MATH 243 Worksheet 4: Extrema and Optimization

Note: Problems 1 & 2 are leftover problems from slides. Problems 3-6 are brand new.

1: Find and classify all critical points:
a. $f(x, y) = x^2 + xy + y^2 + x + y + 1$
b. $f(x, y) = x^3 + y^3 - 3xy + 06232025$
c. $f(x, y) = y^3 - 3y^2 + 3x^2y - 3x^2 + 1$
d. $f(x, y) = |x - 2| + |y - 3|$
e. $f(x, y, z) = x^2 + y^2 + z^2 + xy + yz + zx + x + y + z + 1$
f. $f(x, y) = x^4 - y^4 - 4xy^2 - 2x^2$
g. $f(x, y) = x^{2024} + y^{2026}$

2: Find the min and max, or show they don't exist:
a. $f(x, y) = |x| + |y| + |z|$ on $x^2 + y^2 + z^2 < 1$
b. $f(x, y) = 4x^2 + 10y^2$ on $x^2 + y^2 \leq 4$
c. $f(x, y) = 2x^2 - y^2 + 6y$ on $x^2 + y^2 \leq 16$
d. $f(x, y) = xyz$ on $x^2 + y^2 + z^2 = 1$

3: Find the local extrema and saddle points for the following functions:
(a) $f(x, y) = x^2 + xy + y^2 + y$

(b) $f(x, y) = x^3 + y^3 + 3xy$

(c) $f(x, y) = x^4 - 2x^2 + y^3 - 3y$

4: Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set D, where D is the closed triangular region with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$.

5: Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

6: Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.