Equation of Lines and Planes

Lecture for 6/10

other sources gen. follow this too

$\underbrace{x \times x}_{\text{bad}} = 0$

$a_1, a_2, a_3, \dots$ const.

$v_1, v_2, \dots$ vectors

General remark:

$a, b, c, d, e, \dots, r, s, t$ are typically scalars

u, v, w are vectors

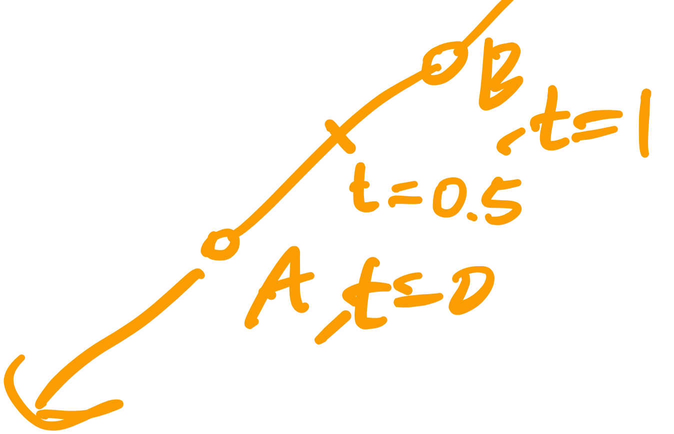
x, y, z up to the situation

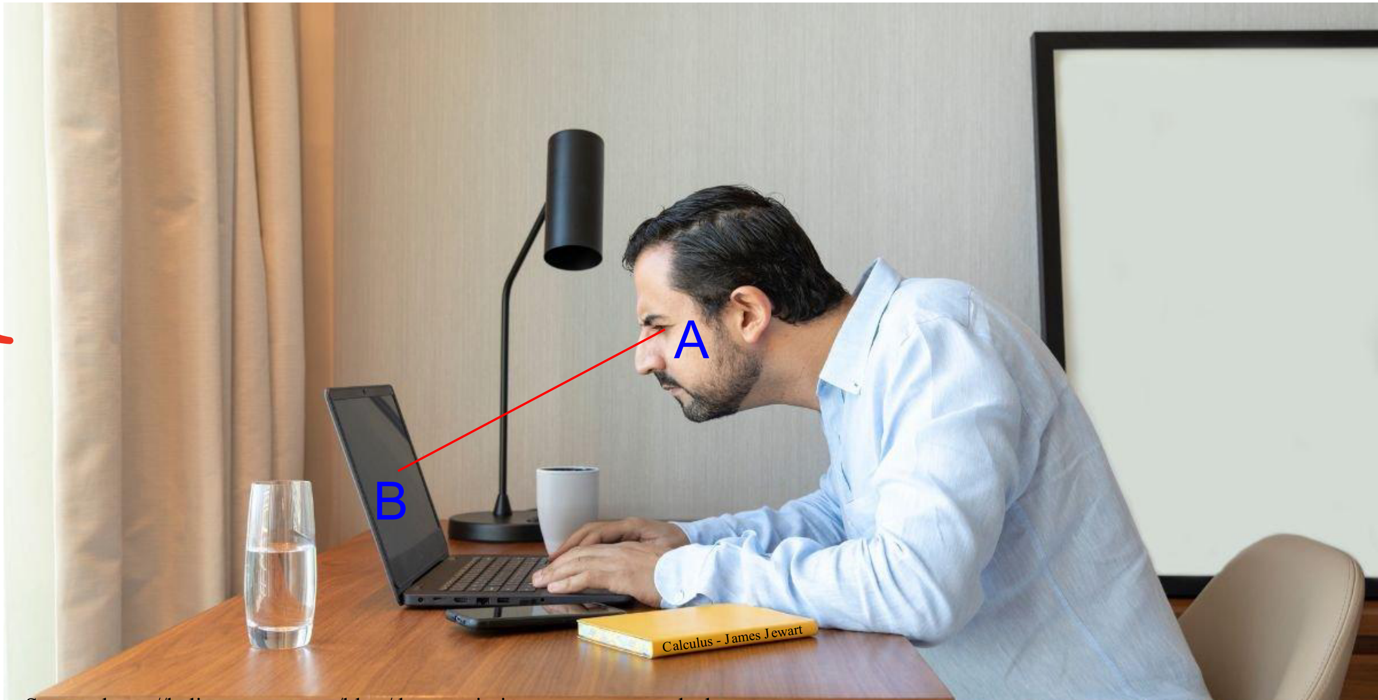
Equation of Lines

vec

- In 3D, impossible with one normal equation
- So how do we do it?
- $\underbrace{A \text{ to } B \text{ is } B-A}_{}$ vec
- Do $A+(B-A)t$
- $0 \leq t \leq 1$

