MATH 243 Quiz 4

1. Select all vector fields that are conservative

A. Reverse alphabet: $\mathbf{F}(v, w, x, y, z) = \langle z, y, x, w, v \rangle$

B. Garfield: $\mathbf{F}(x, y, z) = \langle z, z, z \rangle$

C. $\mathbf{F}(x, y) = \langle e^x \cos(y) + e^{x-y}, e^{y-x} - e^x \sin(y) \rangle$

D. $\mathbf{F}(x, y) = \langle y^2(1 + \cos(x + y)), 2xy - 2y + y^2 \cos(x + y) + 2y \sin(x + y) \rangle$

2. The Fundamental Theorem of Line Integrals and its consequences have been a benefit to the human race. Select all of the following that are true:

A. If C is some path starting at $\mathbf{a}$ and ending at $\mathbf{b} \neq \mathbf{a}$, $-C$ is the same path but in the reverse direction, and $\mathbf{F}$ is not conservative, then $\int_C \mathbf{F} \cdot d\mathbf{r} = -\int_{-C} \mathbf{F} \cdot d\mathbf{r}$

B. If $\mathbf{F}$ is conservative, C_1 is the upper semicircle $y = \sqrt{1 - x^2}$ taken counterclockwise, and C_2 is the lower semicircle $y = -\sqrt{1 - x^2}$ taken clockwise, then $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$

C. If $\mathbf{F}$ is conservative, C is some closed curve parametrized by $\mathbf{r}$, and ds is the arc length differential, then $\int_C (\mathbf{F} \cdot \mathbf{r}) ds = 0$

D. If $\mathbf{F}$ is conservative, T is the triangle with vertices $(0, 0), (1, 0), (0, 1)$ traversed counterclockwise, and $2T$ is the same triangle but doubled in size (so $(0, 1), (1, 0)$ are sent to $(0, 2), (2, 0)$ respectively), then $\int_{2T} \mathbf{F} \cdot d\mathbf{r} = 2 \int_T \mathbf{F} \cdot d\mathbf{r}$

3. Find $\iiint_E 11xy\, dV$ where E is the region bound by $z = 7$ and $z = x^2 + y^2 - 9$

4. Let C be the helix represented by $x^2 + y^2 = 2, z = \tan^{-1}(\frac{y}{x})$. Let γ be half a turn of this helix, starting at $(1, -1, -\frac{\pi}{4})$ and ending at $(1, 1, \frac{\pi}{4})$. For $\mathbf{F} = (x^2, y^2, z)$, let $L = \int_\gamma \mathbf{F} \cdot d\mathbf{r}$.
We have $L = \frac{a}{b}$ in reduced form for integers a, b. Find $10a + b$

5. Let B_1, B_2 be balls with radii 1 and centers $(4, 5, 6), (5, 6, 7)$ respectively. Find the volume of the intersection of B_1 and B_2

6. Let E be the region bound by $x = z, x = z + 1$, and $x^2 + y^2 + 2z^2 = 2xz - 2yz + 1$. Find $\iiint_E y\, dV$

7. Extra Credit: Calculate $\iint_S \frac{dx\, dy}{1 - xy}$ by any means necessary, where $S = [0, 1]^2$ is the unit square
Hint: one way is using the substitution $(x, y) = (u + v, u - v)$