

Monday 3rd November, 2025

Instructions:

1. (9 points) Let $W = xy\sqrt{z}$,

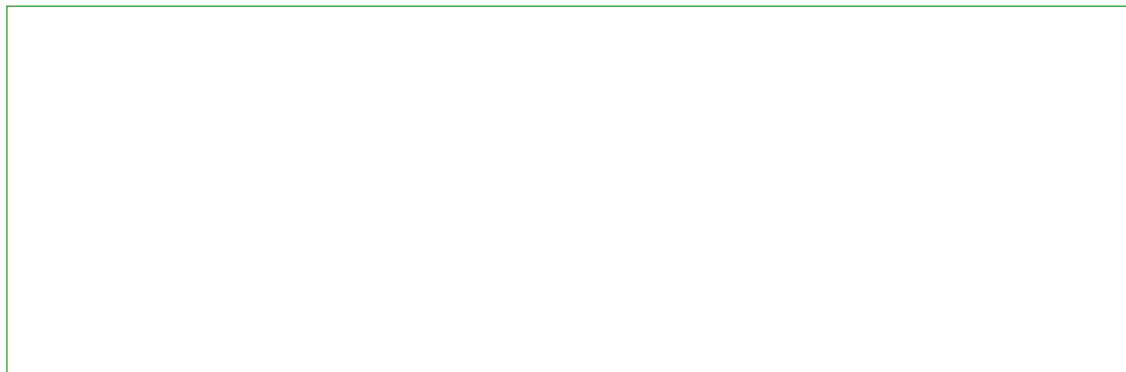
$$\text{where } x = \ln(st), \quad y = \tan(s + 2t) \quad \text{and} \quad z = s^2 e^{2t}.$$

Find $\frac{\partial W}{\partial t}$. Your final answer **must be** in terms of only s and t . Do NOT simplify.

2. Consider the function

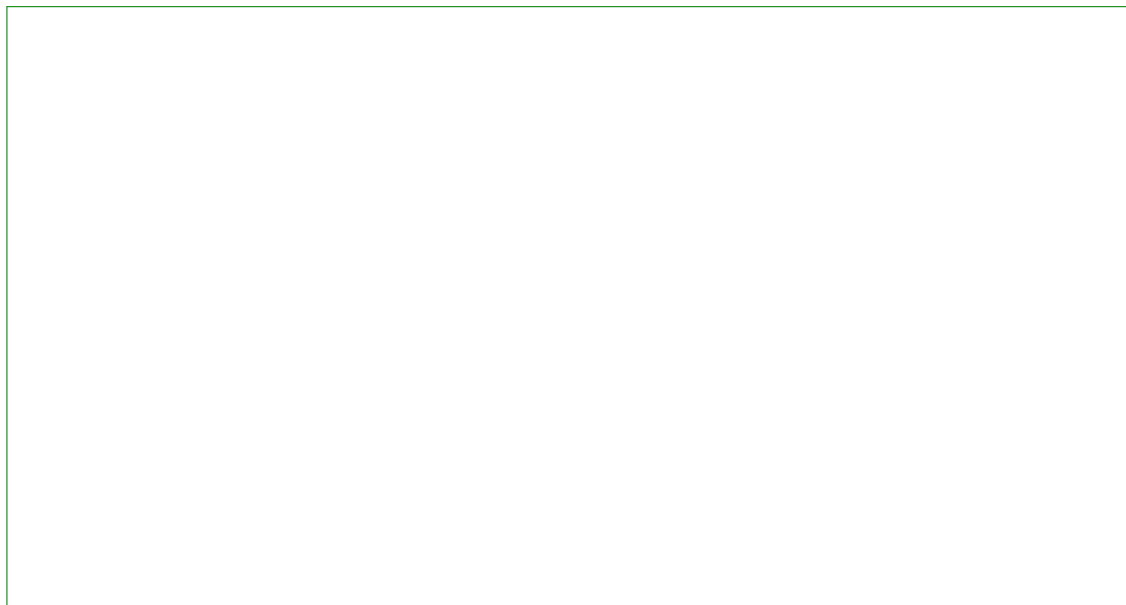
$$f(x, y) = 2x^2 + 3y^2 - 4x - 5.$$

- (a) (5 points) Find the critical point(s) of f that lie in the region $x^2 + y^2 < 16$.



- (b) (8 points) Use the method of Lagrange multipliers to find the extreme values of the function subject to the constraint

$$g(x, y) = x^2 + y^2 = 16.$$



- (c) (5 points) Using the results from part (a) and (b), determine the absolute maximum and absolute minimum of the function

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5 \text{ on the disk } D = \{(x, y) \mid x^2 + y^2 \leq 16\}.$$

3. (8 points) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2xy + y^2}{x^2 + y^2}$ does not exist. Justify your answer.

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Midterm Exam 2

4. (10 points) Determine the **average value** of the function $f(x, y) = e^x + 2y$ over the rectangular region R ,

$$R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 3\}.$$



5. Let $f(x, y) = xe^y + x^2 \sin(3y)$.

(a) (4 points) Find all of the first partial derivatives of f .

