4T1: The Short-Time Fourier Transform (1 of 2)

Xavier Serra
Universitat Pompeu Fabra, Barcelona

Index

- Short-time Fourier Transform equation
- Analysis window

Short-time Fourier Transform

$$X_l[k] = \sum_{n=-N/2}^{N/2-1} w[n]\,x[n+lH]\,e^{-j2\pi kn/N} \qquad l = 0,1,\ldots,$$

Output is a matrix, not an array

The sample at the middle of the window, is considered to be at time 0 (for 0 phase windowing)

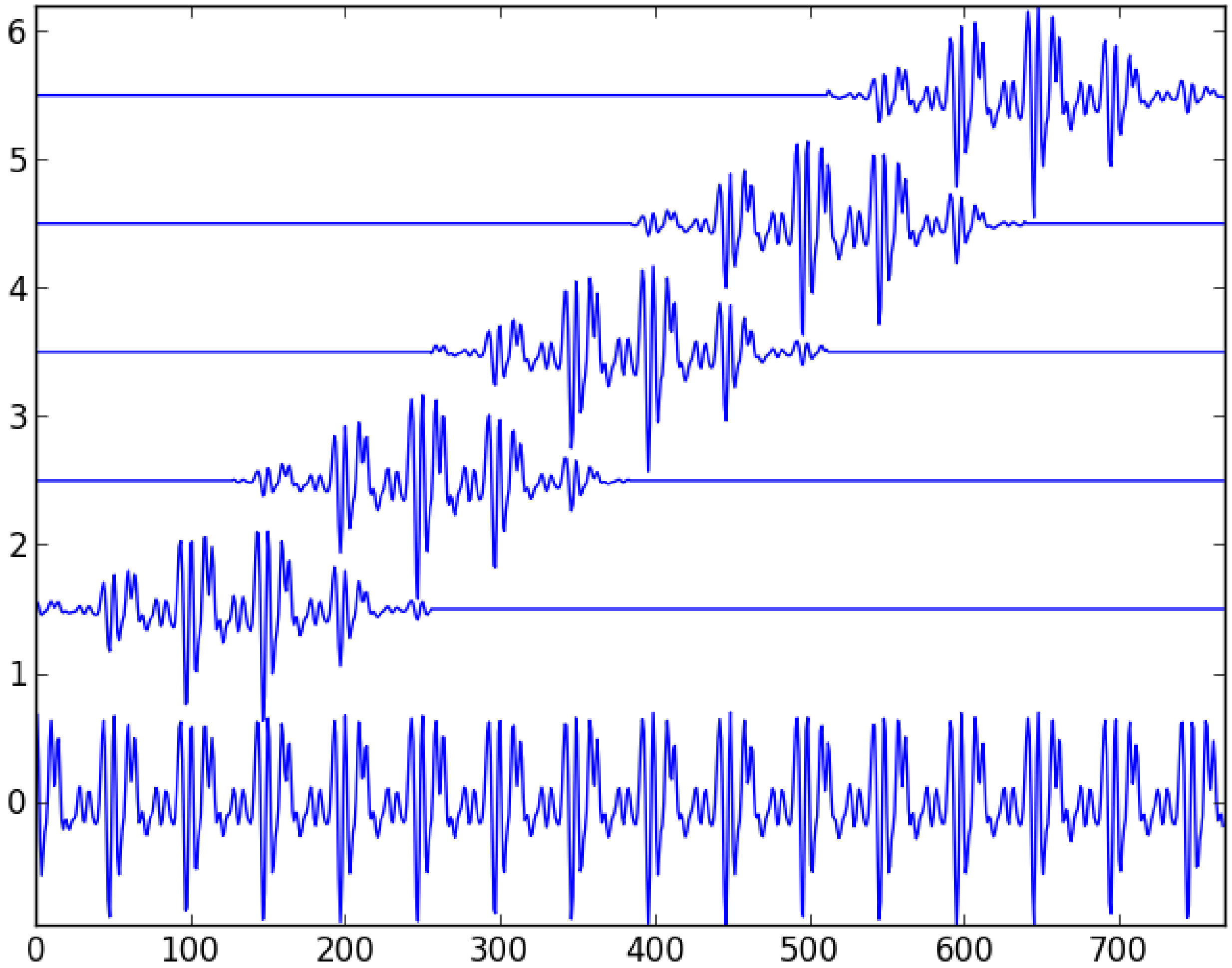
w: analysis window w

l: frame number l (elle)

H: hop-size H (hop size)

