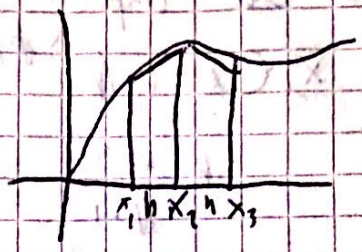
① Reglas intermedias regla trapecio simple

$$I=\int_a^b f(t)dt \qquad A_{Trapecio}=h\frac{(B_1+B_2)}{2}=h\frac{(f(x_1)+f(x_{n+1}))}{2}$$

$$=\int_a^b f(x)dx=\sum_{n=x_0}^{b} h\frac{(f_n+f_{n+1})}{2}$$

- Para * igualmente espaciados $h=x_i+x_{i+1}$



$$=h\sum_{n=x_0}^{N}\frac{f_n+f_{n+1}}{2}$$

$$=\frac{h}{2}\left[f_{x_0}+f_{x+1}+f_{x+1}+f_{x+2}+f_{x+2}+f_{x+3}+\ldots+f_{n-1}+f_n\right]$$

*Observe que después de f_{x_0} hasta f_{n-1} los numeros se repiten 1 vez

$$=\frac{h}{2}\left[f_{x_0}+2f_{x_0}+2f_{x_1}+2f_{x_2}+\ldots+2f_{n-1}+f_n\right]$$

$$=\frac{h}{2}\left[f_{x_0}+2f_{x_1}+2f_{x_2}+\ldots+2f_{n-1}+f_n\right]=h\left[\frac{f_{x_0}}{2}+\sum_{n=0}^{N}f_n+\frac{f_n}{2}\right]$$

② Error regla trapecio

$$\varepsilon=\left|I_n-I_n^{trapecio}\right|$$

$$f(x)=f(x_n)+f'(x_n)(x-x_n)+\frac{f''(x_n)}{2}(x-x_n)^2+\frac{f'''(x_n)}{6}(x-x_n)^3+\ldots$$

$$I_n=\int_{x_n-h}^{x_n+h}f(x)dx=2f(x_n)h+f'(x_n)\int_{x_n-h}^{x_n+h}(x-x_n)dx+\frac{f''(x_n)}{2}\int_{x_n-h}^{x_n+h}(x-x_n)^2dx$$

$$I_n=2f(x_n)h+\frac{f''(x_n)}{3}h^3+O(h^5)$$

$$I_n^{\text{trapezio}} = \frac{h}{2}\left[f(x_n) - f'(x_n)h + \frac{f''(x_n)}{2}h^2 - \frac{f'''(x_n)}{6}h^3 + \frac{f''''(x_n)}{24}h^4 + O(h^5)\right.$$
$$\left. + h\,f(x_n) + \frac{h}{2}\left[f(x_n) + f'(x_n)h + \frac{f''(x_n)}{2}h^2 + \frac{f'''(x_n)}{6}h^3 + \frac{f''''(x_n)}{24}h^4 + O(h^5)\right]\right]$$

$$= 2f(x_n)h + \frac{f''(x_n)}{2}h^3 + O(h^5)$$

$$E_n = |I_n - I_n^{\text{trapezio}}| = \frac{f''(x_n)}{3}h^3 - \frac{f''(x_n)}{2}$$

$$= \frac{f''(x_n)}{6}h^3 \sim O\left(\frac{1}{N^3}\right)$$

$$h \sim O\left(\frac{1}{N}\right)$$