Rectangular 2D Integrals

Pre-lecture for 6/25

Volume under a Curve

- In Calc 1, we have area under curve $y = f(x)$
- Concept is now volume under curve $z = f(x, y)$
- If region is R, let $\iint_R f(x, y)\, dA$ be this volume

Riemann Sums

- Suppose R is a rectangle $a \leq x \leq b$, $c \leq y \leq d$
- Can approximate $\iint_R f(x, y)\, dx\, dy$ with Riemann sums

Fubini's Theorem

- $\iint_R f(x, y)\, dx\, dy = \int\left(\int f(x, y)\, dx\right) dy = \int\left(\int f(x, y)\, dy\right) dx$
  - If f is Riemann integrable

Practice Problems

Evaluate $\iint_R f(x, y)\, dA$ for these functions and regions:

- $f(x, y) = x\cos^2(y)$, $R = [0, 3] \times [0, \pi/2]$
- $f(x, y) = 2x - 4y^3$, $R = [4, 5] \times [0, 3]$
- $f(x, y) = xy + \cos(x) + \sin(y)$, $R = [0, 1] \times [0, 1]$

Scratchwork