3) Interpolación

lagrange

con tuntos $((a, f(a)), (x_m, f(x_m)), (b, f(b)))$

$$f(x) = \sum_{k}^{n} L_k f(x_k)$$

$$L_k = \prod_{i \neq j} \frac{x - x_j}{x_i - x_j}$$

$$L_a = \frac{(x - x_m) \cdot (x - b)}{(a - x_m)(a - b)}$$

$$L_{x_m} = \frac{(x - a)(x - b)}{(x_m - a)(x_m - b)}$$

$$L_b = \frac{(x - x_m)(x - a)}{(b - a)(b - x_m)}$$

$$f(x) = f(a) \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} + f(x_m) \cdot \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} + f(b) \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$$

Integral Polinomio Aproximado

$$f(a)\cdot\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} + f(x_m)\int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} + f(b)\int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$$

1) 2) 3)

$$\frac{f(a)}{(a-b)(a-x_m)}\cdot\int_a^b x^2-bx-x\cdot x_m+bx_m\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)}\cdot\int_a^b x^2-bx-ax+ab\,dx + \frac{f(b)}{(b-a)(b-x_m)}\int_a^b x^2-x\,x_m-ax+ax_m$$

$Z=1x$

$XM = Z$

7) $\frac{f(a)}{(a-b)(a-z)} \int_a^B x^2 - bx - xz + bz\,Dx$

$H = \frac{b-a}{2}$

$A - b = -2H$

$a - z = \frac{2a}{2} - \frac{a}{2} - \frac{b}{2} = \frac{a-b}{2} = -H$

$\frac{f(a)}{2H^2} \cdot \left(\frac{b^3 - a^3}{3} + b^2 z - bza - \left((b^2 - a^2)\left(\frac{b+z}{2}\right)\right)\right.$

$\frac{a}{H}$

$\frac{f(a)}{2H^2} \left(\frac{(b-a)(b^2 + ab + b^2)}{3} + bz(b-a) - (b-a)\left(\frac{b+a}{2}\right)(b+z) \right)$

$z^2 = \left(\frac{a+b}{2}\right)^2 = \frac{a^2 + 2ba + b^2}{4}$

$\frac{(b-a)f(a)}{2H^2} \left(\frac{b^2 + ab + b^2}{3} + bz - zb - z^2 \right.$

$\frac{f(a)}{3H} \cdot \left(b^2 + ab + b^2 - \frac{3}{4}a^2 - \frac{3}{2}ab - \frac{3}{4}b^2 \right)$

$\frac{f(a)}{3H} \left(\frac{b^2}{4} - \frac{1}{2}ab + \frac{a^2}{4} \right)$

$\frac{f(a)}{H} \cdot \left(\frac{b-a}{2} \right)^2$

$I7 = f(a) \cdot H$

2)

$$\frac{f(z)}{(z-a)(z-b)} \int_a^b (x-a)(x-b)\,dx$$

$$= \int_a^b x^2 - bx - ax + ab\,dx$$

$$z = \frac{a+b}{2}$$

$$z-a = \frac{a+b}{2} - \frac{2a}{2} = \frac{b-a}{2} = H$$

$$z-b = \frac{a+b}{2} - \frac{2b}{2} = \frac{a-b}{2} = -H$$

$$-\frac{f(z)}{H^2} \Big|_a^b \frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx = \frac{b^3}{3} - \frac{b^3}{2} - \frac{ab^2}{2} + ab^2 - \frac{a^3}{3} + \frac{ba^2}{2} + \frac{a^3}{2} - a^2b$$

$$-\frac{f(z)}{H^2}\left(\frac{b^3-a^3}{3} - \frac{b^3}{2} - \frac{ab^2}{2} + ab^2 + \frac{ba^2}{2} + \frac{a^3}{2} - a^2b\right)$$

$$-\frac{f(z)}{H^2}\left(-\frac{b^3}{6} + \frac{1}{2}ab^2 + \frac{a^3}{6} - \frac{1}{2}a^2b\right) \cdot \frac{-3}{-3}$$

$$\frac{4 \cdot f(z)}{4 \cdot 3\ H^2}\left(\frac{b^3}{2} - \frac{3}{2}b^2a + \frac{3}{2}a^2b + \frac{a^3}{2}\right)$$

$$H = \frac{b-a}{2}$$

$$\frac{4}{3}\frac{f(z)}{H^2}\left(\frac{b^3}{8} - \frac{3}{8}b^2a + \frac{3}{8}a^2b + \frac{a^3}{8}\right)$$

$$\frac{4}{3}\frac{f(z)}{H^2}\left(\frac{b-a}{2}\right)^3 = \frac{4}{3}f(z)H$$

$$\frac{b-a}{2} = H$$

$$3)\ \frac{f(b)}{(b-a)(b-z)} \cdot \int_a^b x^2 - xz - ax - az \qquad H = \frac{b-a}{2}$$

$$\hookrightarrow \left(\frac{x^3}{3} - \frac{x^2}{2}z - \frac{ax^2}{2} - azx \right)\Big|_a^b \qquad b-a = 2H$$

$$b - z = \frac{2b}{2} - \frac{a-b}{2} = H$$

$$= \frac{f(b)}{2H^2} \cdot \left(\frac{a^3-b^3}{3} + a^2z - bza - (a^2-b^2)\left(\frac{a+z}{2}\right) \right)$$

$$= \frac{f(b)}{2H^2} \cdot \frac{(a-b)(a^2+ab+b^2)}{3} + az(a-b) - (a-b)\frac{(a+b)}{2}(a+z)$$

$$= \frac{(a-b)f(b)}{2H^2} \left(\frac{a^2+ab+b^2}{3} \right) + az - z(a+z)$$

$$z^2 = \left(\frac{a+b}{2}\right)^2 = \frac{a^2+2ab+b^2}{4}$$

$$= \frac{f(b)}{H} \frac{a^2+ab+b^2}{3} + \cancel{az} - \cancel{az} - z^2$$

$$= \frac{f(b)}{3H} \left(a^2 + ab + b - \frac{3}{4}a^2 - \frac{3}{2}ab - \frac{3}{4}b^2 \right)$$

$$\frac{f(b)}{3H} \left(\frac{a^2}{4} - \frac{1}{2}ab + \frac{b^2}{4} \right)$$

$$\frac{f(b)}{3H} \left(\frac{b-a}{2} \right)^2 \qquad \frac{f(b)\cdot H}{3}$$

$$\frac{f(b)}{3\cancel{H}} H^{\cancel{2}} =$$

conclucion

como $1) \rightarrow \frac{f(a)}{3} \cdot H$

y $2) \rightarrow \frac{4}{3} f(x_m) \cdot H$

y $3) \rightarrow \frac{f(b)}{3} \cdot H$

entonces

$$\int_a^b f(x) = \frac{1}{3}H \left(f(a) + 4f(m_x) + f(b) \right)$$