Class site: methyne.github.io/243s26
will be updated with more content by next week

3: $f(1)=7$, $f'(1)=4$, find tangent line to $y=\sqrt{4+3f(x)}$ at $x=1$.

Need slope & some point on line

If $x=1$, $y=\sqrt{4+3f(1)}=\sqrt{25}=5$, so $(1,5)$ is on the line.

Slope $= \frac{dy}{dx} = \frac{3f'(x)}{2\sqrt{4+3f(x)}}$. At $x=1$, this is $\frac{3f'(1)}{2\sqrt{4+3f(1)}} = \frac{12}{10} = \frac{6}{5}$,

now use point-slope form to get the tangent line is $\boxed{y-5=\frac{6}{5}(x-1)}$

42: $\int e^{\tan x}\sec^2 x\,dx$

Notice: $(\tan x)' = \sec^2 x$, so let

$u = \tan x \qquad du = \sec^2 x\,dx$

$$= \int e^u \, du = e^u + C = \boxed{e^{\tan x} + C}$$

6: $a(t) = 3\cos t - 2\sin t$, $s(0) = 0$, $v(0) = 4$, find the position $s(t)$.

$$\begin{tikzcd} s \arrow[d, bend left, "d"] \\ v \arrow[u, bend left, "\int"] \arrow[d, bend left, "d"] \\ a \arrow[u, bend left, "\int"] \end{tikzcd}$$

$$v(t) = \int a(t)\,dt = \int (3\cos t - 2\sin t)\,dt = 3\sin t + 2\cos t + C,$$

Now use the info on initial values

$$4 = v(0) = 3 \cdot 0 + 2 \cdot 1 + C = 2 + C \Rightarrow C = 2 \Rightarrow v = 3\sin t + 2\cos t + 2.$$

$$s(t) = \int v(t)\,dt = \int (3st + 2ct + 2) = -3\cos t + 2\sin t + 2t + C.$$

$$0 = s(0) = -3 + C \Rightarrow C = 3,$$

so $s(t) = -3\cos t + 2\sin t + 2t + 3$

52: $\int_0^{\pi/2} \sin^2\theta \cos\theta \, d\theta$

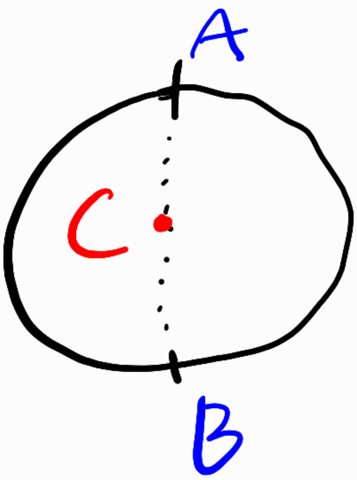
$u = \sin\theta$, $du = \cos\theta \, d\theta$

$\theta = 0 \Rightarrow u = \sin 0 = 0$

$\theta = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$

$$= \int_0^1 u^2 \, du = \frac{u^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

9: Find sphere equation if endpoints $\underbrace{(1,2,4)}_{A}$ & $\underbrace{(4,3,10)}_{B}$



Recall! we need center & radius for equation.

Diameter $= \text{length}(AB) = |\overrightarrow{AB}|$

$= |A-B| = |B-A| = |\langle 3,1,6 \rangle|$

$= \sqrt{3^2+1^2+6^2} = \sqrt{46}$, so

$$\text{radius} = \frac{1}{2}d = \frac{1}{2}\sqrt{46} = \sqrt{\frac{23}{2}}$$

Let C be center of sphere.

C is midpoint of A & B, so

$$C = \frac{1}{2}(A+B) = \frac{1}{2}(5, 5, 14)$$
$$= (2.5, 2.5, 7).$$

Now plug into sphere equation:

$$\boxed{(x-2.5)^2 + (y-2.5)^2 + (z-7)^2 = \frac{23}{2}}$$

46: $\int (2x^4+1)\ln x\,dx$

To get rid of $\ln x$, let $u = \ln x$.

Then $x = e^u \Rightarrow dx = e^u du$, so

$(2x^4+1)\ln x\,dx = (2e^{4u}+1)ue^u du$,

and the integral becomes

$$2\int ue^{5u}du + \int ue^u du.$$

$$\int ue^{5u}\,du = \int v\,dw =$$

$$\boxed{\begin{array}{l} v=u,\ dw=e^{5u}du \\ dv=du,\ w=\frac{1}{5}e^{5u} \end{array}}\quad vw-\int w\,dv$$

$$= \frac{1}{5}ue^{5u} - \int \frac{1}{5}e^{5u}\,du =$$

$$\frac{1}{5}ue^{5u} - \frac{1}{25}e^{5u}+C =$$

$$\frac{1}{25}(5u-1)e^{5u}+C$$

$$\int ue^{u}\,du = ue^{u} - \int e^{u}du$$

$$\boxed{\begin{array}{l} v=u,\ dw=e^{u}du \\ dv=du,\ w=e^{u} \end{array}}\quad \begin{array}{l} = ue^{u}-e^{u} \\ = (u-1)e^{u} \end{array}$$

So $\text{int} = \frac{1}{25}(5u-1)e^{5u}+(u-1)e^{u}+C$

$$= \boxed{\frac{1}{25}(5\ln x-1)x^5+(\ln x-1)x+C}$$

Note: $e^{5u}=(e^{u})^5=(e^{\ln x})^5=x^5$

$$\begin{tikzcd} ue^u \arrow[r, "d"] & (u+1)e^u \arrow[r, "d"] & (u+2)e^u \arrow[d] \\ (u-1)e^u \arrow[u, "d"] & & \vdots \end{tikzcd}$$

$$ue^{5u} \to (5u+1)\cdot e^{5u}$$

$$1e^{5u} \to 5 \cdot e^{5u}$$

$$\frac{u}{5}e^{5u} \xrightarrow{d} \left(u + \frac{1}{5}\right)e^{5u}$$

$$-\frac{1}{25}e^{5u} \xrightarrow{d} -\frac{1}{5}e^{5u}$$

$$\left(\frac{u}{5} - \frac{1}{25}\right)e^{5u} \xrightarrow{d} ue^{5u}$$

Note! try this with $e^x\cos x$, $e^x\sin x$, x^2e^x, ...

