

This worksheet covers selected topics in Sections 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 15.1, 15.2, 15.3. **This worksheet does NOT cover all problems and situations on the Exam**

2. Problems on the exam may not necessarily look exactly like problems on this worksheet. For more practice problems, please see the list of practice problems on the instructor Syllabus under "Suggested List of Textbook Problems"; on the lecture notes; on the discussion worksheets for weeks 6, 7, 8, 9, and 10; on previous quizzes, and on the WebAssign homework.

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1. Show that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^4 + y^2}$$

does not exist. Justify your answer.

2. Show that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{4xy}{3x^2 + y^2}$$

does not exist. Justify your answer.

3. Find the first partial derivatives and second partial derivatives of $f(x, y) = x^3 - 2xy + xy^3 + 3y^2$.

4. Given an implicit function $e^z = xyz$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

5. Consider the function $f(x, y) = \ln(x - 4y)$.

- (a) Find the linearization of f at the point $(5, 1)$.
- (b) Use your answer to part (a) to approximate $f(4.9, 0.8)$.

6. Consider the function $g(x, y, z) = y \arctan(x^2 + z)$.

- (a) Find the linearization of g at the point $(1, 2, -1)$.
- (b) Use your answer to part (a) to approximate $g(0.9, 2.1, -1.1)$.

7. Let $z = f(x, y)$ with $x = g(t)$ and $y = h(t)$. Find $z'(1)$, assuming all functions involved are differentiable, and

$$g(1) = 3, \quad g'(1) = 4, \quad h(1) = 7, \quad h'(1) = \frac{1}{3}, \quad f_x(3, 7) = \frac{1}{2}, \quad f_y(3, 7) = 3.$$

8. Let $F(t) = f(x, y)$ and $x = x(u, v)$, $y = y(u, v)$, $u = u(t)$, and $v = v(t)$. We assume all functions involved are differentiable. What is an expression for $\frac{dF}{dt}$?

9. The radius of a right circular cone is increasing at a rate of 2 m/s while its height is decreasing at a rate of 3 m/s.

- (a) Identify the mathematical interpretation of each given quantity and write an expression for $\frac{dV}{dt}$, where $V = \frac{\pi}{3}r^2h$ is the volume of the cone.

- (b) What is the physical interpretation of the quantity $\frac{dV}{dt}$?
- (c) How fast is the volume of the cone changing when the radius is 1 m and the height is $\frac{3}{\pi}$ m?
Specify the unit measure.
10. The gradient of a function of two variables is
- a vector function
 - a line
 - a negative number
 - a positive number
11. The rate of change of a function of two variables at a point and in the direction of a unit vector is
- a function
 - a line
 - a scalar
 - a vector
12. (a) Find the rate of change of the function $v(x, y) = y \cos x$ at the point $P(0, \pi)$, in the direction of the vector $\mathbf{w} = \mathbf{i} - \mathbf{j}$.
- (b) In which direction does the function $v(x, y) = y \cos x$ change most rapidly at $P(0, \pi)$? Justify your answer.
- (c) What is the maximum rate of change of v at the point $P(0, \pi)$? Justify your answer.
13. Find the directional derivative of the function
- $$f(x, y) = x \ln y$$
- at the point $(2, e)$ in the direction making an angle of $\frac{\pi}{6}$ radians above the horizontal.
14. Let $P(1, 2, 1)$ be a point on the ellipsoid $4x^2 + y^2 + 4z^2 = 12$.
- Find an equation of the tangent plane to the ellipsoid at the point $P(1, 2, 1)$.
 - Find parametric equations of the normal line to the ellipsoid at the point $P(1, 2, 1)$.
 - At what points does the normal line through the point P on the ellipsoid intersect the sphere $x^2 + y^2 + z^2 = 18$?
15. Let $f(x, y) = x^2 \ln(y)$ and $\mathbf{u} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$
- Find the gradient vector field of f .
 - Find the gradient of f at the point $(1, e)$.
 - Find the rate of change of f at the point $(1, e)$ in the direction of the unit vector \mathbf{u} .
 - Find an equation of the tangent plane to the surface $z = x^2 \ln(y)$ at the point $P(1, e, 1)$.
 - Find parametric equations of the normal line to the surface $z = x^2 \ln(y)$ at the point $P(1, e, 1)$.

