

Stellar Spectra A. - Basic Line Formation

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INTRODUCTION

Spectral lines in stellar spectra make out the foundation of astrophysics. They provide a wealth of knowledge about the studied object and are therefore one of the most valuable sources of information to an astrophysicist. In order to get an understanding of spectral lines we will follow the work of Cecilia Payne and Marcel Minnaert which are some of the early pioneers of spectral astronomy. The report is split into two parts. The first part considers Saha-Boltzmann modeling with Cecilia Payne, while the second part deals with Fraunhofer lines and their strenghts with Marcel Minnaert.

1. THE BOLTZMANN AND SAHA LAWS

Cecilia Payne (1900 - 1979) was a British-American astronomer and astrophysicist famous for showing that most stars are made of mainly hydrogen and helium. She applied the Saha distribution which was newly derived to stellar spectra. With this she was able to show that the empirical Harvard classification primarily represented a temperature scale.

In thermodynamical equilibrium, the equipartition laws Saha and Boltzmann describe the division of particles of a specific element. With the temperature as the only major parameter these laws show us how the different ionization stages and discrete energy levels of a specific element are distributed.

1.1 Boltzmann distribution

The Boltzmann distribution deals with the energy levels and is given by

$$\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r} e^{-\chi_{r,s}/kT}, \quad (1)$$

where $n_{r,s}$ is the number density (also called level population) and $g_{r,s}$ the statistical weight of the level (r, s) where r is the ionization stage, and s is the state or level. $N_r = \sum_3 n_{r,s}$ is the total particle density over all levels of ionization. $\chi_{r,t}$ is the excitation energy measured from the ground state $(r, s = 1)$. The partition function U_r is defined by

$$U_r = \sum_s g_{r,s} e^{-\chi_{r,s}/kT}. \quad (2)$$

1.2 Strength ratio of α lines in hydrogen

Payne was studying the absorption lines in stellar spectra when she made the assumption that the strength of the absorption lines scaled with the population density of the lower level of the corresponding

transition. If one assumes that most of the hydrogen resides in the lower energy levels, it follows that most transitions would result in higher energy levels. The strength of the lines should therefore scale with the population density of these lower levels. We know today that this assumption is completely correct, but stellar absorption lines do generally scale with larger lower-level populations. We will therefore proceed by assuming that Payne's assumption holds and that the scaling is linear.

This assumption allows us to estimate the strength ratios of the α lines of hydrogen, where α denotes that the excitation is from s to $s + 1$. For a neutral hydrogen atom ($r = 1$), the statistical weight goes as $g_{1,s} = 2s^2$, and the excitation energy goes as $\chi_{1,s} = 13.6(1 - 1/s^2)$. The ratio between Lyman α ($s = 1$) and Balmer α ($s = 2$) is then given from the Boltzmann equation (1) as

$$\frac{n_{1,1}}{n_{1,2}} = \frac{g_{1,1} e^{-\chi_{1,1}/kT}}{g_{1,2} e^{-\chi_{1,2}/kT}}, \quad (3)$$

where U_r and N_r have canceled out. We only need to insert for s and g in order to compare. By using the formula in equation (3) we can compute the strength ratio of the α lines in H I Lyman, Balmer, Paschen and Brackett series. For neutral hydrogen, the information need is tabulated in table 1. Using the information in this table we find that the following strength ratios for the solar temperature of 5770 K: (Lyman α / Balmer α) $\approx 2 \cdot 10^8$, (Balmer α / Paschen α) ≈ 20 , and (Balmer α / Brackett α) ≈ 42 . We see that Lyman α line strength is enormous compared to the other lines. This is a result of the excitation energy $\chi_{1,1} = 0$, making it temperature independent when computing ratios.

Line α	s	$\chi_{1,s}$ [eV]	$g_{1,s}$
Lyman	1	0	2
Balmer	2	10.20	8
Paschen	3	12.09	18
Brackett	4	12.75	32

Table 1: Neutral hydrogen information

1.3 Saha distribution

The Saha distribution relates the ionization levels of an element in ionization equilibrium and is given by

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_e} \frac{2U_{r+1}}{U_r} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_r/kT}, \quad (4)$$

where m_e is the electron mass, h is Planck's constant, N_e is the electron density, and χ_r is the threshold ionization energy required to ionize from stage r to $r + 1$.

