F25 MATH 243: Lecture Quiz 1A Friday 5$^{\text{th}}$ September, 2025 **Answer Key**

Try each problem, showing all your work. Partial credit may be given, but unsupported answers will receive no credit. Cellphones, calculators, computers, and other electronic devices are prohibited.

Quiz time: **30** minutes.

1. Differentiate each function with respect to x.

   (a) (2 points) $f(x) = \sinh(3x^2 + e^{2x})$.

   Let $u(x) = 3x^2 + e^{2x}$. Then $f(x) = \sinh(u)$, so $f'(x) = \cosh(u)\, u'(x)$.
   Compute $u'(x) = 6x + 2e^{2x}$. Hence

   $$\boxed{f'(x) = \cosh\big(3x^2 + e^{2x}\big)\,\big(6x + 2e^{2x}\big)}.$$

   (b) (3 points) $g(x) = \sqrt{4 + x\ln(2x)}$.

   Rewrite $g(x) = (4 + x\ln(2x))^{1/2}$. By chain rule,

   $$g'(x) = \tfrac{1}{2}(4 + x\ln(2x))^{-1/2} \cdot \frac{d}{dx}[x\ln(2x)].$$

   Since $\frac{d}{dx}[\ln(2x)] = \frac{1}{x}$, we get

   $$\frac{d}{dx}[x\ln(2x)] = \ln(2x) + x \cdot \frac{1}{x} = \ln(2x) + 1.$$

   Therefore

   $$\boxed{g'(x) = \frac{\ln(2x) + 1}{2\sqrt{4 + x\ln(2x)}}}.$$

   (c) (3 points) $h(x) = 7^{\tan(2x)}$.

   Let $u(x) = \tan(2x)$. Then $h(x) = 7^{u(x)}$, so $h'(x) = 7^{u(x)} \ln 7 \cdot u'(x)$.
   Now $u'(x) = \sec^2(2x) \cdot 2$. Thus

   $$\boxed{h'(x) = 2\,\ln(7)\, 7^{\tan(2x)} \sec^2(2x)}.$$

2. (3 points) Find an equation of the sphere with center $(1, -3, 4)$ and diameter 10.

   Radius $r = \frac{10}{2} = 5$. Standard form: $(x-1)^2 + (y+3)^2 + (z-4)^2 = r^2 = 25$. Hence

   $$\boxed{(x-1)^2 + (y+3)^2 + (z-4)^2 = 25}.$$

3. (3 points) Find the unit vector in the same direction as $\overrightarrow{PQ}$, where $P(-1,4,0)$ and $Q(5,1,8)$.

$$\overrightarrow{PQ} = \langle 5-(-1),\ 1-4,\ 8-0\rangle = \langle 6,-3,8\rangle.$$

Magnitude: $|\overrightarrow{PQ}| = \sqrt{6^2+(-3)^2+8^2} = \sqrt{36+9+64} = \sqrt{109}.$

Unit vector:

$$\boxed{\mathbf{u} = \frac{1}{\sqrt{109}}\langle 6,-3,8\rangle = \left\langle \tfrac{6}{\sqrt{109}}, -\tfrac{3}{\sqrt{109}}, \tfrac{8}{\sqrt{109}} \right\rangle}.$$

4. Evaluate the following integrals.

(a) (3 points) $\displaystyle\int_1^3 \sqrt[3]{2+4t}\,dt$

Let $u = 2+4t$, so $du = 4\,dt$ and $dt = \frac{1}{4}du$. Limits: $t=1 \Rightarrow u=6$, $t=3 \Rightarrow u=14$.

$$\int_1^3 (2+4t)^{1/3}\,dt = \int_6^{14} \frac{u^{1/3}}{4}\,du = \frac{1}{4}\cdot\frac{3}{4}u^{4/3}\Big|_6^{14} = \boxed{\frac{3}{16}\left(14^{4/3}-6^{4/3}\right)}.$$

(b) (3 points) $\displaystyle\int 3x\cos(4x)\,dx$

Integration by parts: $u = 3x \Rightarrow du = 3\,dx$, $dv = \cos(4x)\,dx \Rightarrow v = \frac{1}{4}\sin(4x)$.

$$\int 3x\cos(4x)\,dx = \frac{3x}{4}\sin(4x) - \int \frac{3}{4}\sin(4x)\,dx = \frac{3x}{4}\sin(4x) + \frac{3}{16}\cos(4x) + C.$$

$$\boxed{\int 3x\cos(4x)\,dx = \frac{3x}{4}\sin(4x) + \frac{3}{16}\cos(4x) + C}.$$