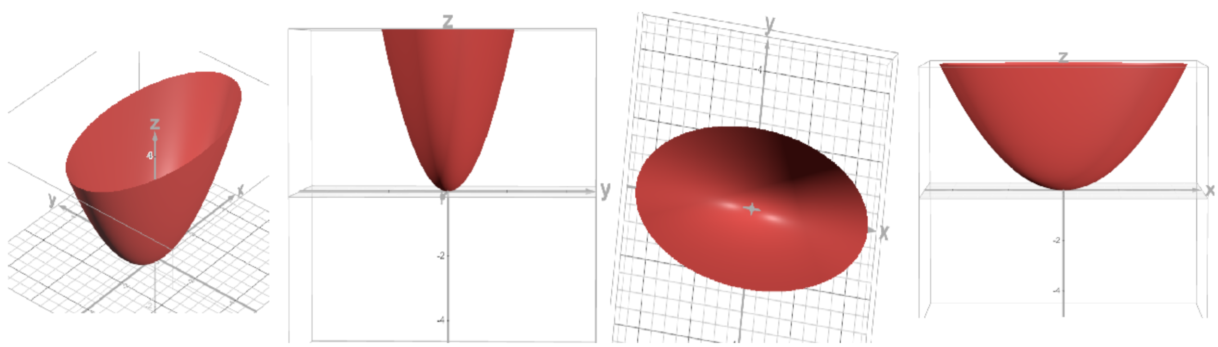
MATH 243: Review Midterm Exam 1 **Discussion Section:** ____________

This worksheet covers the material of the textbook sections 12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 13.1, 13.2, 13.3, 13.4. **This worksheet does NOT cover all topics/types of problems on the Exam 1 material.** Problems on the exam may not necessarily look exactly like the problems on this list. More practice problems can be found on the following resources: the instructor's Syllabus under **Suggested List of Textbook Problems**; the lecture notes; the discussion worksheets; the previous quizzes; the WebAssign homework.

1. Observe the following graphs:



   Which equation below gives the surface shown above?

   A. $z = \frac{x^2}{4} + y^2$

   B. $\frac{z^2}{9} = \frac{x^2}{4} - y^2$

   C. $1 - \frac{z^2}{9} = \frac{x^2}{4} + y^2$

   D. $z = \frac{x^2}{4} - y^2$

2. Given 2 vectors $\mathbf{u} = \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{k}$.

   (a) Compute $\mathbf{u} \cdot \mathbf{v}$

   (b) Compute $\mathbf{u} \times \mathbf{v}$

   (c) Determine the angle between $\mathbf{u}$ and $\mathbf{v}$

3. Consider the vectors $\mathbf{a} = \langle -1, 4, 8 \rangle$ and $\mathbf{b} = \langle 18, 2, 1 \rangle$.

   (a) Find the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

   (b) Find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

4. Determine whether the given vectors are orthogonal, parallel, or neither.

   (a) $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 12\mathbf{j} - 2\mathbf{k}$

   (b) $\mathbf{a} = \langle 6, 5, -2 \rangle$ and $\mathbf{b} = \langle 5, 0, 9 \rangle$

   (c) $\mathbf{a} = \langle -18, 15 \rangle$ and $\mathbf{b} = \langle 12, -10 \rangle$

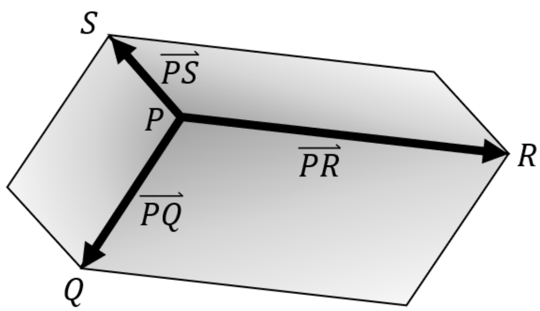
5. Consider points $P(1, 2, 1), Q(2, 5, 4), R(6, 9, 12)$ and $S(5, 6, 9)$ in $\mathbb{R}^3$.

