Scribe

1. Integrador de Beeman 1976.

Partimos de Verlet con

$$\vec{x}(t+\Delta t) = \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \frac{\vec{a}'(t)\cdot\Delta t^3}{6} + O(\Delta t^4)$$

donde

$$\vec{a}'(t) = \frac{a(t+\Delta t) - a(t-\Delta t)}{2\Delta t}$$

Con lo cual tenemos

$$\vec{x}(t+\Delta t) = \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \left(\frac{\vec{a}(t+\Delta t) - \vec{a}(t-\Delta t)}{2\Delta t}\right)\frac{\Delta t^3}{6} + O(\Delta t^4)$$

$$= \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \frac{2\vec{a}(t) - \vec{a}(t-\Delta t) - \vec{a}(t-\Delta t)}{2\cdot 6\cdot\Delta t}\Delta t^3$$

$$= \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \frac{2\vec{a}(t) - 2\vec{a}(t-\Delta t)}{2\cdot 6\cdot\Delta t}\Delta t^3$$

$$= \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \frac{\vec{a}(t) - \vec{a}(t-\Delta t)}{6}\Delta t^2$$

$$= \vec{x}(t) + \vec{v}(t)\Delta t + \frac{\vec{a}(t)\cdot\Delta t^2}{2} + \frac{\vec{a}(t)\ \Delta t^2}{6} - \frac{\vec{a}(t-\Delta t)}{6}\Delta t^2$$

$$= \vec{x}(t) + \vec{v}(t)\Delta t + \frac{4\vec{a}(t)\cdot\Delta t^2}{6} - \frac{\vec{a}(t-\Delta t)}{6}\Delta t^2$$

en la notacion del punto como nos lo ponen

$$r_{n+1} = r_n + v_n\Delta t + \frac{4}{6}a_n h^2 - \frac{a_{n-1}}{6}h^2$$

$$= r_n + v_n\Delta t + \frac{h^2}{6}(4a_n - a_{n-1})$$

nos mantendremos en la notacion

$$r_{n+1} = r_n + v_n h + \frac{h^2}{2} a_n + \frac{h^3}{6} \frac{(a_{n+1} - a_{n-1})}{2h}$$

$$r_{n+1} = r_n + v_n h + \frac{h^2}{2} a_n + \frac{h^3}{6} \frac{(a_{n+1} - (2a_n - a_{n+1}))}{2h}$$

$$= r_n + v_n h + \frac{h^2}{2} a_n + \frac{h^2 \cancel{h}}{6} \frac{\cancel{2} a_{n+1} - \cancel{2} a_n}{\cancel{2} \cancel{h}}$$

$$= r_n + v_n h + \frac{h^2}{2} a_i + \frac{a_{n+1} - a_n}{6} h^2$$

$$= r_n + v_n h + \frac{h^2}{2} a_n - \frac{a_n}{6} h^2 + \frac{a_{n+1}}{6} h^2$$

$$= r_n + v_n h + \frac{2}{6} h^2 a_n + \frac{a_{n+1}}{6} h^2$$

finalmente

$$r_{n+1} = r_n + v_n h + \frac{h^2}{6} (2a_n + a_{n+1})$$