# MATH 243 Worksheet 5: 2D integrals

**Note:** Problems 1-4 are leftover problems from slides. Problems 5-10 are brand new.

**1:** Evaluate $\iint_R f(x,y)\,dA$ for these functions and regions:
**a.** $f(x,y) = x\cos^2(y), R = [0,3]\times[0,\pi/2]$
**b.** $f(x,y) = 2x - 4y^3, R = [4,5]\times[0,3]$
**c.** $f(x,y) = xy + \cos(x) + \sin(y), R = [0,1]\times[0,1]$

**2:** Evaluate $\iint_D f(x,y)\,dA$ for these functions and regions:
**a.** $f(x,y) = 4xy - y^3$, $D$ is region bound by $y = \sqrt{x}$ and $y = x^3$
**b.** $f(x,y) = x^2 - 2y$, $D$ is triangle with vertices $(0,3),(1,1),(5,3)$
**c.** $f(x,y) = e^{x/y}, D = \{(x,y) : 1 \le y \le 2, y \le x \le y^3\}$

**3:** Find the surface area of the portion of $z = xy$ in the cylinder given by $x^2 + y^2 = 1$

**4:** Find the center of mass of the following regions:
**a.** Portion of the unit disk lying in the 1st quadrant
**b.** Square $0 \le x, y \le \pi$ with weight function $f(x,y) = x\sin(x)y^3$

**5:** Find the area of the surface $S$ that is the part of the paraboloid $y = x^2 + z^2$ that lies within the cylinder $x^2 + z^2 = 16$

**6:** Evaluate $\displaystyle\iint_D y^2 e^{xy}\,dA$, where $D$ is the region bounded by $y = x,\ y = 4,\ x = 0$

**7:** Find the average value of $f(x,y) = xy$ over the triangle $D$ with the vertices $(0,0),(1,0),(1,3)$

**8:** Use polar coordinates to find the volume of the solid $z = \sqrt{x^2+y^2}$ and above the ring $1 \le x^2 + y^2 \le 4$

**9:** Calculate $\displaystyle\int_0^{1/2}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\,dx\,dy$ by changing to polar coordinates

**10:** Use polar coordinates to combine the sum

$$\int_{\frac{1}{\sqrt{2}}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_1^{\sqrt{2}}\int_0^x xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_0^{\sqrt{4-x^2}} xy\,dy\,dx$$

into one double integral and evaluate the double integral