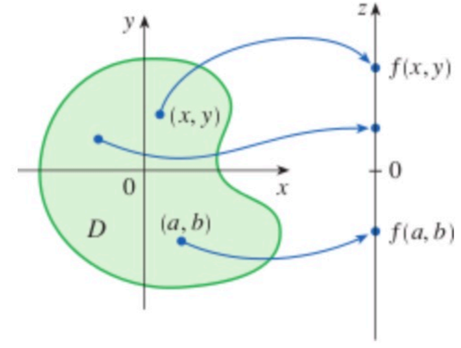
Definition

A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.



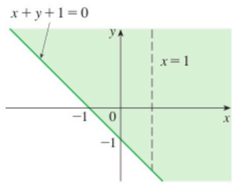
Example 1

For each of the following functions, evaluate $f(3, 2)$ and find and sketch the domain.

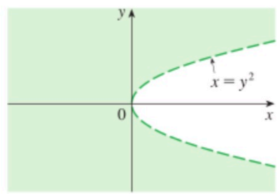
(a) $f(x, y) = \dfrac{\sqrt{x+y+1}}{x-1}$

(b) $f(x, y) = x \ln(y^2 - x)$

Domain of $f(x, y) = \dfrac{\sqrt{x+y+1}}{x-1}$



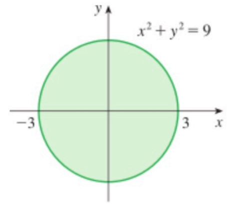
Domain of $f(x, y) = x \ln(y^2 - x)$



Example 2

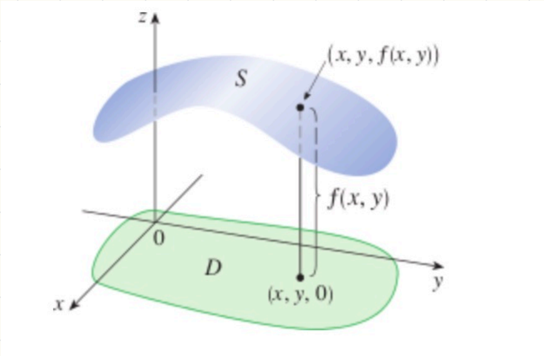
Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

Domain of $g(x, y) = \sqrt{9 - x^2 - y^2}$



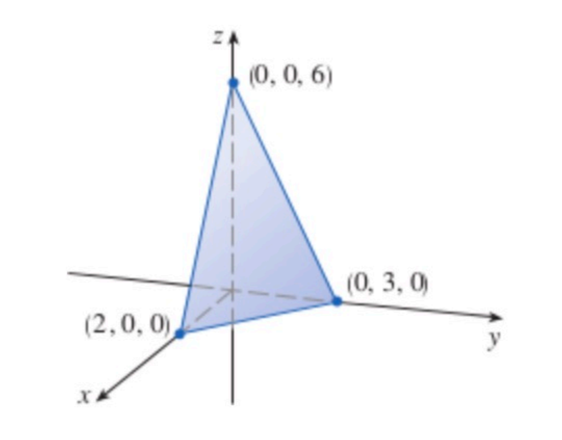
Definition

If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.



Example 5

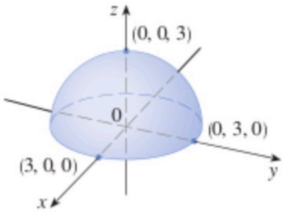
Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.



Example 6

Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

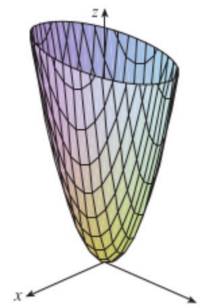
Graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$



Example 8

Find the domain and range and sketch the graph of $h(x, y) = 4x^2 + y^2$.

