# Logistical Announcements

0. Reminder: Don't forget about class site Github, worksheets post ≈2 days before class, solutions Th/Fri etc.

1. Site upgrades: include Arnab notes, clean formatting, update documents, etc. coming soon
done done
A few more things before next week

2. Feedback survey on discussions released end of week. Will put on site & also annouce on Canvas

Blue = Update for Thursday

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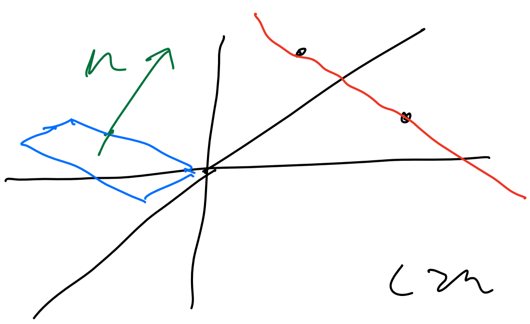
## Discussion Worksheet 4

#1: Find equation for plane through $\overbrace{(0,-2,5)}^{A}$ and $\underbrace{(-1,3,1)}_{B}$ perp. to $2z = 5x+4y$

There is never one golden method which works for all "find the line|plane" equations (except for write down general form $ax+by+cz=1$ and solve for each coefficient, but this is algebra heavy), so you must adapt on a per-problem basis.




$2z = 5x+4y$ has normal vector $\langle 5,4,-2\rangle$ because can rewrite the plane as

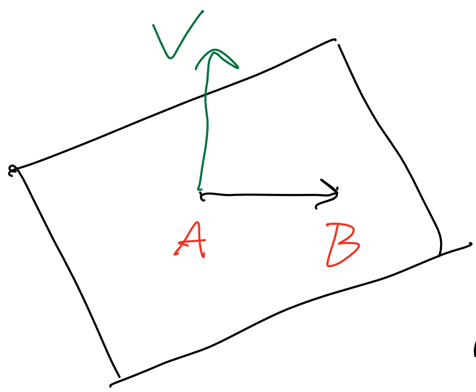
$$\underbrace{\langle 5,4,-2\rangle}_{\vec{n}} \cdot \langle x,y,z\rangle = 0.$$

Observe: perpendicular planes have perpendicular normal vectors & vice versa.

Why is this useful? Let $\vec{v}$ be normal vector to plane we seek. Then $\vec{v}\cdot\vec{n}=0$

If $v=(a,b,c)$ & $P=(x_0,y_0,z_0)$ on the plane we seek, the plane equation

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0.$$




$v$ is perpendicular to $\overrightarrow{AB}$ because $\overline{AB}$ is inside the plane & $v$ is a normal.

$\vec{v}$ is perpendicular to $\overrightarrow{AB}$ & $\vec{n}$, so we may take $\vec{v}=\overrightarrow{AB}\times\vec{n}$.

Note: normal vectors to a plane are not unique. If $\langle 1,0,2\rangle$ is a normal vector, so is $\langle -2,0,-4\rangle$.

$\overbrace{(0,-2,5)}^{A}$ $\underbrace{(-1,3,1)}_{B}$. $\underbrace{\langle 5,4,-2\rangle}_{\vec{n}}$.

$$\overrightarrow{AB}=B-A=\langle -1,5,-4\rangle.$$

$$\begin{vmatrix} i & j & k \\ -1 & 5 & -4 \\ 5 & 4 & -2 \end{vmatrix}$$

$\vec{v} = \vec{AB} \times \vec{n} = \langle 6, -22, -29 \rangle$

Plane equation is

$$6(x-0) - 22(y+2) - 29(z-5) = 0$$

8: Evaluate $\lim_{t \to 1} \left\langle \frac{t^2-1}{t^2-3t+2}, \frac{t-1}{\sqrt{t+3}-2}, \frac{\sin(t-1)}{t-1} \right\rangle$.

Take limit of each component. If any limit DNE, whole limit DNE. If all exist, then $\lim \langle \cdot\cdot, \cdot\cdot, \cdot\cdot \rangle = \langle \lim \cdot\cdot, \lim \cdot\cdot, \lim \cdot\cdot \rangle$.