2: find $\frac{dy}{dx}$ when $x\ln y + e^y = 3$.

Apply $\frac{d}{dx}$ to both sides:

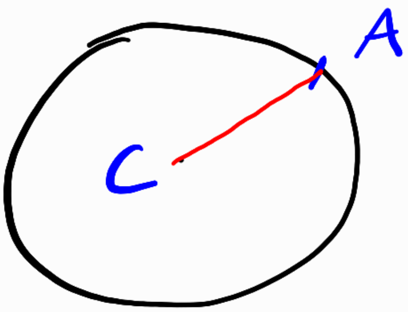
$$0 = \frac{d}{dx}(3) = \frac{d}{dx}(x\ln y + e^y) = \ln y + x\frac{d}{dx}\ln y + e^y\frac{dy}{dx} = \ln y + \frac{dy}{dx}\left(\frac{x\ln y}{y} + e^y\right).$$

$$\boxed{\frac{d}{dx}\ln y = \frac{1}{y}\frac{dy}{dx}}$$

Now we can solve for dy/dx:

$$\frac{dy}{dx} = \frac{-\ln y}{\frac{x\ln y}{y} + e^y} = \boxed{\frac{-y\ln y}{x\ln y + ye^y}}$$

82: Find sphere equation if center $= \underbrace{(3,1,-3)}_{C}$ & sphere contains $\underbrace{(1,8,5)}_{A}$



Standard sphere equation requires center & radius.

Notice $r = |\overrightarrow{CA}| = |C-A| = |\langle 2,-7,-8\rangle|$
$= \sqrt{2^2+(-7)^2+(-8)^2} = \sqrt{53+64} = \sqrt{117}$

Recall: $(x-a)^2+(y-b)^2+(z-c)^2 = r^2$ is sphere with radius r & center (a,b,c).

So eq. is $\boxed{(x-3)^2+(y-1)^2+(z+3)^2=117}$

(b) find eq. for intersection of the sphere with yz-plane.

Recall: yz-plane is just $x=0$.

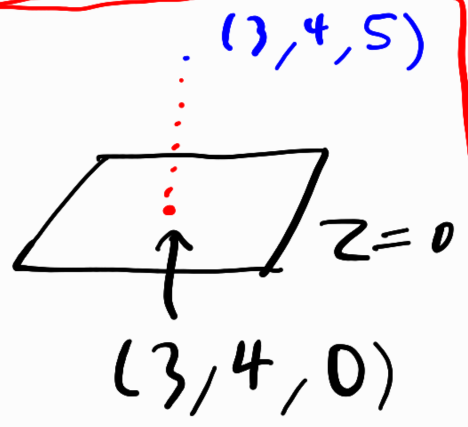
To get intersection of sphere & plane, consider solving the system of equations for the plane & sphere. Since $x=0$ is the simpler equation, plug it into the other equation:

$$117=(-3)^2+(y-1)^2+(z+3)^2 \Rightarrow$$

$$\boxed{(y-1)^2+(z+3)^2=108}$$

7: $P=(3,4,5)$. Find its projection to xy-plane, xz-plane. Find $|\overline{OP}|$. Lastly, find position vector for P in ijk-form.

(a) Recall that xy-plane is $z=0$.



Since $z=0$ on the plane, the projection is $\boxed{(3,4,0)}$

(b) xz-plane is $y=0$, so proj $= \boxed{(3,0,5)}$

(c) Recall $O=(0,0,0)$ is the origin,

$|\overrightarrow{OP}| = |P-O| = |\langle 3,4,5\rangle| = \sqrt{3^2+4^2+5^2} = \sqrt{50} = \boxed{5\sqrt{2}}$

(d) To convert a point\vector in coordinate form to ijk form, recall $\hat{i} \leftrightarrow x$, $\hat{j} \leftrightarrow y$, $\hat{k} \leftrightarrow z$, so $P=(3,4,5) = 3\hat{i}+4\hat{j}+5\hat{k}$.

Note: $\hat{i}, \hat{j}, \hat{k}$ are really just the names for the vectors $\langle 1,0,0\rangle$, $\langle 0,1,0\rangle$, $\langle 0,0,1\rangle$ respectively.

56: $$\int_0^3 \frac{x}{\sqrt{x^2+1}}\,dx$$

Notice $(x^2+1)' = 2x$ is almost x.

Let $u = x^2+1 \Rightarrow du = 2x\,dx$, so

$\frac{1}{2}du = x\,dx$. $x=0 \Rightarrow u=1$, $x=3 \Rightarrow u=10$

New integral is $\displaystyle\int_1^{10} \frac{\frac{1}{2}du}{\sqrt{u}} = \int_1^{10} \frac{du}{2\sqrt{u}}$

$= \sqrt{u}\Big|_1^{10} = \boxed{\sqrt{10} - 1}$