Linear Approximations

Lecture for 6/17

Idea Behind Linear Approximation

- Recall tangent line at x_0 good approximation for $y = f(x)$
- If line equation is $y = L(x)$, then $L(x) \approx f(x)$ for $x \approx x_0$
- So let's use tangent plane to $z = f(x,y)$
- Express plane equation as $z = P(x, y)$ and use it

The Approximation

- Recall plane equation: $z = f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0) + f(x_0,y_0)$
- So $f(x_0+\Delta x, y_0+\Delta y) \approx f(x_0,y_0)+ f_x(x_0,y_0)\Delta x+f_y(x_0,y_0)\Delta y$
- What happens if f_x and f_y not continuous?

Differentiability

Let's define differentiability:

- Let $\mathbf{v} = (x,y)$, write plane equation as $z = P(\mathbf{v})$
- $f(\mathbf{v})$ is differentiable at $\mathbf{v} = \mathbf{c}$ if $\lim_{\mathbf{v}\to\mathbf{c}} [f(\mathbf{v}) - P(\mathbf{v})]/||\mathbf{v}-\mathbf{c}|| = 0$
  - Same when f has more than 2 variables

Relation to partial derivatives:

- If f_x, f_y exist and are continuous, then f differentiable
- If f diff, then f_x and f_y exist
- But f diff. does not mean f_x and f_y exist

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Differentials

- Recall $dy = f'(x)\,dx$ when $y = f(x)$
- If x and y slightly change, how does $z = f(x,y)$ change?
- $dz = df = f_x\,dx + f_y\,dy$

Practice Problems

Explain why $(x+y)/(x^2+y^4+1)^{1/2}$ is differentiable everywhere

A sphere has center (2, 1, -1) and contains (4.01, 3.02, 0.99). Find a good approximation for the radius of the sphere without using a calculator

Find a function f where f_x, f_y always exist, but f is not differentiable

Scratchwork