MATH 243 Quiz 1: Vectors and 3D geometry

1. Let $\mathbf{T}, \mathbf{N}$ be the unit tangent, normal respectively to $\mathbf{r}$, let $\mathbf{v} = \mathbf{r}'$, let $\mathbf{u} = \mathbf{r} \times \mathbf{T}$, and let $\mathbf{w} = \mathbf{u}'$. Which is true about $\mathbf{w}$?

A. $\mathbf{w}$ will represent the unit tangent to the vector $\mathbf{v}$

B. $\mathbf{w}$ is never going to be perpendicular to $\mathbf{r}$

C. $\mathbf{w}$ will always be parallel to $\mathbf{r}$

D. $\mathbf{w}$ will always be perpendicular to $\mathbf{N}$

2. Let $\mathbf{r}(t) = \langle 2t, t^2, 2 \rangle$ and $\mathbf{s}(t) = \mathbf{r}'(t)$. Select all of the following that are true:

A. $\int \mathbf{r}(t)dt = \langle t^2, \frac{t^3}{3}, 2t \rangle$

B. $\mathbf{s}(t) = \langle 2, 2t, 0 \rangle$

C. $\|\mathbf{r}(t)\| = t^2 + 2$

D. The arc length of the curve described by $\mathbf{s}(t)$ from $t = 0$ to $t = 1$ is 4.

3. $\mathbf{u}, \mathbf{v}$ are such that $||\mathbf{u} \cdot \mathbf{v}|| = 3$ and $||\mathbf{u} \times \mathbf{v}|| = 6 + \sqrt{27}$. Let θ be the angle between u and v. Give θ in degrees.

Note: angles should be between 0 and 180 inclusive.

4. Let C be the curve described by $x = e^{4t}\sin(3t), y = e^{4t}\cos(3t), z = 12$. Let L be the arc length of the portion of C from $(0, 1, 12)$ to $(e^4 \sin(3), e^4 \cos(3), 12)$. We can write $L = \frac{p}{q}(e^r - s)$ for positive integers p, q, r, s with the fraction p/q in lowest terms. Find $1000p + 100q + 10r + s$.

5. Let $\mathbf{r}(t) = \langle e^t \sin t, 17.38, e^t \cos t \rangle$. Find the unit tangent, normal, binormal vectors, then find curvature.

6. Let P_1 be the plane described by $2x - y + z = 4$ and P_2 the plane described by $x + y + 2z = 7$. Find the angle of intersection between P_1 and P_2. Give your answer in degrees. Finally, write an equation for the plane that passes through the origin and is perpendicular to both planes.

Note: The angle of intersection is the smaller of the two angles the intersection forms. For example, for the classic high school geometry situation of two lines intersecting below, your answer would be 74 and not 106.