MATH 243 Worksheet 3: Review, Limits, Partial Derivatives

0: Bonuses:
a. When the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular, $\mathbf{0}, \mathbf{u}, \mathbf{v}$ as points form a right triangle. Use this observation to prove the dot product property. Use the law of cosines to provide an alternative proof for the formula for the angle between $\mathbf{u}$ and $\mathbf{v}$.
b. Use the volume of the parallelepiped formed by $\mathbf{a}, \mathbf{b}, \mathbf{a} \times \mathbf{b}$ and the law of sines to prove the formula $\|a \times b\|$.

1: Leftover problems from lecture slides:
a. If the acceleration is given by $\mathbf{a} = \langle 1, 2, 6t \rangle$, find the position $\mathbf{r}$ given that $\mathbf{v}(0) = \langle 0, 1, -1 \rangle$ and $\mathbf{r}(0) = \langle 1, -2, 3 \rangle$. Also, find the tangential and normal components of acceleration.
b. Find the tangential and normal components of acceleration for the object whose position is $\mathbf{r}(t) = \langle \cos(2t), -\sin(2t), 4t \rangle$.
c. Find all partial derivatives for these functions: $x^y + y^x + e^{xy}$, $x^4 \sin(3y) - \frac{x}{y} + \cos(\frac{x}{y})$, $\frac{xyz}{x+y+z}$
d. Let c be a constant and $g(x) = f(x, c)$. Show that $f_x(x, c) = g'(x)$. This vindicates our idea that the partial derivative produces the same value as plugging in constants and taking an ordinary derivative.

1: Find $\displaystyle\lim_{(x,y)\to(3,2)} e^{\sqrt{2x-y}}$ or show it doesn't exist

2: Find $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2y^3 - x^3y^2}{x^2 - y^2}$ or show it doesn't exist

3: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 3y^2}$ or show it doesn't exist

4: Show these limits exist and find them, or show they don't exist

(a) $\displaystyle\lim_{(x,y)\to(2,3)} \frac{3x - 2y}{4x^2 - y^2}.$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x - 2y}{4x^2 - y^2}.$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}.$

5: Determine the domain of $F(x, y) = \dfrac{1 + x^2 + y^2}{1 - x^2 - y^2}$ and find where it is continuous

6: Find the first partial derivatives of the function $g(u, v) = (u^2v - v^3)^5$

7: Given the function $f(x, y) = y \arcsin(xy)$, find $f_y(1, \frac{1}{2})$