All prior materials (6/9 - 6/12) up

Pre-lecture slides were up

Video uploaded in a few minutes

now you can see this

Cylindrical and Spherical

DW3 for 6/17 will be up tomorrow

Pre-lecture for 6/13

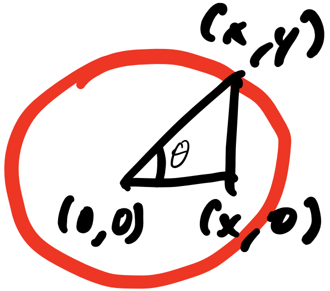
Quiz postponed to 6/14

HW extended 1 day to 6/15

Geometry of Cylinder

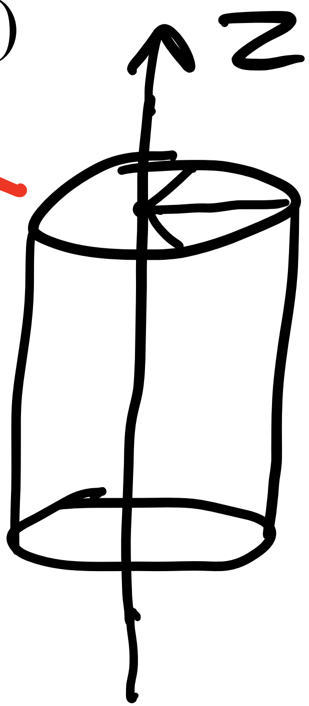
- Cylinder around z-axis has radius $r > 0$ and infinite height
- Any given height is a circle, parametrized by θ
- Cylindrical coordinates: $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$
- Bounds: $0 \leq \theta < 2\pi$ $\longrightarrow$ 1 lap around circle

For any circle cross section, z constant



$x = r\cos\theta$

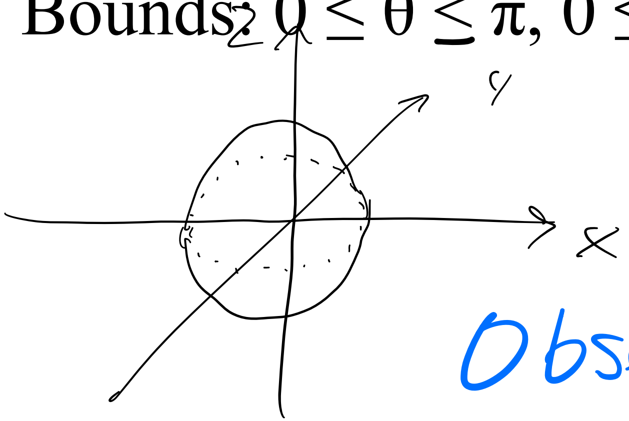
$y = r\sin\theta$



Geometry of Sphere

ρ is factored out

- Consider sphere of radius $\rho > 0$ centered at origin
- For any $\cancel{-r}\ {-\rho} \leq c \leq \cancel{r}\ \rho$, $z = c$ makes a circular cross section
- Spherical Coordinates: $(x, y, z) = \rho(\cos(\varphi)\sin(\theta), \sin(\varphi)\sin(\theta), \cos(\theta))$
- Bounds: $0 \leq \theta \leq \pi$, $0 \leq \varphi < 2\pi$



If $-\rho \leq c \leq \rho$, might as well let $c = \rho\cos\theta$

Observe that sphere equation is

$x^2 + y^2 + z^2 = \rho^2$. Plug in $z = \rho\cos\theta$

Scratchwork

$$x^2+y^2 = \rho^2 - z^2 = \rho^2(1-\cos^2\theta) = \rho^2\sin^2\theta$$

Fixing ρ & θ, this describes a circle of radius $r = \rho\sin\theta$

This means we can write $x = r\cos\varphi$, $y = r\sin\varphi$

$0 \le \varphi < 2\pi$

Sub for r: $x = r\cos\varphi = \rho\sin\theta\cos\varphi$

$y = r\sin\varphi = \rho\sin\theta\sin\varphi$

Converting between coordinates

- Call (x, y, z) the standard coordinates for R^3
- CYL -> ST or SP -> ST: just plug in
- ST -> CYL: $r = \sqrt{(x^2+y^2)}$, $\theta = \tan^{-1}(y/x)$
- ST-> SP: $\rho = \sqrt{(x^2+y^2+z^2)}$, $\theta = \cos^{-1}(z/\cancel{x}\ \rho)$, $\varphi = \tan^{-1}(y/x)$

