$$\psi_n(\xi) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\xi^2/2} H_n(\xi)$$

$$\psi_1(\xi) = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\xi^2/2} H_1(\xi)$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\, x \sim \sqrt{\frac{m\omega}{\hbar}} = 1$$

$$\psi_1(\cancel{\xi}) = \frac{1}{\sqrt{2}} \pi^{-1/4} e^{-x^2/2} H_1(x)$$

$$\psi_1(x) = 2^{-1/2} \pi^{-1/4} \cdot e^{-x^2/2} H_1(x)$$

$$u = \frac{x}{\sqrt{2}} \qquad du = \frac{1}{\sqrt{2}} dx$$

$$u^2 = \frac{x^2}{2}$$

$$\psi_1(x) = 2^{-1/2} \pi^{-1/4} \cdot e^{-u^2} H_1(u)\, 2^{1/2} du$$

$$\psi_1(x) = \pi^{-1/4} e^{-u^2} H_1(\underset{\xi}{u})$$

$$f(x) = \pi^{-1/4} H_1(u)$$

$$\psi_1(x) = 2^{-1/2} \cdots$$

$$\psi_1(x) = \pi^{-1/4} e^{-u^2} H_1(\underset{\xi}{u})$$

$$f(x) = \pi^{-1/4} H_1(u)$$

Regla del trapecio simple.

$$I = \int_a^b f(x)\,dx \cong \int_a^b P_1(x)\,dx = \frac{b-a}{2}\left(f(a) + f(b)\right)$$

$$\circ\ P_1(x) = \sum_{n=0}^{1} f(x_i)\, S_i(x)$$

$$S_i(x) = \prod_{j=0,\, j\neq i}^{1} \frac{x - x_j}{x_i - x_j}$$

$$S_0 = \frac{x - x_1}{x_0 - x_1}$$

$$S_1 = \frac{x - x_0}{x_1 - x_0}$$

$$P_1 = f(x_0)\frac{(x - x_1)}{x_0 - x_1} + f(x_1)\frac{(x - x_0)}{(x_1 - x_0)}$$

Para $x_0 = a$ y $x_1 = b$

$$P_1 = f(a)\left(\frac{x-b}{a-b}\right) + f(b)\frac{(x-a)}{(b-a)}$$

$$= \frac{f(a)(x-b)(b-a) + f(b)(x-a)(a-b)}{(a-b)(b-a)} = \frac{f(a)(xb - ax - b^2 + ab) + f(b)(ax - bx - a^2 + ab)}{(ab - a^2 - b^2 + ab)}$$

$$I = \int_a^b P_1(x)\,dx = \int_a^b \frac{f(a)(xb - ax - b^2 + ab)}{\ldots}\,dx \int_a^b \frac{f(b)(ax - bx - a^2 + ab)}{\ldots}\,dx$$

$$I = \int_a^b P_1(x)\,dx = \int_a \frac{\ldots}{(ab - a^2 - b^2 + ab)} \int_a \frac{\ldots}{(ab - a^2 - b^2 + ab)}$$

$$= \int_a^b \frac{f(a)(x-b)}{(a-b)}\,dx + \int_a^b \frac{f(b)(x-a)}{(b-a)}\,dx = \frac{f(a)}{(a-b)} \int_a^b x-b\,dx + \frac{f(b)}{(b-a)} \int_a^b x-a\,dx.$$

$$\frac{\ldots}{(a-b)(b-a)} \qquad \frac{\ldots}{(ab - a^2 - b^2 + ab)}$$

$$I = \int_a^b P_1(x)\,dx = \int_a^b \frac{f(a)(xb - ax - b^2 + ab)}{(ab - a^2 - b^2 + ab)}\,dx \int_a^b \frac{f(b)(ax - bx - a^2 + ab)}{(ab - a^2 - b^2 + ab)}\,dx$$

