4. Integrador simplectico I: Muestre que el metodo de Verlet es simplectico, es decir, el Jacobiano inducido en el metodo es igual a uno.

$$J = \frac{\partial X_{n+1}}{\partial X_n}\frac{\partial V_{n+1}}{\partial V_n} - \frac{\partial X_{n+1}}{\partial V_n}\frac{\partial V_{n+1}}{\partial X_n} = 1$$

$$x_{n+1} = x_n + V_n(\Delta t) + \frac{a_n \cdot \Delta t^2}{2}$$

$$V_{n+1} = V_n + \Delta t\left(a_{n+1} + a_n\right)\cdot\frac{1}{2}$$

$$\frac{\partial X_{n+1}}{\partial X_n} = 1 \;;\; \frac{\partial V_{n+1}}{\partial V_n} = 1 \;;\; \frac{\partial X_{n+1}}{\partial V_n} = \Delta t \;;\; \frac{\partial V_{n+1}}{\partial X_n} = 0$$

$$J = \frac{\partial X_{n+1}}{\partial X_n}\frac{\partial V_{n+1}}{\partial V_n} - \frac{\partial X_{n+1}}{\partial V_n}\frac{\partial V_{n+1}}{\partial X_n} = 1\cdot 1 - (\Delta t \cdot 0) = 1$$