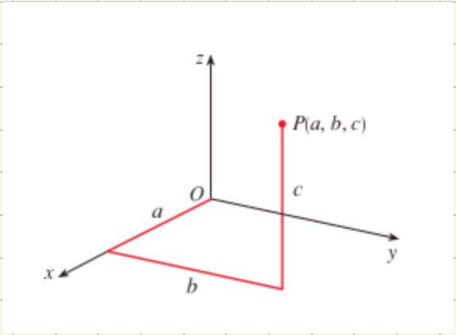
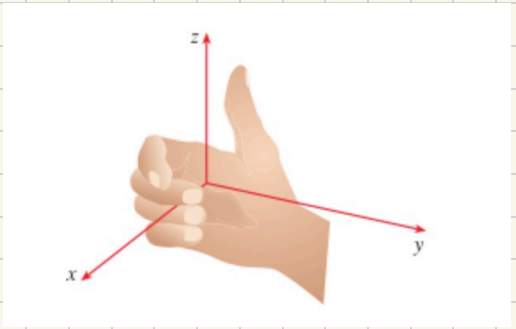
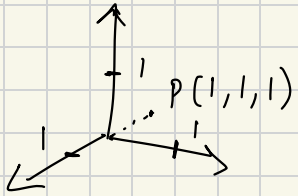
Section 12.1:

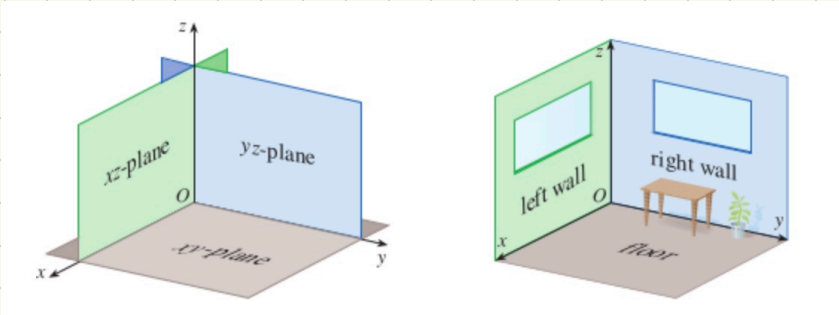
Definition:

- $\mathbb{R}$ is the set of all real #s.
- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}\}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) : x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}\}$





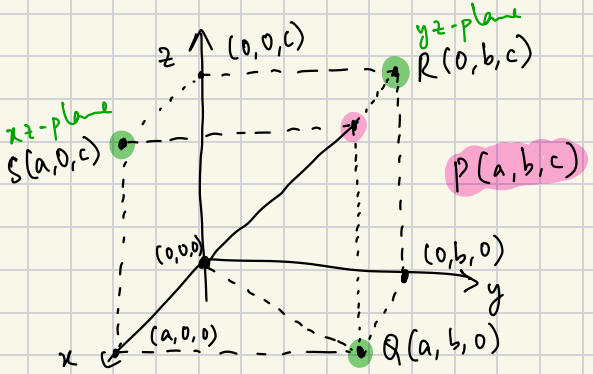




Definition: $xz\text{-plane} = \{(x,0,z) : x \in \mathbb{R}, z \in \mathbb{R}\}$

$xy\text{-plane} = \{(x,y,0) : x \in \mathbb{R}, y \in \mathbb{R}\}$

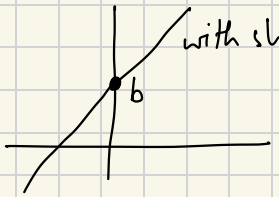
$yz\text{-plane} = \{(0,y,z) : y \in \mathbb{R}, z \in \mathbb{R}\}$



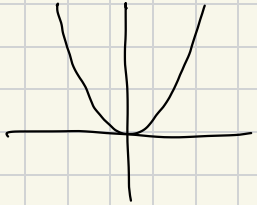
xy-plane

Surfaces: In $\mathbb{R}^2$ a surface is the set of points x and y that satisfy an equation.

