Multivariate Limits

Pre-lecture for 6/16

Directions for One Variable

- Recall $\lim_{x\to a+} f(x) = \lim_{x\to a-} f(x)$ then limit exists at $x = a$
- So we need left and right side directions equal

Also recall alternate forms of the limit:

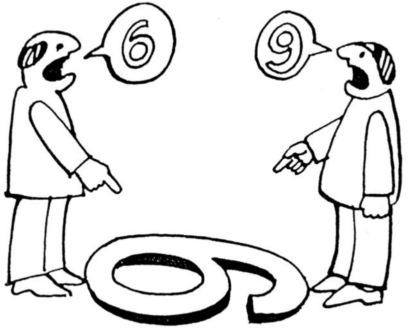
- $\lim_{x\to a} f(x) = \lim_{x\to 0} f(x+a) = \lim_{|x|\to 0} f(x+a)$
- We will use the last alternate form

Directions for Multiple Variables

Definition of multivariate limit:

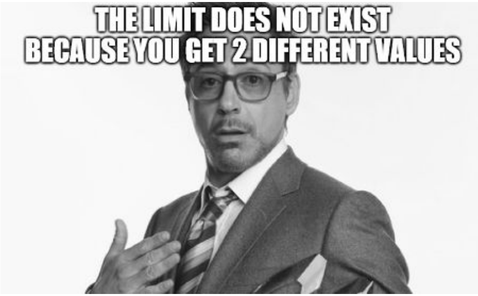
- $\lim_{v \to c} f(v) = L$ if $f(v)$ approaches L regardless how v approaches c

https://donmcminn.com/2018/02/yes-hand-value-dialectic-thinking/



Let's consider an alternate form:

- $\lim_{v \to c} f(v) = \lim_{||v|| \to 0} f(v+c)$
- $||v||$ is a scalar, we can now use 1 variable tactics



Properties of Limits

- Sums: $\lim(f+g) = \lim f + \lim g$
- Products: $\lim fg = (\lim f)(\lim g)$
- Composition: $\lim f \circ g = f \circ (\lim g)$ when f is continuous
- Limits on right side must exist for rule to apply

More Properties

These follow by using the previous slide

- lim cf = c lim f, lim (f-g) = lim f - lim g
- Limit of finite products or sums
- lim f/g = (lim f)/(lim g) provided lim g non-zero

Continuity

Recall continuity for one variable

- If limit exists and $\lim_{x \to a} f(x) = f(a)$, f continuous at $x = a$
  - Function f cont. if it is continuous for every value in domain
- Same definition for multivariable functions
- We now omit writing the variable when it doesn't matter

Properties of Continuity

If f and g are continuous, these are too:

- $f \pm g$, fg, $f \circ g$
- f/g at any point where $g \neq 0$

Continuity is preserved when space gets upgraded:

- If $f(x)$ cont, then $g(a_1, a_2, a_3, \dots) = f(a_i)$ continuous

Checking Limits

- Limits will exist at almost all points (for math 243)
  - Use limit properties to work your way up
- For problem points, try cancelling factors
- Nothing to cancel, try directions
  - Get 2 different values and it doesn't exist
- All directions equal, try substitutions to prove existence
  - Polar: Put $x = r\cos(t)$, $y = r\sin(t)$
  - Linear: Put $y = tx$
- Also try squeeze theorem for existence

Checking for Continuity

Simple recipe:

- Find domain of function
- Check in domain where properties imply continuity
- Check the problem points by considering limits

Practice Problems

Check the following functions are continuous:

- $f(x,y) = (x+y)/(x^2+y^2+1)$
- $f(x,y,z) = 3x^2z + xy\cos(x-y+z) + e^{\tan(x)}$

Show the limit doesn't exist or find its value

- $(2x^2-xy-y^2)/(x^2-y^2)$ at $(1,1)$
- $(x^2+y^2)/(x^4+3y^4)$ at $(0,0)$
- $x^2\ln(x+1)/(x^2+y^2)$ at $(0,0)$

Scratchwork

More Scratchwork