1st limit: $\frac{0}{0}$ when $t=1$, so use L'Hopital

or factor. $L = \lim_{t \to 1} \frac{(t-1)(t+1)}{(t-1)(t-2)} = \lim_{t \to 1} \frac{t+1}{t-2}$

$= \frac{2}{-1} = \underline{-2}$

2nd limit: $\frac{t-1}{\sqrt{t+3}-2} = \frac{\cancel{(t-1)}(\sqrt{t+3}+2)}{\cancel{t+3-4}}$

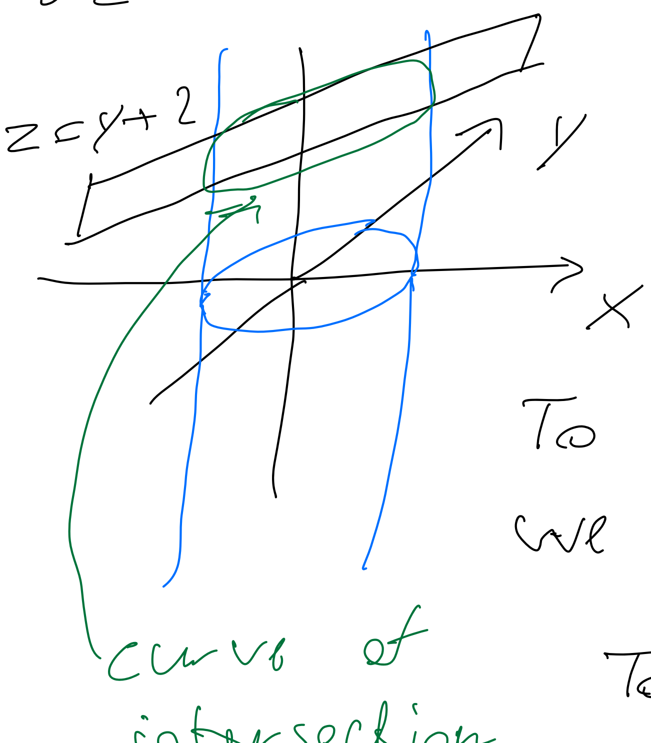
$= \sqrt{t+3}+2 \longrightarrow \sqrt{1+3}+2 = \underline{4}$

3rd limit: $\lim_{t \to 1} \frac{\sin(t-1)}{t-1} = \lim_{t \to 0} \frac{\sin t}{t} = \underline{1}$

All 3 limits exist, so $\text{ans} = \langle -2, 4, 1 \rangle$

General remark: Lecture quizzes only cover material up to the week before the quiz. The stuff the week of quiz isn't tested.

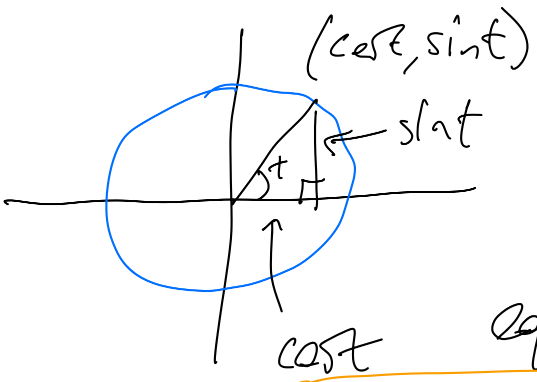
9. Find vector function parametrizing intersection of $x^2+y^2=1$ & $z=y+2$ exactly once and sketch the curve.




$x^2+y^2=1$ is unit circle in xy-plane, so it's a cylinder in 3D.

To parametrize in 3D, first we start with a 2D param.

To cover unit circle exactly once, $(x,y)=(\cos t, \sin t)$ with $0 \le t < 2\pi$.




Plug this into the 2nd equation: $z = y+2 = \sin t + 2$.

Thus, $(x,y,z) = (\cos t, \sin t, \sin t + 2)$, $0 \le t < 2\pi$

describes the curve traversed once.

