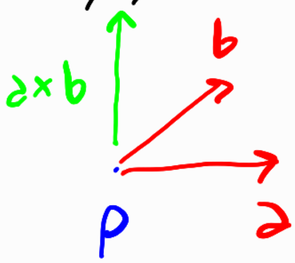
New discussion website available: methyne.github.io/243s26

Attendance is being taken this week + all future weeks

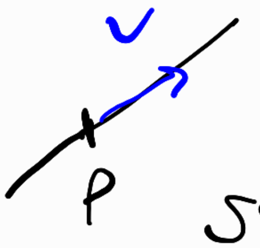
9,

2: Find symmetric equations of the line through $(3,4,0) = P$ perpendicular to $\underbrace{\langle 2,2,0\rangle}_{a}$ & $\underbrace{\langle 0,1,1\rangle}_{b}$.



$a\times b = \langle 2,-2,2\rangle$, which means

$\begin{vmatrix} 2 & 2 & 0 \\ 0 & 1 & 1 \end{vmatrix}$ $\quad \underbrace{\langle 1,-1,1\rangle}_{v}$ also works.



Line generated by $\vec{v}$ through P, so $P+t\vec{v}$ for $t\in\mathbb{R}$ parametrizes the line. Now let's expand:

$$P+tv = (3,4,0)+t\langle 1,-1,1\rangle = (3+t,\ 4-t,\ t).$$

To get symmetric equations, solve for linear expressions in x,y,z such that all expressions equal a common factor.

Notice: $x=3+t$, $y=4-t$, $z=t$.

Solve for all equalling t: $t = \frac{z}{1} = \frac{x-3}{1} = \frac{4-y}{1}$

(b) Find intersection with yz-plane.

Recall yz-plane is just $x=0$.

So to get the intersection, set $x=0$

in line equation & solve for other variables. So $0 = x = 3+t \Rightarrow t = -3$, which we plug in for y & z:

$y = 4-(-3) = 7$, $z = t = -3$.

So intersection point is $(0, 7, -3)$.

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3 (a): Determine if parallel, intersecting, or skew. $L_1: r(t) = \langle -1+3t, 2+4t, 3-2t \rangle$,

$$L_2: \quad \frac{x-1}{2} = \frac{y}{-3} = \frac{z+1}{-3}.$$

Recall: 2 lines are parallel iff their direction vectors parallel.

For r, we decompose into constant & t:

$r(t) = \langle -1, 2, 3 \rangle + t\langle 3, 4, -2 \rangle$,

so L_1 has direction $\langle 3, 4, -2 \rangle$.

For L_2, direction will just be given by denominators, so it's $\langle 2, -3, -3 \rangle$.

$\langle 3,4,-2\rangle$ & $\langle 2,-3,-3\rangle$ not parallel, so L_1 & L_2 not parallel.

Check intersections: $x=-1+3t$, $y=2+4t$, $z=3-2t \Rightarrow \dfrac{-2+3t}{2}=\dfrac{x-1}{2}=\dfrac{y}{-3}=\dfrac{2+4t}{-3}$

$\Rightarrow -3(-2+3t)=2(2+4t) \Rightarrow$

$6-9t=4+8t \Rightarrow 2=17t \Rightarrow t=\dfrac{2}{17}$

Now let's check y vs z:

$\dfrac{y}{-3}=\dfrac{z+1}{-3} \Rightarrow 2+4t=y=z+1=4-2t$

$\Rightarrow 6t=2 \Rightarrow t=\dfrac{1}{3}$.

But t can't have 2 different values, so there is no solution for t that satisfies L_2 equations

$\Rightarrow$ L_1 & L_2 don't intersect, and also not parallel $\Rightarrow$ L_1 & L_2 skew

9: Det. if $L: r(t) = \langle -2t, 2+7t, -1-4t \rangle$ intersects $4x+9y-2z+8=0$.

We have 2 equations, 1 for line & 1 for plane. To solve a system of equations, one common method is taking one equation, plugging it into another, solving that one, and repeating until you run out of equations. The line equation is simpler since it's in terms of t, so let's plug that one into the plane equation.