same exact as previous red

$$x = rc,\quad y = rs \Rightarrow x^2+y^2 = r^2(c^2+s^2) = r^2$$

$$y/x = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

$$x^2+y^2+z^2 = \rho^2(\sin^2\theta(\cos^2\varphi+\sin^2\varphi)+\cos^2\theta) =$$

$$\rho^2(\sin^2\theta + \cos^2\theta) = \rho^2$$

$\rho^2(\sin^2\theta + \cos^2\theta) = \rho^2$

$z = \rho\cos\theta \Rightarrow \cos\theta = \frac{z}{\rho}$

Disclaimers

- No need to plug in if r = 0 $\rightarrow$ will have $x=0$ & $y=0$
- If x = 0, take $\underbrace{\tan^{-1}(y/x)}_{\text{division by } 0} = \pi/2 * \text{sgn}(y)$ $= \begin{cases} +1, & y>0 \\ -1, & y<0 \end{cases}$

$y>0 \Rightarrow \frac{+}{0} = +\infty \quad \tan^{-1}(\infty) = \frac{\pi}{2}$

$y<0 \Rightarrow \quad \tan^{-1}(-\infty) = -\frac{\pi}{2}$

(x, y, z) = (rcos(θ), rsin(θ), z) $\rightarrow$ + if $\frac{\pi}{2}$, − if $-\frac{\pi}{2}$

$x=0 \Rightarrow r\cos\theta = 0,\ r>0 \Rightarrow \cos\theta = 0 \Rightarrow$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$. Want $y>0 \Rightarrow \theta = \frac{\pi}{2}$

Practice Problems

$y<0 \Rightarrow \theta = -\frac{\pi}{2}$

because $\sin\frac{\pi}{2} = 1 > 0$ $\quad \sin(-\frac{\pi}{2}) = -1 < 0$

Convert these points into both cylindrical and spherical

- (0, 1, 0)
- (1, -2, 2) $\Leftarrow$ all in equation with x, y, z
- (3, 4, -5)

Identify the following surfaces

- Cylindrical: $r = 3$, $r^2+z^2 = 9$, $z = r$
- Spherical: $\rho = 0$, $\varphi = 2\pi/3$, $\theta = \pi/3$, $\rho(\sin(\varphi) + \cos(\varphi)) = 1$

Describe all geometrically except

Scratchwork

Q1: 45+10 = 55m total

Q2: 50m

3: 50m

4: 50m

Only take once

1/2 of the points

questions: 2 mc, 2 short, 2 long

1/2 of the points

shown on
Canvas quiz landing page
before you press begin

also discussions &
also lecture practice problems

Everything from HW & is fair game
except for spherical & cylindrical coord.

For Q2, 3, 4 (past Q1), day before the
quiz will also be fair game

Write however you want as long as you
can submit the images of
what you're writing on, whether it is
physical or digital

Upload formats preferred: pdf, notes doc,
jpg, png, webp, heic

try to avoid

On Quiz
landing page before
you click on start

Convert these points into both cylindrical and spherical

1 • (0, 1, 0)
2 • (1, -2, 2)
3 • (3, 4, -5)

Identify the following surfaces

- Cylindrical: $r = 3$, $r^2+z^2 = 9$, $z = r$
- Spherical: $\rho = 0$, $\varphi = 2\pi/3$, $\theta = \pi/3$, $\rho(\sin(\varphi) + \cos(\varphi)) = 1$

