Site is fixed, pre-lec &lec slide links now point to right places

New site feature: link clicking goes to preview instead of download. You can then download the preview if you want to

6/12 pre-lec video not up yet, ran into problem with youtube flagging the video. Will delete and reupload by end of today

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1: $\dfrac{dT/ds}{\|dT/ds\|} = \dfrac{dT/dt}{\|dT/dt\|}$ 2: $\|r'\|' = \dfrac{r' \cdot r''}{\|r'\|}$

$$\frac{\frac{dT}{ds}}{\left\|\frac{dT}{ds}\right\|} = \frac{\frac{dT}{dt}\overbrace{\frac{dt}{ds}}^{\text{scalar}}}{\left\|\frac{dT}{dt}\underbrace{\frac{dt}{ds}}_{\text{scalar}}\right\|} = \frac{\frac{dT}{dt}}{\left\|\frac{dT}{dt}\right\|}\,\frac{\frac{dt}{ds}}{\underbrace{\left|\frac{dt}{ds}\right|}}$$

Inverse function rule, $\dfrac{dt}{ds} = \dfrac{1}{ds/dt}$

$$= \frac{1/(ds/dt)}{1/|ds/dt|} = \frac{|ds/dt|}{ds/dt} = 1$$

because $\dfrac{ds}{dt}$ represents arc length & arc length is non-negative

Key idea for #2: taking derivative of $\|\cdot\|$ is annoying, so we are going to try to sidestep this

$$\|r'(t)\|^2 = r'(t) \cdot r'(t)$$

$$\frac{d}{dt}\|r'(t)\|^2 = \cancel{2}\|r'(t)\| \underbrace{\|r'(t)\|'}_{\text{goal}}$$

$$\frac{d}{dt}\left(r'(t) \cdot r'(t)\right) = \cancel{2} r''(t) \cdot r'(t)$$

$$\|r'\| \underbrace{\|r'\|'} = r'' \cdot r'$$

$$\|r'\|' = \frac{r'' \cdot r'}{\|r'\|} = \frac{r' \cdot r''}{\|r'\|}$$

because dot prod. commutative

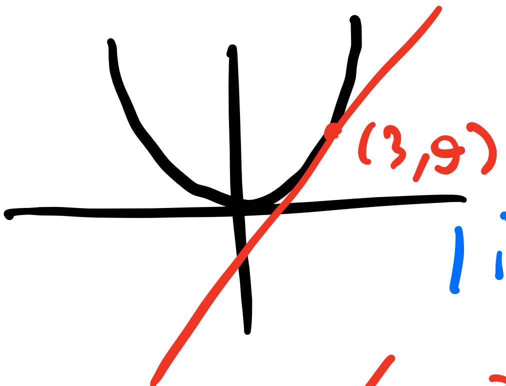
DW1, Q4:

Choose vector parallel to the tangent line to the curve $y=x^2$ at $(\underbrace{3}_{x},\underbrace{9}_{y})$

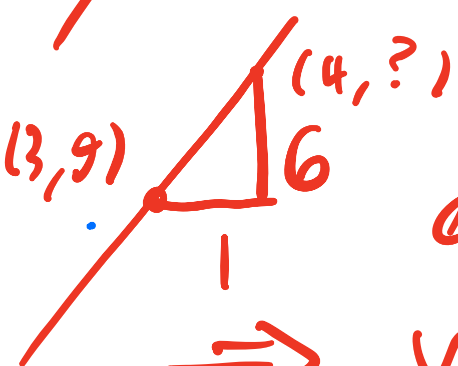
Step 1: find tangent line slope

slope of tangent = derivative of function @ that point $= \left.\frac{dy}{dx}\right|_{x=3}$

$= 2x\big|_{x=3} = 6$



Step 2: choose a vector lying on the tangent line



$? = 9+6 = 15$

vector starts at (3,9), ends at (4,15)

$\Rightarrow$ vector $(4,15)-(3,9)=(1,6)$

In general, $\left(1, \frac{dy}{dx}\right)$ works as a vector pointing the same direction

Step 3: see answer choices, compare

them to $(1,6)$

A. $\langle 3,6\rangle$ B. $\langle 1,2\rangle$ C. $\langle 1,6\rangle$, D. $\langle 1,9\rangle$

we can also check these aren't parallel, so answer is C.

