MATH 243 Worksheet 6: Triple Integrals, Basic Line Integrals

Note: Problems 1-4 are leftover problems from slides, problems 5-11 are brand new.

1: Evaluate $\iiint_E f(x,y,z)\,dV$ for these functions and regions:
a. $f(x,y,z) = x, E$ is region under $2x+3y+z=6$ in the first octant.
b. $f(x,y,z) = \sqrt{3x^2+3z^2}, E$ is region bound by $y = 2x^2+2z^2$ and $y=8$
c. $f(x,y,z) = yz, E$ is region bound by $x = 2y^2+2z^2-5, x=1$
d. $f(x,y,z) = z, E$ is region inside $y^2+z^2=1$ and between $x+y+z=2, x=0$
e. $f = x^2+y^2, E$ is portion of $x^2+y^2+z^2=4$ with $y \geq 0$
f. $f = x^2, E$ is region inside $x^2+y^2+z^2=36$ and $z = -(3x^2+3y^2)^{1/2}$

2: Find the volume of the solid bound by $z = 8-x^2-y^2, z = -2\sqrt{x^2+y^2}$, and $x^2+y^2=4$

3: Evaluate $\int_C f ds$ for $f(x,y) = 16y^5$ where C is $x=y^4$ from $y=0$ to $y=1$, followed by a segment from $(1,1)$ to $(1,-2)$, followed by a segment from $(1,-2)$ to $(2,0)$

4: Evaluate $\int_C (x^2dy - yzdz)$ where C is the segment from $(4,-1,2)$ to $(1,7,-1)$

5: Evaluate $\iiint_E x^{-3}\,dV$ where $E = \{(x,y,z) | 0 \leq y \leq 1, 0 \leq z \leq y^2, 1 \leq x \leq z+1\}$

6: Evaluate $\iiint_E x dV$ where E is bounded by the paraboloid $x = 4y^2+4z^2$ and $x=4$

7: Find the volume of the solid enclosed by the cylinder $x^2+z^2=4$ and the planes $y=-1$ and $y+z=4$

8: Evaluate $\iiint_E z\,dV$ where E is enclosed by the paraboloid $z = x^2+y^2$ and the plane $z=4$

9: Evaluate $\int_C (x/y) ds$, where $C: x=t^3$, $y=t^4$, $1 \leq t \leq 2$

10: Evaluate $\int_C ydx + zdy + xdz$, where $C: x=\sqrt{t}$, $y=t$, $z=t^2$, $1 \leq t \leq 4$

11: Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = xy^2\mathbf{i} - x^2\mathbf{j}$, $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}$, and $0 \leq t \leq 1$