expression of segment between A & B





Source: https://heliooptometry.ca/blog/does-squinting-mean-you-need-glasses

Parametric and Vector Forms

$A + (B-A)t$

- Previous slide gives $\boxed{(a+bt,\ c+dt,\ e+ft)}$ for constants a-f $\in \mathbb{R}$, scalars
- This is the vector form
- Parametric form: $x = a+bt$, $y = c+dt$, $z = e+ft$
- Seems identical, but difference will be useful later
  - End of class: surface integrals, parameterizations

$2 \leq t \leq 6$ bigger line segment

$A = (a, c, e)$

$B - A = (b, d, f) \neq 0$

$0 \leq t \leq 1 \Rightarrow$ line segment

$-\infty < t < \infty \Rightarrow$ line

Later: $x = f(s,t)$?

Lets: $x = f(s,t)$

$y = g(s,t)$

$z = h(s,t)$

Equation of Planes Idea

surface

no t constraint ⟹ line

Surf int.

- We can do one standard equation to describe
  - x = 0, y = 0, x+y = z etc. are all planes
- Set of points perpendicular to given vector is a plane
- Q: Can we reverse this to get vector for plane?

conclusion: 1 equation in x, y, z can describe a plane

$\geq s?$

$\geq t \leq ?$

(x,y,z)

(x',y',z')

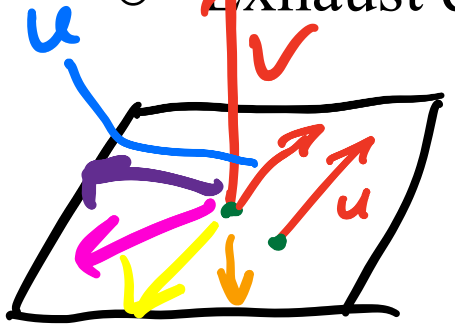
Some students may ask, who cares where things are coming from? Just give us the formulas to plug everything in

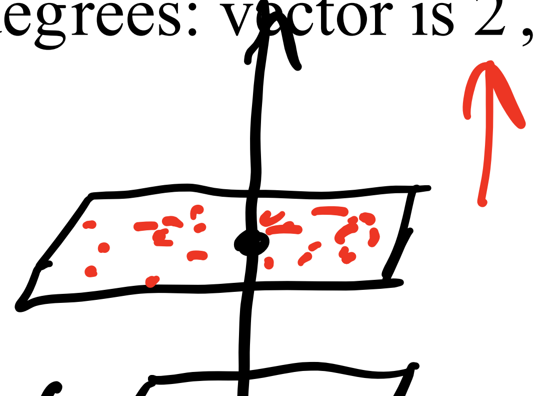
A: you may not care, but others do.
Also, your knowledge will break on tiny changes. Many problems in math are small variations of given examples

