MATH230 - Finite Mathematics with Applications

Last Name: **First Name:** **DIS Sec #:** ____

1. (1+2=3 points) Give one example each of

   a) a *non-linear* equation in two variables. **Answer:** $x^2 + y = 2$

   b) a system of two linear equations representing *distinct parallel* lines and slope equal to -2. **Answer:** $y = -2x + 1, \; y = -2x + 5$

2. (3 points) Circle the option that represents all solutions *(if they exist)* of the linear system:

$$x - z = 1, \; y + 2z = 0$$

a) $(2, -2, 1)$ b) $(1 + t, -2t, t)$ c) $(-1 - t, 2t, t)$ d) DNE

(Optional) show work for Question 2 for partial credits:

$x = 1 + z, \; y = -2z, \; z = t$

3. (2x5 = 10 points) True/False Questions. **Circle ONE option** each.

   (a) The inverse of a square matrix, if it exists, is unique. **True** **False**

   (b) In a row-reduced matrix, all leading entries must be equal to 0. **True** **False**

   (c) If A and B are two dependent events, then $p(A|B) \geq p(A)$. **True** **False**

   (d) If the feasible set of a system of linear inequalities is bounded, then the number of solutions of the system is finite. **True** **False**

   (e) A and B are independent if $p(A) = .1$, $p(B) = .1$, and $p(A \cup B) = .2$. **True** **False**

(Optional) show work for Item (e) for partial credits:

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$.2 = .1 + .1 - p(A \cap B) \Rightarrow p(A \cap B) = 0 \neq p(A)p(B)$

4. (2 points) Give an example of a *strict* linear inequality in two variables: $x + y < 1$

5. (2 points) Given the matrix A, write a matrix B such that AB exists.

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 5 & 4 & 2 \end{bmatrix}^T, \qquad B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}_{2 \times 1}$$

$$= \begin{bmatrix} 1 & 5 \\ -3 & 4 \\ 0 & 2 \end{bmatrix}_{3 \times 2}$$

6. (6 points) Solve for x, y:

$$\begin{bmatrix} 2 & -1 \\ 7 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ x \end{bmatrix}$$

Answers: $x =$ 3 $\quad y =$ 2

(Optional) show work for Question 6 for partial credits:

$$\frac{1}{-8+7}\begin{bmatrix} -4 & 1 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 7 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 4-y \\ 7-2y \end{bmatrix}$$

$$4-y=2 \Rightarrow y=2, \quad x=7-2y=3$$

---

7. In a survey to determine the opinions of Americans on health insurers, 300 baby boomers and 200 pre-boomers were asked this question: Do you believe that insurers are very responsible for high health costs? Of the baby boomers, 200 answered in the affirmative, whereas 100 of the pre-boomers answered in the affirmative.

(1 point) Introduce sets to describe various events related to this probability experiment.

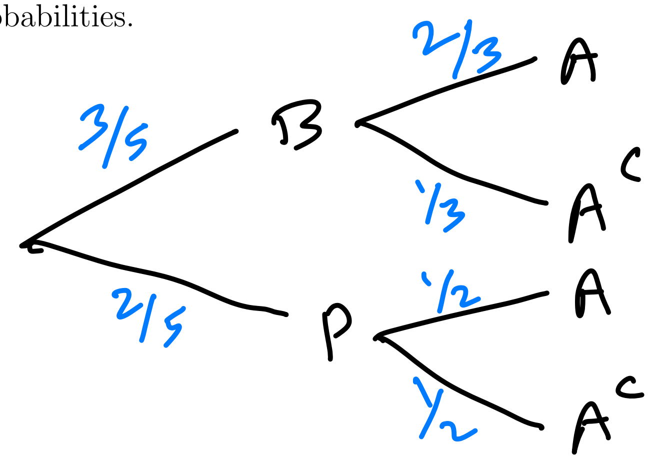
Let,

B = baby boomers

P = pre-boomers

A = respondents who answered affirmatively

(2 points) Draw a tree-diagram showing the probabilities.