THE OUTPUT OF THE STFT IS A WINDOWED VERSION OF THE MAGNITUDE/PHASE SPECTRA,

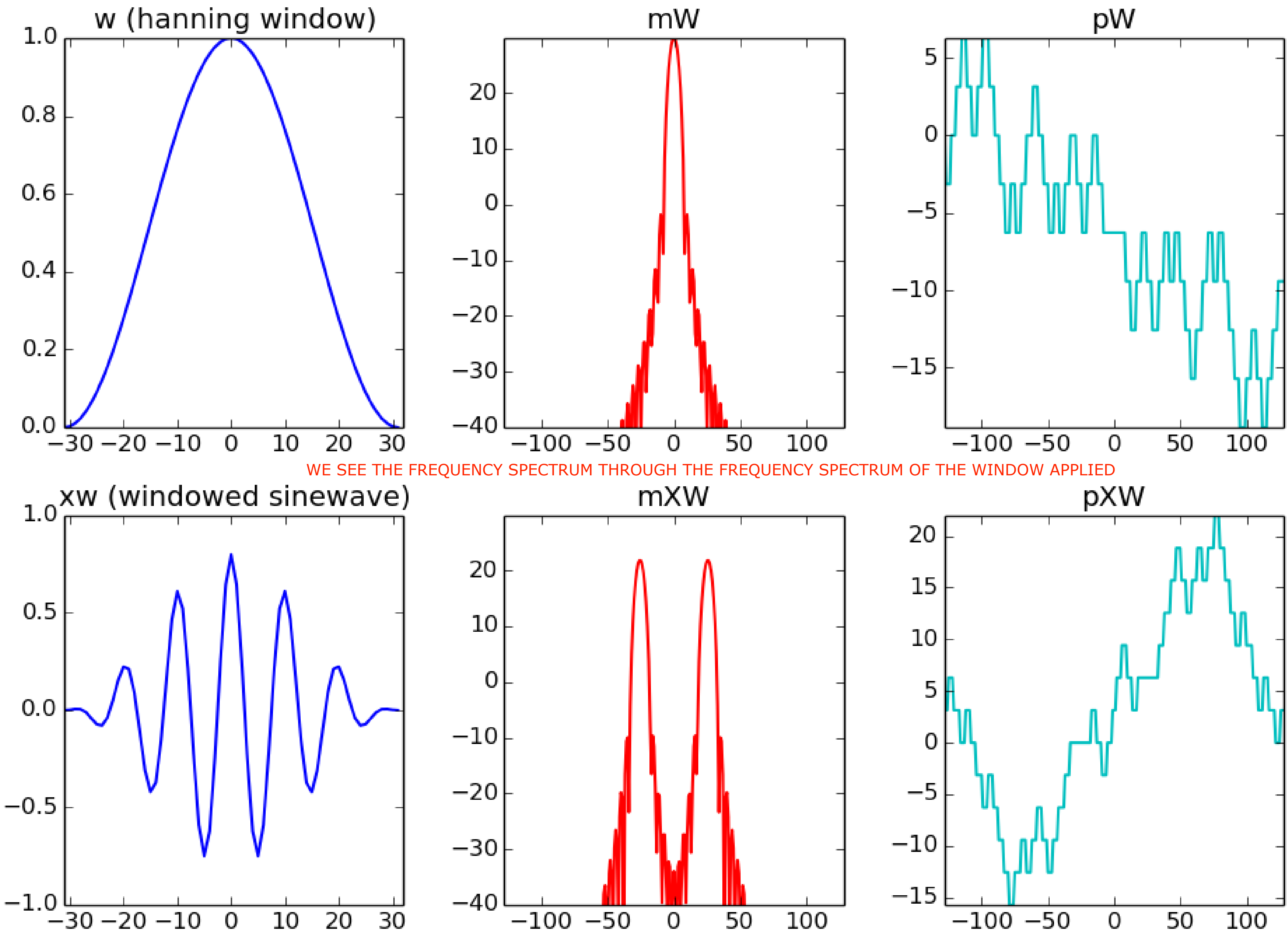
$$xw_l[n]=w[n]x[n+lH] \qquad l=0,1,\ldots,$$



