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General concept: manipulating
equations, simplifying expressions,
canceling out things, factoring things
most fundamental concept

If you understand this concept
well, you will be poised for suc-
cess in math 243

ability of success: anything in Calc 1
done with 12 variables, you need
→ be comfortable with n
x do it with 23 variables resp.

Exceptions are one-off units and
last half of Calc 2. See 6/9
lecture for more info

other sources gen.
follow this too

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a_1, a_2, a_3, \dots const.
General remark. v_1, v_2, \dots vectors
 $z, b, c, d, e, \dots, r, s, t$ are typically
scalars

u, v, w are vectors

x, y, z up to the situation

Q: Some students may ask, who cares
where things are coming from?
Just give us the formulas to
plug everything in

A: You may not care, but other
also, your knowledge will be
tiny changes. Many problems in
math are small variations of
given examples.

See 6/10 discussion for more com-
mentary on this remark

general remark. If you miss class,
download notes, and notice a remark
in the notes is confusing, search the

Segment of the recording used to
create the remark
If that still
leaves you confused, ask me by
email writing what day & slide &
remark quote you need help on

General remark: choose your own
variable names. As long as you
can do the work to solve the
problem and it is clear to
anyone reading the work how
the calculation is going, no
problem. There is some personal
preference on whether to use
 u & v for a problem that needs
2 vectors or use v & w
for example

General remark: don't worry

about whether a vector is v or w as long as it's defined clearly. Focus on computation mistakes and conceptual errors instead, like $2(u+v) = 2u+v$ or taking $\|\sqrt{v}\|$ or $\|u\| = -\|u\|$

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ALL scratch work for exams and future quizzes must be uploaded

Report typos & mistakes whenever you see them.

Dont hesitate if something
seems to be broken

Make sure to state bounds
on my variables that
you use for a paramet-
rization or equation.

Bounds are part of the
answer. If there are no
constraints on the variables
given, then we assume
they can be any real num-
ber. But if there should
be constraints and you
leave them off, then

your answer is wrong.

For example: line has no constraint, so leaving off saying "at any real number" is ok. But line segment does, so include it.

You can ignore derivative rules, just simplify everything into one vector, and only then take a derivative, going component by component. But then it

might be more algebra
and more work because
rules can simplify
certain calculations.

Generalization of the
previous remark: Know
your rules for combin-
ing multiple different
operations on functions
and vectors. Using these
rules often leads to
less work, less

calculations, less room for mistakes than manually simplifying everything as much as possible and only then taking the final operation in your calculation.

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Definitions are given without explanation & don't need justification, except perhaps an explanation of how people came across them & why they're useful.

Formulas are equations, inequalities, results etc. which follow from definitions

For example: $f'(t) = \lim_{dt \rightarrow 0} \frac{f(t+dt) - f(t)}{dt}$

is \approx definition of f' , not a formula

$\frac{df}{dt} = f'(t)$ is \approx definition

of notation.

$\frac{d}{dt}(t^n) = nt^{n-1}$ is a

formula however

When a problem says "find vector\ integral (derivative)\ equat.
etc.", you must simplify

the answer as much as possible.

If answer to problem is a vector, you can leave a constant factored out or brought in, and either is fine in regards to the final form of the answer

If you're not at the end of a problem, you might save some trouble by leaving the constant outside.

If you want to check your work for unit vectors, check at the end the magnitude is

Actually 1.

For speed and convenience,
sometimes I will write
 c, s to denote \cos, \sin
respectively.

Chain rule allows you to
cancel differentials like
 dt, ds, dr etc. legally

For any rule or idea
(you want to apply, but
you are not sure if the
rule/idea works because
you haven't seen it covered)

plug in easy values like 0, 1 to make sure it makes sense. Plug in $f = g$ if it depends on $f & g$, plug in basic vectors like $(0, 0, 1)$, $(1, 0, 0)$ etc.

About delays & deadlines: if deadlines are pushed back by 1-2 days for homeworks and quizzes, don't automatically blow off the extra time and then scramble at the end of the new

deadline to do something.
Try to do the homeworks
at the original deadline
and consider extra time
as a gift rather than
something for granted
that will happen every
single homework

Some seemingly random
problems like equation
of sphere or intersection
of plane & plane, line & plane,
region & plane etc. are
application of existing tools.

preparation for doing double
and triple integrals in the
last $1/2$ of 2f3. More
generally, geometry from the
 $1/2$ half of 243 will come
in at the last $1/2$.

For 2 geometry problems
when in doubt, draw the
diagram of relevant objects.

Take useful 1D or 2D
slice of the region

Still can draw 3D, but
only abstractly

bits with inverse trig
expressions like

$\sin^{-1}\left(\frac{1}{6}\right)$, $\tan^{-1}(3)$, $\cos^{-1}\left(\frac{1}{4}\right)$

that have no nice closed form, you can leave them as is. Don't go for decimal values.

