6/9

General concept: manipulating equations / simplifying expressions, canceling out things / factoring things

most fundamental concept

If you understand this concept well, you will be poised for success in math 243

analogy of analogy: anything in Calc 1/2 done with 1/2 variables, you need

→ be comfortable with so you

→ do it with 2/3 variables resp.

Exceptions are one-off units and last half of Calc 2. See 6/9 lecture for more info

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6/10

other sources gen. follow this too

$a_1, a_2, a_3, \dots$ const.

General remark: $v_1, v_2, \dots$ vectors

$a, b, c, d, e, \dots, r, s, t$ are typically scalars

$u, v, w$ are vectors

$x\ y\ z$ up to the situation

Q: Some students may ask, who cares where things are coming from? Just give us the formulas to plug everything in

A: you may not care, but other

Also, your knowledge will break under tiny changes. Many problems in math are small variations of given examples.

See 6/10 discussion for more commentary on this remark

general remark. If you miss class, download notes and notice a remark in the notes is confusing, search the segment of the recording used to create the remark

If that still leaves you confused, ask me by email writing what day & slide & remark quote you need help on.

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General remark. choose your own variable names. As long as you can do the work to solve the problem and it is clear to anyone reading the work how the calculation is going, no problem. There is some personal preference on whether to use u & v for a problem that needs 2 vectors or use v&w for example

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General remark: don't worry

about whether a vector is $v$ or $w$ as long as it's defined clearly. Focus on computation mistakes and conceptual errors instead, like $2(u+v) = 2u+v$ or taking $\|\sqrt{v}\|$ or $\|-u\| = -\|u\|$

6/11

ALL scratchwork for exams and future quizzes must be uploaded

Report typos & mistakes whenever you see them.

Don't hesitate if something seems to be broken

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Make sure to state bounds on any variables that you use for a parametrization or equation. Bounds are part of the answer. If there are no constraints on the variables given, then we assume they can be any real number. But if there should be constraints and you leave them off then your answer is wrong.

For example: line has no constraint, so leaving off saying "t any real number" is ok. But line segment does, so include it.

You can ignore derivative rules, just simplify everything into one vector, and only then take a derivative, going component by component. But then it might be more algebra and more work because rules can simplify certain calculations.

Generalization of the previous remark: know your rules for combining multiple different operations on functions and vectors. Using these rules often leads to less work , less calculations, less room for mistakes than manually simplifying everything as much as possible and only then taking the final operation in your calcula-tion.

6/12

Definitions are given without explanation & don't need justification, except perhaps an explanation of how people came across them & why they're useful.

Formulas are equations, inequalities, results etc. which follow from definitions

For example: $f'(t) = \lim_{dt \to 0} \frac{f(t+dt) - f(t)}{dt}$ is a definition of $f'$, not a formula

$\frac{df}{dt} = f'(t)$ is a definition of notation.

$\frac{d}{dt}(t^n) = nt^{n-1}$ is a formula however

When a problem says "find vector\ integral\ derivative\ equat." etc., you must simplify the answer as much as the answer as much as possible.

If answer to problem is a vector, you can leave a constant factored out or brought in, and either is fine in regards to the final form of the answer

If you're not at the end of a problem, you might save some trouble by leaving the constant outside.

If you want to check your work for unit vectors, check at the end the magnitude is actually 1.

For speed and convenience, sometimes I will write $c, s$ to denote $\cos, \sin$ respectively.

Chain rule allows you to cancel differentials like $dt, ds, dr$ etc. legally

For any rule or idea you want to apply, but you are not sure if the rule/idea works because you haven't seen it covered, plug in easy values like 0, 1 to make sure it makes sense. Plug in $f = g$ if it depends on $f$ & $g$, plug in basic vectors like $(0,0,1)$, $(1,0,0)$ etc.

About delays & deadlines: if deadlines are pushed back by 1-2 days for homeworks and quizzes, don't automatically blow off the extra time and then scramble at the end of the new

deadline to do something.
Try to do the homeworks at the original deadline and consider extra time as a gift rather than something for granted that will happen every single homework

Some seemingly random problems like equation of sphere or intersection of plane & plane, line & plane, region & plane etc. are generalization of [illegible] preparation for doing double and triple integrals in the last 1/2 of 243. More generally, geometry from the 1/2 half of 243 will come in at the last 1/2.

For a geometry problem, when in doubt, draw the diagram of relevant objects. Take useful 1D or 2D slice of the region. Still can draw 3D, but only abstractly

6/12 | with inverse trig

6/13 with inverse trig expressions like

$\sin^{-1}(\frac{1}{6})$, $\tan^{-1}(3)$, $\cos^{-1}(\frac{1}{4})$

that have no nice closed form, you can leave them as is. Don't go for decimal values.

