University of Delaware

Department of Mathematical Sciences

MATH 243 Midterm Exam 1

Fall 2025

Monday 29th September, 2025

Instructions:

- The time allowed for completing this exam is **90** minutes in total.
- Check your examination booklet before you start. There should be **12** questions on **9** pages.
- Turn off your cell phone and put it away. Headsets, and any other electronic devices are prohibited.
- No calculators.
- Answer the questions in the space provided. If you need more space for an answer, continue your answer on the back of the page and/or the margins of the test pages. No extra paper. *Do not separate the pages from the exam booklet.*
- For full credit, sufficient work must be shown to justify your answer.
- Partial credit will not be given if appropriate work is not shown.
- Write legibly and clearly; indicate your final answer to every problem. Cross out any work that you do not want graded. If you produce multiple solutions for a problem, indicate clearly which one you want graded.
- **Any form of academic misconduct will result in a failing grade.**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	5	7	8	2	8	8	8	8	17	13	6	10	100
Score:													

1. (5 points) Given two vectors $\mathbf{u} = \langle 2, -1, 2 \rangle$ and $\mathbf{v} = \langle 1, 8, 3 \rangle$. Find the angle formed by these two vectors.

2. Consider the vectors $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

   (a) (4 points) Compute the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$ ($\text{comp}_{\mathbf{u}}\, \mathbf{v}$).

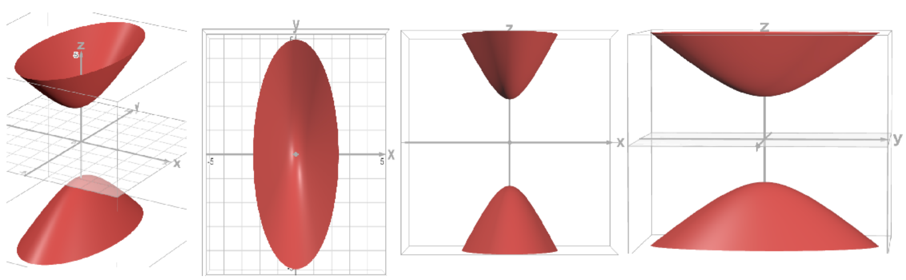
   (b) (3 points) Compute the vector projection of $\mathbf{v}$ onto $\mathbf{u}$ ($\text{proj}_{\mathbf{u}}\, \mathbf{v}$).

3. Given three vectors, $\mathbf{a} = \langle 1, 4, -7 \rangle$, $\mathbf{b} = \langle 2, -1, 4 \rangle$ and $\mathbf{c} = \langle 0, -3, 6 \rangle$.

(a) (6 points) Find the volume of the parallelepiped determined by vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.

(b) (2 points) Are these three vectors coplanar? Justify your answer.

4. (2 points) Observe the following graphs:



Which equation below gives the surface shown above?

A. $\frac{z^2}{4} = x^2 + \frac{y^2}{4} + 1$

B. $z^2 + \frac{x^2}{4} + y^2 = 1$

C. $z = \frac{x^2}{4} - y^2$

D. $\frac{y^2}{4} + 1 = \frac{x^2}{4} + z^2$

5. Consider the following two points: $A(1,-5,1)$ and $B(3,2,-1)$.

   (a) (2 points) Find the vector $\overrightarrow{AB}$.

   (b) (2 points) Find a vector equation for the line containing A and B using $\overrightarrow{AB}$ as a direction vector.

   (c) (2 points) Express the vector equation of the line as parametric equations.

   (d) (2 points) Express the parametric equations of the line as symmetric equations.

6. (8 points) Find a vector equation for the line of intersection between the planes

$$x + y + z = 2 \quad \text{and} \quad x + 2y - z = 1.$$

7. (8 points) Find a vector equation of the line tangent to the vector function

$$\mathbf{r}(t) = te^t\,\mathbf{i} + t^3\,\mathbf{j} + \ln(t)\,\mathbf{k}$$

at the point corresponding to $t = 1$.

8. (8 points) Find an equation of the plane passing through the point $(1, -1, 0)$ and containing the line defined by the parametric equations

$$x = 1 - t, \quad y = 3t - 1, \quad z = t + 2.$$

9. Let $\mathbf{r}(t) = \langle 6\sin(2t),\, 3t,\, 6\cos(2t)\rangle$ and $P = \left(6, \frac{3\pi}{4}, 0\right)$.

(a) (2 points) Verify that P lies on the graph of $\mathbf{r}(t)$.

(b) (6 points) Evaluate the unit tangent vector $\mathbf{T}$ at the point P .

(c) (6 points) Find the unit normal vector $\mathbf{N}$ at the point P.

(d) (3 points) Determine the curvature κ at the point P.

10. A particle moves in space with acceleration

$$\mathbf{a}(t) = \langle 6t,\ 4e^t,\ \frac{1}{t} \rangle.$$

You are given the following data:

$$\mathbf{v}(1) = \langle 5,\ 7,\ -3 \rangle, \qquad \mathbf{r}(1) = \langle 1,\ 0,\ 2 \rangle.$$

(a) (6 points) Find the velocity $\mathbf{v}(t)$.

(b) (7 points) Find the position vector $\mathbf{r}(t)$.

11. (6 points) Find the arc length of the curve given by

$$\mathbf{r}(t) = \langle 2\cos t, 2\sin t, t \rangle\,, \quad 0 \le t \le \frac{\pi}{2}.$$

12. For the following statements, please clearly mark **True** or **False**.

(a) (2 points) The curvature of the circle of radius $r = \frac{1}{3}$ is $\kappa = \frac{1}{9}$.

Check one: **True** **False**

(b) (2 points) If the magnitude of the vector function $\mathbf{r}(t)$ is constant, then the vector $\mathbf{r}'(t)$ is orthogonal to the vector $\mathbf{r}(t)$ for all t.

Check one: **True** **False**

(c) (2 points) If $\mathbf{T}(t)$ is a unit tangent vector and $\mathbf{N}(t)$ is a unit normal vector to a curve at the same point P, then $\mathbf{T}(t) \times \mathbf{N}(t)$ is a unit vector.

Check one: **True** **False**

(d) (2 points) For all vectors $\mathbf{u}$ and $\mathbf{v}$ in three dimensional space, $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 0$.

Check one: **True** **False**

(e) (2 points) Any two lines in three dimensional space that are not parallel must intersect.

Check one: **True** **False**