

Local Extrema

Pre-lecture for 6/23

Types of Extrema

In Calc 1, we saw 2 types of extrema

- Local minimum
- Local maximum

Definition for multivariable functions:

- If $f(\mathbf{x}) \geq f(\mathbf{x}_0)$ for \mathbf{x} around \mathbf{x}_0 , f has a local minimum at \mathbf{x}_0
- If $f(\mathbf{x}) \leq f(\mathbf{x}_0)$ for \mathbf{x} around \mathbf{x}_0 , f has a local maximum at \mathbf{x}_0

Critical Points

Calc 1 definition:

- c is a critical point if $f'(c) = 0$ or f' not defined at $x = c$

New definition:

- \mathbf{v} is a critical point if $\nabla f(\mathbf{v}) = \mathbf{0}$ or ∇f not defined at \mathbf{v}

Just like Calc 1:

- If \mathbf{v} is an extrema and ∇f is defined at \mathbf{v} , then $\nabla f(\mathbf{v}) = \mathbf{0}$

Saddle Points

In Calc 1, critical points are almost always extrema

- If function continuously differentiable, “neither” is rare

Now, they are much more common

- If \mathbf{x}_0 is a critical point but not an extrema, call it a saddle point
- Will have $f(\mathbf{x}) > f(\mathbf{x}_0)$ for some \mathbf{x} near \mathbf{x}_0 and $f(\mathbf{x}) < f(\mathbf{x}_0)$ for others

2nd Derivative Test

How can we tell between local min, local max, or saddle?

- Define $D = f_{xx}f_{yy} - (f_{xy})^2$
- If $D(\mathbf{v}) > 0$ and $f_{xx}(\mathbf{v}) > 0$, then \mathbf{v} is a local min
- If $D(\mathbf{v}) > 0$ and $f_{xx}(\mathbf{v}) < 0$, then \mathbf{v} is a local max
- If $D(\mathbf{v}) < 0$, \mathbf{v} is a saddle point
- If $D(\mathbf{v}) = 0$, test inconclusive; try other methods

We shall see explanation of test and “other methods” in lecture

Practice Problems

Find and classify all critical points

- $f(x, y) = x^2 + xy + y^2 + x + y + 1$
- $f(x, y) = x^3 + y^3 - 3xy + 06232025$
- $f(x, y) = y^3 - 3y^2 + 3x^2y - 3x^2 + 1$

Find and classify the critical points of $f(x, y) = |x-2| + |y-3|$

Scratchwork

