Parte 2

Simpson 3/8 → Subintervalos $m=4$

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} c_i f(x_i) \approx \sum_{i=0}^{n-1} f(x_i) \prod_{\substack{j=0\\ j\neq i}}^{n-1} \frac{x-x_j}{x_i-x_j}$$

$$\approx f(x_0)\int \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + f(x_1)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ f(x_2)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + f(x_3)\int \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$\approx f(x_0)\int \frac{(x-x_1)(x-x_2)(x-x_3)}{(-h)(-2h)(-3h)} + f(x_1)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{(h)(-h)(-2h)}$$

$$+ f(x_2)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{(2h)(h)(-h)} + f(x_3)\int \frac{(x-x_0)(x-x_1)(x-x_2)}{(3h)(2h)(h)}$$

$$\approx f(x_0)\int \frac{(x-x_1)(x-x_2)(x-x_3)}{-6h^3} + f(x_1)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{2h^3}$$

$$+ f(x_2)\int \frac{(x-x_0)(x-x_1)(x-x_3)}{-2h^3} + f(x_3)\int \frac{(x-x_0)(x-x_1)(x-x_2)}{6h^3}$$

Como $f(x_i)=cte$, evaluamos x entre $x_0 \le x \le x_3$ de modo que

$x = x_0 + th$

$\frac{dx}{dt} = h$

$x - x_0 = th$

$x - x_1 = h(t-1)$

$x - x_2 = h(t-2)$

$x - x_3 = h(t-3)$

$$\approx f(x_0)\int_0^3 \frac{(t\cancel{h})(\cancel{h}(t-2))\cancel{h}(t-3)}{-6\cancel{h^3}} + f(x_1)\int_0^3 \frac{(t\cancel{h})\cancel{h}(t-2)\cancel{h}(t-3)}{2\cancel{h^3}}$$

$$+ f(x_2)\int_0^3 \frac{(t\cancel{h})\cancel{h}(t-1)\cancel{h}(t-3)}{-2\cancel{h^3}} + f(x_3)\int_0^3 \frac{(t\cancel{h})\cancel{h}(t-1)\cancel{h}(t-2)}{6\cancel{h^3}}$$

$$\approx \frac{f(x_0)h}{-6}\int_0^3 t(t-2)(t-3)\,dt + \frac{f(x_1)h}{2}\int_0^3 t(t-2)(t-3)\,dt + \frac{f(x_2)h}{-2}\int_0^3 t(t-1)(t-3)\,dt$$

$$+ \frac{f(x_3)h}{6}\int_0^3 t(t-1)(t-2)\,dt$$

$$\approx \frac{f(x_0)h}{6}\left[\frac{t^4}{4}-\frac{6t^3}{3}+\frac{11t^2}{2}-6t\,\Big|_0^3\right]+\frac{f(x_1)}{2}\left[\frac{t^4}{4}-\frac{5t^3}{3}+\frac{6t^2}{2}\Big|_0^3\right]$$

$$+\frac{f(x_2)}{-2}\left[\frac{t^4}{4}-\frac{4t^3}{3}+\frac{3t^2}{2}\Big|_0^3\right]+\frac{f(x_3)}{-6}\left[\frac{t^4}{4}-\frac{\cancel{3}t^3}{\cancel{3}}+\frac{\cancel{2}t^2}{\cancel{2}}\Big|_0^3\right]$$

$$\approx f(x_0)\left(\frac{3h}{8}\right)+f(x_1)\left(\frac{9h}{8}\right)+f(x_2)\left(\frac{9h}{8}\right)+f(x_3)\left(\frac{3h}{8}\right)$$

$$\approx \frac{3h}{8}\Big(f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\Big)$$