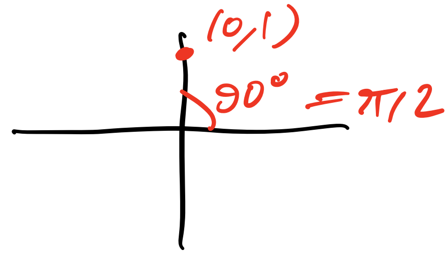
$1:\ r = \sqrt{x^2+y^2} = \sqrt{0^2+1^2} = 1,$

$z = z = 0$

$\tan\theta = \frac{y}{x} = \frac{1}{0}$

$\theta = \frac{\pi}{2}\,\mathrm{sgn}(1) = +\frac{\pi}{2}$

$(\theta, r, z) =$
$(\pi/2, 1, 0)$



$= \pi/2$

$\rho = \sqrt{0^2+1^2+0^2} = 1$

$\tan\varphi = \frac{1}{0}$ × $\Rightarrow \varphi = \pi/2$ actually

$\cos\theta = \frac{z}{\rho} = \frac{0}{1} = 0 \Rightarrow \theta = \pi/2$

$(\theta, \varphi, \rho) = (\frac{\pi}{2}, \frac{\pi}{2}, 1)$

2: $r = \sqrt{1^2+(-2)^2} = \underline{\sqrt{5}}$ $\quad \underline{z=2}$

$\tan\theta = y/x = -2/1 = -2$

$\Rightarrow \theta = \tan^{-1}(-2) = \underline{-\tan^{-1}2}$

$\rho = \sqrt{1^2+(-2)^2+2^2} = \sqrt{9} = \underline{3}$

$\tan\varphi = -2 \Rightarrow \varphi = \underline{-\tan^{-1}2}$

$\cos\theta = z/\rho = 2/3 \Rightarrow \theta = \underline{\cos^{-1}\frac{2}{3}}$

$r = \sqrt{3^2+4^2} = 5, \quad z = 5, \quad \theta = \tan^{-1}\frac{4}{3}$

$\rho = \sqrt{5^2+5^2} = 5\sqrt{2}, \quad \varphi = \tan^{-1}\frac{4}{3},$

$\theta = \cos^{-1}\frac{5}{5\sqrt{2}} = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

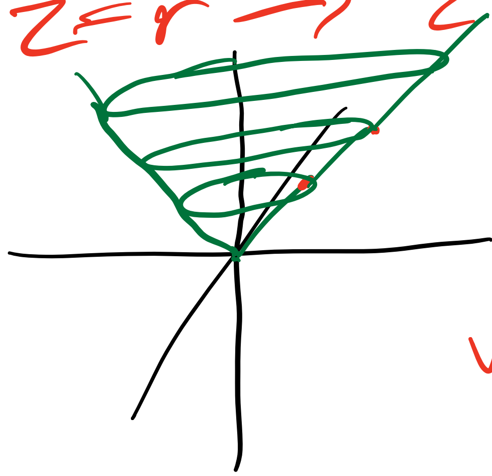
Identify the following surfaces

- Cylindrical: $r = 3$, $r^2+z^2 = 9$, $z = r$
- Spherical: $\rho = 0$, $\varphi = 2\pi/3$, $\theta = \pi/3$, $\rho(\sin(\varphi) + \cos(\varphi)) = 1$

$r=3 \Rightarrow 9 = r^2 = x^2+y^2 \Rightarrow$ cylinder of radius 3 because it's a circle for any value of z

$9 = r^2+z^2 = x^2+y^2+z^2 \Rightarrow$ sphere of radius 3 w/ center $(0,0,0)$

$z=r \Rightarrow z^2 = r^2 = x^2+y^2$,



At any value of z, cross section is a circle of radius z

which is a cone

~~Identify the following surfaces~~

- ~~Cylindrical: $r = 3$, $r^2+z^2 = 9$, $z = r$~~
- Spherical: $\rho = 0$, $\varphi = 2\pi/3$, $\theta = \pi/3$, $\rho(\sin(\varphi) + \cos(\varphi)) = 1$

