2) Taller Integración.

Error regla trapecio simple.

$f''(\xi) = cte$

$$\begin{cases} h = b-a \\ -h = a-b \end{cases}$$

$$\epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$$

$$\int_a^b \epsilon(x)\,dx = \frac{f''(\xi)}{2}\int_a^b (x-a)(x-b)\,dx = \frac{f''(\xi)}{2}\cdot\frac{1}{6}(a-b)^3$$

$$= \frac{-1}{12}h^3 f''(\xi)$$

4).

$$E = \int_a^b \epsilon(x)\,dx = \int_a^b \frac{f'''(\xi)}{4!}(x-x_0)(x-x_1)(x-x_2)\,dx$$

$$\int_a^b \frac{f'''(\xi)}{4!}(x-a)(x-b)\left(x-\left(\frac{a+b}{2}\right)\right)dx$$

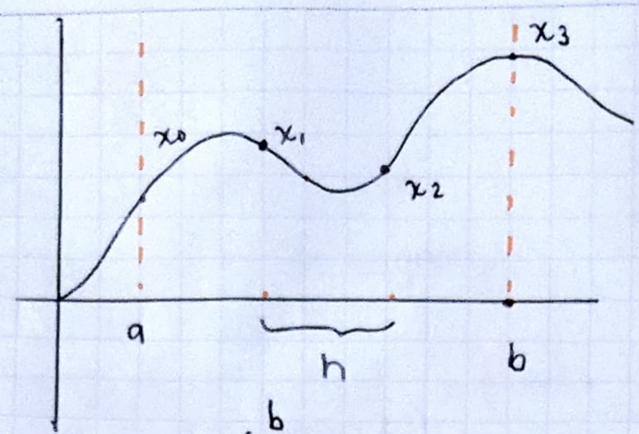
La integral indefinida equivale a

$$\int (x-a)(x-b)\left(x-\frac{a+b}{2}\right)dx = \frac{1}{4}x\left(a^2(x-2b)-2a(b-x)^2+x(b-x)^2+c\right.$$

$$\left[\frac{1}{4}b\left(a^2(b-2b)-2a(\cancel{b}-b)^2+x(\cancel{b}-b)^2\right]-\left[\frac{1}{4}a\left(a^2(a-2b)-2a(b-a)^2\right.\right.$$

$$\left.+\,a(b-a)^2\right] = 0$$

8



$$\begin{cases} x_0 = a \\ x_1 = 2a+b/3 \\ x_2 = a+2b/3 \\ x_3 = b \end{cases}$$

$$h = \frac{b-a}{3}$$

$$E = \frac{f''''(\xi)}{4!}\int_a^b (x-x_0)(x-x_1)(x-x_2)(x-x_3)\,dx$$

$$= \frac{f''''(\xi)}{4!}\int_a^b (x-a)\left(x-\left(\frac{2a+b}{3}\right)\right)\left(x-\left(\frac{a+2b}{3}\right)\right)(x-b)\,dx$$

$$= \frac{f''''(\xi)}{4!}\int_0^{3h} (x-0)(x-h)(x-2h)(x-3h)\,dx$$

$$= \frac{-f''''(\xi)}{80}(3h^5)$$

$$\int_0^{3h} (x-0)(x-h)(x-2h)(x-3h)\,dx$$

$$= \int_0^{3h} -6x^3h + 11x^2h^2 + x^4 - 6xh^3\,dx$$

$$= -6h\int_0^{3h} x^3\,dx + 11h^2\int_0^{3h} x^2\,dx + \int_0^{3h} x^4\,dx - 6h^3\int_0^{3h} x\,dx$$

$$= h^5\left(-\frac{243}{2} + 99 + \frac{243}{5} - 27\right) = h^5\left(-\frac{9}{10}\right)$$

Tal que

$$\frac{f''''(\xi)}{24}\left(h^5 \cdot -\frac{9}{10}\right) = -\frac{3}{80}h^5 f''''(\xi)$$

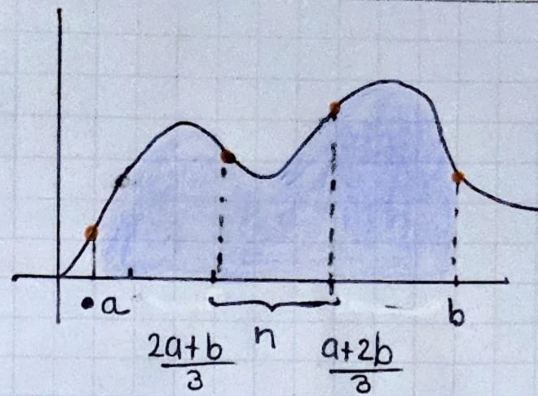
9)

a)

$$I=\int_a^b f(x)\,dx$$

$$f_3(x)=a_0+a_1x+a_2x^2+a_3x^3$$

Usando la interpolacion de Lagrange



$$f_3(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\times f(x_0)+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\times f(x_1)+$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\times f(x_2)+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\times f(x_3)$$

$$I=\int_a^b f(x)\,dx \qquad h=\frac{b-a}{3}$$

$$\approx\int_a^b f_3(x)\,dx$$

$$\begin{cases} x_0=a \\ x_1=a+h=\frac{b-a}{3}+a \\ x_2=a+2h=a+\frac{2(b-a)}{3} \\ x_3=b \\ x_3=a+3h=a+\frac{3b-3a}{3}=b \end{cases}$$

al remplazar en $f_3(x)$ obtenemos.

$$f_3(x)=\frac{[x-(a+h)][x-(a+2h)][x-b]}{(a-a+h)(a-(a+2h))(a-b)}f(a)+\frac{[x-a][x-(a+2h)][x-b]}{[(a+h)-a][(a+h)-(a+2h)][a+h-b]}f(a+h)$$

$$\frac{[x-a][x-(a+h)][x-b]}{[(a+2h)-a][a+2h-a][a+2h-a]}f(a+2h)+\frac{[x-a][x-(a+h)][x-(a+2h)]}{[b-a][b-a+h][b-a+2h]}f(b)$$

Simplificando $f_3(x)$...

$$f_3(x)=-(a-b)\cdot\frac{f(a)+3f(a+h)+3f(a+2h)+f(b)}{8}$$

$$f_3(x)=(b-a)\,\frac{f(x_0)+3f(x_1)+3f(x_2)+f(x_3)}{8}$$

Por lo tanto.

$$\int_a^b f_3(x)\,dx=\frac{3h}{8}\left[f(a)+3f\left(\frac{2a+b}{3}\right)+3f\left(\frac{a+2b}{3}\right)+f(b)\right]$$

b) como se puede ver en el grafico los puntos medios equivalen a $\frac{2a+b}{3}$ y $\frac{a+2b}{3}$