MATH 243: Analytic Geometry & Calculus C **Name:** ________________
Practice Problem Set – Solutions (Corrected 1–5, Detailed Q3–Q4) Date: Friday 17$^{\text{th}}$ October, 2025

Instructions: These are detailed solutions to Problems 1–5. Final answers are highlighted in **boldface** and enclosed in $\boxed{}$.

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1. (2 points) Which of the following paths is **NOT** appropriate to use for showing that
$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$$
does not exist? Justify your answer briefly.
   A. $y = x$
   B. $y = x^2$
   C. $x = 0$
   D. $y = -x^2$
   E. $y = x + 1$

Solution: To use a path to study $\lim\limits_{(x,y)\to(0,0)}$, the path *must* pass through $(0,0)$. The first four paths ($y = x,\ y = x^2,\ x = 0,\ y = -x^2$) all pass through $(0,0)$ and are therefore appropriate. The path $y = x + 1$ does *not* pass through $(0,0)$ (as $x \to 0$, $(x, y) \to (0, 1)$), so it cannot be used to test a limit at $(0,0)$.

$$\boxed{\textbf{Answer: } y = x + 1.}$$

2. (3 points) Show that
$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$
does not exist. (Hint: Compare the limits along at least two different paths.)

Solution: Along $y = 0$:
$$\frac{x^2-0}{x^2+0} = 1 \quad \Rightarrow \quad \lim_{x\to0} \frac{x^2-y^2}{x^2+y^2} = 1.$$
Along $x = 0$:
$$\frac{0-y^2}{0+y^2} = -1 \quad \Rightarrow \quad \lim_{y\to0} \frac{x^2-y^2}{x^2+y^2} = -1.$$
Since the limits along two paths differ, the two-variable limit does not exist.

$$\boxed{\textbf{Final: The limit does not exist.}}$$

3. (3 points) The limit exists. Find its value:

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2+y^2+1}-1}{x^2+y^2}.$$

Show your steps clearly.

Solution: Step 1 (Notation). Let $S := x^2+y^2 \geq 0$ (with $S>0$ for $(x,y)\neq(0,0)$). Define

$$L(x,y) = \frac{\sqrt{x^2+y^2+1}-1}{x^2+y^2} = \frac{\sqrt{S+1}-1}{S} \quad (S>0).$$

Step 2 (Conjugate trick).

$$\frac{\sqrt{S+1}-1}{S}\cdot\frac{\sqrt{S+1}+1}{\sqrt{S+1}+1} = \frac{(\sqrt{S+1}-1)(\sqrt{S+1}+1)}{S(\sqrt{S+1}+1)} = \frac{S}{S(\sqrt{S+1}+1)} = \frac{1}{\sqrt{S+1}+1}.$$

Thus, for $(x,y)\neq(0,0)$,

$$L(x,y) = \frac{1}{\sqrt{x^2+y^2+1}+1}.$$

Step 3 (Limit). As $(x,y)\to(0,0)$, $S\to 0$, and by continuity,

$$\lim_{(x,y)\to(0,0)} L(x,y) = \frac{1}{\sqrt{0+1}+1} = \boxed{\frac{1}{2}}.$$

4. (3 points) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$$

by reasoning directly from simple bounds (no polar coordinates).

Solution: Step 1 (Start from $x^2+y^2\geq x^2$). For all (x,y), $x^2\geq 0$, $y^2\geq 0$, hence

$$x^2+y^2 \;\geq\; x^2 \qquad\Longrightarrow\qquad 0 \;\leq\; \frac{x^2}{x^2+y^2} \;\leq\; 1 \quad \text{for } (x,y)\neq(0,0).$$

