AST4320 COSMOLOGY AND EXTRAGALACTIC ASTRONOMY
ASSIGNMENT 3

Deadline: Thuesday, November 1

Exercise 1 The Lyα forest (20 points) Use cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$, $\Omega_b = 0.048$ and $\Omega_r = 0$ for the following questions.

- Assuming the intergalactic medium (IGM) contains only hydrogen and helium, and the hydrogen and helium mass fractions are $X = 0.76$ and $Y = 0.24$, respectively, compute the mean molecular weight of the IGM, μ.

- Compute the Jeans length of the IGM as a function of redshift. You can assume that the IGM temperature is $T = 10^4$ K. What value for k does this correspond to?

- In redshift-space, the Jeans length would correspond to a velocity width. What is this velocity width?

- What does the finite width of the Lyα extinction profile do to absorption features in the Lyα forest?

- In lectures we discussed that the intrinsic Lyα extinction profile function (i.e. the Lorentz profile) is broadened by gas thermal motions, and the thermal broadening scale is $v_{th} = \sqrt{(2k_bT/m)}$, where T is the temperature of the gas and m is the mass of the particle. Compare the thermal broadening scale to the Jeans length.

Exercise 2 Optical depth of the ionised IGM (10 points)
In the lectures we derived that the total optical depth of the ionised IGM due to electron scattering is given by

$$\tau_{\rm e}(z) = c\int_0^z \frac{n_e(z)\sigma_{\rm T} dz}{(1+z)H(z)},$$

where $\sigma_{\rm T} = \frac{8\pi}{3}r_e^2 = 6.65\times10^{-25}$ cm^2 denotes the cross-section for electron scattering. Assuming the IGM is highly ionised, and the average number density of hydrogen evolves as $\bar{n}_{\rm H}(z) \sim 1.9\times10^{-7}(1+z)^3$ cm $^{-3}$, compute the optical depth as a function of z under cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$ and $\Omega_r = 0$). Plot $\tau_{\rm e}(z)$ as function of z for z = 0-10.

Exercise 3 Isothermal density profile (10 points)
In the lectures we obtained a second order differential equation for $\rho(r)$ if assuming the halo is "isothermal" (i.e. dark matter particles have the same velocity dispersion everywhere in the halo):

$$-\frac{k_\mathrm{b}T}{m_\mathrm{DM}r^2}\frac{d}{dr}r^2\frac{d}{dr}\ln\rho = 4\pi G\rho(r),$$

- Show that

$$\rho(r) = \frac{A}{r^2}, \quad A = \frac{k_\mathrm{b}T}{2\pi G m_\mathrm{DM}}.$$

  provides a solution to this equation.

- For gas in hydrostatic equilibrium with gravity we have

$$\frac{dp}{dr} = -\frac{GM(<r)\rho}{r^2}.$$

  Show that isothermal gas settles into a similar state.