MATH 243 Quiz 2

1. Select all of the following which is true about directional derivatives:

A. The directional derivative of f in the direction of v is $(\nabla f) \cdot v$

B. The directional derivative can never be larger than the greater of $|f_x|$ and $|f_y|$

C. There is always some direction in which the directional derivative is 0

D. The directional derivative for the direction $\theta = \frac{2\pi}{3}$ can be found by doubling the directional derivative for $\theta = \frac{\pi}{3}$

E. The smallest directional derivative is in the direction of $-\nabla f$

2. Select all of the limits that exist:

A. $\lim_{(x,y)\to(2,3)} \frac{\sin((x-2)^2+(y-3)^2)}{\sqrt{x^2+y^2-13}}$

B. $\lim_{(x,y,z)\to(0,0,0)} \frac{e^{|x|+|y|+|z|}}{\sqrt{x+1}-\cos(y)+e^z}$

C. $\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^3+y^3}$

D. $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x+y)}{\sin(x+y)}$

3. Let $f(a,b,c,d,r) = aarabcrabdab$. Find $f_{abracadabra}(1,1,1,1,1)$

4. Let $f(x,y) = x^2+y^2, x = (r-1)^3+(s+1)^2, y = (r+1)^2+(s-1)^3$, and $r = \tan(t), s = \cos(t)$. Let $g(t) = \frac{df}{dt}$. Find $g(0)$.

5. Find two points v_1, v_2 on the graph of $z = x^2+y^2$ such that the two resulting tangent planes to that graph intersect, and the angle of intersection is $60°$. Once you have found two points that work, show that your answer is correct.

6. Let S be the sphere which contains $(1.03, 4.98, 10.04)$ and has center $(4,5,6)$. Find a good approximation for the radius of S and show your work.

If you use an unorthodox (not covered in class) method, note that "good" means within 0.0005 of the actual radius, and you must explain your method.

7. Extra credit: let $f(x,y,z) = \frac{xyz}{x^2+y^2+z^2}$ Determine whether the limit $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$ exists or not, and if it exists, find its value. You won't get any credit for guessing the answer without proper justification.

8. Extra extra credit: let $f(x,y,z,w) = \frac{xyzw}{x^3+y^3+z^3+w^3}$ Determine whether $\lim_{(x,y,z,w)\to(0,0,0,0)} f(x,y,z,w)$ exists or not, and if it exists, find its value. You won't get any credit unless your explanation is correct.