Set

$$c(x,y) := \frac{x^2}{x^2+y^2} \quad\Rightarrow\quad 0\leq c(x,y)\leq 1.$$

Step 2 (Compare y with $|y|$ and scale by $c\geq 0$). We always have the two-sided bound

$$-|y| \;\leq\; y \;\leq\; |y|.$$

Multiplying the entire inequality by the nonnegative factor $c(x,y)$ preserves order:

$$-c(x,y)\,|y| \;\leq\; c(x,y)\,y \;\leq\; c(x,y)\,|y|.$$

Since $0 \le c(x,y) \le 1$, we also have $-|y| \le -c(x,y)|y|$ and $c(x,y)|y| \le |y|$. Chaining these gives the desired two-sided bound:

$$-|y| \ \le \ c(x,y)\,y \ \le \ |y|.$$

Step 3 (Rewrite $c(x,y)\,y$). By definition of c,

$$c(x,y)\,y = \frac{x^2}{x^2+y^2}\,y = \frac{x^2 y}{x^2+y^2}.$$

Therefore we have shown, for all $(x,y) \neq (0,0)$,

$$\boxed{-|y| \ \le \ \frac{x^2 y}{x^2+y^2} \ \le \ |y|}.$$

Step 4 (Apply the Squeeze Theorem). As $(x,y) \to (0,0)$, both bounding functions $-|y|$ and $|y|$ approach 0. Hence, by the Squeeze Theorem,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2} = \boxed{\mathbf{0}}.$$

5. (3 points) Find an example of a function $f(x,y)$ for which $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ along every line $y = mx$ but not 0 along some nonlinear path (e.g. $y = x^2$).

Solution: Let

$$f(x,y) = \frac{x^2 y}{x^4+y^2}, \qquad f(0,0) = 0.$$

Along any line $y = mx$:

$$f(x,mx) = \frac{mx^3}{x^4+m^2x^2} = \frac{mx}{x^2+m^2} \xrightarrow[x\to 0]{} 0.$$

Along the nonlinear path $y = x^2$:

$$f(x,x^2) = \frac{x^4}{x^4+x^4} = \frac{1}{2}.$$

Different path limits $\Rightarrow$ the overall limit does not exist.

$$\boxed{\textbf{Example works; overall limit DNE.}}$$

6. Let $f(x,y) = e^{xy^2}$.

(a) (2 points) Find an equation of the tangent plane to $z = f(x,y)$ at the point $(1,1,f(1,1))$.

Solution: We have

$$f_x(x,y) = y^2 e^{xy^2}, \qquad f_y(x,y) = 2xye^{xy^2}.$$

At $(1,1)$:

$$f_x(1,1) = 1 \cdot e^1 = e, \qquad f_y(1,1) = 2 \cdot 1 \cdot 1 \cdot e = 2e.$$

Since $f(1,1) = e^{1 \cdot 1^2} = e$, the tangent plane formula is

$$z - f(1,1) = f_x(1,1)(x-1) + f_y(1,1)(y-1),$$

so

$$z - e = e(x-1) + 2e(y-1).$$

$$\boxed{\textbf{Tangent plane: } z = e(x+2y-2).}$$

(b) (2 points) Determine the linearization $L(x,y)$ of f at the point $(1,1)$.

Solution: Linearization is

$$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) = e + e(x-1) + 2e(y-1).$$

$$\boxed{L(x,y) = e(x+2y-2).}$$

(c) (2 points) Use $L(x,y)$ to approximate $f(1.02, 0.98)$.

Solution: Substitute $x = 1.02$, $y = 0.98$:

$$L(1.02, 0.98) = e[1.02 + 2(0.98) - 2] = e(1.02 + 1.96 - 2) = e(0.98) = 0.98e.$$

$$\boxed{f(1.02, 0.98) \approx 0.98e \approx 2.664.}$$

7. Let $F(x,y,z) = x^2 + 2y^2 - 3z^2 = 3$ describe a surface S.

(a) (2 points) Verify if the point $(2,1,1)$ lies on the surface.

Solution: Compute

$$F(2,1,1) = 2^2 + 2(1)^2 - 3(1)^2 = 4 + 2 - 3 = 3.$$

Hence $(2,1,1)$ satisfies $F(x,y,z) = 3$.

$$\boxed{\textbf{Yes, the point lies on the surface.}}$$

(b) (4 points) Find the equation of the tangent plane to this surface at the point $(2,1,1)$.

