ejercicio 12

$$E = n_0 \epsilon_0 + n_1 \epsilon_1$$

$$N = \epsilon_0 + \epsilon_1$$

configuraciones posibles, escoger n_0 particulas que tengan configuracion ϵ_0

esto es una combinacion

$$\frac{N!}{n_0!(N-n_1)!} = \frac{N!}{n_0!\, n_1!}$$

b)

$$\ln(\Omega) = \ln\left(\frac{N!}{n_0!\, n_1!}\right) = N\ln(N) - N - \ln(n_0!) - \ln(n_1!) =$$

$$N\ln(N) - N - \ln(n_0)n_0 + n_0 - n_1\ln(n_1) + n_1$$

$$= N\ln(N) - (N + n_0 + n_1) - \ln(n_0)\cdot n_0 - \ln(n_1)\cdot n_1$$

$$= N\ln(N) - N - \sum_{i=0}^{1} n_i \ln(n_i)$$

entonces

$$S = k_b\cdot\left(N\ln(N) - N - \sum_{i=0}^{1} n_i\ln(n_i)\right)$$

c)

$$x = \frac{n_1}{N} \qquad n_1 = xN \qquad n_0 = N - n_1 = N - xN = N(1-x)$$

sustituimos

$$S = k_b\left[N\ln(N) - N(1-x)\ln((x-1)N) - Nx\ln(Nx)\right)$$

$$S = k_b [N\ln(N) - N(1-x)(\ln(x-1) + \ln(N)) - Nx\ln(N) - Nx\ln(x)$$

$$S = k_b[N\ln(N) - N(1-x)\ln(x-1) - N(1-x)\cdot\ln(N) - Nx\ln(N) - Nx\ln(x)$$

$$\ln(N)(N - N(1-x) - Nx) = N(1-1-x) - Nx = 0$$

$$S = k_b[N\ln(N) - N\ln(N) + Nx\ln(N) - Nx\ln(N) - N(1-x)\ln(1-x) - Nx\ln(x)$$

$$S = k_b[-N(1-x)\ln(1-x) - Nx\ln(x)]$$

$$S = k_b N[-(1-x)\ln(1-x) - x\ln(x)]$$

$$S = [k_b[-N[x\ln(x) + (1-x)\ln(1-x)]]]$$

$$S = -k_b N[x\ln(x) + (1-x)\ln(1-x)]$$

$$E = n_0\epsilon_0 + n_1\epsilon_1$$

$$E = N(1-x)\epsilon_0 + n_1\epsilon_1$$

$$E = N(1-x)\epsilon_0 + xN\epsilon_1$$

$$E = N\epsilon_0 - Nx\epsilon_0 + xN\epsilon_1$$

$$E = N\epsilon_0 + Nx(\epsilon_1 - \epsilon_0)$$

$$E - N\epsilon_0 = Nx(\epsilon_1 - \epsilon_0)$$

$$x = \frac{(E - N\epsilon_0)}{N(\epsilon_1 - \epsilon_0)}$$

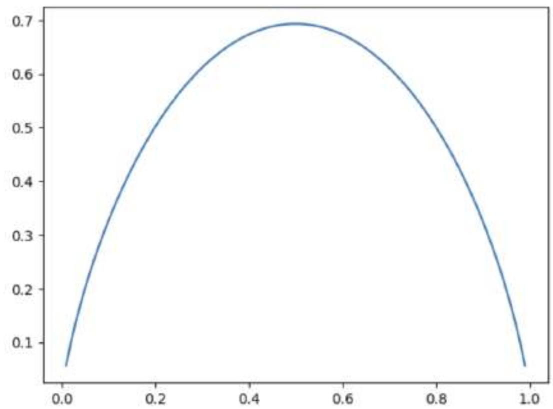
[5]
```python
import numpy as np
import matplotlib.pyplot as plt
```