DWI, Q5: which of these is parallel to $\langle 1,-2,3\rangle$?

Step 1: Recognize only vectors parallel to $v=\langle 1,-2,3\rangle$ are cv for some c.

Step 2: Match answer choices to $cv=\langle c,-2c,3c\rangle$ to see which ones can work

match

Start with E: $\langle -\frac{10}{14}, \frac{2}{14}, -\frac{3}{14}\rangle$

$c=-\frac{10}{14}$. Plug into 2nd component:

$\frac{2}{14} = -2c = \frac{20}{14}$, $2 \neq 20$

Repeat process for A, B, C, ...

→ A, B, C, D work, E fails, so

answer = F : "more than 1 above"

Q10c, Q1

If $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$, why is this is so?

10c. Find u, v, w with : $u \cdot v = u \cdot w$, $v \neq w$
$u \times v = u \times w$, $v \neq w$
$(u \times v) \times w \neq u \times (v \times w)$

Dot: Take $w = 0$, find $u \cdot v = 0$, $v \neq 0$

Take any u & v which are perp.

$u = (1,0)$, $v = (0,1)$, $w = (0,0)$

Cross: Take $w = 0$. Then $u \times w = 0$,
need $u \times v = 0$ with $v \neq 0$

Take $v = u$, then $u \times v = u \times u = 0$

$u = (1,0,0)$, $v = (1,0,0)$, $w = (0,0,0)$

Assoc: take $w = v$, then

$u\times(v\times w) = u\times(v\times v) = u\times 0 = 0$

But $(u\times v)\times w = (u\times v)\times v \neq 0$

because $u\times v$, v are perpendicular

$u=(1,0,0)$, $v=(0,1,0)$, $w=(0,1,0)$

Q1: center & radius of $2x^2+2y^2+2z^2 = 4x-24z+1$

$x^2+y^2+z^2 = 2x-12z+0.5$

$x^2-2x+1^2+y^2+z^2+12z+6^2 = 0.5+1^2+6^2$

$(x-1)^2+y^2+(z+6)^2 = 37.5$

center $= (1,0,-6)$

radius $= \sqrt{37.5} = \sqrt{\frac{75}{2}} = \frac{5\sqrt{6}}{2}$

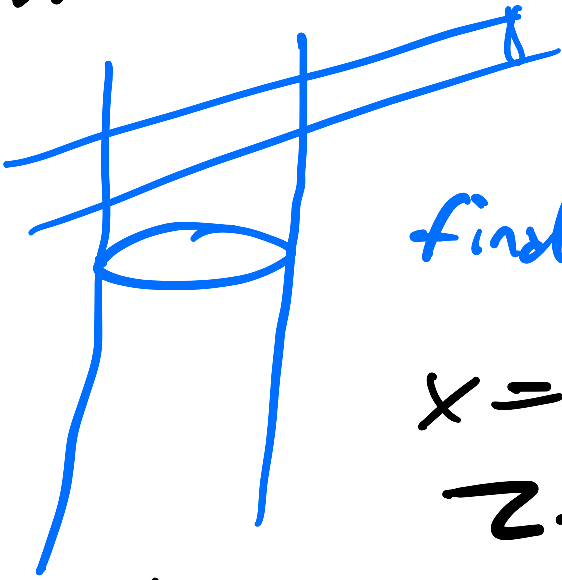
"$\frac{\sqrt{150}}{2}$"

