

# Linear Approximations

Lecture for 6/17

# Idea Behind Linear Approximation

- Recall tangent line at  $x_0$  good approximation for  $y = f(x)$
- If line equation is  $y = L(x)$ , then  $L(x) \approx f(x)$  for  $x \approx x_0$
- So let's use tangent plane to  $z = f(x,y)$
- Express plane equation as  $z = P(x, y)$  and use it

# The Approximation

- Recall plane equation:  $z = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + f(x_0, y_0)$
- So  $f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y$
- What happens if  $f_x$  and  $f_y$  not continuous?

# Differentiability

Let's define differentiability:

- Let  $\mathbf{v} = (x,y)$ , write plane equation as  $z = P(\mathbf{v})$
- $f(\mathbf{v})$  is differentiable at  $\mathbf{v} = \mathbf{c}$  if  $\lim_{\mathbf{v} \rightarrow \mathbf{c}} [f(\mathbf{v}) - P(\mathbf{v})]/\|\mathbf{v}-\mathbf{c}\| = 0$ 
  - Same when  $f$  has more than 2 variables



# Relation to Partial Derivatives

- If  $f_x$ ,  $f_y$  exist and are continuous, then  $f$  differentiable
- If  $f$  diff, then  $f_x$  and  $f_y$  exist
- But  $f$  diff. does not mean  $f_x$  and  $f_y$  are continuous









# Differentials

- Recall  $dy = f'(x) dx$  when  $y = f(x)$
- If  $x$  and  $y$  slightly change, how does  $z = f(x,y)$  change?
- $dz = df = f_x dx + f_y dy$

# Practice Problems

Explain why  $(x+y)/(x^2+y^4+1)^{1/2}$  is differentiable everywhere

A sphere has center  $(2, 1, -1)$  and contains  $(4.01, 3.02, 0.99)$ . Find a good approximation for the radius of the sphere without using a calculator

Find a function  $f$  where  $f_x, f_y$  always exist, but  $f$  is not differentiable

# Scratchwork



