Global Extrema

Pre-lecture for 6/24

Summary

Suppose we want to find max or min for an entire region:

- Check that the max and min exist
- Any global extrema is either a local extrema or on boundary
- Solve $\nabla f = \mathbf{0}$, check points within the region
- Find the boundary
- Check value of f on the boundary

Extrema Existence

Topology Facts:

- Maximum of continuous function on compact set exists
- Subsets of R^n are compact iff they are closed and bounded
    - Lookup Heine-Borel Theorem for more info
- A set is closed iff it contains its boundary points

Boundaries of Sets

Open sets in R^n

- Let $B_r(x) = \{y : ||x-y|| < r\}$ be the open ball of radius r
- Any open neighborhood of x contains a ball, conversely any ball counts as a neighborhood

Boundary Points

- A boundary point of a set S is a point x such that every neighborhood of x contains a point in S and a point outside of S

Tips to Remember

- If region described by $g \geq 0$ or $g>0$ with g continuous, then $g = 0$ is the boundary
- When described by $g \geq 0$, region is closed
- When described by $g > 0$, region is open
- R^n and empty set are both closed and open
- If the region isn't closed, try to show max and min don't exist
- Only use solutions to $\nabla f = \mathbf{0}$ in the region

Practice Problems

Find the maximum and minimum, or show they don't exist:

- $f(x, y) = x^2+y^2-xy^2+1$ on $-1 \leq x, y \leq 1$
- $f(x, y, z) = |x|+|y|+|z|$ on $x^2+y^2+z^2 < 1$
- $f(x, y) = 2x^2-y^2+6y$ on $x^2+y^2 \leq 16$

Scratchwork