(3 points) If a respondent chosen at random from those surveyed answered the question in the affirmative, what is the probability that he or she is a baby boomer? **(Show all work)**

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{P(B)\cdot P(A|B)}{P(B)\cdot P(A|B) + P(P)\cdot P(A|P)}$$

$$= \frac{\frac{3}{5}\cdot\frac{2}{3}}{\frac{3}{5}\cdot\frac{2}{3} + \frac{2}{5}\cdot\frac{1}{2}} = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{5}} = \frac{2}{3}$$

Therefore, the probability that a respondent who answered in the affirmative is a baby boomer is 2/3.

8. (8 points) Find the inverse of matrix A using the **Gauss-Jordan** method. **Show all steps. No credit will be given for the final answer only without the work.**

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 9 \\ 1 & 2 & 6 \end{bmatrix}$$

Solution

$$[A|I] = \left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ 4 & 0 & 9 & 0 & 1 & 0 \\ 1 & 2 & 6 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 0 & 0 & 1 \\ 4 & 0 & 9 & 0 & 1 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 \end{array}\right]$$

$R_2 - 4R_1$
$R_3 - 2R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 0 & 0 & 1 \\ 0 & -8 & -15 & 0 & 1 & -4 \\ 0 & -5 & -9 & 1 & 0 & -2 \end{array}\right]$$

$4R_1 + R_2$
$8R_3 - 5R_2$
$$\left[\begin{array}{ccc|ccc} 4 & 0 & 9 & 0 & 1 & 0 \\ 0 & -8 & -15 & 0 & 1 & -4 \\ 0 & 0 & 3 & 8 & -5 & 4 \end{array}\right]$$

$R_1 - 3R_3$
$R_2 + 5R_3$
$$\left[\begin{array}{ccc|ccc} 4 & 0 & 0 & -24 & 16 & -12 \\ 0 & -8 & 0 & 40 & -24 & 16 \\ 0 & 0 & 3 & 8 & -5 & 4 \end{array}\right]$$

$R_1/4$
$R_2/-8$
$R_3/3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & 4 & -3 \\ 0 & 1 & 0 & -5 & 3 & -2 \\ 0 & 0 & 1 & \frac{8}{3} & -\frac{5}{3} & \frac{4}{3} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -6 & 4 & -3 \\ -5 & 5 & -2 \\ 8/3 & -5/3 & 4/3 \end{bmatrix}$$

or

$$\frac{1}{3}\begin{bmatrix} -18 & 12 & -9 \\ -15 & 15 & -6 \\ 8 & -5 & 4 \end{bmatrix}$$

9. **Consider the problem:** Joan and Dick spent 2 weeks (14 nights) touring four cities on the East Coast: Boston, New York, Philadelphia, and Washington. They paid \$200, \$300, \$150, and \$180 per night for lodging in each city, respectively, and their total hotel bill came to \$3170. The number of days they spent in New York was the same as the total number of days they spent in Boston and Philadelphia, and the couple spent four times as many days in Washington as they did in Philadelphia. How many days did Joan and Dick stay in each city?

   (a) (1 point) Introduce the variables in order to formulate the problem as a system of linear equations. Use complete sentence(s) in English language.

   Let the no. of days spent by Joan and Dick in Boston, New York, Philadelphia, and Washington be x, y, z, and w respectively.

   (b) (4 points) Formulate a system of linear equations for the problem. If necessary, rewrite the equations so that **they are in standard form**. (DO NOT solve)

$$\boxed{x+y+z+w=14}$$
$$\boxed{200x+300y+150w+180z=3170}$$
$$y=x+z \Rightarrow \boxed{x-y+z=0}$$
$$w=4z \Rightarrow \boxed{-4z+w=0}$$

Bonus Question (2 points): Prove that in any probability experiment, an event and its complement are two *dependent* events.

If $p(A)\neq 0$, then $p(A|A^c) = \dfrac{p(A\cap A^c)}{p(A^c)} = 0$

$\Downarrow$

$\boxed{p(A^c)\neq 0 \text{ as well}}$ i.e. $p(A|A^c)\neq p(A) \Rightarrow$ dependent events.

* The case when $A=\phi$ or $A=S$, the sample space is not considered because $p(S|\phi)$ cannot be defined meaningfully.