Worksheet 7: The Curled Diverging Green Line Surfaces

1: Show the following vector calculus identities:
a. $\nabla \times fG = \nabla f \times G + f(\nabla \times G)$
b. $\nabla \times \nabla f = 0$
c. $\nabla \cdot (F \times G) = (\nabla \times F) \cdot G - F \cdot (\nabla \times G)$

2: Let C be the triangle with vertices $(-3, 0), (0, 0), (0, 3)$ oriented clockwise. Verify Green's Theorem for $\int_C (xy^2 + x)\, dx + (4x - 1)\, dy$ by computing both the line integral and the corresponding double integral

3: Find a formula for $\nabla \times (\nabla \times F)$ and justify your claim

4: Evaluate $\iint_S f\, dS$ for the following functions and surfaces:
a. $f(x, y, z) = 6xy, S$ is upper half of sphere of radius 1
b. $f(x, y, z) = y + z, S$ is the surface with sides given by the cylinder $x^2 + y^2 = 3$, bottom given by the disk $x^2 + y^2 \leq 3$, and $z = 4 - y$ on top

5: To be continued