Local Extrema

Pre-lecture for 6/23

Types of Extrema

In Calc 1, we saw 2 types of extrema

- Local minimum
- Local maximum

Definition for multivariable functions:

- If $f(\mathbf{x}) \geq f(\mathbf{x}_0)$ for $\mathbf{x}$ around $\mathbf{x}_0$, f has a local minimum at $\mathbf{x}_0$
- If $f(\mathbf{x}) \leq f(\mathbf{x}_0)$ for $\mathbf{x}$ around $\mathbf{x}_0$, f has a local maximum at $\mathbf{x}_0$

Critical Points

Calc 1 definition:

- c is a critical point if f'(c) = 0 or f' not defined at x = c

New definition:

- $\mathbf{v}$ is a critical point if $\nabla f(\mathbf{v}) = \mathbf{0}$ or ∇f not defined at $\mathbf{v}$

Just like Calc 1:

- If $\mathbf{v}$ is an extrema and ∇f is defined at $\mathbf{v}$, then $\nabla f(v) = 0$

Saddle Points

In Calc 1, critical points are almost always extrema

- If function continuously differentiable, “neither” is rare

Now, they are much more common

- If $\mathbf{x}_0$ is a critical point but not an extrema, call it a saddle point
- Will have $f(\mathbf{x}) > f(\mathbf{x}_0)$ for some $\mathbf{x}$ near $\mathbf{x}_0$ and $f(\mathbf{x}) < f(\mathbf{x}_0)$ for others

2nd Derivative Test

How can we tell between local min, local max, or saddle?

- Define $D = f_{xx}f_{yy} - (f_{xy})^2$
- If $D(\mathbf{v}) > 0$ and $f_{xx}(\mathbf{v}) > 0$, then $\mathbf{v}$ is a local min
- If $D(\mathbf{v}) > 0$ and $f_{xx}(\mathbf{v}) < 0$, then $\mathbf{v}$ is a local max
- If $D(\mathbf{v}) < 0$, $\mathbf{v}$ is a saddle point
- If $D(v) = 0$, test inconclusive; try other methods

We shall see explanation of test and “other methods” in lecture

Practice Problems

Find and classify all critical points

- $f(x, y) = x^2+xy+y^2+x+y+1$
- $f(x, y) = x^3+y^3-3xy + 06232025$
- $f(x, y) = y^3-3y^2+3x^2y-3x^2+1$

Find and classify the critical points of $f(x, y) = |x-2|+|y-3|$

Scratchwork