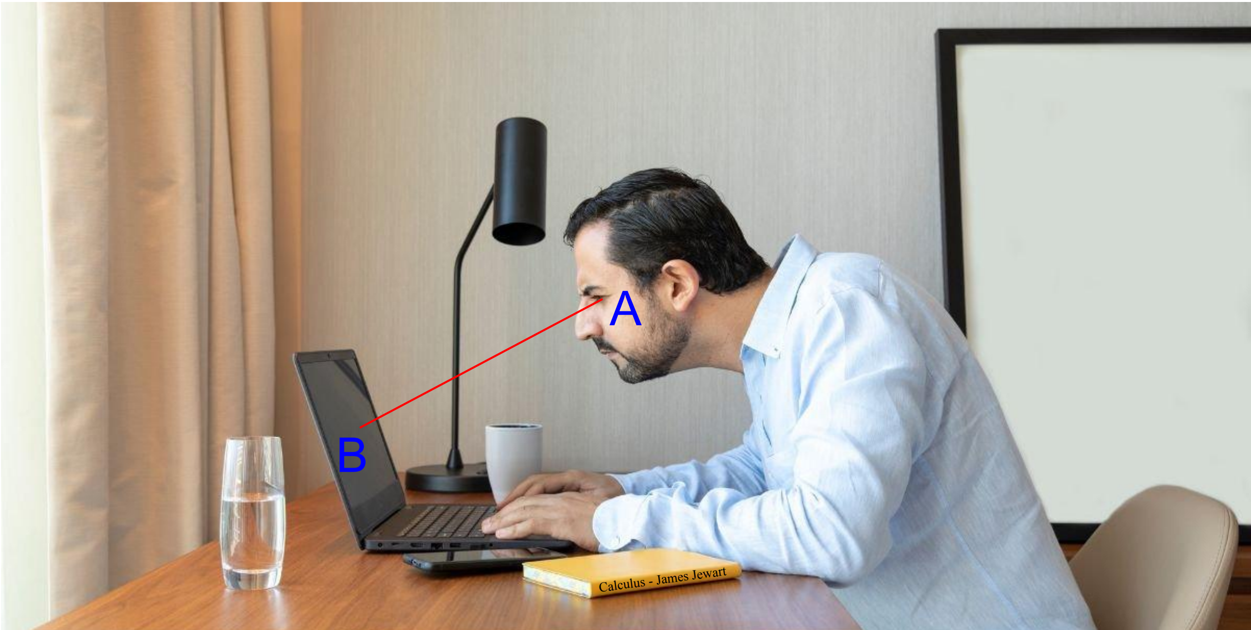
# Equation of Lines and Planes

Lecture for 6/10

# Equation of Lines

- In 3D, impossible with one normal equation
- So how do we do it? 🧐
- A to B is B-A
- Do A+(B-A)t
- Segment: $0 \leq t \leq 1$




Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

# Parametric and Vector Form of Line

- Suppose $\mathbf{r}(t)$ parametrizes a line
- Previous slide gives $\mathbf{r}(t) = \langle a+bt, c+dt, e+ft\rangle$ for constants a-f
- This is the vector form
- Parametric form: $x = a+bt$, $y = c+dt$, $z = e+ft$
- Seems identical, but difference will be useful later
  - End of class: surface integrals, parameterizations

# Equation of Planes Idea

- We can do one standard equation to describe
  - $x = 0$, $y = 0$, $x+y = z$ etc. are all planes
- Set of points perpendicular to given vector is a plane
- Can we reverse this to get vector for plane?

# General Derivation of Equation

- Suppose a normal vector is $\mathbf{v}$
- Consider vector $\mathbf{u}$ in the plane
- Equation $\mathbf{u} \cdot \mathbf{v} = 0$
- In practice, need to find what $\mathbf{u}$ and $\mathbf{v}$ are
    - Plane has 3 degrees of freedom
    - Exhaust degrees: vector is 2, point is 1

# Example Case: vector and point

- $2+1 = 3$, so it's possible
- Let given normal be $\mathbf{v} = (a, b, c)$
- Let $\mathbf{v}_0 = (x_0,y_0,z_0)$ be given point in the plane
- Let $\mathbf{w} = (x, y, z)$ be any point in the plane
- Obtain $0 = \mathbf{v} \cdot (\mathbf{w}-\mathbf{v}_0)$
- Simplify: $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$

# Practice problems

Understand your lines

- Find the line passing through (2, -1, 3) and (1, 4, -3)

Try the point point point case

- Find the equation of the plane containing (1, -2, 0), (3, 1, 4), and (0, -1, 2)

# Scratch Work

# Extra Problem

Extra problem on spatial awareness

- Determine if $-x+2z = 10$ and $\langle 5, 2-t, 10+4t\rangle$ are perpendicular, parallel, or neither