The Matrix

has you

MATH 230 Week 9 Worksheet

0: You have the right to a blank page of paper. Everything you write may not fit on this side. If you do not have a blank page of paper, one will be appointed for you before solving any problems if you wish. For a blank page, flip over to the back of this worksheet. If you decide to answer questions now without a blank sheet of paper, you may run out of room.
1: Multiply the following matrices:

A. $\begin{bmatrix}2\end{bmatrix} \cdot \begin{bmatrix}3\end{bmatrix}$, **B.** $\begin{bmatrix}3\\4\end{bmatrix} \cdot \begin{bmatrix}5 & 6\end{bmatrix}$, **C.** $\begin{bmatrix}\cos(1) & \pi\\ e & \sqrt{3}\end{bmatrix} \cdot \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$, **D.** $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} \cdot \begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$

2: Answer the following true or false questions
a. The zero matrix has the magic property that $A \cdot \mathbf{0} = \mathbf{0} \cdot A = A$ for any A
b. For any square matrix A, the cube $A^3 = AAA$ exists
c. The exists a matrix A such that AA^T is not defined
d. The identity matrix of size 2 has dimensions 2×2 and is $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$
3: The dimensions of A are 3×5 and the product ABA is defined. Find the dimensions of B and explain why $ABBA$ doesn't exist anymore.
4: You may have heard matrix multiplication is not commutative, but perhaps you never knew why. Find A, B such that $AB \neq BA$.
5: In algebra, you can cancel terms. If a, b, c are real numbers with $a \neq 0$, then $ab = ac \Rightarrow b = c$. Is this true for matrices? Prove it, or find A, B, C with $A \neq \mathbf{0}$ such that $AB = AC$ but $B \neq C$.
6: Prove that transposing reverses products: $(AB)^T = B^T A^T$

We now turn our attention to inverses of matrices

7: The textbook immediately jumps into square matrices. Why? Show a non-square matrix A may have a left inverse B such that $BA = I$ or a right inverse C such that $AC = I$, but no matrix can serve as both.
8: Show that an inverse of a matrix is unique. That is, if B, C are both inverses of A, then $B = C$. From now on, we shall speak of "the" inverse.
9: Pick any four distinct integers $1 - 9$ and use these to form a 2×2 matrix. Determine if the inverse exists, and if so find it.
10: Try again with a different choice of four integers for good measure.
11: Prove that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

If you have nothing better to do, try these challenge problems:

12: Prove the associative law for matrices: $(AB)C = A(BC)$ assuming that AB, BC exist
13: For square matrices A, B, prove that if $AB = I$, then $BA = I$
14: Show that if a non-square matrix A has a right inverse, it can't have a left inverse