Substitute $x=-2t$, $y=2+7t$, $z=-1-4t$:

$0 = 4x+9y-2z+8 = 4(-2t) + 9(2+7t)$
$-2(-1-4t)+8 = 63t+28 \Rightarrow$

$t = -\frac{28}{63} = -\frac{4}{9}$.

So L indeed intersects the plane,

and point of intersection is $r\left(-\frac{4}{9}\right)$.

11: Evaluate $\lim_{t\to 1}\left\langle \frac{t^2-1}{t^2-3t+2}, \frac{t-1}{\sqrt{t+3}-2}, \frac{\sin(t-1)}{t-1}\right\rangle$

Limit will exist iff every individual component limit exists.

$$\lim_{t\to 1}\frac{t^2-1}{t^2-3t+2} = \lim_{t\to 1}\frac{\cancel{(t-1)}(t+1)}{\cancel{(t-1)}(t-2)} =$$

$$\lim_{t\to 1}\frac{t+1}{t-2} = \frac{1+1}{1-2} = \frac{2}{-1} = \underline{-2}.$$

$$\lim_{t\to 1}\frac{t-1}{\sqrt{t+3}-2} = \lim_{t\to 1}\frac{(t-1)(\sqrt{t+3}+2)}{\sqrt{t+3}^2-2^2}$$

$$= \lim_{t\to 1}\frac{\cancel{(t-1)}(\cdots)}{\cancel{t-1}} = \lim_{t\to 1}\left(\sqrt{t+3}+2\right)$$

$$= \sqrt{1+3}+2 = 2+2 = \underline{4}$$

$$\lim_{t\to 1}\frac{\sin(t-1)}{t-1} = \lim_{u\to 0}\frac{\sin u}{u} = \underline{1}$$

since this is a famous Calc 1 limit.

So vector limit exists & is $\boxed{\langle -2, 4, 1\rangle}$

10: Domain of $\langle \ln(t+1), \frac{t}{4-t^2}, \sqrt{4-t}\rangle$

A vector is defined only when every component exists. Let D_1, D_2, D_3 be domains of x, y, z components resp., then the domain we seek is $D_1 \cap D_2 \cap D_3$.

1st: $\ln(t+1)$ defined if $t+1 > 0 \Rightarrow$
$t > -1 \Rightarrow t \in \underline{D_1 = (-1, \infty)}$.

2nd: $\frac{t}{4-t^2}$ defined if $4 - t^2 \neq 0$
$\Rightarrow 4 \neq t^2 \Rightarrow t \neq -2, 2 \Rightarrow$
$t \in \underline{D_2 = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)}$.

3rd: $\sqrt{4-t}$ defined if $4 - t \geq 0$
$\Rightarrow 4 \geq t \Rightarrow t \in \underline{D_3 = (-\infty, 4]}$.

To find intersection, begin at $-\infty$ & make your way to ∞.

$D_1 \cap D_2 \cap D_3 = (-1,2) \cup (2,4]$

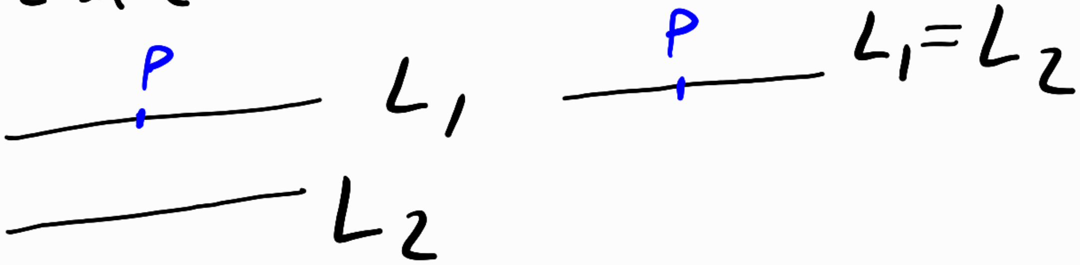
3b: $L_1: \langle x, y, z \rangle = \langle 2t, -3+t, 5-t \rangle$
$= \langle 0, -3, 5 \rangle + t\langle 2, 1, -1 \rangle$, so direction of L_1 is $\langle 2, 1, -1 \rangle$.