3: Param line of intersection of $\begin{cases} 2x+3y+5z=7 \\ x-y+2z=3 \end{cases}$

Motivation for solution: we have a system of 2 linear equations & 3 variables

Solve 2nd equation for x to get

$x = 3+y-2z$

Note: there are many ways of solving system, any method is ok as long as there's no mistakes & method produces right answer.

Plug this into the 1st equation:

$7 = 3y+5z+2(3+y-2z) = 6+5y+z \Rightarrow$

$z = 1-5y$. Then

$x = 3+y-2(1-5y) = 1+11y$

$(x,y,z) = (1+11y, y, 1-5y), \; y \in \mathbb{R}$

is a parametrization of the line.

$(1+11t, t, 1-5t), t \in \mathbb{R}$

Use whatever letter you want...

There are $\geq 12$ ways of solving this problem: 2 choices for 1st equation, 3 choices for 1st variable to solve, 2 choices for final variable to eliminate, and $2 \cdot 3 \cdot 2 = 12$.

WK3 #8: (a) $L_1$: $r(t) = \langle -1+3t, 2+4t, 3-2t \rangle$

$$L_2: \frac{x-1}{2} = \frac{y}{-3} = \frac{z+1}{-3}$$

(b) $L_1$: $x = 2t$, $y = -3+t$, $z = 5-t$

$L_2$: $x = 3-3s$, $y = 2-1.5s$, $z = 1.5s$

For each part, are they parallel, intersecting, or skew? If intersect, find angle of intersection.

(a): In order for lines to intersect, same point needs to be on both lines. So plug 1st equation into 2nd equation & solve.

$$L_2: \frac{x-1}{2} = \frac{y}{-3} = \frac{z+1}{-3}$$

$$r(t) = \langle -1+3t, 2+4t, 3-2t \rangle$$

Direction of $r(t)$:

$$r(t) = \langle -1, 2, 3 \rangle + \boxed{\langle 3, 4, -2 \rangle} t$$

$$L_2: \frac{1}{2}(x-1) \quad -\frac{1}{3}y \quad -\frac{1}{3}(z+1)$$

$$\boxed{\langle 2, -3, -3 \rangle}$$

$$\frac{-1+3t-1}{2} = \frac{2+4t}{-3}$$

$$-3(-2+3t) = 2(2+4t)$$

$$6 - 9t = 4 + 8t$$

$$2 = 17t$$

$$t = 2/17$$

$-3(2+4t) = -3(3-2t+1) \Rightarrow 2+4t = 4-2t$

$\Rightarrow 6t = 2 \Rightarrow t = 1/3$

But $\frac{1}{3} \neq \frac{2}{17}$, so no solution, so $L_1$ & $L_2$ don't intersect.

Note: 2 lines are parallel iff their direction vectors are parallel.

Direction vectors of $L_1$ & $L_2$ are $\langle 3,4,-2\rangle$ and $\langle 2,-3,-3\rangle$, which are not parallel. Thus, $L_1$ & $L_2$ not parallel. So they're skew.

(b): Direction vectors are $\langle 2,1,-1\rangle$ & $\langle -3,-1.5,1.5\rangle$ which are parallel since $-1.5\langle 2,1,-1\rangle = \langle -3,-1.5,1.5\rangle$

$x = 2t$, $y = -3+t$, $z = 5-t$
$x = 3-3s$, $y = 2-1.5s$, $z = 1.5s$

$L_1: \langle 2,1,-1\rangle$
$L_2: \langle -3,-1.5,1.5\rangle$

$\Rightarrow L_1$ & $L_2$ are parallel.

Do $L_1$ & $L_2$ intersect?

Look for a point on both lines

$$\begin{cases} 2t = 3-3s \\ -3+t = 2-1.5s \\ 5-t = 1.5s \end{cases}$$

$$\begin{array}{r} 2t = 3-3s \\ -\;\; -6+2t = 4-3s \\ \hline 6 = -1 \end{array}$$

, so no solution to system of equations, so $L_1$ & $L_2$ don't intersect.