25. $L_n(x) = \frac{e^x}{n!} \cdot \frac{\partial^n}{\partial x^n} (x^n e^{-n})$

$L_2(x) \quad \frac{e^x}{2!} \cdot \frac{\partial^2}{\partial x^2} (x^2 e^{-x})$

$L_2(x) = \frac{e^x}{2!} \cdot 2e^{-x} - 4xe^{-x} + e^{-x}x^2$

$= \frac{1}{2}(x^2 - 4x + 2)$

Raicez

$x^2 - 4x + 4 - 2$

$(x-2)^2 - 2$

$(x - 2 - \sqrt{2}) \quad (x - 2 + \sqrt{2})$

$x - 2 - \sqrt{2} = 0 \qquad x - 2 + \sqrt{2} = 0$

$x_1 = 2 + \sqrt{2} \qquad x_2 = 2 - \sqrt{2}$

$\frac{\partial}{\partial x} \; x^2 e^{-x}$

$2xe^{-x} + (-e^{-x}x^2)$

$= 2xe^{-x} - x^2 e^{-x}$

$\frac{\partial x^2}{\partial x^2} = 2e^{-x} - 2xe^{-x} - 2xe^{-x} + e^{-x}x^2$

$w_1 = \int_0^\infty \frac{x - \sqrt{2} - 2}{2 - \sqrt{2} - 2 - \sqrt{2}} \cdot e^{x}$

$w_1 = \int_0^\infty \frac{x - \sqrt{2} - 2}{-2\sqrt{2}\, e^{-x}}$

$w_1 = \int_0^\infty \frac{1}{-2\sqrt{2}} \left(\frac{x}{e^{-x}} - \frac{\sqrt{2}}{e^x} - \frac{2}{e^x} \right)$

$w_1 = \frac{1}{-2\sqrt{2}} \int_0^\infty xe^{-x} - \int_0^\infty -\sqrt{2}e^{-x} - \int_0^\infty 2 \cdot e^{x}$

$w_1 \frac{1}{-2\sqrt{2}} \lim_{x\to\infty} \; xe^{-x} - e^{-x} + \sqrt{2}e^{-x} + 2e^{-x} \quad \Big|_0^\infty$

$\lim_{x\to\infty} \; \frac{xe}{e^x} - \frac{1}{e^x} + \frac{\sqrt{2}}{e^x} + \frac{2}{e^x} \quad 0$

$\Big|^0 - \frac{x}{2\sqrt{2}e^x} + \frac{1}{2\sqrt{2} \cdot e^x} + \frac{1}{2e^x} \qquad e^0 = 1$

$\frac{0}{2\sqrt{2}} + \frac{1}{2} - \frac{1}{2\sqrt{2}}$

$0{,}5 + 0{,}35355 = 0{,}85355.$

$$w_1 = \int_0^\infty e^{-x} \frac{(x-2+\sqrt{2})}{2+\sqrt{2}-2+\sqrt{2}} = -\frac{1}{2\sqrt{2}}\int_0^\infty \frac{x-2+\sqrt{2}}{e^x}$$

$$= \frac{1}{2\sqrt{2}}\int_0^\infty x e^{-x} - 2\int_0^\infty e^{-x} + \sqrt{2}\int_0^\infty e^{-x}$$

$$\frac{1}{2\sqrt{2}} \Big|_0^\infty -e^{-x}x - e^{-x} + 2e^{-x} - \sqrt{2}e^{-x}$$

$$\lim_{x\to\infty} -\cancel{\frac{x}{e^x}}^{0} - \cancel{\frac{1}{e^x}}^{0} + \cancel{\frac{2e}{e^x}}^{0} - \cancel{\frac{\sqrt{2}}{e^x}}^{0} + \cancel{\frac{0}{e^x}}^{0} + \frac{1}{e^0} + \frac{2e}{e^0} + \frac{\sqrt{2}}{e^0}$$

$$\left(+\frac{1}{1} + 2 + \sqrt{2}\right)\frac{1}{2\sqrt{2}}$$

$$= \frac{-1+1{,}414}{2\sqrt{2}} = 0{,}14644 66$$

Regla cuadratura Gauss Laguerre

$$\int_0^\infty e^{-x}\cdot f(x) = 0.8536 f(\overset{2-\sqrt{2}}{0.5858}) + 0.14664 f(2+\sqrt{2})$$

$w_0 = \frac{2+\sqrt{2}}{4} = 0,8535$

$$\int_0^{\infty} e^{-x} x^3 \, dx = \Gamma(4) = 3! = 6$$

$w_1 = \frac{2-\sqrt{2}}{4} = 0.14644$

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} \, dt$$

$x_0 = 2-\sqrt{2}$

$x_1 = 2+\sqrt{2}$

$\Gamma(x) = (x-1)!$

Con la Regla

$$\int_0^{\infty} e^{-x} x^3 \, dx = \left(\frac{2+\sqrt{2}}{4}\right) \cdot (2-\sqrt{2})^3 + \left(\frac{2-\sqrt{2}}{4}\right) \cdot (2+\sqrt{2})^3$$

$$= \frac{(2+\sqrt{2})(2-\sqrt{2})(2-\sqrt{2})^2}{4} + \frac{2-\sqrt{2}}{4} \cdot (2+\sqrt{2})(2+\sqrt{2})^2$$

$$= \left(\frac{4-2}{4}\right)\left((2-\sqrt{2})^2 + (2+\sqrt{2})^2\right)$$

$$= \frac{1}{2}\left(4-2\sqrt{2}+2 + 4+2\sqrt{2}+2\right)$$

$$\frac{1}{2}\left(4+4+2+2\right)$$

$$\frac{1}{2} \cdot 12$$

$$= 6 = 6$$