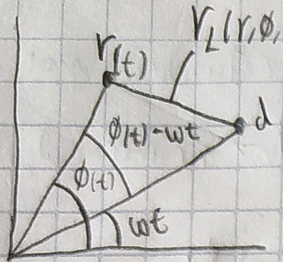
4) C-



Note el triangolo formormado por $r_{(t)}$, d, 0

usando ley de coseno tendremos:

$$r_L^2(r,\phi,t) = r_{(t)}^2 + d^2 - 2r_{(t)}d\cos(\phi_{(t)}-\omega t)$$

$$\Rightarrow r_L(r,\phi,t) = \sqrt{r_{(t)}^2 + d^2 - 2r_{(t)}d\cos(\phi_{(t)}-\omega t)} \quad //$$

D- Energía cinética: $T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) = \frac{1}{2}m(\dot{r}^2+\dot{\phi}^2 r^2)$

Energía Potencial: $V = V_{nave-Tierra} + V_{nave-luna} = -G\dfrac{m_n M_T}{r_{(t)}} - G\dfrac{m_n M_L}{r_L(r,\phi,t)}$

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2+\dot{\phi}^2 r^2) + G\frac{m_n M_T}{r_{(t)}} + G\frac{m_n M_L}{r_L(r,\phi,t)}$$

$$p_r = \frac{\partial \mathcal{L}}{\partial \dot{r}} = m\dot{r} \Rightarrow \dot{r} = \frac{p_r}{m}$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m\dot{\phi}r^2 \Rightarrow \dot{\phi} = \frac{p_\phi}{mr^2}$$

$$\Rightarrow H = \sum p_{q_i}\dot{q}_i - \mathcal{L} = \frac{p_r^2}{m} + \frac{p_\phi^2}{mr^2} - \frac{1}{2}m\left(\frac{p_r^2}{m^2} + \frac{p_\phi^2}{r^2m^2}\right) - Gm_n\left(\frac{M_T}{r_{(t)}} + \frac{M_L}{r_L(r,\phi,t)}\right)$$

$$\Rightarrow H = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - Gm_n\left(\frac{M_T}{r_{(t)}} + \frac{M_L}{r_L(r,\phi,t)}\right) \quad //$$

$$e-\ \dot{q}_i=\frac{\partial H}{\partial p_{q_i}}\ ;\ \dot{p}_{q_i}=-\frac{\partial H}{\partial q_i}$$

$$\Rightarrow \dot{r}=\frac{\partial H}{\partial p_r}=\frac{p_r}{M}\ ;\ \dot{\phi}=\frac{\partial H}{\partial p_\phi}=\frac{p_\phi}{Mr^2}$$

$$\Rightarrow \dot{p}_r=-\frac{\partial H}{\partial r}=Gm_n\left(-\frac{M_T}{r^2}-\frac{M_L}{2}\left[2r-d\cos(\phi-\omega t)\right]\left[r^2+d^2+2rd\cos(\phi-\omega t)\right]^{-3/2}\right)$$

$$\dot{p}_r=-Gm_n\left[\frac{M_T}{r^2}+\frac{M_L}{r_L^3}\left(r-d\cos(\phi-\omega t)\right)\right]$$

$$\Rightarrow \dot{p}_\phi=\frac{-\partial H}{\partial \phi}=-\frac{Gm_nM_L}{2}\left[2rd\sin(\phi-\omega t)\right]\left[r^2+d^2-2rd\cos(\phi-\omega t)\right]^{-3/2}$$

$$\dot{p}_\phi=\frac{-GmM_L}{r_L^3}\left[rd\sin(\phi-\omega t)\right]$$

F: $\tilde{r} = \frac{r}{d}$, ϕ, $\tilde{p}_r = \frac{p_r}{md}$, $\tilde{p}_\phi = \frac{p_\phi}{md^2}$

