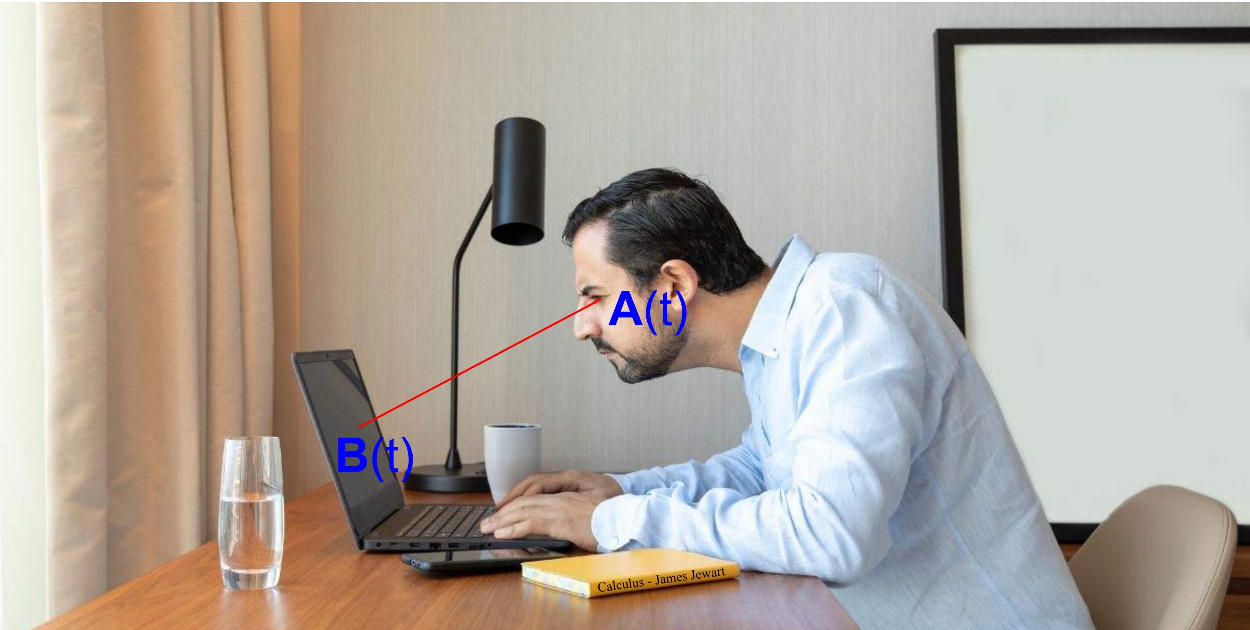
Vector Functions

Lecture for 6/11

Definition of Vector Functions

- Write $\mathbf{r}(t) = \langle f(t), g(t) \rangle$ or $\langle f(t), g(t), h(t) \rangle$
- Same B-A trick to figure out $\mathbf{r}$ when given 2 vectors
- Can restrict domain



Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Limits, derivatives, integrals

- Limits are taken component-wise:
  - $\lim \mathbf{r}(t) = \langle \lim f(t), \lim g(t), \lim h(t) \rangle$
- Vector function limit exists iff each component limit exists
- Derivatives and indefinite integrals also taken component-wise
- Constant of integration +C becomes vector $+\mathbf{c} = +\langle c_1, c_2, c_3 \rangle$
- Definite integrals evaluated using antiderivatives as usual

Derivative Rules

- Let $\mathbf{r}$, $\mathbf{s}$ be vectors, f scalar, c constant
- Basic properties still hold
  - Linearity: $(c\mathbf{r})' = c\mathbf{r}'$, $(\mathbf{r}+\mathbf{s})' = \mathbf{r}'+\mathbf{s}'$
- Product rules
  - $(f\mathbf{r})' = f'\mathbf{r} + f\mathbf{r}'$
  - $(\mathbf{r}\cdot\mathbf{s})' = \mathbf{r}'\cdot\mathbf{s} + \mathbf{r}\cdot\mathbf{s}'$
  - $(\mathbf{r} \times \mathbf{s})' = \mathbf{r}' \times \mathbf{s} + \mathbf{r} \times \mathbf{s}'$
- Chain rule: $[\mathbf{r}(f(t))]' = f'(t)\mathbf{r}'(f(t))$

Arc Length

- Can't reduce to components easily
- Call ds a tiny bit of the arc
- Line segment for ds is **r**(t) to **r**(t+dt)
- Use this to get ds = $\|\mathbf{r}'(t)\|$ dt
- $L = \int ds = \int \sqrt{(f')^2+(g')^2+(h')^2}\, dt$
- Now you have a basic integral

Practice problems

Understand your segments

- Find a vector equation for the line segment between (a, b, c) and (d, e, f)

Mixing vector products and derivatives

- Let $\mathbf{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$ and $\mathbf{s}(t) = \langle \sin(t), -\cos(t), 1 \rangle$. Compute $(\mathbf{r} \times \mathbf{s})'$, $(\mathbf{r} \cdot \mathbf{s})'$ with and without the product rule

Scratch Work

Extra Problem

Arc length of helix

- Let $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$, $0 \le t \le 2\pi$ represent one twist of a helix. Find the arc length of this curve