Assignment 1

AST4320: Cosmology and Extragalatic Astronomy

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Exercise 1

The continuity equation for an unperturbed universe driven by hubble expansion is given as

$$\frac{d\bar{\rho}}{dt} + \bar{\rho}\nabla \cdot \mathbf{v}_0 = 0, \tag{1}$$

where $\bar{\rho}$ is the average density of the universe, and $\mathbf{v}_0$ comes from the Hubble law today which states that

$$\mathbf{v_0} = H_0\mathbf{r}. \tag{2}$$

Here H_0 is the Hubble parameter today, given as $H_0 = \dot{a}(t = t_0)/a(t = t_0)$, where a is the scale factor, and is defined so that $a(t = t_0) = 1$.

Inserting for (2) into the continuity equation we find

$$\frac{d\bar{\rho}}{dt} + \bar{\rho}\nabla \cdot \mathbf{r}\frac{da}{dt}\frac{1}{a} = 0,$$

where the del operator only works on $\mathbf{r}$ as its the scale factor is only a function of time. The operation results in $\nabla \cdot \mathbf{r} = 3$. We can then swap the right term over to the right hand side of the equation which leaves us with

$$\frac{d\bar{\rho}}{dt} = -3\bar{\rho}\frac{da}{dt}\frac{1}{a}.$$

Further, we can separate the equation

$$\frac{d\bar{\rho}}{\rho} = -3\frac{da}{a}.$$

This is then solved in the following way

$$\int_{\bar{\rho}(t=t_0)}^{\bar{\rho}(t)} \frac{d\bar{\rho}}{\rho} = \int_{a(t=t_0)}^{a(t)} -3\frac{da}{a},$$

$$\Rightarrow \ln\left(\frac{\bar{\rho}(t)}{\bar{\rho}(t = t_0)}\right) = \ln\left(\frac{a(t)}{a(t = t_0)}\right)^{-3}.$$

This can then be reduced to

$$\bar{\rho}(t) = \bar{\rho}(t = t_0)a(t)^{-3}, \tag{3}$$

where we have used that $a(t = t_0) = 1$.

Exercise 2

The unperturbed Poisson equation is given as

$$\nabla^2\phi_0 = 4\pi G\rho_0. \tag{4}$$

By rewriting in terms of the perturbation terms, we get the following equation

$$\nabla^2(\phi + \delta\phi) = 4\pi G(\rho + \delta\rho)$$

We can then write this out to the form

$$\nabla^2\phi + \nabla^2\delta\phi = 4\pi G\rho + 4\pi G\delta\rho.$$

Since the two terms $\nabla^2\phi$ and $4\pi G\rho$ simply makes up the Possion equation which is equal to 0, we are left with the perturbed Possion equation

$$\nabla^2\delta\phi = 4\pi G\delta\rho. \tag{5}$$