MATH 243: Worksheet 8 **Discussion Section:** _____

This worksheet covers Chap14.6 Directional Derivatives and the Gradient Vector, Chap14.7 Maximum and Minimum Values.

**Directional Derivatives and the Gradient Vector**

1. Find the directional derivative of $f(x, y) = xy^3 - x^2$ at the given point $(1, 2)$ in the direction indicated by the angle $\theta = \pi/3$.
2. Find the directional derivative of the function $f(x, y) = \dfrac{x}{y^2}$ at the point $P(3, -1)$ in the direction of the point $Q(-2, 11)$.
3. Given a surface of equation $xy^2z^3 = 8$ and a point $P(2, 2, 1)$ on this surface, find the equations of the tangent plane and parametric equations of the normal line to the given surface at the specified point $P$.
4. Determine the gradient of the function $f(x, y, z) = x^2y^3 - 4xz$ and use the gradient to determine the directional derivative of $f(x, y, z)$ at $(1, 1, 1)$ in the direction of $\mathbf{v} = \langle -1, 2, 0\rangle$.
5. Find the maximum rate of change of $f(x, y) = \sin(xy)$ at the point $(1, 0)$ and the direction in which it occurs.
6. Find the maximum rate of change of $f(x, y, z) = e^{2x}\cos(y - 2z)$ at the point $(1, \pi, 0)$ and the direction in which this maximum rate of change occurs.

**Maximum and Minimum Values**

7. Find all the critical points of the following function.
$$f(x, y) = (y - 2)x^2 - y^2.$$
8. Let $f(x, y) = 3x - x^3 - 2y^2 + y^4$.
   (a) Check if the points $(1, -1)$ and $(2, 3)$ are critical points.
   (b) Determine whether the points $(1, -1), (1, 0), (1, 1)$ are local extreme points or not. If yes, explain what kind of points and justify your answer.
9. Find the local maximum and minimum values and saddle point(s) for the following functions.
   (a) $f(x, y) = x^2 + xy + y^2 + y$
   (b) $f(x, y) = x^3 + y^3 + 3xy$
   (c) $f(x, y) = x^4 - 2x^2 + y^3 - 3y$
10. Find the absolute maximum and minimum values of $f$ on the set $D$.
$$f(x, y) = x^2 + y^2 + x^2y + 9, \quad D = \{(x, y) \,|\, |x| \le 1, \quad |y| \le 1\}$$
11. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set $D$, where $D$ is the closed triangular region with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$.

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**Suggested Textbook Problems**

Chapter 14.6: 5, 7-17, 19-26, 28-35, 41-46, 54-61, 63, 64a

Chapter 14.7: 1-5, 7, 11, 13, 25, 27, 30, 31, 33-36, 41, 43, 45, 46, 51-53