Ejercicio 10.

Tenemos: $\displaystyle\int_0^{3h} (x)(x-h)(x-2h)(x-3h)\,dx.$

Es decir:

$$\int_0^{3h} x^4 - 6h x^3 + 11h^2 x^2 - 6h^3 x \quad dx$$

$$\Rightarrow \frac{x^5}{5} - \frac{3}{2}h\,\cancel{(3h)}\,x^4 + \frac{11\,h^2 x^3}{3} - \frac{6h^3x^2}{2}\Bigg|_0^{3h}$$

Calculando obtenemos:

$$\frac{3^5h^5}{5} - \frac{3^5h^5}{2} + \frac{11\cdot 27}{3}h^5 - 27h^5$$

$$\Rightarrow h^5\left(\frac{3^5}{5} - \frac{3^5}{2} + \frac{11\cdot 27}{3} - 27\right)$$

$$\Rightarrow h^5\left(\frac{243}{5} - \frac{243}{2} + \frac{257}{3} - 27\right)$$

$$\Rightarrow h^5\left(\frac{243\cdot 2\cdot 3 - 243\cdot 5\cdot 3 + 257\cdot 5\cdot 2 - 27\cdot 5\cdot 2\cdot 3}{5\cdot 2\cdot 3}\right)$$

$$\Rightarrow h^5\left(\frac{1458 - 3645 + 2570 - 810}{30}\right)$$

$$\Rightarrow h^5\left(-\frac{9}{10}\right).$$

Sabemos entonces que:

$$\frac{f^{4}(\xi)}{4!}\int_{0}^{3h} x(x-h)(x-2h)(x-3h)\,dx = \frac{f^{4}(\xi)}{4!}\cdot\left(-\frac{9}{10}\right)h^{5}.$$

Es decir; $f^{4}(\xi)\cdot\left(-\frac{9}{10\cdot(4!)}\right)h^{5}.$

$$\Rightarrow -\frac{9}{80\cdot 3}f^{4}(\xi)h^{5} \Rightarrow -\frac{3}{80}f^{4}(\xi)h^{5}.$$

Por lo tanto, demostramos que el error es siempre:

$$\boxed{E = -\frac{3}{80}\cdot f^{4}(\xi)\cdot h^{5}}$$