$$= \int_a^b \frac{f(a)(x-b)}{(a-b)}\,dx + \int_a^b \frac{f(b)(x-a)}{(b-a)}\,dx = \frac{f(a)}{(a-b)} \int_a^b x-b\,dx + \frac{f(b)}{(b-a)} \int_a^b x-a\,dx.$$

$$\frac{f(a)}{(a-b)} \underbrace{\left(\frac{x^2}{2} - bx\right)\Big]_a^b} + \frac{f(b)}{(b-a)} \underbrace{\left(\frac{x^2}{2} - ax\right)\Big]_a^b}$$

$$\frac{b^2}{2} - b^2 - \frac{a^2}{2} + ab \qquad \frac{b^2}{2} - ab - \frac{a^2}{2} + a^2.$$

$$= -\frac{b^2}{2} - \frac{a^2}{2} + ab. \qquad \frac{1}{2}(b^2 - 2ab + a^2)$$

$$\frac{1}{2}(a-b)^2$$

$$= \frac{1}{2}(-b^2 - a^2 + 2ab)$$

$$= -\frac{1}{2}(a-b)^2$$

$$= \frac{f(a)}{(a-b)} \cdot -\frac{1}{2}(a-b)^2 + \frac{1}{2}\frac{f(b)}{(b-a)}(a-b)^2$$

$$= -\frac{f(a)}{2}(a-b) - \frac{1}{2} f(b)(a-b)$$

$$= -\frac{1}{2}(a-b)\,(f(a) + f(b))$$

$$= \frac{b-a}{2}\,(f(a) + f(b)) = \int_a^b P_1(x)\,dx \cong \int_a^b f(x)\,dx = I$$

$$= \frac{b-a}{2}\,(f(a) + f(b)) = \int_a^b P_1(x)\,dx \cong \int_a^b f(x)\,dx = I$$

$$\int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \frac{h}{3}(f(a) + 4f(x_m) + f(b))$$

Para $x_m = \frac{a+b}{2}$ $\Omega = \{(a, f(a)), (x_m, f(x_m)), (b, f(b))\}$

$$P_2(x) = \sum_{i=0}^{n} f(x_i) S_i(x)$$

$$l_1(x) = \prod_{\substack{j=0 \\ j \neq 1}}^{2} \frac{x - x_j}{x_1 - x_j}$$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$l_2(x) = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

$$P_2 = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$I = \int_a^b P_2(x)\,dx = \int_a^b f(a)\frac{(x-x_m)(x-b)}{(a-x_m)(a-b)}\,dx + \int_a^b f(x_m)\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}\,dx + \int_a^b f(b)\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}\,dx$$

$$= \frac{f(a)}{(a-x_m)(a-b)}\int_a^b x^2 - xb - x_m x + x_m b\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b (x-a)(x-b)\,dx + \frac{f(b)}{(b-a)(b-x_m)}\int_a^b (x-a)(x-x_m)\,dx$$

- $a - x_m = a - \frac{a+b}{2} = \frac{2a - a+b}{2} = \frac{a+b}{2}$
- $x_m - a = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{-a+b}{2}.$
- $x_m - b = \frac{a+b}{2} - b = \frac{a+b-2b}{2} = \frac{-b+a}{2}.$
- $b - x_m = b - \frac{a+b}{2} = \frac{2b-a+b}{2} = b-a$

$$a - x_m = a - \frac{a-b}{2} = \frac{2a - a - b}{2} = \frac{a-b}{2}$$

$$x_m - a = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2}$$

$$x_m - b = \frac{a+b}{2} - b = \frac{a+b-2b}{2} = \frac{a-b}{2}$$

$$b - x_m = b - \frac{a-b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}$$

$$= \frac{f(a)}{\frac{(a-b)^2}{2}}\int_a^b x^2 - x(b + x_m) + x_m b\,dx + \frac{f(x_m)}{\frac{(b-a)(a-b)}{2}}\int_a^b x^2 - x(b+a) + ab\,dx$$

