University of Delaware - Department of Mathematical Sciences

MATH 243 Midterm Exam 1 - Spring 2024

Tuesday 12^{th} March, 2024

Instructions:

- The time allowed for completing this exam is **75** minutes in total.
- Check your examination booklet before you start. There should be **4** questions on **5** pages.
- Turn off your cell phone and put it away. Headsets, earbuds and any other electronic devices are prohibited.
- No calculators.
- Answer the questions in the space provided. If you need more space for an answer, continue your answer on the back of the page and/or the margins of the test pages. No extra paper. *Do not separate the pages from the exam booklet.*
- For full credit, sufficient work must be shown to justify your answer.
- Partial credit will not be given if appropriate work is not shown.
- Write legibly and clearly; indicate your final answer to every problem. Cross out any work that you do not want graded. If you produce multiple solutions for a problem, indicate clearly which one you want graded.
- **Any form of academic misconduct will result in a failing grade.**

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

1. Let the curve $\mathcal{C}$ be given by the vector function $\mathbf{r}(t) = \dfrac{1}{t^2+1}\mathbf{i} + \sin\left(2t + \dfrac{\pi}{3}\right)\mathbf{j} + (\sqrt[3]{t+8})\,\mathbf{k}$.

(a) (6 points) Find the coordinates of the point P on the curve $\mathcal{C}$ corresponding to $t = 0$.

(b) (6 points) Determine the vector function $\mathbf{r}'(t)$. Fully simplify your answer.

(c) (6 points) Find a scalar equation of the **normal plane** to the curve $\mathcal{C}$ at the point where $t = 0$.

(d) (7 points) Write **parametric equations** of the tangent line to the curve $\mathcal{C}$ at the point $P\left(1, \frac{\sqrt{3}}{2}, 2\right)$.

2. Given that the **velocity** vector function of a particle is $\mathbf{v}(t) = 2\mathbf{i}+(te^{3t-3})\mathbf{j}+(4t^3-2t-5)\mathbf{k}$, and that the **initial position** of the particle is $\mathbf{r}(0) = \mathbf{i}+\mathbf{j}+\mathbf{k}$, find the following.

(a) (8 points) The speed of the particle at the point corresponding to $t = 1$.

(b) (8 points) The acceleration of the particle at the point corresponding to $t = 1$.

(c) (9 points) The position of the particle at any time t, given that $\mathbf{r}(0) = \mathbf{i}+\mathbf{j}+\mathbf{k}$.

3. Given the points

$$A(2,0,1), \quad B(-1,1,2), \quad C(4,1,-1),$$

find the following.

(a) (3 points) The vector $\overrightarrow{AB}$.

(b) (3 points) The vector $\overrightarrow{AC}$.

(c) (6 points) A unit vector that has the same direction as the vector $\overrightarrow{AC}$.

(d) (7 points) The vector projection of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$. Fully simplify your answer.

(e) (6 points) The area of the triangle ABC.

4. Let $z = f(x, y) = 2x \ln\big(x + y^2\big)$ be a function of two variables. Find the following.

(a) (5 points) $f(e, 0)$

(b) (5 points) $\dfrac{\partial z}{\partial y}$

(c) (5 points) The rate of change of $f(x, y)$ with respect to x when y is held fixed.

(d) (5 points) $\dfrac{\partial^2 z}{\partial y \partial x}$

(e) (5 points) The **slope** of the tangent line to the curve of intersection of the surface $z = f(x, y) = 2x \ln(x + y^2)$ with the vertical plane $y = 0$, at the point $P(e, 0, 2e)$.