

## MATH230 Week 2 Worksheet - Basics of Sets

The following problems are on notation and single-set properties. For 1-3, convert to set-builder notation. For 4-6, roster notation. For 7-8, determine true or false;  $A$  is an arbitrary set.

- 1: The set of football teams in the NFL
- 2:  $\{3,4,5,6,7\}$
- 3:  $\{1,3,5,7,\dots,99\}$
- 4:  $\{x \mid x \text{ is a digit in the number } 352646\}$
- 5:  $\{x \mid x \text{ is a letter in HIPPOPOTAMUS}\}$
- 6:  $\{x \mid 2-x = 4 \text{ and } x \text{ is an integer}\}$
- 7a:  $\{a, b, c\} = \{b, c, a\}$ , 7b:  $A \in A$ , 7c:  $0 \in \emptyset$ , 7d:  $\emptyset \in \emptyset$ , 7e:  $0 = \emptyset$ , 7f:  $\{\emptyset\} = \emptyset$
- 8a:  $\{a,b\} \in \{a,b,c\}$ , 8b:  $\{a,b\} \subseteq \{a,b,c\}$ , 8c:  $\emptyset \subseteq A$ , 8d:  $\{a,b\} \subseteq \{a, \{b,a\}\}$
- 9: List all subsets of  $\{1,2,3,4\}$ . How many are there? How many are proper?
- 10: Determine all pairs of sets among  $\emptyset, \{1\}, \{1,2\}, \{1, a\}, \{a, b, 2\}$  which are disjoint.

The following problems cover interactions between multiple sets.

- 11: Simplify the following:  $(A^c)^c \cap A$ ,  $((A^c \cap B^c)^c \cup B)^c$ ,  $(A \cup B^c \cup C)^c \cup (C^c \cup C)^c$
- 12: Let  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 2, 3\}$ ,  $B = \{3, 4, 5, 6\}$ , and  $C = \{2, 3, 4\}$ . Find the following sets:  $A^c$ ,  $A \cup B$ ,  $B \cap C$ ,  $(A \cup B) \cap C$ ,  $(A \cap B) \cup C$ ,  $A^c \cap (B \cup C)^c$ .
- 13: If  $|A| = 4$ ,  $|B| = 5$ , and  $|A \cup B| = 9$ , find  $|A \cap B|$ .
- 14: Let  $A, B$  be subsets of a universal set  $U$  and suppose  $|U| = 100$ ,  $|A| = 60$ ,  $|B| = 40$ , and  $|A \cap B| = 20$ . Compute  $|A \cup B|$ ,  $|A \cap B^c|$ ,  $|A^c \cap B|$ .

If you want extra practice, here are some harder problems

- 15: How many subsets does  $\{a, b, c, \dots, x, y, z\}$  have?
- 16: Prove  $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
- 17: Find exactly when  $|A \cup B \cup C| = |A| + |B| + |C|$  is true
- 18: Expand out, then simplify  $(x-a)(x-b)(x-c)\dots(x-z)$

Even more problems. If you have nothing better to do, try 20.

- 19: If  $|A| = |B| = 12$ ,  $|A \cap B| = |A \cap C| = 5$ ,  $|B \cap C| = 4$ ,  $|A \cap B \cap C| = 2$ ,  $|A \cup B \cup C| = 25$ , find  $|C|$ .
- 20:  $A_1, \dots, A_n \subseteq \{1, 2, \dots, 420\}$  are  $n$  subsets with  $|A_i|$  odd and  $|A_i \cap A_j|$  even for any  $i \neq j$ . What's the largest value of  $n$  that's possible?