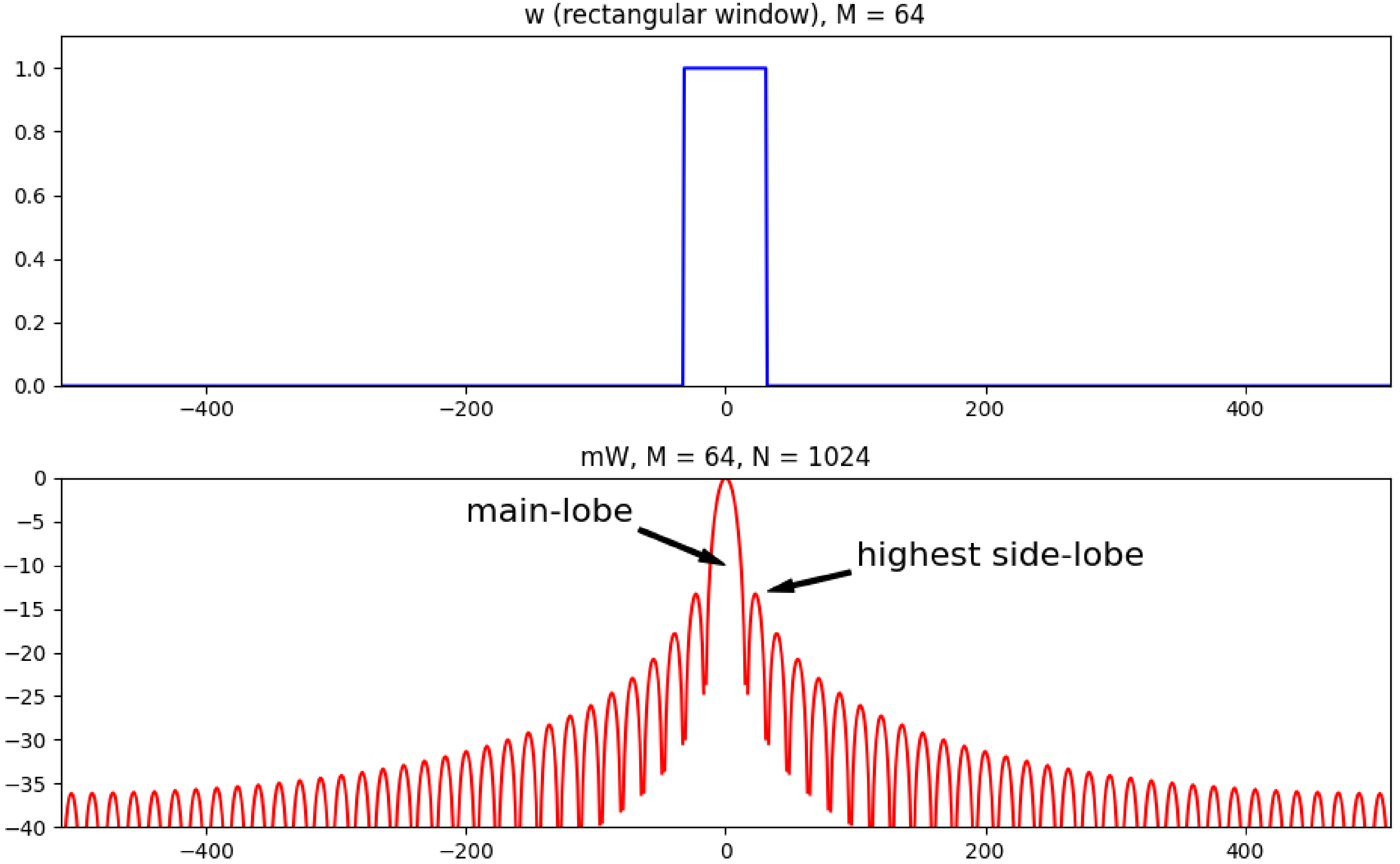
Transform of a windowed sinewave

$$x[n] = A_0 \cos(2\pi k_0 n/N) = \frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N}$$

$$\begin{aligned}
X[k] &= \sum_{n=-N/2}^{N/2-1} w[n] x[n] e^{-j2\pi kn/N} \\
&= \sum_{n=-N/2}^{N/2-1} w[n] \left(\frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N} \right) e^{-j2\pi kn/N} \\
&= \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{j2\pi k_0 n/N} e^{-j2\pi kn/N} + \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{-j2\pi k_0 n/N} e^{-j2\pi kn/N} \\
&= \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi(k-k_0)n/N} + \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi(k+k_0)n/N} \\
&= \frac{A_0}{2} W[k-k_0] + \frac{A_0}{2} W[k+k_0]
\end{aligned}$$


w (hanning window)
mW
pW
WE SEE THE FREQUENCY SPECTRUM THROUGH THE FREQUENCY SPECTRUM OF THE WINDOW APPLIED
xw (windowed sinewave)
mXW
pXW

