F25 MATH 243-012: Lecture Quiz 3 Friday 17th October, 2025

Try each of the following problems, show the detail of your work. Partial credit will be given. No credit for unsupported results.
Cellphones, calculators, computers and any other electronic devices are prohibited.

Quiz time: **30** minutes.

1. (1 point) Which of the following paths is NOT an appropriate path to use for showing that $\lim_{(x,y)\to(0,0)} \dfrac{6x^2y^2}{3x^4+y^4}$ does not exist.

   A. $y = \frac{1}{2}x$

   B. $x = 0$

   C. $y = x + 5$

   D. $y = 0$

   E. $y^2 = x$

2. (3 points) Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^2+6xy-2y^2}{2x^2+4y^2}$ does not exist. Justify your answer.

3. Given the function $f(x, y) = x^2y^3$

   (a) (2 points) Find an equation of the tangent plane to the given surface $z = x^2y^3$ at the point $(2, 1, 4)$.

   (b) (2 points) Use your answer from part (a) to estimate $f(2.1, 0.9)$.

4. (2 points) Given that $x^3 + y^2z - 3xyz = 4xy$, use implicit differentiation to find $\dfrac{\partial z}{\partial x}$.

5. If $z = z(x, y, t) = yx^2e^{t^2}$, where $x = x(u, w) = w \sin u$, $y = y(u, w) = u \tan(w)$ and $t = t(u, w) = u \ln(w)$, find the following.

   (a) (1 point) $\dfrac{\partial y}{\partial u}$

   (b) (1 point) $\dfrac{\partial z}{\partial t}$

   (c) (2 points) $\dfrac{\partial z}{\partial u}$

   (d) (2 points) $\dfrac{\partial^2 z}{\partial y \partial x}$

6. Given a function $f(x, y) = x^2 \ln(y)$

   (a) (2 points) Find the gradient of f.

   (b) (1 point) Evaluate the gradient at the point $P(3, e^2)$.

   (c) (1 point) Find the rate of change of f at P in the direction of the unit vector $\mathbf{u} = \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$.