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2: Find vector in opposite direction as $\langle 9,-6,-2\rangle$ with length 5.

Let's begin with same direction for simplicity. Note $c\langle 9,-6,-2\rangle$ is in the same direction for any $c>0$.

So $-c\langle 9,-6,-2\rangle$ is in the opposite direction. Now check magnitude:

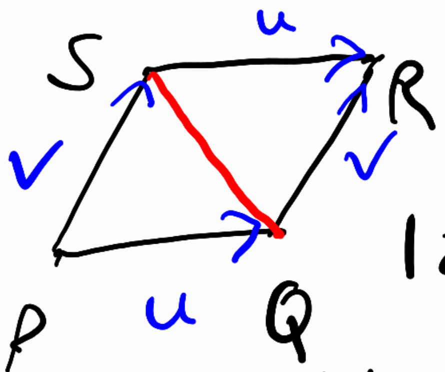
$$5 = |-c\langle 9,-6,-2\rangle| = |-c|\,|\langle 9,-6,-2\rangle|$$
$$= c\sqrt{9^2+(-6)^2+(-2)^2} = c\sqrt{121} = 11c$$
$$\Rightarrow c = \frac{5}{11} \text{ works.}$$

So $\boxed{-\frac{5}{11}\langle 9,-6,-2\rangle \text{ or } \langle -\frac{45}{11}, \frac{30}{11}, \frac{10}{11}\rangle}$

is the desired vector.

10: $P=(1,2,1)$, $Q=(2,5,4)$, $R=(6,9,12)$, $S=(5,6,9)$, find $[PQRS]$ & $[PQS]$. Show $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{PS}$ are coplanar.

(a) Just to be sure, let's check PQRS is actually a parallelogram. A quadrilateral is a parallelogram if the opposite sides are described by the same vector. Let $u = \overrightarrow{PQ}$, $v = \overrightarrow{PS}$.



$u = Q - P = \langle 1,3,3 \rangle$, $v = S - P = \langle 4,4,8 \rangle$.

Check: $\overrightarrow{SR} = R - S = \langle 1,3,3 \rangle = u$, $\overrightarrow{QR} = R - Q = \langle 6,9,12 \rangle - \langle 2,5,4 \rangle = \langle 4,4,8 \rangle = v$.

Recall: Area of parallelogram formed by u & v is $|u \times v|$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 3 & 3 & 1 & 3 \\ 4 & 4 & 8 & 4 & 4 \end{array}\right|$$

$i: 3 \cdot 8 - 3 \cdot 4 = 12$

$j: 3 \cdot 4 - 1 \cdot 8 = 4$

$k: 1 \cdot 4 - 3 \cdot 4 = -8$

Note: Look up Rule of Sarrus

So $u \times v = \langle 12, 4, -8 \rangle = 4\langle 3, 1, -2 \rangle \Rightarrow$

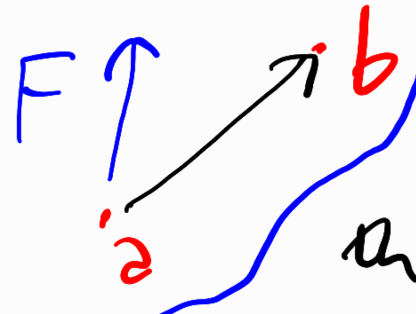
$|u \times v| = |4\langle 3,1,-2 \rangle| = 4\sqrt{14}$, so

$[PQRS] = \boxed{4\sqrt{14}}$

(b) Triangle is $\frac{1}{2}$ of parallelogram, so $[PQS] = \frac{1}{2}[PQRS] = \boxed{2\sqrt{14}}$

(c) Note $\overrightarrow{PQ} = u$, $\overrightarrow{PS} = v$, and $\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{SR} = v + u$. The vectors $u, v, u+v$ are coplanar because the set $\{u, v, u+v\}$ is linearly dependent.

7: Find work done by $F = 8i - 6j + 5k = \langle 8, -6, 5 \rangle$ moving obj. from $\underbrace{(0,6,4)}_{a}$ to $\underbrace{(4,14,22)}_{b}$.



Recall: If work is W, then $W = \vec{F} \cdot \vec{d}$ for a constant force F moving an object in a straight line $\vec{d}$.

$d = b - a = \langle 4, 8, 18 \rangle$. So

$W = F \cdot d = \langle 8, -6, 5 \rangle \cdot \langle 4, 8, 18 \rangle \ N \cdot m =$

$(32-48+90)\,J = (90-16)\,J = \boxed{74\,J}$

4: For vectors $a, b, c \in V_3$, find which of $(a\cdot b)\cdot c$, $(a+b)\cdot c$, $|a|\cdot|c|$ make sense.

$(a\cdot b)\cdot c =$ scalar · vector = syntax error

$(a+b)\cdot c =$ vector · vector = scalar ✓

$|a|\cdot|c| =$ scalar · scalar $= \begin{cases} \text{se}, & \text{`}\cdot\text{' is dot p.} \\ \text{scal}, & \text{`}\cdot\text{' is mult.} \end{cases}$

So (b) meaningful, (a) invalid, and (c) meaningful iff · = multiplication.

3: Determine whether the following are orthogonal, parallel, or neither.

Recall: a, b orthogonal iff $a\cdot b = 0$.

Recall: $\vec{a}, \vec{b} \neq 0$ parallel if there is some constant c such that $\vec{a} = c\vec{b}$.

(a) $a\cdot b = \langle 4,-1,4\rangle \cdot \langle 5,12,-2\rangle = 20-12-8 = 0$, so a & b perpendicular.

But they are not parallel.

Note: 2 non-zero vectors can't be simultaneously orthogonal & parallel.