Solution: For a level surface $F(x,y,z)=3$, the gradient ∇F is normal to the tangent plane:

$$\nabla F = (F_x, F_y, F_z) = (2x, 4y, -6z).$$

At $(2,1,1)$:

$$\nabla F(2,1,1) = (4,4,-6).$$

Equation of tangent plane:

$$4(x-2) + 4(y-1) - 6(z-1) = 0.$$

Simplify:

$$4x - 8 + 4y - 4 - 6z + 6 = 0 \Rightarrow 4x + 4y - 6z - 6 = 0.$$

$$\boxed{\textbf{Tangent plane: } 2x + 2y - 3z = 3.}$$

(c) (4 points) Find the parametric equations of the normal line to the surface at the same point.

Solution: A normal line at (x_0, y_0, z_0) has direction $\nabla F(x_0, y_0, z_0)$. At $(2,1,1)$, direction vector $= \langle 4, 4, -6 \rangle$.

$$x = 2 + 4t, \quad y = 1 + 4t, \quad z = 1 - 6t.$$

8. Let $z = x^3 - 3xy^2$.

(a) (3 points) Find the equation of the tangent plane to this surface at the point $(1,1,z_0)$, where $z_0 = f(1,1)$.

Solution: Compute partial derivatives:

$$f_x = 3x^2 - 3y^2, \qquad f_y = -6xy.$$

At $(1,1)$:

$$f_x(1,1) = 3 - 3 = 0, \qquad f_y(1,1) = -6.$$

Since $z_0 = f(1,1) = 1^3 - 3(1)(1)^2 = -2$, the tangent plane is

$$z - z_0 = f_x(1,1)(x-1) + f_y(1,1)(y-1) \Rightarrow z + 2 = 0(x-1) - 6(y-1).$$

$$\boxed{\textbf{Tangent plane: } z = -6y + 4.}$$

(b) (2 points) Interpret geometrically how the coefficients of the plane relate to $\nabla f(1,1)$.

Solution: For the surface $z = f(x, y)$, the tangent plane at (x_0, y_0, z_0) is

$$z - z_0 \;=\; f_x(x_0, y_0)\,(x - x_0) \;+\; f_y(x_0, y_0)\,(y - y_0).$$

Rewriting,

$$f_x(x_0, y_0)\,(x - x_0) \;+\; f_y(x_0, y_0)\,(y - y_0) \;-\; (z - z_0) = 0,$$

so a normal vector to the plane in $\mathbb{R}^3$ is

$$\langle\, f_x(x_0, y_0),\; f_y(x_0, y_0),\; -1\,\rangle.$$

At $(1, 1)$ we have $f_x(1, 1) = 0,\; f_y(1, 1) = -6$, hence a normal vector is

$$\langle 0,\, -6,\, -1\rangle,$$

while the (2D) gradient of f at $(1, 1)$ is

$$\nabla f(1, 1) = \langle 0,\, -6\rangle.$$

Geometric interpretation: the *coefficients of x and y* in the plane are exactly $f_x(1, 1)$ and $f_y(1, 1)$ (i.e. the components of $\nabla f(1, 1)$), and together with the coefficient -1 of z they form a normal vector $\langle f_x(1, 1), f_y(1, 1), -1\rangle$ to the plane. Meanwhile, $\nabla f(1, 1)$ lies in the xy-plane and points in the direction of steepest increase of the height function f.

$$\boxed{\textbf{Normal to the plane: } \langle 0, -6, -1\rangle, \quad \textbf{and} \;\; \nabla f(1, 1) = \langle 0, -6\rangle.}$$

9. Let $f(x, y) = x^2y + 3y^2$.

(a) (2 points) Compute the gradient $\nabla f(x, y)$.

Solution:

$$f_x = 2xy, \qquad f_y = x^2 + 6y.$$

$$\boxed{\nabla f(x, y) = \langle 2xy,\; x^2 + 6y\rangle.}$$

(b) (1 point) Evaluate ∇f at $P(1, 2)$.

Solution:

$$\nabla f(1, 2) = \langle 2(1)(2), 1^2 + 6(2)\rangle = \langle 4, 13\rangle.$$

$$\boxed{\nabla f(1, 2) = \langle 4, 13\rangle.}$$

(c) (2 points) Find the rate of change of f at $P(1, 2)$ in the direction of $\mathbf{u} = \langle 3/5,\, 4/5\rangle$.

