MATH 243 Quiz 3

1. Select all choices for which the global minimum or the global maximum exist

A. $f(x,y) = \cos(x) + \frac{\sin(y)}{1+x^2}$ on $D = (-\pi/2, \pi) \times (-\pi/2, \pi)$

B. $f(x,y) = \frac{1}{1+(x-y)^2} e^{-|x+y|}$ on the entire plane

C. $f(x,y,z) = x^3 + y^3 + z^3$ on $D = \{(x,y,z) : x^2 + y^2 + z^2 < 1\}$

D. $f(x,y,z) = e^x + e^{y-x} + e^{z-y-x}$ on $D = \{(x,y,z) : 0 \le x, y, z \le 1\}$

2. Select the integral with the largest value. Hint: many of the integrals may be equivalent

A. $\int_0^1 \int_{x^4}^{\sqrt{x}} dy\, dx$

B. $2\int_0^1 \int_{x^4}^{x} dy\, dx$

C. $\int_0^1 \int_{y^2}^{y^{1/4}} dx\, dy$

D. $2\int_0^1 \int_{\tan^{-1}(r^3)}^{\pi/4} r\, d\theta\, dr$

3. Let M be the maximum of $x + y^2 + z$ given $x^2 + y^2 + z^2 = 1$. We can express $M^2 = \frac{a}{b}$ where a, b are positive integers and the fraction is in lowest terms. Find $10a + b$.

4. Let V be the volume of the solid bounded by $z = x^2 + y^2, z = 12$, and $x^2 + y^2 = 9$. We can express $V = \frac{a}{b}\pi$ where a, b are positive integers and the fraction is in lowest terms. Find $10a + b$.

5. Classify all critical points of $f(x,y) = -x^5 + x^2y + yx^3 - y^2$

6. Once again, we run into $f(x,y) = x^2 + y^2$. Find the surface area of the portion of $z = f(x,y)$ over the unit disk. Next, find the center of mass of the square $\{(x,y) : 0 \le x, y \le 1\}$ with weight function f.