Integrals in Polar

Pre-lecture for 6/26

General Idea

What if we convert $f(x, y)$ into polar coordinates?

- Things can simplify if we have terms like x^2+y^2 and y/x
- Recall: $x = r\cos(\theta)$, $y = r\sin(\theta)$
- But we need to figure out what dA becomes
    - Normally, $dA = dx\ dy$

Converting dA

- Split the region R we’re integrating into radial slices
- Find the area of each slice
- Conclusion: $dA = r\,dr\,d\theta$

The Conversion Process

- $\iint_R f(x, y)\, dA = \iint_R f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$
  - Now we need to figure out the bounds on r, θ
- Draw R
- For r: closest and furthest points in R from origin
- For θ: smallest & largest angle for which angle θ line intersects R

Usage Warning

- Polar coordinates are useful on circular regions
- Converting bounds is messy or impossible for general regions
- Use your judgement to see if polar is worth it
- Even a simple rectangle becomes messy

Practice Problems

Problems

- Let R be the annulus centered at the origin with inner radius 2 and outer radius 3. Find $\iint_R (1+x+y+xy)\,dx\,dy$
- For the same R, find $\iint_R (|x|+|y|+x^2+y^2)\,dx\,dy$
- Find the volume inside $z = x^2+y^2$ and below $z = 16$
- Find the volume under the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 5$

Scratchwork