WS1 done

DW2 Q2, Q9

Q9: Find vector function representation

Q 9. Find vector function representing intersection of $\underbrace{x^2+y^2=1}_{\text{cylinder}}$ & $\underbrace{z=y+2}_{\text{plane}}$

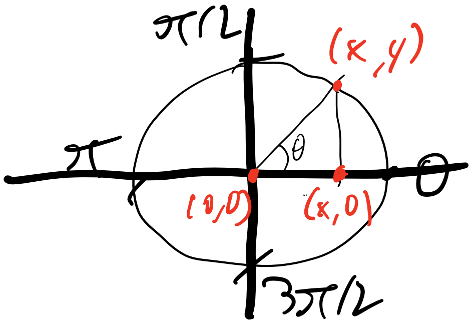


find shape:

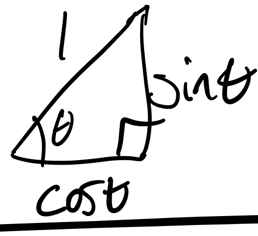
$x=\cos t,\ y=\sin t,\ 0\le t<2\pi$

$z=y+2=\sin t+2$

$\langle \cos t, \sin t, \sin t+2\rangle,\ 0\le t<2\pi$



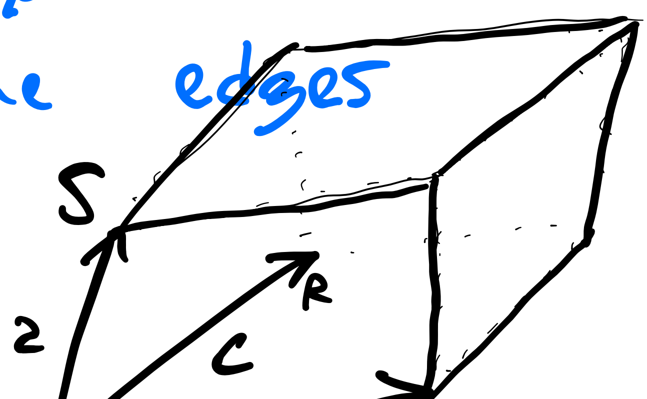
$x^2+y^2=1$



always find θ w/ $x=\cos\theta,\ y=\sin\theta$

Q2: Volume of parallelepiped with adjacent edges PQ, PR, PS, $P=(3,0,1)$, $Q=(-1,2,5)$, $R=(5,1,-1)$, $S=(0,4,2)$

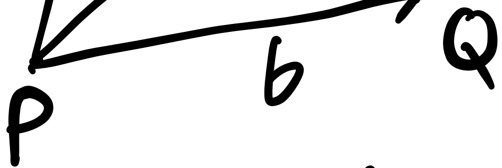
Step 1: draw diagram & find the edges



$PS = S-P = (-3,4,1)$

$PQ=(-4,2,4)$

$PR=(2,1,-2)$



$PR = (2,1,-2)$

$a = (-3,4,1),\ b = (-4,2,4),\ c = (2,1,-2)$

Step 2: Recall that volume of parallelepiped spanned by u, v, w is $|u \cdot (v \times w)|$

$$\text{Volume} = |a \cdot (b \times c)| =$$

$$b \times c = -2(2,-1,-2) \times (2,1,-2) =$$

$$-2\big((c + (0,-2,0)) \times c\big) =$$

$$-2\big(\underbrace{c \times c}_{0} + (0,-2,0) \times c\big) =$$

$$4\big((0,1,0) \times c\big) = 4(-2,0,-2)$$

$$= (-8,0,-8)$$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & -2 & 2 & 1 \end{array}$$

$$a \cdot (b \times c) = (-3,4,1) \cdot (-8,0,-8)$$

$$= 24 + 0 - 8 = 16$$

$$\text{Volume} = |16| = 16$$

WS3 ~~for~~ 6\17 will be posted
by end of 6\13

14, 15, 16, 1/4 of 13, 1/2 of 17

$\approx$ 3.5