More generally: you don't need a calculator for this class to spit out decimal values (with exception of WebAssign problems requiring decimals, which will be rare), so anything can be simplified and fully given by pencil & paper

Be careful with cancelling

trig functions by doing inverses. For example:

$\tan^{-1}(\tan x) \neq x$ although $\tan(\tan^{-1}x) = x$

Problems about $\kappa$, $a$, $v$, $t$, $a_T$, $a_N$, arc length are generally all calculation when it comes to their main difficulty. Students struggle with them not because of calculus, but because of algebra: square roots, cross products, simplifying sums

---

Some problems have complicated fractions as the final answer. This is ok, doesn't mean you made a mistake. You could have a mistake, but only for the same reasons you make mistakes on math problems in general.

However, if you have a complicated fraction in the middle of the problem and need to do something which makes that fraction even worse, like derivative or integral, stop right there and check your work.

6/16

The reason you see many miraculous cancellations when doing problems with trig, arc length, T, N, B, and any other concepts where the formulas might lead to a crazy answer if you just picked a function at random is because the problems have to be solvable. Many problems do not have closed form solutions in

$\sqrt{\ }$, $^{\wedge}$, $n!$, ln, e, a, a/b,

+, −, /, ·, cos, sin etc.,

the problems in this class and other math classes in your whole life are rigged to have solutions.

In particular, if you end up at an impossible integral or overly complicated derivative, you probably made a mistake and should check your work before

continuing to solve the problem

Continuing to solve the problem:

$\int \sqrt{x^2+1}$ = possible

$\int e^{e^x}$ = imp $\int e^{x^2}$ = imp

$\int e^{-x^2}$ = imp, $\int \sqrt{x+\sqrt{x^2+1}}$

= tricky to tell

$\int \sqrt{\cos^2 x + \sqrt{x}} = \int \sqrt{\cos^2 x} + \sqrt{\sqrt{x}} = \ldots$

wrong

$\int fg = (\int f)(\int g)$ not forgivable

missing $+C$, $\int 2f = 2\int f$

↑ accidentally dropping 2

more forgivable

Sometimes, I solve problems in a slower way & include steps or attempts that don't end up mattering. This is to imitate how a successful student who doesn't know the problem but eventually figures it out might go about things. If I just jumped to the fastest solution right away, you wouldn't gain any understanding on the problem solving process from seeing me solve the problems.

Therefore, if you miss class and view the problems on the class site by checking the notes, or checking the answers (when discussion worksheet solutions are uploaded), you will not learn as much as if you had seen the recording of class or had come to class because you won't see the process and ideas; it is these methods & ideas that you need to succeed on tests.

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$\boxed{6/17}$

$dx$ is used for infinitesimal quantities, but $\Delta x$ is used for any kind of increment. Do not confuse these 2 concepts.

$\left[\begin{matrix} f_x \& f_y \\ \text{cont.} \end{matrix} \Rightarrow \begin{matrix} f \\ \text{diff} \end{matrix}\right]$ is a long problem, don't worry if you can't do it yourself. This & other long derivations like finding formulas for $a_T$, $a_N$, $\kappa$ are lecture material because they would be impossible/unfair for homework, quizzes, tests. I only prove them so you know where they come from & don't have to worry about any truth issues when using them on homeworks & tests

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3 ways to notate a domain (set of continuity | set of diffability | set where limit exists etc.

"$\{(x,y,\ldots) \in \mathbb{R}^n \mid$ algebraic conditions on $x,y,\ldots\}$"

"all $(x,y,\ldots)$ with [insert alg cond]"

"all $(x,y,\ldots)$ except those where [insert opposite of that alg cond]"

Any notation which is unambiguous & defined & I can understand what you are saying

general general general remark: this is how math language works in general

purple remark

So same gen gen gen [illegible]
applies for me determining whether you solved a problem that requires an "explain why" or "show your work"

Your work must be unambiguous, readable, and symbols must be defined if they aren't ordinary \ pre-existing Calculus symbols.

6\18

If you watch a video only after the class in which it is recapped, practice problems done, and lecture done, and you have questions about the content or would like some algebra step to be explained more, don't hesitate to contact me by email.

General general remark: It is never too late to ask for more help on some material, even if you have a question about week 1 and it's already the 4th week of class.

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Even if you did well, you can still learn from other people's mistakes. Maybe you don't make some mistake category on a quiz but then make it on next one due to lack of awareness on the type of mistake.

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In case you forget a formula for multivariable differentiation, write out a dependency tree and pretend that your goal derivalue is a fraction. This will tell you what it ought to be

Some students may complain, "I don't want to see more than 1 way. I want everything to be done as the book does it" (whenever there is a book involved)

It is ultimately your choice which way to solve a problem. On quizzes & exams, do any way which follows the general general remark in purple above.