$$\rightarrow \dot{\tilde{r}} = \frac{\dot{r}}{d} = \frac{p_r}{md} = \tilde{p}_r \quad ; \quad \dot{\phi} = \frac{p_\phi}{mr^2} = \frac{\tilde{p}_\phi d^2}{\tilde{r}^2 d^2} = \frac{\tilde{p}_\phi}{\tilde{r}^2}$$

$$\rightarrow \dot{\tilde{p}}_r = \frac{\dot{p}_r}{md} = \frac{1}{md}\left[\frac{p_\phi^2}{mr^3} - \frac{GmM_T}{r^2} - G\frac{mM_L}{r_L^3}(r - d\cos(\phi - \omega t))\right]$$

$$= \frac{1}{d}\left[\frac{md^4\tilde{p}_\phi}{mr^3} - GM_T\left(\frac{1}{r^2} - \frac{M_L[r - d\cos(\phi - \omega t)]}{M_T d^3\left(\sqrt{1 + (\frac{r}{d})^2 - 2\frac{r}{d}\cos(\phi - \omega t)}\right)^3}\right)\right]$$

$$= \frac{\tilde{p}_\phi}{\tilde{r}^3} - \Delta d^2\left[\frac{1}{r^2} - \frac{\mu}{d^2\tilde{r}'^3}(\tilde{r} - \cos(\phi - \omega t))\right]$$

$$\dot{\tilde{p}} = \frac{\tilde{p}}{\tilde{r}^3} - \Delta\left\{\frac{1}{\tilde{r}^2} - \frac{\mu}{\tilde{r}'^3}[\tilde{r} - \cos(\phi - \omega t)]\right\} //$$

$$\rightarrow \dot{\tilde{p}}_\phi = \frac{\dot{p}_\phi}{md^2} = \frac{1}{md^2}\left[-G\frac{mM_L}{r_L^3}\, rd\cos(\phi - \omega t)\right] = -G\frac{M_L M_T\, r\cos(\phi - \omega t)}{M_T d^4\left(\sqrt{1 + (\frac{r}{d})^2 - 2\frac{r}{d}\cos(\phi - \omega t)}\right)^3}$$

$$= -\frac{\Delta\mu\,\tilde{r}}{\tilde{r}'^3}\cos(\phi - \omega t) //$$

9. $$\tilde{p}_r = \frac{p_r}{m d_{T_L}} = \frac{\dot{r}}{d_{T_L}} = \frac{1}{d_{T_L}} \frac{\partial}{\partial t}\sqrt{x^2+y^2} = \frac{1}{2 d_{T_L}} (x^2+y^2)^{-1/2}(2x\dot{x}+2y\dot{y})$$

$$= \frac{1}{d_{T_L} r}(x\dot{x}+y\dot{y}) = \frac{r\cos\phi\, v\cos\theta + r\sin\phi\, v\sin\theta}{d_{T_L} r} = \frac{v}{d_{T_L}}\cos(\theta-\phi) = \tilde{v}\cos(\theta-\phi) //$$

$$\tilde{p}_\phi = \frac{m r^2 \dot{\phi}}{m d_{T_L}^2} = \tilde{r}^2 \frac{d}{dt}\phi = \tilde{r}^2 \frac{\partial}{\partial t}\arctan(y/x) = \frac{\tilde{r}^2}{1+\frac{y^2}{x^2}} \cdot \frac{\partial}{\partial t}\left(\frac{y}{x}\right) = \frac{\tilde{r}^2}{1+\frac{y^2}{x^2}}(\dot{y}x^{-1} - yx^{-2}\dot{x})$$

$$= \frac{\tilde{r}^2}{x^2+y^2}(\dot{y}x - y\dot{x}) = \frac{\tilde{r}^2}{r^2}(vr\sin\theta\cos\phi - rv\sin\phi\cos\theta) = \frac{\tilde{r}^2 v}{r}\sin(\theta-\phi)$$

$$= \frac{\tilde{r}^2 v}{\tilde{r} d_{T_L}}\sin(\theta-\phi) = \tilde{r}\tilde{v}\sin(\theta-\phi) //$$