Tangencies and Curvature

Pre-lecture for 6/12

Tangent Vector

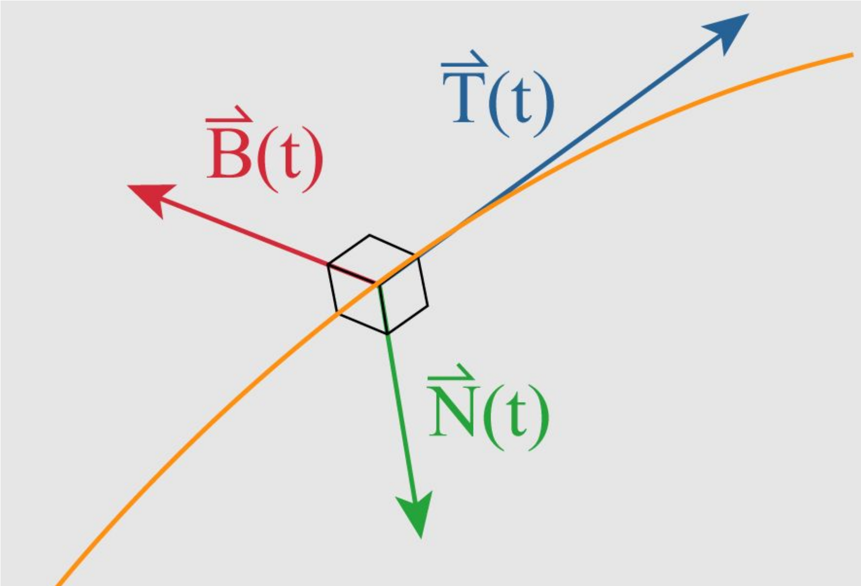
- Tangent to $\mathbf{r}(t)$ is $\mathbf{r}'(t)$
- Unit tangent is $\mathbf{T}(t) = \mathbf{r}'(t)/||\mathbf{r}'(t)||$

Normal Vector

- Define **N**(t) = **T**'(t)/||**T**'(t)||
- Fact: If **u**(t) is a unit vector, then **u**' and **u** are orthogonal
- Fact: **N** is orthogonal to **T**

Binormal Vector

- Define $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$
- Now **T**, **N**, **B** are pairwise orthogonal



Source: https://calcworkshop.com/vector-functions/unit-tangent-vectors/

Curvature

- Measures how fast a curve is changing direction
- Defined by $\kappa = ||d\mathbf{T}/ds||$ where s is arc length
  - **T** measures direction, so d**T** measures its change
  - d**T** gets compared to ds so κ is independent of parametrization

Reformulating κ for Calculations

To find κ, we need a convenient formula

- $\kappa = \|\mathbf{T}'(t)\| / \|\mathbf{r}'(t)\|$
- $\kappa = \|\mathbf{r}'(t) \times \mathbf{r}''(t)\| / \|\mathbf{r}'(t)\|^3$

Scratch Work

Practice Problems

Let $\mathbf{r}(t) = \langle t, 3\sin(t), 3\cos(t)\rangle$. Find the tangent, normal, and binormal vectors for r. Then determine the curvature of r.

Curvature of single-variable function

- Use one of the reformulations to show that the curvature of the graph of $y = f(x)$ is $|f''(x)|/(1+f'(x)^2)^{3/2}$