Graph of $h(x, y) = 4x^2 + y^2$

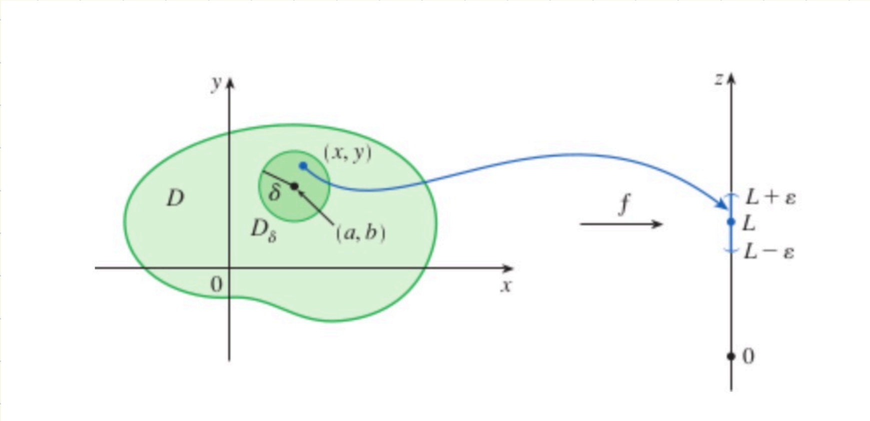


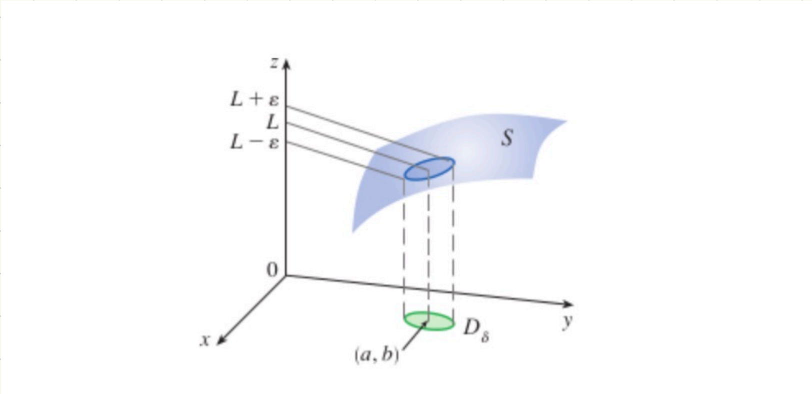
Values of $f(x, y)$

$x \backslash y$	−1.0	−0.5	−0.2	0	0.2	0.5	1.0
−1.0	0.455	0.759	0.829	0.841	0.829	0.759	0.455
−0.5	0.759	0.959	0.986	0.990	0.986	0.959	0.759
−0.2	0.829	0.986	0.999	1.000	0.999	0.986	0.829
0	0.841	0.990	1.000		1.000	0.990	0.841
0.2	0.829	0.986	0.999	1.000	0.999	0.986	0.829
0.5	0.759	0.959	0.986	0.990	0.986	0.959	0.759
1.0	0.455	0.759	0.829	0.841	0.829	0.759	0.455

Values of $g(x, y)$

x \ y	−1.0	−0.5	−0.2	0	0.2	0.5	1.0
−1.0	0.000	0.600	0.923	1.000	0.923	0.600	0.000
−0.5	−0.600	0.000	0.724	1.000	0.724	0.000	−0.600
−0.2	−0.923	−0.724	0.000	1.000	0.000	−0.724	−0.923
0	−1.000	−1.000	−1.000		−1.000	−1.000	−1.000
0.2	−0.923	−0.724	0.000	1.000	0.000	−0.724	−0.923
0.5	−0.600	0.000	0.724	1.000	0.724	0.000	−0.600
1.0	0.000	0.600	0.923	1.000	0.923	0.600	0.000


y
(x, y)
D
δ
D_δ
(a, b)
0
x
f
z
L + ε
L
L − ε
0


z
L + ε
L
L − ε
S
0
x
y
D_δ
(a, b)

Different paths approaching (a, b)