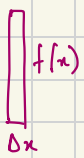
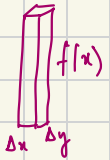
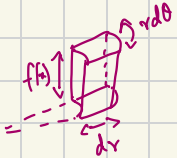
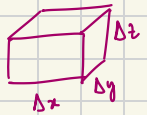
$\int f(x)\,dx$ $\iint_R f(x,y)\,dA$ $\iint_R f(r,\theta)\,r\,dr\,d\theta$ $\iiint_E f(x,y,z)\,dV$

generalisation of polar coordinates







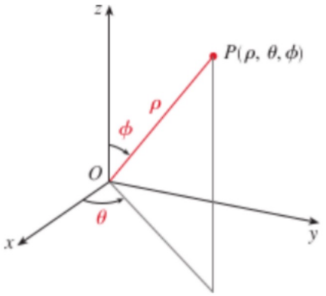


.....

spherical wedge.

1. Spherical Coordinates

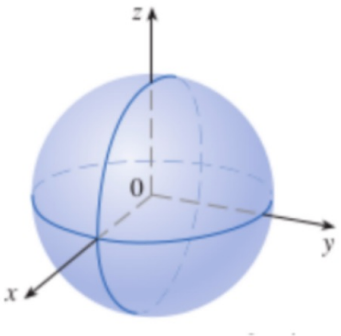
The spherical coordinates (ρ, θ, ϕ) of a point P in space, where $\rho = |OP|$ is the distance from the origin to P, θ is the same angle as in cylindrical coordinates, ϕ is the angle between the positive z-axis and the line segment OP.

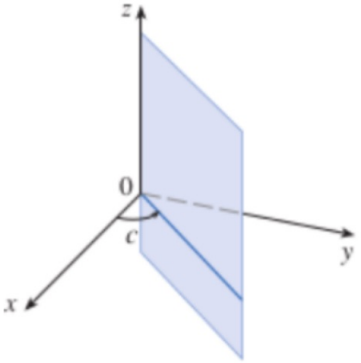


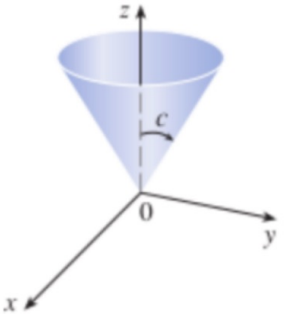
Note that

$$\rho \geq 0 \qquad 0 \leq \phi \leq \pi$$

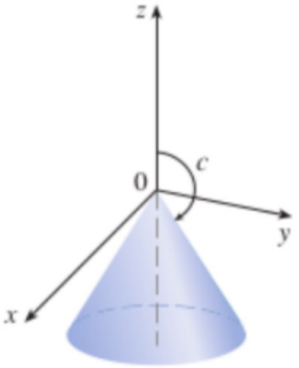
The spherical coordinate system is especially useful in problems where there is symmetry about a point, and the origin is placed at this point.



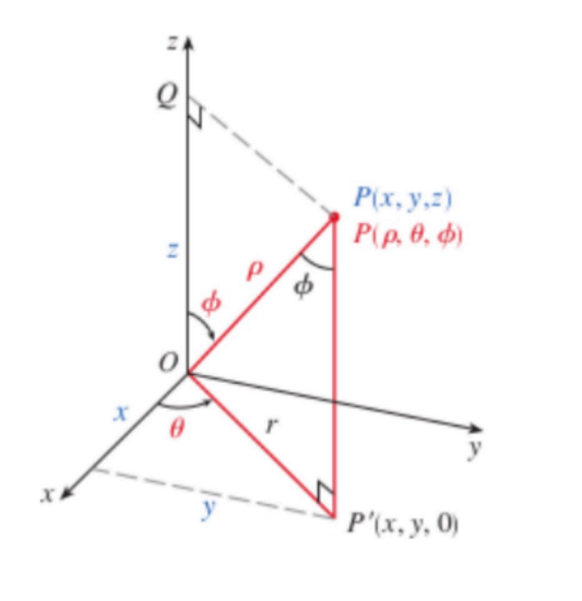




$0 < c < \pi/2$



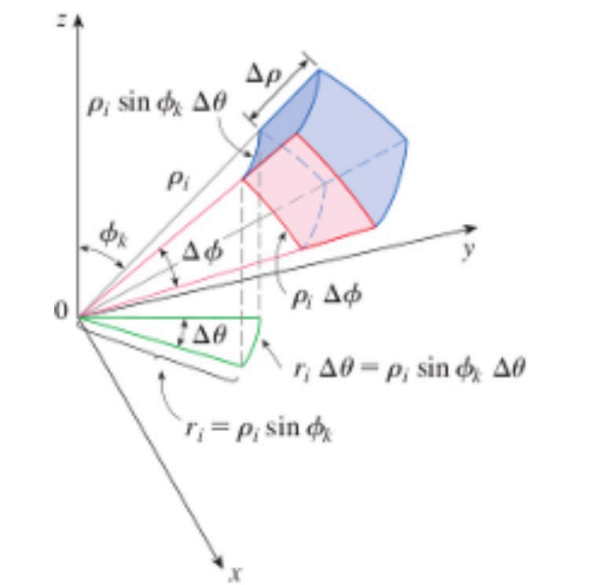
$\pi/2 < c < \pi$


z
Q
P(x, y, z)
P(ρ, θ, φ)
z
ρ
φ
φ
O
x
θ
r
y
x
y
P′(x, y, 0)

Example: Convert from Cartesian Coordinate $(x, y, z) = (-1, 1, \sqrt{\frac{2}{3}})$ to spherical coordinate (ρ, θ, ϕ).

Example: Convert from Cartesian Coordinate $(\rho, \theta, \phi) = (2, \frac{\pi}{2}, \frac{\pi}{4})$ to spherical coordinate (x, y, z).

Triple integrals in spherical coordinates:



(a) A spherical wedge

Example: Evaluate $\iiint_B e^{-(x^2+y^2+z^2)^{3/2}}\,dV$ where B is the ball $x^2+y^2+z^2 \leq 1$.

Example: Find the volume of the region that lies above $z = 0$, below $x^2 + y^2 + z^2 = 4$ and outside the cone $z = \sqrt{x^2 + y^2}$.

Hints for WebAssign:

10. Write the equation x^2-y^2-z^2=1 in spherical coordinates

11. E lies above the cone \phi=\pi/3 and below the sphere \rho=1

14. E lies above the cone z=\sqrt{x^2+y^2} and between the spheres x^2+y^2+z^2=4 and x^2+y^2+z^2=16

15 Part of the ball \rho\leq a that lies between the cones \phi=\pi/6 and \phi=\pi/3