(b) (4 points) Find the gradient of f at the point $(1, 0)$.

(c) (4 points) Find the **rate of change** of f at the point $(1, 0)$ in the direction of the vector $3\mathbf{i} + 4\mathbf{j}$.

- (d) (6 points) Find the **linearization** of $f(x, y) = xe^y + x^2 \sin(3y)$ at the point $(1, 0)$ and use it to **approximate** $f(0.9, 0.1)$.

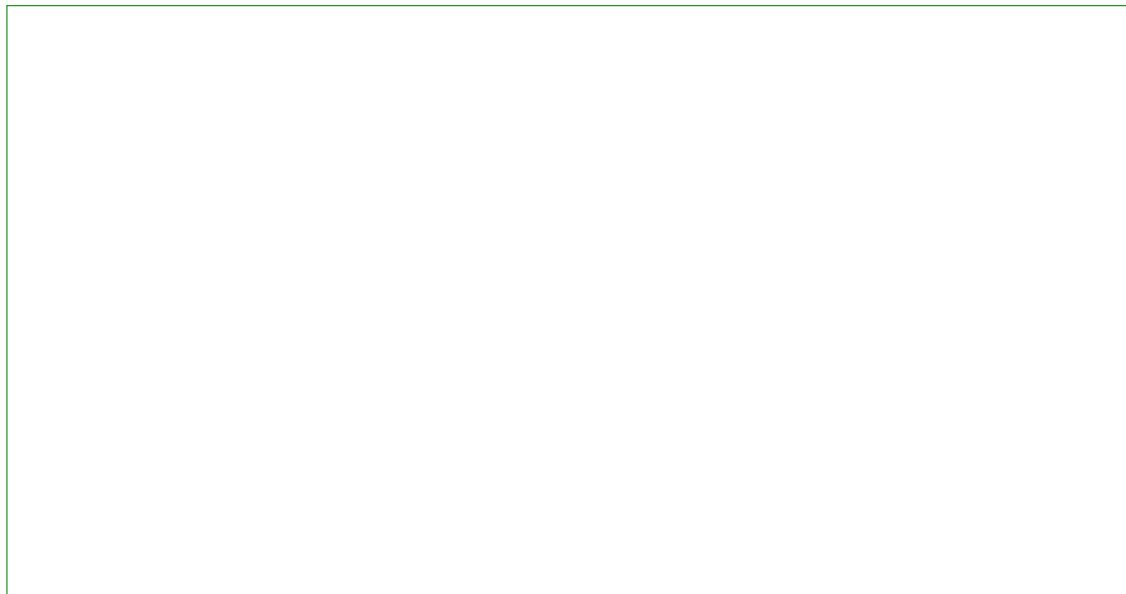
- (e) (6 points) Find **parametric equations of the normal line** to the surface

$$z = xe^y + x^2 \sin(3y)$$

at the point $(1, 0, 1)$.

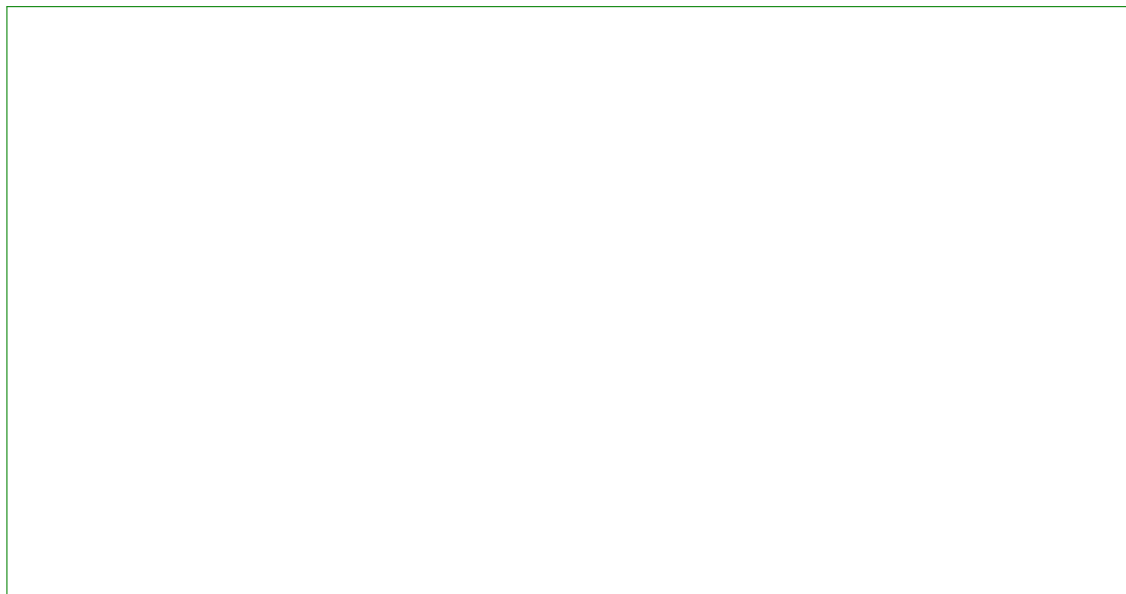
6. (a) (5 points) Change the order of integration for the iterated integral