$\frac{\quad}{\frac{(b-a)^2}{2}}\Big|_a \; x \; -x(x_m+a)+ax_m \; a$

$$\frac{2f(a)}{(a-b)^2}\Big[\frac{x^3}{3}-\frac{x^2}{2}(b+x_m)+x(x_m b)\Big]_a^b+\frac{2f(x_m)}{(b-a)(a-b)}\Big[\frac{x^3}{3}-\frac{x^2}{2}(a+b)+x(ab)\Big]_a^b$$

$$+\frac{2f(b)}{(b-a)^2}\Big[\frac{x^3}{3}-\frac{x^2}{2}(x_m+a)+x(ax_m)\Big]_a^b$$

$$\frac{2f(a)}{(a-b)^2}\left(\frac{b^3}{3}-\frac{b^2}{2}(b+x_m)+b(x_m b)-\frac{a^3}{3}+\frac{a^2}{2}(b+x_m)-a(x_m b)\right)$$

$$+\frac{2f(x_m)}{(b-a)(a-b)}\left(\frac{b^3}{3}-\frac{b^2}{2}(a+b)+b(ab)-\frac{a^3}{3}+\frac{a^2}{2}(b+a)-a(ab)\right)$$

$$+\frac{2f(b)}{(b-a)^2}\left(\frac{b^3}{3}-\frac{b^2}{2}(x_m+a)+b(ax_m)-\frac{a^3}{3}+\frac{a^2}{2}(x_m+a)-a(ax_m)\right)$$

$=$

$$\frac{2f(a)}{(a-b)^2}\left(\frac{b^3}{3}-\frac{b^3}{2}-\frac{b^2x_m}{2}+b^2x_m-\frac{a^3}{3}+\frac{a^2b}{2}+\frac{a^2x_m}{2}-ax_mb\right)$$

$$\frac{2f(a)}{(a-b)^2}\left(\frac{b^3}{3}-\frac{b^2}{2}(b+x_m)+b(x_m b)-\frac{a^3}{3}+\frac{a^2}{2}(b+x_m)-a(x_m b)\right)$$

$$+\frac{2f(x_m)}{(b-a)(a-b)}\left(\frac{b^3}{3}-\frac{b^2}{2}(a+b)+b(ab)-\frac{a^3}{3}+\frac{a^2}{2}(b+a)-a(ab)\right)$$

$$+\frac{2f(b)}{(b-a)^2}\left(\frac{b^3}{3}-\frac{b^2}{2}(x_m+a)+b(ax_m)-\frac{a^3}{3}+\frac{a^2}{2}(x_m+a)-a(ax_m)\right)$$

$=$

$$\frac{2f(a)}{(a-b)^2}\left(\frac{b^3}{3}-\frac{b^3}{2}-\frac{b^2x_m}{2}+b^2x_m-\frac{a^3}{3}+\frac{a^2b}{2}+\frac{a^2x_m}{2}-ax_mb\right)$$

$$+\frac{2f(x_m)}{(b-a)(a-b)}\left(\frac{b^3}{3}-\frac{b^2a}{2}-\frac{b^3}{2}+ab^2-\frac{a^3}{3}+\frac{a^2b}{2}+\frac{a^3}{2}-a^2b\right)$$

$$+\frac{2f(b)}{(b-a)^2}\left(\frac{b^3}{3}-\frac{b^2x_m}{2}-\frac{b^2a}{2}+bax_m-\frac{a^3}{3}+\frac{a^3}{2}-a(ax_m)+\frac{a^2x_m}{2}\right)$$

$$=\frac{2f(a)}{(a-b)^2}\left(-\frac{b^3}{6}+\frac{1}{2}b^2x_m-\frac{a^3}{3}+\frac{a^2}{2}(b+x_m)-ax_mb\right)$$

$$=\frac{h}{3}\left(f(a)+4f(x_m)+f(b)\right).$$