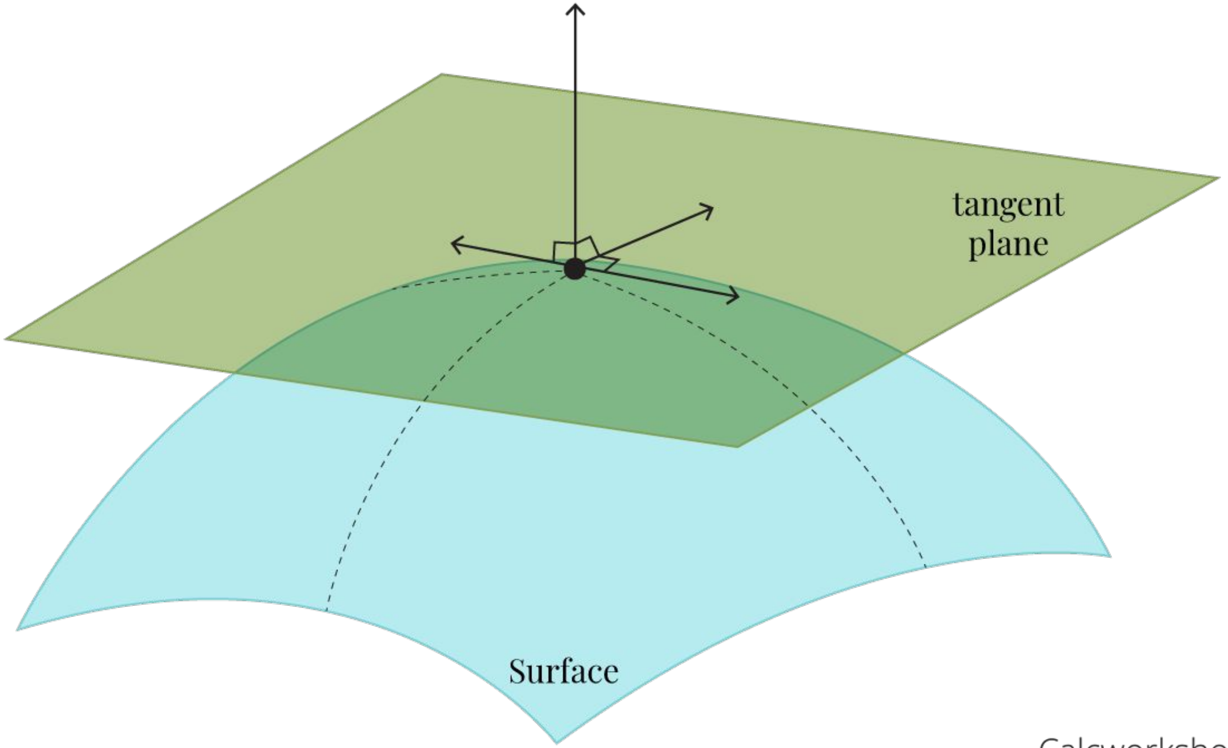
Tangent Plane to Graph

Lecture video for 6/11

What is the tangent plane? 🤔



Calcworkshop.com

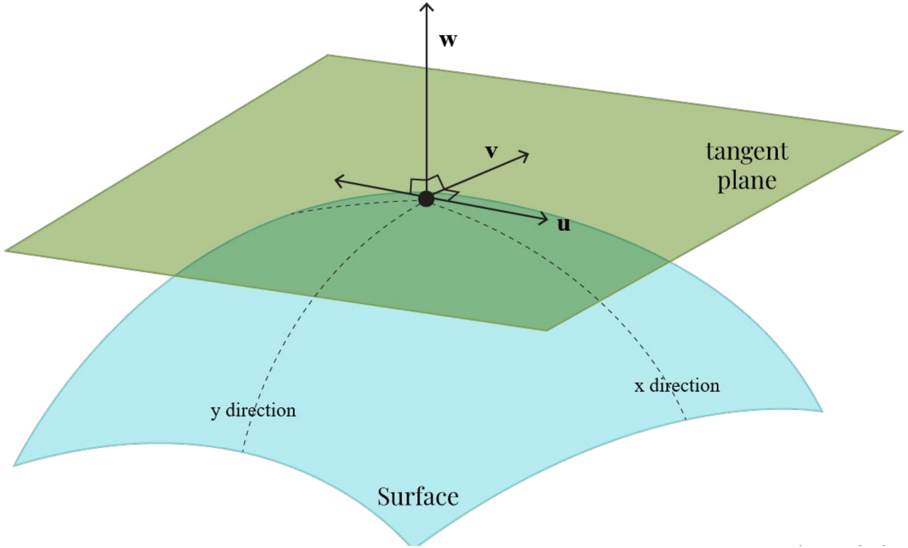
Sourced from https://calcworkshop.com/partial-derivatives/tangent-plane/

