos \theta < 0$

For discussion worksheet:  
in addition to the problems  
from the pdf, also do the  
review problems from the  
6/10 pre-lecture slides

Notes for 6/10 lecture



vector through  
 $A$  &  $B$  has



direction  $B - A$

A

Start  $A \Rightarrow A$

end at B by starting at A  
and adding the vector

check that this is actually true:

$$\underbrace{A}_{\text{start}} + \underbrace{B-A}_{\text{vector}} = B - \underbrace{A+A}_{=0} = \underbrace{B}_{\text{end}}$$

$$\underbrace{A}_{\text{start}} + \underbrace{\frac{1}{2}(B-A)}_{\frac{1}{2} \text{ way in between}} = \underbrace{\frac{1}{2}(B+A)}$$

$[A + t(B-A)]$  is  $t$  of  
the way between A & B

$0 \leq t \leq 1 \Rightarrow$  line segment

$t=0 \Rightarrow A$  ,  $t=1 \Rightarrow B$

Will turn over slides for lectures now

Website now shows slide downloads for pre-lecture video slides and lecture itself slides

Office hours Zoom link is updated

Notify me ASAP if you see any links break

Zoom recordings show up in mediz gallery

Start time for recordings is broken, end time still good

2 things left to fix, hopefully  
all 2 fixed by end of today:

1. Gallery recording start time
2. Upload notes on tablet
3. ?