MATH 243 Midterm 1

1. Consider an infinitely differentiable function $f(x(r,s), y(r,s), z(r,s))$. Select the right expression for $\frac{\partial f}{\partial r}$

A. $\frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$

B. $\frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r} + \frac{\partial f}{\partial r}\frac{\partial r}{\partial r}$

C. $\frac{\partial f}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial f}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial f}{\partial r}\frac{\partial r}{\partial z}$

D. $\frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial s}\frac{\partial s}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}\frac{\partial s}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}\frac{\partial s}{\partial r}$

2. General knowledge bonanza. Select all of the following which are true

A. $4x + 3y - z = 5$ is an equation of the tangent plane to $z = x^3 + y^4$ at $(1, 1, 2)$

B. For $f(x, y, z) = e^{ye^z} + y^{\cos(x)} + (2z + x)^4 \sin(\sin(y))$, we have $f_{zzyzx} = 192 \cos(\sin(y)) \cos(y)$

C. The arc length of $\mathbf{r}(t) = (-1, t^2, 420+t)$ from $(-1, 0, 420)$ to $(-1, 4, 422)$ is given by $\int_0^2 \sqrt{4t^2 + 1}\, dt$

D. $x - 2y + z = 0$ is an equation of the plane containing $(1, 2, 3), (6, 5, 4), (7, 8, 9)$

3. Three of these statements are equivalent conditions, select the odd one out

A. $\mathbf{u}, \mathbf{v}, \mathbf{w}$ lie on the same plane

B. The parallelepiped formed by $\mathbf{u}, \mathbf{v}, \mathbf{w}$ has volume 0

C. The vectors $\mathbf{u} \times \mathbf{v}$ and $\mathbf{w}$ are parallel

D. If M is the 3×3 matrix where the three rows are $\mathbf{u}, \mathbf{v}, \mathbf{w}$, then swapping any two rows of M doesn't change its determinant

4. Let $\mathbf{a} = (3, 0, 1), \mathbf{b} = (0, 2, -1)$. Compute $\mathbf{v} = (\mathbf{a} \times \mathbf{b}) \times \mathbf{a}) \times \mathbf{a}) \times \cdots \times \mathbf{a})$ where there are 50 $\mathbf{a}$'s following the $\mathbf{b}$. If the sum of the coordinates of $\mathbf{v}$ is $r^s t$ for integers r, s, t where r is positive squarefree and r, t have no common factors, find $r + s + t$.

5. Convert the point $(r, \theta, z) = (\sqrt{3}, \frac{\pi}{5}, 3)$ in cylindrical coordinates to spherical coordinates (ρ, θ, ψ), then find the following values: $\rho^2, \theta + \psi$.

6. Let $f(x, y) = \frac{(y^3 - x^2 y)e^{\cos(\ln(1+|y|))}}{(1+x^2+y^6)(8x^3 - 6x^2 y - 3xy^2 + y^3)}$. The set of points (a, b) where $\lim_{(x,y)\to(a,b)} f(x, y)$ doesn't exist is composed of n distinct lines. Find n.

Note: this is for grading convenience. You won't get any credit for guessing the answer without actually doing the problem.

7. A, B, C, D, E, F, G are collinear points evenly spaced apart in that order with $B = (1, 2, 3), E = (-4, 5, -6)$. Parametrize the line through all of the points. Parametrize the line segment from A to G.

8. Let $\mathbf{r}(t) = \langle t, 2t^{3/2}, 1 - t\rangle$. Compute acceleration, then find the tangential and normal components of acceleration.

9. Consider a differentiable function $f(x, y)$ and let $p = (2, 3)$. The directional derivative of f at the point p in the direction of $(3, -4)$ is 9 while the derivative of f at p in direction $\theta = 225°$ is $-4\sqrt{2}$. Find the derivative of f at p in the direction of $(5, 12)$, and find the maximum rate of change of f at p.