Challenge Problems in RED

MATH 243: Worksheet 7 **Discussion Section:** ____________

This worksheet covers Chap 14.3 Partial Derivatives, Chap 14.4 Tangent planes and linear approximations and Chap14.5 The Chain Rule

PARTIAL DERIVATIVES

1. Find all the first order partial derivatives of the following function.

$$f(x,y,z) = 4x^3y^2 - e^zy^4 + \frac{z^3}{x^2} + 4y - x^5$$

2. Consider $g(x,y,z) = \dfrac{x\sin(y)}{z^2}$.

   (a) Find all of the first order partial derivatives for the function $g(x,y,z)$.

   (b) Find all of the second order partial derivatives for the function $g(x,y,z)$.

TANGENT PLANES AND LINEAR APPROXIMATIONS

3. Find an equation of the tangent plane to the given surface at the specified point.

   (a) $z = 4x^2 + y^2 - 9y$ at the point $(1,4)$

   (b) $z = y\tan(x)$ at the point $\left(\frac{\pi}{4}, 6\right)$

4. Find the (linear) approximation (tangent plane approximation) of each function at the specified point.

   (a) $f(-0.99, 1.01)$, where $f(x,y) = \dfrac{5\sqrt{y}}{x}$ at the point $(-1,1)$.

   (b) $f(2.01, 0.99)$, where $f(x,y) = \ln(x+y^7)$ at the point $(2,1)$

THE CHAIN RULE

5. If $w = f(x,y,z,t)$ and $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$, $t = t(u,v)$, then use the Chain Rule to find the partial derivative of w with respect to u. Show a tree diagram of the dependent variable, the intermediate variables and the independent variables.

6. Use the chain rule to find $\dfrac{dz}{dt}$, where $z = \dfrac{x-y}{x+2y}$, $\quad x = e^{\pi t}$, $\quad$ and $\quad y = e^{-\pi t}$.

7. Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$,

$$\text{where} \quad z = \arctan\left(x^2+y^2\right) \quad \text{with} \quad x(s,t) = s\ln(t), \quad y(s,t) = te^s.$$

8. Given that $yz + x\ln(y) = z^2$, use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

9. Suppose f is a differentiable function of x and y, and $f(x, y) = f(g(t), h(t))$, where

$$g(2) = 4,\ g'(2) = -3,\ h(2) = 5,\ h'(2) = 6,\ f_x(4, 5) = 2,\ \text{and}\ f_y(4, 5) = 8.$$

Find the derivative of $f(x, y)$ with respect to t at $t = 2$.

Suggested Textbook Problems
Chapter 14.4: 1-6, 11, 17
Chapter 14.5: 1-13, 15, 17, 18, 21-23, 25, 27-35, 39-43, 45, 50