Cylindrical and Spherical

Pre-lecture for 6/13

Geometry of Cylinder

- Cylinder around z-axis has radius $r > 0$ and infinite height
- Any given height is a circle, parametrized by θ
- Cylindrical coordinates: $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$
- Bounds: $0 \leq \theta \leq 2\pi$

Geometry of Sphere

- Consider sphere of radius $\rho > 0$ centered at origin
- For any $-\rho \leq c \leq \rho$, the plane $z = c$ cuts off a circle
- Spherical Coordinates: $(x, y, z) = \rho(\sin(\varphi)\cos(\theta), \sin(\varphi)\sin(\theta), \cos(\varphi))$
- Bounds: $0 \leq \theta \leq 2\pi$, $0 \leq \varphi \leq \pi$

Scratchwork

Converting between coordinates

- Call (x, y, z) the standard coordinates for R^3
- CYL -> ST or SP -> ST: just plug in
- ST -> CYL: $r = \sqrt{(x^2+y^2)}$, $\theta = \tan^{-1}(y/x)$
- ST-> SP: $\rho = \sqrt{(x^2+y^2+z^2)}$, $\theta = \cos^{-1}(z/r)$, $\varphi = \tan^{-1}(y/x)$

Disclaimers

- Arctan formula applies to 1st quadrant
  - Draw diagram to get the signs right for generic point
- No need to plug in if $r = 0$
- If $x = 0$, take $\tan^{-1}(y/x) = \pi/2 * \text{sgn}(y)$

Practice Problems

Convert these points into both cylindrical and spherical

- (0, 1, 0)
- (1, -2, 2)
- (3, 4, -5)

Identify the following 7 surfaces

- Cylindrical: $r = 3$, $r^2+z^2 = 9$, $z = r$
- Spherical: $\rho = 0$, $\varphi = 2\pi/3$, $\theta = \pi/3$, $\rho(\sin(\varphi) + \cos(\varphi)) = 1$

Scratchwork