1: First we should set up the variables and determine what equations or inequalities we obtain from the constraints. Let x, y be the number of ounces of food A, food B respectively.

Vitamin constraints: The meal will contain $30x+25y$ mg of calcium. Since there is a 400 mg minimum, we get $30x+25y \geq 400$, which reduces to $6x+5y \geq 80$. Similarly, the iron and vitamin C constraints translate to $2x+y \geq 20$ and $2x+5y \geq 40$ respectively.

We wish to minimize the amount of cholesterol, which is $2x+5y$ mg. Since $2x+5y \geq 40$, we will get at least 40 mg. Is 40 possible? Yes. Set $x = 20$, $y = 0$ to satisfy the iron and vitamin C constraints. Thus, we should use 20 ounces of food A and 0 ounces of food B.

2: First we should set up the variables and determine what equations or inequalities we obtain from the constraints. Let g, d be the number of pounds of gold, diamond respectively that Ali Baba carries away

Sack weight constraint: Every pound of gold takes up 1/200th of the sack by volume while every pound of diamonds takes up 1/40th of the sack by volume. You can't go beyond the sack, so $g/200 + d/40 \leq 1 \rightarrow g + 5d \leq 200$.

Carrying constraint: Since Ali Baba can only carry 100 pounds, $g + d = 100 \rightarrow g = 100 - d$.

We want to maximize the number of coins, which is $20g + 60d = 20(g+3d) = 20(100+2d)$. Thus, we should maximize d. This makes sense as diamonds are more valuable pound for pound. The inequality from weight becomes $4d + 100 \leq 200 \rightarrow d \leq 25$, so the maximum number of coins is $20(100+2*25) = 3000$, attained when $d = 25$ and $g = 75$.

3: (a) True. A feasible solution is just one which satisfies all the constraints. An optimal solution must be feasible to make sense by definition, but a feasible solution doesn't have to be optimal. Example: maximize x when x is a real number in [0,1]. Then 1.5, 0.5, 1 are not feasible, feasible but not optimal, optimal respectively.

(b) False. By convexity, an optimal solution always occurs at the boundary of the feasibility region. If there was a solution in the region's interior, we could find* a better solution by moving slightly.

*Rigorous proof: we need to show that you can always move slightly to improve the solution