MATH 230 Week 14 Worksheet

1: Check that $\begin{bmatrix} 0.2 & 0 & 0.3 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.4 & 0.3 \end{bmatrix}$ represents a regular Markov chain and find its steady state vector.

2: The absorbing stochastic matrix $\begin{bmatrix} a & 0 & b \\ c & 1 & d \\ e & 0 & f \end{bmatrix}$ has only one absorbing state. Given this information, find the steady-state matrix.

3: A rat starts at point A. Each second, it randomly chooses to move or stay. At A, it has a probability 0.4 of moving to B. At B, it has a probability 0.4 of moving back to A. Scientists have figured out how to make sure the rat doesn't go anywhere besides A or B. Find the probability p_n that the rat will be at point A after n seconds elapse. Use this to find $\lim_{n\to\infty} p_n$.

4: Claim: all Markov chains either have an absorbing state or their stochastic matrix is regular. Justify this claim or find a counterexample.

5: In a zero-sum game, the 1st player makes one of 4 choices publicly, after which the 2nd player makes one of 3 choices. The game has pay-off matrix $\begin{bmatrix} -1 & 1 & 2 \\ 3 & 1 & 1 \\ -1 & 0 & 3 \\ 3 & 2 & -1 \end{bmatrix}$ with respect to the 1st player. Determine the optimal strategy for player 1.