$\rho=0 \Rightarrow 0=\rho^2 = x^2+y^2+z^2 \Rightarrow$ point $(0,0,0)$

$\varphi = \frac{2\pi}{3} \Rightarrow \frac{2\pi}{3} = \tan^{-1}\left(\frac{y}{x}\right) \Rightarrow$

$\tan\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = \tan\left(\frac{2\pi}{3}\right) \Rightarrow \frac{y}{x} = \tan\frac{2\pi}{3} \ldots$

trick question! arctan always lies between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$, so this equation has no solutions as $\frac{2\pi}{3} > \frac{\pi}{2}$

Empty set / no shape at all

$$\frac{\pi}{3} = \theta = \cos^{-1}\left(\frac{z}{\rho}\right) = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\cos\cos^{-1}\left(\underbrace{\frac{z}{\sqrt{x^2+y^2+z^2}}}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$

in $(-1,1)$ because $|z| = \sqrt{z^2} \le \sqrt{x^2+y^2+z^2}$

$$\frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{1}{2}$$

$$2z = \sqrt{x^2+y^2+z^2} \ge 0 \Rightarrow z \ge 0$$

$$4z^2 = x^2+y^2+z^2$$

$$x^2+y^2 = 3z^2,$$ which is

once again a cone because each cross section is circle of radius $\sqrt{3}z$

$$1 = \rho(\sin\varphi + \cos\varphi)$$

Might try: $1 = \rho^2(1+2s\varphi c\varphi) = (x^2+y^2+z^2)(1+2s\varphi c\varphi)$, but then you need $s\varphi$ & $c\varphi$

$$x = \rho\cos\varphi\sin\theta \Rightarrow x+y = \rho(\cos\varphi+\sin\varphi)\sin\theta$$

$$y = \rho\sin\varphi\sin\theta$$

$$1 = \frac{x+y}{\sin\theta} \Rightarrow x+y = \sin\theta \Rightarrow \sin^2\theta = (x+y)^2$$

$$x^2+y^2 = \rho^2\sin^2\theta$$

$$\sin^2\theta = \frac{x^2+y^2}{\rho^2} = \frac{x^2+y^2}{x^2+y^2+z^2}$$

$$(x^2+y^2+z^2)(x+y)^2 - (x^2+y^2) = 0$$

$$\underbrace{(x+y)^2 = \frac{x^2+y^2}{x^2+y^2+z^2}}_{\text{also fine}}$$

find eq.

lecture used for: ~~time buffer~~, student review, sneak peek next week

Webassign HW 1 Q7

$|u|=|v|=1$, $u+v+w=0$, what is $|w|$?

$w = -u-v \Rightarrow |w| = |-u-v| = |-1|\,|u+v| = |u+v|$

Hint: use $|w|^2 = w\cdot w$

Example w/ equation of a plane

6/10 Lec uncovered problem: find eq. of plane containing $(1,-2,0)$, $(3,1,4)$, $(0,-1,2)$, i.e. example of 3 points case of finding a plane

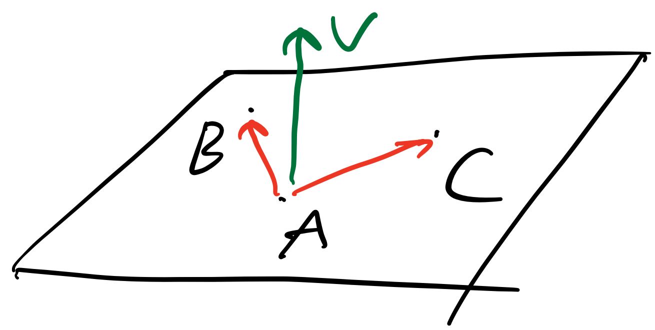
Previously, we did 1 normal vector

and 1 point case.

