Gradient

Lecture for 6/18

Motivation for Gradient

- For $f(x, y)$, we have seen f_x and f_y
  - They represent x and y direction
- Can we take derivatives along other directions?
- If so, how do we write them and work with them?

Directional Derivatives

- Let $\mathbf{u} = \langle a, b \rangle$ represent a direction, $\mathbf{v} = (x, y)$
- Define $D_{\mathbf{u}} f(\mathbf{v}) = \lim_{h \to 0} [f(\mathbf{v}+h\mathbf{u})-f(\mathbf{v})]/h$
- We'll only consider $D_{\mathbf{u}}$ for unit vectors $\mathbf{u}$
  - Bigger $\mathbf{u}$ in same direction give larger derivatives
- Generalizes to any number of variables

Directional Derivatives Formula

- If $u = \langle a, b \rangle$, then $D_u f(x,y) = af_x(x,y) + bf_y(x,y)$
- Define the operator ∇ by $\nabla f = \langle f_x, f_y \rangle$
- $D_u f = \langle a, b \rangle \cdot \langle f_x, f_y \rangle = u \cdot \nabla f$
- Notice what happens if $u = (1,0)$ or $(0,1)$
- Everything repeats in more dimensions

Rates of Change

- Maximum value of $D_u f$ occurs when u, ∇f are parallel
- Minimum value occurs in opposite direction

Gradients vs Graphs

Can we say anything about ∇f graphically?

- For constant c, let G_c be graph of $f(x, y) = c$
- Suppose $p = (x_0, y_0)$ lies on G_c
- $\nabla f(v)$ is normal to the graph of G_c at p
- Same exact deal with more variables

Practice Problems

Find these directional derivatives

- $f(x, y) = x\cos(y)$ in the direction of $(2, -1)$
- $f(x, y, z) = e^z + \sin(xy)$ in direction of $(3, 4)$

You are sitting on a hill whose height is $100-x^2-y^2$ at any point (x,y). If you are at $(4, 7)$, which direction is the descent steepest? Which direction would you be climbing the steepest?

Scratchwork