Worksheet 6 Bonus: Extra Spherical and Cylindrical Practice

1: Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$

2: Find the volume of the solid that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, the planes $z = 0$ and $z = 4$, and in the first octant

3: Evaluate the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2} \, dz \, dy \, dx$$

by changing to cylindrical coordinates

4: Evaluate $\iiint_E (x^2 + y^2) \, dV$ where E is the region between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.

5: Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$

6: Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\phi = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

7: Evaluate the following integrals by switching to spherical coordinates.

(a) $\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 z + y^2 z + z^3) \, dz \, dx \, dy$

(b) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2)^{3/2} \, dz \, dy \, dx$