Analysis window



Window functions in Scipy

barthann (M[, sym])	Return a modified Bartlett-Hann window.
bartlett (M[, sym])	Return a Bartlett window.
blackman (M[, sym])	Return a Blackman window.
blackmanharris (M[, sym])	Return a minimum 4-term Blackman-Harris window.
bohman (M[, sym])	Return a Bohman window.
boxcar (M[, sym])	Return a boxcar or rectangular window.
chebwin (M, at[, sym])	Return a Dolph-Chebyshev window.
flattop (M[, sym])	Return a flat top window.
gaussian (M, std[, sym])	Return a Gaussian window.
general-gaussian (M, p, sig[, sym])	Return a window with a generalized Gaussian shape.
hamming (M[, sym])	Return a Hamming window.
hann (M[, sym])	Return a Hann window.
kaiser (M, beta[, sym])	Return a Kaiser window.
nuttall (M[, sym])	Return a minimum 4-term Blackman-Harris window according to Nuttall.
parzen (M[, sym])	Return a Parzen window.
slepian (M, width[, sym])	Return a digital Slepian window.
triang (M[, sym])	Return a triangular window.

With an infinite number of samples, all windows would approach to being the delta function.

Rectangular window

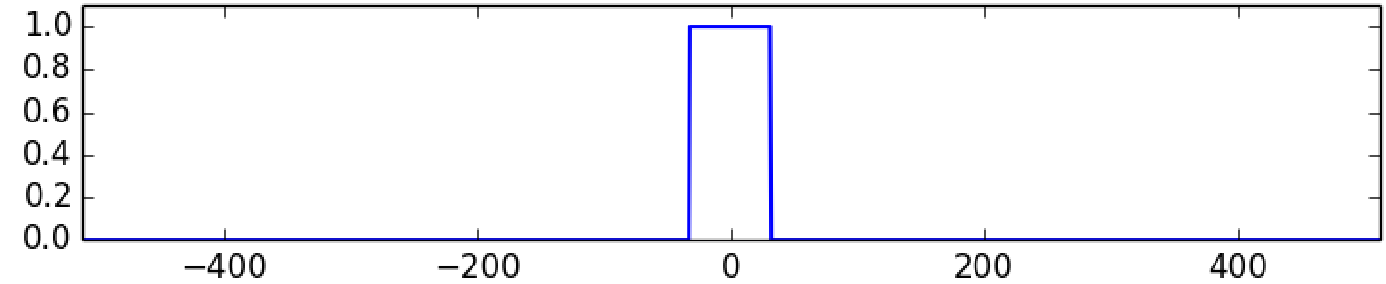
These windows' frequency spectra are the frequency response -dfferent for each window- that we have for each pure sinusoid in a -real world- signal, centred at the corresponding frequency. Basically, the amplitude of each magnitude spectrum's frequency bin is not a perfect peak (delta function) with 0 width, but a more complex shape, as of below.

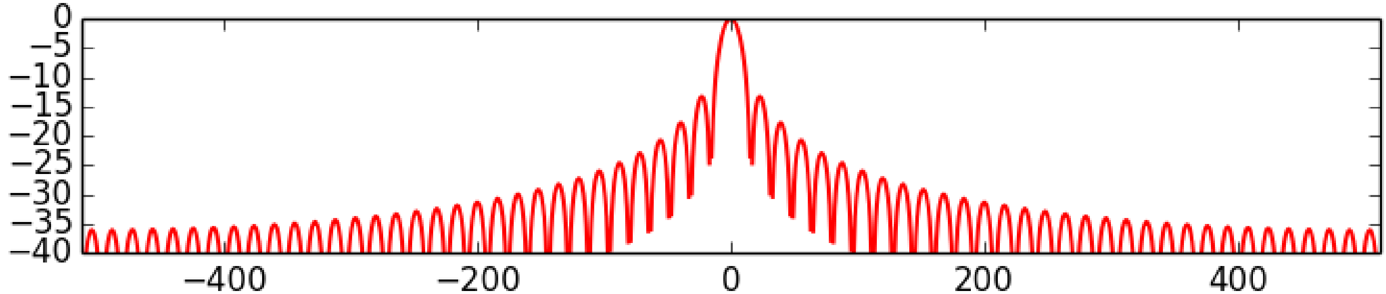
This is why we have to be careful; keep in mind that the final effect is like MANY window functions frequency response are summing up together.

$$w[n] = 1, \quad n = -M/2, \ldots, 0, \ldots M/2$$
$$\quad\quad = 0, \quad n = \text{elsewhere}$$

$$W[k] = \frac{\sin(\pi k)}{\sin(\pi k / M)}$$

Window length does not affect the window's frequency response shape (the same window time domain shape will always have the same frequency domain shape). But the longer the window, the more interpolation (detail) we have in the frequency spectrum. ALWAYS BEAR IN MIND THAT WE ARE LOOKING AT DISCRETE TIME DOMAIN/FREWUENCY DOMAIN SIGNALS, WITH FINITE (SOMETIMES TOO LITTLE) RESOLUTION BETWEEN SAMPLES.