(a) Find the area of the parallelogram with vertices $P(1,2,1)$, $Q(2,5,4)$, $R(6,9,12)$ and $S(5,6,9)$.

(b) Find the area of the triangle PQS.

(c) Show that the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{PS}$ are coplanar.

6. Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS. The points are given by $P(3,0,1)$, $Q(-1,2,5)$, $R(5,1,-1)$, and $S(0,4,2)$.



7. Consider the following vectors.

$$\mathbf{u} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \quad \mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 3\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$$

(a) Find the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

(b) Find the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Are the given vectors coplanar?

8. Find the parametric equations for the line of intersection of the planes $2x + 3y + 5z = 7$ and $x - y + 2z = 3$.

9. Find the vector equation, parametric equations and symmetric equations of the line passing through the points $A(2,1,1)$ and $B(3,2,-2)$.

10. Find an equation of the plane that passes through the point $P(1,1,3)$ and contains the line given by the symmetric equations $\dfrac{x+1}{2} = y + 2 = \dfrac{z-3}{2}$.

11. Find an equation for the plane that passes through the points $(0,-2,5)$ and $(-1,3,1)$ and is perpendicular to the plane $2z = 5x + 4y$.

12. Find an equation for the plane that passes through the points $(0,-2,5)$ and is parallel to the plane $2z = 5x + 4y$.

13. Write an equation of the plane containing the points

$$P(4,-3,1), \quad Q(-3,-1,1), \quad R(4,-2,8).$$

14. Let $\mathcal{C}$ be the curve given by the vector function $\mathbf{r}(t) = \cos(t)\mathbf{i} + \ln(t)\mathbf{j} + \dfrac{1}{t-3}\mathbf{k}$.

(a) Find the the domain of $\mathbf{r}(t)$. Use the interval notation.

(b) Find $\lim_{t\to\pi} \mathbf{r}(t)$.

(c) Find the point P on the curve at $t = 1$.

15. Consider the vector function $\mathbf{r}(t) = 2\,\mathbf{i} + 2\sin(t)\,\mathbf{j} + 2\cos(t)\,\mathbf{k}$.

    (a) Find the length of the curve of C with $\mathbf{r}(t)$, where $-2 \le t \le 2$.

    (b) Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter $t = \pi/6$. Simplify the answer completely.

    (c) Find the principal unit normal vector $\mathbf{N}(t)$ at the point with the given value of the parameter $t = \pi/6$. Simplify the answer completely.

    (d) Use the formula $\kappa = \dfrac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||}$ to find the curvature.

    (e) Find the binormal vector $\mathbf{B}(t)$ at the point with the given value of the parameter $t = \pi/6$. Simplify the answer completely.

    (f) Find the tangential and normal components of acceleration $\mathbf{a}(t)$.

16. Consider the position function $\mathbf{r}(t) = 8\sqrt{2}\,t\mathbf{i} + e^{8t}\mathbf{j} + e^{-8t}\mathbf{k}$.

    (a) Find the velocity of a particle with the given position function $\mathbf{r}(t)$.

    (b) Find the acceleration of a particle with the given position function $\mathbf{r}(t)$.

    (c) Find the speed of a particle with the given position function $\mathbf{r}(t)$.

17. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

$$\mathbf{a}(t) = 2\mathbf{i} + 2t\mathbf{k}, \quad \mathbf{v}(0) = 5\mathbf{i} - \mathbf{j}, \quad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}.$$

18. Given a vector function

$$\mathbf{r}(t) = (\arctan t)\,\mathbf{i} + 2t^2\,\mathbf{j} + t\ln(t)\,\mathbf{k}$$

    (a) Find a vector equation of the line tangent to the vector function at the point $\left(\frac{\pi}{4}, 2, 0\right)$

    (b) Find the unit tangent vector $\mathbf{T}(t)$ at the point $\left(\frac{\pi}{4}, 2, 0\right)$.