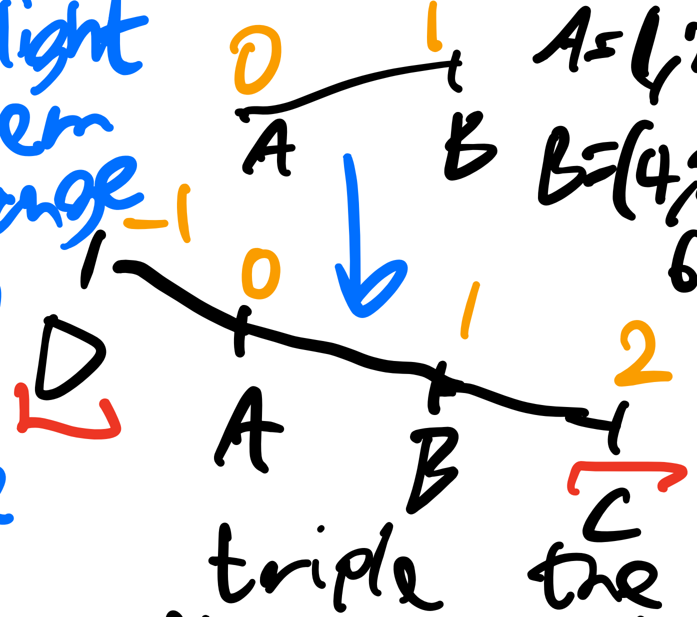
General Derivation of Equation

- Suppose a normal vector is v ← not unique
- Consider vector u in the plane
- Equation $u \cdot v = 0$
- In practice, need to find what u and v are
  - Plane has 3 degrees of freedom
  - Exhaust degrees: vector is 2 , point is 1

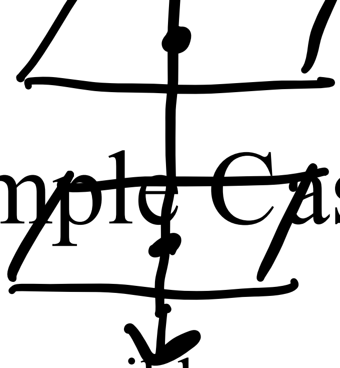
slight problem change is also fair game







choose: normal line, height of intersection



line segment. what is the new eq.

Example Case: vector and point

height = 1
normal = 2
uniq. vect = 3
vect w/ scaling = 3 − 1 = 2

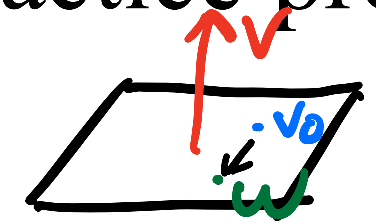
- 2+1 = 3, so it's possible
- Let given normal be v = (a, b, c)] 2
- Let $v_0 = (x_0, y_0, z_0)$ be given point in the plane] 1
- Let w = (x, y, z) be any point in the plane
- Obtain $0 = v \cdot (w - v_0)$] because v & $w - v_0$ perp.
- Simplify: $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$

point, line, plane has 0, 1, 2 degrees of movement] mov. within fixed plane = # deg freedom of ≥ direction

"plane has a point w"

degrees of freedom are degrees on what an object could be

Practice problems



$w - v_0$ is a vector in the plane, so it's perp to v

Understand your lines

- Find the line passing through (2 , -1, 3) and (1, 4 , -3)

Try the point point point case

- Find the equation of the plane containing (1, -2 , 0), (3, 1, 4), and (0, -1, 2)

Extra problem on spatial awareness

- Determine if -x+2 z = 10 and (5 , 2 -t, 10+4 t) are perpendicular, parallel, or neither

Scratch Work

$v_0 = (x_0, y_0, z_0)$

$v = (a, b, c)$ norm.

$w = (x, y, z)$

$0 = v\cdot(w - v_0)$

$= (a,b,c)\cdot(x-x_0, y-y_0, z-z_0)$

$= a(x-x_0) + b(y-y_0) + c(z-z_0)$

$\Rightarrow ax+by+cz=d$

Standard equation of plane

$rx+sy+tz=1$

$b' = b/d$

$a' = a/d$

$c' = c/d$

$x + b'y + c'z = d'$

$dx+ey+fz=1$ $a'x+b'y+c'z=1$

bad

general remark: if you miss class –

download notes, and notice a remark in the notes is confusing, search the segment of the recording used to create the remark. If that still leaves you confused, ask me by email writing what day & slide & remark quote you need help on

Extra Problem

Extra problem on spatial awareness

- Determine if $-x+2z = 10$ and $(5, 2-t, 10+4t)$ are perpendicular, parallel, or neither

General remark: choose your own

variable names. As long as you can do the work to solve the problem and it is clear to anyone reading the work how the calculation is going, no problem. There is some personal preference on whether to use u & v for a problem that needs 2 vectors or use v & w for example.

General remark: don't worry about whether a vector is v or w as long as it's defined clearly. Focus on computation mistakes and conceptual errors instead, like $2(u+v) = 2u+v$ or taking $\|\sqrt{v}\|$ or $\|-u\| =$

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