Problem Statement

Given a continuous function f: R^2 -> R whose partial derivatives exist everywhere, find the tangent plane to the graph of f at the point when $x = x_0$ and $y = y_0$

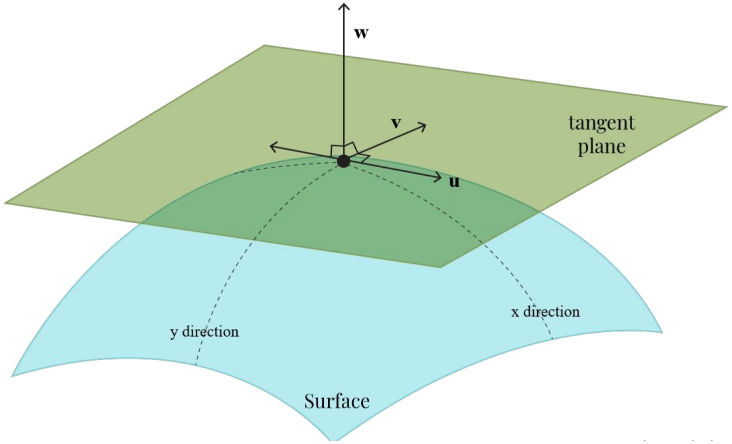
Recall: the graph of f is given by $z = f(x,y)$

Investigating the Plane



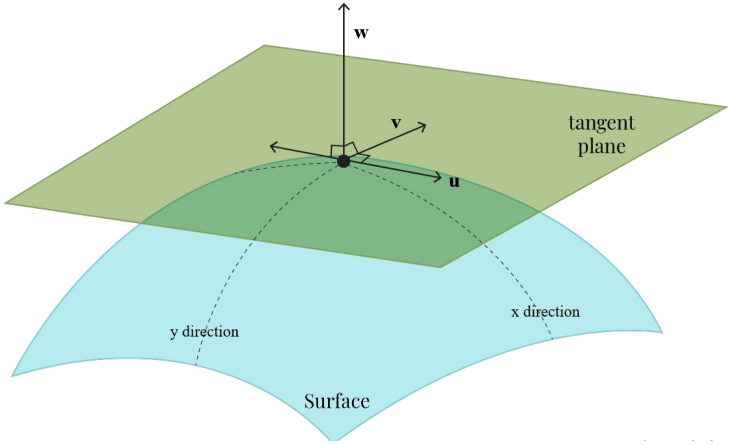
- Let u, v be tangent vectors in plane in x, y direction respectively
- Let w be the normal to plane
- Let a, b be slope of u, v resp.
- In x direction, x changes while y is held constant
- y is constant while traveling along u
- If x changes by dx, then z = f(x,y) changes by ≈ a*dx
- So u = (1, 0, a)
- Similarly, v = (0, 1, b)

Continuing the Investigation



- Note: w is perpendicular to u and v
- Also, **u** x **v** is perpendicular to u and v
- We may take w = **u** x **v**
- Compute w = (1,0,a) x (0,1,b) = (-a, -b, 1)
- If t is perpendicular to w, then z·w = 0
- Let c = $(x_0,y_0,f(x_0,y_0))$ be point of tangency
- If t on tangent plane, vector from t to c is perpendicular to w
- We get 0 = -0 = -w·(t-c) = -w · (t-c)
- Let t = (x, y, z)
- $a(x-x_0)+b(y-y_0) - (z-f(x_0,y_0)) = 0$
- But what are a and b?

Concluding the Investigation



- Recall that a is defined to be the slope of u
- As x changes by dx, z changes by a dx
- Thus, dz = a dx
- So $a = dz/dx = d/dx\, f(x,y) = f_x(x,y)$
- Thus, $a = f_x(x_0,y_0)$
- Similarly, $b = f_y(x_0,y_0)$
- Combine everything together:

$$\mathbf{f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0) = z-f(x_0,y_0)}$$

is the equation of the tangent plane to the graph of f when $(x,y) = (x_0,y_0)$

Review for Understanding

Use this formula to find the equation of the tangent plane to $z = x^2+y^2$ at the point (3, 4, 25)

We know how to get tangent lines from Calculus 1. Explain how you can also find b by viewing v as a tangent line to a restricted graph of f

Find the formula for normal line to graph of f(x,y)