16. Find the critical points of the function

$$g(x, y) = (x + y^2)e^x$$

Determine whether each critical point is a local maximum, local minimum, or saddle point.

17. Find the extreme values of the function $f(x, y) = xy$ on the ellipse $36x^2 + y^2 = 72$.

18. Find the extreme values of

$$f(x, y) = x^2 + y^2 + 4x - 4y$$

on the region described by the inequality $x^2 + y^2 \leq 18$.

19. Evaluate $\int_0^1 \int_0^1 (x - y)^2 dx dy$

20. Evaluate $\iint_R e^{-y} \cos(x) dA$, where $R = \{(x, y) \mid 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq 1\}$

21. The volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + \alpha$ and above the rectangle $R = [-1, 1] \times [0, 1]$ is $\frac{10}{3}$. Find the value of the constant α , where $\alpha \geq 1$.

22. Find the average value of $f(x, y) = xe^y$ over the rectangle R with the vertices $(0, 0), (4, 0), (4, 1)$, and $(0, 1)$.

23. Evaluate the double integral $\iint_D e^{x^3} dA$, where $D = \{(x, y) \mid 0 \leq y \leq x^2 \text{ and } 0 \leq x \leq 4\}$.

24. Evaluate the double integral

$$\iint_D x \sqrt{y^2 - x^2} dA, \text{ where } D = \{(x, y) \mid 0 \leq x \leq y \text{ and } 0 \leq y \leq 3\}.$$

25. Consider the region D , where D is bounded by $y = \cos x$ where $0 \leq x \leq \pi/2$, $y = 0$, and $x = 0$.

- (a) Iterate the double integral $\iint_D \sin^2 x dA$ over a type I region D . DO NOT EVALUATE.

- (b) Iterate the double integral $\iint_D \sin^2 x dA$ over a type II region D . DO NOT EVALUATE.

- (c) Evaluate the integral you iterated in part (a).

26. Change the order of integration for the iterated integral (Do NOT evaluate):

$$\int_1^e \int_0^{\ln x} f(x, y) dy dx.$$

27. Evaluate the double integral

$$\iint_R 3x^2 y dA$$

where R is the region enclosed by the triangle with vertices $(0, 0), (-1, 1)$, and $(1, 1)$.

28. Evaluate the iterated integral by changing the order of integration:

$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy.$$

29. Which of the following answers correctly rewrites the iterated integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} x^3 dy dx$ in polar coordinates?

- A. $\int_0^{\pi/2} \int_0^2 e^{-r^2} r^3 \cos^3(\theta) dr d\theta$
- B. $\int_0^{\pi/2} \int_0^2 e^{-r^2} r^4 \cos^3(\theta) dr d\theta$
- C. $\int_0^\pi \int_{-2}^2 e^{-r^2} r^3 \cos^3(\theta) dr d\theta$
- D. $\int_0^2 \int_0^{\sqrt{4-r^2}} e^{-r^2} r^4 \cos^3(\theta) dr d\theta$

30. Find the volume of the solid region bounded by $z = 0$, $z = 1$, $x^2 + y^2 = 1$, and $x^2 + y^2 = 9$.

31. Compute the value of the iterated integral

$$\int_0^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (x^2 + y^2) dx dy$$

by converting to polar coordinates.

32. Let $f(x, y) = 100e^{-(x^2+y^2)/4}$, and D is the disk of radius 4 centered at the origin $(0, 0)$.

- (a) Set up an iterated integral to compute $\iint_D f(x, y) dA$ in polar coordinates.
 - (b) Compute the iterated integral you set up in part (a).
33. Let D be the region in the first quadrant of the xy -plane enclosed by the lines $x = 0$, $y = x$, and the curve $y = \sqrt{4 - x^2}$. Use a double integral in polar coordinates to compute the area of this region D .