Textbook Sections: 16.9
Topics: The Divergence Theorem
Instructions: Try each of the following problems, show the detail of your work.

Cellphones, graphing calculators, computers and any other electronic devices are not to be used during the solving of these problems. Discussions and questions are strongly encouraged.

The Divergence Theorem

1. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = \langle x^2, -y, z\rangle$ where E is the solid cylinder $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$, by computing the surface integrals on the boundary and by applying the divergence theorem.

2. Calculate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x, y, z) = \langle xye^z, xy^2z^3, -ye^z\rangle$ and S is the surface of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$.

3. Calculate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x, y, z) = \langle xe^y, z - e^y, -xy\rangle$ and S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 4$.

4. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where

$$\mathbf{F}(x, y, z) = \langle xy + 2xz, x^2 + y^2, xy - z^2\rangle$$

   and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = y - 2$ and $z = 0$.

5. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$
   (a) Compute *curl* $\mathbf{F}$
   (b) Compute *div* $\mathbf{F}$
   (c) Calculate the **flux** of the vector field $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$ across the surface S of the solid region E which is the upper half of the ball of radius 1 given by the equations $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$.

Suggested Textbook Problems

Section 16.9	1-12, 17-19
Chapter 16 Concept Check	1-13, 15, 16
Chapter 16 True-False	1-7, 11, 12
Chapter 16 Review	2-18, 27-30, 34, 35

SOME USEFUL DEFINITIONS, THEOREMS AND NOTATION:

The Divergence Theorem
Let E be a simple solid region and let S be the boundary of E, given with positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. Then

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\,dV.$$