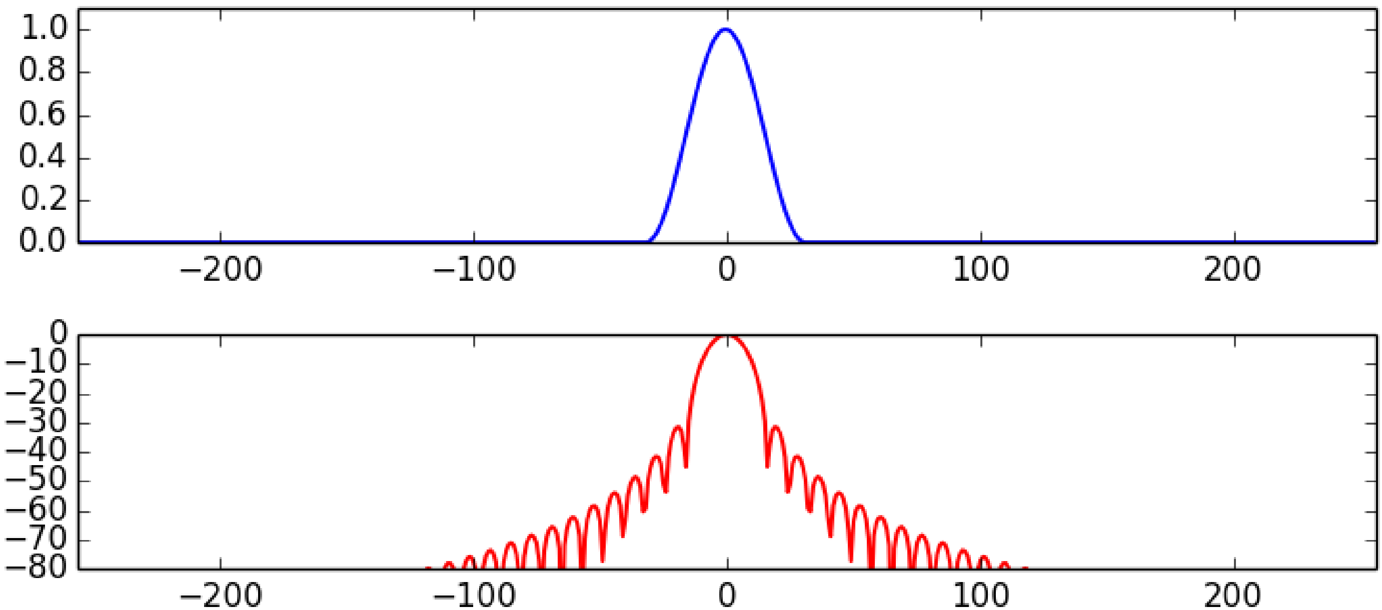
Hanning window

Lower pitch -> bigger N (larger window) (fewer periods in the same number ofinpiut smaples)