1.4 Element E (Schadeenium)

We will now consider the imaginary element E, or "Schadeenium". E is a simple atom which lets us evaluate Saha-Boltzmann statistics without having to bother about complex atomic data. It has the following properties:

- The statistical weight is $g_{r,s} = 1$ for all levels.
- For neutral E, the ionization energy is $\chi_1 = 7$ eV. For E^+ (ionized once) the energy is $\chi_2 = 16$ eV, and similarly $\chi_3 = 31$ eV and $\chi_4 = 51$ eV.
- The excitation energies increase incrementally by 1 eV: $\chi_{r,s} = s - 1$ eV.

We begin by computing the partition function U_r of E found in equation (2). The results is seen in figure 1.

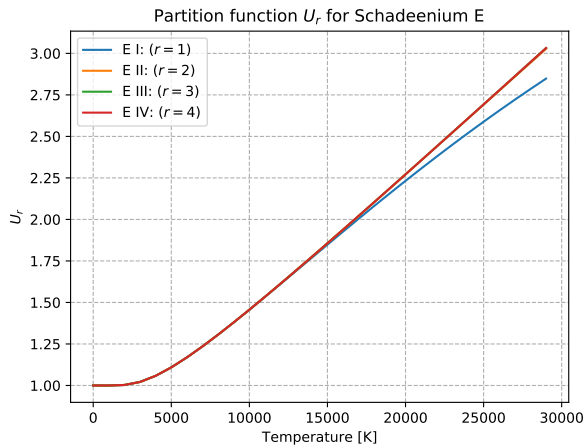


Figure 1: Partition function U_r for the imaginary element E.

We find that all ionizations mostly behave the same way for temperatures below 15000 K. For higher temperatures, U_1 splits from the other ionizations and keeps a lower value. Generally we note that the partition function of element E is of order unity and appears to be weakly sensitive to temperature.

Further we can compare our calculation with Schadee's first table. We find that for $T = 5000$ K all ionizations have the value $U_r = 1.109$, which corresponds well to Schadee's $U_r = 1.11$. For $T = 10000$ K we find that again all ionizations have more or less the same value of $U_r = 1.456$, which can be rounded up to Schadee's $U_r = 1.46$. Finally for $T = 20000$ K we find that $U_1 = 2.232$ and $U_2 = U_3 = U_4 = 2.271$, where as Schadee tabulated $U_1 = 2.27$, and $U_2 = U_3 = U_4 = 2.27$. We see that these values correspond nearly perfectly to those tabulated by Schadee.

1.4.1 Boltzmann distribution for E

With the partition function calculated, we can study the Boltzmann distribution for E. This is done by computing the boltzmann equation (1) for varies levels s . The result can be seen in figure 2.

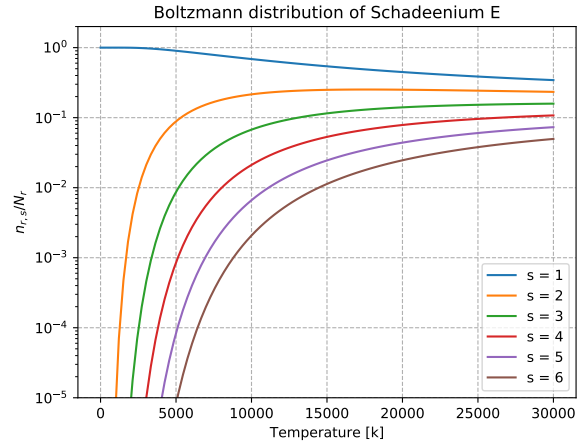


Figure 2: Boltzmann distribution of the imaginary element E.

By studying figure 2 we notice that in thermal equilibrium, for all temperatures, the ground state always has the largest population. At $T = 5000$ K we have $n_{r,s}/N_r = 0.9$ for $s = 1$ and $n_{r,s}/N_r \approx 0.009$ for $s = 2$ and so on for the lower levels which perfectly corresponds to those tabulated by Schadee. We see that all these add up to unity for all temperatures, which is a results off the scaling by N_r . We can conclude that the lowest levels are the most important ones which is a result of the rapid decay of the Boltzmann factor $e^{-\chi_{r,s}/kT}$. This can also explain the insensitivity of U_r to temperature.

