FYS 4150 - Computational Physics
Project 1: Solving Poisson's equation in one dimension

Maren Rasmussen   Markus Leira Asprusten   Metin San

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Abstract

1. Introduction

n	General Algorithm	Special Algorithm	LU-Decomposition
10	0.030738	0.023905	
100	0.028017	0.024106	
1000	0.028026	0.024124	
10 000	0.028299	0.023557	
100 000	0.028742	0.024067	

2. Theoretical Background

3. Algorithm & Implementation

3.2 **LU-decomposition**

Solving the linear algebra problem with an LU decomposition is relatively simple. By decomposing the matrix $\hat{\mathbf{A}}$ into the lower triangular matrix $\hat{\mathbf{L}}$ and the upper triangular matrix $\hat{\mathbf{U}}$, where all the diagonal elements in $\hat{\mathbf{L}}$ is 1, in such a way that $\hat{\mathbf{A}} = \hat{\mathbf{L}}\hat{\mathbf{U}}$. We can then rewrite the linear algebra problem into

$$
\begin{aligned}
\hat{\mathbf{A}}\hat{\mathbf{u}} &= \hat{\mathbf{f}} \\
\hat{\mathbf{L}}\hat{\mathbf{U}}\hat{\mathbf{u}} &= \hat{\mathbf{f}} \\
\hat{\mathbf{U}}\hat{\mathbf{u}} &= \hat{\mathbf{L}}^{-1}\hat{\mathbf{f}} = \hat{\mathbf{y}} \\
\implies \hat{\mathbf{L}}\hat{\mathbf{y}} &= \hat{\mathbf{f}}\,, \; \hat{\mathbf{U}}\hat{\mathbf{u}} = \hat{\mathbf{y}}.
\end{aligned} \tag{1}
$$

The problem is then to solve two equations, firstly for $\hat{\mathbf{y}}$ and lastly for $\hat{\mathbf{u}}$. Suppose the matrices have dimension $(n \times n)$. The solution can then be found by iterating n-times for each equation, to a total of $2n$ iterations. The Armadillo library solves this problem simply with the `solve`-function.

4. Results

4.5 **LU-decomposition**