Solution: Directional derivative:

$$D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u} = 4\left(\frac{3}{5}\right) + 13\left(\frac{4}{5}\right) = \frac{12+52}{5} = \frac{64}{5}.$$

$$\boxed{\textbf{Rate of change } = \frac{64}{5} = 12.8.}$$

(d) (1 point) In which direction does f increase most rapidly at P? What is the maximum rate of increase?

Solution: The direction of maximum increase is that of $\nabla f(1,2) = \langle 4, 13\rangle$. Maximum rate of increase is its magnitude:

$$|\nabla f(1,2)| = \sqrt{4^2 + 13^2} = \sqrt{185} \approx 13.6.$$

$$\boxed{\textbf{Direction: } \langle 4, 13\rangle, \quad \textbf{Max rate: } \sqrt{185} \approx 13.6.}$$

10. Let $f(x,y,z) = xyz + x^2z^2$.

(a) (2 points) Compute $\nabla f(x,y,z)$.

Solution:

$$f_x = yz + 2xz^2, \qquad f_y = xz, \qquad f_z = xy + 2x^2z.$$

$$\boxed{\nabla f = \langle yz + 2xz^2,\ xz,\ xy + 2x^2z\rangle.}$$

(b) (2 points) Find the directional derivative of f at the point $(1,-1,2)$ in the direction of $\mathbf{v} = \langle 2, -1, 2\rangle$.

Solution: First normalize $\mathbf{v}$:

$$|\mathbf{v}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3, \quad \hat{\mathbf{v}} = \frac{1}{3}\langle 2, -1, 2\rangle.$$

Compute $\nabla f(1,-1,2)$:

$$f_x(1,-1,2) = (-1)(2) + 2(1)(2)^2 = -2 + 8 = 6,$$

$$f_y(1,-1,2) = 1 \cdot 2 = 2,$$

$$f_z(1,-1,2) = 1(-1) + 2(1)^2(2) = -1 + 4 = 3.$$

So $\nabla f(1,-1,2) = \langle 6, 2, 3\rangle$. Directional derivative:

$$D_{\hat{\mathbf{v}}}f = \nabla f \cdot \hat{\mathbf{v}} = \langle 6,2,3\rangle \cdot \frac{1}{3}\langle 2,-1,2\rangle = \frac{1}{3}(12 - 2 + 6) = \frac{16}{3}.$$

$$\boxed{\textbf{Directional derivative } = \frac{16}{3} \approx 5.33.}$$

(c) (2 points) Verify that the magnitude of ∇f at that point equals the maximum rate of change of f there.

Solution:

$$|\nabla f(1,-1,2)| = \sqrt{6^2+2^2+3^2} = \sqrt{49} = 7.$$

Hence, the maximum rate of change is 7, attained in the direction of $\nabla f(1,-1,2)$.

$$\boxed{\textbf{Max rate of change } = |\nabla f(1,-1,2)| = 7.}$$

11. Suppose $z = x^2y + \sin(y)$, where $x = u^2 - v$ and $y = e^{uv}$.

(a) (2 points) Find $\dfrac{\partial z}{\partial u}$ using the Chain Rule.

Solution: By the multivariable Chain Rule,

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}.$$

Compute each derivative:

$$z_x = 2xy, \qquad z_y = x^2 + \cos(y),$$

$$x_u = 2u, \qquad y_u = ve^{uv}.$$

Substitute:

$$\frac{\partial z}{\partial u} = (2xy)(2u) + (x^2 + \cos y)(ve^{uv}) = 4uxy + ve^{uv}(x^2 + \cos y).$$

Now express everything in terms of u and v using

$$x = u^2 - v, \quad y = e^{uv}.$$

Thus

$$\frac{\partial z}{\partial u} = 4u(u^2-v)e^{uv} + ve^{uv}\left[(u^2-v)^2 + \cos(e^{uv})\right].$$

$$\boxed{\frac{\partial z}{\partial u} = e^{uv}\left[4u(u^2-v) + v\big((u^2-v)^2 + \cos(e^{uv})\big)\right].}$$

(b) (2 points) Find $\dfrac{\partial^2 z}{\partial v \partial u}$, expressed only in terms of u, v.