viceversa

$$w[n] = .5 + .5\cos(2\pi n / M), \quad n = -M/2, \ldots, 0, \ldots M/2$$

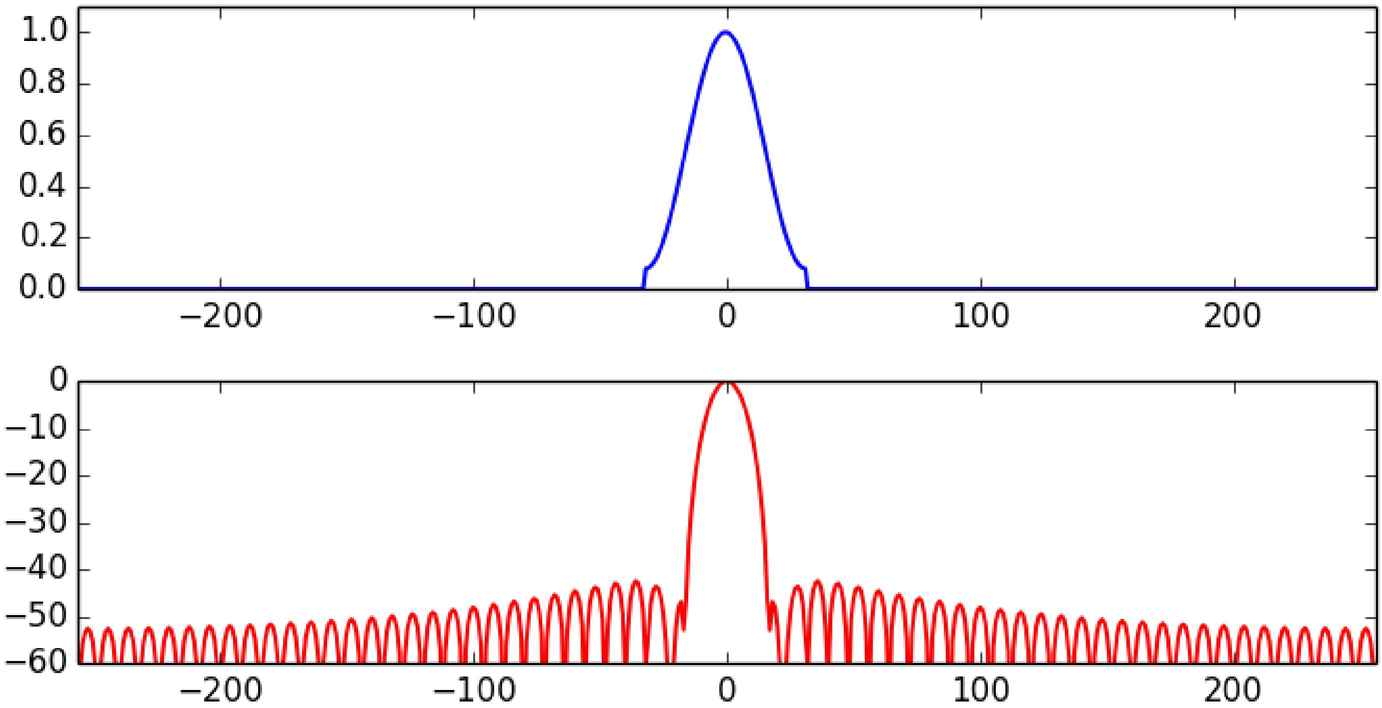
$$W[k] = .5\,D[k] + .25\left(D[k-1] + D[k+1]\right) \quad \text{where } D[k] = \frac{\sin(\pi k)}{\sin(\pi k / M)}$$



Hamming window

Default for speech

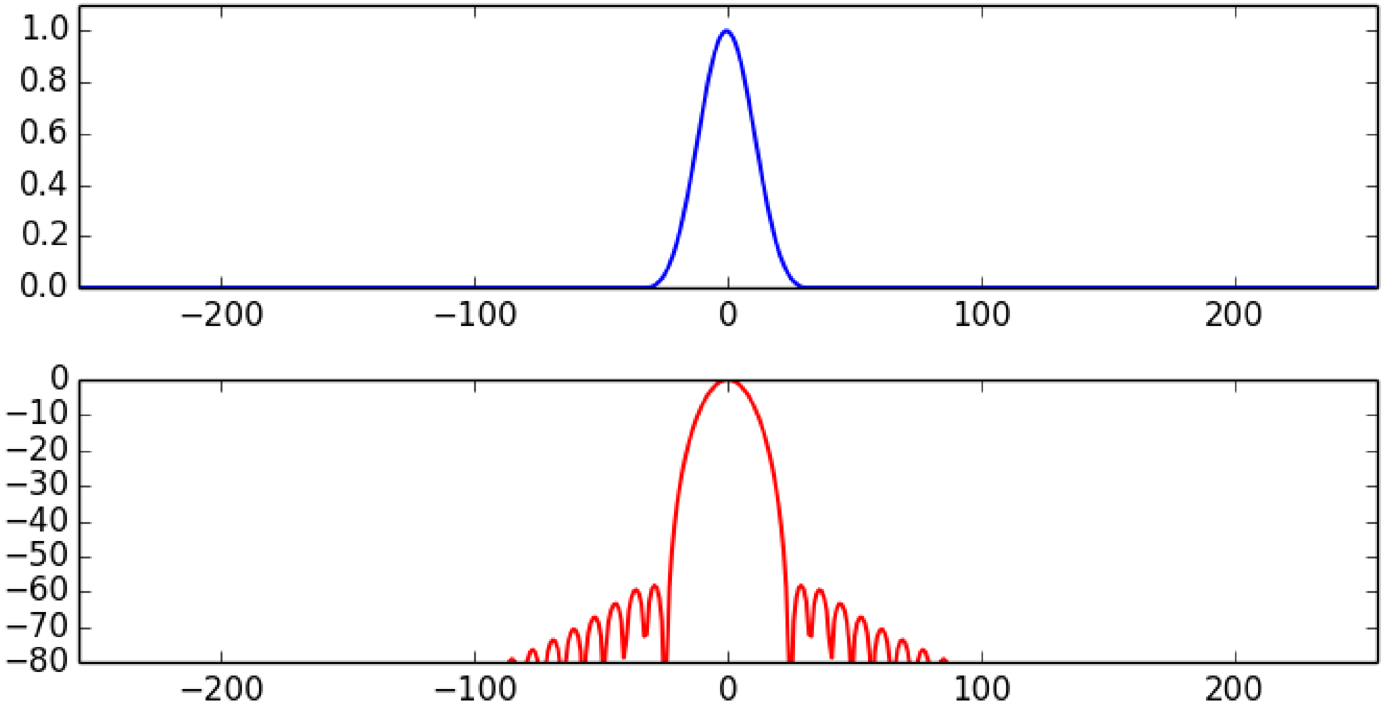
$$w[n] = .54 + .46\cos(2\pi n/M), \quad n = -M/2, \ldots, 0, \ldots M/2$$



