MATH 243 Worksheet 1: Vectors, Basic 3D Geometry

0. These are problems for the 2nd day of class. If you want problems for the 1st day of class, which covered only prerequisites, try the prerequisite quiz.

1. Find the center and the radius of the sphere of equation $2x^2 + 2y^2 + 2z^2 = 4x - 24z + 1$

2. Describe in words the region of $\mathbb{R}^3$ represented by the equations:

(a) $x^2 + y^2 = 4$

(b) $x = -1$

3. Find an equation of a sphere if one of its diameters has the endpoints at $(1, 2, 4)$ and $(4, 3, 10)$.

4. Choose a vector that is parallel to the tangent line to the curve $y = x^2$ at the point $(3, 9)$.

A. $\langle 3, 6\rangle$

B. $\langle 1, 2\rangle$

C. $\langle 1, 6\rangle$

D. $\langle 1, 9\rangle$

5. Which of the following is a vector that is parallel to the vector $\langle 1, -2, 3\rangle$?

A. $\langle \frac{1}{6}, -\frac{2}{6}, \frac{3}{6}\rangle$

B. $\langle \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$

C. $\langle \frac{1}{14}, -\frac{2}{14}, \frac{3}{14}\rangle$

D. $\langle -1, 2, -3\rangle$

E. $\langle -\frac{10}{14}, \frac{2}{14}, -\frac{3}{14}\rangle$

F. More than one of the above.

6. Find a vector with magnitude 5 that points in the same direction as the vector $\langle 2, 1, 2\rangle$.

7. Given the vectors $\mathbf{a} = \langle \mathbf{8}, \mathbf{1}, \mathbf{-4}\rangle$ and $\mathbf{b} = \langle \mathbf{5}, \mathbf{-2}, \mathbf{1}\rangle$, find the following:

(a) $\mathbf{a} + \mathbf{b}$

(b) $4\mathbf{a} + \mathbf{2b}$

(c) $||\mathbf{a}||$

(d) $||\mathbf{a} - \mathbf{b}||$

8. Let $\mathbf{u} = (t, 0, 6), \mathbf{v} = (1, 0, 2), \mathbf{w} = (0, 2, 5)$. Find all t such that $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = 0$

9. Compute $(5\sin(t), 0.5e^{\ln(t)}, \sqrt{t^2}) \times (10\sin(t), t, 2|t|)$

10. Extra problems from slides or mentioned in lecture:

a. Verify that $v \cdot v = ||v||^2$ for any vector v.

b. Verify that $u \times v$ is perpendicular to both u and v for any choice of u and v.

c. Find u, v, w such that $u \cdot v = u \cdot w$ but $v \neq w$. Similarly, find u, v, w such that $u \times v = u \times w$ but $v \neq w$. Lastly, find u, v, w with $(u \times v) \times w \neq u \times (v \times w)$.