teniendo en cuenta la 4

$$D^2 f(x) = \frac{f(x+1) - 2f(x) + f(x-1)}{h^2}$$

$$f(x) = \frac{\dfrac{f(x+1+1) - 2f(x+1) + f(x-1+1)}{h^2} - 2\left(\dfrac{f(x+1) - 2f(x) + f(x-1)}{h^2}\right) + \left(\dfrac{f(x+1-1) - 2f(x-1) + f(x-1-1)}{h^2}\right)}{h^2}$$

$$D^4 f(x) = \frac{f(x+2) - 2f(x+1) + f(x) - 2f(x+1) + 4f(x) - 2f(x-1) + f(x) - 2f(x-1) + f(x-2)}{h^4}$$

$$D^4 f(x) = \frac{f(x+2) - 4f(x+1) + 6f(x) - 4f(x-1) + f(x+2)}{h^4}$$

Desarrollo Taylor

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f''''(x)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f''''(x)$$

sumamos desarrollos

$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \frac{h^4}{4!} f''''(x)$$

$$f''''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{h^4\, 4!} - O(h^2)$$

orden $O(h^2)$