Blackman window

Default for music

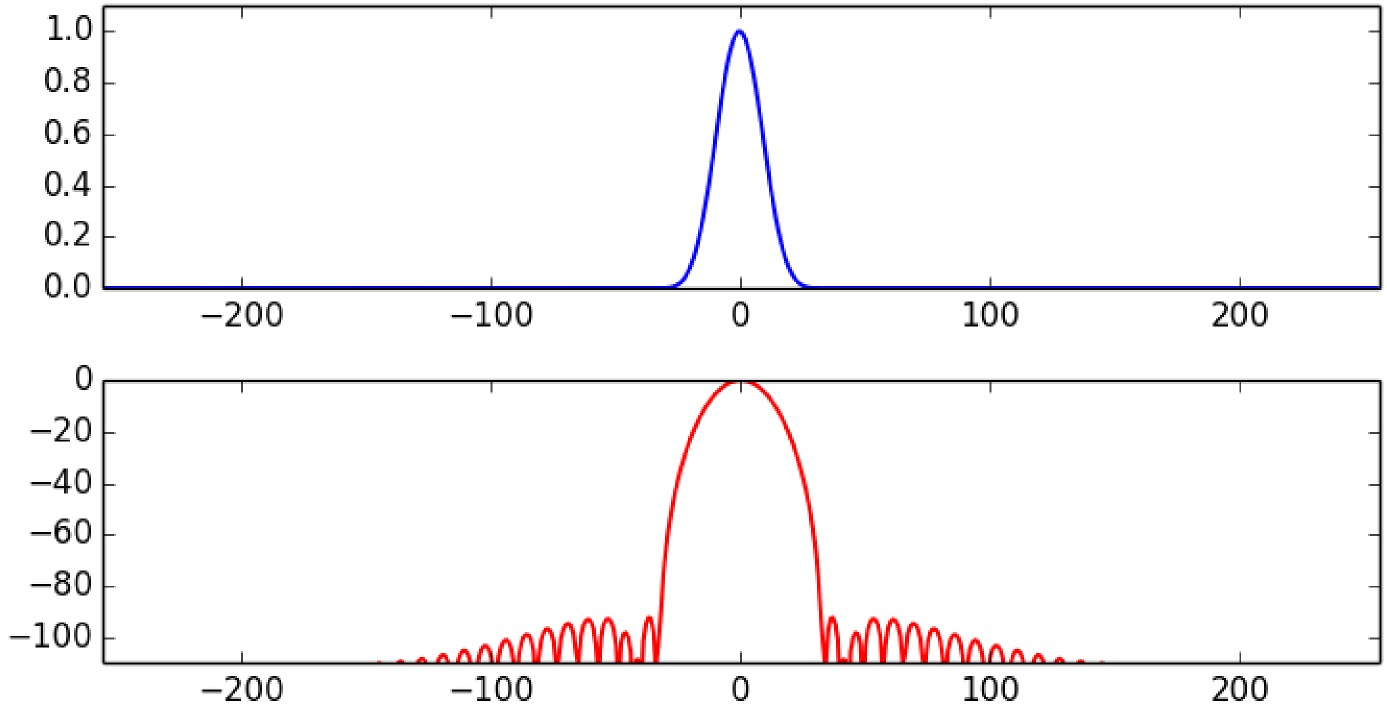
$$w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$$