Solution: Differentiate the previous expression with respect to v:

$$\frac{\partial z_u}{\partial v} = \frac{\partial}{\partial v}\left[e^{uv}\big(4u(u^2-v) + v\big((u^2-v)^2 + \cos(e^{uv})\big)\big)\right].$$

Apply product rule:

$$\frac{\partial^2 z}{\partial v \partial u} = e^{uv}(u)\big(4u(u^2-v) + v\big((u^2-v)^2 + \cos(e^{uv})\big)\big) + e^{uv}\frac{\partial}{\partial v}(4u(u^2-v)$$
$$+v\big((u^2-v)^2 + \cos(e^{uv})\big)\big).$$

Compute the inner derivative step by step.

(i) Derivative of $4u(u^2-v)$ w.r.t v:

$$\frac{\partial}{\partial v}[4u(u^2-v)] = 4u(-1) = -4u.$$

(ii) Derivative of $v\big((u^2-v)^2 + \cos(e^{uv})\big)$ w.r.t v:

$$\frac{\partial}{\partial v}\left[v((u^2-v)^2 + \cos(e^{uv}))\right] = ((u^2-v)^2 + \cos(e^{uv})) + v\left[2(u^2-v)(-1) - \sin(e^{uv})(ue^{uv})\right].$$

Simplify:

$$= (u^2-v)^2 + \cos(e^{uv}) - 2v(u^2-v) - uve^{uv}\sin(e^{uv}).$$

Now combine:

$$\frac{\partial^2 z}{\partial v \partial u} = e^{uv}\{u\big(4u(u^2-v) + v\big((u^2-v)^2 + \cos(e^{uv})\big)\big)$$
$$+\big[-4u + (u^2-v)^2 + \cos(e^{uv}) - 2v(u^2-v) - uve^{uv}\sin(e^{uv})\big]\}.$$

12. Let $w = x^2y + yz^3$, where $x = t^2$, $y = e^t$, and $z = \sin t$.

(a) (3 points) Find $\frac{dw}{dt}$ using the multivariable Chain Rule.

Solution: We have

$$\frac{dw}{dt} = w_x x' + w_y y' + w_z z',$$

where $x' = \frac{dx}{dt},\ y' = \frac{dy}{dt},\ z' = \frac{dz}{dt}$.

Compute each:

$$w_x = 2xy, \quad w_y = x^2 + z^3, \quad w_z = 3yz^2.$$

Also,

$$x' = 2t, \quad y' = e^t, \quad z' = \cos t.$$

Hence

$$\frac{dw}{dt} = (2xy)(2t) + (x^2+z^3)(e^t) + (3yz^2)(\cos t) = 4txy + e^t(x^2+z^3) + 3yz^2\cos t.$$

Now substitute $x = t^2,\ y = e^t,\ z = \sin t$:

$$\frac{dw}{dt} = 4t(t^2)e^t + e^t((t^2)^2 + (\sin t)^3) + 3(e^t)(\sin^2 t)(\cos t).$$

Simplify:

$$\frac{dw}{dt} = 4t^3e^t + e^t(t^4 + \sin^3 t) + 3e^t \sin^2 t \cos t.$$

$$\boxed{\frac{dw}{dt} = e^t(t^4 + 4t^3 + \sin^3 t + 3\sin^2 t \cos t).}$$

(b) (2 points) Evaluate $\dfrac{dw}{dt}$ at $t = \pi/4$.

Solution: At $t = \pi/4$,

$$\sin(\tfrac{\pi}{4}) = \frac{\sqrt{2}}{2}, \qquad \cos(\tfrac{\pi}{4}) = \frac{\sqrt{2}}{2}, \qquad e^t = e^{\pi/4}.$$

Plug into the formula:

$$\frac{dw}{dt} = e^{\pi/4}\Big[\Big(\frac{\pi}{4}\Big)^4 + 4\Big(\frac{\pi}{4}\Big)^3 + \Big(\frac{\sqrt{2}}{2}\Big)^3 + 3\Big(\frac{\sqrt{2}}{2}\Big)^2\Big(\frac{\sqrt{2}}{2}\Big)\Big].$$