More generally: you don't need a calculator for this class to spit out decimal values (with exception of WebAssign problems requiring decimals, which will be rare), so anything can be simplified and fully given by pencil & paper

Be careful with cancelling

trig functions by doing
inverses. For example:
 $\tan^{-1}(\tan x) \neq x$ although
 $\tan(\tan^{-1}x) = x$

Problems about K , a , v , t , ωT , ωN ,
arc length are generally all calc-
ulation when it comes to their
main difficulty. Students strug-
gle with them not because
of calculus, but because of
algebra: square roots, cross
products, simplifying sums

Some problems have complicat-
ed fractions as the final

answer. This is ok, doesn't mean you made a mistake. You could have a mistake but only for the same reasons you make mistakes on math problems in general.

However, if you have a complicated fraction in the middle of the problem and need to do something which makes that fraction even worse, like derivative or integral, stop right there and check your work.

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The reason you see many
miraculous cancellations when
doing problems with trig,
arc length, T, N, B, and
many other concepts where
the formulas might lead
to a crazy answer if you
just picked a function at
random is because the
problems have to be solvable.
Many problems do not have
closed form solutions in

the problems in this class
and other math classes in
your whole life are rigged
to have solutions.

In particular, if you end
up at an impossible integ-
ral or overly complicated
derivative, you probably
made a mistake and should
check your work before
submitting to the problem

containing to solve the problem.

$$\int \sqrt{x^2 + 1} = \text{possible}$$

$$\int e^{e^x} = \text{imp} \quad \int e^{x^2} = \text{imp}$$

$$\int e^{-x^2} = \text{imp}, \quad \int \sqrt{x + \sqrt{x^2 + 1}}$$

= tricky to tell

$$\int \sqrt{\cos^2 x + \sqrt{x}} = \int \sqrt{\cos x} +$$

$\sqrt{\sqrt{x}} = \dots$ ↑ wrong

$$\int f g = (\int f)(\int g) \text{ not } \text{forgivable}$$

$$\text{missing } +C, \quad \sqrt{2f} = 2\sqrt{f}$$

$$= \dots = 245$$

↑ accidentally
dropping 2

more forgivable

Sometimes, I solve problems in 2 slower ways & include steps or attempts that don't end up mattering. This is to imitate how a successful student who doesn't know the problem but eventually figures it out might go about things. If I just jumped to the fastest solution right away, you wouldn't gain anything.

understanding on the problem solving process from seeing me solve the problems.

Therefore, if you miss class and view the problems on the class site by checking the notes, or checking the answers (when discussion worksheet solutions are uploaded), you will not learn as much as if you had seen the recording of class or had come to class

because you want see the process and ideas; it is these methods & ideas that you need to succeed on tests.

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δx is used for infinitesimal quantities, but Δx is used for any kind of increment. Do not confuse these 2 concepts.

[$f(x+\delta x) \rightarrow f(x)$ $\Rightarrow \delta x \rightarrow 0$] is a long problem, don't worry if you can't do it yourself. This & other long derivations like finding formulas for ΔT , ΔN , K are lecture material because they would be impossible (impossible) for homework, quizzes, tests. I only prove them so you know where they come from & don't have to

worry about any truth issues when
using them on homeworks & tests

3 ways to note \geq domain (set
of continuity) / set of diffability /
set where limit exists etc.

" $\{(x, y, \dots) \in \mathbb{R}^n \mid \text{algebraic conditions } \}$ "
on x, y, \dots
"all (x, y, \dots) with [insert alg cond]"
"all (x, y, \dots) except those where
[insert opposite of that alg cond]"

Any notation which is unambiguous
to us & defined & I can understand
what you are saying

general general general remark:
this is how math language works
in general

purple
remark

So same goes for a math problem
applies for me determining whether
you solved a problem that
requires an "explain why" or
"show your work"

Your work must be unambiguous,
readable, and symbols must be defined
if they aren't ordinary\ pre-existing
calculus symbols.

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If you watch a video only
after the class in which it is
recapped, practice problems done,
and lecture done, and you
have questions about the content
or would like some algebra step

to be explained more, don't hesitate to contact me by email.

General remark: It is never too late to ask for more help on some material, even if you have a question about week ℓ and it's already the 4th week of class.

Even if you did well, you can still learn from other people's mistakes. Maybe you don't make some mistake ℓ on a quiz but then make it on next one due to lack of awareness on the type of mistake.

In case you forget a formula for multivariable differentiation, write out a dependency tree and pretend that your goal is

value is a fraction. This will tell you what it ought to be

Some students may complain, "I don't want to see more than 1 way. I want everything to be done as the book does it"
(Whenever there is a book involved)

It is ultimately your choice which way to solve a problem. On quizzes & exams, do any way which follows the general general remark in purple above.

However, if you see more than 1 way, you will understand more and it will be easier to solve problems. It will be more work of course to understand more than 1 way, but that

extra work is worth the benefit.

Don't forget step 4 ("plug in" for leftover variables") on 6/18 notes pre-lec exercises solutions, or you will lose points because your answer isn't simplified

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For MATH 243, complex numbers will not appear inside answers to problems. If you got a complex number from using quadratic formula or taking square root, you made a mistake. Should check your work