Surface Area

Lecture for 6/26

General Idea

Suppose we have some 2D region R and graph $z = f(x, y)$

- Portion of the graph constrained by the region is a surface S
- If $f = c$, then S is just a translation of R
  - Area(S) = Area(R)
- Can we find the surface area if f is non-constant?
- Can we find other quantities like center of mass?

Derivation of Surface Area

- Consider small patch of the surface from (x, y) to $(x+dx, y+dy)$
- For infinitesimal dx, dy, patch resembles a parallelogram P
  - In fact, P is a piece of the tangent plane
- Consider vectors u and v describing the parallelogram
- Let’s find the area of P and call it dS

Center of Mass

- Assign a weight $f(x, y)$ to every point (x,y) in R
- To find center of mass, let's try to average the weights
- Recall: average value of some g is $(\text{area}(R))^{-1} \iint_R g(x, y)\, dA$

Summary

Let R be the 2D region, S the portion of $z = f(x,y)$ above R

- area(S) = $\iint_R f\, dS = \iint_R (f_x^{\,2}+f_y^{\,2}+1)^{1/2}\, dA$
- $x_{COM} = (\iint_R x\, f(x, y)\, dA)/(\iint_R f(x, y)\, dA)$
- $y_{COM} = (\iint_R y\, f(x, y)\, dA)/(\iint_R f(x, y)\, dA)$

Practice Problems

Find the surface area of the following regions

- Portion of plane $3x+2y+z = 6$ lying in the 1st octant
- Portion of $z=xy$ in the cylinder given by $x^2+y^2 = 1$

Find the center of mass of the following regions

- Portion of the unit disk lying in the 1st quadrant
- Square $0 \leq x, y \leq \pi$ with weight function $f(x,y) = x\sin(x)y^3$

Scratchwork