Tarea 2. Métodos Computacionales

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1 (Theoretical)

$$\frac{d^2 f(x_i)}{dx^2} = \frac{f(x_i+2) - 2f(x_i) + f(x_i-2)}{4h^2}$$

M Demuestre la fórmula alternativa para la estimación de la segunda derivada discreta.

$$f'(x_j) = \frac{f(x_j+1) - f(x_j-1)}{2h} \Rightarrow \frac{d}{dx}\left(\frac{f'(x)}{x}\right) \equiv \frac{d}{dx}\left(\frac{f(x_j+1) - f(x_j-1)}{2h}\right)$$

Derivada de un cociente tal que.

$$f''(x_j) = \frac{\frac{d}{dx}\left(f(x_j+1) - f(x_j-1)\right)(2h) - (f(x_j+1) - f(x_j-1))\frac{d}{dx}(2h)}{(2h)^2}$$

Usando la definición de $f'(x_j)$ entonces

$$f'(x_j+1) = \frac{f(x_j+2) - f(x_j)}{2h}, \quad f(x_j-1) = \frac{f(x_j) - f(x_j-2)}{2h}$$

Remplazando

$$f''(x_j) = \frac{\cancel{2h}\left(\frac{f(x_j+2) - f(x_j)}{\cancel{2h}}\right) - \cancel{2h}\left(\frac{f(x_j) - f(x_j-2)}{\cancel{2h}}\right) - 0\cancel{(f(x_j+1) - f(x_j-1))}}{4h^2}$$

$$f''(x_j) = \frac{f(x_j+2) - 2f(x_j) - f(x_j-2)}{4h^2}$$ como se quería demostrar

5) (Theoretical)

$$D^4 f(x_j) \cong \frac{f(x_j+2) - 4f(x_j+1) + 6f(x_j) - 4f(x_j-1) + f(x_j-2)}{h^4}$$

Usando las anteriores definiciones de $f'(x_i)$ y $f''(x_i)$

Es cero

$$\frac{d}{dx}\left(\frac{f''(x_j)}{dx}\right) = \frac{4h^2\frac{d}{dx}(f(x_i+2) - 2f(x_i) - f(x_i-2)) - \frac{d(\cancel{4h^2})}{dx}(\dots)}{4h^2}$$

$$\frac{d(f''(x_i))}{dx}$$

$$\frac{d}{dx}f(x_i+2) = \frac{f(x_i+3) - f(x_i-1)}{2h}, \quad \frac{df(x_i-2)}{dx} = \frac{f(x_i-1) - f(x_i-3)}{2h}$$

$$\frac{df''(x_i)}{dx} = \frac{\cancel{4}^{2} h^2\left(\frac{f(x_i+3)-f(x_i-1)}{\cancel{2}h} - \cancel{2}\left(\frac{f(x_i+1)-f(x_i-1)}{\cancel{2}h}\right) + \left(\frac{f(x_i-1)-f(x_i-3)}{2h}\right)\right)}{16h^4}$$

$$\frac{df''(x_i)}{dx} = \frac{f(x_i+3)-3f(x_i+1)+3f(x_i-1)-f(x_i-3)}{8h^3} = D^3 f(x_i)$$

Para encontrar $D^4 f(x_i)$ hace falta derivar $D^3 f(x_i)$ una vez más

$$D^4 f(x_i) = \frac{d}{dx} f'''(x_i)$$

$$\left\langle \frac{d^2 f''(x_i)}{dx^2} = \frac{d}{dx}\left(\frac{d}{dx} f''(x_i)\right) \right\rangle$$

$$\frac{df'''(x_i)}{dx} = \frac{1}{8h^3}\frac{d}{dx}\left(f(x_i+3)-3f(x_i+1)+3f(x_i-1)-f(x_i-3)\right)$$

$$= \frac{1}{8h^3}\left(\left[\frac{f(x_i+4)-f(x_i-2)}{2h}\right]-\left[\frac{3f(x_i+2)+3f(x_i)}{2h}\right]+3\left[\frac{f(x_i)-f(x_i-2)}{2h}\right]-\left[\frac{f(x_i-2)-f(x-4)}{2h}\right]\right)$$

$$= \frac{\frac{1}{2h}}{8h^3}\; f(x_i+4)-f(x_i-2)-3f(x_i+2)-3f(x_i)+3f(x_i)-3f(x_i-2)-f(x_i-2)-f(x_i-4)$$

$$= \frac{1}{16h^4}\; f(x_i+4)-4f(x_i+2)+6f(x_i)-4f(x_i-2)+f(x_i-4) = f''''(x_i)$$

$(O(h^k)) = O(h^5)$ para el operador $D^4 f$.