MATH 243: Discussion Worksheet 1

Topics: Review of differentiation and integration; 12.1 Three-Dimensional Coordinate Systems
Please put away all electronic devices, including cell phones and calculators.

1. Find the derivative of $f(x) = \cos(x^2 + e^x)$.

2. Given that $x \ln(y) + e^y = 3$, find $\frac{dy}{dx}$ by implicit differentiation.

3. Suppose that $f(x)$ is a differentiable function such that $f(1) = 7$ and $f'(1) = 4$. Find an equation of the tangent line to $y = \sqrt{4 + 3f(x)}$ at $x = 1$.

4. Evaluate the indefinite integrals.

   (a) $\displaystyle\int e^{\tan x} \sec^2(x)\, dx$

   (b) $\displaystyle\int (2x^4 + 1) \ln(x)\, dx$

5. Evaluate the definite integrals.

   (a) $\displaystyle\int_0^{\pi/2} \sin^2\theta \cos\theta\, d\theta$

   (b) $\displaystyle\int_0^3 \frac{x}{\sqrt{x^2+1}}\, dx$

6. A particle is moving with acceleration at time t given by

$$a(t) = 3\cos t - 2\sin t.$$

   Given that $s(0) = 0$ and $v(0) = 4$, determine the position of the particle $s(t)$ at any time t.

7. Consider the point $P(3, 4, 5)$.

   (a) What is the projection of the point onto the xy-plane?

   (b) What is the projection of the point onto the xz-plane?

   (c) Find the length of the line segment $\overline{OP}$.

   (d) Find the position vector for the point $P(3, 4, 5)$ in **ijk**-form.

8. (a) Find an equation of the sphere that passes through the point $(1, 8, 5)$ and has center $(3, 1, -3)$.

   (b) Using your answer from part (a), find an equation describing the intersection of the sphere with the yz-plane. If the sphere does not intersect the yz-plane, write DNE.

9. Find an equation of a sphere if one of its diameters has endpoints at $(1, 2, 4)$ and $(4, 3, 10)$.

SOME USEFUL DEFINITIONS, THEOREMS, AND NOTATION:

Differentiation Formulas

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(b^x) = b^x \ln b$

$\frac{d}{dx}(cf(x)) = cf'(x)$

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$

The Product Rule $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

The Quotient Rule $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

The Chain Rule $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

Derivatives of Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

The Chain Rule for the power of a function

$\frac{d}{dx}[f(x)^n] = n[f(x)]^{n-1} f'(x)$

Derivatives of Inverse Trigonometric Functions

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\text{arccot}\, x) = -\frac{1}{1+x^2}$

Derivatives of Logarithmic Functions

$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$

Integration Formulas

a. If $n \neq -1$, $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$

b. $\int e^x \, dx = e^x + C$

c. $\int \sin x \, dx = -\cos x + C$

d. $\int \sec^2 x \, dx = \tan x + C$

e. $\int \sec x \tan x \, dx = \sec x + C$

f. $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

g. $\int \tan x \, dx = \ln|\sec x| + C$

i. $\int \frac{1}{x} \, dx = \ln|x| + C$

j. $\int b^x \, dx = \frac{b^x}{\ln b} + C$

k. $\int \cos x \, dx = \sin x + C$

l. $\int \csc^2 x \, dx = -\cot x + C$

m. $\int \csc x \cot x \, dx = -\csc x + C$

n. $\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

o. $\int \cot x \, dx = \ln|\sin x| + C$

where C is any real constant.

The Substitution Rule: This rule is useful when the integrand contains a function and (a constant multiple of) its derivative. Let $u = g(x)$, so $du = g'(x)\,dx$. Then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du.$$

Integration by Parts: This rule is useful for integrals that look like a product of two functions. Choose u and dv from the integrand so that $du = u'\,dx$ and $v = \int dv$. Then

$$\int u\, dv = uv - \int v\, du.$$

Suggested Textbook Problems

Section 12.1: 1-46