Blackman-Harris window

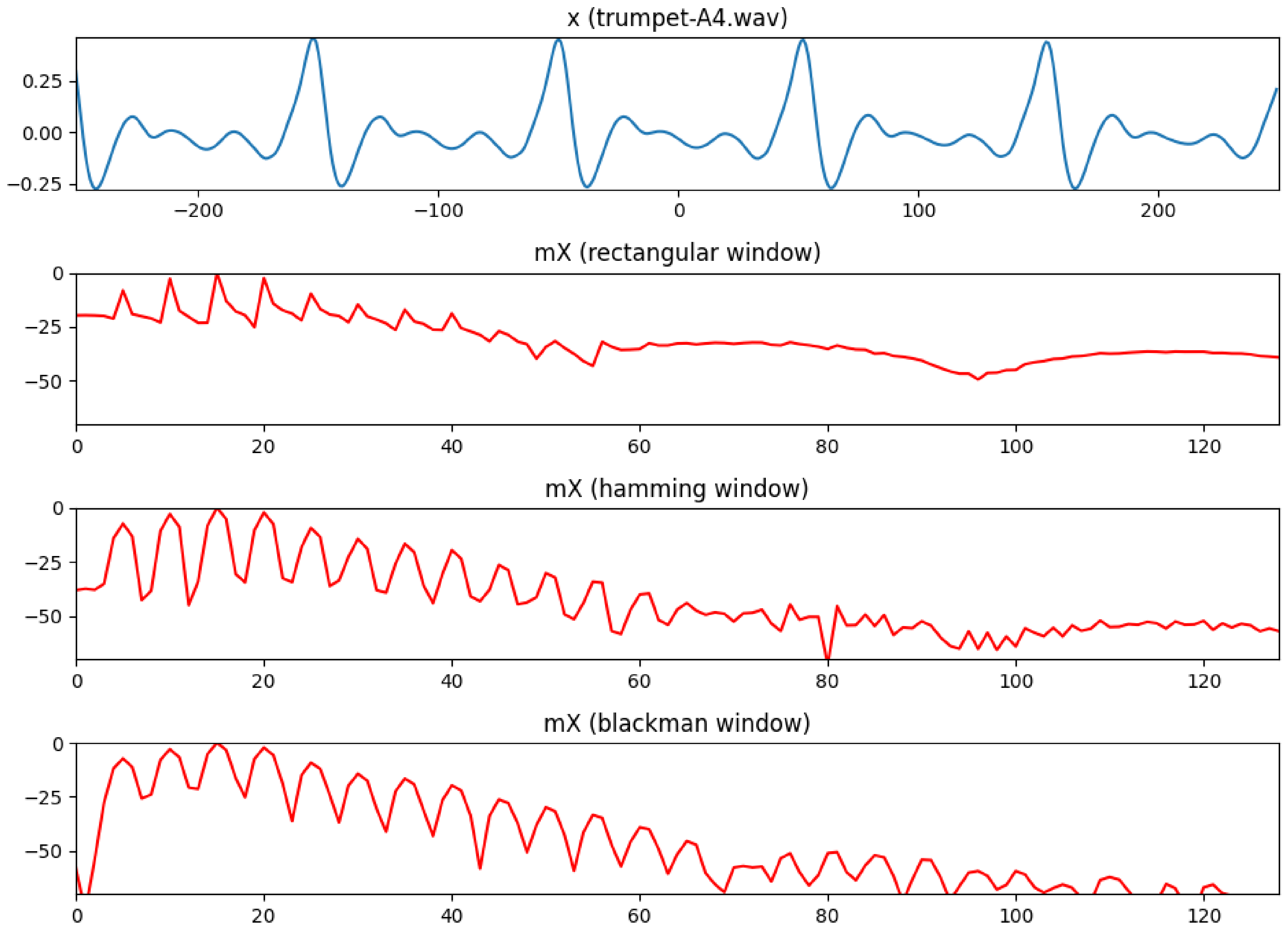
$$w(n)=\frac{1}{M}\sum_{l=0}^{3}\alpha_l \cos(2nl\pi/M), \quad n=-M/2,\ldots 0,\ldots M/2$$

where $\alpha_0=0.35875, \alpha_1=0.48829, \alpha_2=0.14128, \alpha_3=0.01168$



main lobe width : 8 bins

side-lobe level : −92dB


x (trumpet-A4.wav)
0.25
0.00
−0.25
−200
−100
0
100
200
mX (rectangular window)
0
−25
−50
0
20
40
60
80
100
120
mX (hamming window)
0
−25
−50
0
20
40
60
80
100
120
mX (blackman window)
0
−25
−50
0
20
40
60
80
100
120

References and credits

- More information in:
  https://en.wikipedia.org/wiki/STFT
  https://en.wikipedia.org/wiki/Window_function
- Reference on the STFT by Julius O. Smith:
  https://ccrma.stanford.edu/~jos/sasp/
- Sounds from:
  http://www.freesound.org/people/xserra/packs/13038/
-

4T1: The Short-Time Fourier Transform (1 of 2)

Xavier Serra
Universitat Pompeu Fabra, Barcelona