$$\int_0^{\ln(4)} \int_{e^x}^4 f(x, y) \, dy \, dx.$$



- (b) (5 points) Change the order of integration for the iterated integral

$$\int_0^2 \int_{y^3}^8 g(x, y) \, dx \, dy.$$



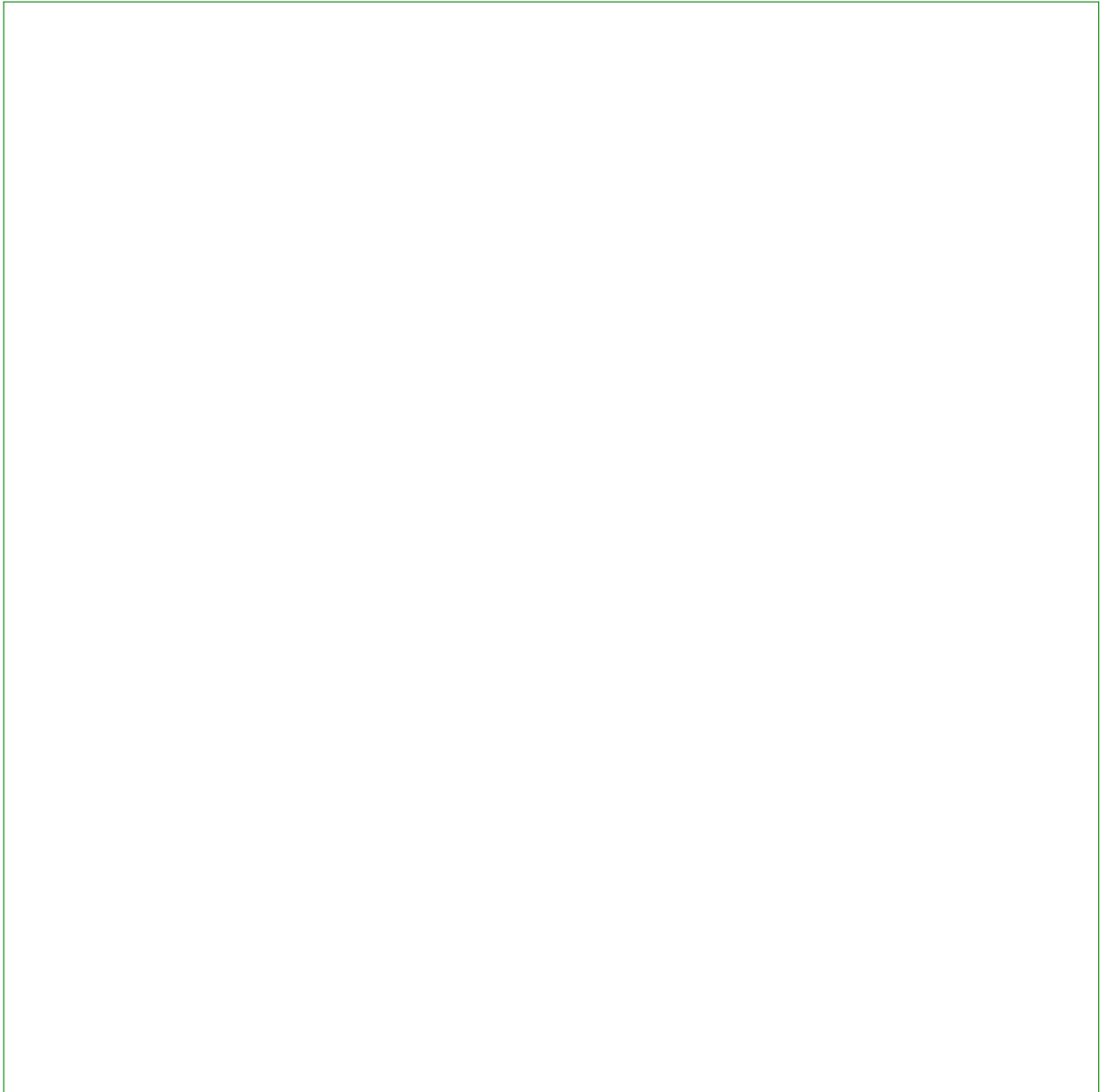
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Midterm Exam 2

7. (10 points) Evaluate the double integral

$$\iint_D (2xy + 1) \, dA,$$

where the plane region D is bounded by the parabola $x = y^2$ and the vertical line $x = 1$.



8. (11 points) Use **polar coordinates** to **evaluate** the double integral

$$\iint_D (x^2 + y^2)^{\frac{3}{2}} dA,$$

where D is the plane region in the first quadrant bounded by the circle $x^2 + y^2 = 1$, the line $y = \frac{x}{\sqrt{3}}$ and the x -axis.