Compute the trigonometric terms:

$$\Big(\frac{\sqrt{2}}{2}\Big)^3 = \frac{\sqrt{2}}{4}, \quad 3\Big(\frac{\sqrt{2}}{2}\Big)^2\Big(\frac{\sqrt{2}}{2}\Big) = 3 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{4}.$$

Add those: $\frac{\sqrt{2}}{4} + \frac{3\sqrt{2}}{4} = \sqrt{2}$. Hence

$$\frac{dw}{dt} = e^{\pi/4}\Big[\frac{\pi^4}{4^4} + 4\frac{\pi^3}{4^3} + \sqrt{2}\Big] = e^{\pi/4}\Big[\frac{\pi^4}{256} + \frac{\pi^3}{16} + \sqrt{2}\Big].$$

$$\boxed{\frac{dw}{dt}\Big|_{t=\pi/4} = e^{\pi/4}\left(\frac{\pi^4}{256} + \frac{\pi^3}{16} + \sqrt{2}\right).}$$

13. (2 points) **Conceptual (Multiple Choice):** Which of the following statements about the Chain Rule is true?

A. If $z = f(x, y)$ and x, y are functions of t, then $\dfrac{dz}{dt} = f_x\dfrac{dx}{dt} + f_y\dfrac{dy}{dt}$.

B. If $z = f(x, y)$ and x, y are functions of u, v, then $\dfrac{\partial z}{\partial u} = f_x + f_y$.

C. $\dfrac{dz}{dt}$ can be found only if z is a linear function.

D. $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ are always equal.

Solution: The multivariable chain rule for $z = f(x, y)$ with $x = x(t)$, $y = y(t)$ is

$$\frac{dz}{dt} = f_x\frac{dx}{dt} + f_y\frac{dy}{dt}.$$

This corresponds to option (A).

$\boxed{\textbf{Correct answer: (A)}}$

14. (3 points) Warm-up: Given $x^2 + yz = 4$, find $\dfrac{\partial z}{\partial x}$ in terms of x, y, z.

Solution: Differentiate implicitly with respect to x, treating y as constant:

$$2x + y\frac{\partial z}{\partial x} = 0 \quad \Rightarrow \quad \frac{\partial z}{\partial x} = -\frac{2x}{y}.$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{2x}{y}}.$$

15. (5 points) Given that $x^2 + y^2 + z^2 = 3xyz$, find $\dfrac{\partial z}{\partial x}$ using implicit differentiation. (Simplify your result as much as possible.)

Solution: Differentiate both sides with respect to x, treating y as constant:

$$2x + 0 + 2z\frac{\partial z}{\partial x} = 3\left(yz + xy\frac{\partial z}{\partial x}\right)$$

(since $\frac{\partial}{\partial x}(xyz) = yz + xy\frac{\partial z}{\partial x}$). Now collect terms with $\frac{\partial z}{\partial x}$:

$$2x + 2z\frac{\partial z}{\partial x} = 3yz + 3xy\frac{\partial z}{\partial x}.$$

Group the $\frac{\partial z}{\partial x}$ terms to one side:

$$(2z - 3xy)\frac{\partial z}{\partial x} = 3yz - 2x.$$

Hence

$$\boxed{\frac{\partial z}{\partial x} = \frac{3yz - 2x}{2z - 3xy}}.$$

16. (2 points) **Multiple Choice:** Which of the following is true about the gradient $\nabla f(x, y)$ at a point (x_0, y_0)?

   A. $\nabla f(x_0, y_0)$ points in the direction of the fastest increase of f at that point.

   B. $\nabla f(x_0, y_0)$ is tangent to the level curve of f at that point.

C. $\nabla f(x_0, y_0)$ always has magnitude 1.

D. $\nabla f(x_0, y_0)$ is undefined for differentiable functions.

Solution: By definition, the gradient $\nabla f(x_0, y_0) = \langle f_x, f_y \rangle$ points in the direction of greatest increase of f, and is perpendicular to the level curve $f(x, y) = f(x_0, y_0)$. Therefore, the correct choice is (A).

Correct answer: (A)