Multivariable Chain Rule

Pre-lecture for 6/18

Ordinary Chain Rule

- Recall: $h(t) = f(g(t))$ implies $h'(t) = f'(g(t))g'(t)$
- If $y = f(x)$, $x = g(t)$, then $dy/dt = (df/dx)(dx/dt)$
- We will extend to more variables by using fraction form

Multivariable Chain Rule

- Let $z = f(x, y)$, $x = g(t)$, $y = h(t)$
- $dz/dt = (\partial f/\partial x)(dx/dt) + (\partial f/\partial y)(dy/dt)$

https://www.arizonadiamondcenter.com/blog/2023/Jan/30/gold-or-silver-chain-what-should-you-wear/

Even More Variables

- Let $z = f(x_1, x_2, \ldots)$, x_i a function of $t_1, t_2, \ldots, t_m$
- $\partial z/\partial t_i = (\partial z/\partial x_1)(\partial x_1/\partial t) + (\partial z/\partial x_2)(\partial x_2/\partial t) + \ldots$

https://www.arizonadiamondcenter.com/blog/2023/Jan/30/gold-or-silver-chain-what-should-you-wear/

Implicit Differentiation Shortcut

Recall trying to find dy/dx in Calc 1:

- Usually given a long equation in x and y
- Have to do a bunch of algebra to find dy/dx

Witness the power of Calc 3:

- Let equation be $F(x, y) = 0$
- $dy/dx = -F_x/F_y$

Practice Problems

Find all of the derivatives

- dz/dt when $z = xe^{xy}$, $x = t^2$, $y = 1/t$
- $\partial z/\partial s$, $\partial z/\partial t$ for $z = e^{2r}\sin(\theta)$, $r = st-t^2$, $\theta = (s^2+t^2)^{1/2}$
- $f_{\theta\theta}$ for $f(x,y)$ if $x = r\cos(\theta)$, $y = r\sin(\theta)$

Use same idea behind shortcut to find a formula for $\partial z/\partial x$, $\partial z/\partial y$ when z depends on x and y according to $F(x, y, z) = 0$

- Apply on $x^2\sin(y-z) = 1+y\cos(xz)$ to find $\partial z/\partial x$, $\partial z/\partial y$

Scratchwork