MATH 243-012 — Lecture Quiz 2: Solution Sheet

1. Consider $\mathbf{u} = \langle -5, 0, 4\rangle$ and $\mathbf{v} = \langle 2, -4, 4\rangle$.

(a) *Determine whether* $\mathbf{u}$ *and* $\mathbf{v}$ *are perpendicular. Justify.*

$\mathbf{u}\cdot\mathbf{v} = (-5)(2) + 0(-4) + 4(4) = -10 + 16 = 6 \neq 0$, so they are *not* perpendicular.

Final Answer: $\boxed{\text{Not perpendicular (since } \mathbf{u}\cdot\mathbf{v} = 6 \neq 0).}$

(b) *Find* $\operatorname{proj}_{\mathbf{v}} \mathbf{u}$.

$$\operatorname{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{6}{2^2 + (-4)^2 + 4^2}\mathbf{v} = \frac{6}{36}\mathbf{v} = \frac{1}{6}\mathbf{v} = \left\langle \tfrac{1}{3}, -\tfrac{2}{3}, \tfrac{2}{3}\right\rangle.$$

Final Answer: $\boxed{\left\langle \tfrac{1}{3}, -\tfrac{2}{3}, \tfrac{2}{3}\right\rangle}$.

2. *Find parametric equations of the line through* $A(5, 2, 3)$ *and* $B(4, -2, -3)$.
Direction $\mathbf{d} = B - A = \langle -1, -4, -6\rangle$. Using point A:

$$\boxed{x = 5 - t, \quad y = 2 - 4t, \quad z = 3 - 6t \quad (t \in \mathbb{R})}.$$

Final Answer: $\boxed{x = 5 - t,\ y = 2 - 4t,\ z = 3 - 6t}$.

3. *(True/False) The planes* $2x + 4y - 3z + 4 = 0$ *and* $2x + 3y = -5z - 5$ *are parallel.*
Normals: $\mathbf{n}_1 = \langle 2, 4, -3\rangle$, $\mathbf{n}_2 = \langle 2, 3, 5\rangle$. Since $\mathbf{n}_1$ is not a scalar multiple of $\mathbf{n}_2$, the planes are not parallel.
Final Answer: $\boxed{\text{False}}$.

4. *Find an equation of the plane through* $P(1, 2, 3)$ *that contains the line*

$$\frac{x+1}{2} = \frac{y-2}{3}, \qquad z = 3.$$

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$$\text{Line: } x = -1 + 2t,\ y = 2 + 3t,\ z = 3.$$

Direction vector of the line:

$$\mathbf{v} = \langle 2,\ 3,\ 0\rangle.$$

Choose a point on the line (e.g., at $t = 0$):

$$P_1 = (-1,\ 2,\ 3).$$

Vector connecting P to P_1:

$$\mathbf{u} = \overrightarrow{PP_1} = P_1 - P = \langle -2,\, 0,\, 0 \rangle.$$

A normal to the plane is the cross product:

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \langle 0,\, 0,\, -6 \rangle$$

Thus the plane through $P(1,2,3)$ with normal $\mathbf{n}$ satisfies

$$\mathbf{n} \cdot \langle x-1,\, y-2,\, z-3 \rangle = 0 \;\Rightarrow\; 0(x-1) + 0(y-2) + 6(z-3) = 0 \;\Rightarrow\; \boxed{z = 3}.$$

Final Answer (in the box): $\boxed{z = 3}$.

5. *Which equation matches the pictured surface?*
Final Answer: B

A. $\dfrac{z^2}{9} = \dfrac{x^2}{4} + y^2 \quad \to$ double cone (opens in $\pm z$).

B. $z = \dfrac{x^2}{4} + y^2 \quad \to$ elliptic paraboloid (upward bowl).

C. $z = \dfrac{x^2}{4} - y^2 \quad \to$ hyperbolic paraboloid (saddle).

D. $\dfrac{z^2}{9} = \dfrac{x^2}{4} - y^2 \quad \to$ hyperbolic cone (ruled; hyperbolic cross-sections).