14.2 Saha distribution for E

We will now study the Saha distribution of the element E. By computing the saha equation (4) for varying ionizations we obtain the results seen in figure 3.

Since this is a normalized distribution in the same way as the Boltzmann distribution, the ionizations always add up to one. We also notice that for solar temperatures, only 2 ionizations are significantly present. At $T = 5000$ we see that $N_{r+1}/N_r \approx 0.91$ for $r = 1$, while $N_{r+1}/N_r \approx 0.089$ for $r = 2$, while the higher ionization stages are negligible. This means that for solar temperatures, E is mostly neutral. For higher temperatures around $T = 10000$ K, we find that $N_{r+1}/N_r \approx 0.95$ for $r = 2$ and $N_{r+1}/N_r \approx 0.05$ for $r = 3$, while the other ionizations are again negligible. These values correspond nicely to those found by in Schadee's table. This suggests that there are at all times mainly 2 ionizations stages present in a gas at a given temperature.

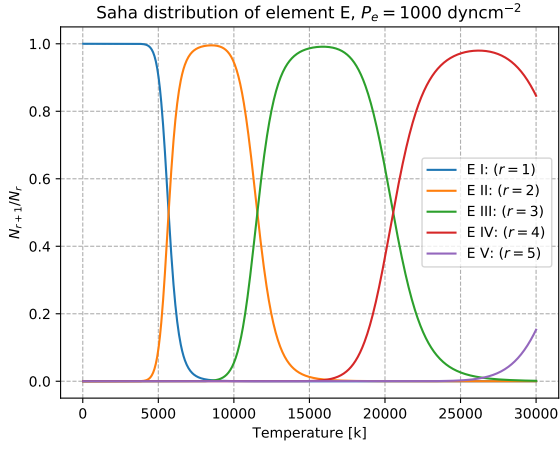


Figure 3: Saha distribution of the imaginary element E with an electron pressure of $P_e = 1000 \text{ dyn cm}^{-2}$.

We can also study what effects the electron pressure has on the saha distribution. We do so by computing the saha equation (4) for two values of P_e . The results of this can be seen in figure figure 4.

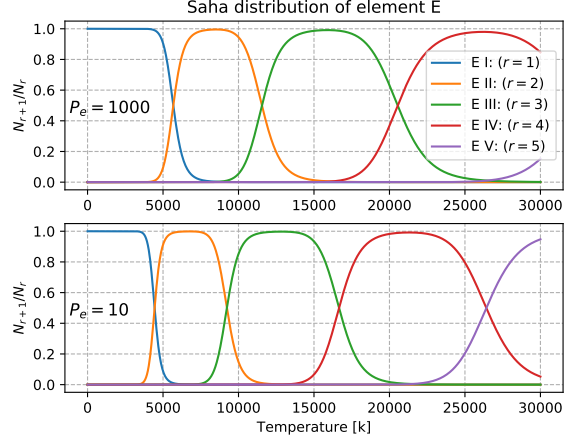


Figure 4: Saha distribution of the imaginary element E for two different electron pressures. Upper plot: $P_e = 1000$. Lower plot: $P_e = 10$, where P_e has units [dyn cm^{-2}].

Figure 4 shows the difference P_e makes when computing the Saha distribution. We note that in the lower subplot with $P_e = 10$ the distribution seems to have been squished towards the left. This means that ionization occurs at earlier temperatures for low values of electron pressure.

We note that the Boltzmann and Saha distributions behave differently for increasing temperatures. The Boltzmann equation converges to $g_{r,s}/U_r$ as $T \rightarrow \infty$. The same is not true for the Saha distribution. The two distributions share the term $e^{-\chi/kT}$ term which converges to 1 as $T \rightarrow \infty$. However, the Saha distribution has a factor $T^{3/2}$ which grows exponentially as T increases. This is the factor that separates the two distributions. The scaling with $1/\text{Ne}$ says that ionization is easier if there is room for the resulting free electron or, reversedly, that recombination from stage $r + 1$ to stage r requires catching a free electron.