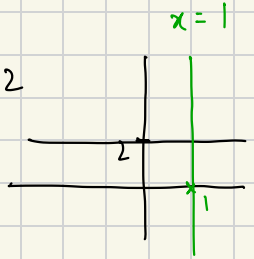
Eg. $y = mx + b$ with slope m



Eg: $y = x^2$

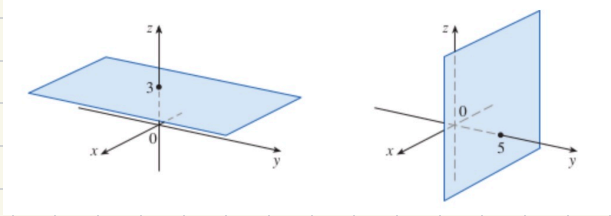


Eg: $y = 2$



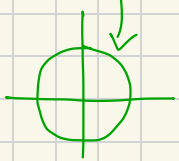
Example: What surfaces in $\mathbb{R}^3$ are represented by the following equations?

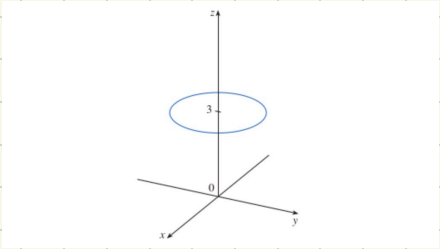
(a) $z = 3$ (b) $y = 5$



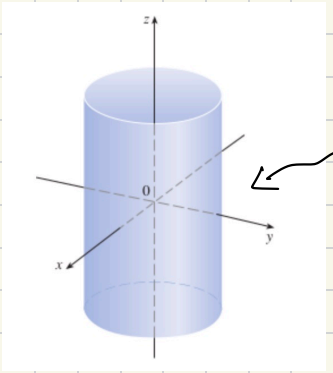
Example: (a) Which points (x, y, z) satisfy the equations

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 3$$





(b) What does the equation $x^2 + y^2 = 1$ represent in $\mathbb{R}^3$?

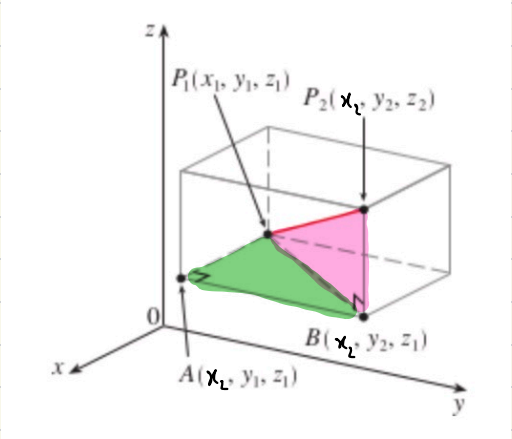


$\{(x, y, z) : x^2 + y^2 = 1, z \in \mathbb{R}\}$

Distances and spheres:

Distance formula in Three dimensions: The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

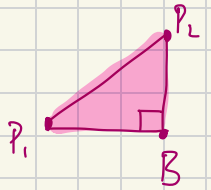
$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} \iff |P_1P_2|^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$$



$$|P_1A| = |x_2 - x_1|$$

$$|AB| = |y_2 - y_1|$$

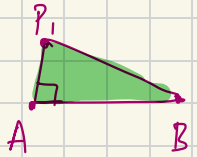
$$|P_2B| = |z_2 - z_1|$$



$$|P_1P_2|^2 = |P_1B|^2 + |P_2B|^2$$

$$= |P_1B|^2 + |z_2 - z_1|^2$$

$$= |P_1A|^2 + |AB|^2 + |z_2 - z_1|^2$$



$$= |x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Example 4: The distance from point $P(2,-1,7)$ to the point $Q(1,-3,5)$ is

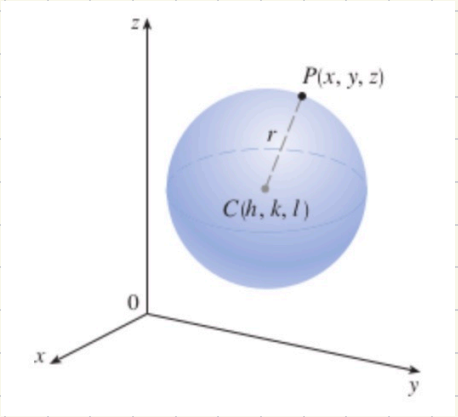
$$|PQ|^2 = 1^2 + 2^2 + 2^2 = 1 + 4 + 4 = 9 \Rightarrow |PQ| = \sqrt{9} = 3$$

Example 5: Find an equation of a sphere with radius r and center $C(h,k,l)$

(h,k,l)

Sphere with center $C = \{ P(x,y,z) : |PC| = r \}$

$$|PC|^2 = (x-h)^2 + (y-k)^2 + (z-l)^2$$

we want $|PC| = r \Leftrightarrow |PC|^2 = r^2$

so we want $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$