Ecuacion de onda 2D en coordenadas cilindricas:

$$\lambda = \frac{\Delta \rho}{\Delta \phi} \; ; \; \gamma = \frac{\alpha \Delta t}{\Delta \rho}$$

$$\frac{\partial^2 u(\rho,\phi,t)}{\partial t^2} = \alpha^2 \frac{\partial^2 u(\rho,\phi,t)}{\partial \rho^2} + \beta^2 \frac{\partial^2 u(\rho,\phi,t)}{\partial \phi^2}$$

De Laplace en coordenadas cilindricas tenemos que

$$\nabla^2 u(\rho,\phi) = \frac{u_{i+1}^{j} - 2u_i^j + u_{i-1}^{j}}{(\Delta \rho)^2} + \frac{1}{\rho[i]} \frac{u_i^j - u_{i-1}^j}{\Delta \rho} + \frac{1}{\rho[i]^2}\left(\frac{u_i^{j+1} - 2u_i^j + u_i^{j-1}}{(\Delta \phi)^2}\right)$$

Adaptando tenemos

$$\nabla^2 u(\rho,\phi,t) = \frac{u^l_{i+1,j} - 2u^l_{i,j} + u^l_{i-1,j}}{(\Delta\rho)^2} + \frac{1}{\rho[i]}\frac{u^l_{i,j} - u^l_{i-1,j}}{\Delta\rho}$$

$$+ \frac{1}{\rho[i]^2}\left(\frac{u^l_{i,j+1} - 2u^l_{i,j} + u^l_{i,j-1}}{(\Delta\phi)^2}\right)$$

$$\rho[i] \quad (\quad (\Delta \phi) \quad)$$

II

Si $\alpha = \beta$

$$\alpha^2 \left(\frac{\partial^2 u(\rho,\phi,t)}{\partial \rho^2} + \frac{\partial^2 u(\rho,\phi,t)}{\partial \phi^2} \right) = \alpha^2 \left(\frac{1}{(\Delta \rho)^2} \left(u^l_{i+1,j} - 2u^l_{i,j} + u^l_{i-1,j} \right. \right.$$

$$\left. \left. + \frac{(\Delta \rho)}{\rho[i]} \frac{u^l_{i,j} - u^l_{i-1,j}}{1} + \frac{(\Delta \rho)^2}{\rho[i]^2} \left(\frac{u^l_{i,j+1} - 2u^l_{i,j} + u^l_{i,j-1}}{(\Delta \phi)^2} \right) \right) \right)$$

III

$$\frac{\partial^2 u(\rho,\phi,t)}{\partial t^2} = \frac{u^{l+1}_{i,j} - 2u^l_{i,j} + u^{l-1}_{i,j}}{(\Delta t)^2}$$

VII

$$\frac{u_{i,j}^{l+1}-2u_{i,j}^{l}+u_{i,j}^{l-1}}{(\Delta t)^2}=\frac{\alpha^2}{(\Delta p)^2}\left(\left(u_{i+1,j}^{l}-2u_{i,j}^{l}+u_{i-1,j}^{l}\right)+\frac{\Delta p}{P[i]}\frac{u_{i,j}^{l}-u_{i-1,j}^{l}}{1}\right.$$

$$\left.+\frac{(\Delta p)^2}{P[i]^2}\left(\frac{u_{i,j+1}^{l}-2u_{i,j}^{l}+u_{i,j-1}^{l}}{(\Delta \phi)^2}\right)\right)$$

$$u_{i,j}^{l+1}=\frac{\alpha^2(\Delta t)^2}{(\Delta p)^2}\left(\left(u_{i+1,j}^{l}-2u_{i,j}^{l}+u_{i-1,j}^{l}\right)+\frac{\Delta p}{P[i]}\frac{u_{i,j}^{l}-u_{i-1,j}^{l}}{1}\right.$$

$$\left.+\frac{(\Delta p)^2}{(\Delta\phi)^2\,P[i]^2}\left(u_{i,j+1}^{l}-2u_{i,j}^{l}+u_{i,j-1}^{l}\right)\right)+2u_{i,j}^{l}-u_{i,j}^{l-1}.$$

VIII Recordemos

$$\lambda^2=\left(\frac{\Delta p}{\Delta \phi}\right)^2 \qquad v^2=\frac{\alpha^2\,\Delta t^2}{(\Delta p)^2}$$

IX

$$u_{i,j}^{l+1}=v^2\left(\left(u_{i+1,j}^{l}-2u_{i,j}^{l}+u_{i-1,j}^{l}\right)+\frac{\Delta p}{P[i]}\frac{u_{i,j}^{l}-u_{i-1,j}^{l}}{1}\right.$$

$$\left.+\frac{\lambda^2}{P[i]^2}\left(u_{i,j+1}^{l}-2u_{i,j}^{l}+u_{i,j-1}^{l}\right)\right)+2u_{i,j}^{l}-u_{i,j}^{l-1}.$$