

**Textbook Sections:** 14.1, 14.2 and 14.3

**Topics:** Functions of several variables; limits and continuity; partial derivatives

**Instructions:** Try each of the following problems, show the detail of your work.

Clearly mark your choices in multiple choice items. Justify your answers.

Cellphones, graphing calculators, computers and any other electronic devices are not to be used during the solving of these problems. Discussions and questions are strongly encouraged.

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**FUNCTIONS OF SEVERAL VARIABLES:**

1. If  $g(x, y) = x \sin y + y \sin x$ , find

- (a)  $g(\pi, 0)$ .
- (b)  $g(\pi/2, \pi/4)$ .
- (c)  $g(0, y)$ .
- (d)  $g(x, y + h)$ .

2. Let  $F(x, y, z) = \sqrt{y} - \sqrt{x - 2z}$ .

- (a) Evaluate  $F(3, 4, 1)$ .
- (b) Find and describe the domain of  $F$ .

3. Find and sketch the domain of the function

$$g(x, y) = \frac{\ln(2 - x)}{1 - x^2 - y^2}.$$

**LIMITS AND CONTINUITY:**

4. Evaluate the following limits, if it exists, otherwise show it does not exist.

- (a)  $\lim_{(x,y) \rightarrow (3,2)} e^{\sqrt{2x-y}}$ .
- (b)  $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2y^3 - x^3y^2}{x^2 - y^2}$ .
- (c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + 3y^2}$ .
- (d)  $\lim_{(x,y) \rightarrow (2,3)} \frac{3x - 2y}{4x^2 - y^2}$ .
- (e)  $\lim_{(x,y) \rightarrow (0,0)} \frac{3x - 2y}{4x^2 - y^2}$ .
- (f)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$ .

5. Determine the set of points at which the function is continuous.

(a)  $F(x, y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2}.$

(b)  $G(x, y) = \ln(1 + x - y).$

### PARTIAL DERIVATIVES:

6. Find the first partial derivatives of the function  $g(u, v) = (u^2v - v^3)^5.$

7. Given the function  $f(x, y) = y \arcsin(xy)$ , find  $f_y(1, \frac{1}{2}).$

8. Verify that the conclusion of Clairaut's Theorem holds for the function  $u(x, y) = e^{xy} \sin(y).$

9. Given the function  $f(x, y) = x^4y^2 - x^3y$ , find  $f_{xxx}(x, y)$  and  $f_{xyx}(x, y).$

### Suggested Textbook Problems

Section 14.1	9-22, 25, 27, 29, 30, 32-34, 36, 47, 49, 61-66
Section 14.2	5-22, 29, 33
Section 14.3	15-41, 47, 51-53, 56, 59, 63, 68, 73, 82, 83, 90

### Some Useful Definitions and a Strategy:

**A Function of Two Variables** A function  $f$  of two variables is a rule that assigns to each ordered pair of real numbers  $(x, y)$  in a set  $D$  a unique real number denoted by  $f(x, y)$ . The set  $D$  is the domain of  $f$ , and its range is the set of values it takes on, namely  $\{f(x, y) : (x, y) \in D\}.$

**A Continuous Function** A function  $f$  of two variables is called continuous at  $(a, b)$  if

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = f(a, b).$$

We say that  $f$  is continuous on  $D$  if it is continuous at every point of  $D.$

**Showing a Limit Does Not Exist** If  $f(x, y) \rightarrow L_1$  as  $(x, y) \rightarrow (a, b)$  along a path  $C_1$  and  $f(x, y) \rightarrow L_2$  as  $(x, y) \rightarrow (a, b)$  along a path  $C_2$ , and  $L_1 \neq L_2$ , then the limit

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y)$$

does not exist.

**Notations for Partial Derivatives** If  $z = f(x, y)$ , we write

$$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$
$$f_y(x, y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x, y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f.$$

### Rules For Finding Partial Derivatives

- To find  $f_x$ , regard  $y$  as a constant and differentiate  $f(x, y)$  with respect to  $x$ .
- To find  $f_y$ , regard  $x$  as a constant and differentiate  $f(x, y)$  with respect to  $y$ .

**Clairaut's Theorem** Suppose  $f$  is defined on a disk  $D$  that contains the point  $(a, b)$ . If the functions  $f_{xy}$  and  $f_{yx}$  are both continuous on  $D$ , then

$$f_{xy}(a, b) = f_{yx}(a, b).$$