[6]
```python
funcion=lambda x:-1*(x*np.log(x)+(1-x)*np.log(1-x))
```

[25]
```python
y=np.linspace(0,1,100)
```

```python
plt.plot(y,funcion(y))
```

```
<ipython-input-6-9347c17d1bec>:1: RuntimeWarning: divide by zero encountered in log
  funcion=lambda x:-1*(x*np.log(x)+(1-x)*np.log(1-x))
<ipython-input-6-9347c17d1bec>:1: RuntimeWarning: invalid value encountered in multiply
  funcion=lambda x:-1*(x*np.log(x)+(1-x)*np.log(1-x))
[<matplotlib.lines.Line2D at 0x7852066948b0>]
```



Conectado a del backend de Google Compute Engine que utiliza Python 3

$$\frac{\partial S}{\partial x} = -k_b N \left[\ln(x) + 1 - \ln(1-x) - 1\right]$$

$$\frac{\partial S}{\partial x} = -k_b N \left[\ln(x) - \ln(1-x)\right]$$

$$x = \frac{(E - N\epsilon_0)}{(\epsilon_1 - \epsilon_0)N} \qquad \Delta E = \epsilon_1 - \epsilon_0$$

$$\frac{\partial x}{\partial E} = \frac{1}{\Delta E N}$$

$$T = \frac{1}{-k_b N\left[\ln\left(\frac{x}{x-1}\right)\right]} \Delta E \cdot N \qquad \ln\left(\frac{x}{1-x}\right) = \frac{N\Delta E}{-k_b N T}$$

$$\left(\frac{x}{1-x}\right)^{-1} = \left(e^{\frac{N\Delta E}{k_b T}}\right)^{-1}$$

$$\frac{1-x}{x} = e^{\frac{N\Delta E}{k_b T}}$$

$$\frac{1}{x} - 1 = e^{\frac{N\Delta E}{k_b T}}$$

$$\frac{1}{x} = e^{\frac{\Delta E N}{k_b T}} + 1$$

$$x = \frac{1}{e^{\frac{N\Delta E}{k_b T}} + 1}$$

$$x(T) = \frac{1}{e^{\frac{N\Delta E}{k_b T}} + 1}$$

$$\lim_{T \to 0} \frac{1}{e^{\frac{N\Delta E}{k_b T}} + 1} = \frac{1}{\infty + 1} = 0$$

$$\lim_{T \to 0} e^{\frac{\Delta E N}{k_b T}} \to \infty$$

$$\lim_{T \to \infty} \frac{1}{e^{\frac{\Delta E N}{k_b T}} + 1} =$$

$\lim_{T \to \infty} \frac{1}{T} = 0$ entonces $e^0 = 1$

$$\lim_{T \to \infty} \frac{1}{1+1} = \frac{1}{2}$$

$$S(0) = N k_b \sum [0 \cdot \ln(0) + 1 \ln(1)]$$

$$= 0$$

cuando $T \to \infty$ $x \to \frac{1}{2}$

$$S(\tfrac{1}{2}) = -k_b N [\tfrac{1}{2} \ln(\tfrac{1}{2}) + (\tfrac{1}{2}) \ln(\tfrac{1}{2})]$$

$$= -k_b N [\ln(\sqrt{\tfrac{1}{2}} \cdot \sqrt{\tfrac{1}{2}})]$$

$$= -k_b N [\ln(\tfrac{1}{2})]$$

$\ln(\frac{1}{2}) = -1 \ln(2)$

$$+ k_b N [\ln(2)]$$

$$S(\tfrac{1}{2}) = k_b N \ln(2)$$

f) $\Delta S = nR \ln\left(\frac{V_2}{V_1}\right)$

$$\Delta S = nR \ln\left(\frac{2V}{V}\right)$$

$$\Delta S = nR \ln(2)$$

Comparamos

en un gas ~~Intti~~ ideal el cambio depende de la cantidad de moles, mientras que la entropia en una T infinita depende de K_B, podemos decir que su comportamiento es similar