From that case, we already know that if $v = (a, b, c)$ is a normal vector and $v_0 = (x_0, y_0, z_0)$ is point on the plane, $a(x-x_0) + b(y-y_0) + c(z-z_0)$ is one equation ~~for~~ the plane

Goal: find a normal vector so that we can use the green equation

$A = (1, -2, 0)$
$B = (3, 1, 4)$
$C = (0, -1, 2)$



Observation: normal through A will be perpendicular to $\overrightarrow{AB}$ & $\overrightarrow{AC}$

So whatever v is, it must be perpendicular to $\overrightarrow{AB} = (2, 3, 4)$, $\overrightarrow{AC} = (-3, -2, -2)$

Note: $u \times w$ perp. to both u & w, so one possible choice for v is

So the possible

$$\overrightarrow{AB} \times \overrightarrow{AC} = (2,3,4) \times (-3,-2,-2) =$$

$$-(2,3,4) \times (3,2,2)$$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 2 & 3 & 4 & 2 & 3 \\ 3 & 2 & 2 & 3 & 2 \end{array}$$

$i: \ 3 \cdot 2 - 4 \cdot 2 = -2$

$j: \ 12 - 4 = 8$

$k: \ 4 - 9 = -5$

$$-(-2, 8, -5) = (2, -8, 5)$$

So $v = (2, -8, 5)$. And v is pointing out of A, we'll take

$$\underline{a(x-x_0) + b(y-y_0) + c(z-z_0)}$$

$$(x_0, y_0, z_0) = A = (1, -2, 0)$$

for the green formula

$$(a, b, c) = v = (2, -8, 5)$$

Sub in all values:

$$2(x-1) - 8(y+2) + 5z = 0$$

$$2x - 2 - 8y - 16 + 5z = 0$$

$$\underline{2x - 8y + 5z = 18}$$

an equation of the plane

Example of applying K

Find K for $r(t) = \langle 4t, -t^2, 2t^3 \rangle$

Recall $K = \dfrac{\|r' \times r''\|}{\|r'\|^3}$ ← num ← den

$r'(t) = \langle 4, -2t, 6t^2 \rangle = 2\langle 2, -t, 3t^2 \rangle$

$r''(t) = \langle 0, -2, 12t \rangle = 2\langle 0, -1, 6t \rangle$

$\|r' \times r''\| = 4\|\langle 2, -t, 3t^2 \rangle \times \langle 0, -1, 6t \rangle\|$

$$\begin{array}{ccc|cc} i & j & K & i & j \\ 2 & -t & 3t^2 & 2 & -t \\ 0 & -1 & 6t & 0 & -1 \end{array}$$

i: $-6t^2 - (-3t^2) = -3t^2$

j: $0 - 12t = -12t$

k: $-2 - 0 = -2$

$\|\langle -3t^2, -12t, -2 \rangle\| =$

$\sqrt{(-3t^2)^2 + (-12t)^2 + 4} = \sqrt{9t^4 + 144t^2 + 4}$

$\|r'\| = \|2\langle 2, -t, 3t^2 \rangle\| =$

$[2\|\langle 2, -t, 3t^2 \rangle\| = 2\sqrt{4 + t^2 + 9t^4}$

$\dfrac{\|r' \times r''\|}{\|r'\|^3} = \dfrac{4\sqrt{9t^4 + 144t^2 + 4}}{8(4 + t^2 + 9t^4)^{3/2}} =$

$$\frac{\sqrt{9t^4+144t^2+4}}{2(9t^4+t^2+4)^{3/2}}$$

impossible to integrate with respect to t, 2nd derivative of this is insane

Upload quiz by end of Friday, so that you have a >24 hour window to start the quiz

HW moved to due on 6/15

Quiz 6/14 11:59pm
HW 6/15 11:59pm