Worksheet 7: The Curled Diverging Green Line Surfaces

1: Show the following vector calculus identities:
a. $\nabla \times fG = \nabla f \times G + f(\nabla \times G)$
b. $\nabla \times \nabla f = 0$
c. $\nabla \cdot (F \times G) = (\nabla \times F) \cdot G - F \cdot (\nabla \times G)$

2: Let C be the triangle with vertices $(-3,0),(0,0),(0,3)$ oriented clockwise. Verify Green's Theorem for $\int_C (xy^2 + x)\,dx + (4x - 1)\,dy$ by computing both the line integral and the corresponding double integral

3: Find a formula for $\nabla \times (\nabla \times F)$ and justify your claim

4: Evaluate $\iint_S f\,dS$ for the following functions and surfaces:
a. $f(x,y,z) = 6xy, S$ is upper half of sphere of radius 1
b. $f(x,y,z) = y + z, S$ is the surface with sides given by the cylinder $x^2 + y^2 = 3$, bottom given by the disk $x^2 + y^2 \leq 3$, and $z = 4 - y$ on top

5: Determine whether or not $\mathbf{F}$ is conservative, and if so find f with $\mathbf{F} = \nabla f$
(a) $\mathbf{F}(x,y) = (xy + y^2)\mathbf{i} + (x^2 + 2xy)\mathbf{j}$

(b) $\mathbf{F}(x,y) = ye^x\mathbf{i} + (e^x + e^y)\mathbf{j}$

6: Find a function f such that $\mathbf{F} = \nabla f$, where $\mathbf{F}(x,y) = (1+xy)e^{xy}\mathbf{i} + x^2e^{xy}\mathbf{j}$, then evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C parametrized by $\mathbf{r}(t) = \langle \cos t, 2\sin t \rangle$, $0 \leq t \leq \pi/2$

7: Show that the line integral $\int_C \sin y dx + (x\cos y - \sin y)dy$ is independent of the path, where C is any path from $(2,0)$ to $(1,\pi)$. Then, evaluate the integral.

8: Use Green's Theorem to evaluate the line integral

$$\int_C (y + e^{\sqrt[3]{x^5}})dx + (2x + \cos(y^2))dy$$

along the positively oriented curve C that is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$

9: Find parametric representations for the following surfaces:
(a) The part of the hyperboloid $4x^2 - 4y^2 - z^2 = 4$ that lies in front of the yz-plane

(b) The part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$

10: Find an equation of the tangent plane at $(5,2,3)$ to the surface S given by the parametric equations $x = u^2 + 1,\ y = v^3 + 1,\ z = u + v$