(b) $a \cdot b = \langle 6,5,-2\rangle \cdot \langle 5,0,9\rangle = 30+0-18$
$= 12 \neq 0$, so a & b not orthogonal.

Now let's try to solve $\vec{a} = c\vec{b}$.
Consider the 2nd component on both sides since 0 is convenient: $5 = c \cdot 0 = 0$, which is impossible.
so $\vec{a} = c\vec{b}$ no solutions $\Rightarrow$ not parallel,
so answer is neither.

(c) $(-18,15) = \vec{a} = c\vec{b} = (12c, -10c)$.
Take 1st component: $-18 = 12c \Rightarrow c = -\frac{3}{2}$.
Does this work? $-10c = 10 \cdot \frac{3}{2} = 15$, so it does, $a = -\frac{3}{2}b$, and they're parallel.

5: Find the angle between $a = \langle 2,0,-3\rangle$ and $b = \langle -2,3,1\rangle$.

Recall: $\cos\theta = \dfrac{a \cdot b}{\|a\|\|b\|} = \dfrac{-4+0-3}{\sqrt{2^2+3^2}\sqrt{2^2+3^2+1^2}}$

$= \dfrac{-7}{\sqrt{13}\sqrt{14}} = -\dfrac{7\sqrt{7}}{\sqrt{13}\,7\sqrt{2}} = -\dfrac{\sqrt{7}}{\sqrt{26}} =$

$$-\frac{\sqrt{182}}{26} \Rightarrow \boxed{\theta = \cos^{-1}\left(-\frac{\sqrt{182}}{26}\right)}.$$

$$\frac{-7}{\sqrt{13}\sqrt{7}\sqrt{2}} = -\frac{\cancel{7}\sqrt{7}}{\sqrt{13}\sqrt{2}\cancel{\sqrt{7}\sqrt{7}}} = -\frac{\sqrt{7}}{\sqrt{26}}$$

6: $a = \langle -1, 4, 8 \rangle$, $b = \langle 18, 2, 1 \rangle$.

(a): Recall: scalar projection of b onto a is

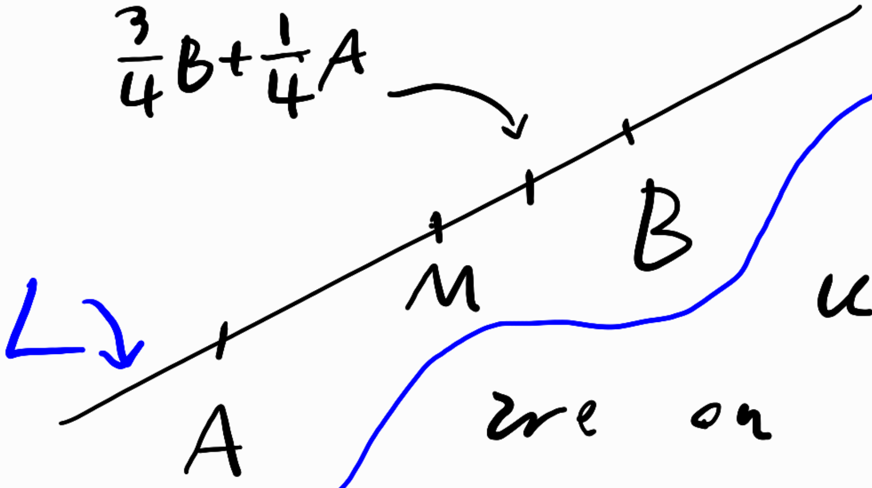
$$\frac{a \cdot b}{\|a\|} = \frac{\langle -1, 4, 8\rangle \cdot \langle 18, 2, 1\rangle}{\sqrt{(-1)^2+4^2+8^2}} = \frac{-2}{\sqrt{81}}$$

$$= \boxed{\frac{-2}{9}}.$$

(b): Recall: vector projection is just the unit vector being projected onto, scaled by the scalar projection. So

$$\text{proj}_b a = \left(\frac{a \cdot b}{\|a\|}\right)\hat{a} = -\frac{2}{9}\frac{a}{\|a\|} = -\frac{2}{81}a$$

$$= \boxed{-\frac{2}{81}\langle -1, 4, 8\rangle \text{ or } \left\langle \frac{2}{81}, \frac{-8}{81}, \frac{-16}{81} \right\rangle}$$



Suppose we know A, B are on the line L and we want to find a parametric equation describing L. We also know $M = \frac{1}{2}(A+B) = \frac{1}{2}A + \frac{1}{2}B$ is also on the line. In general, any point between A & B can be considered as a weighted average of A & B.

Assigning a weight of $t, 1-t$ to A & B respectively gives us the point $tA + (1-t)B = B + t(A-B)$ for $t \in [0,1]$ lying on L.

As t varies from 0 to 1, we go from B to A, covering the line segment $\overline{AB}$.

But you can go beyond 1 or before 0 and keep traveling along the line, so the line is described by

$$\boxed{B + t(A-B),\ t \in \mathbb{R}.}$$

Example: $B=(1,2,3)$, $A=(4,5,6)$.

$A-B = \langle 4,5,6\rangle - \langle 1,2,3\rangle = \langle 3,3,3\rangle$,

so $B + t(A-B) =$

$\langle 4,5,6\rangle + t\langle 3,3,3\rangle = \langle 4+3t, 5+3t, 6+3t\rangle$

is a parametrization of $\overline{AB}$.

xy-plane $=$ plane $z=0$, so we see when $z=0$ on $\overline{AB}$.

Solve $0 = 6 + 3t \Rightarrow t = -2 \Rightarrow$ pt. of intersection is when $t = -2$, which is $\langle -2, -1, 0 \rangle$.