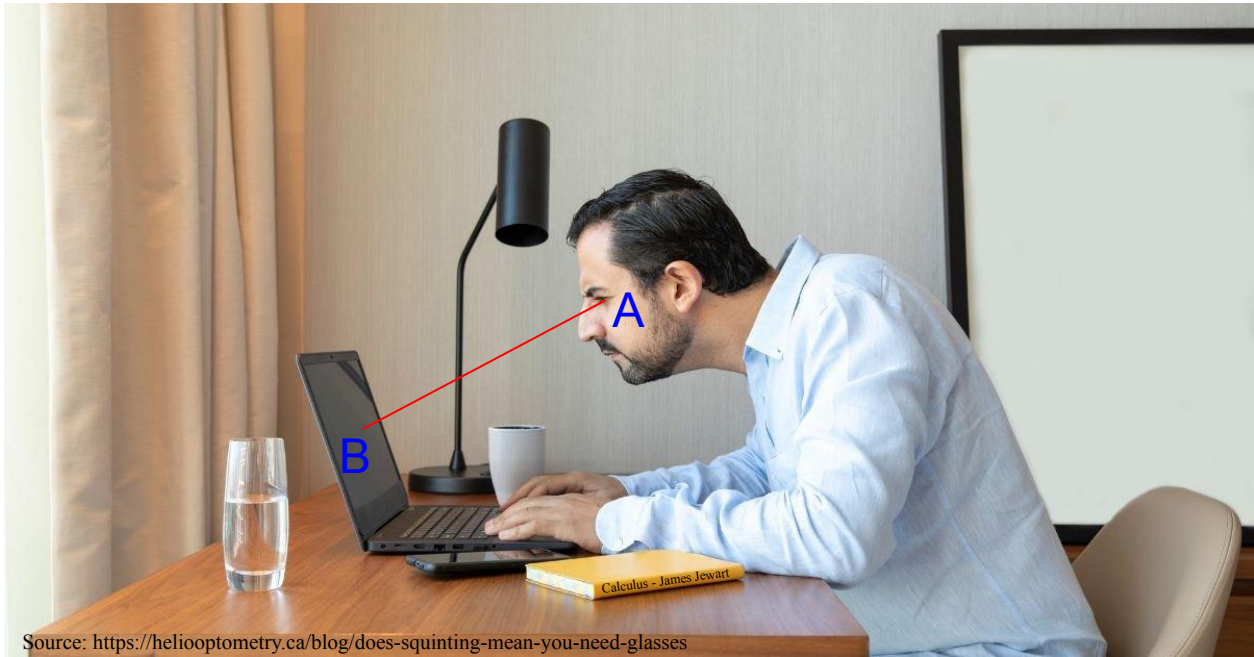
Equation of Lines and Planes

Lecture for 6/10

Equation of Lines

- In 3D, impossible with one normal equation
- So how do we do it? 🧐
- A to B is B-A
- Do A+(B-A)t
- $0 \leq t \leq 1$



Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Parametric and Vector Forms

- Previous slide gives (a+bt, c+dt, e+ft) for constants a-f
- This is the vector form
- Parametric form: $x = a+bt$, $y = c+dt$, $z = e+ft$
- Seems identical, but difference will be useful later
  - End of class: surface integrals, parameterizations

Equation of Planes Idea

- We can do one standard equation to describe
  - $x = 0$, $y = 0$, $x+y = z$ etc. are all planes
- Set of points perpendicular to given vector is a plane
- Can we reverse this to get vector for plane?

General Derivation of Equation

- Suppose a normal vector is v
- Consider vector u in the plane
- Equation $u \cdot v = 0$
- In practice, need to find what u and v are
  - Plane has 3 degrees of freedom
  - Exhaust degrees: vector is 2, point is 1

Example Case: vector and point

- 2+1 = 3, so it's possible
- Let given normal be v = (a, b, c)
- Let $v_0 = (x_0,y_0,z_0)$ be given point in the plane
- Let w = (x, y, z) be any point in the plane
- Obtain $0 = v \cdot (w-v_0)$
- Simplify: $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$

Practice problems

Understand your lines

- Find the line passing through (2, -1, 3) and (1, 4, -3)

Try the point point point case

- Find the equation of the plane containing (1, -2, 0), (3, 1, 4), and (0, -1, 2)

Extra problem on spatial awareness

- Determine if $-x+2z = 10$ and $(5, 2-t, 10+4t)$ are perpendicular, parallel, or neither

Scratch Work

Extra Problem

Extra problem on spatial awareness

- Determine if $-x+2z = 10$ and $(5, 2-t, 10+4t)$ are perpendicular, parallel, or neither