Cylindrical and Spherical Integrals

Lecture for 6/30

Converting Integrals

Cylindrical: $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$

- Bounds: $0 \leq \theta \leq 2\pi$

Spherical: $(x, y, z) = \rho(\cos(\theta)\sin(\varphi), \sin(\theta)\sin(\varphi), \cos(\varphi))$

- Bounds: $0 \leq \varphi \leq \pi$, $0 \leq \theta \leq 2\pi$

We can convert functions, but what about $dV = dx\,dy\,dz$?

- Cylindrical: $dV = r\,dr\,d\theta\,dz$
- Spherical: $dV = \rho^2 \sin(\varphi)\,d\rho\,d\varphi\,d\theta$

Differential Derivation

Practice Problems

Evaluate $\iiint_E f \, dV$ for the following functions and regions:

- $f = z$, E is region inside $y^2+z^2 = 1$ and between $x+y+z = 2$, $x = 0$
- $f = x^2+y^2$, E is portion of $x^2+y^2+z^2 = 4$ with $y \geq 0$
- $f = x^2$, E is region inside $x^2+y^2+z^2 = 36$ and $z = -(3x^2+3y^2)^{1/2}$

Scratchwork