Dividimos el intervalo en 4. Esto quiere decir. $h_m = \frac{b-a}{3}$

por lo tanto $\int_{x_i}^{x_{i+3}} f(x)\,dx = \int_{x_1}^{x_{m3}} P_3(x)$

$$\frac{(x-b)(x-x_{m1})}{(a-b)(a-x_{m1})} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}$$

$$P_{i3}(x) = \sum_{i=0}^{3} f(x_i)\,\ell_i(x)$$

$$f(x_0)\,\ell_0(x) = f(x_0)\frac{(x-x_1)}{(x_0-x_1)}\frac{(x-x_2)}{(x_0-x_2)}\frac{(x-x_3)}{(x_0-x_3)}$$

$$f(x_1)\,L_1(x) = f(x_1)\frac{(x-x_0)}{(x_1-x_0)}\frac{(x-x_2)}{(x_1-x_2)}\frac{(x-x_3)}{(x_1-x_3)}$$

$$f(x_2)\,L_2(x) = f(x_2)\frac{(x-x_0)}{(x_2-x_0)}\frac{(x-x_1)}{(x_2-x_1)}\frac{(x-x_3)}{(x_2-x_3)}$$

$$f(x_3)\,L_3(x) = f(x_3)\frac{(x-x_0)}{(x_3-x_0)}\frac{(x-x_1)}{(x_3-x_1)}\frac{(x-x_2)}{(x_3-x_2)}$$

$\approx f(x_0)(x-x_1)$

$$f(x_i)\left(\frac{(x-x_{i+1})}{(x_i-x_{i+1})}\frac{(x-x_{i+2})}{(x_i-x_{i+2})}\frac{(x-x_{i+3})}{(x_i-x_{i+3})}\right) + f(x_{i+1})\left(\frac{(x-x_i)}{(x_{i+1}-x_i)}\frac{(x-x_{i+2})}{(x_{i+1}-x_{i+2})}\frac{(x-x_{i+3})}{(x_{i+1}-x_{i+3})}\right) + f(x_{i+2})\frac{(x-x_i)}{(x-x_i)}\frac{(x-x_{i+1})}{(x-x_{i+1})}\frac{(x-x_{i+3})}{(x_{i+1}-x_{i+3})}.$$

$$+ f(x_{i+3})\frac{(x-x_i)}{(x_{i+3}-x_i)}\frac{(x-x_{i+1})}{(x_{i+3}-x_{i+1})}\frac{(x-x_{i+2})}{(x_{i+3}-x_{i+2})}.$$

$$\frac{(x-x_1)(x-x_2)(x-x_3)}{-6h^3} = (x^2 - x_2x - x_1x + x_1x_2)(x-x_3)$$

$$\left(x^3 - x_2x^2 - x_1x^2 + x_1x_2x - x^2x_3 + x_2x_3x + x_1x_3x - x_1x_2x_3\right)\cdot\frac{1}{-6h^3}$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{h\cdot -h\cdot -2h} = \frac{(x^2 - x_1x - x_0x + x_0x_1)(x-x_3)}{2h^3}$$

$$= \frac{x^3 - x_1x^2 - x_0x^2 + x_0x_1x - x^2x_3 + x_1x_3x + x_0x_3x - x_0x_1x_3}{2h^3}$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{2h\cdot h\cdot -h} = \frac{(x^2 - x_1x - x_0x + x_0x_1)(x-x_3)}{-2h^3} = \left(x^3 - x_1x^2 - x_0x^2 + x_0x_1x - x_3x^2 + x_1x_3x + x_0x_3x - x_0x_1x_3\right)\frac{1}{-2h^3}$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{} = \frac{(x^2 - x_1x - x_0x + x_0x_1)(x-x_2)}{6h^3} = \left(x^3 - x_1x^2 - x_0x^2 + x_1x_0x\right)\frac{1}{6h^3}$$

$$\frac{1}{h^3}\left(\frac{x^2x_3}{6} - \frac{x_2x_3x}{6} - \frac{x_1x_3x}{6} + \frac{x_1x_2x_3}{6} - \frac{x_3x^2}{2} + \frac{x_0x_2x}{2} + \frac{x_2x_3x}{2} - \frac{x_0x_2x_3}{2}\right.$$

$$\left. + \frac{x_1x^2}{2} + \frac{x_0x_1x}{2} - \frac{x_1x_3x}{2} + \frac{x_0x_1x_3}{2} - \frac{x_0x^2}{6} + \frac{x_1x_0x}{6} + \frac{x_0x_3x}{6} - \frac{x_0x_1x_2}{6}\right)$$

$$\frac{1}{h^3}\left(x^2\left(\frac{x_3}{6} - \frac{x_2}{2} + \frac{x_1}{2} - \frac{x_0}{6}\right) + x\left(-\frac{x_2x_3}{6} - \frac{x_1x_3}{6} + \frac{x_0x_2}{2} + \frac{x_2x_3}{2} - \frac{x_0x_3}{2} - \frac{x_1x_3}{2} + \frac{x_1x_0}{6} + \frac{x_0x_3}{6}\right)\right.$$

$$\left. - \frac{x_0x_2}{2}\left(x_3 + \frac{x_1}{3}\right)\right].$$

$$\frac{1}{h^3}\left(x^2\left(\frac{3h}{6} - \frac{h}{2}\right) + x\left(\frac{-x_2x_3 - x_1x_3 + x_1x_0 + x_0x_2}{6} + \frac{x_0x_2 + x_2x_3 - x_0x_1 - x_1x_3}{2}\right)\right.$$

$$\simeq \frac{3h}{8}\left(f(x_i) + 2f(x_{i+1}) + 2f(x_{i+2}) + f(x_{i+3}) + f(x_{i+1}) + f(x_{i+2})\right.$$

$$\simeq \frac{3h}{8}\left(f(x_i) + 3f(x_{i+1}) + 3f(x_{i+2}) + f(x_{i+3})\right.$$