

$$1) \quad I = \int_a^b f(x) dx \cong \int_a^b p_1(x) dx = \frac{b-a}{2} (f(a) + f(b))$$

$$\Omega = \{(a, f(a)), (b, f(b))\}$$

$$f(x) \approx p_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b p_1(x) dx$$

$$\int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) dx$$

$$f(a) \int_a^b \frac{x-b}{a-b} dx + f(b) \int_a^b \frac{x-a}{b-a} dx$$

$$\frac{f(a)}{a-b} \int_a^b x-b dx + \frac{f(b)}{b-a} \int_a^b x-a dx$$

$$\frac{f(a)}{a-b} \left(\frac{1}{2} x^2 - bx \right) \Big|_a^b + \frac{f(b)}{b-a} \left(\frac{1}{2} x^2 - ax \right) \Big|_a^b$$

$$\frac{f(a)}{a-b} \left(\left(\frac{b^2}{2} - b^2 \right) - \left(\frac{a^2}{2} - ab \right) \right) + \frac{f(b)}{b-a} \left(\left(\frac{b^2}{2} - ab \right) - \left(\frac{a^2}{2} - a^2 \right) \right)$$

$$\frac{f(a)}{a-b} \left(-\frac{b^2}{2} - \frac{a^2}{2} + ab \right) + \frac{f(b)}{b-a} \left(\frac{b^2}{2} - ab + \frac{a^2}{2} \right)$$

$$\frac{f(a)}{a-b} \left(-\frac{(a-b)^2}{2} \right) + \frac{f(b)}{b-a} \left(-\frac{(a-b)^2}{2} \right)$$

$$f(a) \left(-\frac{(a-b)}{2} \right) + f(b) \left(-\frac{(a-b)}{2} \right)$$

$$f(a) \left(\frac{b-a}{2} \right) + f(b) \left(\frac{b-a}{2} \right)$$

$$\frac{b-a}{2} (f(a) + f(b))$$