However, if you see more than 1 way, you will understand more and it will be easier to solve problems. It will be more work of course to understand more than 1 way, but that extra work is worth the benefit.

Don't forget step 4 ("plug in for leftover variables") on 6/18 notes pre-lec exercises solutions, or you will lose points because your answer isn't simplified

6/23

For MATH 243, complex numbers will not appear inside answers to problems. If you got a complex number from using quadratic formula or taking square root, you made a mistake & should check your work

6/24

Don't confuse ∂ in boundaries & ∂ in

partial derivatives. $\partial S$ = boundary of set $S$, $\partial f$ ~~for~~ for function $f$ doesn't make sense, $\frac{\partial}{\partial x_i} f$, $f_{x_i}$, $\frac{\partial f}{\partial x_i}$ are defined notation

---

It is possible for a set to be not closed and/or the function to be not continuous, yet the min or max still exist. So you must rule out existence in the case you think a global extreme doesn't exist.

---

This course obeys Chekov's gun. Anything that is mentioned early on but not used will come up later. So if something seems useless, just be patient.

---

If you are going for global min or max, you don't need to apply 2nd derivative or Hessian test to every point which solves $\nabla f = 0$. It doesn't matter if some candidate is a local min, local max, or saddle. The truth will be revealed after plugging back into $f$.

---

WLOG = without loss of generality

Just like for limits in Calc 1, we treat $-\infty$, $\infty$, DNE as different concepts.

For example:

max of $f(x,y) = x^2+y^2$ is $\infty$

min of $f(x,y) = x+y$ is $-\infty$

min of $f(x,y) = e^{x+y}$ DNE because $e^{x+y} > 0$ but never $0$.

$\overline{R} := R \cup \partial R$ denotes the closure of $R$. $\overline{R}$ is always closed, so $\overline{(\overline{R})} = \overline{R}$.

6/26

Hardest part of solving 2D or 3D integrals is not solving the integral. Solving is just doing 2 or 3 nested 1D integrals using techniques from single variable calculus. The hardest part is typically finding the bounds. If we have a 2D integral

$$\int_{?}^{?}\int_{?}^{?}[\ ?\ ]\,d?\,d?$$, the hardest part is filling in the question mark. Things to consider when computing these integrals:

1. Do we keep $dxdy$ / $dxdydz$, or switch to another coordinate system?
2. How does the region look like? (When in doubt, draw a graph)
3. Using what the region looks like, what are the upper & lower bounds for each coordinate? This can be tricky if the bounds aren't constant, but rather there is some dependency in the inner bounds.
4. Do I need to split the region into multiple pieces and do a separate integral for each piece?

Only after setup should you worry about taking antiderivatives, but with one caveat:

If the antiderivatives look impossible to compute, check for algebra mistakes, then check if switching order of integration would eliminate the issue, and lastly consider a different coordinate system. You should never have something like $\int_0^1 e^{x^2}$, $\int\left(\int_0^1 e^{r^2}\cos\theta\right)$ ... in the course of solving.

---

In Calc I, you shouldn't forget extra terms modify the differential when doing u-substitutions. Similarly, in math 243 you shouldn't forget extra terms when converting coordinates or your answer will be wrong & you may get stuck when doing algebra, wasting extra time.

Examples: $dx\,dy \neq dr\,d\theta$, $dx\,dy\,dz \neq dr\,d\theta\,dz$, $dx\,dy \neq du\,dv$, $dx\,dy\,dz \neq dr\,d\theta\,d\varphi$

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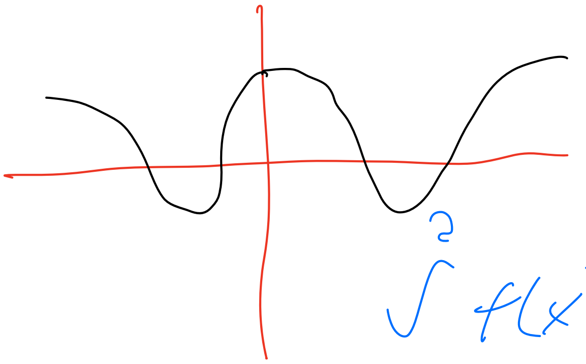
Make sure you understand arguments from geometric symmetry and how to apply them. It will save a lot of time if you can show certain integrals are 0 by some reflection symmetry, or an integral over the sphere is just 8 times the integral over the 1st octant.



You have seen this idea in Calc 1 before, like

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \quad \text{if}$$

$f$ is even & $\int_{-a}^{a} f(x)\,dx = 0$ if $f$ odd

[$f$ even $:= f(x) = f(-x)$, $f$ odd $:= f(-x) = -f(x)$]