$L_2: \langle 3-3s, 2-1.5s, 1.5s \rangle = \langle 3, 2, 0 \rangle + s\langle -3, -1.5, 1.5 \rangle$, so direction of L_2 is $\langle -3, -1.5, 1.5 \rangle$.

Notice that $\langle -3, -1.5, 1.5 \rangle = -1.5\langle 2, 1, -1 \rangle$, so the direction vectors are parallel, so the lines L_1 & L_2 are parallel.

2 parallel lines can intersect only if they are the same exact line.

Let's pick a point P on L_1. If P is on L_2, then $L_1 = L_2$. Else, L_1 & L_2 don't intersect.



Plug in $t=0 \Rightarrow P=\langle 0,-3,5\rangle \in L_1$.

Now let's solve $P=L_2(s)=\langle 3-3s, 2-1.5s, 1.5s\rangle$

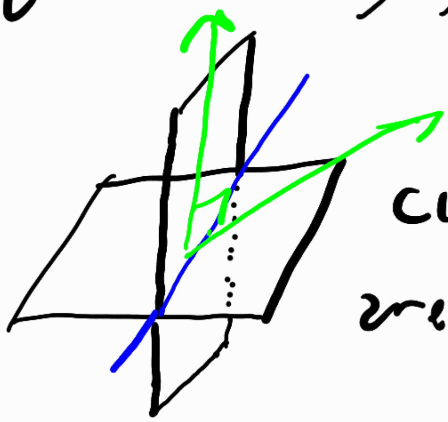
$\Rightarrow \begin{cases} 0=3-3s \\ \vdots \end{cases} \Rightarrow s=1$, but

$L_2(1)=\langle 0, 0.5, 1.5\rangle \neq P$, so

$P \notin L_2 \Rightarrow L_1 \& L_2$ don't intersect

$\Rightarrow L_1 \& L_2$ are merely parallel.

5: Find plane through $A=(0,-2,5)$ and $B=(-1,3,1)$ perp. to $P: 5x+4y-2z=0$.

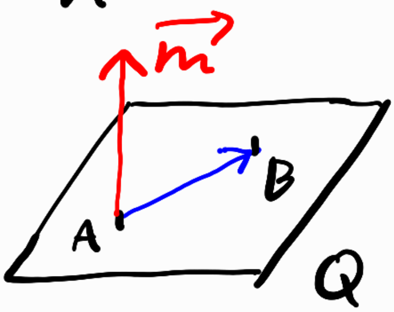


Recall: 2 planes are perpendicular iff their normal vectors are perpendicular.

Notice that we can rewrite P eq. as

$0=\langle 5,4,-2\rangle \cdot \langle x,y,z\rangle$, so

$\vec{n}=\langle 5,4,-2\rangle$ is normal of P.



Let the plane we are looking be Q & $\vec{m}$ a normal to Q.

Notice now that $\vec{m}$ is perpendicular to $\vec{n}$ (by earlier remark on perpendicular planes), and perp. to $\vec{AB}$ (since $\vec{AB}$ is a vector lying inside Q and $\vec{m}$ is normal to Q). So we can just take $\vec{m} = \vec{n} \times \vec{AB} = \langle 5,4,-2\rangle \times \langle 1,-5,4\rangle$

$$= \langle 6, -22, -29 \rangle.$$

$$\begin{vmatrix} 5 & 4 & -2 \\ 1 & -5 & 4 \end{vmatrix}$$

Recall: if $\langle a,b,c\rangle$ is a normal to a plane through (x_0,y_0,z_0), the plane equation is

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0.$$

So an equation for Q is

$$6x - 22(y+2) - 29(z-5) = 0.$$